Gaussian Joint Source-Channel Coding for the Strong Interference Channel

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***Abstract*—Transmission of correlated Gaussian sources over a Gaussian interference channel is studied. Each terminal has one source available, which has to be reconstructed at the corresponding destination with the minimum average distortion. Focusing on the strong interference setting, we first derive necessary conditions on the achievable distortion pairs. Then, focusing on the symmetric scenario, we present achievable distortion pairs considering several transmission strategies. We compare the achievable distortion by the proposed schemes with the lower bound. In particular, we consider separate source and channel coding, uncoded transmission, a vector quantization (VQ) scheme which uses the quantization codewords as channel inputs, and finally a superposition of two quantization codewords. We show that the VQ scheme is optimal in the high SNR regime. We also show that the proposed superposition scheme performs very close to the lower bound in certain regimes.**

I. Introduction

Many emerging wireless applications, such as multimedia streaming and Internet of things (IoT), require lossy transmission of source signals over noisy channels. With the increasing number of wireless terminals, signals at different nodes typically exhibit statistical correlation due to the nature of the underlying random processes; e.g., video signals recorded by cameras observing the same scene, or measurements at nearby sensors. Lossy transmission of source signals over a noisy channel is a joint source-channel coding (JSCC) problem. Shannon's separation theorem reduces it, in point-to-point scenarios, to separate rate-distortion and channel encoding problems, without loss of optimality. Nevertheless, in many multi-terminal problems, the optimality of separation breaks down, since JSCC can exploit source correlation to generate correlated channel inputs despite the distributed nature of the encoders, potentially improving the overall performance [1]–[5].

We consider the JSCC problem in which two correlated Gaussian sources, S_1 and S_2, are available at two separate terminals, which transmit these observations to their destinations over a Gaussian interference channel (IC). Receiver i, $i=1,2$, is interested in reconstructing source S_i with the minimum average distortion (see Fig. 1). Interference is typically considered as an impairment, and communication systems are designed to minimize and combat interference. On the other hand, in our model, due to the correlation among the sources, the problem is significantly more intriguing. The interference signal may carry useful information for the unintended receiver, and at the same time, the underlying correlation may allow new techniques to combat interference. In particular, we will consider the *JSCC problem for the Gaussian strong IC*, in which the interference is stronger than the direct channel signal, i.e., coefficients in Fig. 1 satisfy $c_i^2 \geq 1$, $i = 1, 2$.

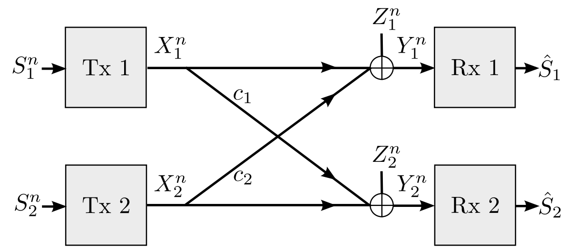

Fig. 1. Gaussian interference joint source-channel coding problem.

The JSCC IC problem has previously been studied in the lossless setup in [3], [6]–[8], showing the suboptimality of separation, and characterizing the sufficient and necessary conditions for reliable transmission in certain cases. Achievable schemes for lossy transmission over the IC based on JSCC are considered in [2] and [4]. The strong IC is also considered in [4] and separation is shown to be suboptimal in this regime. However, the characterization of the optimal achievable distortion region remains open in general.

The capacity of the IC is a longstanding open problem. It has been fully characterized only in some special cases; in [9] for Gaussian strong ICs, and in [10] for discrete memoryless ICs. The main idea in [9] and [10] is that the interference caused at the unintended terminal is strong enough to allow each receiver to decode the unintended message without loss of optimality. Thus, the capacity region reduces to the intersection of the capacity regions of two multiple access channels (MACs). However, the JSCC problem is significantly harder, and the set of achievable distortion pairs remain open even for the strong ICs. We will see that, since the encoders transmit correlated sources, techniques employed to characterize the capacity of the Gaussian strong IC cannot be applied here.

