

Mobility and Popularity-Aware Coded Small-Cell Caching

Emre Ozfatura and Deniz Gündüz

Abstract—In heterogeneous cellular networks with caching capability, due to mobility of users and storage constraints of small-cell base stations (SBSs), users may not be able to download all of their requested content from the SBSs within the delay deadline of the content. In that case, the users are directed to the macro-cell base station (MBS) in order to satisfy the service quality requirement. Coded caching is exploited here to minimize the amount of data downloaded from the MBS taking into account the mobility of the users as well as the popularity of the contents. An optimal distributed caching policy is presented when the delay deadline is below a certain threshold, and a distributed greedy caching policy is proposed when the delay deadline is relaxed.

Index Terms—Heterogeneous cellular network, content caching, user mobility.

I. INTRODUCTION

CACHING popular video contents at the network edge, closer to the end users, is a promising method to cope with increasing video traffic. In heterogeneous cellular networks, popular video files are stored at SBS caches to reduce latency as well as backhaul traffic [1]–[3]. The goal in these works is to maximize the hit rate, or equivalently, to minimize the amount of data downloaded from MBSs under given SBS cache capacity constraints. A common assumption is that a user stays connected to the same set of SBSs during the whole duration of video download. However, in ultra dense networks, where there is a large number of operating SBSs with limited coverage areas, it is indispensable to take user mobility into consideration to meet the prescribed quality of service (QoS) requirements [4].

Our aim in this paper is to provide a caching policy, which, for given video popularity profile and user mobility patterns, minimizes the average amount of data downloaded from the MBS. We adopt the *delayed offloading* scheme in heterogeneous networks; that is, each video content request has a deadline, and if the mobile user cannot download all the fragments of the content from the SBSs it has connected to by the deadline, the remainder of the request is satisfied by the MBS [5]. We consider maximum distance separable (MDS) coding for storing the video contents in the SBS caches. To the best of our knowledge, [6] is the only work that considers mobility-aware caching with delayed offloading. The goal in [6] is to minimize the probability of a request being served by the MBS. In this work, as in [7], we assume that the MBS serves the remaining fragments as MDS coded packets at a higher cost. Hence, unlike [6], our goal is to minimize the

amount of data downloaded from the MBS. We show that, if the request deadline is below a certain threshold there is an optimal distributed solution. If the request deadline does not meet this condition, we introduce a sub-optimal greedy caching policy.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a heterogeneous network that consists of one MBS and N SBSs, SBS_1, \dots, SBS_N , with disjoint coverage areas. A mobile user (MU) is served by only one SBS at any particular time. We consider a video library $\mathbb{V} = \{v_1, \dots, v_K\}$ consisting of K distinct video files, each of length B bits, indexed according to their popularity profile, i.e., file v_k is the k th most popular file with request probability p_k . Further, we assume that video files are encoded by rateless MDS coding [3], so that a video file can be retrieved when B parity bits are collected in any order, from any SBSs.

We consider equal-length time slots, whose duration corresponds to the minimum time that a MU remains in the coverage area of a SBS. Therefore, although a MU cannot connect to more than one SBS during one time slot, it may stay connected to the same SBS over several consecutive time slots. We assume that SBS_n is capable of transmitting R_n bits within a time slot to a MU within its coverage area, and it has a storage capacity of C_n bits. Due to the QoS requirement, a video file must be downloaded within T time slots once it is requested. Thus, if a MU is not able to collect B bits from SBSs in T slots, the remaining bits are provided by the MBS at a higher cost.

We define the *mobility path* of a user as the sequence of small-cells visited over T time slots after requesting a video file. For instance, for $T = 5$, $SBS_1, SBS_2, SBS_2, SBS_3, SBS_4$ is a possible mobility path. We remark that the MU may remain connected to the same SBS more than one time slot, and can connect to at most T different SBSs. Hence, there is a finite number of distinct mobility paths, denoted by M . We denote the m th mobility path by I_m , and its realization probability by q_m . Realization probabilities can be obtained from empirical observations, or via modeling mobility paths as random walks on a Markov chain.

