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A Leland Model for Delta-Hedging in Central Risk Books

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Motivation

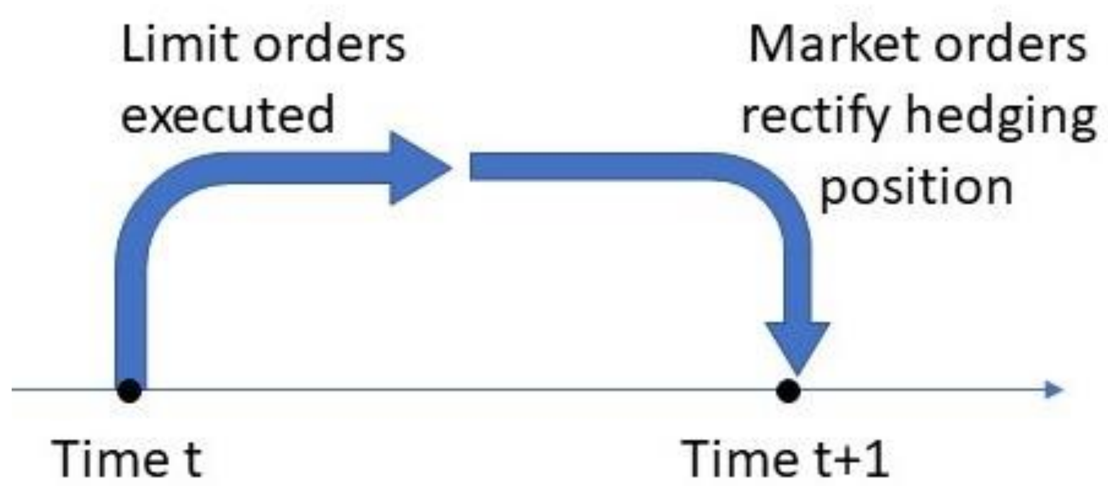
Central risk books record net trading exposure on any single derivative. It largely reduces the amount of hedging needed and also promotes the usage of limit orders in hedging.

An **operationalized** hedging strategy is given together with derivative price.

Model Setup

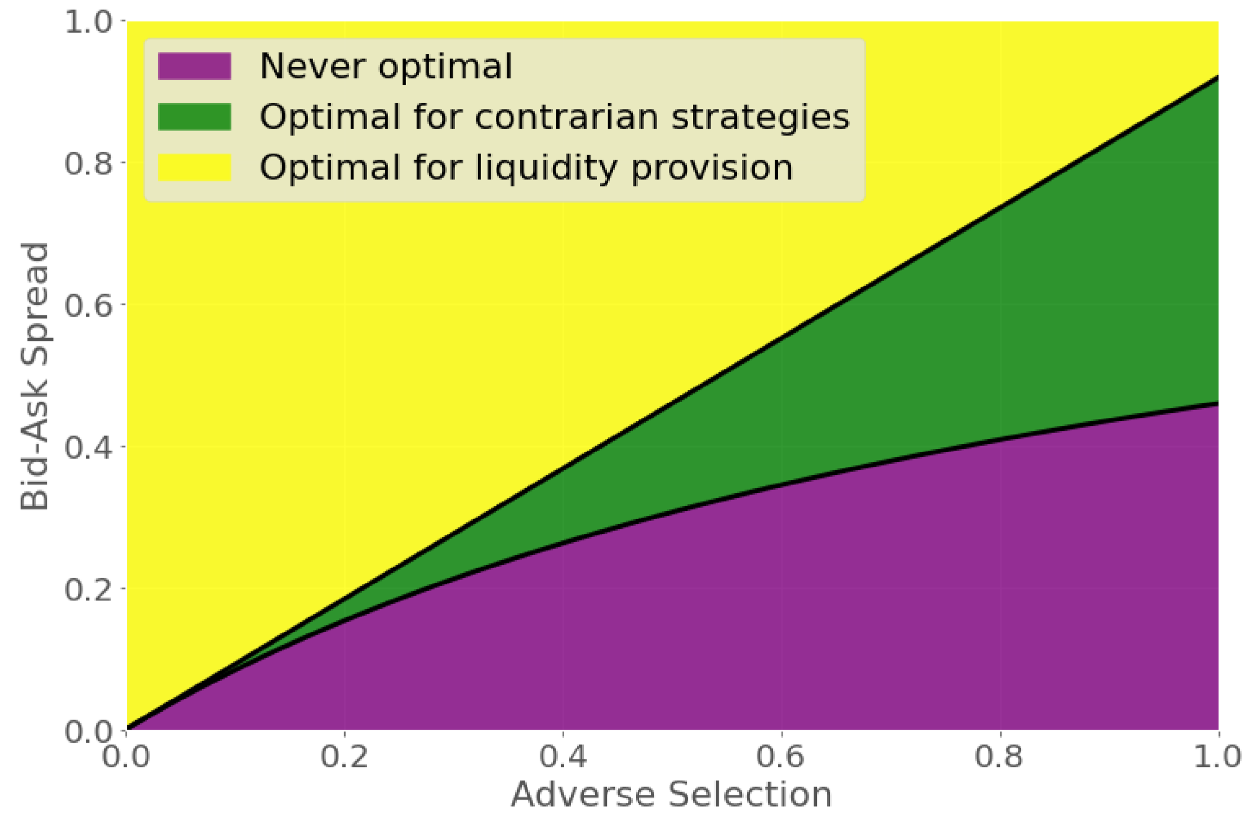
Spread $s \sim \sqrt{\Delta t}$ for time step Δt.