In this paper we first derive an outer bound on the achievable distortion pairs in a strong Gaussian IC, which is significantly tighter than the cut-set bound. Then, we consider achievable schemes based on separate source and channel coding (SSCC) and uncoded transmission. We also consider three JSCC schemes based on hybrid coding [2]. First, we consider a scheme in which each source is optimally quantized at the corresponding transmitter, and the quantization codewords are scaled and used as channel inputs, same as the vector quantizer (VQ) scheme proposed for the MAC in [1]. We show that, similarly to the MAC scenario, this scheme achieves the optimal distortion pairs in the high SNR regime. Then, we consider a generalized VQ scheme in which an uncoded layer is superposed on the quantized codeword. Finally, we consider a novel VQ strategy in which two VQ codewords are superposed, and we numerically show that, this superposition scheme significantly outperforms the previous schemes and approaches the proposed outer bound in certain regimes.

II. System Model

Consider a length-n sequence of independent identically distributed (i.i.d.) zero mean bivariate Gaussian source $\{S_{1k}, S_{2k}\}_{k=1}^n$ with a covariance matrix

$$\mathbf{K}_{S_1,S_2} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}. \quad (1)$$

Without loss of generality, we assume that the sources have equal variances, i.e., $\sigma_1^2 = \sigma_2^2 = \sigma^2$, and $\rho \in [0,1]$, as one of the transmitters can always multiply its source by -1 if $\rho < 0$.

Transmitter i observes the i-th source sequence and encodes it with function $f_i^n : \mathbb{R}^n \to \mathbb{R}^n$ such that $X_i^n = f_i^n(S_i^n)$ for $i = 1,2$. The corresponding channel input vectors X_i^n are subject to individual average power constraints

$$\mathrm{E}[|X_i^n|^2] = \frac{1}{n}\sum_{k=1}^n \mathrm{E}[|X_{ik}|^2] \le P_i, \quad i = 1,2. \quad (2)$$

The additive memoryless IC is characterized by

$$Y_{1k} = X_{1k} + c_2 X_{2k} + Z_{1k}, \quad (3)$$
$$Y_{2k} = c_1 X_{1k} + X_{2k} + Z_{2k}, \quad (4)$$

where $c_i^2 \ge 1$, $i = 1,2$, and Z_{ik} is the i.i.d. zero-mean Gaussian noise term at the i-th terminal with variance N. The decoding function at the i-th receiver, $\phi_i^n : \mathbb{R}^n \to \mathbb{R}^n$, for $i = 1,2$, estimates S_i^n, i.e., $\hat{S}_i^n = \phi_i^n(Y_i^n)$.

Given $(\sigma^2, \rho, P_1, P_2, N)$, we say that an average distortion pair (D_1, D_2) is *achievable* if there exists a sequence $\{f_1^n, f_2^n, \phi_1^n, \phi_2^n\}$ satisfying the power constraints in (2), and achieve a mean square-error distortion of

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^n \mathrm{E}[(S_{ik} - \hat{S}_{ik})^2] \le D_i, \quad i = 1,2. \quad (5)$$

A *symmetric strong IC* (SS-IC) refers to the strong IC problem with $D_1 = D_2 = D$, $P_1 = P_2 = P$ and $c_1 = c_2 = c$. We define $\log^+(\cdot) \triangleq \max\{0, \log(\cdot)\}$ and $i^c = \{1,2\}\backslash i$.

III. Two outer bounds for the strong IC

In this section we derive two outer bounds on the region of achievable distortion pairs (D_1, D_2) for the strong IC. Our first bound is based on the idea that, unlike in the channel coding problem with independent messages, when the two sources S_1^n and S_2^n are correlated, the encoders can generate correlated channel inputs exploiting the available correlation among the source samples observed at different transmitters. This brings an additional degree-of-freedom to the system, which potentially improves the performance. However, the amount of correlation that can be created is limited by the source correlation [1]. We use this fact, together with the cut-set bound arguments to obtain the following set of necessary conditions on the achievable distortion pairs. This bound is tighter than the one in [4].

