Dynamic mean-variance indifference valuation

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actuarial variance principle or mean-variance criterion.

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Question: How to use Π^γ in order to hedge and value H?

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Start with initial capital $x_0 \in \mathbb{R}$ and call $\pi^\gamma(H, x_0) \in \mathbb{R}$ a Π^γ-indifference value if

$$\sup_{\vartheta \in \Theta} \Pi^\gamma \left(x_0 + \sum_{k=1}^{T} \vartheta_k \Delta S_k \right) \stackrel{!}{=} \sup_{\vartheta \in \Theta} \Pi^\gamma \left(x_0 + \pi^\gamma(H, x_0) + \sum_{k=1}^{T} \vartheta_k \Delta S_k - H \right),$$

where $\Theta := \{\text{all predictable } \vartheta \text{ such that } \vartheta_k \Delta S_k \in L^2\}$.

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Denote by $\vartheta^0, \vartheta^H \in \Theta$ the solutions, then $\vartheta^H - \vartheta^0$ is called Π^γ-*indifference hedging strategy*

Static Mean-Variance indifference valuation

Theorem (Mercurio (2000), Schweizer (2001))

If $\vartheta^0 \in \Theta$ (NA condition) and

$$H = c^H + \sum_{k=1}^{T} \xi_k^H \Delta S_k + N^H \in G_T(\Theta) \oplus (G_T(\Theta))^\perp$$

where $c^H \in \mathbb{R}$, $\xi^H \in \Theta$ and $N^H \in L^2$ with mean zero and orthogonal wrt to all stochastic integrals of S, then

$$\pi^\gamma(H, x_0) = \pi^\gamma(H) = c^H + \frac{\gamma}{2} Var(N^H) = \widetilde{E}[H] + \frac{\gamma}{2} Var(N^H).$$

Moreover, ξ^H is the Π^γ-indifference hedging strategy.

A posteriori valuation $\pi^\gamma \rightarrow$ *Financial variance principle.*

Static Mean-Variance indifference valuation

Message:

1. $$\Pi^\gamma(H) := E[H] - \frac{\gamma}{2} Var(H) \mapsto \pi^\gamma(H) = \widetilde{E}[H] + \frac{\gamma}{2} Var(N^H).$$
2. *Davis price* (here $\widetilde{E}[H]$), bid-ask spread, extension to continuous-time.
3. Π^γ-indifference hedging strategy is the *mean-variance hedging strategy* ξ^H.
4. An explicit scheme for valuation and hedging a general H.
   - Determine $H = c^H + \sum_{k=1}^T \xi_k^H \Delta S_k + N^H$.
   - (OTC value) π^γ and hedging strategy ξ^H.

Dynamic Mean-Variance indifference valuation

Dynamic Mean-Variance Indifference Valuation

Let $T \in \mathbb{N}$ and let $\mathbb{F} = (\mathcal{F}_k)_{k=0,\ldots,T}$, $S = (S_k)_{k=0,\ldots,T}$ and $H \in L^2(\mathcal{F}_T)$

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Extend Π^γ to $(\Pi^\gamma_k)_{k=0,\dots,T}$ where

$$\Pi^\gamma_k(H) := E[H|\mathcal{F}_k] - \frac{\gamma}{2} Var(H|\mathcal{F}_k).$$

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Extend Π^γ to $(\Pi_k^\gamma)_{k=0,\dots,T}$ where

$$\Pi_k^\gamma(H) := E[H|\mathcal{F}_k] - \frac{\gamma}{2} Var(H|\mathcal{F}_k).$$

Think of $\Pi_k^\gamma(X)$ as the utility of X at time k.

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How do we define $\pi_k^\gamma(H)$?

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How do we define $\pi_k^\gamma(H)$?

