Distortion Exponent of Parallel Fading Channels

Deniz Gunduz, Elza Erkip
Department of Electrical and Computer Engineering,
Polytechnic University,
Brooklyn, NY 11201, USA
dgundu01@utopia.poly.edu, elza@poly.edu

***Abstract*— We consider the end-to-end distortion achieved by transmitting a continuous amplitude source over M parallel, independent quasi-static fading channels. We analyze the high SNR expected distortion behavior characterized by the distortion exponent. We first give an upper bound for the distortion exponent in terms of the bandwidth ratio between the channel and the source assuming the availability of the channel state information at the transmitter. Then we propose joint source-channel coding schemes based on layered source coding and multiple rate channel coding. We show that the upper bound is tight for large and small bandwidth ratios. For the rest, we provide the best known distortion exponents in the literature. By suitably scaling the bandwidth ratio, our results would also apply to block fading channels.**

I. Introduction

Transmitting continuous amplitude sources such as voice, video, sensor measurements, over randomly varying wireless channels is one of the key problems of wireless communications systems. In a point-to-point channel, when we are not constrained by complexity or delay, we can apply Shannon's source-channel separation theorem which states that there is no loss by separate design of source and channel coders. However, this is not the case for time-sensitive applications where the delay requirement is short compared to the channel coherence time, for example the quasi-static fading channel or the block fading channel with small number of blocks. In that case we need to take a joint source-channel approach and design the overall system by considering the source and channel coder parameters jointly. The performance measure we use in this paper is the end-to-end distortion.

We assume that M parallel channels are available between the transmitter and the receiver where each of these channels can be modeled as having independent quasi-static Rayleigh fading. We assume that the fading states are constant for a block of N channel uses and change independently from one block to another. The transmitter wishes to send K source samples in N channel uses over M parallel channels. We define the corresponding *bandwidth ratio* as $b = N/K$, and analyze the system performance with respect to b. We assume that K is large enough to consider the source as ergodic and N is large enough to design codes that can achieve all rates below the instantaneous channel capacity.

We assume that the receiver can track the channels perfectly while the transmitter only has access to the statistics of the fading process. Lack of channel state information at the transmitter prevents the design of a channel code that achieves the instantaneous channel capacity and requires a code that performs well on the average. We aim to design a joint source-channel code that achieves the minimum expected end-to-end distortion. Our main focus is the high SNR behavior of this expected distortion (ED). This behavior is characterized by the *distortion exponent* denoted by Δ and defined as

$$\Delta = -\lim_{SNR\to\infty} \frac{\log ED}{\log SNR}. \tag{1}$$

When we consider digital transmission strategies that first compress the source and then transmit over the channel at a specific channel rate, we need to decide the transmission rate that gives the best average performance. However, digital transmission in general suffers from the threshold effect, i.e., when the channel quality falls under a certain threshold, channel code can no longer recover the transmitted information reliably which leads to maximal distortion. Furthermore, digital transmission lacks the ability to utilize the increased channel quality when it is above the target threshold.

In order to overcome the threshold effect and achieve a graceful degradation with varying channel quality, we propose layered source coding concatenated with multi-rate channel coding. Using successive refinement of the source, we divide the source code into layers and transmit each layer over the channel at a different rate. We allocate system resources among the layers such that the reliability of transmission over the channel decreases from the first layer to the last, hence increasing the reliability of the first layers at the expense of the succeeding layers. We argue that, this strategy improves the average distortion performance due to the exponential decay of the distortion-rate function in general.

We consider two different channel coding schemes. In the first scheme, which we call layered source with progressive coding (LS), we divide the channel block of N channel uses among the layers and transmit them progressively in time within their respective portions at full power. In the second scheme, we allocate transmit power among the layers and transmit a superposition of the codewords of all layers simultaneously spanning the whole N channel uses. We name this scheme as broadcast strategy with layered source (BS).

