Acoustics of friction[a)]

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This article presents an overview of the acoustics of friction by covering friction sounds, friction-induced vibrations and waves in solids, and descriptions of other frictional phenomena related to acoustics. Friction, resulting from the sliding contact of solids, often gives rise to diverse forms of waves and oscillations within solids which frequently lead to radiation of sound to the surrounding media. Among the many everyday examples of friction sounds, violin music and brake noise in automobiles represent the two extremes in terms of the sounds they produce and the mechanisms by which they are generated. Of the multiple examples of friction sounds in nature, insect sounds are prominent. Friction also provides a means by which energy dissipation takes place at the interface of solids. Friction damping that develops between surfaces, such as joints and connections, in some cases requires only microscopic motion to dissipate energy. Modeling of friction-induced vibrations and friction damping in mechanical systems requires an accurate description of friction for which only approximations exist. While many of the components that contribute to friction can be modeled, computational requirements become prohibitive for their contemporaneous calculation. Furthermore, quantification of friction at the atomic scale still remains elusive. At the atomic scale, friction becomes a mechanism that converts the kinetic energy associated with the relative motion of surfaces to thermal energy. However, the description of the conversion to thermal energy represented by a disordered state of oscillations of atoms in a solid is still not well understood. At the macroscopic level, friction interacts with the vibrations and waves that it causes. Such interaction sets up a feedback between the friction force and waves at the surfaces, thereby making friction and surface motion interdependent. Such interdependence forms the basis for friction-induced motion as in the case of ultrasonic motors and other examples. Last, when considered phenomenologically, friction and boundary layer turbulence exhibit analogous properties and, when compared, each may provide clues to a better understanding of the other. [DOI: 10.1121/1.1456514]

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I. INTRODUCTION

Friction develops between sliding surfaces regardless of the magnitude of relative motion between them. Always acting as a resistance to relative motion, friction fulfills a dual role by transmitting energy from one surface to the other and by dissipating energy of relative motion. The physical processes that contribute to friction have a wide range of length and time scales. At the length scale that corresponds to the interatomic distance in solids, which forms the upper limit of acoustics, friction acts as a dissipation mechanism, converting kinetic energy to thermal energy. This means of conversion, arguably the most fundamental aspect of friction, is an acoustical process that involves oscillations of atoms and relates to the first principles.

In practice, at longer length scales, the dual roles of friction, both transmitting and dissipating energy, almost always coexist. The conditions under which friction provides more energy to a system than that system can dissipate constitute the basis for most of the instabilities observed in friction-excited vibrations and a prime source of resulting sound radiation.

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Examples of sounds that result from friction-excited vibrations and waves appear frequently. They include squeaks and squeals in the interior of automobiles, the squeal of sneakers on parquet floors, the squeak of snow when walking on it, door hinges, chalk on a blackboard, turkey friction calls that hunters use, aircraft and automotive brake squeals, belts on pulleys, rail-wheel noise, and retarders in rail yards. These and many others typify the annoying aspects of friction sounds. String instruments exemplify the musical dimensions of friction sounds.

The acoustics of friction extends beyond noise and music and includes such phenomena as friction damping, friction-assisted assembly, and friction motion. Contact damping of inserted blades in gas turbines and braided wires exemplify friction dampers. In manufacturing assembly processes, friction has an important role, such as in vibratory conveyors and in reducing friction during the cold drawing of wires by vibrating them. Friction motion refers to vibromotion devices such as the ultrasonic motor that relies on bending waves and friction to develop precise motion. A historical example demonstrated by Tyndall[1] shown in Fig. 1 suggests that rubbing a glass tube along its length can break off rings at its free end. (Admittedly, the author has been unsuccessful trying to re-enact this experiment.) These examples and others involve both acoustics and friction. This article considers acoustics

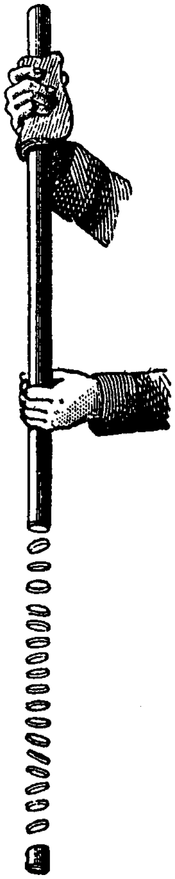

FIG. 1. Tyndall's experiment breaking rings off a glass tube by friction. From Ref. 1.

and friction each in its broader meaning, rather than their conventional engineering implications, while keeping the focus narrow to consider aspects of friction relevant to acoustics. As such, the article leaves out many rigid-body oscillation problems associated with friction-induced instabilities.

A major challenge in studying the acoustics of friction relates to the ability to predict sound and vibration responses of systems subject to friction. The challenge rests with the modeling of friction as well as the modeling of real boundary conditions that often involve friction. A friction force interacts with and often depends on the response of the system within which it develops. Such interaction sets up a feedback between the friction force and the waves on the surfaces, making them interdependent. In addition to such dynamic effects with short time scales, friction also undergoes changes over a longer time that correspond to, for example, the deformation and wear of surfaces. Consequently, friction, when acting on even a simple and an otherwise linear system, can still produce very complicated responses.

This paper treats several aspects of the acoustics of friction. The next section describes friction-induced vibrations and sounds starting with several pedagogical examples to illustrate fundamental concepts. It follows these with detailed descriptions of two engineering applications that involve complex friction and vibration interaction, namely bowed string and brake noise. The second section concludes with examples of sounds in nature. The third section of the paper treats macroscopic interactions between friction and vibrations to demonstrate friction-induced motion, followed in Sec. IV by a description of friction-induced damping phenomena. The paper then presents a brief review of the issues pertinent to measuring and modeling of friction in dynamic systems, followed by a description of friction and its components which includes a description of friction dissipation in terms of atomic oscillations. The final part of the paper draws an analogy between two little-understood phenomena: friction-excited vibrations in solid mechanics and boundary-layer turbulence in fluid mechanics.

II. FRICTION-EXCITED VIBRATIONS AND SOUNDS

The range of sounds a bow and a string can produce bespeaks the extent of the many different forms friction sounds may take. Friction sounds are rarely, if at all, ergodic and stationary. Rather, they are mostly unsteady or transient. Sounds emanate from either one or both components of the friction pair or from other parts of the system to which the friction pair transmits unsteady forces. In some cases, transient radiation originates from sudden deformation of the surfaces near the contact areas, which may also be accompanied by waves that develop within the components under friction excitation. Rubbing of viscoelastic materials often exhibits a transient sound radiation, as in the case of the heel of a sneaker sliding on a polished parquet floor. Rubbing an inflated balloon with a finger also produces a transient sound from near the contact area, in this case, followed by a pseudo-steady-state radiation. Similarly, rubbing the surface of a small ($D=1$ cm) steel ball with a moist finger also demonstrates radiation from local acceleration near a contact. Since the fundamental natural frequency of the ball is in the ultrasonic range, the sound produced in the sonic range results solely from the rapid movement of the contact area of the finger and rigid-body radiation of the ball.

Sliding, whether through a continuous or transient contact, is an unsteady process. Transient sliding, the source of many squeaks and squeals, develops intermittently or cyclically, as in *spiccato* bowing of violin that requires a sliding stroke of a bow onto and off a string. Continuous sliding, on the other hand, can produce a broad range of responses including transient response, for example, by a slight variation in the normal contact load.

For a particular friction pair, the differences in the sounds they radiate, and their governing vibrations, largely arise from the variation of contact forces at the interface where the sliding surfaces meet. In turn, contact forces depend on the interface properties and the external forces that maintain the sliding contact. In cases where the friction pair is attached to other components, the response of the entire system can also modify the interface behavior and thus the resulting acoustic response.

The strength of contact governs the type of waves and oscillations that develop during sliding. Weak contacts, with effects localized to the interface region, produce response in each component at its own natural frequencies, nearly independently of other components. Weak contacts that involve rough surfaces can produce light impulses as asperities come into contact, and again produce a response at the natural frequencies of each component. Such sliding conditions often produce sounds, referred to here as roughness noise or surface noise. For example, two pieces of sandpaper, when rubbed against each other, produce a random sound from the vibrating paper that results from the interaction among the granules on opposing surfaces. Another example of surface or roughness noise involves bending waves of a rod with a rough surface when lightly rubbed against another rough surface. Similarly, corrugated surfaces sliding over each other under light normal loads produce impulsive contact forces that develop at the ''corrugation frequency'' with components in both tangential and normal directions to the interface. Such contact forces produce response in each direction at a combination of corrugation frequency and natural frequencies of each component.

When sliding takes place under strong contact conditions, the influence of the contact force reaches beyond the interface and friction; the friction pair becomes a coupled system and produces a more complex and often nonlinear response. Under such conditions, instabilities develop and frequently lead to a condition called mode lock-in, where the coupled system responds at one of its fundamental frequencies and its harmonics.[2] Development of mode lock-in and the selection of the mode at which the system responds depends on the normal load, sliding velocity, and the contact geometry.

The external normal force that maintains contact between surfaces not only influences the strength of the contact, and thus the friction force, but it can also modify the dynamic response of the components under friction. As discussed later, the splitting of modes in beams and disks is but one example that also exhibits the influence of the normal force.

A. Pedagogical examples

The following examples present a hierarchy of friction sounds that helps illustrate the fundamental concepts associated with friction-excited vibrations and the ensuing sound radiation described above. They start with the simple cases involving only two components with distinctly different impedances followed by those that have comparable impedances. In the former cases, response of the system is closer to the component with ''weaker'' impedance, whereas in the latter, system response is different than that of either component.

1. Wineglass

It is well-known that rubbing the rim of a glass, preferably with a moist finger, makes it radiate sound from its bending waves at one of its natural frequencies and its harmonics.[3,4] In cases of crystal glasses with very small internal damping, even light rubbing of the glass surface produces a sound. Friction, although applied to the rim in the circumferential direction, excites bending waves. When approximated as a cylinder of height H and radius R that has a rigid base, its fundamental vibration frequency is given as[5]

$$\omega_0=\frac{h_g}{R}\sqrt{\frac{3E_g}{5\rho_g}}\left[1+\frac{4}{3}\left(\frac{R}{H}\right)^4\right]^{1/2}, \qquad (1)$$

where h_g, ρ_g, and E_g represent the thickness, density, and Young's modulus of glass cylinder, respectively. Rubbing a glass around its perimeter can also excite the torsional oscillations of the wine glass about its axis of symmetry, but at a much higher frequency than the bending natural frequencies.[5]

A different mechanism takes place when the rim of a glass is gently rubbed in the radial direction by a dry finger. This light radial application of friction to the edge of the glass primarily excites the bending waves of the glass wall by light impulses. Impulses develop at a corrugation frequency corresponding to the speed and the spacing between ridges on the fingertip, and sometimes described as the ''picket fence'' or ''washboard'' effect. As illustrated in Fig. 2, such impulses excite the natural frequencies of the glass.

A more complex response results from exciting a wineglass with a violin bow applied radially to its rim. The response usually, but not always, shows a dominance by only one of the natural frequencies of the glass and its harmonics. Frequently, the first or the second mode appears depending on the application of the bow. By imposing a nodal point on the surface of the glass, for example with a finger tip, even the third mode and its integer harmonics can result from a seemingly identical application of the bow. This behavior, where the bow and the glass respond, in this case, at or near a particular natural frequency of the glass to the exclusion of its other modes, illustrates the concept of mode lock-in described earlier.[2]

The presence of water in the glass helps demonstrate the corresponding mode shapes of the glass, similar to Galileo's observations[6] (cited in Ref. 7). Under high excitation amplitudes, water in the glass shoots up, similar to the Chinese sprouting fish basin, which consists of a pot, usually bronze, that fountains the water it is filled with when rubbed at its handles.

2. Cantilever beam and plate

Another demonstration of mode lock-in results from applying a violin bow to the free edge of a cantilever beam or a plate. Chladni also happened to demonstrate mode lock-in[8,9] when presenting the first visualization of mode shapes in 1787 by sprinkling sand on a plate and bowing its free edge to create different mode shapes.

Similarly, bowing the free end of a cantilever beam can also produce mode lock-in. Depending on the application of the bow, the spectra of bending vibrations and radiated sound display either the first, the second, or even the third natural frequency of the beam and its harmonics. In these experiments, only infrequently does more than one natural frequency appear simultaneously. Mostly, the bow and beam lock into a particular mode and oscillate in a rather stable manner. Because bow impedance in the direction of excitation differs significantly from that of the beam, the dynamics of the two does not critically interact, and the beam responds at one of its natural frequencies with corresponding harmonics, as shown in the radiation spectrum given in Fig. 3.

3. Cuica

The cuica, the Brazilian friction drum, presents an example of a friction system in which friction excitation takes

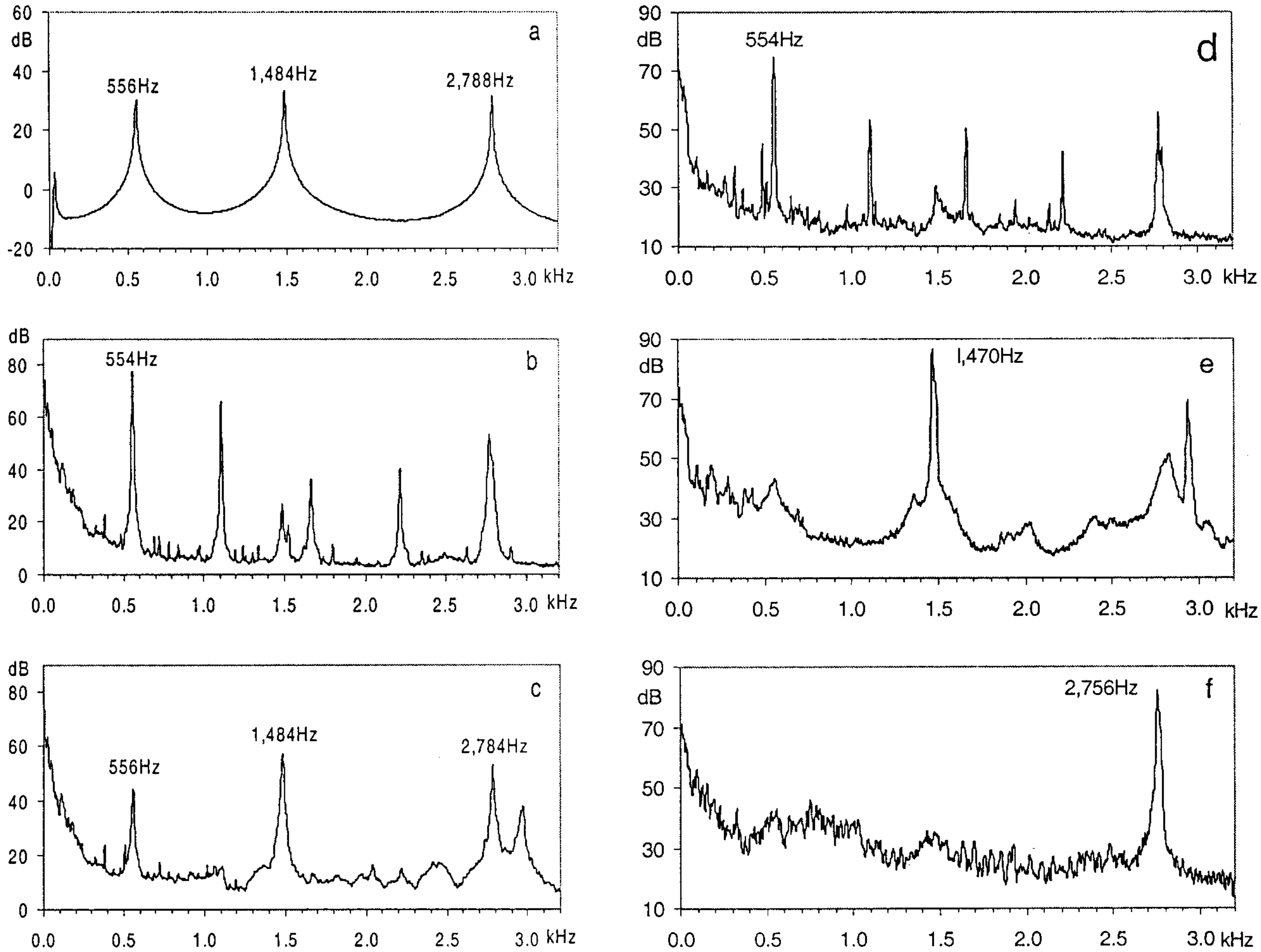

FIG. 2. Demonstration of mode lock-in with a wine glass half-filled with water. (a) Frequency response; (b) response to excitation to a finger rubbed along its edge; (c) excitation by dry finger across its rim; (d)–(f) response to excitation with a bow; (e)–(f) show the effect of placing a finger on different positions on the glass to suppress the other modes thus creating a mode lock-in.

place away from the radiation source. The cuica consists of a cylindrical metal shell covered at one end with a membrane, as in a drum. Inside the cylinder, a thin bamboo stick suspends from the center of the membrane where one end of the stick is tied with a knot. The player continuously strokes the stick with a piece of moist cloth, thereby exciting the membrane at its center with the resulting stick–slip motion. The player produces music by controlling the radiation response by judiciously placing a finger on various spots on the membrane.

Mechanically, the operation of the cuica has similarities to that of a bowed string instrument. In the case of the cuica, the bamboo stick vibrates the membrane as a bow does with a string, and the player's finger on the membrane selects the mode of vibration as does placing a finger on the string.