What we expect is

$$\text{“}\pi_k^\gamma(H) = \widetilde{E}[H|\mathcal{F}_k] + \frac{\gamma}{2} Var(N^H|\mathcal{F}_k)\text{”}.$$

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$$x_k + \sum_{j=k+1}^{T} \vartheta_j^0 \Delta S_j$$

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Second alternative: Sell H at time k for $\pi_k^\gamma(H)$ and trade optimally with initial capital x_k

$$x_k + \pi_k^\gamma(H) + \sum_{j=k+1}^{T} \vartheta_j^H \Delta S_j - H$$

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1) Suppose that $\vartheta^0_{k+2}, \ldots, \vartheta^0_T$ have already been prescribed. At time k, we choose ϑ^0_{k+1} such that

$$\Pi^\gamma_k\left(x_k + \sum_{j=k+1}^{T} \vartheta^0_j \Delta S_j\right)$$

$$= \operatorname{ess\,sup} \Pi^\gamma_k\left(x_k + \vartheta_{k+1}\Delta S_{k+1} + \sum_{j=k+2}^{T} \vartheta^0_j \Delta S_j\right),$$

where the essential supremum is taken over all $\mathcal{F}_k$-measurable ϑ_{k+1} such that $\vartheta_{k+1}\Delta S_{k+1} \in L^2(P)$.

Dynamic Mean-Variance Indifference Valuation

The solution to the first alternative (if it exists) is

$$\vartheta^0_{k+1} = \frac{1}{\gamma}\frac{E[\Delta S_{k+1}|\mathcal{F}_k]}{Var(\Delta S_{k+1}|\mathcal{F}_k)} - \frac{Cov\left(\Delta S_{k+1}, \sum_{j=k+2}^{T} \vartheta^0_j \Delta S_j \middle| \mathcal{F}_k\right)}{Var(\Delta S_{k+1}|\mathcal{F}_k)}.$$

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2) Suppose that $\vartheta_{k+2}^H, \dots, \vartheta_T^H$ have already been prescribed. At time k, we choose ϑ_{k+1}^H such that

$$\Pi_k^\gamma\left(x_k + \pi_k^\gamma(H) + \sum_{j=k+1}^{T} \vartheta_j^H \Delta S_j - H\right)$$

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Dynamic Mean-Variance Indifference Valuation

The solution to the second alternative (if it exists) is

$$\vartheta_{k+1}^{H} = \frac{1}{\gamma}\frac{E[\Delta S_{k+1}|\mathcal{F}_k]}{Var(\Delta S_{k+1}|\mathcal{F}_k)} + \frac{Cov\left(\Delta S_{k+1}, H - \sum_{j=k+2}^{T} \vartheta_j^H \Delta S_j \middle| \mathcal{F}_k\right)}{Var(\Delta S_{k+1}|\mathcal{F}_k)}.$$

Dynamic Mean-Variance Indifference Valuation

We define the *dynamic mean-variance indifference value* $\pi_k^\gamma(H)$ at time k by

$$\Pi_k^\gamma\left(x_k + \sum_{j=k+1}^{T} \vartheta_j^0 \Delta S_j\right) \stackrel{!}{=} \Pi_k^\gamma\left(x_k + \pi_k^\gamma(H) + \sum_{j=k+1}^{T} \vartheta_j^H \Delta S_j - H\right)$$

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$\rightarrow$ Write $\xi^H := \vartheta^H - \vartheta^0$ for the Π^γ-indifference hedging strategy.

$\rightarrow$ How can ϑ^0 and ξ^H be described?

Denote by

$$\Theta := \left\{ \vartheta = (\vartheta_k)_{k=1,\dots,T} \text{ pred. s. t. } \vartheta_k \Delta S_k \in L^2(P)\, \forall k = 0, \dots, T \right\}.$$

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Lemma (Characterization of ξ^H and ϑ^0)

Assume that $\lambda_k := \frac{E[\Delta S_{k+1} | \mathcal{F}_k]}{Var(\Delta S_{k+1} | \mathcal{F}_k)}$ *is well-defined and that* $\vartheta^H, \vartheta^0 \in \Theta$.

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- *Then* ξ^H *is the integrand in the FS decomposition of* H *with respect to* S*:*

$$H = \widehat{H}_0 + \sum_{k=1}^{T} \xi_k^H \Delta S_k + L_T^H,$$

$\rightarrow$ *local risk-minimization*

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- *Then* ϑ^0 *is the integrand in the FS decomposition of* $1/\gamma(\lambda \bullet M)_T$, *i.e.,*

$$\frac{1}{\gamma} \sum_{k=1}^{T} \lambda_k \Delta M_k = \widehat{X}_0 + \sum_{k=1}^{T} \vartheta_k^0 \Delta S_k + L_T,$$

where the process M denotes the martingale part of S.