Another possible strategy that would result in graceful degradation is hybrid digital-analog transmission [1], where an analog (uncoded) portion is transmitted together with the

[1]This work is partially supported by NSF Grant No. 0430885.

digital (coded) portion. We consider this analog approach together with the LS scheme and hence call it hybrid-LS (HLS).

In [2], we analyzed the single quasi-static fading channel case and showed that BS scheme can achieve the optimal performance in the limit of infinite layers. We also showed that pure analog transmission is optimal in the distortion exponent sense when the bandwidth ratio is greater than 1, i.e., the channel bandwidth is greater than the source bandwidth. However, pure analog transmission results in $\Delta = 0$ when $b < 1$. Later in [3], authors showed that a hybrid digital-analog scheme can achieve the optimal performance for $b < 1$ case using only a single digital layer superimposed with analog transmission.

In this paper, we first find an upper bound to the distortion exponent for parallel channels. We then characterize the distortion exponents achieved by LS and HLS schemes. For the BS strategy we show that a specific resource allocation policy we use is optimal for bandwidth ratios greater than the number of channels. Comparison of our results with single rate coding, single-layer hybrid scheme of [3] and hybrid scheme of [7] for 2-parallel channels and pure analog transmission is given in Section VIII. Distortion exponent analysis for block fading MIMO systems can be found in [5], [6]. In a concurrent work [4], authors have also extended their hybrid scheme to parallel channels.

II. System Model

We consider an analog source denoted by $\mathbf{s}$. For the analysis, we focus on a memoryless, complex Gaussian source with independent real and imaginary components each with variance 1/2. Generalizations to other memoryless sources follows as discussed in [6]. The distortion-rate function for the complex Gaussian source with unit variance is $D(R) = 2^{-R}$. Here we use compression strategies that meet the distortion-rate bound.

We assume the availability of M parallel independent quasi-static Rayleigh fading channels. The channel model is

$$\mathbf{Y} = \sqrt{\frac{SNR}{M}}\mathbf{H}\mathbf{X} + \mathbf{Z}, \tag{2}$$

where $\mathbf{X} \in \mathcal{C}^{M\times N}$ is the transmitted codeword, $\mathbf{Z} \in \mathcal{C}^{M\times N}$ is the complex Gaussian noise with i.i.d entries $\mathcal{CN}(0,1)$, and $\mathbf{H} \in \mathcal{C}^{M\times M}$ is the diagonal channel matrix which has i.i.d. entries with $\mathcal{CN}(0,1)$. This models a channel which is constant over a block of N channel uses while independent from block to block. $\mathbf{H}$ is assumed to be known by the receiver and unknown by the transmitter. The transmitted codeword is normalized in power so that it satisfies $tr(E[\mathbf{X}^\dagger\mathbf{X}]) \leq MN$.

The decoder maps the received output of each block $\mathbf{Y}$ to an estimate $\hat{\mathbf{s}} \in \mathcal{C}^K$ of the source. Average distortion $ED(SNR)$ is defined as the average mean squared error between $\mathbf{s}$ and $\hat{\mathbf{s}}$ where the expectation is taken with respect to $\mathbf{s}$, $\mathbf{H}$ and $\mathbf{Z}$. Note that this average distortion is due to both the lossy compression of the source and errors that occur over the channel.

Since we are interested in the high SNR regime, we use outage probability, P_{out}, instead of the channel error probability as it forms a tight lower bound and has the same exponential behavior [8]. For a family of codes with rate $R = r\log SNR$, r is defined as the multiplexing gain of the family, and

$$d(r) = -\lim_{SNR\to\infty}\frac{\log P_{out}(SNR)}{\log SNR} \tag{3}$$

as the diversity advantage. The diversity gain $d^*(r)$ is defined as the supremum of the diversity advantage over all possible code families with multiplexing gain r. The tradeoff between multiplexing and diversity gains for the parallel channel model is characterized as follows.

