

# A General Framework for the Optimization of Energy Harvesting Communication Systems with Battery Imperfections

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**Abstract**—Energy harvesting has emerged as a powerful technology for complementing current battery-powered communication systems in order to extend their lifetime. In this paper a general framework is introduced for the optimization of communication systems in which the transmitter is able to harvest energy from its environment. Assuming that the energy arrival process is known non-causally at the transmitter, the structure of the optimal transmission scheme, which maximizes the amount of transmitted data by a given deadline, is identified. Our framework includes models with continuous energy arrival as well as battery constraints. A battery that suffers from energy leakage is studied further, and the optimal transmission scheme is characterized for a constant leakage rate.

**Index Terms**—Battery leakage, battery size constraint, broadcast channel, continuous energy arrival, energy efficient communications, energy harvesting, rechargeable wireless networks, throughput maximization.

## I. INTRODUCTION

Energy efficiency is a key challenge in the sustainable deployment of battery-powered communication systems. Applications such as wireless sensor networks depend critically on the lifetime of individual sensors, whose batteries are limited due to physical constraints as well as cost considerations. Power management is essential in optimizing the energy efficiency of these systems in order to get the most out of the available limited energy in the battery. A complementary approach has recently been made possible by introducing rechargeable batteries that can harvest energy from the environment. Several different technologies have been proposed and implemented for harvesting ambient energy such as solar, radio-frequency, thermoelectric or solar (see [1]–[5] and references therein for various examples of energy harvesting technology).

Harvesting energy from the environment is an important alternative to battery-run devices to extend their lifetime. However, it is important to design the system operation based on the energy harvesting process to increase the efficiency. Energy harvesting systems have received a lot of recent attention [6]–[10]. Node and system level optimization have been considered

from both practical and theoretical perspectives. The previous work that are most relevant to the problems studied in this paper are [11]–[14]. In [11], the problem of transmission time minimization is studied when the data and the energy arrives at the transmitter in packets; and the transmission power is optimized when the data and energy arrival times and amounts are known in advance. In [12], the amount of transmitted data is maximized for an energy harvesting system under deadline and finite battery capacity constraints. Reference [12] also shows that the transmission time minimization problem studied in [11] and the transmitted data maximization problem are duals of each other and their solutions are identical for the same parameters. The problem is extended to the broadcast channel in [13]–[16], to the relay channel in [17], and to the multiple access channel in [18].

The problem considered in this work is that of maximizing the amount of data that is transmitted within a given deadline constraint under various assumptions regarding the energy harvesting model as well as the battery limitations. Our focus is on the offline optimization of the energy harvesting communication system, that is, we assume that the energy arrival profile is known in advance. We first introduce a general framework for transmitted data maximization by adjusting the transmit power in an energy harvesting system with battery limitations. Our model includes continuous energy harvesting, generalizing the packetized energy arrival model considered in [11] and [12]. Moreover, different from the previous work, our model also includes the realistic scenario of battery degradation over time by considering a time-varying battery capacity. We show that these constraints can be modeled through cumulative harvested energy and minimum energy curves, which are then used to obtain the optimal transmission policy. The framework introduced for the energy harvesting system optimization is similar to the calculus approach introduced by Zafer and Modiano for energy-efficient data transmission in [19]. We later show that the proposed framework also applies to a broadcast channel with an energy harvesting transmitter.

We then consider a more realistic battery model with energy leakage. Assuming a constant leakage rate, we identify the optimal transmission strategy for the case of a packetized energy arrival model.

The paper is organized as follows. Section II presents the system model. Optimal transmission scheme for a point-to-point system under battery size constraints is derived in Sections III. In Section IV, it is shown that the proposed

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framework can be used to characterize the optimal transmission scheme in an energy-harvesting broadcast channel. We consider battery leakage in Section V and find the optimal transmission scheme for a linear leakage rate. Finally, conclusions are provided in Section VI.

## II. SYSTEM MODEL

We consider a continuous-time model for both the harvested and the transmitted energy, that is, the harvested energy is modeled as a continuous-time process, while the transmitter is assumed to be able to adjust its transmission power, and hence, the transmission rate, instantaneously. This continuous-time model generalizes the discrete-time arrival model considered in [11] and [12]. A cumulative curve approach is used to described the flow of energy in the system.

*Definition 2.1 (Harvested Energy Curve):* The harvested energy curve  $H(t)$  is a right-continuous function of time  $t$ ,  $t \in \mathbb{R}^+$ , that denotes the amount of energy that has been harvested in the interval  $[0, t]$ .

*Definition 2.2 (Transmitted Energy Curve):* The transmitted energy curve  $E(t)$  is a continuous function with bounded right derivative, that denotes the amount of energy that has been used for data transmission in the interval  $[0, t]$ ,  $t \in \mathbb{R}^+$ .

Naturally, we require  $E(t) \leq H(t)$ , i.e., the transmitter cannot use more energy than that has arrived. We also consider a “minimum energy curve” that might model, for example, a battery size constraint.

*Definition 2.3 (Minimum Energy Curve):* Given an harvested energy curve  $H(t)$ , a minimum energy curve  $M(t)$  is a function satisfying  $M(t) \leq H(t)$ ,  $\forall t \geq 0$ , and denotes the minimum amount of energy that needs to be used by the system until time  $t$ .

Given the harvested energy curve and the minimum energy curve, a feasible transmitted energy curve should satisfy the conditions  $M(t) \leq E(t) \leq H(t)$ ,  $\forall t \geq 0$ . Among all feasible transmitted energy curves, our goal is to characterize the one that transmits the highest amount of data over a given finite time interval  $[0, T]$ . We consider offline optimization, that is, the harvested and the minimum energy curves are assumed to be known in advance<sup>1</sup>.

We assume that the instantaneous transmission rate relates to the power of transmission at time  $t$  through a rate function  $r(\cdot)$ , which is a non-negative strictly concave increasing function of the power with  $r(0) = 0$ . We note here that many common transmission models, such as the capacity of an additive white Gaussian noise channel, satisfy these conditions [19]. The total transmitted data corresponding to a given curve  $E(t)$  over the interval  $[0, T]$  is found by

$$\mathcal{D}(E(t)) \triangleq \int_0^T r(E'(t))dt, \quad (1)$$

<sup>1</sup>This is an accurate assumption for systems in which the energy harvesting process can be modeled as a deterministic process. For example, in solar based systems the amount of energy that can be harvested at various times of the day can be modeled quite accurately. In some other systems, harvested energy depends on the operating schedule of the harvesting device rather than the energy source, such as shoe-mounted piezoelectric devices; and the harvested energy curve can be modeled accurately in advance.

where  $E'(t)$  is the derivative of function  $E(t)$  at time  $t$ , and it gives the power of transmission at that instant while  $r(E'(t))$  is the corresponding transmission rate.

## III. OPTIMAL TRANSMISSION SCHEME UNDER BATTERY SIZE CONSTRAINTS

In our problem formulation we assume that the transmitter always has data to transmit. Hence, the minimum energy curve can be used to model a constraint on the battery size, forcing the system to use any energy that cannot be stored in the battery for transmission of additional data before it is discarded. For a fixed energy curve  $E(t)$  and unlimited battery size, the energy that is available in the battery at time instant  $t$  is given by  $H(t) - E(t)$ . However, if the battery size is  $b$ , we should have  $H(t) - E(t) \leq b$ . Consequently, the associated minimum energy curve is given by  $M(t) = \max\{H(t) - b, 0\}$ .

