Opportunistic Cooperation by Dynamic Resource Allocation
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Abstract—We consider a Rayleigh fading wireless relay channel where communication is constrained by delay and average power limitations. Assuming partial channel state information at the transmitters and perfect channel state information at the receivers, we first study the delay-limited capacity of this system and show that, contrary to a single source-single destination case, a non-zero delay-limited capacity is achievable. We introduce opportunistic decode-and-forward (ODF) protocol which utilizes the relay depending on the channel state. Opportunistic cooperation significantly improves the delay-limited capacity of the system and performs very close to the cut-set bound. We also consider the system performance in terms of minimum outage probability. We show that ODF provides performance close to the cut-set bound from the outage probability perspective as well. Our results emphasize the importance of feedback for cooperative systems that have delay sensitive applications.

Index Terms—Delay-limited capacity, fading, resource allocation, outage probability, user cooperation.

I. INTRODUCTION

User cooperation has emerged as a spatial diversity technique to provide robustness against channel fluctuations by utilizing the broadcast nature of the wireless transmission [1], [2]. Cooperation provides performance improvements through the use of available resources in the network, and is especially important when the size of mobile devices limits the number of antennas that can be deployed. Recent years have witnessed an increased interest on the topic and cooperative diversity systems have been extensively studied in terms of different techniques and different performance measures (see [1]-[8], [12]-[19] and references therein).

Cooperative system model builds upon the relay channel (see Fig. 1). In this paper, we model the links among the terminals as independent, quasi-static, frequency non-selective Rayleigh fading channels and impose strict transmission delay constraint where each codeword is to be transmitted over one realization of the network. In this case, it is no more possible to average over all realizations of the fading channel, thus the ergodic capacity is not a meaningful performance measure. Instead, outage probability and delay-limited capacity are used. It is well known that from the outage probability perspective, even simple protocols that do not utilize channel state information at the transmitter (CSIT) -while assuming perfect channel state information at the receiver (CSIR)- can achieve the full diversity level of two in the case of two cooperating users [5]. However, without any CSIT only limited improvement can be achieved by statistical channel resource and power allocation (e.g. [6]).

In this paper, we assume partial CSIT in the form of channel state amplitudes, and perfect CSIR. This can be accomplished by separate low rate feedback channels. We restrict the feedback information to the amplitude of the channel state, since obtaining and utilizing the phase information by symbol synchronizing the source and the relay is a practically demanding task [8]. This restriction will also reduce the resources used by the feedback channel.

We assume a long-term average power constraint which limits the average power of all codewords over all channel realizations. Short-term power constraint on the other hand, limits the power of each codeword, thus is less dynamic than the long-term constraint and cannot overcome channel impairments especially in deep fades [9]. Our long-term power constraint is imposed on the total power of the source and the relay since we view the relay power not as a free resource for the source terminal but something that is to be paid for. A constraint on the weighted sum of average powers, assigning higher weight/price for relay power, may also be used.

We consider half-duplex relays that cannot receive and transmit simultaneously, and analyze decode-and-forward (DF) type strategies [5], where the relay decodes and re-encodes the message using an independent codebook. DF strategy makes it possible to optimize over channel resources (bandwidth/time) by varying the rate and prevents error propagation. Independent codebook usage at the relay results in higher rates compared to the suboptimal repetition based schemes.

We first analyze the delay-limited capacity of the cooperative system [3]. Delay limited capacity corresponds to the highest achievable rate that can be sustained independent of
the channel states [10]. This approach is justified by the delay sensitive applications such as real-time voice and video communications. Here the channel state information is essential, since otherwise for any pre-assigned rate and power values, there is a non-zero probability of outage. We dynamically optimize the allocation of system resources, namely time and power, depending on the partial CSIT within the power constraint in order to maximize the delay-limited capacity.

We next consider the case where the average total system power limitation makes it impossible to guarantee zero outage for the specified target rate, i.e., the required transmission rate might be above the maximum delay-limited capacity that is achievable with the available average power. In this case, we minimize the outage probability by dynamic resource allocation [4].

Presence of CSIT has also been assumed in some other recent works on user cooperation with various performance criteria and constraints. In [12] and [13] ergodic capacity of the system is explored with short-term and long-term power constraints, respectively. Reference [12] also explores outage capacity with short-term power constraint for both synchronous and asynchronous relays. In [13], the analysis is based on separate power constraints on the terminals where terminals transmit over orthogonal channels. In [14] the authors consider a multi-hop system with short-term power constraint and a fixed target SNR. In [15] outage performance of a user cooperation system is explored with a short-term power constraint for various collaboration and resource allocation schemes. Reference [16] considers power allocation for outage minimization with long-term total power constraint with fixed time slot allocation and repetition coding. In [17], outage performance with long-term power constraint is investigated where terminals cooperate independent of the channel state, and relays operate in the full-duplex mode, thus no channel resource allocation needed. Further in [18] authors show that even 1 bit feedback is effective in decreasing the outage probability for an amplify-and-forward (AF) cooperation scheme. Recently, [19] extended the opportunistic scheme of [3], [4] to two-source amplify-and-forward cooperation protocol.