Lemma 1. *The achievable distortion pairs for a Gaussian IC are included in the region formed by the pairs (D_1, D_2) satisfying, for some $0 \le \rho_x \le \rho$, and $i = 1,2$,*

$$R_{S_i}(D_i) \le \frac{1}{2}\log\left((P_i + c_{i^c}^2 P_{i^c} + 2c_{i^c}\rho_x\sqrt{P_1P_2} + N)/N\right),$$
$$R_{S_i|S_{i^c}}(D_i) \le \frac{1}{2}\log((P_i(1-\rho_x^2) + N)/N),$$

where $R_{S_i|S_{i^c}}(D_i) \triangleq 1/2\log^+\left(\sigma^2(1-\rho^2)/D_i\right)$ is the Wyner-Ziv rate-distortion function for S_i when S_{i^c} is available at the receiver, and $R_{S_i}(D_i) \triangleq 1/2\log^+\left(\sigma^2/D_i\right)$, is the classical rate-distortion function.

Next, we derive a tighter outer bound. We highlight the difficulty in characterizing the optimal set of achievable distortion pairs even in the case of strong interference. The converse proof in [9] hinges on the idea that, after decoding the intended message, each receiver is capable of reconstructing a statistically equivalent signal available at the other receiver and, therefore, decode the non-intended message as well. However, in the JSCC Gaussian IC problem, the receivers do not necessarily decode the channel inputs, and this argument cannot be used anymore. Instead, we use the following information flow inequality implied by the strong interference condition.

Lemma 2. *If $I(X_i;Y_i|X_{i^c}) \le I(X_i;Y_{i^c}|X_{i^c})$, for $i = 1,2$, holds for all $p(x_1,x_2)$, or equivalently $c_i^2 \ge 1$ for the Gaussian IC, then*

$$I(X_i^n;Y_i^n|S_{i^c}^n X_{i^c}^n) \le I(X_i^n;Y_{i^c}^n|S_{i^c}^n X_{i^c}^n), \quad i = 1,2. \quad (6)$$

Proof: The proof follows similarly to [11, Lemma 5], in which the strong interference conditions of [10] are extended to the IC with correlated channel inputs. ■

We apply Lemma 2 together with cut-set bound arguments and the limitation on the maximum channel input correlation to derive the following necessary conditions.

Lemma 3. *The achievable distortion pairs for the Gaussian strong IC are included in the region formed by the pairs (D_1, D_2) satisfying, for some $0 \le \rho_x \le \rho$, and $i = 1,2$,*

$$R_{S_i}(D_i) + R_{S_{i^c}|S_i}(D_{i^c}) \quad (7)$$
$$\le \frac{1}{2}\log\left((P_i + c_{i^c}^2 P_{i^c} + 2c_{i^c}\rho_x\sqrt{P_1P_2} + N)/N\right),$$
$$R_{S_i|S_{i^c}}(D_i) \le \frac{1}{2}\log((P_i(1-\rho_x^2) + N)/N). \quad (8)$$

Proof: A proof sketch can be found in Appendix A. ■

In the SS-IC we denote by D_{cs} the lower bound on the distortion resulting from Lemma 1; and by D_l, the lower bound resulting from the application of Lemma 3, given by

$$D_l \triangleq \min_{0\le\rho_x\le\rho} \max\left\{\sigma^2\sqrt{\frac{N(1-\rho^2)}{(1+c^2+2c\rho_x)P+N}}, \frac{\sigma^2 N}{(1+c^2+2c\rho_x)P+N}, \frac{\sigma^2 N(1-\rho^2)}{(1-\rho_x^2)P+N}\right\}.$$

Next, we show that the latter bound is tight in certain regimes.