Theorem 1: ([9]) Consider M-parallel channels with i.i.d. Rayleigh coefficients. The optimal diversity- multiplexing gain tradeoff curve $d^*(r)$ for $0 \leq r \leq M$ is given by $d^*(r) = M - r$.

III. Distortion Exponent Upper Bound

In this section we find an upper bound for the distortion exponent. In the general case there is no channel state information at the transmitter, and due to the delay limitations imposed by the application, it is not possible to transmit long codewords to average out the effects of fading. Hence Shannon's source-channel separation theorem is not applicable here. However, if we assume that the instantaneous channel state information is available to the transmitter, then source-channel separation applies to each block of transmission, and the minimum source distortion at the receiver is obtained by transmitting at the instantaneous channel capacity. We would have no outage in this case, and the distortion at the receiver would be due to the lossy compression of the source only. The average of this distortion over the channel state distribution obviously constitutes an upper bound to the minimum average end-to-end distortion we are interested in.

Consider the channel matrix $\mathbf{H}$ in Section II where $\mathbf{H}\mathbf{H}^\dagger \in \mathbb{R}^{M\times M}$ is a diagonal matrix with entries $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_M$. For the instantaneous channel capacity $C(\mathbf{H})$, we can write

$$\begin{aligned} C(\mathbf{H}) &= \sup_{\mathbf{Q}\succeq 0, tr(\mathbf{Q})\leq M} \log\det\left(\mathbf{I} + \frac{SNR}{M}\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\right), \\ &\leq \log\det\left(\mathbf{I} + SNR\mathbf{H}\mathbf{H}^\dagger\right), \\ &= \log\prod_{i=1}^{M}(1+SNR\mu_i). \end{aligned} \tag{4}$$

Let $\alpha_i = -\ln\mu_i / lnSNR$. Then we can bound the minimum distortion that can be achieved transmitting at the instantaneous channel capacity as

$$\begin{aligned} D(\mathbf{H}) &= D\big(bC(\mathbf{H})\big), \\ &\geq \left[\prod_{i=1}^{M}(1+SNR^{1-\alpha_i})\right]^{-b}. \end{aligned} \tag{5}$$

We consider $E[D(\mathbf{H})]$, where the expectation is taken over all channel realizations and analyze its high SNR exponent to find the corresponding distortion exponent upper bound. The result is expressed in the following theorem whose proof can be found in the Appendix.

Theorem 2: For transmission of memoryless, complex Gaussian source over M independent quasi-static Rayleigh fading channels, the distortion exponent is upper bounded by

$$\Delta = \begin{cases} bM & \text{if } b < 1, \\ M & \text{if } b \geq 1. \end{cases} \quad (6)$$

IV. Layered Source with Progressive Transmission (LS)

The main idea here is to do source coding in layers, where each layer is a refinement of the previous ones, and to transmit layers successively in time over M parallel channels using codes at different rates. For n layers, each layer is transmitted at rate R_i bits per channel use in $t_i N$ channel uses for $i = 1, 2, \ldots, n$. Transmission power is kept constant across layers. Each layer utilizes all M parallel channels. This rate allocation corresponds to source coding rates of bt_iR_i bits per sample. The ith layer is composed of the successive refinement bits for the previous $i-1$ layers. This enables the receiver to get as many layers as it can depending on the current fading state.

Let P_{out}^R denote the outage probability when the transmission rate is R, i.e., $P_{out}^R = Pr\{C(\mathbf{H}) > R\}$. The distortion achieved when i layers are successfully decoded is found as

$$D_i^{LS} = D\left(b\sum_{k=1}^{i} t_k R_k\right), \quad (7)$$

with $D_0^{LS} = 1$. Due to successive refinement source coding, a layer is useless unless all the preceding layers are received successfully. This imposes a non-decreasing rate allocation among the layers, i.e., $R_i \leq R_j$ for $i < j$. Then the expected distortion (ED) for such a rate allocation can be written as

$$ED(\mathbf{R}, \mathbf{t}, SNR) \doteq \sum_{i=0}^{n} D_i^{LS} \cdot \left(P_{out}^{R_{i+1}} - P_{out}^{R_i}\right), \quad (8)$$

where we define $P_{out}^{R_0} = 0$ and $P_{out}^{R_{n+1}} = 1$ and $\doteq$ is used for exponential equality as defined in [8].