Tilted execution scheme:



Symmetrically placed limit orders capture a fraction of order flow. Adverse selection is modelled as the negative correlation between order flow and underlying price.

Regimes



Due to the disadvantage in speed, most traders are not able to profit from liquidity provision directly, but executing contrarian strategies for derivative hedging are still profitable.

Order Placement

In regime optimal for contrarian strategies, the optimal fraction of order flow to capture is given by:

$$\hat{\lambda} = -\frac{\gamma}{\eta}\left(\rho - \operatorname{sgn}(\gamma) y\sqrt{\frac{1-\rho^2}{1-y^2}}\right)$$

$$\lambda^* = \min\left(\max\left(\hat{\lambda}, 0\right), 1\right), y = 1 - \frac{\rho\sigma\sqrt{2\pi}}{s}$$

where γ is the product of option gamma and underlying volatility, η is the volatility of order flow, ρ is adverse selection level.

Option Pricing

With limit orders, the cheaper option price $C(t,p)$ is described by such PDE:

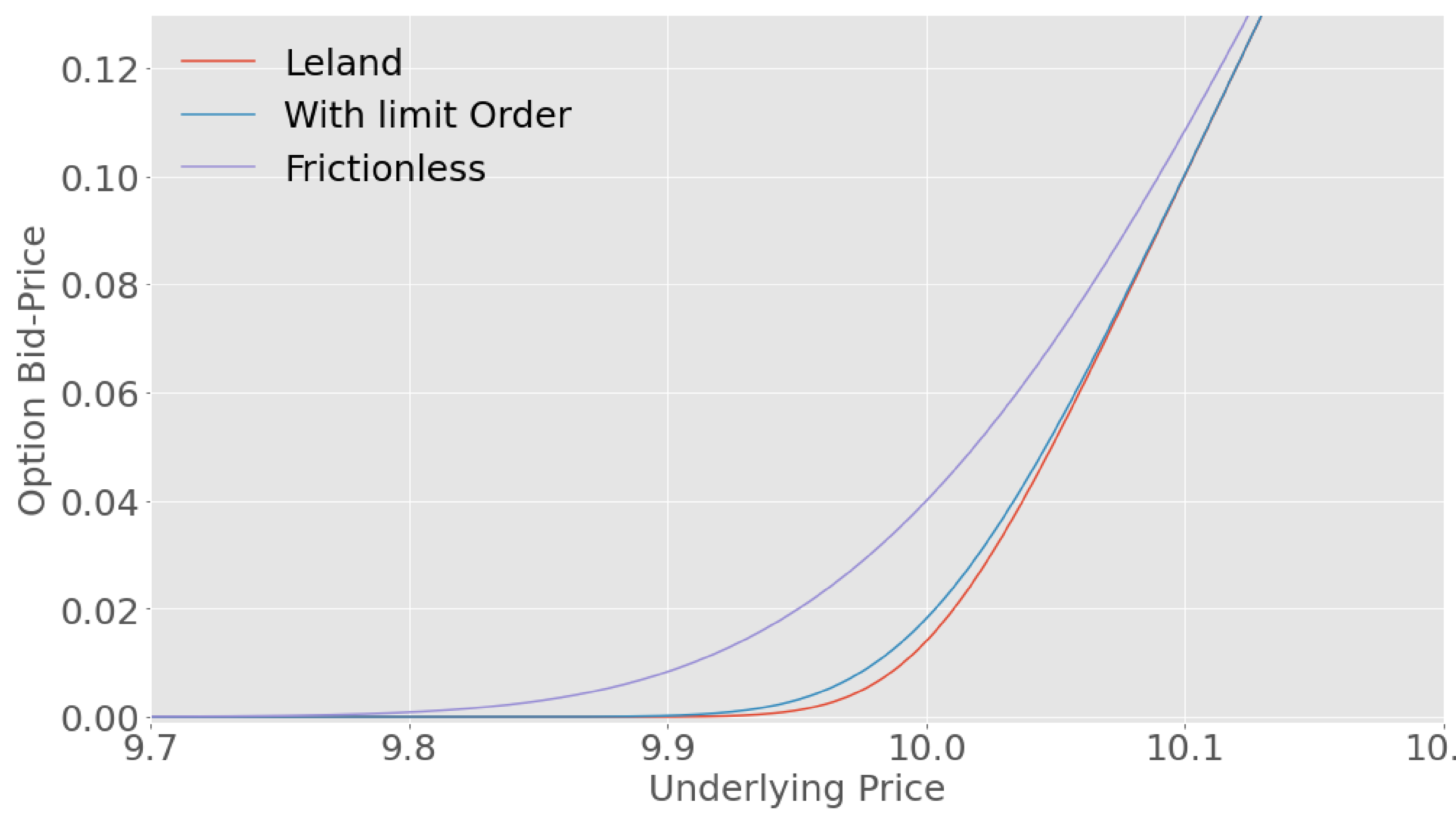
$$0 = \partial_t C + \frac{\sigma^2}{2}\partial_{pp} C + \sup_{\lambda \in [0,1]}\left\{\frac{s}{\sqrt{2\pi}}\left[y\lambda\eta - \sqrt{\left(\sigma\partial_{pp}C + \rho\lambda\eta\right)^2 + (1-\rho^2)(\lambda\eta)^2}\right]\right\}$$

with terminal condition $C(T,p) = \psi(p)$ for payoff function ψ,

which is a generalization of the classical Leland equation [1]:

$$0 = \partial_t C + \frac{\sigma^2}{2}\partial_{pp} C - \frac{s\sigma}{\sqrt{2\pi}}\left|\partial_{pp} C\right|$$

- Coincidence occurs at limit order placement rate $\lambda = 0$.
- There exists a unique solution to the nonlinear PDE.
- Same parameter constraint shared: $\sigma > s\sqrt{2/\pi}$



For parameters $\rho = 0.45, s = 0.11, \sigma = 0.1$, the bid-prices for European call option with strike price 10.0 for different underlying prices are plotted. The three curves are respectively: 1. Frictionless option price (purple); 2. Classical Leland option bid-price (red); 3. Option bid-price by hedging with both market and limit orders (blue). The improvement with limit orders is about 10%.

Acknowledgement

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References

[1] H. E. Leland. Option pricing and replication with transactions costs. J. Finance, 40(5):1283–1301,1985.

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