IV. Achievable schemes for the strong IC

In this section we propose a number of transmission schemes, and derive the corresponding achievable distortion values.

A. Separate Source and Channel Coding (SSCC)

In SSCC, the sources are first compressed using optimal distributed source compression, and the compressed bits are transmitted over the IC using a capacity achieving channel code. Under strong interference, the capacity is achieved by jointly decoding both channel inputs at both receivers [12]. Since both destinations have the transmitted messages, the source coding problem reduces to the distributed compression problem. For Gaussian sources, the optimal distribued source coding scheme is given by the Berger-Tung scheme [13]. Therefore, the set of achievable distortion pairs is given by the intersection of the capacity region of the strong IC with the Berger-Tung rate-distortion region. In the SS-IC, the minimum distortion achievable by SSCC is given by

$$D_s = \sigma^2\sqrt{2^{-4R_*}(1-\rho^2)+\rho^2 2^{-8R_*}}, \tag{9}$$

where the rate minimizing the distortion can be shown to lie on the boundaries of the two MAC capacity regions, i.e.,

$$R_* = \frac{1}{2}\log(\min\{\sqrt{(1+c^2)P+1}, P+1\}). \tag{10}$$

When the sources are uncorrelated, i.e., $\rho = 0$, SSCC achieves all the distortion pairs satisfying the necessary conditions from Lemma 2, as also shown in [14]. However, for $\rho > 0$, as we will see below, SSCC is suboptimal.

B. Uncoded Transmission

In uncoded transmission, each transmitter sends scaled source samples directly over the channel, i.e., $X_i^n = \sqrt{\beta_i P_i/\sigma^2}S_i^n$, where $\beta_i \in [0,1]$ is the scaling factor. Minimum mean square estimator (MMSE) is used at each receiver. In general, better distortion pairs can be achieved by not employing full power at both transmitters to reduce interference [5]. However, in SS-IC, $D_u^i(\beta_1,\beta_2)$ is minimized by transmitting at full power, i.e., $\beta_1=\beta_2=1$, and it is given by

$$D_u = \sigma^2\frac{c^2P(1-\rho^2)+N}{(1+c^2+2c\rho)P+N}. \tag{11}$$

When $\rho=1$, uncoded transmission achieves any distortion pair satisfying the necessary conditions in Lemma 3, and therefore, is optimal, while SSCC is strictly suboptimal.

C. Hybrid Coding with Common Message (HC-CM)

The best known JSCC scheme for the general IC is the hybrid coding scheme proposed in [2]. In this scheme, similarly to the Han-Kobayashi scheme for channel coding, each transmitter generates a common message, decoded at both destinations, and a private message, decoded only at the corresponding destination, but these are transmitted using the hybrid coding approach. Here, we consider the special case of this scheme, which we call hybrid coding with common message (HC-CM), in which both transmitters only send common information.

In HC-CM, each source sequence S_i^n is mapped to one of the 2^{nR_i} codewords $W_i^n(m_i)$, which serve as common information. Then, $W_i^n(m_i)$ are mapped symbol-by-symbol to generate the channel input as $X_i = x_i(W_i,S_i)$. Upon receiving Y_i^n, receiver i recovers $W_1^n(m_1)$ and $W_2^n(m_2)$ using a joint typicality decoder, and reconstructs $\hat{S}_i$ by mapping symbol-by-symbol the channel output Y_i^n and the decoded codewords $W_1^n(m_1), W_2^n(m_2)$. The distortion pairs achievable by this scheme follow from [2] and are given next.