Minimal martingale measure

The *intrinsic value process* of H, denoted by $\widehat{V}(H) = (\widehat{V}_k(H))_{k=0,\dots,T}$, is defined by

$$\widehat{V}_k(H) := \widehat{H}_0 + \sum_{\ell=1}^{k} \xi_\ell^H \Delta S_\ell + L_k^H.$$

Define

$$\widehat{Z}_k^T := \prod_{j=k+1}^{T} (1 + \lambda_j \Delta M_j) \quad \text{and} \quad \widehat{E}[H|\mathcal{F}_k] := E[\widehat{Z}_k^T H|\mathcal{F}_k].$$

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Then we have

$$\widehat{E}[H|\mathcal{F}_k] = \widehat{V}_k(H).$$

Main result

Theorem (Dynamic Mean-Variance Indifference Valuation)

Let $1/\gamma(\lambda \bullet M)_T$ and H admit a Föllmer-Schweizer decomposition. Then the dynamic mean-variance indifference value process $\pi^\gamma(H)$ is given by the formula

$$\pi_k^\gamma(H) = \widehat{E}[H|\mathcal{F}_k] + \gamma Cov(L_T, L_T^H|\mathcal{F}_k) + \frac{\gamma}{2} Var(L_T^H|\mathcal{F}_k),$$

for $k = 0, \ldots, T$.

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for $k = 0, \ldots, T$.

A posteriori valuation rule $\pi_k^\gamma(H)$.

Some examples

P is already martingale measure for S

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Let S be a P-martingale. Since $\Delta S_k = \Delta M_k + \lambda E[(\Delta M_k)^2 | \mathcal{F}_{k-1}]$, we have $\lambda = 0$.

Since

$$0 = \frac{1}{\gamma} \sum_{k=1}^{T} \lambda_k \Delta M_k = \widehat{X}_0 + \sum_{k=1}^{T} \vartheta_k^0 \Delta S_k + L_T \Rightarrow \vartheta^0 = 0, \quad L_T = 0.$$

Moreover, the FS-decomposition coincides with the KW decomposition.

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$$\pi_k^{\gamma}(H) = E[H|\mathcal{F}_k] + \frac{\gamma}{2} Var(L_T^H|\mathcal{F}_k)$$

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Moreover, the FS-decomposition coincides with the KW decomposition.

$$\pi_k^\gamma(H) = E[H|\mathcal{F}_k] + \frac{\gamma}{2} Var(L_T^H | \mathcal{F}_k)$$

$\pi^\gamma(H)$ fulfills the following recursion (DPP substitute):

$$\begin{cases} \pi_T^\gamma(H) = H, \\ \pi_k^\gamma(H) = E[\pi_{k+1}^\gamma(H)|\mathcal{F}_k] - \frac{\gamma}{2} Var(\Delta L_{k+1}^H | \mathcal{F}_k), \quad k = T-1, \dots, 0. \end{cases}$$

Deterministic mean-variance tradeoff (i.i.d. models)

Suppose that the *mean-variance tradeoff process*

$$\sum_{k=1}^{j} \frac{(E[\Delta S_k | \mathcal{F}_{k-1}])^2}{Var(\Delta S_k | \mathcal{F}_{k-1})} = \sum_{k=1}^{j} \lambda_k \Delta A_k$$

is deterministic.

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is deterministic. Then

$$\sum_{k=1}^{T} \lambda_k \Delta S_k = \sum_{k=1}^{T} \lambda_k \Delta M_k + \sum_{k=1}^{T} \lambda_k \Delta A_k$$

and hence $L_T = 0$. Moreover we have $\widehat{P} = \widetilde{P}$ (non trivial). Therefore

$$\pi_k^{\gamma}(H) = \widetilde{E}[H|\mathcal{F}_k] + \frac{\gamma}{2} Var(L_T^H|\mathcal{F}_k)$$

$\rightarrow$ Dynamic financial variance principle.

Conclusion

1. Construction of an a posteriori valuation rule $\pi_k^\gamma(H)$ at time k from Π_k^γ through an indifference argument.
2. Π^γ-indifference hedging strategy is the local risk-minimization stratetgy ξ^H.
3. An explicit scheme for valuation and hedging a general claim H:
    - Determine the FS-decompositions of H and $1/\gamma(\lambda \bullet M)_T$.
    - (OTC) value at time k is $\pi_k^\gamma(H)$ and hedging strategy is the local risk-minimization strategy ξ^H.

Thank you