Source and Channel Coding for Quasi-Static Fading Channels

Deniz Gunduz, Elza Erkip
Dept. of Electrical and Computer Engineering
Polytechnic University, Brooklyn, NY 11201, USA
dgundu01@utopia.poly.edu elza@poly.edu

***Abstract*— We consider transmission of a continuous amplitude source over a quasi-static Rayleigh fading channel. We analyze three different source and channel coding strategies in terms of overall expected distortion (ED). Our goal is to maximize the distortion exponent ($\triangle$), which is the exponential decay rate of ED with increasing SNR. In each case, by adjusting the system parameters we find the best $\triangle$ as a function of the bandwidth expansion. We also find an upper bound for $\triangle$ and illustrate how this upper bound can be achieved for all bandwidth expansions even with reasonably simple strategies. Although we focus on a Gaussian source for brevity, we demonstrate that our results can be extended to more general source distributions.**

I. Introduction

In this paper we consider transmission of continuous amplitude sources over quasi-static Rayleigh fading channels. We impose stringent delay requirements and assume that the instantaneous channel state information is only available at the receiver. Thus any predetermined transmission rate might result in an outage and the appropriate performance measure to use is the overall expected source distortion at the destination. For this scenario Shannon's source-channel separation theorem does not apply and a joint optimization of source and channel coding strategies is necessary. Our objective is to minimize the overall expected distortion which depends on the source characteristics, the channel model, the distortion metric, the power constraint of the transmitter, the joint compression, channel coding and transmission techniques used.

We use compression strategies that meet the rate-distortion bound and channel codes with Gaussian codebooks. We are particularly interested in the high SNR behavior of the expected distortion (ED). This is captured by the distortion exponent, $\triangle$ which is defined as [1]

$$\triangle = -\lim_{SNR\to\infty}\frac{\log ED}{\log SNR}. \tag{1}$$

A distortion exponent of $\triangle$ means that the optimal expected distortion achieved by the system decays as $SNR^{-\triangle}$ when SNR is high.

During each transmission block, which corresponds to N channel uses, a sequence of K source samples are compressed and sent over the channel. This corresponds to a bandwidth expansion ratio of

$$b = N/K. \tag{2}$$

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K is large enough to consider the source as ergodic, but slow channel variations result in nonergodic channel. Throughout the paper we allow for arbitrary bandwidth expansion ratio b.

We note that increasing source coding rate would result in a decreased distortion; however this would also increase the outage probability. This trade-off tells us that there is an optimal operating rate in the average distortion sense for a given SNR value [3]. Alternatively, instead of transmitting the compressed signal at a single rate, we can compress the source into multiple layers and transmit them at different rates. We will show that this variable rate transmission enables us to adapt to the channel variations without the need of channel state information at the transmitter (CSIT) and improves the expected distortion considerably.

We consider two strategies that utilize a layered source coder. In the first one, called layered source with progressive transmission (LS), each layer is successively transmitted in time. The second strategy, called broadcast strategy with layered source (BS), uses broadcast codes to superimpose each source layer. In both cases we optimize the source and channel coder parameters including the rates to maximize $\triangle$. We compare our performance results with uncoded transmission, which is known to be optimal for the AWGN channel without bandwidth expansion, and with an upper bound that we calculate by assuming the availability of perfect channel state information at the transmitter. Our results indicate the benefits of layered source coding for slowly fading environments. In fact, BS strategy with infinite layers is able to achieve optimum Δ for all bandwidth expansions b. In [3], the techniques used in this paper for a single source-single destination pair are applied to the relay channel in the user cooperation context and shown to improve the end-to-end distortion.

We note here that in Sections II-VII we consider a memoryless, complex Gaussian source, however in Section VIII we prove that our results can also be extended to any complex source with finite second moment and finite differential entropy, and with squared-error distortion.

II. System Model

We consider a continuous source that is to be transmitted over a quasi-static flat Rayleigh fading channel. We first focus on a memoryless, complex Gaussian source with independent real and imaginary components each with variance $1/2$. We then discuss extensions to other sources in Section VIII.

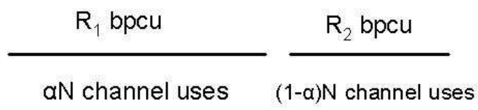

Fig. 1. Channel allocation for two-layered source coding strategy (LS).