As can be seen from the expected distortion expression, there is a tradeoff between the outage probability and the distortion of the corresponding layer. There exists an optimal rate vector $\mathbf{R} = [R_1, \ldots, R_n]^T$ and a time allocation vector $\mathbf{t}$ which result in the lowest average distortion for any specific SNR. In the high SNR regime, to get the optimal distortion exponent we need $\mathbf{R} = \mathbf{r} \log SNR$, where $\mathbf{r} = [r_1, \ldots, r_n]^T$ is the multiplexing gain vector.

It is possible to prove that the more layers we have, the higher the distortion exponent is. For the highest possible distortion exponent achievable by this strategy we consider the limit of infinite layers. In the limit, we can prove that equal channel allocation among the layers gives us the best asymptotic performance, i.e., we take $t_i = 1/n \ \forall i$. Then the following set of equations are the necessary conditions for the optimal multiplexing gain vector. Note that these equations make SNR exponents equal in all the terms of Eqn. (8).

$$\frac{b}{n} r_n = d^*(r_n), \quad (9)$$

$$d^*(r_n) + \frac{b}{n} r_{n-1} = d^*(r_{n-1}), \quad (10)$$

$$\cdots$$

$$d^*(r_2) + \frac{b}{n} r_1 = d^*(r_1), \quad (11)$$

where $\Delta = d^*(r_1)$. This corresponds to each layer operating at a different point on the diversity-multiplexing tradeoff of Theorem 1. Solving these equations for infinite layers, we find

$$\Delta = M\big(1 - e^{-b}\big). \quad (12)$$

The distortion exponent achieved by LS with single and infinite layers with respect to bandwidth ratio for 2-parallel channels can be seen in Fig. 1. The distortion exponent of single layer ($n = 1$) case is found to be $\Delta = 2b/(b+1)$ and is also included in the figure for comparison.

V. Hybrid Digital-Analog Transmission with Layered Source

A natural compromise between digital and analog transmission is using an hybrid scheme, which can better adapt to various bandwidth ratios due to the digital portion, and has a graceful degradation due to the analog portion [1]. We combine our LS scheme with uncoded (analog) transmission and call it hybrid-LS (HLS). We consider two cases depending on the bandwidth ratio.

When $b > 1/M$, the channel bandwidth is large enough to transmit an analog signal corresponding to each source sample. We allocate K/M channel uses for analog transmission. We divide the rest of the $N - K/M$ channel uses to transmit digital source layers progressively as in LS scheme.

Let $\bar{\mathbf{s}} \in \mathcal{C}^K$ be the reconstruction of the source $\mathbf{s}$ upon successful reception of all layers of LS scheme. We denote the reconstruction error as $\mathbf{e} \in \mathcal{C}^K$ where $\mathbf{e} = \mathbf{s} - \bar{\mathbf{s}}$. We map this error to the M parallel channels where each component of the error vector is transmitted in an analog fashion. Receiver first tries to decode all the digital layers, and in case of successful reception of all the layers, it forms the estimate $\hat{\mathbf{s}} = \bar{\mathbf{s}} + \tilde{\mathbf{e}}$, where $\tilde{\mathbf{e}}$ is the linear MMSE estimate of $\mathbf{e}$ based on the received signal during the K/M channel uses reserved for analog transmission. This analog portion is neglected unless all digitally transmitted layers can be decoded at the destination.