4. Extraction of a nail from wood

Extracting a nail from a plank of wood presents another instructive example of friction, stick-and-slip, and friction sounds. Pulling a nail exhibits stick–slip if it is extracted from hard wood. Both the hardness of the wood and the average diameter of the nail determines the pressure, and thus the friction force, on the surface of the nail. Larger-size nails produce more friction force and sounds. Speed with which a nail is extracted also affects the dynamic response, but probably because of the temperature effects. Rapid extraction of a nail can easily char the adjacent surface of the wood and change the characteristics of friction.

5. Smooth versus corrugated surfaces

Two simple experiments using polished and corrugated surfaces can demonstrate the effects of surface texture on sound radiation from friction excitation. Rubbing along the length of a steel bar with a smooth surface produces longitudinal vibrations and radiates sound at its fundamental frequencies as determined by the position at which the bar is constrained. Friction dominates the dynamic component of the contact force without a significant perturbation of its normal component. The friction force, in this case, travels along the rod and gives rise to longitudinal waves, modeled as

$$-E\frac{\partial^2 u}{\partial x^2}+\rho\frac{\partial^2 u}{\partial t^2}=\frac{1}{A}\mu F_N\delta(x-Vt). \tag{2}$$

The use of a smooth rod excited by friction dates back to 1866, when it became the excitation source in what is now known as the Kundt's tube to measure the speed of sound. In that experiment, longitudinal waves in the stroked rod vi-

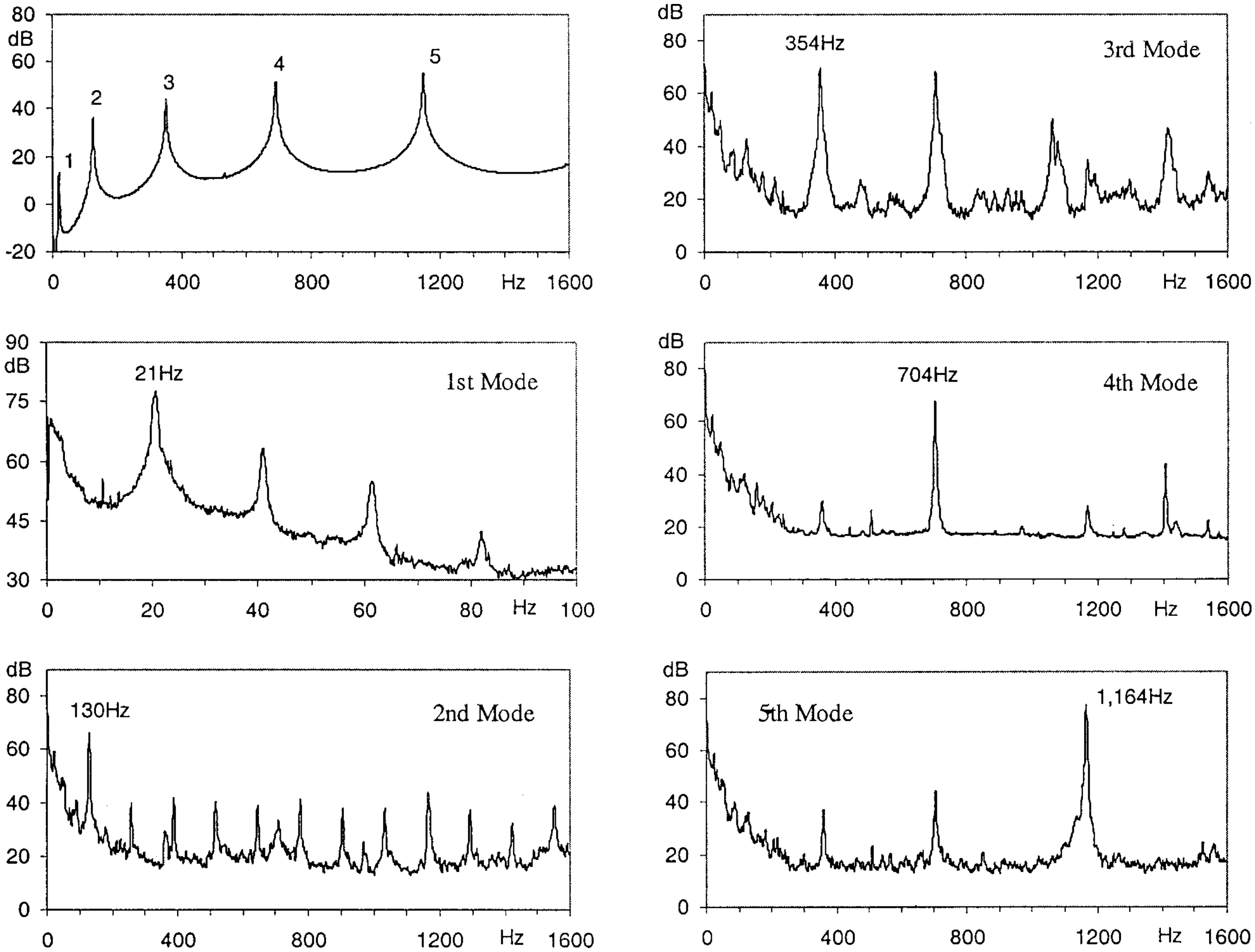

FIG. 3. Demonstration of mode lock-in obtained by applying friction to a clamped–free beam using a violin bow. Response lock-in to each of the natural frequencies separately, although higher modes require suppression of lower modes.

brates a disk attached to its end, which acts like a loudspeaker inside a tube.

A rod with a corrugated surface, such as a threaded bar, when rubbed along its length with a rough surface, produces both longitudinal and bending vibrations of the bar. Unlike the case for a smooth rod, forces that develop at the corrugations have an impulsive character with components both in tangential and normal directions, thus simultaneously exciting longitudinal and bending waves in the rod. Assuming that impulsive contact forces dominate over friction forces, the longitudinal response of a rod with corrugations Δx apart is governed by

$$-E\frac{\partial^2 u}{\partial x^2}+\rho\frac{\partial^2 u}{\partial t^2} = \frac{1}{A}\sum_{n=1}^{m} F_n \cos\phi_n \prod\ [(n\Delta x - Vt)/b]\,\delta(x-n\Delta x), \quad (3)$$

where $n=\text{integer}(L/\Delta x)$, and m represents the number of simultaneous contacts that have a length b traveling at a sliding velocity V. The function $\Pi[(n\Delta x - Vt)/b]$ represents a rectangle of width b positioned at $n\Delta x$ and traveling at a velocity V. F_n represents the discrete impulsive point forces that develop during motion at an angle ϕ with the axis of the rod. Similarly, the bending vibrations of the rod can be described by

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2} = \sum_{n=1}^{n} F_n \sin\phi_n \prod\ [(n\Delta x - Vt)/b]\,\delta(x-n\Delta x), \quad (4)$$

where ρ, E, I, and A represent the density, Young's modulus, the cross-sectional moment of inertia, and the cross-section area of the beam, respectively.

A reed rubbed along a guitar string presents another example in which both bending and longitudinal vibrations are excited. The contact force characteristics again involve primarily the spatial period of the winding on the string and the speed, V, with which the reed moves along the string

$$-c_s^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial t^2}=F_N\,\delta(x-n\Delta x), \quad (5)$$

where $n=\text{Integer}(V\cdot t/\Delta x)$ indicates the position of excitation at time t, $c_s=\sqrt{T/\rho}$ represents the speed of sound for a string of density ρ and under tension T, and $F_N=F_i\cos\phi$ corresponds to the normal component of the contact force. This "picket-fence effect," where the reed excites the string with impulses at the corrugation frequency, has a similar mechanism as friction sounds from insects (described later), but is a simpler form of what occurs among the asperities between rough surfaces in sliding contact. In the latter case, impulses arising from impacts of asperities have more complex spatial and temporal distributions.

6. Rod on rotating pulleys

A rod placed in the grooves of two pulleys, set apart a distance of $2a$ and rotating toward each other, oscillates along its axis with a frequency that depends on the friction coefficient between the pulleys and the rod, $\Omega=\sqrt{\mu g/a}$. In addition to this rigid-body response, the rod develops bending and axial vibrations and radiates sound as the pulleys slip at their respective contacts with the rod. The average intensity of sound reaches a maximum as the oscillatory motion of the rod on the pulleys diminishes and energy from slipping pulleys feeds the rod vibrations. Sound from the rod radiates from its bending vibrations and from longitudinal vibrations through its flat ends. The fundamental longitudinal natural frequency of the rod dominates the sound spectrum, which indicates that, as described earlier, for smooth surfaces excitation is predominantly in a tangential direction. The spatial distribution of sound intensity has a peak along the axis of the rod, in part, because it radiates from its ends due to longitudinal vibrations. When any of its natural frequencies in longitudinal and bending directions coincide, the corresponding modes readily couple thereby increasing sound radiation from bending waves of the rod. This result suggests the significance of in-plane waves in rotating disks where they couple with and promote bending waves under tangential (friction) forces.

As the rigid-body oscillations of the bar cease, its axial vibrations that develop from friction excitation at the two supports can be described with

$$-E\frac{\partial^2 u}{\partial x^2}+\rho\frac{\partial^2 u}{\partial t^2}=[F_N(t)\mu]_{-a}\delta(x+a)-[F_N(t)\mu]_{+a}\delta(x-a), \tag{6}$$

where $F_N(t)$ represents the normal component of the contact force and $\mu_{\pm a}$ represent the coefficients of friction at the contacts the rod makes with the pulleys.

Although the nominal direction of friction excitation is tangential (parallel to the rod axis), surface roughness gives rise to transverse excitation of the rod due to light impacts among the asperities on the surfaces, described by

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=F_N^-(t)\,\delta(x+a)-F_N^+(t)\,\delta(x-a), \tag{7}$$

where $F_N^{\pm}(x,t)$ represent the impulsive normal forces that develop where the rod rests on the pulleys. For elastic contacts, Hertz formulation provides a convenient approximation for $F_N^{\pm}(t)$.

As surface roughness increases, the transverse component of the excitation also increases. Higher rotation speeds often cause contact loss and exacerbate the development of impacts. On the other hand, high normal loads imposed on the beam can inhibit its transverse motion while maintaining tangential excitation. Under such conditions response of the rod primarily consists of axial vibrations. Thus, the partitioning of energy between bending and axial waves in the rod depends on the texture of the rod and pulley surfaces as well as the contact load in the normal direction.

A rod in the V-grooves of a double-pulley system also experiences torsional vibrations that result from the difference in the friction it experiences at its contact areas on the walls of the V-grooves. In this case, sound radiation at torsional frequencies accompanies the rotation of the rod about its axis.

Friction in the grooves also develops waves within the pulleys. However, in the absence of any coupling mechanisms such as impacts due to asperities, these waves confine themselves to in-plane vibrations and produce very little sound radiation. In most systems, however, coupling mechanisms always exist as discussed in the next section.

B. Beam and rotating disk

Friction between the free end of a cantilever beam and a rotating disk provides an example that engenders the different modes of friction behavior and system response, including mode lock-in and surface noise. Different configurations of sliding contact between a beam and disk generate different combinations of waves within each. For instance, in the seemingly simple case where the flat tip of a beam rubs against a planar side of a rotating disk, the friction force, acting tangentially on the disk surface, excites the in-plane waves of the disk. Also, as expected, friction at its tip simultaneously develops bending waves of the beam. As a result of these bending vibrations the beam undergoes, the contact force at the interface develops a fluctuating component normal to the surface of the disk and easily excites the disk-bending vibrations. (The impulses due to the surface texture discussed earlier further contribute to the bending wave excitation of the disk.) This type of geometric coupling that results from the dynamic response of the beam and disk, a potential cause for instabilities, is exacerbated when the beam contacts the disk surface at an acute angle against the direction of motion. Furthermore, should the disk have its in-plane and bending vibration frequencies in close proximity, energy transfer through friction becomes enhanced, thus making it easier to generate bending waves. The examples described below present evidence of different friction sounds that can develop from the sliding contact of a beam tip and a disk and the relationship such sounds have with the disk and beam natural frequencies.

1. Experiments

The demonstrations described here consist of a stationary clamped beam with its free end in sliding contact with the flat surface of a rotating disk during which both the beam and disk simultaneously respond to contact forces. Figures 4(a) and (b) show a free vibration response of the beam and disk, respectively. Figure 4(c) shows the transfer function of the disk when in stationary contact with the beam. Note the splitting of frequency that corresponds to the (0,2) mode of the disk when in contact with the beam. The spectrum of sound during sliding contact of the beam tip and disk, shown in Fig. 4(d), exhibits a mode lock-in at the higher of the split

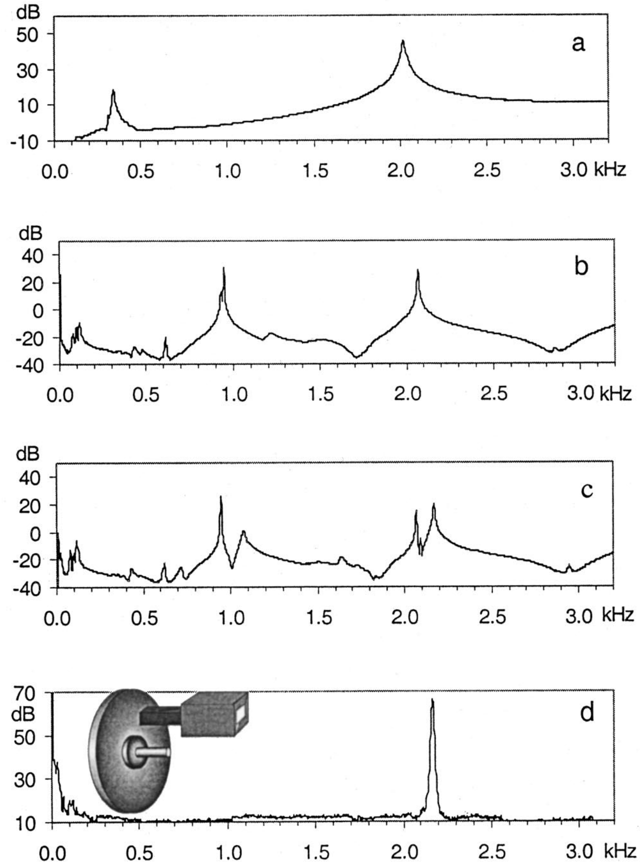

FIG. 4. Development of mode lock-in between a beam and a disk (a) transfer function of disk; (b) transfer function of beam; (c) transfer function of beam and disk in stationary contact; (d) response during sliding contact.

frequencies, which represents the cosine mode, whereas the lower of the pair corresponds to the sine mode for which the beam contact takes place at its nodal line.

Figure 5 demonstrates the diversity of examples of sound radiation from the same beam–disk system. The conditions that lead to sound spectra shown in Figs. 5(a) and (b) differ from each other only by the direction of the disk rotation and produce these very different sounds. Although in both cases the frequency of the second mode of the beam coincides with the (0,2) mode natural frequency of the disk, the first one represents a mode lock-in, but the second does not. For cases shown in Figs. 5(c) and (d), the second natural frequency of the beam coincides with the (0,3) mode natural frequency of the disk. The conditions that lead to the responses shown in Figs. 5(c) and (d) have very little difference between them except for a slight change in the normal pressure. Yet, one produces a pure mode lock-in, Fig. 5(c), and the other, Fig. 5(d) leads to beats (when observed over a longer time than shown here).

2. Models

In general, response of systems to friction, such as those illustrated above for a beam and disk, can be described as forced vibrations, resonant vibrations, or instabilities. Instabilities in the presence of friction usually develop through one of four mechanisms: (i) geometric instabilities (*viz.*, Refs. 10 and 11); (ii) material nonlinearities; (iii) thermoelastic instabilities[12,13] and (iv) instabilities due to decreasing friction with increasing velocity. In the first two types of instabilities, friction has a passive role. For example, in geometric instabilities, such as brush and commutator contact in electric motors, friction has a necessary but passive role.[10] Friction also has a passive role in instabilities caused by system nonlinearities, e.g., in cases where material properties exhibit nonlinear contact stresses (*viz.*, Refs. 14 and 15). On the other hand, friction has an active role in instabilities caused, for example, by a decreasing friction force with increasing velocity that lead to mode lock-in.[2]

A thorough review of unstable vibrations of disks by frictional loading given by Mottershead[16] includes an analysis of instabilities due to frictional follower forces and friction-induced parametric resonances, and their extension to the case with a negative friction-velocity slope.[17–22] Using a distributed frictional load, Mottershead and Chan[17] showed that follower friction force led to flutter instability indicated by the coalescing of eigenvalues, and they compared their results with previous studies.[23,24] Mottershead *et al.*[18] also investigated parametric resonances under a sector load with friction rotating on an annular disk and identified combination resonances and how they change when friction has a negative slope with respect to velocity. Other studies also address the propensity of a disk to generate noise when the natural frequencies of in-plane and bending vibrations exist close to each other[18,25–28] as well as self-excited vibrations of a circular plate with friction forces acting on its edge to model squeal in railway brakes and drum brakes in automobiles.[34,35] Studies addressing friction-induced rigid-body instabilities, such as those encountered in braking systems, are adequately covered elsewhere (*viz.*, Refs. 7, 29–33).