The quasi-static Rayleigh fading channel is assumed to remain constant over a block of N channel uses. The corresponding fading is circularly symmetric complex Gaussian with variance $1/2$ in each dimension. Then the amplitude square of the fading coefficient denoted as a, is exponentially distributed. The additive noise is modelled as complex Gaussian with variance σ^2. There is an average power constraint of P, and thus the received average SNR is $SNR = P/\sigma^2$. The fading coefficients are known to the destination, but not known to, or not exploited by the source. Thus, any transmission rate over the channel might result in an outage.

We consider expected distortion for all bandwidth expansion ratios b defined in (2). Although we use the term bandwidth expansion to abide by the general usage, we will consider the cases where $b \leq 1$ as well.

We will consider three different compression and communication strategies. The first one, which we call layered source with progressive transmission (LS), is based on dividing the transmission block into smaller portions in time and transmitting at different rates during each portion. The second one is the broadcast strategy with layered source (BS) where different rate channel codes are superimposed to transmit each source layer simultaneously. The third strategy is the uncoded transmission strategy (UT) where no compression or channel coding is used and the source samples are transmitted by appropriately scaling according to the transmitter power constraint, P.

III. Layered Source with Progressive Transmission

We first introduce layered source with progressive transmission (LS) with two layers. We compress the source into two layers: Base and enhancement. We divide the whole transmission block, or N channel uses, into two portions (Fig. 1). In the first portion which corresponds to αN channel uses ($0 \leq \alpha \leq 1$), we transmit the base layer at a channel rate of R_1 bits per channel use (bpcu). In the second portion, we transmit the enhancement layer consisting of the successive refinement bits [5] of the source at a rate of R_2 bpcu. Here we should note that the reason that we prefer to send successive refinement bits instead of sending another description of the same source such as the one obtained by multiple description coding, is due to the fact that the channel state remains constant for all of the N channel uses, so the description that has higher transmission rate would always be in outage when the other one is. This means that this description would never be received on its own.

For the transmission rates, we can impose the constraint $R_1 \leq R_2$ since the enhancement layer is useless by itself.

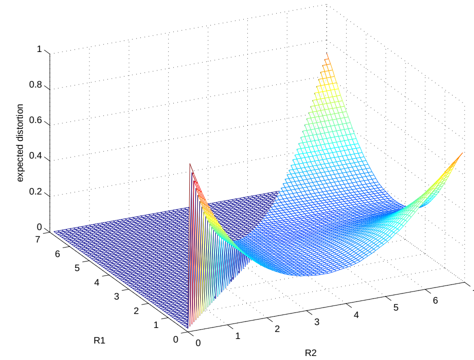

Fig. 2. Expected distortion vs. varying rates for LS with 2 layers (SNR=30 dB and $b = 1$).

This constraint also guarantees successful reception of the base layer when the enhancement layer is received. Upon successful reception of both portions, destination achieves a source description rate $\alpha b R_1 + (1-\alpha) b R_2$ bits per source sample. However, in case of an outage in the second portion only, it gets $\alpha b R_1$ bits per source sample. Using the successive refinability property, these correspond to distortions of $D(\alpha b R_1 + (1-\alpha) b R_2)$ and $D(\alpha b R_1)$, respectively, where $D(.)$ is the distortion rate function of the given source. In case of an outage at the base layer, the achieved distortion is $D(0)$. Let $P_{out}(R, SNR)$ be the outage probability at rate R and average received signal-to-noise ratio SNR, which we will denote as P_{out}^{R}. Then we can write the expected distortion expression for 2-level LS as:

$$\begin{aligned} ED(R_1, R_2, SNR) &= (1 - P_{out}^{R_2}) D(\alpha b R_1 + (1-\alpha) b R_2) \\ &+ (P_{out}^{R_2} - P_{out}^{R_1}) D(\alpha b R_1) + P_{out}^{R_1}. \end{aligned} \quad (3)$$

LS with one layer corresponds to direct transmission, and in [2] it is shown that in case of direct transmission there is an optimal choice of an operating rate that results in minimum expected distortion. Similarly, an optimal rate pair exists for two-layered LS. Fig. 2 shows the expected distortion for varying transmission rates R_1 and R_2 and for received $SNR = 30$ dB using optimal value of α for each (R_1, R_2) pair. As expected, we observe that there is an optimal (R_1, R_2) pair that results in minimum expected distortion. For the case shown here $(4.50, 6.35)$ pair results in a minimum average distortion of 0.0543 at the destination.