Our aim is to minimize the expected amount of data downloaded from the MBS for given SBS storage capacities $\mathbf{C} \triangleq \{C_n\}_{n=1}^N$, data transmission rates $\mathbf{R} \triangleq \{R_n\}_{n=1}^N$, video popularity profiles $\mathbf{P} \triangleq \{p_k\}_{k=1}^K$, and mobility paths with realization probabilities $\mathbf{I}_T \triangleq \{(I_m, q_m)\}_{m=1}^M$. Let $\mathbf{X}_T = \{x_{n,k}\}_{n=k=1}^{N,K}$ denote the caching policy over T time slots, where $x_{n,k}$ indicates the number of parity bits for file v_k stored in SBS_n . A caching policy is *feasible* if $\sum_{k=1}^K x_{n,k} \leq C_n, \forall n$. The average amount of data downloaded from the MBS is denoted by $d_{av}(\mathbf{X}_T, \mathbf{I}_T)$, or simply by d_{av} .

Let $d_{k,m}$ denote the amount of coded data downloaded from

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the MBS for video v_k following mobility path I_m . For given $\mathbf{C}, \mathbf{R}, \mathbf{P}, \mathbf{I}_T$, and caching policy \mathbf{X}_T , we have

$$d_{k,m} = \max \left\{ B - \left(\sum_{n=1}^N \min \{x_{n,k}, R_n S_{m,n}\} \right), 0 \right\}, \quad (1)$$

where $S_{m,n}$ denotes the total number of time slots the MU connected to SBS_n in mobility path I_m . Then, taking the average over all mobility paths and video files, d_{av} can be written in terms of $d_{k,m}$ as $d_{av} = \sum_{m=1}^M \sum_{k=1}^K q_m p_k d_{k,m}$. Our goal is to find the optimal feasible caching policy \mathbf{X}_T^* that minimizes d_{av} , formulated as follows:

$$\begin{aligned} \mathbf{P1:} \quad & \min_{\mathbf{X}_T} d_{av} \\ \text{subject to:} \quad & \sum_{k=1}^K x_{n,k} \leq C_n, \quad \forall n. \end{aligned} \quad (2)$$

$$x_{n,k} \geq 0, \quad \forall n, k. \quad (3)$$

In order to avoid the max operator in Eqn. (1), problem **P1** is reformulated as follows. We treat $d_{k,m}$ as decision variables, and add the following constraint:

$$d_{k,m} \geq \max \left\{ B - \left(\sum_{n=1}^N \min \{x_{n,k}, R_n S_{m,n}\} \right), 0 \right\}. \quad (4)$$

We note that, for a given feasible caching policy the objective function is monotonically increasing with $d_{k,m}$; hence, for the optimal solution, constraint (4) must be satisfied with equality. The equivalent optimization problem is obtained as follows.

$$\begin{aligned} \mathbf{P2:} \quad & \min_{\mathbf{X}_T, \mathbf{D}_T} d_{av} \\ \text{subject to:} \quad & \sum_{k=1}^K x_{n,k} \leq C_n, \quad \forall n, \end{aligned} \quad (5)$$

$$\sum_{n=1}^N \min \{x_{n,k}, R_n S_{m,n}\} + d_{k,m} \geq B, \quad \forall k, m, \quad (6)$$

$$d_{k,m} \geq 0, \quad \forall k, m, \quad (7)$$

where $\mathbf{D}_T \triangleq \{d_{k,m}\}_{k=m=1}^{K,M}$. Notice that constraints (6) and (7) together imply constraint (4). Recall that a MU cannot connect to more than T different SBSs. Hence, for each (k, m) pair, constraint (6) can be replaced by at most 2^T linear constraints. To clarify, let $T = 4$ and $N = 6$, and consider the mobility path $I_m = \{\text{SBS}_1, \text{SBS}_1, \text{SBS}_2, \text{SBS}_2\}$. For this specific mobility path and file v_k , (6) can be written as

$$\min \{x_{1,k}, 2R_1\} + \min \{x_{2,k}, 2R_2\} + d_{k,m} \geq B. \quad (8)$$

Equivalently, (8) can be replaced by the following set of linear constraints:

$$x_{1,k} + x_{2,k} + d_{k,m} \geq B, \quad (9)$$

$$x_{1,k} + 2R_2 + d_{k,m} \geq B, \quad (10)$$

$$2R_1 + x_{2,k} + d_{k,m} \geq B, \quad (11)$$

$$2R_1 + 2R_2 + d_{k,m} \geq B. \quad (12)$$

Consequently, the initial optimization problem **P1** can be cast into a linear optimization problem. However, the number of constraints are exponential in time constraint T . In the next subsection we show that, under a certain assumption on T , **P1** can be solved in a distributed manner.