Because different configurations and operating conditions of a clamped beam and a rotating disk can produce the different responses cited above, they provide a useful benchmark for investigating friction sounds and vibrations. As depicted in Fig. 6, considering a beam that makes an angle φ with the normal to the surface of the disk and neglecting the effects of shear and rotatory inertia, its bending and longitudinal vibrations are governed by[36–40]

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}+C\frac{\partial w}{\partial t} =F_N(t)[\mu(v_r)\cos\varphi+\sin\varphi]\delta(x-L), \quad (8)$$

$$EA\frac{\partial^2 u}{\partial x^2}-\rho A\frac{\partial^2 u}{\partial t^2}-C'\frac{\partial u}{\partial t} =F_N(t)[-\mu(v_r)\sin\varphi+\cos\varphi]\delta(x-L), \quad (9)$$

where the beam properties ρ, E, I, and A are as defined before. C and C' are representative damping constants proportional to bending and longitudinal vibration velocities, respectively. The contact force develops at the tip of the beam of length L.

Bending vibrations of a disk excited at a point (r_0,θ_0) on its flat surface are described by

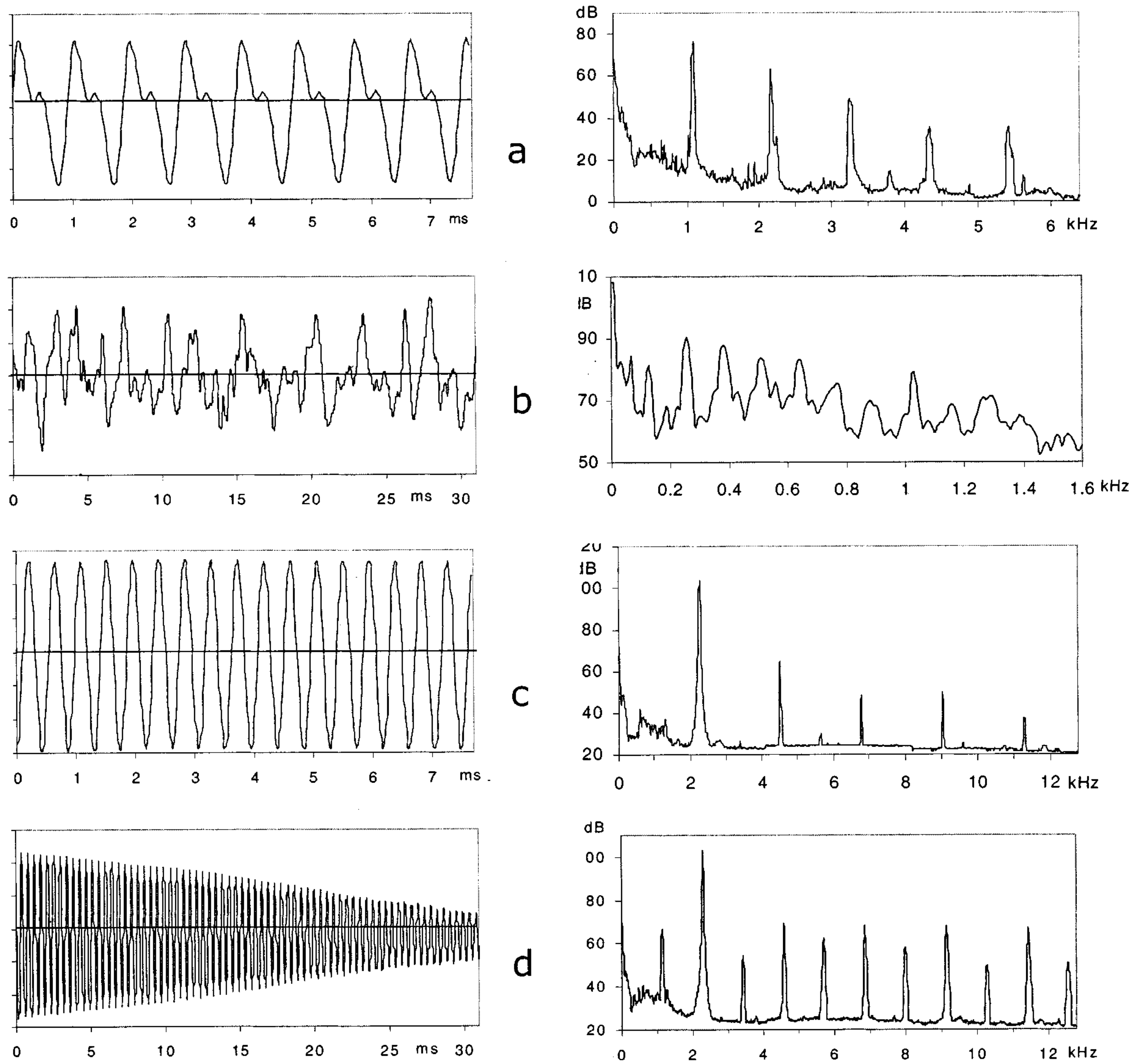

FIG. 5. Time history and spectrum for sounds generated by beam and disk friction. Beam length in (a)–(b) 200 mm, in (c)–(d) 135 mm. (a) CW rotation shows mode lock-in of (0,2) mode of disk; (b) CCW rotation does not generate a lock-in; (c) CW rotation lock-in to (0,3) mode of disk, but a slight change of normal load shows beating in (d).

$$D\nabla^4\eta+\rho_d h_d\frac{\partial^2\eta}{\partial t^2}+C''\frac{\partial\eta}{\partial t}=-F_N(t)\,\delta(r-r_0)\,\delta(\theta-\theta_0), \tag{10}$$

where $D=E_d h_d^3/12(1-\nu^2)$ represents the bending rigidity of the disk, and h_d and ρ_d are its thickness and density, respectively. C'' represents an equivalent damping proportional to velocity.

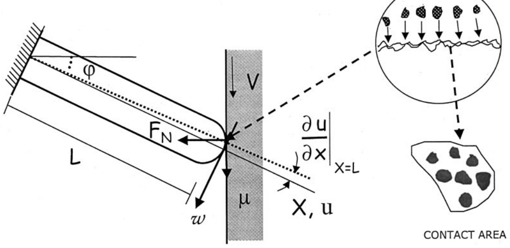

FIG. 6. Schematic of contact of a beam with a flat surface and depiction of the magnified contact area.

Assuming that friction force can be represented with a constant coefficient, $\mu(v_r)F_N(t)$ represents the friction force as a function of the relative sliding velocity, v_r, between the disk and rod tip under a normal contact force $F_N(t)$. The two primary sources of nonlinearities in the equations given above come from the coefficient of friction expressions and the contact forces.

3. Contact force

In the cases described in this section, the normal component of the contact forces relates to the deformation of the contact areas. For instance, in the case of a beam in contact with a disk, the normal contact force $F_N(t)$ can be expressed as one that results from the combined deformation, α, of the beam and disk at the contact area, referred to as the approach. In addition to an initial approach α_0 that results from a static normal load applied to the beam–disk system, the combined deformation between the bending and longitudinal deformations of the beam and the bending deformation of the disk gives rise to a fluctuating contact force that changes with the vibration of the beam and disk.

For example, in case of Eqs. (8)–(10), for perfectly smooth surfaces the combined approach can be expressed as

$$\alpha_H = \alpha_0 + u \cos\varphi - w \sin\varphi + \eta. \tag{11}$$

In cases where surface has a texture that alters the contact force, the approach can include an additional term that describes surface topography which considers both its roughness and waviness.

The Hertz theory describes the contact force $F_N(t)$ that results from elastic deformation in terms of approach α

$$F_N(t) = k_H \alpha_H^{3/2}(t), \tag{12}$$

where for a spherical surface, with radius a, in contact with a planar surface of similar material, $k_H = 2E\sqrt{a}/3(1-\nu^2)$.

When $\alpha_H < 0$, contact ceases and the ensuing impacts lead to a highly nonlinear behavior. A common approach to solving Eqs. (8)–(10) assumes that contact is maintained between the beam and the disk. Under conditions where the friction force linearly depends on the normal load, but independent of sliding velocity, the coupling of Eqs. (8)–(10) eliminates the normal load, leaving only its ratio to friction force, or the coefficient of friction, as a parameter. This model also permits the friction force to follow the displacement and act as a follower force. However, as noted earlier, a normal load more than only modifies the friction force F_f. Within certain regimes of $|F_f/F_N|$, modes of the system coalesce and develop instability. This particular type of instability, sometimes referred to as a flutter instability, is one of the mechanisms that leads to mode lock-in described earlier.

Improvements to this model include consideration of the in-plane vibrations of the disk and an accurate model of the friction force at contact. Just as in the case of longitudinal and bending waves of a beam, in-plane natural frequencies of the disk can have a significant role, particularly when they fall in the neighborhood of its bending wave natural frequencies.

A friction coefficient that increases with decreasing velocity also produces instabilities that generate mode lock-in. Phenomenological expressions for a friction force lead to qualitative results that help bring out the underlying physical phenomena, but not the accuracy required in some applications. Such expressions take several forms, for example[36,2]

$$\mu(v_r) = \alpha_1 e^{\alpha_2 v_r^2} + \alpha_3 v_r + \alpha_4, \tag{13}$$

$$\mu(v_r) = \mathrm{sgn}(v_r)(1 - e^{-\beta_v |v_r|})[1 + (f_r - 1)e^{-\alpha_v |v_r|}], \tag{14}$$

where α_1, α_2, α_3, α_4 in Eq. (13) and α_v and β_v in (14) are constants with f_r representing the ratio of the static to kinetic friction coefficients.

The foregoing equations follow the conventional approach which relies on representing friction as an external contact force. In most cases this approach gives satisfactory answers, but not in all. Some cases require a more detailed and more accurate representation of friction to better predict system response and the development of instabilities. Numerically obtained results of the models described above support the experimental observations, suggesting that at light contact loads, sound sources are confined to the surfaces.[36–40] Other studies also relate sound radiation to the effects of surface roughness, *viz.*, Refs. 41–44.

Analyses of friction sounds that appear in other mechanical systems present different challenges, as in the case of thermally induced noise in roofs,[45,46] roller bearings,[47–50] gear noise,[51] belts,[52] and band brakes.[53,54] Adams,[55–60] in a series of studies that involved friction between elastic half-spaces, discussed instability mechanisms between two half-spaces and showed that the steady sliding generates self-excited waves due to destabilization of slip waves. He also showed that, as a result of unstable sliding, the apparent coefficient of friction measured away from the interface can differ from the interface friction coefficient. Similarly, Ruina and Rice showed the existence of flutter instability during the steady sliding of surfaces where shear stress depends on slip velocity.[61]

The effort required to analyze response to friction in simple systems described above increases with an increase in the dimension of the system. The two examples reviewed below, bowed strings and brakes (both subjects of continuing investigations), represent different but equally complex sources of friction sounds.

C. Bowed string—violin sounds

A bowed string presents a particularly useful example because of the seemingly simple configuration of a bow and a string. Yet, its very complex dynamic response makes it a challenge to accurately predict sound radiation from a string instrument. Such predictions necessitate a high degree of accuracy because the narrow frequency intervals between musical notes (tones) make errors as little as 1% unacceptable.[62] The factors that contribute to the complexity of string dynamics result from the combined interaction among the string, the (violin) structure, the bow, and the friction between the bow and string.

A nondimensional coupling constant was shown to characterize the classical stick–slip example that consists of a mass, m, restrained by a spring of constant k on a conveyor belt that moves at a speed V[62]

$$\Gamma = \frac{F_0/k}{V_0/\omega_0}, \tag{15}$$

where F_0 represents the maximum static friction force and $\omega_0 = \sqrt{k/m}$. The larger the value of Γ, the longer the sticking period of the mass and the longer the period of its oscillation. However, since the frequency of vibration of a bowed string remains the same as for its free vibrations, the coupling function does not correctly describe the bowed-string oscillations. As pointed out earlier,[62] string, and by extension other friction-excited systems, should be considered not as having a single degree of freedom, but as continuum systems with many degrees of freedom. Also, the mode lock-in seen in other examples is not so obvious in the case of a string, for harmonics of a string coincide with its natural frequencies.

The bowing action divides a string into two approximately straight-line sections, on either side of the ''corner,'' caused by the bow contact. During bowing, the string moves with the bow until the tension in the string exceeds the friction force, at which time the string separates from the bow until they reconnect. The corner travels along the string at the transverse speed of string waves, and follows a parabolic path between the two fixed ends of the string. This behavior of the string, named after Helmholtz,[63] who first studied it prior to Raman,[64] describes the free vibrations of the string. At each instant, the string maintains the two approximately straight-line segments between the corner and the fixed ends. Thus, the complete motion of the string during bowing consists of the classic circulating corner of the Helmholtz motion, which is the homogeneous solution of the equations of motion, superposed on the particular integral that describes a stationary corner at the bow contact.

The structure of the violin often modifies the pure Helmholtz motion, i.e., the transverse motion of the string under ideal conditions, through the compliance of pegs at one end and the triangular tailpiece at the other end. As the pegs and tailpiece deform, the tension and length of the string change to make the string vibrations anharmonic. In addition to such nonlinearities, the torsional and longitudinal vibrations of the string, as well as the vibration of the bow, also participate in the transmission of structure-borne sounds through the bridge, thereby adding to the complexity of the problem.

An example of how sound quality depends on the interaction of friction and vibration appears in the phenomenon called *double slip*. Double slip occurs when the actual bowing force falls below the minimum bowing force necessary for ''playability.'' Double slip manifests itself by splitting the fundamental frequency and producing the ''wolf tone.''[65–67]

The difficulties associated with predicting sound radiation from musical instruments with bowed strings, particularly the violin, fall beyond the scope of this paper. Studies continue to uncover the details of what makes one violin sound better than another. Additional references summarizing some of the previous research can be found in *viz.*, Refs. 68–79.

D. Brake noise

Aircraft, rail, and automotive brakes have qualities opposite those of the violin in terms of their structures, purposes, and, albeit unintentionally, the sounds they produce. Within brakes all possible vibrations and classes of waves may develop through several instability mechanisms and the forced vibrations described earlier. Responses range from roughness noise to mode lock-in, which on occasion develops with more than one fundamental frequency. Such a wide range of responses stems from the number of components in a brake system, the range of braking conditions, and the large amount of energy transmitted by friction, compared to a violin or a door hinge.

Records show that as early as 1930 brake noise emerged as one of the top-10 noise problems in a survey conducted in New York City, and it still continues to be a source of annoyance. Industry sources suggest that the warranty costs in North America for brake noise, vibration, and harshness reaches one billion dollars each year.

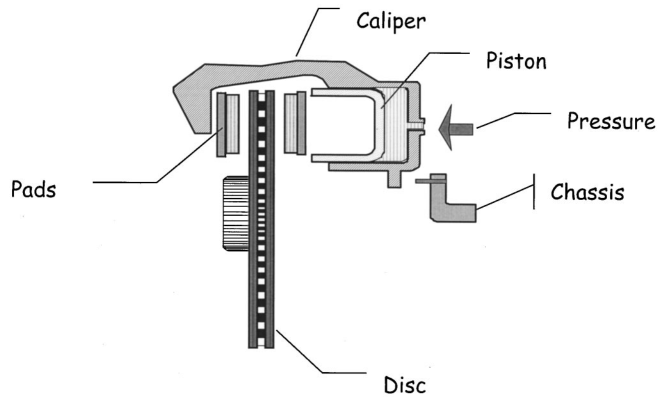

FIG. 7. Components of an automotive brake.

Braking action in an automobile takes place between the rotors and brake pads. A rotor or disk, attached to an axle and usually made of cast iron, carries the wheel (Fig. 7). Each brake pad, one on each side of a rotor, moves freely in a caliper and consists of a layer of friction material attached to a steel backplate. Braking takes place by pushing the pads against the rotor under hydraulic pressure. A steering knuckle attaches the caliper to the frame of the automobile and carries the torque.

Braking converts most of the kinetic energy of a vehicle to thermal energy primarily within the pads and rotors.[80] However, a small fraction of the energy finds its way to vibrational energy within the braking system which can even travel to the suspension of the vehicle. The vibratory energy follows a complex path, and the resulting sound radiation may involve any number of components of the brake system. Aircraft brakes tend to respond at lower frequencies. They ''walk'' (5–20 Hz), chatter (50–100 Hz), and squeal (100–1000 Hz), and sometimes the dynamic instabilities lead to a ''whirl'' (200–300 Hz) of the landing system. Automotive and rail brake noises often reach much higher frequencies. In an effort to display the vibratory energy path (and the types of waves and radiation) braking produces, the following summary presents the sound and vibration sources in a brake in terms of a hierarchy of causes and effects.[81–83]

1. Brake noise generation mechanisms

An idealized brake consists of a pair of pads that squeezes a rotating disk with a constant friction coefficient, with each component having perfect geometry and uniform material properties. Under such conditions, the rotor and the pads experience normal and tangential forces at their interface. These forces, while uniformly distributed during stationary contact, develop a nonuniform distribution during relative motion.

Under a constant normal load, tangential forces acting on the surfaces of a rotor develop in-plane vibrations within it.[25,27,28] A combination of in-plane vibrations, through consequent surface deformations, alters the contact area, thereby changing the presumed constant behavior of the friction force. Even under assumed constant-friction properties at the

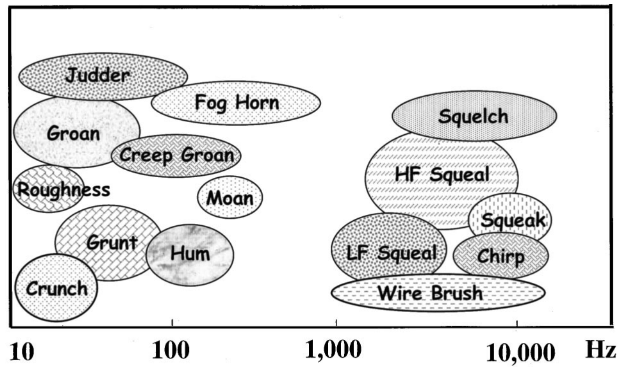

FIG. 8. Different brake noises and their approximate spectral contents.

interface, a friction force induces a moment arm about the supports of the pads that leads to oscillations of the pads. Consequently, oscillatory normal force components always accompany the friction force even in an otherwise ideal braking system, and easily excite bending waves in the rotor and pads.