We can also consider a time-varying battery capacity  $b(t)$ , which can model the degradation in the battery capacity over time. This is a common phenomenon in rechargeable batteries used for energy harvesting applications. See Fig. 1(a) for an illustration of the harvested and minimum energy curves for a battery with continuously decreasing capacity.

Now, the optimization problem can be stated as follows.

$$\max_{E(t) \in \Gamma} \mathcal{D}(E(t)) = \int_0^T r(E'(t))dt \quad (2)$$

$$\text{such that } M(t) \leq E(t) \leq H(t), \forall t \in [0, T], \quad (3)$$

where  $\Gamma$  specifies the set of all non-decreasing, continuous functions with bounded right derivatives for all  $t \in [0, T]$  and with  $E(0) = 0$ .

We first present the optimality conditions for the transmitted energy curve. Similar to previous studies, such as [11], [12], [19] and [20], our main tool is the Jensen’s inequality given in the following lemma (in the integral form).

*Lemma 3.1:* [Jensen’s inequality] Let  $f : [a, b] \rightarrow \mathbb{R}$  be a non-negative real valued function, and  $\phi(\cdot)$  be a concave function on the real line, then

$$\phi \left( \int_a^b f(t)dt \right) \geq \int_a^b \frac{\phi((b-a)f(t))}{b-a} dt, \quad (4)$$

with strict inequality if  $\phi(\cdot)$  is strictly concave,  $a \neq b$ , and  $f$  is not constant over the interval  $[a, b]$ .

Consider the simple setup in which the battery has available energy  $E_0$  at time  $t = 0$ , no further energy is harvested, and the minimum energy curve is given as  $M(t) = 0$  for  $0 \leq t < T$  and  $M(T) = E_0$ . We will prove for this simple setting that the constant power curve transmits the maximum amount of data over the time interval  $[0, T]$ .

For any transmitted energy curve  $E(t)$  with non-constant power, by replacing the function  $f$  in Lemma 3.1 with  $E'(t)/T$ , and letting  $a = 0$ ,  $b = T$  and  $\phi(\cdot) = r(\cdot)$ , we obtain

$$r \left( \int_0^T \frac{E'(t)}{T} dt \right) > \int_0^T \frac{r(E'(t))}{T} dt, \quad (5)$$

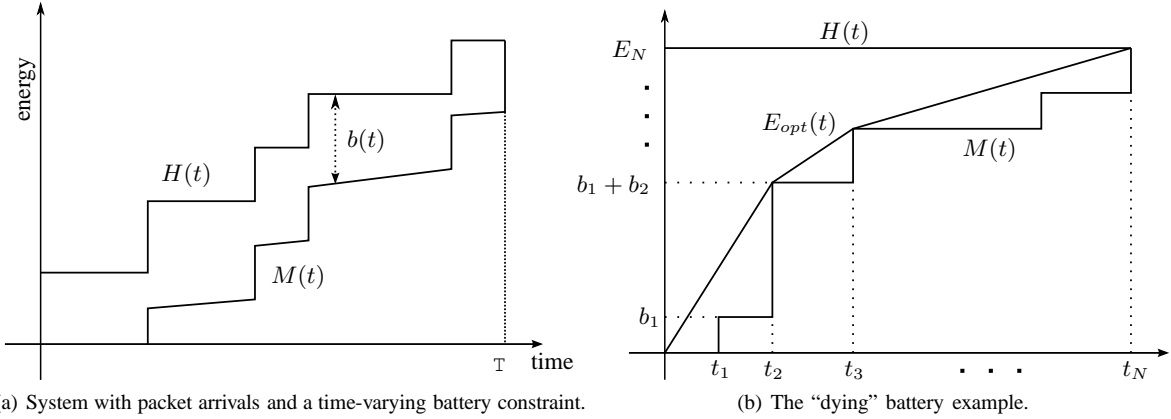


Fig. 1. Illustration of the harvested and minimum energy curves for different examples.

which is equivalent to

$$\int_0^T r(E'(t))dt < Tr\left(\frac{E_0}{T}\right). \quad (6)$$

Note that  $Tr\left(\frac{E_0}{T}\right)$  is the transmitted data by the constant power scheme. Hence, this proves the fact that the maximum data is transmitted by this scheme. We can express this result in a more general context as in the following theorem.

**Theorem 3.2:** Let  $E(t)$  be a feasible transmitted energy curve and  $S(t)$  be a straight line segment over interval  $[a, b]$  that joins  $E(a)$  and  $E(b)$ ,  $0 \leq a < b \leq T$ . If  $S(t)$  satisfies  $M(t) \leq S(t) \leq H(t)$  for  $a \leq t \leq b$ , the transmitted energy curve defined as

$$\hat{E}(t) = \begin{cases} E(t), & t \in [0, a) \\ S(t), & t \in [a, b) \\ E(t), & t \in [b, T] \end{cases} \quad (7)$$

satisfies  $\mathcal{D}(\hat{E}(t)) \geq \mathcal{D}(E(t))$ .

The following theorems state, respectively, the uniqueness of the optimal transmitted energy curve and the optimality conditions. Their proofs follow similarly to those of Theorem 2 and Lemmas 2-4 in [19].

**Theorem 3.3:** For a strictly concave rate function  $r(\cdot)$ , if  $\tilde{E}(t)$  is a feasible transmitted energy curve which does not have any two points that can be joined by a distinct feasible straight line, then  $\tilde{E}(t)$  is unique and it maximizes the transmitted data.

**Theorem 3.4:** Let  $E_{opt}(t)$  be the optimal energy expenditure curve and  $t_0$  be any point at which the power of transmission changes, i.e., the slope of  $E_{opt}(t)$  changes. Then, at  $t_0$ ,  $E_{opt}(t)$  intersects either  $H(t)$  or  $M(t)$ . If  $E_{opt}(t_0) = H(t_0)$ , then the slope change must be positive. If  $E_{opt}(t_0) = M(t_0)$ , then the slope change must be negative.

The optimal transmitted energy curve is also the one that has the minimum length, and hence, the same ‘‘string visualization’’ suggested in [19] can be applied here. The string visualization suggests that, if we tie one end of a string to the origin and connect it to the point  $(T, H(T))$  tightly while constraining it to lie between  $H(t)$  and  $M(t)$ , this string gives us the optimal energy expenditure policy.

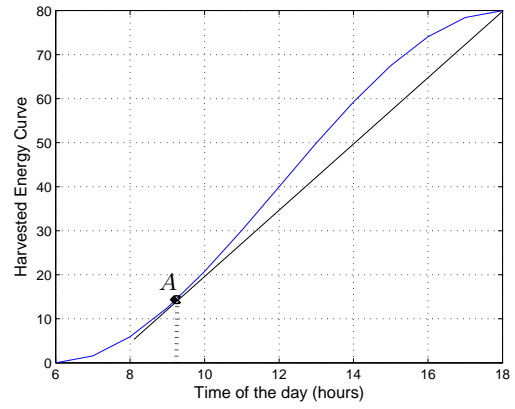


Fig. 2. Continuous energy harvesting curve for a solar panel.