III. DISTRIBUTED SOLUTION

In this section, we consider the case $T \leq T_{min} \triangleq \frac{B}{R_{max}}$, where R_{max} is the maximum data rate across all the cells, i.e., $R_{max} \triangleq \max \{R_1, \dots, R_N\}$. This special case is also instrumental in highlighting the distinction between our problem formulation and that of [6], whose goal is to minimize the probability of downloading any data from the MBS. We note that, with the formulation of [6], when $T < T_{min}$ all caching policies are equivalent since it is not possible to collect B bits in T slots. While [6] ignores the mobility paths when $T < T_{min}$, each caching policy will induce a different d_{av} . Hence, an optimal caching policy \mathbf{X}_T in [6] may lead to a suboptimal d_{av} . Instead, we present the optimal caching algorithm that minimizes d_{av} when $T \leq T_{min}$. We also propose a greedy caching policy, for $T > T_{min}$.

A. Optimal Distributed Solution

When $T \leq T_{min}$, (1) simplifies to

$$d_{k,m} = B - \sum_{n=1}^N \min \{x_{n,k}, R_n S_{m,n}\}. \quad (13)$$

Then, our objective d_{av} can be rewritten as:

$$d_{av} = B - \underbrace{\sum_{m=1}^M \sum_{k=1}^K \sum_{n=1}^N q_m p_k \min \{x_{n,k}, R_n S_{m,n}\}}_{\triangleq \tilde{d}_{av}}. \quad (14)$$

Note that minimizing d_{av} is equivalent to maximizing \tilde{d}_{av} , which denotes the average amount of data downloaded from the SBSs. We change the order of the summations in (14):

$$\tilde{d}_{av} = \sum_{n=1}^N \sum_{k=1}^K p_k \underbrace{\sum_{m=1}^M q_m \min \{x_{n,k}, R_n S_{m,n}\}}_{\triangleq \tilde{d}_{av,n}}, \quad (15)$$

we observe that the optimal caching policy can be obtained via maximizing $\tilde{d}_{av,n}$, defined above, for each SBS_n separately. Let \mathbf{X}_T^n denote the caching policy for SBS_n . For SBS_n , we have the following optimization problem:

$$\mathbf{P3:} \quad \max_{\mathbf{X}_T^n} \tilde{d}_{av,n}$$

$$\text{subject to:} \quad \sum_{k=1}^K x_{n,k} \leq C_n. \quad (16)$$

$$x_{n,k} \geq 0, \quad \forall k. \quad (17)$$

If we group the mobility paths according to the time spent in cell SBS_n , $\tilde{d}_{av,n}$ can be written as

$$\tilde{d}_{av,n} = \sum_{k=1}^K \sum_{t=1}^T \underbrace{\sum_{m: S_{m,n}=t} p_k q_m \min \{x_{n,k}, tR_n\}}_{\triangleq \tilde{d}_{av,n}^k}. \quad (18)$$

The term $\min \{x_{n,k}, tR_n\}$ can be expanded as follows:

$$\min \{x_{n,k}, tR_n\} = \sum_{i=0}^{t-1} \max \{ \min \{x_{n,k} - iR_n, R_n\}, 0 \}, \quad (19)$$

Algorithm 1: Algorithm for optimal distributed caching (γ -based policy)