Geometric instabilities can result in cases where the contact between pads and rotor is not uniform. These instabilities usually lead to rigid-body oscillations of the pads, but they can also cause vibrations within the pads. Pad vibrations, in turn, can have sufficient energy to easily travel to the brake caliper and beyond, and cause caliper resonances, providing yet another source of excitation for the bending waves in the rotor. Rail[84–88] and aircraft[14,15,89–92] brakes have different but analogous designs that also yield geometric instabilities.

In addition to the above, numerous other parameters afford an abundant set of possible sources for brake-noise generation. Examples include the time-dependent nature of friction properties, manufacturing tolerances with respect to flatness and parallelism of rotor surfaces, and variability of material properties. These different noise sources manifest themselves by a variety of brake-noise types described below.

Twenty-five or so different designations describe automotive brake noise and vibrations. Some refer to their alleged mechanisms, and others describe the characteristics of the sounds. Figure 8 shows their approximate distribution in the frequency spectrum. Grunt, hum, groan, and moan have lower frequency content than the family of squeals. Not surprisingly, some of these sounds have similar generation mechanisms. For example, *squeak* describes a short-lived squeal, *wire brush* describes a randomly modulated squeal, and *squelch* describes an amplitude-modulated version of squeak noise.[83] The following sections give synopses of a few of the brake-noise mechanisms.

2. High-frequency squeal

A high-frequency squeal typically involves the higher-order disk modes, with 5 to 10 nodal diameters. Their frequencies range between 5–15 kHz. Nodal spacing between the excited modes is comparable to or less than the length of brake pads. Holograms usually show unconstrained disk mode shapes associated with high-frequency squeal sounds, thus suggesting that the pads do not act as constraints everywhere within their nominal contact areas. Mode shapes, however, remain stationary with respect to the ground, thereby indicating some constraining effect by the pads, although not excessive enough to alter the mode shapes. Furthermore, the spectra show one or more fundamental frequencies closely associated with disk natural frequencies, and their harmonics, whereby exhibiting the mode lock-in phenomenon described earlier.[82,83]

3. Low-frequency squeal

A low-frequency squeal involves modes with 1, 2, 3, or 4 nodal diameters (ND) each with nodal spacing larger than the pad length. Holograms of rotating disks show that squeal occurs at a frequency other than a natural frequency of the rotor. Furthermore, the displacement of the rotor has a fixed shape usually resembling a 2.5-ND mode shape with respect to the laboratory coordinates. Analyses show that this shape is comprised of first, second, third, and fourth modes, and the mode with 3 nodal diameters contributes the most at nearly 50%.[82,83,93]

4. Groan

A typical groan has a spectrum between 10 to 30 Hz, with harmonics reaching 500 Hz. It occurs at low speeds and under moderate braking conditions. A groan appears to result from a geometric instability of pads that gives rise to stick–slip which, in turn, excites the low-frequency resonances in a brake system. In particular, resonances of the rigid-body rotation mode of the caliper and the local suspension parts develop and radiate sound without the participation of the rotor. The position of the pads with respect to the rotor has a significant role: the higher the relative tilt between them, the higher the propensity for groan generation.[83,94]

5. Judder

Judder develops from continuous pulsations between the rotor and pads and manifests itself as a low-frequency vibration with frequencies that are integer multiples of the rotational speed of the wheel. Judder, transmitted through the brake to the chassis and steering, falls into the category of vibration and harshness rather than noise. The conditions that lead to judder generally result from nonuniform friction force between the rotor and pads. The nonuniformity may result from any one or a combination of circumferential thickness variation, uneven coating by friction material, and variation in surface finish.[95,83]

6. Some solutions to brake noise

The engineering solutions to reduce brake noise described below give a few examples that demonstrate methods to prevent waves from developing within a rotor that make it unable to radiate sound. Each of these solutions, however, has obvious drawbacks in terms of its implementation.

Radial slits in a rotor reduce its propensity to radiate sound by preventing the circumferential waves from fully developing. In Fig. 9, the short solid lines on the disks indicate radial slits with numbers referring to the length of each

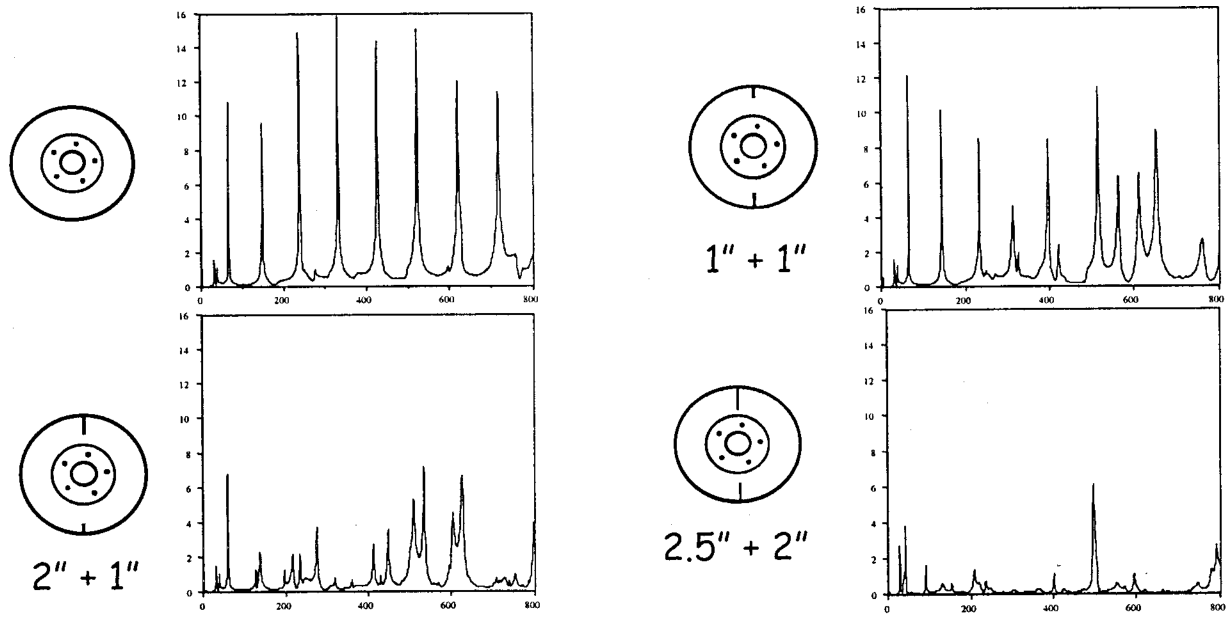

FIG. 9. Transfer functions of a typical automotive rotor with radial slits. The numbers next to lines describing slit locations indicate their lengths in inches.

slit. The corresponding transfer functions of the rotor show a reduction of peak amplitudes with increasing slit length.

The insertion of a metal piece into a radial slit nearly eliminates the sharpness of the peaks in the transfer function of the rotor, thus making it an effective method to reduce high-frequency squeal.

The use of ring dampers around the circumference of a rotor, as shown in Fig. 10, can completely eliminate the peaks in the bending-wave transfer function of a brake rotor. Ring dampers reduce vibrations by means of friction between the ring and the rotor.[96]

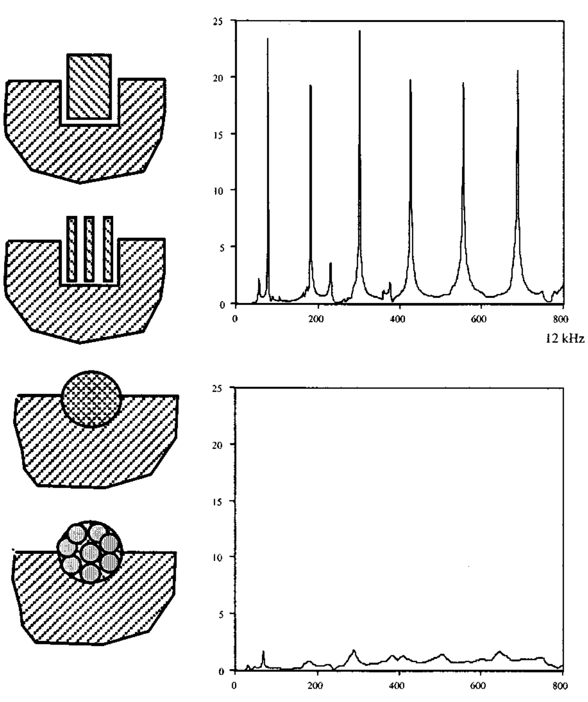

FIG. 10. Transfer function of a rotor with and without ring dampers. The lower figures show the different types of ring cross sections that can be used.

E. Friction sounds in nature

Nature offers its own friction sounds that have developed through evolution, in this case, for the purpose of communication among the animals such as fish, crustacea, and insects. One of the mechanisms by which invertebrates, such as crickets, locusts, and cicadas,[97,98] produce sounds is through friction of differential parts, ordinarily termed stridulation.

1. Stridulatory sounds by insects

The stridulatory apparatus of an insect body has two parts as shown in Figs. 11 and 12. One part, named *pars stridens* (or file), has a surface with ridges, the composition and size of which vary (hairs, spines, tubercles, teeth, ridges, ribs, etc.). The second part, called the *plectrum* (scraper), is a hard ridge or a knob. Morphology and location of *pars stridens* and *plectrum* differ widely with the insects. The *plectrum*, in some cases, is a protrusion with a sharp edge while in others it is composed of the tapering edge of an appendage or a joint. Their locations may be just as varied, on the sides of their heads, under their torso, or at one of many other places, depending on the type of insect.

Insects stridulate by rubbing the scraper (*plectrum*) on the file (*pars stridens*), or vice versa. Stridulation commonly refers to sounds that result from the rubbing of two special bodily structures. However, in some cases, the mechanism by which insects generate sounds may also be described as a succession of impacts made by the rubbing of the scraper and the file, not unlike sounds that result from friction between two periodically rough surfaces exhibiting the "picket-fence effect." As an insect draws the scraper across the file, each tooth impact produces a vibration of the body surface, and generates percussive sounds. Such a series of impact sounds almost always contains many frequencies with complex harmonics. However, in some species (acrid-

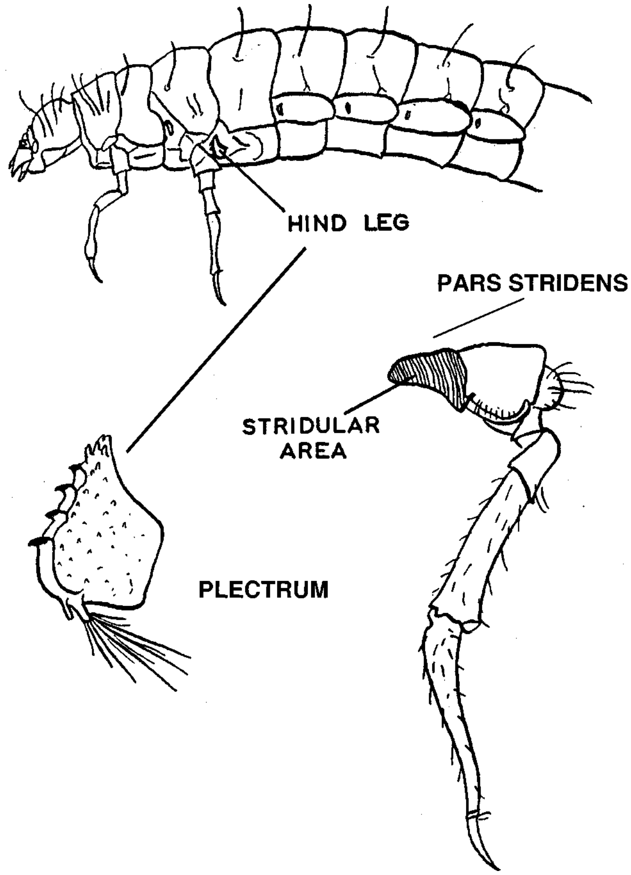

FIG. 11. Drawings that describe stridulatory components of an insect. (a) adult larva; (b) modified hind leg; (c) *pars stridens* on the inner leg [from P. T. Haskell (Ref. 97). After G. C. Schiodte, Nat. Tidsskr. **9**, 227–376 (1874)].

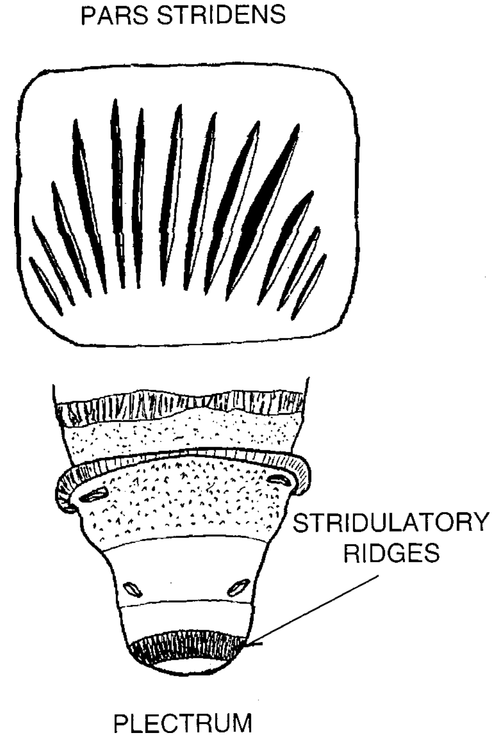

FIG. 12. Stridulatory ridges of a silken cocoon *Eligma hypsoides* with ridges (above) and end of abdomen (below) [from P. T. Haskel (Ref. 97)].

ids), sound emission evokes more of a rubbing sound than a succession of percussions.

In insects with hard exoskeleton, a certain amount of vibrational energy will be transmitted to large resonant surfaces and radiated, even when the actual site of frictional mechanisms is remote and localized, as in the case of insects that produce sound by rubbing their antennae. This transmission is reminiscent of sounds from the cuica and some types of brake noise discussed earlier.

Some of the parameters that affect the emission of sound from insects are inherent in the physical nature of the stridulatory apparatus, for example the spatial distribution of ridges on *pars stridens*. Others concern more the manner by which the insect uses its apparatus: duration, speed of displacement of the parts, and differences in the spacing of movements. During stridulation, the displacement of the point of contact between the file and the scraper excites and produces resonance of various portions of the elytron, thereby modifying the frequency, intensity, modulation, and transients in the sounds insects produce.

Frequency content of friction sounds in insects range from 1–2 kHz up to 90–100 kHz. The breadth of the sound spectrum for a given species is highly variable; for example, some can cover 90 kHz while others are limited to 500 Hz. Often the lower end of the spectrum is used for calling, and the upper end is used for courtship. Warning songs have about 30–35-dB amplitude, whereas calling songs can reach up to more than 50 dB.

To generate sound, an insect moves its limbs back and forth. While observations suggest that sound radiation corresponds to the closing movement, in some cases sound emission occurs in both directions. Some insects employ more complex motions that involve rapid oscillations. It appears that no single law rules the sound emission procedure in insects, not unlike the case for other friction sounds.

Similar sound generation mechanisms exist in ants.[99] Spiny lobsters, however, rub a soft-tissue plectrum over a file, generating a stick-and-slip motion, without the impacts associated with the picket-fence or washboard action described above.[100]

Different groups of fish generate sounds by using skeletal stridulatory mechanisms which are magnified by resonance of the air bladder. An example is stridulation of *pharyngeal* teeth adjacent to the air bladder that amplifies the faint friction sounds. Other underwater insects, such as the water bug, have long been known to generate sounds through stridulation. *Microvelia diluta*, for example, when disturbed, produces a ‘‘shrill scraping sound’’ by stridulation.[98]

2. Pleural and pericardial friction sounds

Friction sounds also develop in humans. Two common forms, pleural and pericardial friction sounds or friction rubs, result from an inflammation of tissues that surround the lungs and the heart, respectively. Pleural friction sounds develop during respiration when inflamed visceral and pulmonary surfaces that cover the lungs and line the thoracic cavity come together. Pericardial friction sounds or friction rubs are caused by an inflammation in the pericardial space or pericardial sac, the fibroserous sac which surrounds the heart, that leads to the parietal (outer) and visceral (inner) surfaces of the roughened pericardium rubbing against each other. A pericardial friction rub produces an extra cardiac sound with both systolic and diastolic components. Up to three components of a pericardial friction rub may be audible. Pericardial friction sounds are said to resemble those of squeaky leather and are often described as grating or scratching.

III. FRICTION AND MOTION

Friction-induced motion, in this paper, refers to flight-free transport of an object on a surface by means of inducing vibrations to the surface and utilizing the friction between the two. The examples described below represent two very different friction motion mechanisms. Flight-free motion, or transportation of objects over a surface without losing contact, represents a useful example of friction-induced motion that involves oscillation of at least one of the surfaces. A flight-free conveyance has uses in the transportation of fragile materials and in motion control that requires accuracy, such as a camera lens. Other forms of motion and force conveyance by friction, such as clutches and belts, do not fall within this category, for they do not require vibration of their components.

The essential elements of flight-free motion involve friction and oscillatory relative motion between the transported object and the platform. Motion results from a nonzero average friction force that develops during oscillatory movements. Such a force and the consequent motion develop through either a rigid-body movement of the platform, or track, or by means of traveling- or standing waves in it. As described below, many of the classical vibratory conveyors rely on the first mechanism, and the ultrasonic motors rely on the second.