The results obtained in this paper prove the importance of feedback regarding channel state information and the considerable increase in the performance shows that feedback, on top of cooperation might help mobile terminals attain improved coverage and/or battery life. Furthermore, we show that the dynamic nature of the proposed cooperation scheme, i.e., to cooperate when it is advantageous, and the ability to decide the amount of cooperation, improves the overall performance compared to the non-cooperative or the fixed type cooperative strategies.

The outline of the paper is as follows: In Section II, the system model is introduced. In Section III, we show that a non-zero delay-limited capacity is achievable with cooperation. We introduce two cooperation protocols, namely multi-hop (MH) and opportunistic decode-and-forward (ODF), and find the corresponding optimal resource allocation policies and numerically calculate their delay-limited capacities. Section IV is devoted to minimum outage probability analysis of the cooperation protocols introduced in Section III. Then the Conclusion and the Appendix follow.

II. SYSTEM MODEL

Our model consists of a single source (S), single destination (D) pair and an available relay (R) as shown in Fig. 1. The links among the terminals are modelled as having independent, quasi-static Rayleigh fading as well as path loss. The overall instantaneous channel gains among terminals are denoted as \( h_i, \ i \in \{1, 2, 3\} \). There is also independent, zero-mean additive white Gaussian noise with unit variance at each receiver. The channel coefficients are constant over a block of \( N \) symbols during which one codeword is transmitted, and are independent from one block to the other. We assume \( N \) is large enough to achieve instantaneous capacity. This is a valid assumption as the corresponding performance can be obtained with moderate length codes in practice.

Amplitude squares of the channel coefficients, denoted by \( a = |h_1|^2 \), \( b = |h_2|^2 \), and \( c = |h_3|^2 \) as in Fig. 1, are exponentially distributed random variables with means \( \lambda_a \), \( \lambda_b \), and \( \lambda_c \), respectively. The means capture the effect of pathloss across the corresponding link. To consider the effect of the relay location on the performance of the network, we follow the model in Fig. 2. In our analysis, we normalize the distance between the source and the destination, and assume that the relay is located between the source and the destination, on the straight line connecting them. For a fixed pathloss exponent, the effect of this normalization is scaling the average power. We denote the source-relay distance as \( d \) and the relay-destination distance as \( 1 - d \), where \( 0 < d < 1 \). Then the overall network channel state, \( s = (a, b, c) \) becomes a 3-tuple of independent exponential random variables with means \( \lambda_a = 1 \), \( \lambda_b = \frac{1}{d} \), and \( \lambda_c = \frac{1}{(1-d)} \), respectively, where \( \alpha \) is the pathloss exponent. For the numerical analysis we assume \( \alpha = 4 \). Although pathloss exponent depends on terrain and other environmental factors, this value approximately models metropolitan areas [20]. We assume that the channel state \( s \) is known at the source, the relay and the destination, while the phase information for \( h_1, h_2 \) and \( h_3 \) is only available at the corresponding receivers. Furthermore, we assume that there is a long-term average total transmit power limitation, \( P_{\text{avg}} \).

We assume a half-duplex relay, that is, it cannot receive and transmit simultaneously. Our cooperation protocol is based on the decode-and-forward (DF) protocol of [5]. In [5], total time slot of the source, which consists of \( N \) symbols is divided into two equal portions. In the first half, the source transmits to both the relay and the destination, and in the second half the relay forwards the message to the destination if it decodes successfully. The destination, receiving two copies of the same message from two independent fading channels combines them. In the DF protocol defined in [5] the relay remains silent if it cannot decode at the end of the first half. However, in our system, due to the availability of the channel state information, when it is feasible for source to utilize the relay, it can transmit at a power level that guarantees decoding at the relay. Another
improvement is the usage of an independent codebook at the relay, which results in an increased spectral efficiency.

We also consider dynamic channel resource allocation among the terminals. Although the same ideas can be generalized to allocating bandwidth or allocating time and bandwidth together, in this paper we consider only the time allocation. We offer an opportunistic cooperation strategy that dynamically adjusts the portion of the time slot that the relay listens and the power allocation among the source and the relay, subject to the long-term average total power constraint $P_{\text{avg}}$. Depending on the network state $s$, the source either directly transmits to the destination, or utilizes the relay through cooperation, hence the name opportunistic cooperation. Direct transmission (DT) is preferable in some channel states as we have a total power constraint for the source and the relay, thus the relay power cannot be utilized without cost.

Note that since the channel state information is limited to the amplitudes of the channel coefficients, the source and the relay do not need to transmit simultaneously to the destination after the relay listens to the source. No channel phase information at the transmitters means that coherent combination of the source and the relay signals (or beamforming) is not possible leading to performance loss in case of simultaneous transmission for fixed total transmit power.