A special case of the framework considered here is the one with packetized energy arrivals and without any battery constraint. This is the energy-harvesting dual of the packet arrival problem considered in [20]. As it is shown in [12], this is equivalent to the problem of transmission time minimization problem studied in [11]. In this problem we have  $M(t) = 0$  for  $t \in [0, T]$ , and  $N$  energy packets arrive at times  $\{t_i\}_{i=1}^N$ . The algorithm that gives the optimal transmitted energy curve for this problem can be obtained following [11] and [19].

Another example that fits into the general structure introduced above is the following. Consider a wireless system with an energy storage unit consisting of  $N$  batteries. Assume that all the batteries are full initially and a total of  $E_N = \sum_{i=1}^N b_i$  energy is available in the system at time  $t = 0$ , where  $b_i$  is the capacity of battery  $i$ . It is assumed that the batteries in the system have finite lifetime, and they die at certain time instants,  $t_i, i = 1, \dots, N$ . The problem is to find the maximum amount of data that can be transmitted until the last battery dies, i.e., until  $t_N$ . In this problem we have  $H(t) = E_N$  for  $t \in [0, t_N]$ , and  $M(t)$  can be obtained as in Fig. 1(b). Note that, since once the battery dies, the energy stored in it is not available for transmission anymore, and since we always have data in the queue to be transmitted, it is always beneficial to use the available energy in a battery before it

dies. In this sense, we can consider the time until a battery dies as a deadline constraint on the time the available energy in this battery should be used. The optimal transmitted energy curve can then be found using the string argument as seen in Fig. 1(b).

As an example of continuous energy arrival, we consider here a model of a solar panel harvesting energy during the day. The amount of energy harvested per unit of time changes during the day. While no energy is harvested when there is no sun, the harvested energy is maximized at noon (see [21]). We model the rate of harvested energy with the function  $h(t) = 5 - \frac{5}{36}(t - 12)^2$  for  $6 \leq t \leq 18$ , and  $h(t) = 0$  elsewhere, where  $t \in [0, 24]$  denotes the time of the day (hours), such that  $H(t) = \int_0^t h(\tau) d\tau$ . The unit of energy depends on the solar panel characteristics. The corresponding harvested energy curve is depicted in Fig. 2.

Assume that we want to maximize the amount of data that can be transmitted up to time  $t = 18$ , i.e., until the panel stops harvesting energy. Based on the above arguments, the optimal transmitted energy curve is identified as follows. First we draw a tangent to the harvested energy curve from the point  $(18, H(18))$ , and denote its intersection with the curve by  $A$ . The transmitted energy curve follows the harvested energy curve from the origin up to  $A$ <sup>2</sup>. Afterwards, it follows the straight tangent line, i.e., it uses constant power transmission. Note that, while it is easy to prove the optimality of this strategy using Theorem 3.3, the discrete energy arrival models studied in [11] and [12] do not apply here.

#### IV. OPTIMAL BROADCAST SCHEME WITH BATTERY CONSTRAINT

In this section, we show that the general approach introduced in Section III can be instrumental in identifying the optimal transmission policy in a broadcast channel (BC) with an energy harvesting transmitter [13], [14]. Consider the same energy harvesting model at the transmitter as before; however, now there are two receivers in the system, and the transmitter has independent data for each receiver.

The BC problem is studied in [13] and [14]; however, the solutions in these papers are elaborated from the basics rederiving the behavior of the optimal transmission policy in the BC scenario. Here, we show that the general approach introduced in previous section for the point-to-point setting can be directly applied to the BC scenario as well. This approach allows to generalize the results in [13] and [14] to continuous energy arrivals, and introduce battery constraints in the problem formulation [15], [16].

We consider an additive white Gaussian BC in which the signal received at receiver  $i$  is given by

$$Y_i = X + Z_i, \quad i = 1, 2, \quad (8)$$

<sup>2</sup>In practice, a continuous adaptation of the transmission rate is unrealistic due to the block structure of channel coding, and the finite number of modulation and coding modes available. However, such practical constraints are out of the scope of this paper.

where  $X$  is the channel input of transmitter and  $Z_i$  is the zero-mean Gaussian noise component with variance  $N_i$ . Without loss of generality, we assume that  $N_2 > N_1 > 0$ <sup>3</sup>. Let  $B_i(t)$  denote the total number of bits transmitted to receiver  $i$  up to time  $t$ . Our goal is to maximize the weighted sum of transmitted bits by time  $T$ ,  $\mu_1 B_1(T) + \mu_2 B_2(T)$  for some  $\mu_1, \mu_2 \geq 0$ .

In the broadcast channel setting, the transmitter not only needs to identify the transmitted energy curve  $E(t)$ , but also has to decide how to allocate the power among the two receivers at each time instant. Accordingly, we denote by  $p_1(t)$  and  $p_2(t)$  the power allocated to each receiver at time  $t$ . The optimization problem can be written as follows.

$$\begin{aligned} \max_{p_1(t), p_2(t) \geq 0} \quad & \mu_1 \int_0^T r_1(t) dt + \mu_2 \int_0^T r_2(t) dt \quad (9) \\ \text{such that} \quad & M(t) \leq \int_0^t p_1(\tau) + p_2(\tau) d\tau \leq H(t), t \in [0, T] \end{aligned}$$

We assume that the rate-power functions are operating on the boundary of the capacity region of the Gaussian BC:

$$r_1(t) = \frac{1}{2} \log_2 \left( 1 + \frac{p_1(t)}{N_1} \right) \quad (10)$$

$$r_2(t) = \frac{1}{2} \log_2 \left( 1 + \frac{p_2(t)}{p_1(t) + N_2} \right). \quad (11)$$

The considered optimization can be decoupled into two maximization problems as follows:

$$\max_{M(t) \leq E(t) \leq H(t)} \int_0^T \left[ \max_{\substack{p_1(t), p_2(t) \geq 0 \\ p_1(t) + p_2(t) = p(t)}} \mu_1 r_1(t) + \mu_2 r_2(t) \right] dt, \quad (12)$$

where we define  $p(t) = E'(t)$ .

First, we consider the maximization problem in between brackets in (12). Defining  $\mu \triangleq \frac{\mu_2}{\mu_1}$ , we can make the following observations on its solution<sup>4</sup>:

- If  $\mu > \frac{N_2}{N_1}$ , no power is allocated to the first receiver, i.e.  $p_1 = 0$ , independent of the total power.
- If  $\mu \leq 1$ , no power is allocated to the second receiver, i.e.  $p_2 = 0$ , independent of the total power.
- When  $1 < \mu \leq \frac{N_2}{N_1}$ , the optimal power allocation behaves as follows. If the available total power is below  $p_{th} \triangleq \frac{N_2 - \mu N_1}{\mu - 1}$ , all the total power is allocated to receiver 1, i.e.,  $p_1 = p$  and  $p_2 = 0$ . On the other hand, if  $p \geq p_{th}$ , then we have  $p_1 = p_{th}$  and  $p_2 = p - p_{th}$ .

Note that, if  $\mu > \frac{N_2}{N_1}$  or  $\mu \leq 1$ , the problem reduces to the point-to-point setting; hence, we assume  $1 < \mu \leq \frac{N_2}{N_1}$  in the remainder. We can write the outcome of the maximization problem in between brackets in (12) as

$$r(p) \triangleq \begin{cases} \frac{\mu_1}{2} \log_2 \left( 1 + \frac{p}{N_1} \right) & \text{if } 0 \leq p \leq p_{th}, \\ \frac{\mu_1}{2} \log_2 \left( 1 + \frac{p_{th}}{N_1} \right) + \frac{\mu_2}{2} \log_2 \left( 1 + \frac{p - p_{th}}{p_{th} + N_2} \right) & \text{if } p_{th} \leq p. \end{cases} \quad (13)$$

Then we can rewrite the optimization problem in (12) in the same form as the point-to-point problem in (2) with a rate

<sup>3</sup>The case with  $N_1 = N_2$  reduces to the single receiver problem.