A. Analysis of vibratory conveyance

A simple vibratory conveyor consists of a planar platform at an angle α with the horizontal and a mass m placed on it. For conveyance, a throw force vibrates the platform in a direction ψ to the axis of the platform at a frequency ω and amplitude ξ_0. Maintaining a positive contact force N normal to the track, or satisfying the condition $\xi_0\omega^2 \sin\psi/g\cos\alpha < 1$, ensures continuous contact or flight-free motion. Motion of the mass along the track that results from the combined vibration and friction force is governed by

$$m\ddot{x} + \mathrm{sign}(\dot{x})\mu N = -mg\sin\alpha, \tag{16}$$

where x describes the relative displacement of the mass on the track. Substituting in Eq. (16) for the normal force on the mass $N = m(g\cos\alpha - \xi_0\omega^2 \sin\omega t \sin\psi)$ explicitly describes the motion of the mass as

$$\ddot{x} = -g\sin\alpha \mp \mu(g\cos\alpha - \xi_0\omega^2 \sin\omega t\sin\psi), \tag{17}$$

where minus and plus signs indicate slip in the negative and positive directions, respectively. During the stick phase the particle, $\ddot{x}$ will have a value between the two slip accelerations.[101,102] However, conveyors sometimes exhibit nonperiodic, and possibly chaotic, motion of the mass on a track that cannot be explained with the conventional friction models.[103–107]

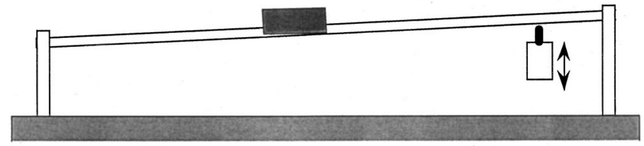

FIG. 13. Description of a flight-free motion of a mass on a beam.

B. Vibratory conveyance: An experiment

A simple laboratory demonstration of friction motion consists of a mass that can move freely on an inclined beam supported at both ends, and a shaker that vibrates the beam and its supports, as shown in Fig. 13. Under a suitable combination of excitation frequency and amplitude, the mass can move up or down the inclined beam without losing contact. The frequencies at which conveyance takes place correspond to a natural frequency of the beam and its supports, whereby each point on the beam surface follows an elliptical trajectory. During oscillations, the mass on the beam experiences a larger contact force, hence a larger friction force, during one-half of the cycle. Such uneven friction forces during each half of a cycle of oscillations produce a nonzero average force on the mass along the beam. This example of flight-free conveyance points to the significance of system dynamics in friction problems and suggests a reconsideration of the classical definition of the coefficient of friction which does not account for the presence of vibrations within the system. A similar demonstration shows a wheel in contact with the beam rotating clockwise or counterclockwise as a result of changing the frequency of excitation that changes the direction of the phasor of surface vibrations.

C. Ultrasonic motors

The principle of motion in ultrasonic motors also relies on elliptical motion of each material point on the surface of the driver that leads to a nonzero average friction force. Ultrasonic motors, whether linear or rotary, produce elliptical motion by utilizing either standing or traveling waves within or on the surface of the driver. Those that rely on standing waves require only one exciter, but without directional control, and operate similarly to the traditional vibratory conveyors described above.[108–115]

Rotating ultrasonic motors normally use traveling bending waves in a circular driver. The method to generate traveling waves involves inducing two standing waves, of equal amplitude that differ in space and in time by $\pi/2$

$$w(r,\theta,t) = W_0 R(r)\cos m\theta \cos\omega t,$$

$$w(r,\theta,t) = W_0 R(r)\cos(m\theta - \pi/2)\cos(\omega t - \pi/2),$$

where the radial component of the displacement given by $R(r)$ represents Bessel functions and θ describes the circular coordinate. Two such standing waves combine to produce a traveling wave

$$w(r,\theta,t) = W_0 R(r)\cos(m\theta - \cos\omega t), \tag{18}$$

that travels at an angular speed of ω/m.

The expression in Eq. (13) describes the motion of the middle plane of the driver disk in the normal direction. The actual displacement of a point on the surface of the disk follows an elliptical path in the $z-\theta$ plane and can be obtained by projecting a surface point onto the midplane.

A rotor placed on such a driver that has traveling waves described above rotates with a speed in the opposite direction to the bending waves[108]

$$\dot{\theta}=-\omega(h_d/2r)m[R(r)/r], \tag{19}$$

which differs from the speed of the traveling waves. By appropriate choice of modes of vibration and disk geometry, the ultrasonic motors rotate at much reduced speeds.[108]

In advanced ultrasonic motors, excitation is usually provided by using an array of piezoelectric transducers distributed near the perimeter of the disk with a spatial distribution that matches the desired mode shape. Alternative methods include use of two thin piezoelectric membranes below the disk, each excited at one of its own resonant frequencies.

D. Granular flow

A different type of friction motion develops with granular materials when they behave like a liquid under vibratory excitation. For example, glass granules of 2–3-mm diameter that fill a cylindrical container move in a regular and coherent manner when the container is vibrated vertically. The granular material flows downward adjacent to and near the wall of the cylindrical container and upward along its center, together forming a toroidal pattern with an axis that coincides with that of the cylinder. The flow originates at the wall of the container where, as a result of wall friction, the granules develop a nonzero average velocity. The downward flow at and near the wall has a narrow thickness reminiscent of a boundary layer. Objects buried among the granules come up to the surface following the flow in the center of the container. But, they cannot follow the boundary layer flow if their size exceeds the layer thickness, a phenomenon which explains the reasons behind size segregation in cylindrical containers (*viz.*, Refs. 116–123). As discussed in the next section, the flow generated in a container of granular materials can have a significant effect on their damping properties.

E. Friction reduction by vibration

Just as vibrations can loosen bolted joints, they can be used to reduce friction. For example, during the cold drawing of wires, vibrations assist in reducing friction between the wire and die. The underlying mechanism of friction reduction is similar to friction-induced motion in the sense that contact forces are manipulated to reduce the average friction force during a cycle of oscillation.[124–126]

IV. FRICTION DAMPING

Friction damping refers to the conversion of kinetic energy associated with the relative motion of vibrating surfaces to thermal energy through friction between them (*viz.*, Refs. 127–148). Applications of friction damping range from damper rings in gears to beanbag dampers consisting of granular materials. Friction damping devices such as the Lanchester damper[149] effectively reduce torsional vibrations. Significant structural damping can also result from friction at joints and connections as well as within braided wire ropes.

A. Granular materials

Granular materials provide an effective mechanism of vibration damping by dissipating energy primarily through inelastic collisions and friction among the granules.[145] For example, sand[150] and lead grains absorb energy largely through inelastic collisions among the grains. On the other hand, low-density granular materials packed adjacent to beam or plate surfaces form a very effective dissipation mechanism mostly due to friction among the particles to which the bending waves of the structure provide the motion.[146,147]

Elastic granules, such as ball bearings, also absorb vibration energy but become effective only when exposed to a vibration field collectively, as in beanbag absorbers. Although each ball bearing may rebound upon impact, when collected in a flexible container such as a bag they behave inelastically, due to friction among them and due to diffusion of the direction of vibratory forces. This collective behavior strongly depends on the packing force that holds them together. If the packing force severely limits relative motion, damping decreases.

A demonstration of applying friction damping to structural vibrations by using granular media consists of a steel tube welded to a beam, illustrated in Fig. 14. A spring-loaded cap adjusts the relative motion among the granules in the tube. Thus, as illustrated in Fig. 15, by changing the pressure (packing force) on the granules through the end cap, damping properties can be adjusted as desired. In the experiments described here, granular materials reduce the response of second and higher modes.[146] Experiments also show that the amplitude of vibrations influences the dynamic behavior of the granules in the container, ranging from a collective plug-like motion to a liquid-like flow. The different behaviors of granules produce different damping effects, thus making granular damping nonlinear and amplitude dependent.[148] Damping properties of granules and optimum packing forces can be evaluated directly by impact tests using a container described above. However, effective impact tests do not address liquefaction that develops during continuous vibrations.

B. Conforming surfaces

Contacts that generate friction damping generally fall into two groups: (a) contact between nominally conforming surfaces that do not have a relative rigid-body motion between the surfaces as in the case of bolted or riveted joints, braided wire ropes, and gas turbine blades, and (b) contacting surfaces that also have a relative whole-body motion as in the case of damper rings in gears and Lanchester dampers. In the first case, relative motion, sometimes referred to as micromotion, may not reach slip conditions, and friction remains in the ''static'' range associated with tangential stiffness. In the second case, full slip can develop between the surfaces.

In any type of contact, friction damping has a preferred range of contact force within which it becomes most effective. Below such an optimum range, excess relative motion

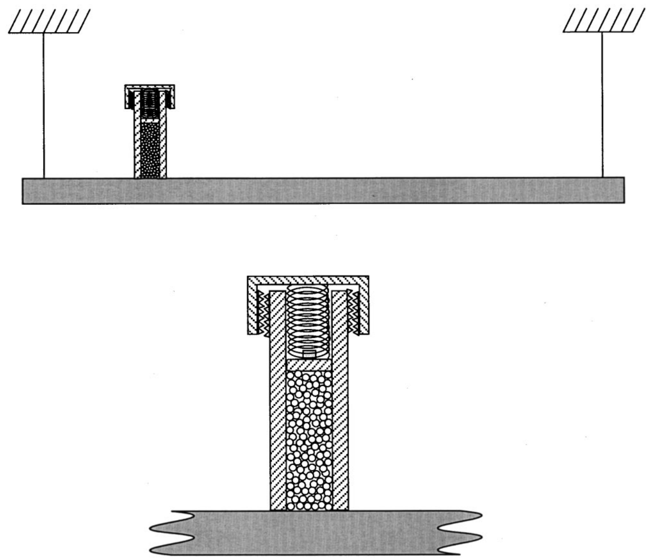

FIG. 14. Granular material in a cylindrical container when oscillated vertically generates a flow within the container.

at the interface develops without significant energy dissipation. Above it, excess pressure limits the development of relative motion for friction to act as an effective damper. Den Hartog[151] provided the case of a simple oscillator with a friction damper solving $m\ddot{x}+kx=F_0\cos\omega t-\mu N\,\mathrm{sgn}(\dot{x})$. More on the specific mechanisms and models of friction damping can be found elsewhere (*viz.*, Refs. 127–148).

The transition from low to high contact forces and the corresponding changes in friction damping and system response can be demonstrated with a simple setup that consists of a cantilever beam with its free end positioned against another surface. Exciting bending vibrations of the beam with a shaker induces the relative motion between the free end of the beam and the opposite surface.[152] The frequency-response functions measured at the tip of the beam, shown in Fig. 16, illustrate the effects of varying normal contact load. The first figure displays the response without friction and clearly shows the clamped–free vibration frequencies. With increased normal load, amplitudes at these frequencies decrease, thus demonstrating damping due to friction. However, at higher normal loads, the response transitions to a different set of frequencies than the clamped–free frequencies and does so without the influence of friction damping. The new frequencies correspond to the clamped–constrained beam natural frequencies, illustrating that, beyond a critical normal load, friction turns into a constraint. Between these two extreme contact forces, the tip of the beam undergoes stick–slip motion. In frequency-domain terms, during stick–slip, both clamped–free and clamped–constrained boundary conditions coexist within each cycle of motion. Such time-dependent boundary conditions form the basis of nonlinearities in systems that involve friction.[152]

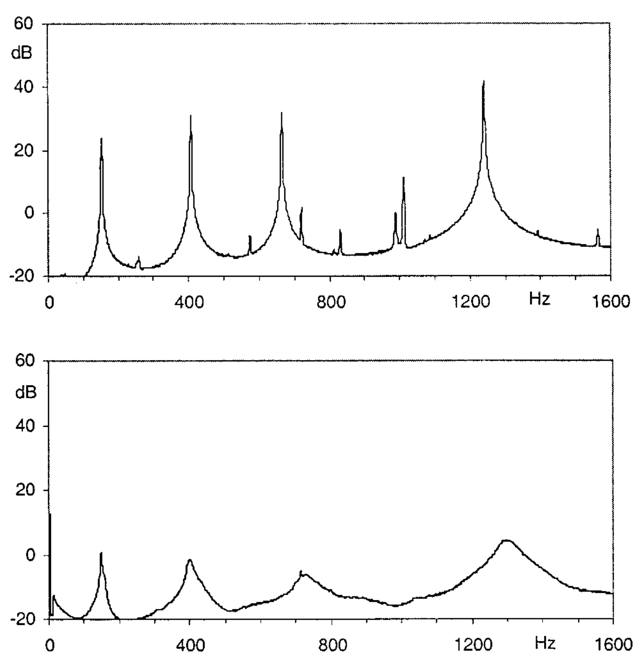

FIG. 15. Transfer functions of a beam with a container carrying granular materials under different pressures show damping of higher frequencies under optimum pressure.

Using thin inserts in the contact area to alter friction makes it possible to isolate the influence of friction from that of the system dynamics. Such inserts do not otherwise affect the system response. In Fig. 17, the measured responses clearly show the significance of friction, regardless of system dynamics or normal load.

V. FRICTION MEASUREMENT AND IDENTIFICATION

A. Measurement of friction

A friction force that results from the sliding contact of two surfaces depends strongly upon the dynamics of the system within which it develops. Since a friction force can only be inferred from measurements of its dynamic effects on a system, the need to also consider the influence of other forces within the system becomes unavoidable. The simplest case of friction force measurement involves an idealized mass on a flat surface moved to and fro by a force F. Under such ideal conditions, the friction force becomes the difference between the force applied to the mass and the resulting inertial force of the mass

$$F_f=F-m\ddot{x}. \tag{20}$$

Friction force relates to a normal load, N, through the conventional coefficient of friction that describes the average friction characteristics of an interface: $F_f=\mu N$. However, in most dynamic systems contact parameters between two sliding surfaces continually change, and the normal contact force does not stay constant. As a result, friction force becomes time dependent, and its estimate requires detailed measurements of the system response and an understanding of the system dynamics.

Figure 18 displays a simple demonstration of friction measurement in a dynamic system that uses a cantilever beam excited harmonically by a shaker similar to that described earlier. The free end of the beam rubs against the spherical surface of a rigid mass attached to a platform that can move freely. A force transducer placed between the rigid mass and the freely moving platform measures the force on

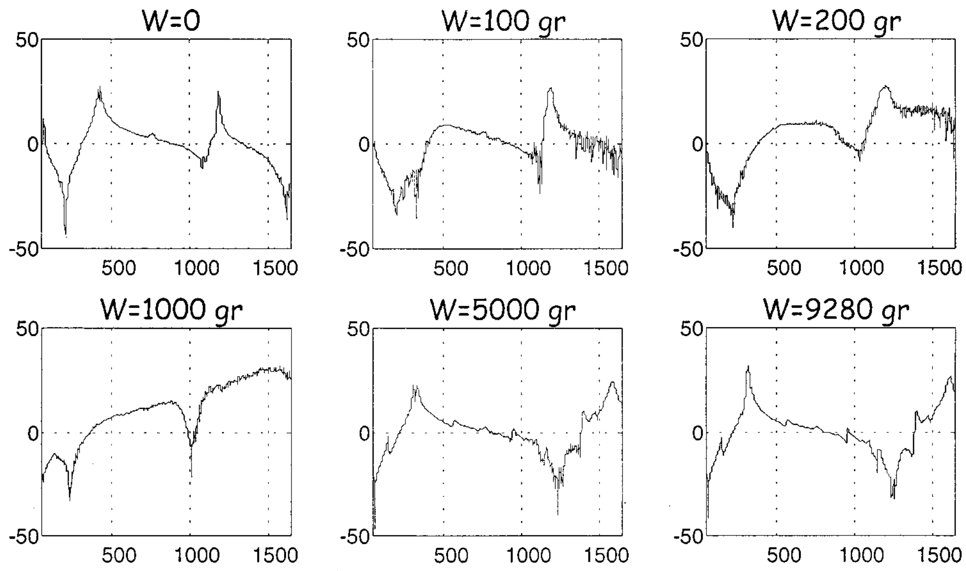

FIG. 16. Transfer function of a clamped–free beam subject to friction at its free end. As normal load increases, response transitions from one of clamped–free to a damped beam and eventually to a clamped beam with a constrained end (Ref. 152).

the mass effected by friction. The sum of the measured force and the inertial force of the mass (obtained by measuring its acceleration) yields the friction force.

B. Identification of friction

Most analyses of dynamic systems represent friction with a constant coefficient or with a function that depends on such parameters as velocity or, infrequently, temperature. The use of such phenomenological expressions gives acceptable results, particularly when modeling a continuous relative motion, as in rotating machinery. In cases of oscillatory relative motion, in particular those with small amplitudes, modeling requires further details of the friction force in terms of its variation with both displacement and velocity. During an oscillatory motion, characteristics of friction change, particularly where relative velocity at the extrema of displacement vanishes. Specifically, in cases where relative motion is very small, as in the contact dampers for turbine blades, an accurate characterization of friction becomes necessary to model system response. The use of the measurement technique described above provides a convenient method to characterize the friction force. As displayed in Fig. 19, an examination of the friction force with respect to time, displacement, and velocity can lead to expressions that fully characterize it.