Let $\mathbf{P}(s) = (P_1(s), P_2(s), t(s))$ be a resource allocation rule defined over the set of all possible network states $s = (a, b, c)$, where $P_1(s)$ and $P_2(s)$ are the transmission powers of the source and the relay, respectively, and $t(s)$ is the ratio of the time slot allocated for source transmission with $0 < t(s) \leq 1$. We define $\Omega$ as the set of all possible resource allocation functions. We have:

$$\Omega = \{\mathbf{P}(s) : P_1(s) \geq 0, P_2(s) \geq 0, 0 < t(s) \leq 1\}$$

Let $F(s)$ be the probability distribution function of the channel states. Then the long-term power constraint can be written as

$$E[\mathbf{P}] \triangleq \int_{s} [t(s)P_1(s) + (1 - t(s))P_2(s)]dF(s) \leq P_{\text{avg}}.$$  

Average power limitation imposes a set of feasible resource allocation functions, $\Omega \leq \Omega$, that is composed of power allocation functions which satisfy the above inequality, i.e.,

$$\Omega = \{\mathbf{P} : E[\mathbf{P}] \leq P_{\text{avg}}, \mathbf{P} \in \Omega\}.$$ 

## III. DELAY-LIMITED CAPACITY

The delay-limited capacity ($C_{DL}$), introduced in [10], is the highest error-free transmission rate for a fading channel when delay-limited is imposed on the transmission. Channel state information is crucial for non-zero delay-limited capacity since for channel distribution functions for which there does not exist a minimum power that guarantees reliable transmission for all possible channel states, outage probability cannot be bounded away from zero. Even for channel distributions that allow non-zero delay-limited capacity, availability of CSIT provides considerable improvement. However, perfect channel state information does not always guarantee non-zero delay-limited capacity. As shown in [10], for Rayleigh fading channels where no time or frequency diversity available, average power required to achieve any non-zero rate with zero outage probability diverges. Thus the delay-limited capacity is zero for any finite average power limitation.

In [9] and [11] it is shown that a non-zero delay-limited capacity with finite average power can be achieved for block-fading and MIMO Rayleigh fading channels, respectively. This stems from the additional diversity introduced by these systems. Motivated from this fact, we will show that non-zero delay-limited capacity can also be achieved by user cooperation. We will also propose an opportunistic cooperation protocol that achieves a delay-limited capacity very close to the half-duplex relay bound.

We first argue that the delay-limited capacity is non-zero for relay assisted communications over slowly fading Rayleigh links. For this purpose, we consider a simple multi-hop (MH) protocol where the time slot is divided into two equal portions $t(s) = 1/2$ for all $s$. As argued in Section II, we have partial CSI at the source and the relay. The source uses the direct channel (i.e., relay is kept silent) in the first half of the time slot whenever $a \geq b$ or $a \geq c$. When $a < b$ and $a < c$, the relay decodes the message and forwards. In this case, the destination only listens to the relay signal. Consider the following power allocation functions for the source and the relay:

$$P_1(s) = \begin{cases} 2^n & , \text{if } a \geq b \text{ or } a \geq c \\ 0 & , \text{if } a < b \text{ and } a < c \end{cases}$$

$$P_2(s) = \begin{cases} 2^n & , \text{if } a \geq b \text{ or } a \geq c \\ 0 & , \text{if } a < b \text{ and } a < c \end{cases}$$

Note that the above power allocation strategy and Gaussian codebooks lead to an achievable rate $R$ irrespective of the channel state. It is not difficult to see that the above power allocation results in minimum total power for MH at any channel state $s$.

**Proposition 3.1:** For the above multi-hop (MH) protocol, given any positive rate $R$, power allocation policy given in (3)-(4) results in finite average total power. Thus, non-zero delay limited capacity can be achieved with a finite average total power constraint.

**Proof:** Proof can be found in Appendix A.

Any protocol that can achieve the same target rate with total power less than the above MH protocol for any given channel state will also have non-zero delay limited capacity. We will describe one such efficient protocol, namely opportunistic decode-and-forward, in the next section.

MH protocol, in particular (32) in Appendix A, can be used to derive a lower bound for the delay-limited capacity of a cooperative system. Defining the constant factor in (32) as $c = [f(a, b) + f(a, c) + f(b, a) + f(c, a)]/2$, we get

$$P_{\text{avg}} < c \cdot \left(2^{cP_{\text{avg}}} - 1\right),$$

$$\Rightarrow C_{DL} > \frac{1}{2} \log_2 (1 + \frac{1}{c} P_{\text{avg}}),$$

$$\sim \frac{1}{2} \log_2 (P_{\text{avg}}) \text{ as } P_{\text{avg}} \rightarrow \infty.$$
DT Mode: \( S \) transmits with power \( P_1 \)
DF Mode: \( S \) transmits with \( P_1 \) \( R \) transmits with \( P_2 \)

\[ t \quad 1-t \]

Fig. 3. The time slot allocation for direct (DT) and decode-and-forward (DF) modes of ODF protocol.

Note that for 2x1 or 1x2 multiple antenna systems the high SNR behavior is \( C_{DL} \sim \log_2(P_{avg}) \) [11]. It is also easy to see that the 2x1 MIMO with transmit antenna selection would also achieve the same high SNR performance. We conclude that the \( 1/2 \) factor in MII protocol is due to spectral efficiency loss resulting from forcing the source to transmit only half of the time slot.