<sup>4</sup>The time variable  $t$  is omitted for conciseness.

function given in (13). We next prove that this rate function is strictly concave.

*Lemma 4.1:* The rate function  $r(p)$  in (13) is a strictly concave function of power  $p$ .

*Proof:* It is easy to show that  $r(p)$  is continuous, differentiable, and its derivative is decreasing with  $p$ ; hence, it is a strictly concave function of  $p$ . ■

Now, based on this form of the optimization problem, we can directly use the results of Section III in the broadcast channel setting in order to identify the optimal transmission scheme for an energy harvesting transmitter. Note that as opposed to [13] and [14], our solution is valid for continuous energy arrivals as well as transmitters with various battery constraints. Once the optimal total transmit power over time is characterized, the power allocation among the users at each instant can be found using (13).

## V. OPTIMAL TRANSMISSION SCHEME WITH BATTERY LEAKAGE

In Sections III and IV and references therein, the battery has been considered to be ideal, that is, there was no energy leakage. In this section, we consider the more realistic scenario of a battery that leaks part of the stored energy.

The leakage rate of a battery depends on the type (Li-ion batteries have a lower leakage rate compared to the nickel-based ones), age and usage of the battery as well as the medium temperature. Moreover, even for a fixed type of battery and medium temperature, the leakage rate changes over time; the batteries leak most right after being charged. However, for simplicity, a constant rate leakage model is considered here. If the battery is non-empty at a given time instant, the energy is assumed to leak from the battery at a constant finite rate denoted by  $\epsilon \geq 0$ . Obviously no leakage occurs if the battery is empty. We use the same cumulative curve approach to model the battery leakage process. Note that the leakage rate  $\epsilon$  can alternatively be interpreted as the constant operation power of the node, that is, the circuit power needed to maintain the node awake.

*Definition 5.1 (Energy Leakage Curve):* The energy leakage curve  $L(t)$  is the amount of energy that has leaked from the battery in the time interval  $[0, t]$ ,  $t \in \mathbb{R}^+$ , with  $L(0) = 0$ . Due to the constant leakage rate assumption,  $L(t)$  is a continuous, non-decreasing function whose right-derivative is given by

$$L'_+(t) = \begin{cases} \epsilon, & \text{if } E(t) < H(t) - L(t), \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

To highlight the effect of leakage, we do not consider any minimum energy curve in this section, i.e.,  $M(t) = 0 \forall t$ , and we focus only on discrete energy packet arrivals. Defining a maximum energy curve as  $U(t) \triangleq H(t) - L(t)$ , the feasibility condition on the transmitted energy curve becomes  $0 \leq E(t) \leq U(t)$ . We tackle again the problem of characterizing the feasible transmitted energy curve that transmits the most data over a given finite time interval  $[0, T]$ . The

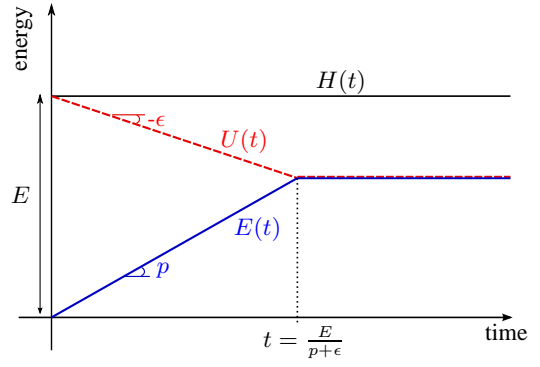


Fig. 3. The  $S_\infty$  problem.

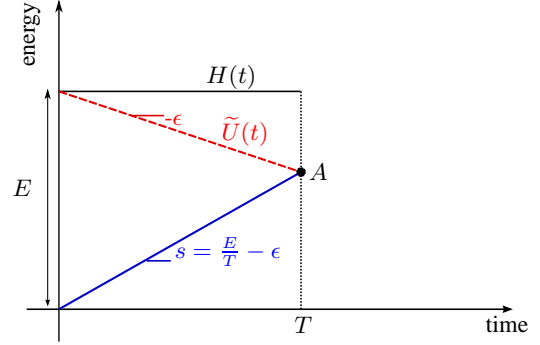


Fig. 4. The  $S_T$  problem.

corresponding optimization problem can be stated as

$$\max_{E(t) \in \Gamma} \mathcal{D}(E(t)) = \int_0^T r(E'(t)) dt \quad (15)$$

$$\text{such that } 0 \leq E(t) \leq U(t), \forall t \in [0, T]. \quad (16)$$

*Remark 5.1:* Unlike the battery size constraint studied in Section III, the battery leakage phenomenon does not translate into a minimum energy curve, but into a maximum energy curve obtained by removing the total leaked energy from the harvested energy curve. More importantly, the leakage curve is a function of the transmitted energy curve. Consequently, the maximum energy curve inherently depends on the transmitted energy curve, and hence, the solution framework presented in Section III does not directly extend to this setup.

Throughout this section, we consider the discrete energy harvesting process in which the  $n$ -th energy packet of size  $E_n$  arrives at time instant  $t_n$  for  $n = 1, \dots, N$ . Without loss of generality, the first packet is assumed to arrive at time  $t = 0$  (i.e.,  $t_1 = 0$ ). We call this general setup the  $N$ -packet problem. As before, we assume that the transmitter always has enough data to transmit. Below, we characterize the optimal transmission scheme first for the single-packet problem (i.e.,  $N = 1$ ), and then for the general  $N$ -packet problem.

### A. The Single-Packet Problem

We consider here the simplified problem consisting of a single energy packet  $E$  harvested at time  $t = 0$ . We refer to it as the single-packet problem. The solution of this problem will serve as a building block for the general  $N$ -packet problem.

First, let us treat the single-packet problem with infinite<sup>5</sup> deadline constraint (i.e.,  $T = \infty$ ), and denote it by  $S_\infty$ . It is depicted in Fig. 3. Following Section III, it is not hard to show that the optimal transmitted energy curve  $E(t)$  has to be piecewise linear, and the slope changes occur only if  $E(t)$  intersects  $U(t)$ . Consequently, the optimal  $E(t)$  for the  $S_\infty$  problem is as shown in Fig. 3: the node transmits at a constant power  $p$  until the battery runs out of energy. One can see that there is a trade-off in the choice of  $p$ : while it is more energy efficient to transmit at lower power for a longer period of time, the longer the transmission time, the more energy will be wasted due to leakage. The optimization problem in (15)-(16) becomes

$$\max_{p \geq 0} \mathcal{D}(E(t)) = \frac{E}{p+\epsilon} r(p). \quad (17)$$

Assuming that  $r(p)$  is a strictly concave increasing function with  $r(0) = 0$ , and a finite leakage rate  $\epsilon$ , the function  $f(p) \triangleq \frac{r(p)}{p+\epsilon}$  achieves its maximum at a finite  $p \in \mathbb{R}^+$ , as shown in Appendix A. We denote the corresponding optimal value by  $p^*$ . Note that while the total amount of transmitted data is proportional to  $E$ ,  $p^*$  is independent of  $E$ . Summarizing, the optimal transmission strategy for the  $S_\infty$  problem is to transmit at constant power  $p^*$  until the battery is empty. The total amount of transmitted data is  $\frac{E}{p^*+\epsilon} r(p^*)$ .