Considering the change of friction with displacement in Fig. 19, the measurements show that the friction force remains largely constant during sliding, but changes with displacement during each reversal of direction. The slope of the change with displacement represents the tangential stiffness associated with contact, usually represented by a constant in friction expressions

$$F_f = \begin{cases} k(d \mp X_0) \pm \mu N & |k(x - X_0)| < \mu N \\ \mu N \operatorname{sign}(v) & \text{otherwise} \end{cases}, \qquad (21)$$

where X_0 is the maximum amplitude displacement x reaches during the oscillatory motion.

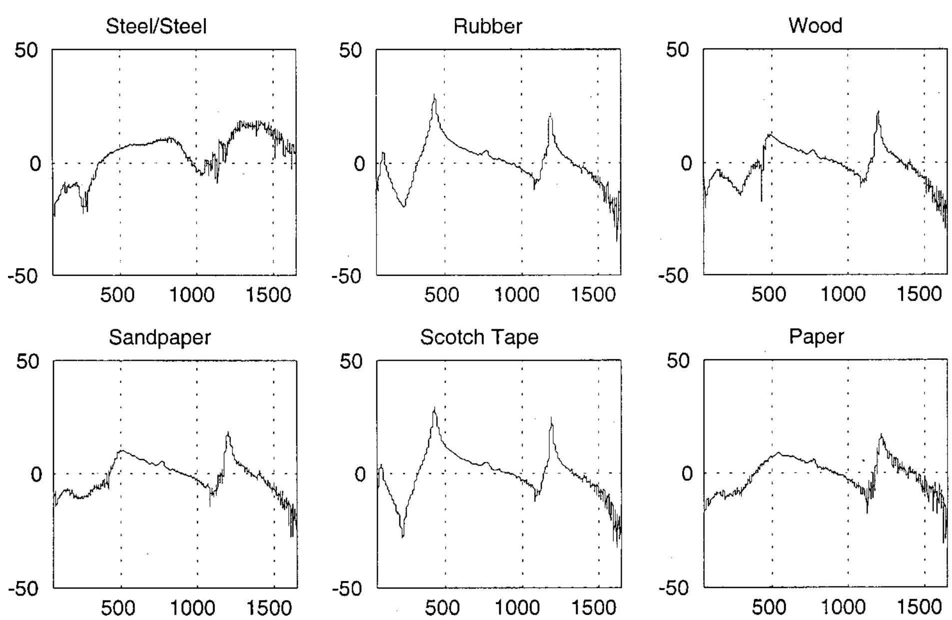

FIG. 17. Transfer functions of a clamped–free beam under a normal load of $N = 9280$ gr with different inserts at the contact area shows how friction affects the dynamic response of the beam (Ref. 152).

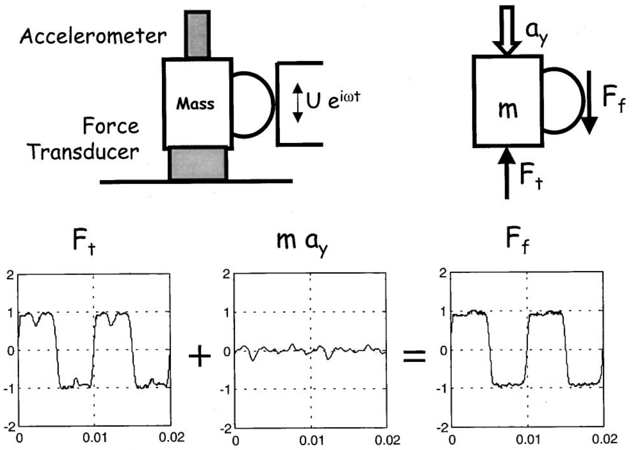

FIG. 18. Description and results of friction measurements between the oscillating tip of a cantilever beam and spherical surface attached to a rigid mass that can move freely.

Similarly, when examined with respect to relative velocity, the friction force remains unchanged during most of the sliding regime, but it shows an exponential dependence on velocity during a reversal of its direction. This velocity dependence can be expressed as

$$F_f = \mu N[1 - e^{-\beta|v \pm V_0|}]\mathrm{sign}(v \pm V_0), \tag{22}$$

where V_0 represents the relative velocity at which the friction force vanishes and β relates to the slope at V_0.

The characteristic coefficients, k and β, that describe the measurements shown here in a sense represent the number of different processes that contribute to friction between two surfaces. The following brief review describes the constituents of friction and relates some of the fundamental aspects of friction in terms of acoustical phenomena.

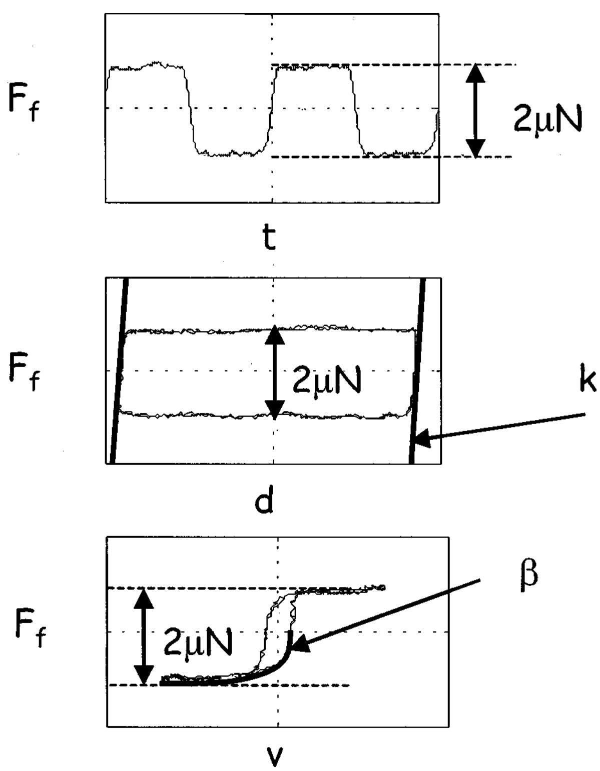

FIG. 19. Variation of a typical friction force with time, displacement, and velocity that shows the constants that characterize it (Ref. 152).

VI. FRICTION

A closer look at a contact area, as depicted in Fig. 6, shows that the true contact between the disk and beam consists of a distribution of small contacts that takes place between asperities of the surfaces. During motion, such true contacts develop at different locations between new asperity pairs, thus making the distribution of the contact area time dependent. The nonuniformity and the resulting time dependence confound accurate modeling of friction. The computation of true contact between surfaces can be modeled through direct numerical simulation (*viz.*, Refs. 153 and 154). However, such detailed simulations have prohibitive computational requirements, at least for now (*viz.*, Refs. 155 and 156). In addition, friction still has aspects that require further understanding before it can be modeled accurately. Consequently, the unavailability of definitive friction models prevents accurate prediction of acoustic response of those friction-excited systems that are sensitive to friction parameters.

A. Friction components

The chart in Fig. 20 describes the processes by which "dc" kinetic energy of a relative motion converts into "ac" oscillations that represent thermal energy that eventually dissipates to the surroundings. During sliding contact, part of the kinetic energy produces waves and oscillations in the bodies, and part of it leads to plastic and anelastic deformation of asperity tips. Some energy expends through viscous dissipation, and the balance through adhesion, fracture, chemical reactions, and photoemission. Distribution of energy conversion through these processes varies for different applications. Each of these processes provides a mechanism for converting the original kinetic energy to an interim one in the form of vibration and sound, deformation energy, surface energy, tribo-chemical energy, and other tribo-emissions. In the end, part of the initial energy remains stored as potential energy, and part of it converts to thermal energy, eventually dissipating to the surroundings. Thus, friction can be viewed as a combination of processes that transforms ordered kinetic energy into either potential energy or a disordered, or thermalized, state of kinetic energy. It then follows that the friction force can be considered as a combination of forces that resist motion during each of these energy conversion processes

$$F_f = F_{\text{elastic deformation}} + F_{\text{plastic deformation}} + F_{\text{fracture}} + F_{\text{adhesion}} + \cdots . \tag{23}$$

The schematic depiction in Fig. 21 summarizes contributions to friction at different scales. The events that take place at each scale and contribute to the unsteady nature of friction require different models. The difficulty continues to center around integration of these length scales, each of which, by necessity, requires expertise from different disciplines of science and engineering. However, each scale of friction involves acoustics. Friction sounds and vibrations

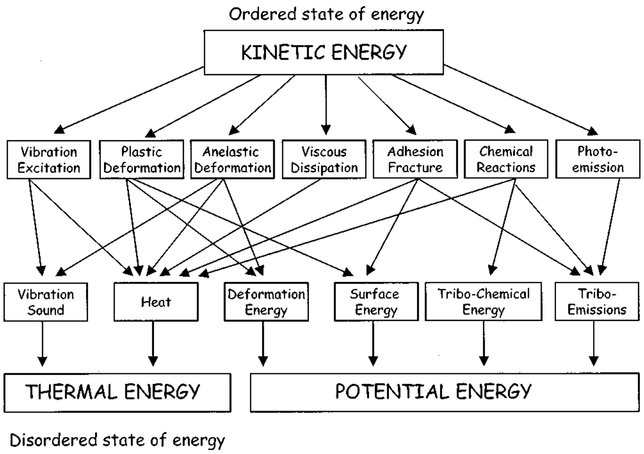

FIG. 20. A description of energy path during friction.

described earlier have engineering time and length scales. Near the interface, however, a continuum scale becomes more suitable for modeling, and the corresponding acoustical phenomena include, for example, ultrasonic emissions from contact areas. At the atomic scale, the primary issues relate to the dissipation and oscillation of atoms that describe thermal energy, examined later in this section.

Many of the current efforts to model friction start at the continuum scale, relating surface roughness to friction (*viz.*, Refs. 155–157). As depicted in Fig. 20, the obvious mechanisms that contribute to friction in such models include elastic/plastic contacts, viscous dissipation, fracture, and adhesion. Each of these processes develops at each true contact region between the surfaces. And, the true contacts take place between asperities on the surfaces or on particles between them. Computational resources required to compute a model that includes all the processes that contribute to friction at each asperity, for the time being, make it prohibitive. Also, in cases where inertial effects and elastic waves may not be ignored, the motion of the surfaces adds one more complication to the modeling of friction.

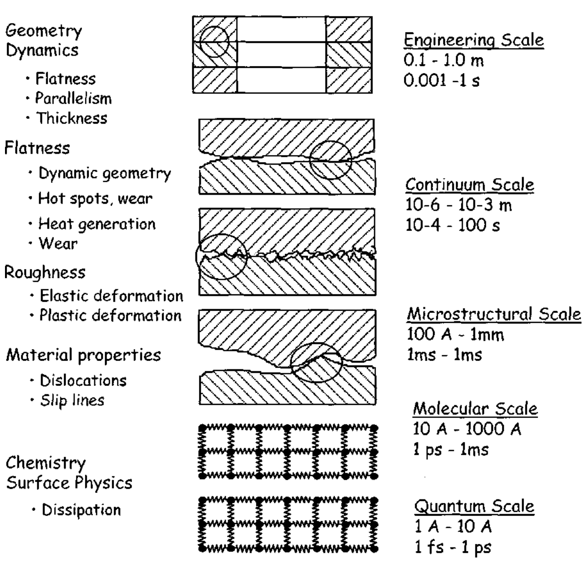

FIG. 21. Length and time scales associated with friction and examples of different events that accompany each.

B. Friction–vibration interaction

A significant, but often overlooked, aspect of friction and the waves and oscillations it causes relates to the feedback relationship between friction and the resulting structure-borne acoustic field. During a sliding contact, whether caused by friction or other forces, waves and oscillations modify the distribution of the true contact areas. This interaction of vibration and friction at the continuum scale forms the basis for a feedback loop between friction and vibrations.

Because friction is an intrinsic part of a dynamic system, accurate models to describe friction-excited vibrations must account for the coupling of system dynamics and the simultaneous development of friction during relative motion. One approach that combines system dynamics with the development of friction relates reactions at asperities (continuum scale) to relative motion (engineering scale) in terms of the deformation of asperities and their resistance to adhesion.[157,158] Forces at each asperity contact depend on the area of that contact which, in turn, depends on the nominal relative position of the surfaces. Such dependence between the individual true contact areas and relative position of surfaces relates friction to the motion of the surfaces. This approach also assumes that contacts take place at asperity slopes. Consequently, normal oscillations develop, even though the nominal motion takes place in the tangential direction.[159,160] Considering the classical case of an oscillator on a conveyor, equations that describe its response in tangential and normal directions to the conveyor couple with equations that, in this case, describe the true area of contact, A, which changes during sliding

$$m\ddot{x}+c\dot{x}+kx=\mathcal{F}_t, \quad m\ddot{y}=-N+\mathcal{F}_n,$$

$$\dot{A}_n^{(r)}+\mathcal{G}(\dot{x},\dot{y},Z,\dot{Z},v)A_n^{(r)}=0,$$

$$\dot{A}_n^{(a)}+\mathcal{G}'(\dot{x},\dot{y},Z,\dot{Z},v)A_n^{(a)}=0.$$

The first two equations above describe the dynamics of a simple oscillator on a conveyor configuration subject to tangential and normal forces $(\mathcal{F}_t,\mathcal{F}_n)$ that result from sliding friction. Both components of the contact forces relate to the instantaneous properties of the footprint of the total true contact area, A_n, on a plane parallel to the interface.[2,158] The second set of equations, using functions $\mathcal{G}$ and $\mathcal{G}'$, describe the variation of the true contact area as a function of the motion of the system in both tangential and normal directions, relative velocity v, average slope Z of the asperities on the surface, as well as time rate of change of slope, $\dot{Z}$. The superscripts (r,a) identify the slopes of asperities that resist or assist sliding. Such an approach to modeling friction together with system response simultaneously yields the friction force and system response that affect each other.[158]

One missing element in friction models relates to friction at the atomic level, sometimes referred to as microfriction. Presently, efforts to simulate materials at the atomic scale continue, but with a different emphasis than those presented here where the focus is on the oscillatory behavior of atoms.

C. Dissipation and atomic level friction

Friction in its most fundamental form occurs at the atomic level. During friction between two atomically flat surfaces, some of the kinetic energy associated with the relative motion propagates (as phonons) beyond the surfaces into the bodies and dissipates. Dissipation here refers to conversion of kinetic energy to thermal energy. The same phenomenon occurs during passage of a sound wave in a solid when some of its energy converts to thermal energy.

Thermal energy relates to the vibrations of atoms in a solid. In solids atoms are held in equilibrium with respect to each other by means of electrostatic forces or interatomic potentials, often described as bonds between atoms. As external forces such as a sound wave or friction on a surface excite atoms, their energy levels increase as represented by their vibration amplitudes. Models that study thermal energy consider vibrations of atoms in a solid as an idealized lattice which has a periodic structure of identical mass-spring cells. In such a lattice, nonlinear springs link the masses together.

Proposed relationships between the kinetic energy of atoms and temperature in a solid assume that vibrations of atoms are in statistical (or thermal) equilibrium and that energy absorption is irreversible. Investigations of thermalization in a lattice look for conditions leading to energy equipartitioning among its modes of vibration. Studies on deterministic models of lattice dynamics point to chaos, instead of ergodicity as believed earlier, as an indicator of thermalization and the means of energy equipartitioning.

If the lattice does not respond chaotically, energy absorbed by the system remains coherent, and periodically returns to the external source. Thus, chaotic behavior ensures both equipartitioning and irreversibility of energy absorbed by a lattice. Although a model based on first principles that describes the mechanism by which this conversion takes place continues to be a research topic, a one-dimensional array of atoms can help elucidate the behavior of the response of atoms. Considering an array of oscillators connected with identical nonlinear springs with a spring constant

$$K(x)=2bD[e^{-b(x-a)}-e^{-2b(x-a)}], \qquad (24)$$

which can be derived by representing the interatomic potential between atoms by the Morse potential[161]

$$V(x)=D[e^{-2b(x-a)}-2e^{-b(x-a)}]. \qquad (25)$$

The equations that describe the response of the array can be expressed as

$$m\ddot{x}_i=2bD[e^{-2b(x_i-x_{i-1}-a)}-e^{-b(x_i-x_{i-1}-a)}] \\ -2bD[e^{-2b(x_{i+1}-x_i-a)}-e^{-b(x_{i+1}-x_i-a)}], \qquad (26)$$

where $i=1,\ldots,n-1$. The equation describing the motion of the atom at its free end is

$$m\ddot{x}_n=2bD[e^{-2b(x_n-x_{n-1}-a)}-e^{-b(x_n-x_{n-1}-a)}]+A\cos\omega t. \qquad (27)$$

Numerically solving these equations allows examination of conditions under which energy becomes thermalized. A detailed explanation of the conditions that lead to dissipation is given in Ref. 162.

VII. FRICTION AND BOUNDARY-LAYER TURBULENCE

The purpose of this section is to describe analogies between solid friction and boundary-layer turbulence. Although friction and boundary-layer turbulence occur in dissimilar continua, they possess notable phenomenological similarities between many of the attributes that describe them. The following analogies assume motion of a solid or a fluid continuum over a semi-infinite solid body (*viz.*, Refs. 163–165).

A. Generation mechanisms

Consider the boundary between the two media having a perfectly flat surface. In a flow over a smooth plane near the leading edge, a laminar boundary layer develops. At higher speeds and large distances downstream, after a region of transition, a turbulent boundary layer fully develops. [Such development depends on the Reynolds number (Re) and the amount of turbulence present in the free stream.] While not commonly addressed, surface friction has a significant role in the development of boundary-layer turbulence, particularly over a perfectly flat surface. An analogous configuration for solids consists of friction between flat surfaces of half-spaces where interface or Stoneley waves develop. In cases of finite-size bodies, the friction force leads to the development of a moment and other types of vibratory behavior as a result of nonuniform contact force distribution.