Lemma 4. *A distortion pair (D_1,D_2) is achievable by HC-CM in the JSCC IC if there exist a pdf $p(w_1|s_1)p(w_2|s_2)$, two encoding functions $x_i(w_i,s_i)$, and two decoding functions $\hat{s}_i(w_1,w_2,y_i)$, for $i=1,2$, such that $\mathrm{E}[d_i(S_i,\hat{S}_i)] \le D_i$ and*

$$R_j \ge I(W_j;S_j), \qquad R_j \le I(W_j;Y_j,W_{j^c}),$$
$$R_1+R_2 \le I(W_1,W_2;Y_j)+I(W_1;W_2), \quad j=1,2$$

Lemma 4 provides a single letter expression for the distortion pairs achievable by HC-CM. However, it does not indicate how to characterise the optimal auxiliary random variables and the channel input mappings. Next, we consider different constructions in order to maximize the performance of HC-CM.

D. Vector Quantizer (VQ)

Here, we consider a special case of HC-CM, called the vector quantizer (VQ) scheme, in which both transmitters quantize their source vector and use the quantized codewords as channel input. VQ is shown to achieve the optimal performance in high SNR asymptotics when transmitting correlated sources over a Gaussian MAC setup [1].

In VQ, each source sequence S_i^n is mapped to one of the 2^{nR_i} codewords $W_i^n(m_i)$, generated with the test channel $W_i=(1-2^{-2R_i})S_i+Q_i$, where $Q_i \sim \mathcal{N}(0,2^{-2R_i}(1-2^{-2R_i}))$ is independent of S_i. Codewords $W_i^n(m_i)$ serve as common information. Then, $W_i^n(m_i)$ are mapped symbol-by-symbol to generate the channel input as $X_i^n = \sqrt{P_i/(1-2^{-2R_i})}W_i^n$, which satisfies the power constraint. Upon receiving Y_i^n, receiver i recovers $W_1^n(m_1)$ and $W_2^n(m_2)$ using a joint typicality decoder, and reconstructs $\hat{S}_i$ using MMSE estimation as $\hat{S}_i^n = \mathrm{E}[S_i^n|W_1^nW_2^n]$. The distortion pairs achievable by VQ follows from Lemma 4 and are characterized next.

Lemma 5. *The distortion pairs (D_1,D_2) satisfying*

$$D_i > \sigma^2 2^{-2R_i}\cdot\frac{1-\rho^2(1-2^{-2R_{i^c}})}{1-\tilde{\rho}^2}, \quad i=1,2, \tag{12}$$

are achievable by VQ, where the rate-pair (R_1,R_2) satisfies

$$R_i < \frac{1}{2}\log\left(\frac{P_i(1-\tilde{\rho}^2)+N}{N(1-\tilde{\rho}^2)}\right), \quad i=1,2,$$
$$R_1+R_2 < \min_{i=1,2}\frac{1}{2}\log\left(\frac{P_{i^c}+c_i^2P_i+2\tilde{\rho}c_i\sqrt{P_1P_2}+N}{N(1-\tilde{\rho}^2)}\right),$$

with $\tilde{\rho} \triangleq \rho\sqrt{(1-2^{-2R_1})(1-2^{-2R_2})}$.

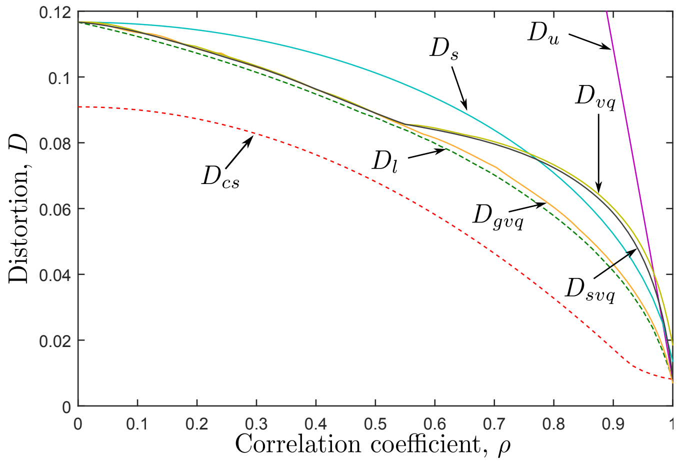

Fig. 2. Distortion bounds for the SS-IC with $P/N = 10$ and $c = 2.5$.