Input : $R, C, \{\gamma^n\}_{n=1}^N$
Output: X_T^γ

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1 for  $n=1, \dots, N$  do
2    $x_{n,k} \leftarrow 0, k \in \{1, \dots, K\}$ ;
3   while  $C_n > 0$  do
4      $\gamma_{k,i}^n \leftarrow \max \gamma^n$ ;
5      $x_{n,k} \leftarrow x_{n,k} + \min(C_n, R_n)$ ;
6      $\gamma^n \leftarrow \gamma^n \setminus \{\gamma_{k,i}^n\}$ ;
7      $C_n \leftarrow C_n - R_n$ ;
8   end
9 end
```

and $\bar{d}_{av,n}^k$ can be rewritten as:

$$\bar{d}_{av,n}^k = \sum_{t=1}^T \sum_{m: S_{m,n} \geq t} p_k q_m \max \{ \min \{ x_{n,k} - (t-1)R_n, R_n \}, 0 \}. \quad (20)$$

Equivalently,

$$\bar{d}_{av,n}^k = \sum_{t=1}^T \gamma_{k,t}^n \max \{ \min \{ x_{n,k} - (t-1)R_n, R_n \}, 0 \}, \quad (21)$$

where $\gamma_{k,t}^n \triangleq p_k P(S_{m,n} \geq t)$. We observe that for each k , $\bar{d}_{av,n}^k$ is a monotonically increasing piecewise linear function of $x_{n,k}$, and its slope is $\gamma_{k,t}^n$ for $x_{n,k} \in ((t-1)R_n, tR_n)$. Consequently, the objective function in **P3** is the sum of N monotonically increasing piecewise linear functions, and sum of its variables are bounded by constraint (16). Accordingly, it is maximized by maximizing the variable that corresponds to the linear function with the maximum slope. We propose Algorithm 1 to maximize the objective function that follows a straightforward procedure using $\gamma_{k,t}^n$ values for each SBS_n . The algorithm starts with increasing the variable $x_{n,k}$ that corresponds to the maximum slope, until the slope of $\bar{d}_{av,n}^k$ changes, then it again searches for the maximum slope, and repeats this process until the sum of the variables satisfies (16) with equality. From a computational point of view, proposed algorithm sorts the elements of set $\Gamma^n \triangleq \{\gamma_{k,t}^n : k = 1, \dots, K; t = 1, \dots, T\}$ for each $n \in \mathbb{N}$. Since $|\Gamma^n| = KT$, the complexity of Algorithm 1 is $O(NKT \log(KT))$. The optimality of the algorithm follows from the fact that $\gamma_{k,t}^n \leq \gamma_{k,t-1}^n$ for any (k, t) pair, which implies that $\bar{d}_{av,n}^k$ is a concave function for each k . The caching policy constructed according to Algorithm 1 is called the γ -based policy, and denoted by X_T^γ . We note that when $T \leq T_{min}$, X_T^γ is the optimal policy, i.e., $X_T^\gamma = X_T^*$.

B. Distributed Greedy Cache Allocation Scheme

When $T > T_{min}$ it is not possible to predict the performance of X_T^γ , or ensure that $d_{av}(X_T^\gamma, I_T) \leq d_{av}(X_{T_{min}}^\gamma, I_{T_{min}})$. We note that $X_{T_{min}}^\gamma$ is the γ -based caching policy explained above for T_{min} . However, we know that for any $T > T_{min}$, $d_{av}(X_{T_{min}}^\gamma, I_T) \leq d_{av}(X_{T_{min}}^\gamma, I_{T_{min}})$. Hence, our aim is to provide a greedy distributed caching policy X_T^g that performs

Algorithm 2: Greedy algorithm for storage reallocation

Input : $C, R, I_T, P, X_{T_{min}}^*$
Output: X_T^g

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1 for  $n=1, \dots, N$  do
2    $\Delta_k^+, \Delta_k^- \leftarrow \text{NULL} : k \in \{1, \dots, K\}, V^{red} \leftarrow \{\}$ ;
3    $V^+ \leftarrow \{\}, V^- \leftarrow \{\}$ ;
4    $x_{max} = \max \{x_{n,1}, \dots, x_{n,K}\}$ ;
5   while  $x_{max} > 0$  do
6      $\hat{k} = \max \{k : x_{n,k} \geq x_{max}\}$ ;
7     if  $\hat{k} \notin V^-$  then
8        $V^- \leftarrow V^- \cup \{\hat{k}\}$ , calculate  $\Delta_{\hat{k}}^-$ ;