For n layers, effect of the analog portion on the analysis in Eqn.(9-11) is to change the slopes of the curves from $\frac{b}{n}$ to $(b - \frac{1}{M})\frac{1}{n}$, and replace the first equation with $1 + \frac{b}{n} r_n = d^*(r_n)$. We find the following distortion exponent for $b > 1/M$ in the limit of infinite layers as

$$\Delta = 1 + (M-1)\left(1 - \exp(\frac{1}{M} - b)\right). \quad (13)$$

For $n = 1$ this strategy boils down to the hybrid scheme of [3], whose performance can also be seen in Figure 1 together with infinite layer case.

For the $b \leq 1/M$ case, we use the hybrid scheme introduced in [3], where MN source samples are transmitted uncoded in N channel uses, and the rest $K - MN$ samples are compressed, channel coded at rate R, and superimposed on the uncoded signals. Analysis of this scheme for M parallel channels results in the following distortion exponent ([6]).

$$\Delta = bM. \tag{14}$$

Comparing this with the upper bound given by (6), we see that HLS scheme is optimal for $b \leq 1/M$. However, the region of optimality of the distortion exponent decreases with increasing number of parallel channels.

VI. Broadcast Strategy with Layered Source (BS)

We combine the broadcast strategy of [10] with layered source coding and call it 'broadcast strategy with layered source' (BS). Similar to LS, information is sent in layers using successive refinability. The codes corresponding to different layers are superimposed, assigned different power levels and sent throughout the whole transmission block over all M channels. We consider successive decoding at the receiver, where the layers are decoded in order from 1 to n although this may be suboptimal in terms of end-to-end distortion.

As in LS we need to scale the transmission rate as $r \log SNR$. The distortion achieved by BS when first i layers are received is

$$D_i^{BS} = D\left(b \sum_{i=1}^{k} R_i\right).$$

We allocate power among layers such that the received SNR of layer i is SNR_i and define $\overline{SNR}_i = \sum_{j=i}^{n} SNR_j$ for $i = 1, \dots, n$ with $\overline{SNR}_1 = SNR$. Then for this power allocation we define the following outage events which correspond to outage when decoding the ith layer after decoding and subtracting the first $i-1$ layers.

$$\mathcal{A}_i = \left\{ \mathbf{H} : \log \frac{|\mathbf{I} + \overline{SNR}_i \mathbf{H}\mathbf{H}^\dagger|}{|\mathbf{I} + \overline{SNR}_{i+1} \mathbf{H}\mathbf{H}^\dagger|} < r_i \log SNR \right\}. \tag{15}$$

Let $P_{out}^k = Pr\{\mathbf{H} : \mathbf{H} \in \mathcal{A}_k\}$. For successive decoding, the overall outage event of layer i, denoted by $\mathcal{B}_i$ can be written in terms of $\mathcal{A}_i$'s as below.

$$\mathcal{B}_i = \bigcup_{j=1}^{i} \mathcal{A}_j, \text{ for } i = 1, \dots, n. \tag{16}$$

Although it might not be optimal, for tractable expected distortion expression in the high SNR limit, we constrain the multiplexing gains as $r_1 + \cdots + r_n < 1$ and let the power allocation be $\overline{SNR}_i = SNR^{1-(r_1+\ldots+r_{i-1}+\epsilon_{i-1})}$ with $0 < \epsilon_1 < \cdots < \epsilon_{n-1}$ for $i = 2, \dots, n$.