A. Delay Limited Capacity of Opportunistic Decode-and-Forward

In opportunistic decode-and-forward (ODF) we again let the terminals to operate in two different modes, direct transmission (DT) mode or decode and forward (DF) mode. In DT mode, the source transmits directly to the destination throughout the whole time slot and the relay neither tries to decode the message nor transmits at any portion of this time slot. In DF mode, however, the source first transmits its message to both the relay and the destination, and the relay decodes and retransmits this message using an independent Gaussian codebook. The destination tries to decode the message by combining the signals from both the source and the relay.

In DF mode, time allocation is not necessarily equal \((0 < t(s) \leq 1)\), while \( t(s) = 1 \) for DT mode. This is illustrated in Fig. 3. Let \( \mathbf{A} \) be the set of network states that DT is used and \( \mathbf{A}^c \) be the set of states that DF is used. How to choose \( \mathbf{A} \) is part of the optimization problem to be solved.

When compared to MH, ODF is a more advanced protocol in four aspects: i) dynamic time allocation, ii) more advanced decision rule among DT and DF modes, iii) higher spectral efficiency for DT mode, and iv) independent codebook at the relay and maximum ratio combining of both the source and the relay signals at the destination. However, ODF still keeps the simple nature of the decode-and-forward protocol in the coding sense. Then the instantaneous capacities of the two modes for resource allocation policy \( \mathbf{P} \) and channel state \( s \) can be written as

\[
C_{DT}(\mathbf{P}, s) = \log(1 + P_1 a),
\]

\[
C_{DF}(\mathbf{P}, s) = \min \{ t \log(1 + P_1 b), t \log(1 + P_1 a) + (1 - t) \log(1 + P_2 c) \},
\]

where the minimization in \( C_{DF}(\mathbf{P}, s) \) is due to the fact that, in DF mode we require both relay and destination to decode the signal. We can define the overall instantaneous capacity for resource allocation \( \mathbf{P} \) at channel state \( s \) as

\[
C_{ODF}(\mathbf{P}, s) = \max\{C_{DT}(\mathbf{P}, s), C_{DF}(\mathbf{P}, s)\}.
\]

Then the delay-limited capacity maximization problem for given \( P_{avg} \) can be stated as

\[
\max_{\mathbf{P}(s) \in \Omega} R,
\]

s.t. \( C_{ODF}(\mathbf{P}, s) \geq R \) for all \( s \)

where \( \Omega \) is the set of feasible resource allocation functions defined in Section II. Note that the choice of the resource allocation function \( \mathbf{P} \) in the above optimization implicitly includes picking the best mode to operate at each channel state.

Now, consider the following equivalent optimization problem which is easier to analyze.

\[
\min_{\mathbf{P}(s) \in \Omega} E[\mathbf{P}],
\]

where \( \Omega \) is the set of feasible resource allocation functions defined as \( \Omega = \{ \mathbf{P} : C_{ODF}(\mathbf{P}, s) \geq R \text{ for all } s, \mathbf{P} \in \Omega \} \) and \( R \) is a given target rate. These two optimization problems result in the same set of \( (R, P_{avg}) \) pairs. Then, consider the following policy \( \mathbf{P}^*(s) = (P_1^*(s), P_2^*(s), t^*(s)) \in \Omega \) which, for every state \( s \), satisfies

\[
P_{req}(R, s) \triangleq t^*(s)P_1^*(s) + (1 - t^*(s))P_2^*(s),
\]

\[
\leq t(s)P_1(s) + (1 - t(s))P_2(s),
\]

for all \( \mathbf{P}(s) \in \Omega \). This means that \( \mathbf{P}^* \) is the policy that achieves instantaneous capacity at least \( R \) using minimum required total power \( P_{req}(R, s) \) for all \( s \). Obviously, \( \mathbf{P}^* \) gives us the solution of (11). In fact the optimal resource allocation policy for any protocol, not just ODF, is the one from the feasible set that results in the minimum required total power at any channel state.

The minimum required total power for ODF at channel state \( s \) is

\[
P_{req}(R, s) = \min\{P_{req}^{DT}(R, s), P_{req}^{DF}(R, s)\},
\]

where

\[
P_{req}^{DT}(R, s) = \min_{P_1} \{ P_1 : C_{DT}(\mathbf{P}, s) \geq R \},
\]

\[
P_{req}^{DF}(R, s) = \min_{P_1, P_2, t} \{ tP_1 + (1 - t)P_2 : C_{DF}(\mathbf{P}, s) \geq R \}.
\]

Then we have

\[
s \in \mathbf{A} \iff P_{req}^{DT}(R, s) \leq P_{req}^{DF}(R, s).
\]

Therefore, given any channel state, DT is preferred over DF when the total power required to achieve rate \( R \) for DT at that channel state is smaller.

**Remark 1.** If \( a \geq b \) or \( a \geq c \), we have \( P_{req}^{DF} > P_{req}^{DT} \). Intuitively, there is no need to ask for the relay’s help when the source-destination channel is better than the source-relay or the relay-destination channels. Thus \( \{s : a \geq b \text{ or } a \geq c\} \subseteq \mathbf{A} \).