As mentioned in Section I, we are interested in the expected distortion for high SNR regime. Evident from the ED expression, in order to have the expected distortion decay to zero with increasing SNR, we need to scale the channel rate as $r \log SNR$. Due to bandwidth expansion this results in a source coding rate of $br \log SNR$. We know $D(R) = 2^{-R}$ for complex Gaussian source with unit variance and the outage probability for direct transmission is

$$P_{out}(R, SNR) = 1 - e^{-(2^R - 1)/SNR}, \quad (4)$$

with high SNR approximation as $P_{out} \approx (2^R - 1)/SNR$.

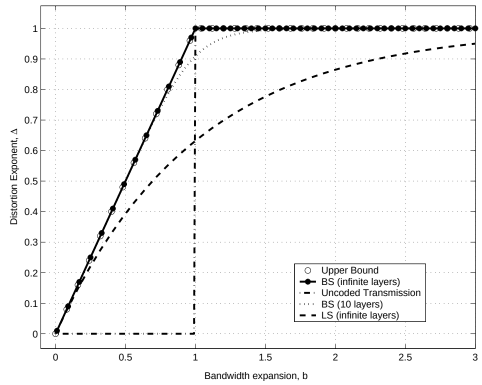

Fig. 3. The relation between the distortion exponent and the bandwidth expansion for different transmission schemes. It can be seen that BS approaches the upper bound even with moderate number of layers.

Then assuming we scale both R_1 and R_2 as $R_1 = r_1 \log SNR$, and $R_2 = r_2 \log SNR$, we can rewrite Eqn. (3) as:

$$ED(r_1, r_2, SNR) = SNR^{-(\alpha b r_1 + (1-\alpha) b r_2)} + SNR^{r_2 - 1 - \alpha b r_1} + SNR^{r_1 - 1}. \quad (5)$$

We want to find the optimal distortion exponent, $\triangle$ of this expression which consists of the sum of three exponential terms. This sum will be dominated by the slowest decay, and thus the optimal $\triangle$ is achieved when all three exponents are equal. Using this we find $\triangle = 1 - \frac{1}{(\alpha b+1)(1+(1-\alpha)b)}$. Optimizing for α gives $\alpha = 1/2$ and $\triangle = 1 - \frac{1}{(b/2+1)^2}$.

We can extend the above argument to n source coding layers by dividing the transmission block into n portions. It can be easily proven that, in general, for n layers of coding, the optimal value of $\triangle$ is

$$\triangle = 1 - \frac{1}{(b/n+1)^n}. \quad (6)$$

We see that $\triangle$ increases with the increasing number of coding layers. In the limit, we get

$$\lim_{n\to\infty} \triangle = 1 - e^{-b}. \quad (7)$$

The relation between the optimal $\triangle$ and the bandwidth expansion for infinite layers can be seen in Fig. 3.

We note that LS is a simple communication strategy as the only requirement for transceivers is the rate adaptation, and layered compression is already a part of image/video compression. It can be observed from Fig. 4 and Table I that even with moderate number of layers, improvements in both the expected distortion and the distortion exponent compared to direct transmission can be obtained.

IV. Broadcast Strategy with Layered Source

Broadcast strategy for slow fading channels was first introduced by Shamai in [4]. The main idea is for the transmitter to view the fading channel as a degraded broadcast channel with a continuum of receivers each experiencing a different received signal-to-noise ratio corresponding to each fading level. It was shown in [4] that BS provides improvement in the expected channel rate that can be supported by the system.

We will combine the broadcast strategy with source coding by utilizing layered source coding. Similar to LS, information is sent in layers, where each layer consists of the successive refinement information for the previous layers. However, in this case the codes corresponding to different layers are superimposed, assigned different power levels and sent throughout the whole transmission block. Compared to LS, power distribution and interference among different layers are traded off for increased multiplexing gain.

We first study 2-level superposition coding. We superimpose a code at rate R_2 for the enhancement layer on rate R_1 code for the base layer. Similar to LS, we scale these rates with increasing SNR as $R_1 = r_1 \log SNR$, and $R_2 = r_2 \log SNR$. Power levels of these layers are $P_1 = \beta(SNR)P$ and $P_2 = (1-\beta(SNR))P$, where P is the power constraint of the transmitter and $\beta(SNR)$ is the power assignment rule which is a function of SNR that satisfies $0 \le \beta(SNR) \le 1$. As a shorthand we will denote it by β.