We next consider the single-packet problem with a fixed transmission deadline  $T$ , and denote it by  $S_T$ . It is depicted in Fig. 4, and the following notations are defined:  $\tilde{U}(t) \triangleq H(t) - \epsilon t$  (we assume  $\tilde{U}(t) > 0$  for all  $0 \leq t \leq T$ , as otherwise the problem is equivalent to the  $S_\infty$  problem). We denote the point  $(T, \tilde{U}(T))$  by  $A$ . Finally, the slope of the line segment from the origin to  $A$  is denoted by  $s$ . We have  $s = E/T - \epsilon$ . As before  $p^*$  denotes the value that maximizes the function  $f(p)$ . Note that, as shown in Appendix A,  $f(p)$  is strictly decreasing for  $p > p^*$ . Hence, building on the solution derived for the  $S_\infty$  problem, the solution of the  $S_T$  is easily derived:

- if  $s < p^*$ , transmit at constant power  $p^*$  until the battery is empty.
- else, transmit at constant power  $s$  during the whole  $[0, T]$  interval.

In short, the optimal transmission strategy for the  $S_T$  problem is to transmit at constant power  $\tilde{p} = \max(p^*, s)$  for a time duration  $\frac{E}{\tilde{p}+\epsilon}$  (that is, until the battery is empty), and remain silent afterwards. The amount of transmitted data is  $\frac{E}{\tilde{p}+\epsilon} r(\tilde{p})$ .

### B. The $N$ -Packet Problem

We consider here the general  $N$  packet problem with finite deadline constraint  $T$ , denoted as the  $N_T$  problem.

We start with the following lemma which proves that the optimal solution of the  $N_T$  problem can be emulated in the equivalent  $S_T$  problem with  $E = \sum_{n=1}^N E_n$ . That is, having all energy packets at time  $t = 0$  is at least as good as having

<sup>5</sup>Note that, in the case of energy leakage, potential transmit time is finite when the number of harvested energy packets is finite as the available energy decays to zero even if no data is transmitted.

them arrive over time. Let us denote by  $D_{N_T}$  and  $D_{S_T}$  the optimal solutions (in terms of total transmitted data) of the  $N_T$  and equivalent  $S_T$  problems, respectively.

*Lemma 5.1:* The optimal solution of the  $N_T$  problem can be obtained in the equivalent  $S_T$  problem with  $E = \sum_{n=1}^N E_n$ . That is, we have  $D_{S_T} \geq D_{N_T}$ .

*Proof:* Consider the optimal curve for the  $N_T$  problem, and divide the  $[0, T]$  time interval into  $N$  sub-intervals:  $[t_1, t_2]$ ,  $[t_2, t_3]$ ,  $\dots$ ,  $[t_n, t_{n+1}]$ ,  $\dots$ ,  $[t_N, T]$ . We denote by  $T_n$  the duration of the  $n^{\text{th}}$  interval, i.e.,  $T_n \triangleq t_{n+1} - t_n$  for  $n = 1, \dots, N-1$ , and  $T_N \triangleq T - t_N$ . From Theorem 3.3, we know that the optimal transmitted energy curve is a piecewise linear function, which is composed of constant power periods possibly separated by silent intervals (i.e. horizontal segments) in case the battery runs out of energy. Accordingly, we define the optimal solution of the  $N_T$  problem by the sequences  $\{\bar{p}_1, \bar{p}_2, \dots, \bar{p}_N\}$  and  $\{\bar{T}_1, \bar{T}_2, \dots, \bar{T}_N\}$ , meaning that the node transmits for time  $\bar{T}_n$  (with  $\bar{T}_n \leq T_n$ ) at power  $\bar{p}_n > 0$  in the  $n^{\text{th}}$  interval. The node is silent in the remainder of the interval, i.e., for time  $T_n - \bar{T}_n$ .

The data transmitted by this transmission strategy is  $D_{N_T} = \sum_{n=1}^N \bar{T}_n r(\bar{p}_n)$ . The total transmit energy is  $\sum_{n=1}^N \bar{T}_n \bar{p}_n$ , while the total energy leakage is  $\epsilon \sum_{n=1}^N \bar{T}_n$ . Since the optimal solution should eventually empty the battery, we have

$$\sum_{n=1}^N \bar{T}_n \bar{p}_n + \epsilon \sum_{n=1}^N \bar{T}_n = \sum_{n=1}^N E_n. \quad (18)$$

We now argue that this optimal solution can be emulated in the  $S_T$  problem with  $E = \sum_{n=1}^N E_n$ . Consider the following transmission strategy  $E(t)$  for the  $S_T$  problem: transmit at constant power equal to  $\bar{p}_1$  for time  $\bar{T}_1$ , followed by  $\bar{p}_2$  for time  $\bar{T}_2$ , and so on, ending with  $\bar{p}_N$  for time  $\bar{T}_N$ . By construction, this strategy transmits the same amount of data  $D_{N_T}$  as the optimal solution of the  $N_T$  problem. We conclude the proof by showing that this strategy is feasible, that is,  $E(t) \leq U(t)$  for all  $t \in [0, T]$ . Since the node is constantly transmitting during the interval  $[0, \sum_{n=1}^N \bar{T}_n]$ , the curve  $U(t)$  is constantly decreasing<sup>6</sup> during this interval at rate  $\epsilon$ , i.e.  $U(t) = \sum_{n=1}^N E_n - \epsilon t$ , for  $t \in [0, \sum_{n=1}^N \bar{T}_n]$ . We have

$$U\left(\sum_{n=1}^N \bar{T}_n\right) = \sum_{n=1}^N E_n - \epsilon \sum_{n=1}^N \bar{T}_n \quad (19)$$

$$= \sum_{n=1}^N \bar{T}_n \bar{p}_n \quad (20)$$

$$= E \left( \sum_{n=1}^N \bar{T}_n \right) \quad (21)$$

where the equality (20) follows from (18). This proves the feasibility of  $E(t)$ .

Having proved the achievability of  $D_{N_T}$  in the equivalent  $S_T$  problem, the inequality  $D_{S_T} \geq D_{N_T}$  naturally follows. ■

<sup>6</sup>We assume  $U(t) > 0$  in the considered time interval, as otherwise the problem can be divided into equivalent subproblems.