**Remark 2.** In DF mode, the optimal policy has the two terms in (8b) equal. This observation is useful to calculate \( P_{req}^{DF} \) as shown below.
For ODF protocol, the expected transmit power \(E[P^*_s]\) of the optimal policy for each mode can be found using:

\[
E[P^*_s | s \in A] = (2^R - 1)E \left[ \frac{1}{a} s \in A \right],
\]

\[\text{(16a)}\]

\[
E[P^*_s | s \in A^c] = E \left[ \frac{2^R/1^*}{b} - 1 \right] s \in A^c,
\]

\[\text{(16b)}\]

\[
E[P^*_2 | s \in A^c] = E \left[ \frac{1}{c} \left( \frac{2^R}{t^*} - 1 \right)^{t^*/(t^*-1)} - 1 \right] s \in A^c,
\]

\[\text{(16c)}\]

\[
E[P^*] = E[P^*_s | s \in A]Pr(s \in A) + E[t^* P^*_s + (1 - t^*) P^*_2 | s \in A^c]Pr(s \in A^c).
\]

\[\text{(16d)}\]

In Section III-C we will present the delay-limited capacity of ODF numerically calculated using the above equivalent formulation.

B. Upper Bound to the Delay-Limited Capacity

In this section, we find an upper bound (UB) to the delay-limited capacity. We use the cut-set bounds for half-duplex relay (referred to as ‘cheap relay’ in [22]), specialized to our scenario. Considering the fact that, for our model beamforming is not possible as the channel phases are not known at the transmitters, only one of the terminals transmits at a given time. Then for any given power allocation scheme, we can upper bound the instantaneous capacity for the half-duplex relay as

\[
C_{UB}(P, s) = \min \left\{ \text{t log}(1 + (a + b)P_1), \text{t log}(1 + aP_1) + (1 - t) \text{log}(1 + cP_2) \right\}.
\]

\[\text{(17)}\]

Here the first term in the minimization corresponds to the cut-set around the source and the second term corresponds to the cut-set around the destination.

We can express the corresponding constrained optimization problem as:

\[
\max_{P(s) \in \mathcal{O}} R_s,
\]

\[\text{s.t. } C_{UB}(P, s) \geq R_s \text{ for all } s.
\]

\[\text{(18)}\]

We will use the equivalent formulation, similar to (11), to numerically calculate the delay-limited capacity upper bound in Section III-C.

C. Numerical Results for Delay-Limited Capacity

Fig. 4 illustrates the delay-limited capacity vs. the average total power constraint of the system for various communication scenarios for a relay location of \(d = 0.2\). The topmost curve corresponds to the half-duplex upper bound. The second curve, which almost coincides with the upper bound corresponds to ODF protocol. The third curve is ODF without time allocation optimization where \(t = 1/2\) independent of the channel state. We see that ODF achieves almost optimal performance, while fixing \(t\) results in an increasing loss with total power.

The MH curve shows that although its performance is inferior to the upper bound and the ODF protocol, it still achieves a nonzero delay-limited capacity, increasing with average power, as proved in Proposition 3.1. This shows the importance of cooperation, even in the simplest form, on the performance of delay-limited systems.

Figure 5 illustrates the variation of the delay-limited capacity with respect to relay location characterized by \(d\) for the three protocols we have considered and the upper bound. We see that ODF protocol follows the delay-limited capacity upper bound closely when the relay is closer to the source. Although the gap between the upper bound and the ODF performance increases as the relay gets closer to the destination, ODF protocol with dynamic time allocation still has a superior performance compared to ODF with fixed time allocation \((t = 1/2)\) or MH protocols.

IV. OUTAGE MINIMIZATION

In some applications the available power of the system may not support the target delay-limited capacity. Depending on the
characteristics of the application, by allowing some amount of outage in cases of deep fading, we can keep the transmission rate constant during successful transmission periods. For such applications the aim is to achieve the lowest outage probability, while transmitting at the constant rate during non-outage periods and satisfying the average power requirement. We start by summarizing the results for direct transmission and multi-hop and then analyze opportunistic cooperation from the outage minimization perspective.

A. Outage Minimization for Direct Transmission and Multi-hop

First consider direct transmission between the source and the destination. For DT, the set of feasible resource allocation functions becomes \( \Omega_{DT} = \{ \mathbf{P}(s) : P_1(s) \geq 0, P_2(s) = 0 \} \) and \( \xi(s) = 1 \) for all \( s \). The outage probability of \( \mathbf{P} \) for an attempted transmission rate \( R \) is \( P_{out}^{DT}(\mathbf{P}) = Pr(C_{DT} < R) \). Then we can formulate the outage minimization problem for given \( R \) and \( P_{avg} \) as

\[
\min_{\mathbf{P} \in \Omega_{DT}} P_{out}^{DT}(\mathbf{P}),
\]

where \( \Omega_{DT} = \{ \mathbf{P} : \mathbf{P} \in \Omega_{DT}, E[\mathbf{P}] \leq P_{avg} \} \). Intuitively, to achieve minimum probability of outage within the average power limitation, it is better to transmit during the better channel states and not to transmit at all when the channel is in deep fade. In [9] it is shown that in the general case a probabilistic power allocation is needed to achieve the optimal performance. This randomized power allocation is necessary when the distribution of the channel state amplitude has point masses, however, deterministic power allocation suffices for our analysis as we concentrate on Rayleigh fading. Then, we can see that the optimal power allocation rule is of the threshold form as follows [9].