For rough surfaces, the nature of contact of a fluid flow over a surface differs from that between two solids. In solids, actual contact takes place at the true contact areas, usually between crests of waves or tips of asperities. At the fluid interface, contact develops everywhere. In the case of solids under light loads, stress waves develop locally around the asperities. As long as the contact load stays light, stress distribution is confined to an area near the interface. At higher contact loads, the stress field around each true contact spreads and starts to interact with the stress fields of the other contacts, eventually developing into a full stress field in the body. Because the system is dynamic, these contact stresses lead to waves in the bodies and transport some of the energy away from the contact areas. (In some cases, the response can rigidly move the entire body.)

Fluid flow over a rough surface exhibits an analogous behavior where local pressure fields develop around the protuberances (asperities) on a surface. At low Reynolds numbers, the influence of pressure fields remains local. As Re increases, the pressure field around each asperity spreads and starts to interact with the pressure fields around the others.

In the case of solids, interactions between asperities of opposite surfaces generate waves which then transmit energy into the body of the solids. In the case of a fluid, eddies develop and carry momentum into the free-stream flow. In both cases, the mean motion of the main body of the solid or the fluid may be unaffected by its behavior at the boundary.

B. Analogies

1. Dissipation

Analogous to friction, turbulent flows also have dual roles. Turbulence transmits energy from the flow to the boundary as well as dissipates the kinetic energy of the turbulent flow as viscous shear stresses perform work increasing the internal energy of the fluid. Consequently, turbulence needs a continuous supply of energy to make up for these viscous losses. Without a continuous energy supply, turbulence decays rapidly. As proposed by Kolmogorov, and explained further by Bachelor, any large-scale individual motion in a fluid cannot persist as such indefinitely, but sooner or later breaks down into smaller eddies. However, as the energy progressively passes on to smaller length scales, Re becomes too low to permit the formation of yet smaller eddies. The energy is then directly converted into heat as it is absorbed into the random motion of the molecules by viscosity. In summary, kinetic energy moves from the mean flow into the large, or energy-containing scales, and from them into the next smaller eddies, and so on until it reaches the scales at which dissipation takes place in terms of increased thermal energy. This process is reminiscent of what takes place during solid friction. As discussed in the earlier sections, while friction also has the role of transferring energy from one body to the other, it also dissipates energy. Unless a constant source of energy makes up for the losses, the relative motion will not last.

2. Continuum

The equations of fluid mechanics govern turbulence as a continuum phenomenon. Except in certain circumstances, even the smallest turbulent length scales (the Kolmogorov microscale) are very much larger than molecular scales. Friction, while commonly treated as a continuum concept, also develops at the molecular scale. In this manner, friction differs from the current understanding of boundary layer turbulence and suggests that turbulence may also have molecular scale events, similar to friction.

3. Inertia versus stiffness

Turbulence is an inertial phenomenon. It has statistically indistinguishable characteristics on energy-containing scales in gases, liquids, slurries, foams, and many non-Newtonian media. These media have markedly different fine structures, and their mechanisms for dissipation of energy are quite different. Friction, in contrast, exhibits stiffness characteristics at low velocities and damping at higher velocities, as discussed earlier.

4. Irregularity

Irregularity and randomness that characterize turbulence make deterministic approaches impossible and necessitate statistical methods. As pointed out earlier, due to the computational requirements required for deterministic simulations of friction, statistical approaches are often used to describe such characteristics as surface roughness.

5. Diffusivity

Diffusivity, another important feature of turbulence, causes rapid mixing and increased rates of momentum, heat, and mass transfer. The diffusivity of turbulence prevents boundary-layer separation on airfoils at large angles of attack. It increases heat transfer rates in machinery and the source of resistance of flow in pipelines. It also increases momentum transfer between winds and ocean waves.

During friction, momentum and heat transfer also take place. Equally important, friction generates heat.

6. Boundary layer turbulence and friction as features of system dynamics

Turbulence originates as an instability of laminar flow as Re becomes large. The instabilities stem from interactions of viscous terms and the nonlinear inertia terms in the equations of motion. Thus, turbulence is a feature not of fluids but of fluid flows. Most of the dynamics of turbulence is common to all fluids if Re of turbulence is large enough. According to current thinking, molecular properties of the fluid in which the turbulence occurs do not control the major characteristics of turbulent flows. Since the equations of motion are nonlinear, each individual flow pattern has certain unique characteristics associated with its initial and boundary conditions.

Analogously, friction is neither a property of a material nor of a surface, but it is a property of a dynamic system. The waves generated at the interface emanate from contact areas into the solid bodies which then change the contact configuration. Their wave numbers range from molecular distance to correlation lengths between material impurities, and to the size of the surface on which friction develops.

VIII. CONCLUDING REMARKS

This paper attempts to bring together acoustics and friction by exposing many of the topics that are common to both fields. When treated as a tangential force acting on a surface, friction becomes the source of all imaginable types of waves within solids, the cause of music or noise, and the source of damping for resonant and unstable vibrations. Conversely, through the use of acoustic perturbations, the influence of friction can be modified, mostly to reduce friction and, when managed, to produce precisely controlled motion. The ultimate intersection of friction and acoustics takes place at the atomic level, where oscillations of atoms provide the mechanism by which the kinetic energy associated with friction converts to thermal energy. This paper excludes many other topics related to the acoustics of friction, such as acoustic emission from contact regions and friction-excited rigid-body behavior. However, the author hopes that this review adequately covers the most common aspects of acoustics and friction, and reaffirms that, like many other dynamic phenomena, even friction down to the atomic level has its foundation in acoustics.

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[1] J. Tyndall, *Sound* (Appleton, New York, 1903) or (Greenwood Press, Westport, CT, 1970).
[2] M. T. Bengisu and A. Akay, ''Stability of friction-induced vibrations in multi-degree-of-freedom systems,'' J. Sound Vib. **171**, 557–570 (1994).
[3] J. W. S. Rayleigh, *The Theory of Sound*, 2nd ed. (Dover, New York, 1894).
[4] R. T. Spurr, ''The ringing of wine glasses,'' Wear **4**, 150–153 (1961).
[5] A. P. French, ''In vino veritas: A study of wineglass acoustics,'' Am. J. Phys. **51**, No. 8, 688–694 (1983).
[6] G. Galileo, *Dialogues Concerning Two New Sciences*, translated by H. Crew and A. DeSalvo (Northwestern University Press, Chicago, 1946).
[7] B. Feeny, A. Guran, N. Hinrichs, and K. Popp, ''A historical review on dry friction and stick–slip phenomena,'' Appl. Mech. Rev. **51**, 321–341 (1998).
[8] E. F. F. Chladni, *Entdeckungen über die Theorie des Klanges* (Breitkopf und Härtel, Leipzig, 1787).
[9] E. F. F. Chladni, *Die Akustik* (Breitkopf und Härtel, Leipzig, 1802 and 1830).
[10] J. Swayze and A. Akay, ''Effects of system dynamics on friction-induced oscillations,'' J. Sound Vib. **173**, 599–610 (1994).
[11] E. J. Berger, C. M. Krousgrill, and F. Sadeghi, ''Stability of sliding in a system excited by a rough moving surface,'' J. Tribol. **119**, 673–680 (1997).
[12] Y.-B. Yi, S. Du, J. R. Barber, and J. W. Fash, ''Effect of geometry on thermoelastic instability in disk brakes and clutches,'' J. Tribol. **121**, 661–666 (1999).
[13] K. Lee, ''Frictionally excited thermoelastic instability in automotive drum brakes,'' J. Tribol. **122**, 849–855 (2000).
[14] S. Y. Liu, J. T. Gordon, and M. A. Ozbek, ''Nonlinear model for aircraft brake-squeal analysis: Model description and solution methodology,'' J. Aircr. **35**, 623–630 (1998).
[15] J. T. Gordon, S. Y. Liu, and M. A. Ozbek, ''Nonlinear model for aircraft brake squeal analysis: Stability analysis and parametric studies,'' J. Aircr. **35**, 631–636 (1998).
[16] J. E. Mottershead, ''Vibration- and friction-induced instability in disks,'' Shock Vib. Dig. **30**, 14–31 (1998).
[17] J. E. Mottershead and S. N. Chan, ''Flutter instability of circular discs with frictional follower loads,'' J. Vibr. Acoust. **117**, 161–163 (1995).
[18] H. Ouyang, J. E. Mottershead, M. P. Cartmell, and M. I. Friswell, ''Friction-induced parametric resonances in discs: Effect of a negative friction-velocity relationship,'' J. Sound Vib. **209**, 251–264 (1998).
[19] H. Ouyang, J. E. Mottershead, M. P. Cartmell, and D. J. Brookfield, ''Friction-induced vibration of an elastic slider on a vibrating disc,'' Int. J. Mech. Sci. **41**, 325–336 (1999).
[20] S. N. Chan, J. E. Mottershead, and M. P. Cartmell, ''Parametric resonances at subcritical speeds in discs with rotating frictional loads,'' Proc. Inst. Mech. Engrs. Part C (J. Mech. Eng. Sci.) **208**, 417–425 (1994).
[21] H. Ouyang, J. E. Mottershead, D. J. Brookfield, S. James, and M. P. Cartmell, ''A methodology for the determination of dynamic instabilities in a car disc brake,'' Int. J. Veh. Des. **23**, 241–262 (2000).
[22] H. Ouyang, S. N. Chan, J. E. Mottershead, M. I. Friswell, and M. P. Cartmell, ''Parametric vibrations in discs: Point-wise and distributed loads, including rotating friction,'' Des. Eng. Tech. Conf., Part A, DE-Vol 84-1 (3-A), 1125–1133 (1995).
[23] J. S. Chen and D. B. Bogy, ''Effects of load parameters on the natural frequencies and stability of a flexible spinning disk with a stationary load system,'' J. Appl. Mech. **59**, 5230–5235 (1992).
[24] D. Lee and A. M. Waas, ''Stability analysis of a rotating multi-layer annular plate with a stationary frictional follower load,'' Int. J. Mech. Sci. **39**, 1117–1138 (1997).
[25] K. I. Tzou, J. A. Wickert, and A. Akay, ''In-plane vibration modes of arbitrarily thick disks,'' J. Vibr. Acoust. **120**, 384–391 (1998).
[26] K. I. Tzou, J. A. Wickert, and A. Akay, ''Frequency clusters in the spectrum of annular cylinders,'' J. Appl. Mech. **65**, 797–803 (1998).
[27] Y. Gui-Lan and W. Yue-Sheng, ''In-plane motion induced by an SH pulse at a frictional contact surface,'' Mech. Res. Commun. **25**, 203–210 (1998).
[28] M. Matsuzaki and T. Izumihara, ''Brake noise caused by longitudinal vibration of the disc rotor,'' SAE Paper 930804 (1993).
[29] V. Aronov, A. F. D'Souza, S. Kalpakjian, and I. Shareef, ''Interactions among frictions, wear and system stiffness, Part 1: Effect of normal load and system stiffness,'' J. Tribol. **106**, 55–58 (1984).
[30] V. Aronov, A. F. D'Souza, S. Kalpakjian, and I. Shareef, ''Interactions among frictions, wear, and system stiffness, Part 2: Vibrations induced by dry friction,'' J. Tribol. **106**, 59–64 (1984).
[31] V. Aronov, A. F. D'Souza, S. Kalpakjian, and I. Shareef, ''Interactions among frictions, wear, and system stiffness, Part 3: Wear model,'' J. Tribol. **106**, 65–69 (1984).
[32] N. Hinrichs, M. Oestreich, and K. Popp, ''On the modeling of friction oscillators,'' J. Sound Vib. **216**, 435–459 (1998).
[33] B. Feeny and F. C. Moon, ''Chaos in a forced dry-friction oscillator: Experiments and numerical modelling,'' J. Sound Vib. **170**, 303–323 (1994).
[34] A. Sueoka, Y. Yoshitake, H. Tamura, and K. Horita, ''Self-excited vibration of a circular plate subjected to frictional forces exerted in two regions on its outer circumference. Part I. The case of a circular plate without effect of internal resonances,'' Bull. JSME **29**, No. 255, 3019–3057 (1986).
[35] A. Sueoka, Y. Yoshitake, H. Tamura, and K. Horita, ''Self-excited vibration of a circular plate subjected to frictional forces exerted in two regions on its outer circumference. Part II. The case of a circular plate with effect of internal resonances,'' Bull. JSME **29**, No. 255, 3058–3065 (1986).
[36] M. Yokoi and M. Nakai, ''A fundamental study on frictional noise, 1st report: The generating mechanism of rubbing noise and squeal noise,'' Bull. JSME **22**, No. 173, 1665–1671 (1979).
[37] M. Yokoi and M. Nakai, ''A fundamental study on frictional noise, 2nd report: The generating mechanism of squeal noise of higher modes,'' Bull. JSME **23**, No. 186, 2118–2124 (1980).
[38] M. Yokoi and M. Nakai, ''A fundamental study on frictional noise, 3rd report: The influence of periodic surface roughness on frictional noise,'' Bull. JSME **24**, No. 194, 1470–1476 (1981).
[39] M. Yokoi and M. Nakai, ''A fundamental study on frictional noise, 4th report: The influence of angle of inclination of the rod on frictional noise,'' Bull. JSME **24**, No. 194, 1477–1483 (1981).
[40] M. Yokoi and M. Nakai, ''A fundamental study on frictional noise, 5th report: The influence of random surface roughness on frictional noise,'' Bull. JSME **25**, No. 203, 827–833 (1982).
[41] C. H. Wang and A. Soom, ''Interpretation of rubbing noise radiated from a pin-on-disc configuration,'' ASME 83-WA-NCA-41-6 (1983).
[42] F. Bergman, M. Eriksson, and S. Jacobson, ''Influence of disk topography on generation of brake squeal,'' Wear **225–229**, 621–628 (1999).
[43] T. Ananthapadmanaban and V. Radhakrishnan, ''An investigation of the role of surface irregularities in the noise spectrum of rolling and sliding contacts,'' Wear **83**, 399–409 (1982).
[44] V. M. Baranov, E. M. Kuddryavtsev, and G. A. Sarychev, ''Modeling of the parameters of acoustic emission under sliding friction of solids,'' Wear **202**, 125–133 (1997).
[45] C. H. Ellen, C. V. Tu, and W. Y. D. Yuen, ''Thermally induced roof noise,'' J. Appl. Mech. **52**, 494–496 (1985).
[46] C. H. Ellen, C. V. Tu, and W. Y. D. Yuen, ''Theory for thermally induced roof noise,'' J. Struct. Eng. **111**, 2302–2319 (1985).
[47] M. Nakai, M. Yokoi, M. Inue, and K. Kawakami, ''Squealing of cylindrical roller bearing,'' JSME Int. J. **34**, 72–81 (1991).
[48] D. G. Evseev, B. M. Medvedev, G. G. Grigoryan, and O. A. Ermolin, ''Description of the rolling friction process by acoustic modeling,'' Wear **167**, 33–40 (1993).
[49] A. I. Krauter, ''Generation of squeal/chatter in water-lubricated elastomeric bearings,'' J. Lubr. Technol. **103**, 406–413 (1981).
[50] B. Bhushan, ''Stick-slip induced noise generation in water-lubricated compliant rubber bearings,'' J. Lubr. Technol. **102**, 201–212 (1980).
[51] S. M. A. Aziz and A. Seireg, ''A parametric study of frictional noise in gears,'' Wear **176**, 25–28 (1994).
[52] H. Tokoro, M. Nakamura, N. Sugiura, H. Tani, K. Yamamoto, and T. Shuku, ''Analysis of high frequency noise in engine timing belt,'' JSAE Paper 9830037 (1998).
[53] M. Nakai, and M. Yokoi, ''Band brake squeal,'' J. Vibr. Acoust. **118**, 190–197 (1996).
[54] M. Oya, T. Masamura, S. Hattori, and T. Ohoki, ''Squealing of band brake for bicycle,'' Trans. Jpn. Soc. Mech. Eng. (Preprint), no. 780-14, 151–153 (1978).
[55] G. G. Adams, ''Self-excited oscillations of two elastic half-spaces sliding