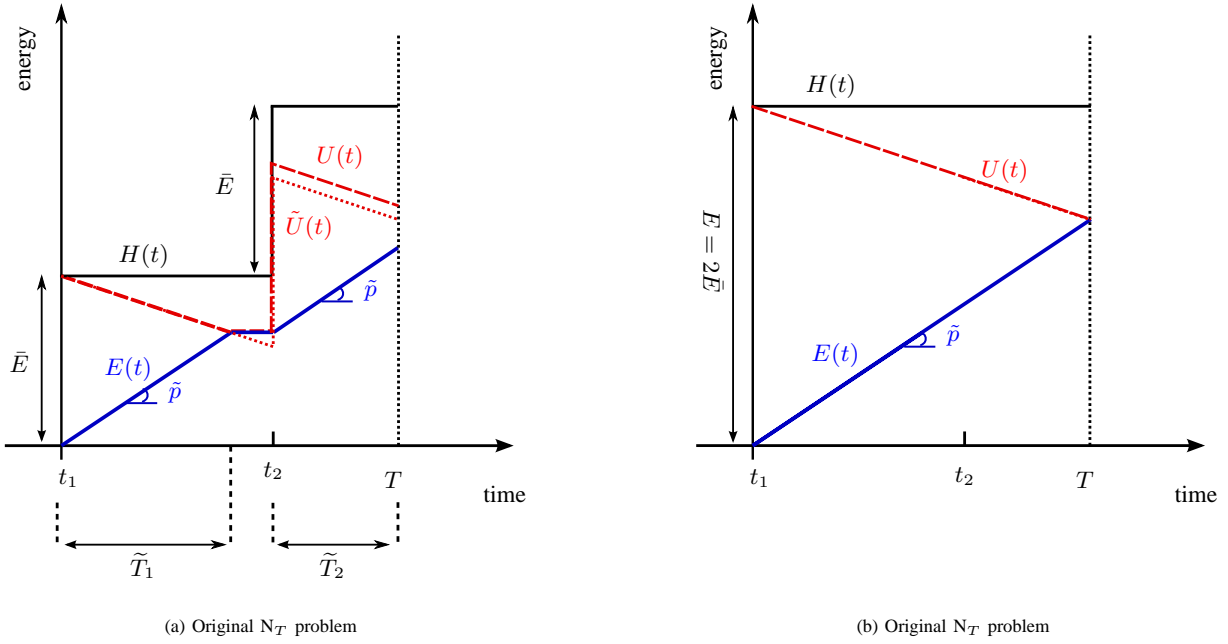


Fig. 5. Counterexample for the equivalence of the  $N_T$  and  $S_T$  problems, with  $N = 2$  energy packets.

The counterpart of Lemma 5.1 in the other direction does not always hold, that is, the optimal solution of the equivalent  $S_T$  problem cannot always be emulated in the original  $N_T$  problem. A counterexample can indeed easily be constructed, which is depicted in Fig. 5. Part (a) of the figure depicts a 2-packet problem with  $E_1 = E_2 = \bar{E}$ ,  $t_1 = 0$ , and  $t_2 > T/2$ . In part (b) the equivalent  $S_T$  problem is depicted. Let the optimal transmission power for the  $S_T$  problem be given by  $\tilde{p} = \frac{2\bar{E}}{T} - \epsilon$ . This solution cannot be emulated in the original  $N_T$  problem. In fact, as shown in part (a), the node cannot transmit a constant power  $\tilde{p}$  during the full  $[0, T]$  time interval as the battery runs out of energy at time  $T/2$ , and the node has to remain silent during the time interval  $[T/2, t_2]$ .

However, in the following lemma, we provide a sufficient condition for the counterpart of Lemma 5.1 to hold. For this, we define  $A_i$  as the point on the  $\tilde{U}(t)$  curve corresponding to the time instant  $t_{i+1}$ , for  $i = 1, 2, \dots, N-1$ , and  $A_N$  as the point corresponding to time  $t = T$ , as illustrated in Fig. 6.

**Lemma 5.2:** If the line segment from the origin to the point  $A_N$  does not cross the curve  $\tilde{U}(t)$  at any other point than  $\{A_1, \dots, A_N\}$ , then the optimal solution of the  $S_T$  problem with  $E = \sum_{n=1}^N E_n$  can be obtained in the  $N_T$  problem, and  $D_{N_T} \geq D_{S_T}$ . This sufficient condition is expressed by the following  $N-1$  inequalities:

$$\frac{\sum_{n=1}^i E_n}{\sum_{n=1}^i T_n} \geq \frac{\sum_{n=1}^N E_n}{T}, \quad i = 1, \dots, N-1. \quad (22)$$

*Proof:* First note that the set of inequalities in (22) expresses that the line segments from origin to points  $A_1, A_2, \dots, A_{N-1}$  have slope which are all greater than that of the segment from origin to  $A_N$ . This requires that the line segments from the points  $A_1, A_2, \dots, A_{N-1}$  to the point  $A_N$

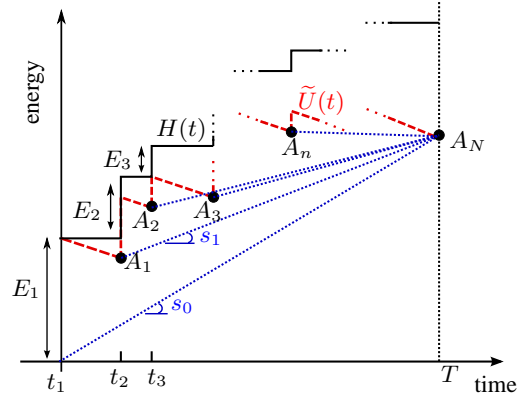


Fig. 6. A  $N_T$  problem satisfying the conditions of Lemma 5.2.

have slopes  $s_1, s_2, \dots, s_{N-1}$  respectively, which all are lower than or equal to the slope  $s_0 = \frac{\sum_{n=1}^N E_n}{T} - \epsilon$  of the segment from the origin to point  $A_N$ :

$$s_i \leq s_0 = \frac{\sum_{n=1}^N E_n}{T} - \epsilon \quad (23)$$

for  $i = 1, \dots, N-1$ . An illustration of an  $N_T$  problem satisfying the conditions of this lemma is given in Fig. 6.

Consider now the  $S_T$  problem with  $E = \sum_{n=1}^N E_n$ . Remember that the optimal scheme for the  $S_T$  problem requires transmitting at constant power  $\tilde{p}$  for a duration of  $\frac{E}{\tilde{p} + \epsilon}$ , with  $\tilde{p} = \max(p^*, s_0) \geq s_0$ . We now argue that this solution can be emulated in the  $N_T$  problem. Consider the following transmission strategy for the  $N_T$  problem: transmit at  $\tilde{p}$  whenever the battery is non-empty, and remain silent otherwise. By construction, this strategy is feasible. Again consider the  $N$  time intervals between energy arrivals  $[t_1, t_2]$ ,

$[t_2, t_3], \dots, [t_N, T]$  of durations  $T_1, T_2, \dots, T_N$ , respectively. We denote by  $\tilde{T}_n$  (with  $\tilde{T}_n \leq T_n$ ) the time for which the node is transmitting in the  $n^{\text{th}}$  interval. The total transmission time is then given by  $T_{tot} \triangleq \sum_{n=1}^N \tilde{T}_n$ . Moreover, combining the inequalities in (23) with the fact that  $\tilde{p} = \max(p^*, s_0) \geq s_0$ , we have that  $\tilde{p} \geq s_i$  for  $i = 1, \dots, N-1$ . This ensures that the considered strategy uses up the whole available energy by time  $T$ , i.e.,  $E(T) = U(T)$ . Then, by the conservation of energy, the transmit and leakage energies must sum to the total harvested energy:

$$T_{tot} \tilde{p} + \epsilon T_{tot} = \sum_{n=1}^N E_n = E, \quad (24)$$

from which we get that  $T_{tot} = \frac{E}{\tilde{p} + \epsilon}$ , just like for the optimal solution of the  $S_T$  problem. This transmission strategy thus transmits the same amount of data  $D_{S_T}$  as the optimal solution of the  $S_T$  problem. Consequently, under the conditions given in the theorem, the inequality  $D_{N_T} \geq D_{S_T}$  holds. ■

Building on the two previous lemmas, the following theorem can be formulated.