\[
P_1 = \begin{cases} \frac{(2R - 1)}{a} & \text{if } a > a_{th}, \\ 0 & \text{if } a < a_{th}, \end{cases}
\]

We can rewrite the outage probability and the average power in terms of \( a_{th} \) as

\[
P_{out}^{DT}(\mathbf{P}) = Pr(a < a_{th}),
\]

\[
E[\mathbf{P}] = (2R - 1)\left[ \frac{1}{a} \right]_{a \geq a_{th}}.
\]

We observe that the outage probability is an increasing function of \( a_{th} \) while the average power is a decreasing function of it. We conclude that the minimum outage probability can be obtained with the power allocation function that satisfies the average power constraint with equality. Since \( a \) has a continuous distribution, and we have \( \lim_{a_{th} \to \infty} E[\mathbf{P}] = 0 \) and \( \lim_{a_{th} \to 0} E[\mathbf{P}] = \infty \), there always exists an \( a_{th} \) such that \( (2R - 1)\left[ \frac{1}{a} \right]_{a \geq a_{th}} = P_{avg} \). Outage probability corresponding to \( a_{th} \) obtained from this equation is the solution to the optimization problem (19).

Remember that for MH protocol, any rate can be sustained with zero outage probability for some large enough, finite average total power. However, when the available power is below that required average power, there is a non-zero outage probability. With the power allocation functions given in (3)-(4), consider the following threshold type policy:

\[
P = \begin{cases} (P_1, P_2, 1/2) & \text{if } \frac{P_1 + P_2}{2} < P_{th}, \\ (0, 0, 1/2) & \text{otherwise}. \end{cases}
\]

This policy results in outage if and only if \( \frac{P_1 + P_2}{2} > P_{th} \). Then the average total power becomes:

\[
E[\mathbf{P}] = E\left[ \frac{P_1}{a} \right]_{a \geq b \text{ or } a \geq c \text{ and } \frac{P_1 + P_2}{2} \leq P_{th}} \times Pr(a \geq b \text{ or } a \geq c \text{ and } \frac{P_1 + P_2}{2} \leq P_{th})
\]

\[
+ E\left[ \frac{P_1 + P_2}{2} \right]_{a < b \text{ and } a < c \text{ and } \frac{P_1 + P_2}{2} \leq P_{th}} \times Pr(a < b \text{ and } a < c \text{ and } \frac{P_1 + P_2}{2} \leq P_{th}).
\]

Similar to the DT case, due to the continuity and the limiting relation of \( \lim_{P_{th} \to 0} E[\mathbf{P}] = 0 \), we can achieve any power constraint with equality by adjusting \( P_{th} \) and the corresponding outage probability can be found by \( Pr(\frac{P_1 + P_2}{2} \geq P_{th}) \).

B. Outage Minimization for Opportunistic Decode-and-Forward

For ODF protocol of Section III-A, the instantaneous capacity corresponding to resource allocation \( \mathbf{P} \) is given in (9). Then the outage probability for a target rate of \( R \) becomes \( P_{out}^{ODF}(\mathbf{P}) = Pr(C_{ODF} < R) \). The outage minimization problem for ODF is

\[
\min_{\mathbf{P}(s) \in \Omega} P_{out}^{ODF}(\mathbf{P}).
\]

Proposition 4.1: The resource allocation policy that achieves the minimum in (25) is of the threshold type given by

\[
P(s) = \begin{cases} \mathbf{P}^*(s) & \text{if } Pr(\text{req}(R,s) \leq P_{th}) < P_{req}(R,s), \\ (0, 0, t) & \text{if } Pr(\text{req}(R,s) < P_{th}) > P_{req}(R,s), \end{cases}
\]

such that \( E[\text{req}(R,s)]P_{req}(R,s) < P_{th} = P_{avg} \).

Recall from Section III-A that \( Pr(\text{req}(R,s) \leq P_{th}) \) as expressed in (12) is the minimum total power that is required to achieve a transmission rate of \( R \) at channel state \( s \) and \( \mathbf{P}^*(s) \) is the resource allocation policy that achieves this. Compared to delay-limited capacity, we observe that for outage minimization the set of channel states for which DT is selected over DF, i.e., \( A \), and the optimal resource allocation function \( \mathbf{P}^*(s) \) remain the same, but now we need to turn off the transmission if the minimum total required power \( P_{req}(R,s) \) is higher than a threshold \( P_{th} \). The proof of Proposition 4.1 is given in Appendix B.

