ENERGY EFFICIENT LOSSY TRANSMISSION OVER SENSOR NETWORKS WITH FEEDBACK

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ABSTRACT

The *energy-distortion function (E(D))* for a network is defined as the minimum total energy required to achieve a target distortion D at the receiver without putting any restrictions on the number of channel uses per source sample. $E(D)$ is studied for a sensor network in which multiple sensors transmit their noisy observations of a Gaussian source to the destination over a Gaussian multiple access channel with perfect channel output feedback. While the optimality of separate source and channel coding is proved for the case of a single sensor, this optimality is shown to fail when there are multiple sensors in the network.

A network with two sensors is studied in detail. First a lower bound on $E(D)$ is given. Then, two achievability schemes are proposed: a separation based digital scheme and a Schalkwijk-Kailath (SK) type uncoded scheme. The gap between the lower bound and the upper bound based on separation is shown to be a constant even as the total energy requirement goes to infinity in the low distortion regime. On the other hand, as the distortion requirement is relaxed, the SK based scheme is shown to outperform separation in certain cases, proving that the optimality of source-channel separation does not hold in the multi-sensor setting.

Index Terms— Sensor networks, energy efficiency, feedback, joint source-channel coding, information theory.

1. INTRODUCTION

We study a sensor network in which the sensors observe noisy versions of a stochastic source and transmit their observations to a destination such that the destination can reconstruct the underlying source with a certain fidelity. If the energy available at the sensors is severely constrained, they could reduce their transmission energy by spreading their transmissions over multiple channel uses (e.g., see [9], [7] and references therein). Furthermore, if the destination is under no power constraint, it could help the sensors by providing them with a perfect feedback of all its receptions. Thus, the feedback allows a sensor to obtain information about the transmissions of the other sensors. This fact could be exploited by the sensors to pass messages to each other over the feedback link, allowing the sensors to cooperate with each other. It is in this framework that we set our problem. In this work, we study the problem of minimizing the energy expenditure per source sample at the sensors, in a sensor network with feedback and with no constraint on the number of channel uses per source observation.

Our model brings together the problems of compression and communication in a distributed setting, and hence it is a joint source and channel coding problem. While this model has been widely studied in the literature (e.g., see [2], [10] and references therein), previous works consider a fixed ratio between the source and channel bandwidths, i.e., a fixed number of channel uses per source sample, and search for the minimum achievable distortion under a power constraint. In this work, we introduce a fundamental information-theoretic energy-distortion function $E(D)$ in which we do not put any constraint on the bandwidth ratio, and find the minimum energy per source sample required to achieve a target distortion.

We assume that perfect channel output feedback is available at the sensors. While we prove that separate source and channel coding achieves $E(D)$ for a single sensor scenario, similarly to the power-distortion tradeoff, optimality of separation fails as the number of sensors increases. Our focus in this paper is on a two-sensor model for which we provide lower and upper bounds on $E(D)$. We study an uncoded achievability scheme based on the well-known Schalkwijk-Kailath (SK) scheme [4] as well as separate source and channel coding.

2. SYSTEM MODEL

We consider a sensor network with perfect channel output feedback as illustrated in Fig. 1. The source S^M is an M-length random vector of independent and identically distributed (i.i.d.) real-valued Gaussian random variables with zero means and variances σ_S^2, i.e., $S \sim \mathcal{N}(0, \sigma_S^2)$. Each of the K sensors observes a noisy version of the underlying source S, i.e., the observation vector at sensor k for $k = 1, \ldots, K$ is denoted by U_k^M, which is defined as

$$U_{k,m} \quad = \quad S_m + W_{k,m} \quad \text{for } m = 1, \ldots, M \qquad (1)$$

where W_k^M is a Gaussian noise vector with i.i.d. $\mathcal{N}(0, \sigma_{W_k}^2)$ components.