*Theorem 5.3:* If the inequalities in (22) hold, then:

- (i)  $D_{N_T} = D_{S_T}$ , that is, the optimal solutions of the  $N_T$  problem and the  $S_T$  problem with  $E = \sum_{n=1}^N E_n$  are equivalent.
- (ii) The optimal transmission strategy for the  $N_T$  problem is to transmit at constant power  $\tilde{p}$  whenever the battery is non-empty, and remain silent otherwise, where the value  $\tilde{p}$  corresponds to the solution of the equivalent  $S_T$  problem:

$$\tilde{p} = \max \left( p^*, \frac{\sum_{n=1}^N E_n}{T} - \epsilon \right) \quad (25)$$

The total amount of transmitted data is  $\left( \frac{\sum_{n=1}^N E_n}{\tilde{p} + \epsilon} \right) r(\tilde{p})$ .

An illustration of the result in Theorem 5.3 is provided in Fig. 7 for  $N = 3$  and  $\tilde{p} = p^* > s_0$ . Part (a) of the figure depicts the  $N_T$  problem, while its equivalent  $S_T$  problem is given in part (b). According to Theorem 5.3, for both problems the optimal strategy is to transmit at constant power  $p^*$ . The only particularity of the  $N_T$  problem is the presence of silent zones in between energy packet arrivals. However, the distribution over time of these silent zones do not affect the total duration of transmission, guaranteeing the equivalence of both solutions in terms of amount of transmitted data.

Now, building on Theorem 5.3, we can provide the optimal solution for any  $N_T$  problem. Consider all line segments connecting the origin to points  $A_i, i = 1, \dots, N$ . Among the segments that do not intersect  $\tilde{U}(t)$  other than at point  $\{A_1, \dots, A_N\}$ , we pick the one with the highest index, i.e., the rightmost end point. We denote this index by  $k$ . We can now consider the  $k$  first energy packets only, and solve the corresponding  $k$  packet problem with deadline  $\sum_{n=1}^k T_n$ , using the equivalence given in Theorem 5.3. We then proceed recursively by considering the remaining  $N-k$  packet problem separately. This recursive algorithm is described next. It takes

as inputs the number of packets  $N$ , the sizes of energy packets  $\{E_n\}_{n=1}^N$ , and the packet interarrival times  $\{T_n\}_{n=1}^N$ . It returns as output the set of optimal transmission powers  $\{\tilde{p}_n\}_{n=1}^N$ , meaning that the optimal solution of the  $N_T$  problem is to transmit at constant power  $\tilde{p}_n$  in the  $n^{\text{th}}$  interval as long as the battery is non-empty. The optimality of the algorithm is proved in Appendix B.

*Algorithm 5.1:*  $N_T$ -problem( $N, E_1, \dots, E_N, T_1, \dots, T_N$ )

**Input:**

- $N$ : number of energy packets
- $\{E_n\}_{n=1}^N$ : amount of energy in each packet
- $\{T_n\}_{n=1}^N$ : interarrival times

**Output:**  $\{\tilde{p}_n\}_{n=1}^N$

**Algorithm:**

- 1) Find the highest  $k \in \{1, \dots, N\}$  such that

$$\frac{\sum_{n=1}^i E_n}{\sum_{n=1}^i T_n} \geq \frac{\sum_{n=1}^k E_n}{\sum_{n=1}^k T_n} \quad (26)$$

for all  $i \in \{1, \dots, k-1\}$ .

- 2)

$$\tilde{p}_i = \max \left( p^*, \frac{\sum_{n=1}^k E_n}{\sum_{n=1}^k T_n} - \epsilon \right) \quad (27)$$

for all  $i \in \{1, \dots, k\}$ .

- 3) If  $k < N$ , find the  $\{\tilde{p}_n\}_{n=k+1}^N$  by running

$N_T$ -problem( $N-k, E_{k+1}, \dots, E_N, T_{k+1}, \dots, T_N$ )

We conclude this section by identifying two special cases of the solution provided here:

- The special case of an  $N$ -packet problem without deadline constraint can be solved by Algorithm 5.1 by setting  $T_N = \infty$ . In this case, the inequalities in (26) hold with  $k = N$ , and (27) reduces to  $\tilde{p}_i = \max(p^*, 0) = p^*$  for all  $i \in \{1, \dots, N\}$ . Hence, the optimal transmission strategy for the  $N$ -packet problem without deadline constraint is to transmit at constant power  $p^*$  whenever the battery is non-empty, and remain silent otherwise.
- The special case of a perfect battery with no leakage is obtained by setting  $\epsilon = 0$ . In this case,  $p^* = 0$  (as detailed in Appendix A), and Algorithm 5.1 reduces to the solution proposed in [11].

## VI. CONCLUSION

We have considered a communication system with an energy harvesting transmitter. Taking into account various constraints on the battery we have optimized the transmission scheme in order to maximize the amount of data transmitted within a given transmission deadline. We have provided a general framework extending the previous work in [11] and [12] to the model with continuous energy arrival as well as time-varying battery size constraints. We have also showed that the proposed framework applies to the optimization of energy harvesting broadcast systems. Moreover we have studied the case of a battery suffering from energy leakage, for which



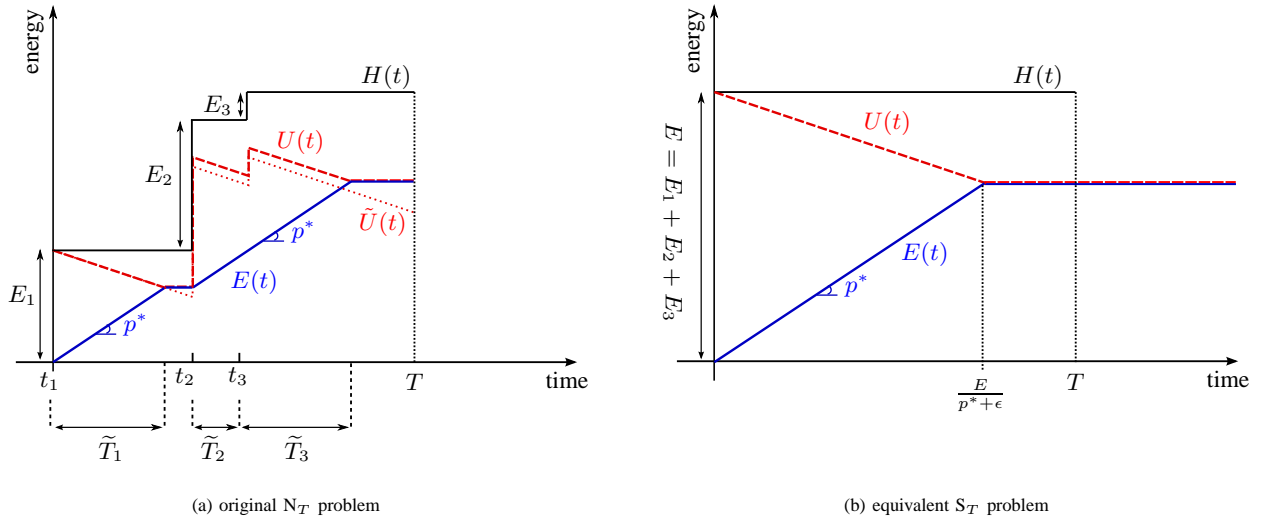


Fig. 7. Illustration of Theorem 5.3, with  $N = 3$  energy packets and  $\tilde{p} = p^*$ .

the optimal transmission scheme has been characterized for a constant leakage rate.