C. Lower Bound to the Outage Probability

We again use the cut-set bounds for the half-duplex relay as in Section III-B. For a target rate of \( R \), the minimization problem for the outage probability lower bound can be expressed as

\[
\min_{\mathbf{P}(s) \in \Omega} P_{out}^{LB}(\mathbf{P}),
\]

with

\[
P_{out}^{LB}(\mathbf{P}) = Pr(C_{UB}(\mathbf{P},s) < R),
\]

where \( C_{UB}(\mathbf{P},s) \) is the upper bound on the communication rate.
where $C_{UB}$ is expressed in (17). From the results of Appendix B, we can conclude that the optimal resource allocation to achieve the lower bound is also threshold type.

D. Numerical Results for Outage Minimization

Fig. 6 illustrates the minimum outage probability vs. average total power of the system for various communication scenarios for $R = 1$ and $d = 0.2$. The results for different rates are similar to this figure, so we limit the presentation to $R = 1$. The topmost curve corresponds to DT with constant power. Comparison of this to DT with dynamic power allocation curve shows that a power reduction of almost 8 dB is possible at $P_{out} = 10^{-1}$ by optimal power allocation. The third curve is for MH. We see that the system performance improves considerably compared to DT. Similar to the delay-limited capacity case, cooperative transmission, even with a very simple protocol like MH, performs much better than DT, proving the importance of cooperation. The fourth curve corresponds to ODF protocol without time allocation optimization, i.e., time slot is divided equally among the source and the relay, while the next curve corresponds to fully optimized ODF. We observe that ODF with $t = 1/2$ can gain more than 4 dB compared to MH for almost all SNR values, where optimal time allocation brings an additional 0.8 dB gain. We see that dynamic power and time allocation brings the performance very close to the lower bound for a relay location of $d = 0.2$, which is the lowest curve in the figure.

To see the effect of the relay location on the performance of the ODF protocol, we plot the minimum required average total power for fixed outage probability and rate vs. the source-relay distance in Fig. 7. Here we observe that, ODF protocol performs almost optimally when the relay is located closer to the source. However, when the relay is closer to the destination, ODF has a loss around 1 dB. Fixing time allocation at $t = 1/2$ for ODF brings a power loss around 0.8 dB, which is approximately constant for all relay locations. MH consistently performs worse than ODF and requires about 4-6 dB more power than the lower bound. Overall our results are consistent with those for the delay-limited capacity.

In a denser network scenario, where multiple relays are available, it is possible to pick the one that requires the least total power for the same target rate. Consider the model where the S-D distance is normalized to 1 as before, and there are multiple relays all having S-R and R-D distances equal to 1. Assuming that all link fading amplitudes are available to the transmitters, by simply extending Section IV, at each channel realization it is possible to pick the relay (or direct transmission) with the minimum required total power. Since no phase information is available at the transmitters, no simultaneous transmission can perform better than this scheme as before. Thus, as opposed to multiple relay strategies such as distributed space-time coding, the coding-decoding complexity of the system does not increase with the number of relays. However, the amount of feedback required is larger. In Fig. 8 we see the minimum outage probability vs. average total power relation for increasing number of relay terminals. As expected, the outage probability decreases with each additional relay.
V. CONCLUSION

In this paper, we explore the effect of partial CSIT on the performance of cooperative communications for delay limited applications. We propose an opportunistic decode-and-forward (ODF) protocol where the relay terminal is utilized depending on the overall network state with dynamic power and time allocation. We show that ODF brings a considerable improvement compared to direct transmission or multi-hop, and performs close to the cut-set bound for the half-duplex relay model in terms of both the delay-limited capacity and minimum outage probability.

APPENDIX

A. Proof of Proposition 3.1

The average total power for MH protocol corresponding to a target rate of $R$ is

$$E[P] = E\left[\frac{P_1}{2}\right]_{a > b \text{ or } a > c} \cdot Pr(a > b \text{ or } a > c) + E\left[\frac{P_1 + P_2}{2}\right]_{a < b \text{ and } a < c} \cdot Pr(a < b \text{ and } a < c),$$

$$= \frac{2^{2R} - 1}{2} \left( E\left[\frac{1}{a}\right]_{a \geq b \text{ and } c \geq b} \cdot Pr(a \geq b \text{ and } c \geq b) + E\left[\frac{1}{c}\right]_{a \geq c \text{ and } b \geq c} \cdot Pr(a \geq c \text{ and } b \geq c)
\right.$$

$$+ E\left[\frac{1}{b}\right]_{a \geq b \text{ and } a \leq c} \cdot Pr(a \geq b \text{ and } a \leq c) + E\left[\frac{1}{c}\right]_{a \leq c \text{ and } b \leq c} \cdot Pr(a \leq c \text{ and } b \leq c) \right),$$

where we substituted the values for $P_1, P_2,$ and $P_3$ from (3) and (4). Then we have

$$E[P] = \frac{2^{2R} - 1}{2} \left( E\left[\frac{1}{a}\right]_{a \geq b \text{ and } c \geq b} \cdot Pr(a \geq b \text{ and } c \geq b) + E\left[\frac{1}{c}\right]_{a \geq c \text{ and } b \geq c} \cdot Pr(a \geq c \text{ and } b \geq c)
\right.$$

$$+ E\left[\frac{1}{b}\right]_{a \geq b \text{ and } a \leq c} \cdot Pr(a \geq b \text{ and } a \leq c) + E\left[\frac{1}{c}\right]_{a \leq c \text{ and } b \leq c} \cdot Pr(a \leq c \text{ and } b \leq c) \right),$$

$$\leq \frac{2^{2R} - 1}{2} \left( E\left[\frac{1}{a}\right]_{a \geq b} + E\left[\frac{1}{a}\right]_{a \geq c} + E\left[\frac{1}{b}\right]_{a < b} + E\left[\frac{1}{c}\right]_{a < c} \right).$$