9     end
10    if  $\hat{k} + 1 \notin V^+$  then
11       $V^+ \leftarrow V^+ \cup \{\hat{k} + 1\}$ , calculate  $\Delta_{\hat{k}}^+$ ;
12    end
13     $x_{max} \leftarrow x_{max} - R_n$ ;
14  end
15   $\Delta_{max}^+ \leftarrow \max \Delta^+, \Delta_{max}^- \leftarrow \max \Delta^-$ ;
16  if  $\Delta_{max}^+ > |\Delta_{max}^-|$  then
17     $\hat{k} \leftarrow \max_k \Delta_k^+, \hat{k} \leftarrow \max_k \Delta_k^-$ ;
18     $\Delta_{\hat{k}}^+, \Delta_{\hat{k}}^- \leftarrow \text{NULL}$ ;
19     $x_{n,\hat{k}} \leftarrow x_{n,\hat{k}} + R_n, x_{n,\hat{k}} \leftarrow x_{n,\hat{k}} - R_n$ ;
20    Go back to line 3
21  end
22 end
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better than $X_{T_{min}}^\gamma$, i.e., $d_{av}(X_T^g, I_T) \leq d_{av}(X_{T_{min}}^\gamma, I_T)$. Our proposed method to construct X_T^g consists of two steps. In the first step, we obtain the optimal caching policy $X_{T_{min}}^*$ by executing Algorithm 1. In the second step, we follow a greedy method for cache reallocation for each cell separately, which is performed by Algorithm 2. Assume that we are reallocating the cache for SBS_n , Algorithm 2 first identifies candidate video files for cache capacity increment and reduction. In this identification process, the main criteria is the popularity of the video files, e.g., if there are several video files that have been allocated the same cache capacity, then the most popular file among those is a candidate for cache capacity increment, whereas the least popular one is a candidate for cache capacity reduction. Accordingly, let V^+ and V^- denote the sets of indices of the video files that are candidates for cache capacity increment and reduction, respectively. After identification of a candidate file v_k , we calculate Δ_k^- if $k \in V^-$, or Δ_k^+ if $k \in V^+$, whose initial values are is NULL. Δ_k^+ and Δ_k^- denote the amount of change in d_{av} when the cache capacity of video file v_k is increased by R_n , or decreased by R_n^{-1} , respectively. In the last step, algorithm compares the values of $\Delta_{max}^+ \triangleq \max_k \Delta_k^+$ and $\Delta_{max}^- \triangleq \max_k \Delta_k^-$. The condition $\Delta_{max}^+ > |\Delta_{max}^-|$ implies that d_{av} can be reduced via storage capacity reallocation. Then, the algorithm performs the following task, storage capacity of video file v_k , where $\Delta_k^+ = \Delta_{max}^+$, is increased by R_n , and the storage capacity of

¹Storage reallocation can be done with smaller sizes to improve the performance of the policy with a cost of complexity; however, due to limited space, we do not study this tradeoff in this letter.

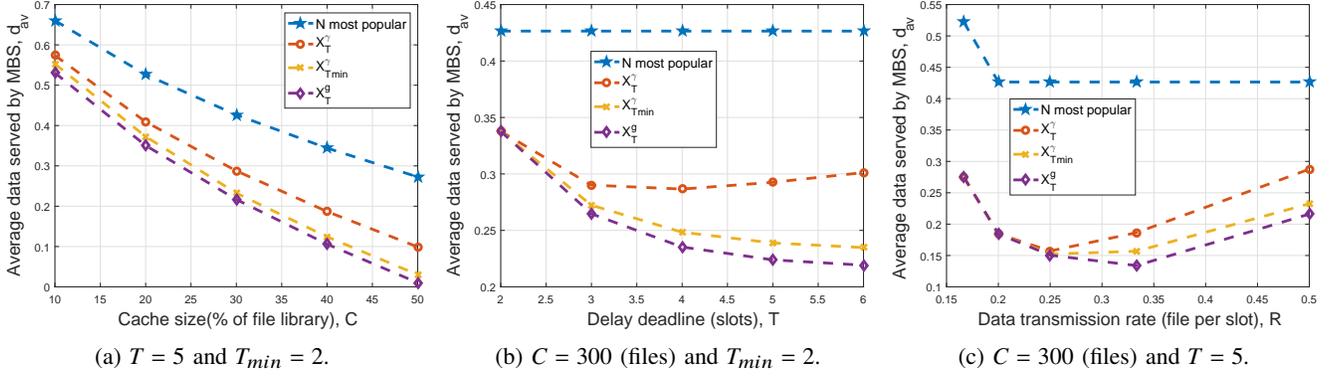


Fig. 1: The average amount of data served by the MBS (normalized by the file size) for different values of: (a) normalized cache size C , (b) delay deadline T , and (c) data transmission rate $R = 1/T_{min}$.

video file v_k where $\Delta_k^- = \Delta_{max}^-$ is decreased by R_n .

IV. NUMERICAL RESULTS