Sensors transmit their observations over a multiple access channel (MAC). Denoting the transmission vector of sensor k as X_k^N, and the corresponding channel output vector as Y^N,

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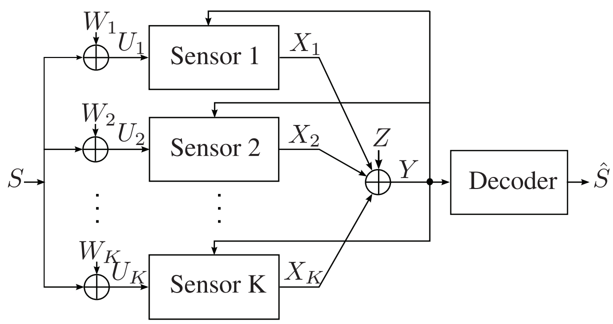

Fig. 1. Gaussian sensor network model with perfect channel output feedback.

the channel is characterized by

$$Y_n = \sum_{k=1}^{K} X_{k,n} + Z_n \quad \text{for } n = 1, \ldots, N \tag{2}$$

where Z^N is the vector of i.i.d. Gaussian channel noise with distribution $\mathcal{N}(0, \sigma_Z^2)$. In this work, we focus on the symmetric scenario in which the observation noises at different sensors have the same variance, i.e., $\sigma_{W_k}^2 = \sigma_W^2$ for $k = 1, \ldots, K$. We denote this system as a $(K, \sigma_S^2, \sigma_W^2, \sigma_Z^2)$ sensor network.

We assume the availability of perfect channel output feedback at all the sensors, and hence the encoding function at each sensor can depend on its noisy observation as well as the previous channel outputs. Considering block encoding from an M-length source vector to an N-length channel vector, the encoder at sensor k is described by a sequence of encoding functions $f_{k,n}^{(M,N)} : \mathbb{R}^M \times \mathbb{R}^{n-1} \to \mathbb{R}$ where $X_{k,n} = f_{k,n}^{(M,N)}(U_k^M, Y^{n-1})$ for $k = 1, \ldots, K$ and $n = 1, \ldots, N$. The decoder is described by the decoding function $g^{(M,N)} : \mathbb{R}^N \to \mathbb{R}^M$ where $\hat{S}^M = g^{(M,N)}(Y^N)$.

Definition 2.1 *For a $(K, \sigma_S^2, \sigma_W^2, \sigma_Z^2)$ sensor network, we say that an energy-distortion pair (E, D) is "achievable" if there exist a sequence (over M and N) of encoding functions*

$$\{f_{1,n}^{(M,N)}\}_{n=1}^N, \ldots, \{f_{K,n}^{(M,N)}\}_{n=1}^N$$

satisfying the total energy (per source sample) constraint

$$\mathbb{E}\left[\sum_{k=1}^{K}\sum_{n=1}^{N} X_{k,n}^2\right] \leq ME, \tag{3}$$

and a sequence of decoding functions $g^{(M,N)}$ such that the corresponding distortion sequence satisfies

$$\limsup_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} \mathbb{E}\left[(S_m - \hat{S}_m)^2\right] \leq D.$$

Definition 2.2 *We define the "energy-distortion function" for a given sensor network as*

$$E(D) \triangleq \inf\{E \geq 0 : (E, D) \text{ is achievable}\}. \tag{4}$$

Our goal is to identify $E(D)$ for a given sensor network. Note that, we do not impose any source or channel bandwidth constraints, hence it is possible to transmit as many channel symbols per source observation as needed as long as the total energy constraint is satisfied.

3. SINGLE SENSOR SCENARIO

We first focus on the point-to-point scenario in which there is only a single sensor transmitting its noisy observation to the destination, i.e., $K = 1$. We show that separate source and channel coding achieves $E(D)$ in this case. To present this result we first define $E_{b\min}$ as the *minimum energy per bit* [8] for the underlying communication channel.