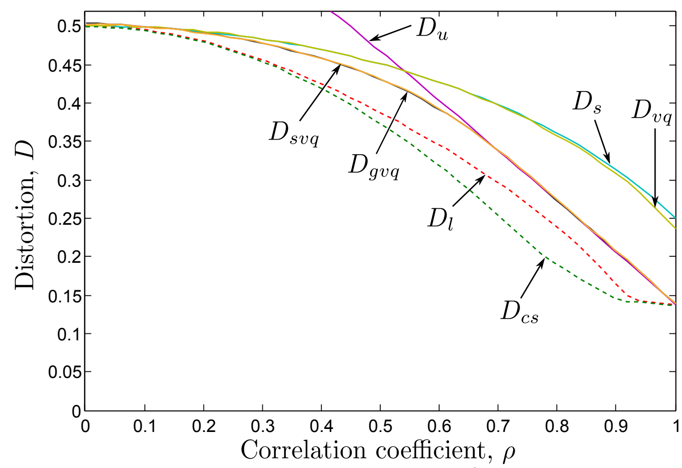

Fig. 3. Distortion bounds for the SS-IC with $P/N = 1$ and $c = 1.5$.

Next, we show that VQ achieves the optimal performance in the high SNR asymptotics, i.e., when $N \to 0$.

Theorem 1. *VQ achieves the optimal performance in the high SNR asymptotics, characterized by the pairs* (D_1^*, D_2^*) *satisfying*

$$\lim_{N\to 0} D_1^* D_2^* \cdot K^* = \sigma^4(1-\rho^2),$$

where $K^* \triangleq \min_{i=1,2}\{P_{i^c} + c_i^2 P_i + 2c_i\rho\sqrt{P_1P_2} + N\}/N$, *provided that* $D_1^* \le \sigma^2$ *and* $D_2^* \le \sigma^2$, *and that*

$$\lim_{N\to 0} \frac{N}{P_1 D_1^*} = 0 \text{ and } \lim_{N\to 0} \frac{N}{P_2 D_2^*} = 0. \quad (13)$$

Theorem 1 indicates that for sufficiently small N, conditions in (7) are tight. We note that each condition coincides with the necessary condition for a MAC, as given in [1, Theorem IV.1]. Therefore, in the high SNR regime, the set of achievable distortion pairs is equivalent to the one formed by the intersection of the achievable pairs in two MACs, in which both destinations have to reconstruct both source sequences (S_1, S_2) at distortion pair (D_1, D_2). This is reminiscent of the strong IC channel coding problem, in which each destination decodes both messages.

A generalization of VQ, denoted by superposed vector quantizer (S-VQ) scheme, is proposed in [1]. In S-VQ, an uncoded version of the source sequence is superposed with a scaled VQ codeword. S-VQ is the best known scheme for the transmission of correlated sources over a Gaussian MAC [1]. The performance of the S-VQ scheme will be included in the numerial analysis in Section V.

E. Generalized Vector Quantizer (G-VQ)

Here, we consider an alternative HC-CM scheme in which each encoder quantizes the source sequence into two quantization codewords and sends a superposition of the two. We denote this scheme as generalized vector quantized (G-VQ).