#### APPENDIX

##### A. Properties of $f(p) \triangleq \frac{r(p)}{p+\epsilon}$

Remember that  $r(p)$  is a non-negative strictly concave increasing function, with  $r(0) = 0$ . We prove here that the function  $f(p) \triangleq \frac{r(p)}{p+\epsilon}$ , with  $p \geq 0$ , achieves its maximum at a finite  $p^* \in \mathbb{R}^+$ , and is strictly decreasing for  $p > p^*$ .

The derivative of  $f(p)$  is calculated as follows:

$$f'(p) = \frac{r'(p)(p+\epsilon) - r(p)}{(p+\epsilon)^2} \quad (28)$$

We distinguish two cases:

1) If  $\epsilon = 0$ , (28) becomes

$$f'(p) = \frac{r'(p)p - r(p)}{p^2}, \quad (29)$$

which is analyzed as follows:

- if  $p = 0$ , both the numerator and the denominator are zero. By l'Hôpital's rule, we get  $\lim_{p \rightarrow 0} f'(p) = r''(0)/2 < 0$ , which follows from the strict concavity of  $r(p)$ .
- if  $p > 0$ , the numerator  $r'(p)p - r(p)$  is a strictly negative function. Indeed, the strict concavity of  $r(p)$  together with the fact that  $r(0) = 0$  guarantees that  $r(p) > r'(p)p$  for all  $p > 0$ .

Overall,  $f'(p)$  is thus strictly negative for all  $p \geq 0$ . Hence,  $f(p)$  finds its maximum at  $p^* = 0$ , and is strictly decreasing for  $p > 0$ .

2) Consider now  $\epsilon > 0$ . The sign of (28) is analyzed by focusing on its numerator only, which is rewritten for clarity as:

$$n(p) = r'(p)\epsilon + [r'(p)p - r(p)] \quad (30)$$

We analyze  $n(p)$  term by term:

- The first term  $r'(p)\epsilon$  is a positive and strictly decreasing function, due to the increasing and strictly concave property of  $r(p)$ , respectively.
- The term in between brackets  $r'(p)p - r(p)$  is equal to zero if  $p = 0$ , and a strictly negative (as shown above), strictly decreasing function for  $p > 0$ . Indeed, the strict concavity of  $r(p)$  guarantees that the derivative of this term  $r''(p)p$  is strictly negative for all  $p > 0$ .

Consequently, overall  $n(p)$  is a strictly decreasing function of  $p$  for  $p \geq 0$ . More precisely, the lower  $\epsilon$ , the more rapid the decrease of  $n(p)$  will be. The initial value at  $p = 0$  is positive and proportional to  $\epsilon$ :  $n(0) = r'(0)\epsilon \geq 0$ . On the other hand, the asymptotic value of  $n(p)$  as  $p \rightarrow \infty$  is negative:  $\lim_{p \rightarrow \infty} n(p) = \lim_{p \rightarrow \infty} [r'(p)p - r(p)] < 0$ , where the inequality follows from the strict concavity of  $r(p)$  together with the fact that  $r(0) = 0$ . Between these two extremes, the strict decrease of  $n(p)$  guarantees that  $f'(p)$  changes sign only once (from positive to negative) at some finite value denoted by  $p^*$ , and that it will remain strictly negative for all  $p > p^*$ . Hence, we have that:

- $f(p)$  has a unique maximum, which is achieved at some finite value of  $p \geq 0$ , denoted by  $p^*$ . The lower the value of  $\epsilon$ , the lower the value of  $p^*$  will be.
- $f(p)$  is strictly decreasing for  $p > p^*$ .

##### B. Proof of Optimality of Algorithm 5.1

If the inequalities in (26) hold with  $k = N$  (as in Fig. 6), the optimal solution provided in Theorem 5.3 is produced by Algorithm 5.1 in (27) with  $k = N$ .

Consider now that the inequalities in (26) do not hold for  $k = N$ . Then, denote by  $k$  the highest  $k < N$  for which (26) holds. This situation is depicted in Fig. 8. We first argue that the optimal solution is such that it empties the battery before receiving the  $(k+1)^{th}$  energy packet, i.e. before  $t_{k+1}$ . Put differently, the optimal transmitted energy curve should intersect  $U(t)$  at a time  $t \leq t_{k+1}$ . Assume that the opposite

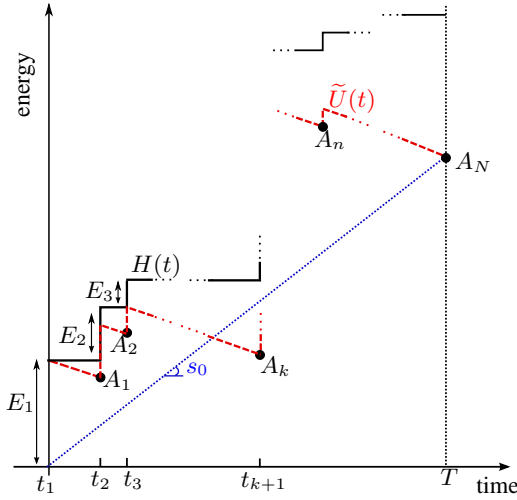


Fig. 8. General  $N_T$  problem.

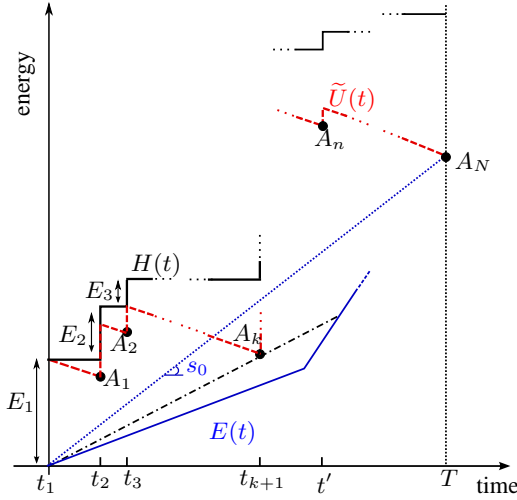


Fig. 9. General  $N_T$  problem: suboptimal solution.

holds, as depicted in Fig. 9. Then, at some time  $t' \geq t_{k+1}$ , the slope of the transmitted energy curve  $E(t)$  has to increase in order to guarantee to empty the battery at time  $t = T$  (which is a necessary condition for optimality). However, it is easy to realize (see the dot-dashed curve in Fig. 9) that such strategy is suboptimal since it violates Theorem 3.3. Note that the feasibility of the dot-dashed curve in Fig. 9 is ensured by considering the largest  $k$  rather than any  $k$  satisfying (26). Now, since the battery has to be emptied before receiving the  $(k+1)^{th}$  energy packet, we can optimally decouple the problem. First, the  $k$  packet problem with deadline  $t_{k+1} = \sum_{n=1}^k T_n$  is solved independently. This subproblem satisfies the inequalities in (22), such that Theorem 5.3 guarantees that its optimal solution is obtained by Algorithm 5.1 in (27). Then, proceeding recursively, the algorithm is run for the remaining  $N - k$  packet problem which can be considered as a new problem with an empty battery at the origin.

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