We define $R_{U_1}(D)$ as the remote rate-distortion function for the given source, that is, the minimum rate required to achieve an average distortion D when the encoder observes U_1. For the Gaussian setup considered here, we have

$$R_{U_1}(D) = \frac{1}{2}\log_2^+\left(\frac{\sigma_S^4}{(\sigma_S^2 + \sigma_W^2)D - \sigma_S^2\sigma_W^2}\right),$$

where $\log^+(x) = \log(x)$ if $x \geq 1$ and 0 otherwise.

Lemma 3.1 *For a single sensor scenario, we have $E(D) = E_{b\min}R_{U_1}(D)$, and this can be achieved by separate source and channel coding*[1].

The proof of Lemma 3.1 is along the lines of proof of the source–channel separation theorem (see also [9, Theorem 2]). For the $(1, \sigma_S^2, \sigma_W^2, \sigma_Z^2)$ network, we have $E_1(D) = \sigma_Z^2 \log_e^+\left(\frac{\sigma_S^4}{(\sigma_S^2+\sigma_W^2)D - \sigma_S^2\sigma_W^2}\right)$.

Remark 3.1 *We note here that the optimality of source and channel separation is not limited to the Gaussian model considered here. Lemma 3.1 holds for a general discrete memoryless stationary source and channel with any additive distortion measure and any separable cost function respectively.*

In the remainder of the paper we focus on the two-sensor scenario, i.e., $K = 2$, for which the optimality of separation fails. We provide lower and upper bounds on $E(D)$.

4. LOWER BOUND ON $E(D)$

A trivial lower bound is obtained by a cut-set argument, assuming the sensors can perfectly cooperate. Under this assumption, the network reduces to a point-to-point link with two noisy observations and perfect channel output feedback. From Lemma 3.1, optimal performance is achieved by source and channel separation.

We define the remote rate-distortion function $R_{U_1,U_2}(D)$ as the minimum rate required to achieve an average distortion

[1]The proofs are omitted due to space limitations.

D when the encoder observes both U_1 and U_2. We have

$$R_{U_1,U_2}(D) = \min_{\substack{P_{T|U_1,U_2}: \\ E[(S-T)^2]\le D}} I(U_1,U_2;T) = \frac{1}{2}\log_2^+\left(\frac{\sigma_S^2}{D\left(1+\frac{\sigma_W^2}{2\sigma_S^2}\right)-\frac{\sigma_W^2}{2}}\right).$$

We have $E_{b\min} = \sigma_Z^2 \log_e 2$ for the underlying Gaussian channel with two transmit antennas. Then the lower bound is found as

$$E_{cutset}(D) = \frac{\sigma_Z^2}{2}\log_e^+\left(\frac{\sigma_S^2}{D\left(1+\frac{\sigma_W^2}{2\sigma_S^2}\right)-\frac{\sigma_W^2}{2}}\right). \quad (5)$$

Next, we identify a tighter lower bound based on the ideas from the converse of the MAC with feedback by Ozarow [6]. To present this lower bound, we first introduce the conditional remote rate-distortion function $R_{U_1|U_2}(D)$, which is defined as the minimum rate required to achieve distortion D when the encoder observes both U_1 and U_2 while U_2 is observed by the decoder as well. We have [11]

$$R_{U_1|U_2}(D) = \min_{\substack{P_{T|U_1,U_2}: \\ E[(S-T)^2]\le D}} I(U_1;T|U_2)$$
$$= \frac{1}{2}\log_2^+\left(\frac{\sigma_W^2\sigma_S^2}{(\sigma_W^2+\sigma_S^2)(\sigma_W^2+2\sigma_S^2)\left(D-\frac{\sigma_W^2\sigma_S^2}{\sigma_W^2+2\sigma_S^2}\right)}\right).$$