Lemma 1: With the above multiplexing gain and power allocation among the layers, for $k = 1, \dots, n$ we have $P_{out}^k \doteq SNR^{-d_k}$, where

$$d_k = \inf_{\boldsymbol{\alpha} \in \tilde{A}_k} \sum_{i=1}^{M} \alpha_i, \tag{17}$$

and

$$\begin{aligned} \tilde{A}_k = \Big\{ & \boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_n] : \alpha_1 \geq \cdots \geq \alpha_M \geq 0, \\ & \sum_{i}^{M} (1 - r_1 - \cdots - r_{k-1} - \epsilon_{k-1} - \alpha_i)^+ \\ & - \sum_{i}^{M} (1 - r_1 - \cdots - r_k - \epsilon_k - \alpha_i)^+ < r_k \Big\}. \end{aligned}$$

Proof: The proof follows along the lines of the proof of Theorem 4 in [8]. ■

Note that the constraint on the multiplexing gain allocation makes the sequence $\{1 - r_1 - \cdots - r_k\}_{k=1}^{n}$ decreasing and bounded above zero. Thus the outage probabilities of the layers P_{out}^k constitute an increasing sequence which in the high SNR regime leads to $Pr\{\mathbf{H} : \mathbf{H} \in \mathcal{B}_k\} \doteq P_{out}^k$.

The minimizing $\tilde{\boldsymbol{\alpha}}$ of Eqn. (17) for each layer can be explicitly found as

$$\tilde{\alpha}_i = 1 - r_1 - \cdots - r_{k-1} - \frac{r_i}{M} - \epsilon_k, \text{ for } i = 1, \dots, M,$$

This results in the following diversity gains as $\epsilon_k \to 0$.

$$d_k = M(1 - r_1 - \cdots - r_{k-1}) - r_k. \tag{18}$$

High SNR approximation for the expected distortion expression can be written as follows.

$$ED \doteq \sum_{k=0}^{n} D_k^{BS} P_{out}^{k+1}, \tag{19}$$

where $P_{out}^{n+1} = 1$, and $D_0^{BS} = 1$. We now find the maximum distortion exponent for the above expected distortion expression by properly choosing the multiplexing gains. For $b \leq M-1$ we have $\Delta = b$ for $r_1 + \cdots + r_n = 1$. For $b > M-1$ the optimal multiplexing gains can be found as below.

$$r_1 = \frac{M(b-M)}{b(b-M+1)^n - M}, \tag{20}$$

$$r_i = (b-M+1)^{i-1} r_{i-1}, \text{ for } i = 2, \dots, n. \tag{21}$$

Here we note that it is possible to prove these multiplexing gains satisfy $r_1 + r_2 + \cdots + r_n \leq 1$ for any bandwidth ratio $b > M-1$. The above multiplexing gain assignment makes SNR exponents of all the terms equal which, in the end, gives us the distortion exponent we are looking for. We find

$$\Delta = M - \frac{M(b-M)}{b(b-M+1)^n - M}. \tag{22}$$

In the limit as $n \to \infty$, the overall distortion exponent is

$$\Delta = \begin{cases} M & \text{if } b > M, \\ b & \text{if } b \leq M. \end{cases} \tag{23}$$

Distortion exponent vs. bandwidth ratio results of BS for 2 parallel channels are illustrated in Fig. 1 along with the previous results.

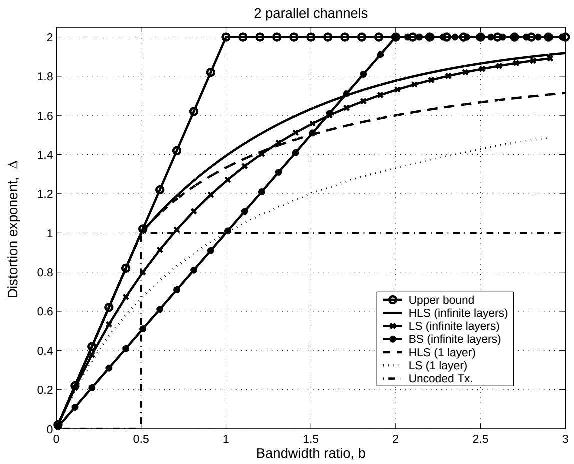

Fig. 1. Distortion exponent vs. bandwidth ratio for the 2 parallel fading channels.

