Certifying Multilevel Coherence in the Motional state of a Trapped Ion

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Certifying coherence

A defining feature of quantum mechanics is the ability for a system to be in a coherent superposition of states. One method of determining the number of states coherently superposed is state tomography. This however scales poorly with the size of the quantum system. It also relies on having confidence that the tomographic procedure can be performed accurately.

The existence of a superposition of two states can be verified if an interference fringe is produced in a Ramsey type experiment, without needing full tomography. For superpositions of k>2 states, it is possible to produce a certifier based on a generalized interference pattern that verifies that k-coherence is present [1].

If we consider a qubit that can be coupled to the state under study, we can produce an interference pattern by combining a period of free evolution of the state (providing a phase evolution) with a mapping operation, then measuring the qubit:

\[ p(\phi) = \langle \chi | \hat{U}_{n+1}^{\dagger} \hat{U}_n (\phi) \hat{U}_n^{\dagger} (\phi) \hat{U}_m | \chi \rangle \]

Our certifier is then defined in terms of the moments of this interference pattern:

\[ C = \frac{M_3}{M_1^2} \quad M_n = \frac{1}{2\pi} \int_0^{2\pi} p(\phi)^n d\phi \]

It can be shown that C has the property that it can only be greater than 1.25 for a 3-coherent state and greater than 1.86 for a 4-coherent state.

Results

A series of four pulses are used to prepare the state \(|\chi, 0\rangle + |\chi, 1\rangle + |\chi, 2\rangle\sqrt{3}\). To certify this state, five pulses are then used to implement the mapping operation before the qubit state is measured. Rather than using a period of free evolution, equivalently a phase offset is applied to all the sideband pulses during the mapping stage. The optimal mapping operation is found using numerical methods. The red line is a simulation of the process - the high frequency oscillations are due to off-resonant carrier excitation during the sideband pulses.

Conclusion

Multilevel coherence can be certified efficiently using an interference-type experiment.

The certification theory makes very few assumptions about the processes used to manipulate the state, meaning it is reliable even in the case of imperfect or unreliable control, unlike state tomography.

We demonstrate this in a trapped ion system, certifying that a given quantum state must be in a superposition of three motional Fock states.