Proposition 4.1 *In a $(2,\sigma_S^2,\sigma_W^2,\sigma_Z^2)$ sensor network, $E(D)$ is lower bounded by*

$$E_{LB}(D) = \max_{\hat\rho\in[0,1]}\min\left\{\frac{\sigma_Z^2}{1+\hat\rho}\log_e^+\left(\frac{\sigma_S^2}{D\left(1+\frac{\sigma_W^2}{2\sigma_S^2}\right)-\frac{\sigma_W^2}{2}}\right),\right.$$
$$\left.\frac{2\sigma_Z^2}{1-\hat\rho^2}\log_e^+\left(\frac{\sigma_W^2\sigma_S^2}{(\sigma_W^2+\sigma_S^2)(\sigma_W^2+2\sigma_S^2)\left(D-\frac{\sigma_W^2\sigma_S^2}{\sigma_W^2+2\sigma_S^2}\right)}\right)\right\}. \quad (6)$$

In (6), $\hat\rho$ denotes the correlation between the transmissions of the two sensors.

5. SEPARATE SOURCE AND CHANNEL CODING

In this scheme, sensors first compress their observations, and then transmit these compressed observations over the MAC using channel codes that are generated independently of the sensor observations.

The source coding component of separate coding is the well-known CEO problem [5]. Assuming equal rates, the minimum rate required at each sensor to achieve distortion D for the symmetric Gaussian CEO problem is given by [5]

$$R_{CEO}(D) = \frac{1}{4}\log_2^+\left(\frac{\sigma_S^2}{D}\right) + r, \quad (7)$$

where r satisfies

$$\frac{1}{\sigma_S^2} + 2\,\frac{1-2^{-2r}}{\sigma_W^2} = \frac{1}{D}. \quad (8)$$

On the other hand, the rate pairs that can be achieved over a MAC with perfect channel output feedback is characterized by Ozarow [6] as follows:

$$\bigcup_{0\le\rho\le1}\left\{(R_1,R_2): 0\le R_1\le\frac{1}{2}\log_2\left(1+\frac{P_1}{\sigma_Z^2}(1-\rho^2)\right),\right.$$
$$0\le R_2\le\frac{1}{2}\log_2\left(1+\frac{P_2}{\sigma_Z^2}(1-\rho^2)\right),$$
$$\left.0\le R_1+R_2\le\frac{1}{2}\log_2\left(1+\frac{P_1+P_2+2\rho\sqrt{P_1P_2}}{\sigma_Z^2}\right)\right\},$$

where $P_k \ge 0$ is the power allocated to user k, $k=1,2$.

For an equal power allocation of $P_1 = P_2 = P/2$, it is possible to achieve the following rate at each sensor:

$$\min\left\{\frac{1}{2}\log_2\left(1+\frac{P}{2\sigma_Z^2}(1-\rho^2)\right), \frac{1}{4}\log_2\left(1+\frac{P}{\sigma_Z^2}(1+\rho)\right)\right\}$$

for any $0\le\rho\le1$.

Assume that M source samples are quantized and transmitted over N channel uses, where $M,N\to\infty$ while $\frac{M}{N}\to 0$. Then the energy-distortion pair (E,D) is achievable if

$$R_{CEO}(D) \le \max_{0\le\rho\le1}\min\left\{\frac{E}{4\sigma_Z^2}(1-\rho^2)\log_2 e,\ \frac{E}{4\sigma_Z^2}(1+\rho)\log_2 e\right\}$$
$$= \frac{E}{4\sigma_Z^2}\log_2 e. \quad (9)$$

Note that the maximum in the last step is achieved by setting $\rho=0$; that is, no beamforming is required to achieve the minimum required energy in separate source and channel coding, and hence, the same performance can be achieved in the absence of feedback as well.