with a constant coefficient of friction,'' J. Appl. Mech. **62**, 867–872 (1995).
[56] G. G. Adams, ''Self-excited oscillations in sliding with a constant friction coefficient—A simple model,'' J. Tribol. **118**, 819–823 (1996).
[57] G. G. Adams, ''Dynamic instabilities in the sliding of two layered elastic half-spaces,'' J. Tribol. **120**, 289–295 (1998).
[58] G. G. Adams, ''Steady sliding of two elastic half-spaces with friction reduction due to interface stick-slip,'' J. Appl. Mech. **65**, 470–475 (1998).
[59] G. G. Adams, ''Self-excited oscillations in sliding with a constant friction coefficient,'' ASME Des. Eng. Tech. Conf., Part A, DE-Vol 84-1, 3, 1171–1177 (1995).
[60] L. Johansson, ''Model and numerical algorithm for sliding contact between two elastic half-planes with frictional heat generation and wear,'' Wear **160**, 77–93 (1993).
[61] J. R. Rice and A. L. Ruina, ''Stability of steady frictional slipping,'' J. Appl. Mech. **50**, 343–349 (1983).
[62] G. Weinreich, ''What science knows about violins—And what it does not know,'' Am. J. Phys. **61**, 1067–1077 (1993).
[63] H. v. Helmholtz, *On the Sensations of Tone* (Dover, New York, 1954) (English translation of the German edition, 1877).
[64] *Scientific Papers of C. V. Raman: Acoustics*, edited by S. Ramaseshan (Oxford University Press, Oxford, 1989).
[65] J. C. Schelleng, ''The violin as a circuit,'' J. Acoust. Soc. Am. **35**, 326–338 (1963).
[66] C. V. Raman, ''On the 'wolf-note' in bowed stringed instruments,'' Philos. Mag. **32**, 391–395 (1916).
[67] M. E. McIntyre and J. Woodhouse, ''The acoustics of stringed musical instruments,'' Interdisciplinary Sci. Rev. **3**, 157–173 (1978).
[68] J. Woodhouse, ''On the playability of violins, Part I: Reflection functions,'' Acustica **78**, 125–136 (1993).
[69] J. Woodhouse, ''On the playability of violins, Part II: Minimum bow force and transients,'' Acustica **78**, 137–153 (1993).
[70] M. E. McIntyre, R. T. Schumacher, and J. Woodhouse, ''On the oscillations of musical instruments,'' J. Acoust. Soc. Am. **74**, 1325–1345 (1983).
[71] M. E. McIntyre and J. Woodhouse, ''On the fundamentals of bowed-string dynamics,'' Acustica **43**, No. 2, 93–108 (1979).
[72] M. E. McIntyre, ''Aperiodicity in bowed-string motion,'' Acustica **50**, 294–295 (1982).
[73] M. E. McIntyre, R. T. Schumacher, and J. Woodhouse, ''Aperiodicity in bowed-string motion,'' Acustica **49**, 13–32 (1981).
[74] R. T. Schumacher, ''Self-sustained oscillations of the bowed string,'' Acustica **43**, 109–120 (1979).
[75] G. Weinreich and R. Caussé, ''Elementary stability considerations for bowed-string motion,'' J. Acoust. Soc. Am. **89**, 887–895 (1991).
[76] J. Woodhouse, R. T. Schumacher, and S. Garoff, ''Reconstruction of bowing point friction force in a bowed string,'' J. Acoust. Soc. Am. **108**, 357–368 (2000).
[77] G. Essl and P. R. Cook, ''Measurements and efficient simulations of bowed bars,'' J. Acoust. Soc. Am. **108**, 379–388 (2000).
[78] L. Cremer, *The Physics of the Violin* (MIT Press, Cambridge, 1984).
[79] C. M. Hutchins and V. Benade. *Research Papers in Violin Acoustics 1975–1993* (ASA, New York, 1997), Vols. I and II.
[80] A. J. Day and T. P. Newcomb, ''The dissipation of friction energy from the interface of annular disc brake,'' Proc. Inst. Mech. Eng., Part D (J. Automob. Eng.) **198D**, No. 11, 201–209 (1984).
[81] M. Nishiwaki, ''Generalized theory of brake noise,'' Proc. Inst. Mech. Eng. Part D (J. Automob. Eng.) **207**, 195–202 (1993).
[82] M. Nishiwaki, ''Review of study on brake squeal,'' Jap. Soc. Automob. Eng. Rev. **11(4)**, 48–54 (1990).
[83] A. M. Lang and H. Smales. ''An approach to the solution of disc brake vibration problems,'' IMechE Paper C37/83, 223–230 (1983).
[84] P. J. Remington, ''Wheel/rail squeal and impact noise: What do we know? What don't we know? Where do we go from here?,'' J. Sound Vib. **116**, 339–353 (1987).
[85] P. J. Remington and J. D. Stahr, ''The effects on noise of changes in wheel/rail system parameters,'' J. Sound Vib. **87**, 221–229 (1983).
[86] M. Nakai and M. Yokoi, ''Railway wheel squeal—3rd report: Squeal of a disk simulating a wheel in internal resonances,'' Bull. JSME **28**, 500–507 (1985).
[87] M. Nakai and S. Akiyama, ''Railway wheel squeal (squeal of disk subjected to random excitation),'' JSME Int. J., Ser. C **41**(3), 608–615 (1998).
[88] M. Nakai, M. Yokoi, and M. Sugiura, ''Railway wheel sqeal (squeal of a rotating disk),'' JSME Int. J., Ser. III **32**, No. 3, 406–412 (1989).
[89] J. T. Gordon, ''A perturbation analysis of nonlinear squeal vibrations in aircraft braking systems,'' Proc. ASME Des. Eng. Tech. Conf., DETC97/VIB-4163 (1997).
[90] M. A. Ozbek, S. Y. Liu, J. T. Gordon, and D. S. Newman, ''Chaotic vibration in aircraft braking systems,'' Proc. ASME Des. Eng. Tech. Conf., Part A, DE-Vol84-1 3, 1233–1240 (1995).
[91] D. J. Feld and D. J. Fehr, ''Complex eigenvalue analysis applied to an aircraft brake vibration problem,'' ASME Des. Eng. Tech. Conf., Part A, DE-Vol 84-1, 3:1135–1142 (1995).
[92] M. H. Travis, ''Nonlinear transient analysis of aircraft landing gear brake whirl and squeal,'' ASME Des. Eng. Tech. Conf., Part A, DE-Vol. 84-1, **3**, 1209–1216 (1995).
[93] I. Kido, T. Kurahachi, and M. Asai. ''A study on low-frequency brake squeal noise,'' SAE Paper 960993 (1996).
[94] M. Gouya and M. Nishiwaki, ''Study on disc brake groan,'' SAE Trans. Paper 900007 (1990).
[95] T. K. Kao, J. W. Richmond, and M. W. Moore. ''The application of predictive techniques to study thermo elastic instability of braking,'' SAE Paper 942087 (1994).
[96] J. A. Wickert and A. Akay, ''Damper for Brake Noise Reduction,'' U.S. Patent No. 5,855,257 (1999), and ''Damper for Brake Noise Reduction (brake drums),'' U.S. Patent No. 6,112,865 (2000).
[97] P. T. Haskell, *Insect Sounds* (Witherby, London, 1961).
[98] R.-G. Busnell, *Acoustic Behavior of Animals* (Elsevier, New York, 1963).
[99] R. Hickling and R. L. Brown, ''Analysis of acoustic communication by ants,'' J. Acoust. Soc. Am. **108**, 1920–1929 (2000).
[100] S. N. Patek, ''Spiny lobsters stick and slip to make sound,'' Nature (London) **411**, 153–154 (2000).
[101] R. M. Nedderman and G. H. L. Harding, ''The flight-free vibrating conveyor: Part I. Basic theory and performance analysis,'' Trans. Inst. Chem. Eng., Part A **68**, 123–130 (1990).
[102] G. H. L. Harding and R. M. Nedderman, ''The flight-free vibrating conveyor: Part II. Stability analysis and criteria for optimal design,'' Trans. Inst. Chem. Eng., Part A **68**, 131–138 (1990).
[103] Y. Hashimoto, Y. Koiki, and S. Ueha, ''Transporting objects without contact using flexural traveling waves,'' J. Acoust. Soc. Am. **103**, 3230–3233 (1998).
[104] P. Vielsack and H. Spiess, ''Sliding of a mass on an inclined driven plan with randomly varying coefficient of friction,'' J. Appl. Mech. **67**, 112–116 (2000).
[105] E. C. Miranda and J. J. Thomsen, ''Vibration induced sliding: Theory and experiment for a beam with a spring-loaded mass,'' Nonlinear Dyn. **16**, 167–186 (1998).
[106] Y. G. Long, K. Nagaya, and H. Niwa, ''Vibration conveyance in spatial-curved tubes,'' J. Vib. Acoust. **116**, 38–46 (1994).
[107] T. Sashida and T. Kenjo, *An Introduction to Ultrasonic Motors* (Clarendon, Oxford, 1993).
[108] P. Hagedorn and J. Wallashek, ''Travelling wave ultrasonic motors. Part I. Working principle and mathematical modeling of the stator,'' J. Sound Vib. **155**, 31–46 (1992).
[109] J. Wallashek, P. Hagedorn, and W. Konrad, ''Travelling wave ultrasonic motors. Part II. A numerical method for the flexural vibrations of the stator,'' J. Sound Vib. **168**, 115–122 (1993).
[110] K. Adachi, K. Kato, and Y. Sasatani, ''The micro-mechanism of friction drive with ultrasonic wave,'' Wear **194**, 137–142 (1996).
[111] S. Ueha and Y. Tomikawa, *Ultrasonic Motors—Theory and Applications* (Clarendon, Oxford, 1993).
[112] T. Sashida, ''Motor device utilizing ultrasonic oscillation,'' U.S. Patent No. 4,562,374.
[113] A. Kumada, ''Piezoelectric revolving resonator and ultrasonic motor,'' U.S. Patent No. 4,868,446 (1989).
[114] T. Nakashima, M. Yishizawa, H. Kuno, M. Tsukui, K. Takatsuka, and T. Akuto, ''Driving characteristics of traveling wave ultrasonic motor (Effect of rotor vibration),'' Proc. ASME Design Eng. Tech. Conf., Pittsburgh, PA, 9–12 September 2001. DETC2001/VIB-21653.
[115] A. Kumada, T. Iochi, and M. Okada, ''Piezoelectric revolving resonator and single-phase ultrasonic motor,'' U.S. Patent No. 5,008,581 (1991).
[116] H. M. Jaeger and S. R. Nagel, ''Physics of the granular state,'' Science **255**, 1523–1531 (1992).
[117] H. M. Jaeger and S. R. Nagel, ''Dynamics of granular material,'' Am. Sci. **85**, 540–545 (1997).

[118] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, ''The physics of granular materials,'' Phys. Today **49**(4), 32–38 (1996).

[119] J. B. Knight, H. M. Jaeger, and S. R. Nagel, ''Vibration-induced size separation in granular media: The convection connection,'' Phys. Rev. Lett. **70**, No. 24, 3728–3731 (1993).

[120] J. M. Montanero, V. Garzo, A. Santos, and J. J. Brey, ''Kinetic theory of simple granular shear flows of smooth hard spheres,'' J. Fluid Mech. **389**, 391–411 (1999).

[121] I. Peterson, ''Dry sand, wet sand—digging into the physics of sandpiles and sand castles,'' Sci. News (Washington, D. C.) **152**, 186–187 (1997).

[122] P. Evesque and J. Rajchenbach, ''Instability in a sand heap,'' Phys. Rev. Lett. **62**, No. 1, 44–46 (1989).

[123] A. Rosato, K. J. Strandburg, F. Prinz, and R. H. Swendsen, ''Why the Brazil nuts are on top: Size segregation of particulate matter by shaking,'' Phys. Rev. Lett. **58**, No. 10, 1038–1040 (1987).

[124] R. I. Zadoks and X. Yu, ''An investigation of the self-loosening behaviour of bolts under transverse vibration,'' J. Sound Vib. **208**, 189–209 (1997).

[125] D. P. Hess and K. Davis, ''Threaded components under axial harmonic vibration, Part I. Experiments,'' J. Vibr. Acoust. **118**, 417–429 (1996).

[126] D. P. Hess, ''Vibration-induced loosening and tightening of threaded fasteners,'' Des. Eng. Tech. Conf., Part A, DE-Vol 84-1, 3, 1165–1170 (1995).

[127] S. Thomas, ''Vibrations damped by solid friction,'' Philos. Mag. **9**, 329 (1930).

[128] J. H. Griffin, ''Friction damping of resonant stresses in gas turbine engine airfoils,'' J. Eng. Power **102**, 329–333 (1980).

[129] C. H. Menq and J. H. Griffin, ''A comparison of transient and steady state finite element analyses of the forced response of a frictionally damped beam,'' J. Vib., Acoust., Stress, Reliability Design **107**, 19–25 (1985).

[130] A. Sinha and J. H. Griffin, ''Effects of static friction on the forced response of frictionally damped turbine blades,'' J. Eng. Power **106**, 65–69 (1984).

[131] L. Gaul and J. Lenz, ''Nonlinear dynamics of structures assembled by bolted joints,'' Acta Mech. **125**, 169–181 (1997).

[132] M. Groper, ''Microslip and macroslip in bolted joints,'' Exp. Mech. **42**, 171–174 (1985).

[133] P. R. Dahl, ''Solid friction damping of mechanical vibrations,'' AIAA J. **14**, 1675–1682 (1976).

[134] K. Yamanaka, T. Kakubari, A. Yamada, and M. Kano, ''Study on the dynamic characteristic of a contact surface (the logarithmic decrement and natural frequency dependent on an amplitude of vibration),'' No. 85-194A.

[135] W. Sextro, ''The calculation of the forced response of shrouded blades with friction contacts and its experimental verification,'' Proceedings of the International Gas Turbine & Aeroengine Congress, Munich (2000-GT-540) (2000).

[136] T. Berruti, S. Filippi, M. M. Gola, and S. Salvano, ''Friction damping of interlocked vane segments: Experimental results,'' Proceedings of the International Gas Turbine & Aeroengine Congress, Munich, 2001-GT-0432 (2001).

[137] E. H. Dowell, ''Damping in beams and plates due to slipping at the support boundaries,'' J. Sound Vib. **105**, 243–253 (1986).

[138] D. M. Tang and E. H. Dowell, ''Damping in beams and plates due to slipping at the support boundaries, Part 2: Numerical and experimental study,'' J. Sound Vib. **108**, 509–522 (1986).

[139] E. H. Dowell and H. B. Schwartz, ''Forced response of a cantilever beam with a dry friction damper attached. Part I: Theory,'' J. Sound Vib. **91**, 255–267 (1983).

[140] E. H. Dowell and H. B. Schwartz, ''Forced response of a cantilever beam with a dry friction damper attached. Part II: Experiment,'' J. Sound Vib. **91**, 269–291 (1983).

[141] A. A. Ferri, ''Friction damping and isolation systems,'' J. Vibr. Acoust. **117**, 196–206 (1995).

[142] W. E. Whiteman and A. A. Ferri, ''Displacement-dependent dry friction damping of a beam-like structure,'' J. Sound Vib. **198**, 313–329 (1996).

[143] W. E. Whiteman and A. A. Ferri, ''Displacement-dependent dry friction damping of a beam-like structure,'' J. Sound Vib. **207**, 403–418 (1997).

[144] K. Y. Sanliturk and D. J. Ewins, ''Modeling two-dimensional friction contact and its application using harmonic balance method,'' J. Sound Vib. **193**, 511–523 (1996).

[145] N. Popplewell and S. E. Semercigil, ''Performance of the bean bag impact damper for a sinusoidal external force,'' J. Sound Vib. **133**, 193–223 (1989).

[146] D. Warwick and D. Feit, ''An evaluation of polyethylene beads as a damping treatment for tubular truss structures,'' Carderock Division Naval Surface Warfare Center, CDNSWC-SIG-94-146-7250, October 1994.

[147] J. R. Fricke, ''Lodengraf damping—An advanced vibration damping technology,'' Sound Vib. **34**, 22–27 (July 2000).

[148] G. Tomlinson, ''Recent developments in particle dampers,'' Proceedings of the 6th National Turbine Engine High Cycle Fatigue Conference (Jacksonville, FL, 5–8 March 2001)

[149] J. N. Macduff and J. R. Curreri, *Vibration Control* (McGraw-Hill, New York, 1958).

[150] S. R. Nagel and H. M. Jaeger, ''Sound propagation in sand,'' Echoes **8**, No. 3, 1, 4 (1998).

[151] J. P. Den Hartog, *Mechanical Vibrations*, 3rd ed. (McGraw-Hill, New York, 1947).

[152] C. E. Celik, ''Modeling and Identification of Friction in Dynamic Systems,'' Ph.D. thesis, Carnegie Mellon University (2000).

[153] J. T. Oden and J. A. C. Martins, ''Models and computational methods for dynamic friction phenomena,'' Comput. Methods Appl. Mech. Eng. **52**, 527–634 (1985).

[154] J. A. C. Martins, J. T. Oden, and F. F. Simões, ''A study of static and kinetic friction,'' Int. J. Eng. Sci. **28**, 29–92 (1990).

[155] Y. Karpenko and A. Akay, ''A numerical model of friction between rough surfaces,'' Tribol. Int. **34**, 531–545 (2001).

[156] Y. Karpenko and A. Akay, ''A numerical method for analysis of extended rough wavy surfaces in contact,'' J. Tribol. (July 2002).

[157] M. T. Bengisu and A. Akay, ''Relation of dry-friction to surface roughness,'' J. Tribol. **119**, 18–25 (1997).

[158] M. T. Bengisu and A. Akay, ''Stick–slip oscillations: Dynamics of friction and surface roughness,'' J. Acoust. Soc. Am. **105**, 194–205 (1999).

[159] A. Soom and C. Kim, ''Interactions between dynamic normal and frictional forces during unlubricated sliding,'' J. Lubr. Technol. **105**, 221–229 (1983).

[160] P. E. Dupont and P. S. Kasturi, ''Experimental investigation of friction dynamics associated with normal load,'' Des. Eng. Tech. Conf., Part A, DE-Vol 84-1 3:1109–1115, 1995.

[161] K. U. Ingard, *Fundamentals of Waves and Oscillations* (Cambridge University Press, New York, 1993).

[162] C. E. Celik and A. Akay, ''Dissipation in solids: Thermal oscillations of atoms,'' J. Acoust. Soc. Am. **108**, 184–191 (2000).

[163] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, 1995).

[164] P. Holmes, J. L. Lumley, and G. Berkooz, *Turbulence, Coherent Structures, Dynamical Systems and Symmetry* (Cambridge University Press, Cambridge, 1996).

[165] M. T. Landahl and E. Mollo-Christensen, *Turbulence and Random Processes in Fluid Mechanics* (Cambridge University Press, Cambridge, 1986).