VII. Block Fading Channel

The results we obtained for M parallel channels can be equivalently used for multiple block fading within a single channel. Consider the channel model with a single link between the transmitter and the receiver, which experiences M independent fading blocks in N channel uses. Now if we scale the bandwidth ratio of this channel with M, we obtain the same model as in the parallel channel case. Thus, by scaling the bandwidth ratio by $1/M$ in the M-parallel channel results, we obtain the corresponding distortion exponents for M block fading channel.

VIII. Discussion and Conclusion

In Fig. 1 we present the distortion exponent vs. bandwidth ratio for the upper bound, BS, LS and HLS schemes all in the infinite layer limit, together with single layer LS, HLS and pure analog transmission.

As seen from our results and Fig. 1, BS scheme is distortion exponent optimal when bandwidth ratio is larger than the number of parallel channels ($b \geq M$). For smaller bandwidth ratios, its performance degrades rapidly and falls below both LS and HLS schemes. HLS dominates LS for all values of bandwidth ratio, but the advantage of HLS compared to LS vanishes with increasing bandwidth ratio. We can also observe that the single layer HLS scheme performs worse than infinite layer LS for a wide range of bandwidth ratios proving the importance of layering. The performance of the hybrid scheme proposed in [7] is equivalent to our LS with one layer as the digital layer constrained to only one of the parallel channels. We also included pure analog transmission to show that it is not able to utilize the diversity of the system. The distortion exponent of analog transmission is limited to 1 no matter how many parallel channels we have.

Although we have a gap between the upper bound and the best achievable performance for $1/M < b < M$, it is not known whether this upper bound is tight for these bandwidth ratios.

Appendix
Proof of Theorem 2

We can write the expected end-to-end distortion as

$$E[D(\mathbf{H})] \;=\; \int D(\mathbf{H}) p(\alpha_1, \ldots, \alpha_n) d\alpha_1, \ldots, d\alpha_n. \quad (24)$$

The joint pdf of μ_i's is

$$p(\mu_1, \ldots, \mu_n) = \prod_{i=1}^{M} e^{-\mu_i}, \mu_i \geq 0 \text{ for } i = 1, \ldots, n. \quad (25)$$

Then the joint pdf of α_i's can be found as

$$p(\alpha_1, \ldots, \alpha_n) \;=\; (-1)^M (\ln SNR)^M \prod_{i=1}^{M} SNR^{-\alpha_i} \cdot \exp\left[-\sum_{i=1}^{M} SNR^{-\alpha_i} \right]. \quad (26)$$

Since we are interested in the distortion exponent in the high SNR regime, we can simplify the expected distortion expression as below.

$$E[D(\mathbf{H})] \;\doteq\; \int_{\mathcal{R}^{n+}} \prod_{i=1}^{M} SNR^{-b(1-\alpha_i)^+} \prod_{i=1}^{M} SNR^{-\alpha_i} d\boldsymbol{\alpha}.$$
$$\doteq\; \int_{\mathcal{R}_{n+}} SNR^{-\left(\sum_{i=1}^{M} \alpha_i + b(1-\alpha_i)^+\right)} d\boldsymbol{\alpha},$$

where $\mathcal{R}_{n+}$ is the non-negative orthant of the n dimensional space. From the proof of Theorem 4 in [8], we have $E[D(\mathbf{H})] \doteq SNR^{-\Delta}$, where

$$\Delta \;=\; \inf_{\boldsymbol{\alpha} \in \mathcal{R}^{n+}} \sum_{i=1}^{M} \alpha_i + b(1-\alpha_i)^+. \quad (27)$$

The minimizing $\tilde{\boldsymbol{\alpha}}$ can be found as

$$\tilde{\alpha}_1 = \cdots = \tilde{\alpha}_M = \begin{cases} 0 & \text{if} \quad b < 1 \\ 1 & \text{if} \quad b \geq 1. \end{cases} \quad (28)$$

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