Using (7) and (9), $E(D)$ is upper bounded by

$$E_{sep}(D) = \sigma_Z^2\log_e^+\left(\frac{\sigma_S^2}{D\left(1-\frac{\sigma_W^2}{2}\left(\frac{1}{D}-\frac{1}{\sigma_S^2}\right)\right)^2}\right). \quad (10)$$

6. UNCODED TRANSMISSION

Next, we describe an achievability scheme based on uncoded transmission. It is well-known that uncoded transmission achieves the optimal power-distortion tradeoff for point-to-point systems when the source and channel bandwidths match [3]. Interestingly, it is shown in [4] that a variant of uncoded transmission achieves the optimal power-distortion tradeoff in a point-to-point system even when multiple channel uses are available per source sample in the presence of perfect

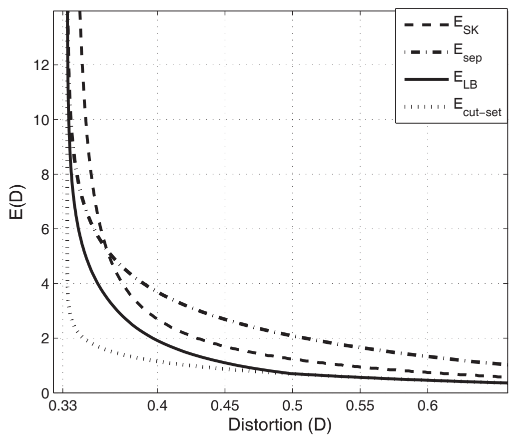

Fig. 2. $E(D)$ bounds for a $(2,1,1,1)$ sensor network.

channel output feedback. This is achieved by the well-known Schalkwijk–Kailath (SK) scheme in which the transmitter sends the estimation error at the receiver in an uncoded fashion. Adapting this scheme to our model, it can also be shown that $E(D)$ in a point-to-point system (i.e., $K = 1$), which can be achieved by separate source and channel coding as shown in Lemma 3.1, can also be achieved using the SK scheme. Note that the SK scheme is much simpler than separation and it operates on a per symbol basis thereby reducing delay.

Much less is known for the case of multiple sensors. In the sensor network model without feedback, when the source and channel bandwidths match, it has recently been shown in [1] that uncoded transmission is optimal in terms of the power-distortion tradeoff. Since perfect feedback is available in our model, we study a variant of the SK scheme. The sensors not only jointly communicate their beliefs about the estimation error of the receiver, but also update their own estimates about the source realization at each step. While we do not have a closed form expression for the minimum required energy for the SK scheme, we are able to characterize it in a recursive manner, which is used to obtain the numerical result in the next section.

7. NUMERICAL RESULTS AND DISCUSSIONS

In Fig. 2, we illustrate the lower and upper bounds on $E(D)$ for a $(2,1,1,1)$ sensor network. We observe that the optimal tradeoff is achieved in the two asymptotic regimes of low or high distortion targets. However, while the separation based scheme performs better in the low distortion regime, the SK scheme outperforms the separation based scheme as the target distortion increases, proving the sub-optimality of source-channel separation for multiple sensors. None of the schemes meet the proposed lower bound, yet it is possible to bound the gap between the energy requirement of the separation based scheme and $E(D)$ in the low distortion regime for which $E(D)$ goes to infinity.

Lemma 7.1 *In a* $(2,1,1,1)$ *sensor network, the gap between* $E(D)$ *and the energy required by the separation scheme is upper bounded by* 1.674 *as* $D \to 1/3$*.*

8. CONCLUSIONS

We have introduced and studied a fundamental energy-distortion function $E(D)$ for sensor networks when perfect channel output feedback is available at the sensors. $E(D)$ identifies the optimal energy-distortion tradeoff in the wideband limit, that is, when there is no constraint on the available channel bandwidth per source sample. We have first proven the optimality of separate source and channel coding in a point-to-point system in terms of $E(D)$. Then, since the optimality fails in the case of multiple sensors, we have provided lower and upper bounds on $E(D)$ when there are two sensors.

9. ACKNOWLEDGMENT

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