Each source sequence S_i^n is mapped to one of the 2^{nR_i} codeword pairs $(\bar{W}_i^n(m_i), U_i^n(m_i))$, generated with the test channels $\bar{W}_i = \gamma_i S_i + Q_i$, where $Q_i \sim \mathcal{N}(0,1)$ is independent of S_i, as in VQ, and $U_i = \eta_i S_i + T_i$, where $T_i \sim \mathcal{N}(0,1)$ is independent of S_i. Then, $\bar{W}_i^n(m_i)$ and $U_i^n(m_i)$ are mapped symbol-by-symbol to generate the channel input as $X_i^n = \sqrt{P_i/\nu_i}(\alpha_i \bar{W}_i^n + \tau_i(U_i - \eta_i S_i))$, where $\nu_i = \alpha_i^2(\eta_i^2+1)+\tau_i^2$ in order to satisfy the power constraint. Upon receiving Y_i^n, receiver i recovers $(\bar{W}_1^n(m_1), U_1^n(m_1))$ and $(\bar{W}_2^n(m_2), U_2^n(m_2))$ using a joint typicality decoder, and reconstructs $\hat{S}_i$ using MMSE estimation as $\hat{S}_i^n = \mathrm{E}[S_i^n|\bar{W}_1^n\bar{W}_2^n U_1^n U_2^n Y_i^n]$. The achievable distortion pairs follow from Lemma 4, and are not given here due to space limitations. Numerical results are provided in Section V.

Note that G-VQ reduces to VQ by considering $\tau_i = 0$ but not to S-VQ, in general. While a superposed uncoded layer in G-VQ, as in S-VQ, could be considered, numerical simulations indicate that this does not improve the achievable distortion.

V. Numerical Results

In Fig. 2, we plot the proposed distortion upper and lower bounds for the SS-IC with respect to the correlation coefficient ρ for $P/N = 10$ and $c = 2.5$, and numerically optimize the performance of VQ, S-VQ and G-VQ, denoted by D_{vq}, D_{svq} and D_{gvq}, respectively. We observe that in general, the proposed lower bound D_l is significantly tighter that the cut-set bound D_{cs}. Up to a certain correlation threshold $\rho_{th} \simeq 0.55$, VQ, S-VQ and G-VQ perform very close to the lower bound. When $\rho \ge \rho_{th}$, the performances of VQ and S-VQ digress from the lower bound, while G-VQ continues to follow the lower bound closely. In our extensive numerical simulations we observe that as the SNR increases, the value of ρ_{th} increases and tends to one. This is in line with Theorem 1, and the optimality of VQ in the high SNR regime. Interestingly, we observe that for high correlation values SSCC outperforms VQ and S-VQ. For very high correlation values, uncoded transmission, together with S-VQ and G-VQ, is again the best transmission scheme, and they all meet D_l for $\rho = 1$.

Fig. 3 shows the proposed distortion upper and lower bounds for the SS-IC with respect to ρ for $P/N = 1$ and $c = 1.5$. We note that the cut-set bound D_{cs} and D_l are very close for low and high correlation values, and D_l is tighter for intermediate values. We observe that SSCC, VQ, S-VQ and G-VQ meet D_l for $\rho = 0$, while uncoded transmission performs far from the lower bounds. On the contrary, uncoded transmission achieves the optimal performance for $\rho = 1$, while SSCC and VQ fall short of the lower bound, and S-VQ and G-VQ behave as uncoded transmission for high correlation values. For intermediate correlation values, S-VQ and G-VQ achieve the best performance. Interestingly, G-VQ achieves the same performance as uncoded transmission despite not using a superposed analog layer.

VI. Conclusion

We have studied the JSCC problem over a two-user Gaussian IC with correlated Gaussian sources. Focusing on the strong interference regime, we have proposed necessary conditions on the achievable distortion pairs. We have shown that separation and uncoded transmission are optimal for independent and fully correlated sources, respectively. Next, we have proposed schemes based on hybrid coding. First, we have considered a scheme in which a quantized source sequence is used as the channel input, and a generalized scheme in which an analog component is superposed on the quantized source sequence. We have shown that these schemes achieve the optimal distortion region in the high SNR regime. Finally, a novel scheme that superposes two quantized versions of the source is considered, and it is numerically shown to outperform the other considered schemes, and perform very close to the lower bound.