The destination first tries to decode the base layer by considering the second layer as noise. This results in a distortion of $D(0)$ in case of outage. If it can decode the base layer, but not the enhancement layer after subtracting the decoded portion, the achieved distortion is $D(bR_1)$. Successful decoding of both layers results in an achieved distortion of $D(bR_1 + bR_2)$. The expected distortion, ED for BS can be written as follows.

$$\begin{aligned} ED(R_1, R_2, \beta, SNR) &= (1 - P_{out}^2) D(bR_1 + bR_2) + \\ &\quad (P_{out}^2 - P_{out}^1) D(bR_1) + P_{out}^1, \end{aligned}$$

where P_{out}^1 is the outage probability of the first layer and P_{out}^2 is the outage probability of the second layer when the first layer is subtracted from the received signal. We have

$$P_{out}^1 = Pr\left(\log(1 + \frac{\beta SNR a}{1 + (1-\beta) SNR a}) < R_1\right), \quad (8)$$

$$P_{out}^2 = Pr\left(\log(1 + (1-\beta) SNR a) < R_2\right). \quad (9)$$

Here we also consider the fact that decoding the second layer reduces distortion only if the first layer can be decoded as well. Further analysis gives us

$$P_{out}^1 = Pr\left(aSNR[\beta - (1-\beta)(2^{R_1 - 1})] < 2^{R_1 - 1}\right). \quad (10)$$

For an outage probability less than 1, we need to have $[\beta - (1-\beta)(2^{R_1-1})] > 0$. This is equivalent to $1 - \beta < 2^{-R_1} = SNR^{-r_1}$. This means that $1-\beta$ should decay exponentially with increasing SNR. Thus the second layer is assigned exponentially small powers for high SNR. Letting $1 - \beta = SNR^{-x}$ where $x > r_1$, we get the high SNR approximation for ED as

$$ED(r_1, r_2, x, SNR) = SNR^{-b(r_1+r_2)} + SNR^{r_2 - 1 + x} SNR^{-br_1} + SNR^{r_1 - 1}. \quad (11)$$

Similar analysis of this exponential form as in the LS case, results in an optimal value of $\triangle = 1 - \frac{1}{b^2+b+1}$. Furthermore, generalization of the result to strategies with n layers of broadcast coding will give us the relation

$$\triangle = 1 - \frac{1}{1 + b + b^2 + ... + b^n}. \quad (12)$$

Comparing Eqn. (6) and Eqn. (12) we conclude that the distortion exponent achieved by BS with the same number of layers is greater than LS. It is also seen that, in the limit of $n \to \infty$, BS achieves $\triangle = 1$ for $b \geq 1$ and $\triangle = b$ for $b < 1$. This dependence can be seen in Fig. 3. In [6] it is argued that most of the performance improvement that is provided by the broadcast strategy in the expected rate sense can be obtained with two layers. However, our results show that it is possible to improve the expected distortion by using more than two layers, especially for bandwidth expansions close to 1.

V. Uncoded Transmission

It is known that for an additive white Gaussian channel and a source with squared-error distortion metric with $b = 1$, uncoded transmission (UT) is optimal [7]. Motivated by this, here we find the distortion exponent of an uncoded system for the quasi-static fading scenario, and compare its performance to LS and BS for different bandwidth expansions.

For $b \geq 1$, we transmit each uncoded source sample in one use of the channel. Note that for $b > 1$, this results in $N - K$ channel uses for which the transmitter is silent. Hence for $b > 1$, we allow the transmitter to scale its power to bP when it is transmitting. For $b < 1$, only the first N source samples are transmitted, and the remaining $K - N$ are assumed to be received with maximum distortion. Using an MMSE estimator at the destination for optimal detection, we find:

$$ED = \begin{cases} E_a \left[\frac{1}{1+bSNRa}\right] & \text{if } b \geq 1; \\ E_a \left[1 - b + \frac{b}{1+SNRa}\right] & \text{if } b < 1, \end{cases} \quad (13)$$

where $E_a[.]$ corresponds to expectation over channel states. Using [8], we can argue that the uncoded transmission scheme explained above in fact results in an optimal linear encoding of the source. The corresponding distortion can not be improved by any other linear transformation of the source vector.