(30a)

(30b)

Now we have four similar expectation expressions which we can calculate as below.

$$f(a, b) = E\left[\frac{1}{a}\right]_{a \geq b} = \int_0^\infty \int_0^\infty \frac{1}{a}\lambda_a e^{-a/\lambda_a} e^{-b/\lambda_b} dadb,$$

$$= \int_0^\infty \frac{1}{\lambda_a}\lambda_b E_1\left(\frac{b}{\lambda_a}\right) e^{-b/\lambda_b} db,$$

$$= \frac{1}{\lambda_a}\ln\left(1 + \frac{\lambda_a}{\lambda_b}\right),$$

(31)

where exponential integral is defined as $E_1(t) = \int_0^\infty \frac{e^{-tw}}{w} dw$ and we use the following exponential integral relation to get (31) [21].

$$\int_0^\infty e^{-zt}E_1(t)dt = \frac{1}{z}\ln(1 + z), \text{ for } z > -1.$$

Substituting this in (30b), we obtain:

$$E[P] \leq \frac{2^{2R} - 1}{2} \times \left[ f(a, b) + f(a, c) + f(b, a) + f(c, a) \right],$$

$$< \infty.$$

Using the continuous and the increasing behavior of the total average power with respect to rate $R$ (due to continuity of the Rayleigh distribution), we can deduce that, non-zero delay-limited capacity can be achieved in a cooperative system for any finite average total power constraint.

B. Proof of Proposition 4.1

In this appendix, we prove that the threshold type resource allocation policy given in (26) is the solution to (25). Let $P \in \Omega$ be any resource allocation function that satisfies the average power constraint. Now, consider

$$P' = \begin{cases} P & \text{if } C_{ODF}(P, s) \geq R, \\ 0 & \text{if } C_{ODF}(P, s) < R. \end{cases}$$

It is easy to see that the outage probabilities corresponding to power allocations $P$ and $P'$ are equal, i.e., $P_{out}^{OOF}(P') = P_{out}^{OOF}(P)$ and $E[P'] = E[P]$. Therefore $P' \in \Omega$ as well and we only need to concentrate on resource allocation functions of the form $P'$. Now, consider the following resource allocation policy:

$$P = \begin{cases} P & \text{if } P_{req}(R, s) \leq P_{th}, \\ 0 & \text{if } P_{req}(R, s) > P_{th}. \end{cases}$$

(34)

where $P_{th}$ is chosen to satisfy $P_{out}^{OOF}(P) = P_{out}^{OOF}(P')$. There always exists such $P_{th}$ since the channel state distribution is continuous, and thus $P_{out}^{OOF}(P)$ is a continuous function of $P_{th}$. Define $M = \{s : C_{ODF}(P', s) \geq R\}$, and $N = \{s : P_{req}(R, s) \leq P_{th}\}$. Then we have

$$E[P'] = \int_{M,N} \left[ t'P'_1 + (1 - t')P'_2 \right] dF(s),$$

$$= \int_{M,N} \left[ t'P_1 + (1 - t')P_2 \right] dF(s) + \int_{M,N} \left[ t'P'_1 + (1 - t')P'_2 \right] dF(s),$$

$$\geq \int_{M,N} \left[ t'h_i dF(s) \right. + \int_{M,N} \left[ t'P_1 + (1 - t')P_2 \right] dF(s),$$

$$= P_{th}Pr(M \cap N'),$$

$$= P_{th}Pr(M \cap N'),$$

$$+ \int_{M,N} \left[ t'P'_1 + (1 - t')P'_2 \right] dF(s),$$

$$\geq \int_{M,N} P_{req}dF(s) + \int_{M,N} P_{req}dF(s),$$

$$= \int_N P_{req}dF(s),$$

$$= E[P].$$
Here, (a) and (c) follow from the definition of set $\mathcal{N}$, and
(b) follows from $P(\mathcal{N}) = 1 - P_{\text{out}}^{\text{ODF}}(\mathbf{P}) = 1 - P_{\text{out}}^{\text{DF}}(\mathbf{P}) = P(\mathcal{M})$. Thus we conclude that $E[\hat{\mathbf{P}}] \leq E[\mathbf{P}^n]$, so $\hat{\mathbf{P}} \in \hat{\Omega}$. This means that, of all the resource allocation functions in $\hat{\Omega}$, the minimum outage probability is achieved by a function of the form $\hat{\mathbf{P}}$.

As we did not use any specific property of ODF protocol in the proof, the optimality of the threshold form in (26) is valid for all the protocols while $P_{\text{eq}}(\tilde{R}, \tilde{s})$ and thus $P_{\text{th}}$ are protocol dependent.

REFERENCES


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