Appendix A
Sketch of the proof of Lemma 3

For $i = 1, 2$, we have

$$\begin{aligned} I(X_1^n X_2^n; Y_i^n) &= I(S_i^n X_1^n X_2^n; Y_i^n) && (14)\\ &\geq I(S_i^n; Y_i^n) + I(X_{i^c}^n; Y_i^n | S_i^n X_i^n) \\ &\geq I(S_i^n; Y_i^n) + I(X_{i^c}^n; Y_{i^c}^n | S_i^n X_i^n) && (15)\\ &\geq I(S_i^n; Y_i^n) + I(S_{i^c}^n; Y_{i^c}^n | S_i^n X_i^n) && (16)\\ &\geq I(S_i^n; Y_i^n) + H(S_{i^c}^n | S_i^n) - H(S_{i^c}^n | Y_{i^c}^n S_i^n) && (17)\\ &= I(S_i^n; Y_i^n) + I(S_{i^c}^n; Y_{i^c}^n | S_i^n) \\ &\geq nR_{S_i}(D_i) + nR_{S_{i^c}|S_i}(D_{i^c}), && (18) \end{aligned}$$

where (14) follows since $(S_1^n S_2^n) - (X_1^n X_2^n) - Y_i^n$ form a Markov chain; (15) follows from Lemma 2; (16) follows due to the data processing inequality; (17) follows since $S_{i^c} - S_i^n - X_i^n$ and conditioning reduces entropy; and (18) follows since, from standard rate-distortion arguments, we have $I(S_i^n; Y_i^n) \geq nR_{S_i}(D_i)$ and $I(S_i^n; Y_i^n | S_{i^c}^n) \geq nR_{S_i|S_{i^c}}(D_i)$.

We also have, for $i = 1, 2$,

$$I(X_1^n X_2^n; Y_i^n) \leq \sum_{k=1}^{n} I(X_{1k} X_{2k}; Y_{ik}). \quad (19)$$

On the other hand, we have for $i = 1, 2$,

$$\begin{aligned} nR_{S_i|S_{i^c}}(D_i) &\leq I(S_i^n; Y_i^n | S_{i^c}^n) \\ &\leq I(X_i^n; Y_i^n | X_{i^c}^n) \leq \sum_{k=1}^{n} I(X_{ik}; Y_{ik} | X_{i^c k}). \quad (20) \end{aligned}$$

Next, we jointly upper bound the mutual information terms on the left hand side of (19) and (20). We will use the following lemma, which is an extension to the interference channel of [15, Converse], derived in the context of a MAC channel with feedback.

Lemma 6. *Let $\{X_{1k}\}$ and $\{X_{2k}\}$ be zero-mean satisfying $\sum_{k=1}^{n} \mathrm{E}[X_{ik}^2] \leq nP_i$, $i = 1, 2$. Let $Y_{ik} = X_{ik} + c_{i^c} X_{i^c k} + Z_{ik}$, where $Z_{ik} \sim \mathcal{N}(0, N)$, and for every k, Z_{ik} independent of (X_{1k}, X_{2k}). Then, for any $\rho_n \in [0, 1]$ we have, for $i = 1, 2$,*

$$\begin{aligned} \sum_{k=1}^{n} I(X_{1k} X_{2k}; Y_{i^c k}) &\leq \quad (21)\\ &\frac{n}{2} \log\left((c_i^2 P_i + P_{i^c} + 2c_i \rho_n \sqrt{P_1 P_2} + N)/N \right), \\ \sum_{i=1}^{n} I(X_{ik}; Y_{ik} | X_{i^c k}) &\leq \frac{n}{2} \log\left((P_i(1 - \rho_n^2) + N)/N \right). \end{aligned}$$

The correlation between the channel inputs, denoted by ρ_n in Lemma 6, is upper bounded by the correlation between the source sequences as $0 \leq \rho_n \leq \rho$, similarly to [1, Lemma B.2]. Using this bound together with Lemma 6, we can upper bound (19) and (20); and finally, combining these with the lower bounds in (14) - (18), we obtain (7) and (8).

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