Imperial – TUM Workshop

20-21 June 2019, Imperial College London

The aim of this workshop is to explore avenues for future joint research projects and funding initiatives involving the mathematics departments at Imperial College London and TU Munich and to strengthen the relationship between both institutions.

The workshop takes place in Room 402, EPSRC Centres for Doctoral Training Suite, which is located on the 4th floor of the ICSM Building with access via Sherfield Building Level 2.

Programme

Thursday, 20 June 2019

10.00-10.15	Introduction
10.15-11.00	Darryl Holm (Imperial) Stochastic Advective Lie Transport (S.A.L.T.) in Geophysical Fluid Dynamics (GFD)
11.00-11.30	Coffee break
11.30-12.15	Folkmar Bornemann (TUM) Nonstandard Preconditioning by Surplus Rigidity
12.15-14.00	Lunch break
14.00-14.45	Almut Veraart (Imperial) Ambit stochastics
14.45-15.30	Massimo Fornasier (TUM) Spatially inhomogeneous evolutionary games
15.30-16.00	Coffee break
16.00-18.00	Discussion session
19.00-22.00	Dinner in Ognisko Polish Club, 55 Exhibition Road, London SW7 2PN

Friday, 21 June 2019

10.00-10.45	Pierre Degond (Imperial) Mathematical models of collective dynamics and self-organization
10.45-11.15	Coffee break
11.15-12.00	Gero Friesecke (TUM) Battling the curse of dimension in PDEs from many-body physics: sparse ansatz methods, effective variables, asymptotics, learning
12.00-14.00	Lunch and discussion session

Abstracts

Talk 1: Darryl Holm (Imperial)

Stochastic Advective Lie Transport (S.A.L.T.) in Geophysical Fluid Dynamics (GFD)

We discuss a data driven approach for parametrising the small fast scales of GFD flows by using temporal homogenization of Lagrangian paths. This approach leads to stochastic transport, instead of stochastic forcing or diffusion. The approach is called SALT (Stochastic Advection by Lie Transport). It is consistent with Newton's force law and Kelvin's circulation theorem. It can also be derived via a stochastic variational principle. The physics packages inherit stochasticity from the stochastically advected material properties. A methodology for calibrating the stochastic parameters using data analysis, quantifying the resulting uncertainty, and then reducing the uncertainty by using a further data assimilation step based on particle filtering will also be described.

In particular, we introduce a class of stochastic fluid equations, whose smooth solutions are characterized by natural extensions of the Kelvin theorems of their deterministic counterparts, which hold along certain noisy flows. Some of these stochastic Euler-Poincaré equations were previously derived from a stochastic variational principle which we briefly review.

Collaborators:
Stochastic GFD team: Colin J Cotter, Dan Crisan, Wei Pan, Igor Shevschenko (London) and Georg A Gottwald (Sydney)
Synergy team: Etienne Mémin (Rennes) and Bertrand Chapron (Brest) + Their PhD Students
London PhD students: A. Bethencourt de Leon, Erwin Luesink and So Takao

References:
Variational principles for stochastic fluid dynamics.
Holm, D. D., Proc. R. Soc. A 471.2176 (2015): 20140963. http://dx.doi.org/10.1098/rspa.2014.0963

Stochastic partial differential fluid equations as a diffusive limit of deterministic Lagrangian multi-time dynamics.
Colin J Cotter, Georg A Gottwald, Darryl D Holm, 2017, Proc Roy Soc A, Vol 473, 20170388
http://dx.doi.org/10.1098/rspa.2017.0388

Solution properties of a 3D stochastic Euler fluid equation.
Dan O. Crisan, Franco Flandoli, Darryl D. Holm, J Nonlinear Sci 29(3): 813-870 (2019).
https://doi.org/10.1007/s00332-018-9506-6

Modelling uncertainty using circulation-preserving stochastic transport noise in a 2-layer quasi-geostrophic model.
Colin Cotter, Dan Crisan, Darryl D. Holm, Wei Pan, Igor Shevchenko. arXiv:1802.05711

Numerically Modelling Stochastic Lie Transport in Fluid Dynamics.
Colin J. Cotter, Dan Crisan, Darryl D. Holm, Wei Pan, Igor Shevchenko. arXiv:1801.09729

Talk 2: Folkmar Bornemann (TUM)

Nonstandard Preconditioning by Surplus Rigidity

The stability and speed of numerical methods can be severely hampered by the ill-conditioning of intermediate steps: the common remedy is to re-phrase or transform some of the computational steps, often attacking the problem in a more direct fashion. In the last decade or so the author has run into several unexpected instances of ill-conditioning that can be solved by using surplus rigidity of the underlying problem, such as analyticity or integrability. As a guiding example we will discuss the calculation of the distribution of the length of the longest increasing subsequence of random permutations where two such instances of ill-conditioning are stacked on top of each other: the calculation of higher derivatives and the evaluation of Toeplitz determinants. The ill-conditioning of the first is treated by the theory of entire functions and that of the second by techniques from integrable systems.

Talk 3: Almut Veraart (Imperial)

Ambit stochastics

In this talk I will give a brief overview of the area of ambit stochastics and discuss some recent developments in the context of estimating stochastic co-volatility for some multivariate ambit processes.

Talk 4: Massimo Fornasier (TUM)

Spatially inhomogeneous evolutionary games

We present a mean-field model for a system of spatially distributed players interacting through an evolutionary game driven by a replicator dynamics. Strategies evolve by a replicator dynamics influenced by the position and the interaction between different players and return a feedback on the velocity field guiding their motion. The description of the whole system is realized by an evolving probability measure Σ on an infinite dimensional state space (pairs (x,σ) of position and distribution of strategies). We provide a Lagrangian and a Eulerian description of the evolution, and we show their equivalence, together with existence, uniqueness, and stability of the solution. As a byproduct of the stability result, we also present convergence of the finite agents model to our mean-field formulation, when the number N of the players goes to infinity.

Talk 5: Pierre Degond (Imperial)

Mathematical models of collective dynamics and self-organization

In this talk, I will review some mathematical challenges posed by the modelling of collective dynamics and self-organization. Then, I will focus on two specific problems, first, the derivation of fluid equations from particle dynamics of collective motion and second, the study of phase transitions and the stability of the associated equilibria.

Talk 6: Gero Friesecke (TUM)

Battling the curse of dimension in PDEs from many-body physics: sparse ansatz methods, effective variables, asymptotics, learning

I will discuss the topics in the title in the context of a central example, the many-electron Schroedinger equation, emphasizing possible points of contact within the emerging TUM-Imperial collaboration.

For recent progress in a strongly interacting limit see G.F., Daniela Voegler, Breaking the curse of dimension in multi-marginal Kantorovich optimal transport on finite state spaces, SIAM J. Math. Anal. 50 (4), 3996-4019, 2018.

Organisers:
Jeroen Lamb, Greg Pavliotis, and Martin Rasmussen