For numerical simulations we consider $K = 1000$ files in the library, and assume that their popularities follow a Zipf distribution with parameter 0.56 [8]. There are 16 SBSs located in a 2D square grid. We fix the transmission rate of each SBS to R , according to parameter T_{min} , i.e., $R = 1/T_{min}$ file per slot. We consider the following Markov mobility model: a MU connected to SBS _{n} remains connected to the same SBS with probability f_n , or connects to one of the neighboring SBSs with equal probability. In the experiment, we consider $f_4 = f_{13} = 0.4$, $f_7 = f_9 = 0.5$, and for the all other SBSs $f_n = 0.3$. As a performance benchmark, we also consider the *N most popular policy*, which simply caches the most popular N files at each SBS. We remark that, when $T > T_{min}$ the value of T has no impact on the performance of the *N most popular policy* since it caches the files as a whole. On the other hand, when $T \leq T_{min}$, the value of d_{av} decreases linearly with increasing T .

In the first experiment, we set $T_{min} = 2$, $T = 5$ and consider the normalized SBS cache sizes (as portion of the entire file library) 10%, 20%, 30%, 40%, 50%. The greedy algorithm X_T^s provides up to 40% further reduction in the amount of data downloaded from the MBS compared to γ -based policy X_T^γ as depicted in Fig. 1(a). We also observe that the gap between the performances of X_T^s and X_T^γ widens with increasing cache sizes. Finally, note that the *N most popular policy* performs quite poorly in general as it ignores the mobility patterns.

In the second experiment, we set $C = 300$, $T_{min} = 2$, and the delay deadline T takes values from 2 to 6 time slots. Performance of the caching policies for different T values are plotted in Fig. 1(b). The key observation from the figure is that, although the average portion of the video file downloaded from the MBS monotonically decreases with increasing T under policy X_T^s and X_T^γ , this is not the always the case for X_T^g . Note that X_T^γ mainly depends on the sojourn statistics, $P(S_{m,n} \geq t)$, over all possible paths, and when $T > T_{min}$ those statistics might be misleading because in certain paths the MU might collect all the parity bits before connecting to SBS _{n} . In that case, storage capacity of the popular files might

be increased due to sojourn statistics even though it is not required.

In the third experiment we set $T = 5$, $C = 300$, and the transmission rate takes values 1/2, 1/3, 1/4, 1/5, 1/6 file per slot, which correspond to T_{min} values of 2, 3, 4, 5, 6 slots respectively. Although it is expected that the MBS usage decreases with the increasing transmission rate, Fig 1(c) illustrates that after a certain point the amount of data downloaded from the MBS increases with the transmission rate under all policies. This is because, T_{min} decreases when the rate increases and the difference between T_{min} and T widens, as a result of which the performance becomes worse.

V. CONCLUSIONS

In this letter, we studied mobility and popularity aware content caching for a heterogeneous network with MDS-coded caching at the SBSs. Assuming a maximum download time requirement T , for each request, we first defined the threshold T_{min} on T , below which some bits of the request must be downloaded from the MBS. Then, we obtained the optimal distributed caching policy when $T \leq T_{min}$, called the γ -based policy, which minimizes the amount of data that need to be downloaded from the MBS. Then, we utilized the parameter T_{min} and the γ -based policy for $T = T_{min}$ to obtain a greedy caching policy for $T > T_{min}$. Consequently, we showed how to design a coded caching policy according to T_{min} and performed various simulations to demonstrate that the utilization of T_{min} improves the performance significantly.

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