We use the exponential integral approximation to find $\triangle$ [11]. For $b \geq 1$ we have:

$$\begin{aligned} ED &= E_a[\frac{1}{1 + bSNRa}], \\ &\approx \frac{e^{1/bSNR}}{bSNR} E_1(\frac{1}{bSNR}), \end{aligned} \quad (14)$$

where $E_1(z) = -\gamma - \ln z - \sum_{n=1}^{\infty} \frac{(-z)^n}{n.n!}$, and γ is the Euler constant. Then in the high SNR regime expected distortion can be expressed as $ED \approx SNR^{-1+\frac{\log(\log SNR)}{\log SNR}}$. Obviously $\triangle$ converges to 1 as SNR increases, however the rate of convergence is slower due to the second term.

For the $b < 1$ case, $\triangle = 0$ since there is a constant term in the ED expression, which means that for UT when the source bandwidth is higher than the channel bandwidth, distortion does not approach 0 with increasing SNR.

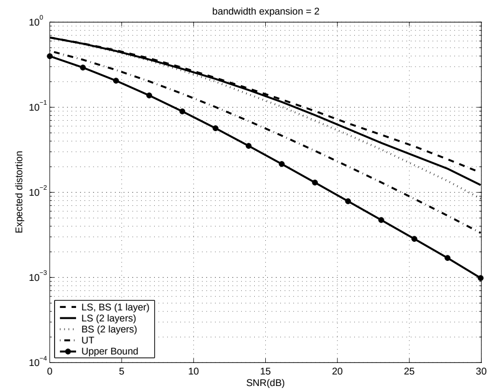

Fig. 4. Expected distortion vs. SNR plots for $b = 2$. The topmost curve LS, BS (1 layer) corresponds to direct transmission without layering.

VI. Upper Bound

To find an upper bound for the distortion exponent, we assume the availability of the perfect channel state information (CSI) at the transmitter. However, we impose that this information is only used for rate adaptation, and not utilized for power adaptation. Since power adaptation is not possible without CSI, this is still an idealization, and gives us an upper bound for $\triangle$.

When the channel state for a given transmission block is known at both the transmitter and the receiver, each transmission block can be seen as an additive white Gaussian noise channel, and thus the source-channel separation theorem applies. The minimum distortion, D achieved at the destination for a given channel state is

$$D = 2^{-b \log(1+aSNR)} = \frac{1}{(1 + aSNR)^b}. \quad (15)$$

Then the minimum expected distortion is

$$\begin{aligned} ED &= E_a \left[\frac{1}{(1 + aSNR)^b}\right], \\ &= \frac{e^{1/SNR}}{SNR} \int_1^{\infty} \frac{e^{-t/SNR}}{t^b} dt. \end{aligned} \quad (16)$$

For integer values of $b = 1, 2, ...$ using the exponential integral approximation for high SNR [11], this can be simplified to

$$\begin{aligned} ED \approx & \frac{e^{1/SNR}}{SNR} [\frac{(-SNR)^{1-b}}{(b-1)!} (\ln SNR + \psi(b)) \\ & - \sum_{m=0, m \neq b-1}^{\infty} \frac{(-SNR)^{-m}}{(m - b + 1)m!}], \end{aligned} \quad (17)$$

where $\psi(n) = -\gamma + \sum_{m=1}^{n-1} \frac{1}{n}$, and γ is the Euler constant. Non-integer values can be dealt similarly with the corresponding Euler expansion [11]. High SNR analysis yields $\triangle = 1$ for $b \geq 1$.

For $0 < b < 1$, we will use the gamma function $\Gamma(z) = k^z \int_0^{\infty} t^{z-1} e^{-kt} dt$ for $z > 0$ and $k > 0$ and the Euler function

$\Gamma(z) = \lim_{n\to\infty} \frac{n! n^z}{z(z+1)...(z+n)}$ to find $\triangle$.

$$ED = \frac{e^{1/SNR}}{SNR}\int_1^\infty \frac{e^{-t/SNR}}{t^b}dt, \quad (18)$$

$$\leq \frac{e^{1/SNR}}{SNR} SNR^{1-b}\Gamma(1-b). \quad (19)$$

High SNR analysis results in $\triangle \leq b$.

VII. Discussion of The Results

The results for the distortion exponent are summarized in Fig. 3 and Table I. We observe that BS with infinite layers is optimal with respect to the distortion exponent as it meets the upper bound. We also see that BS with the same number of layers outperforms LS for given bandwidth expansion. However, the encoder-decoder pair required for LS is simpler than the ones required for BS, because BS requires SNR-dependent power allocation among layers, superimposition of codewords and sequential decoding.

Uncoded transmission for $b \geq 1$ is optimal in terms of distortion exponent. However, as it is shown in [3] this performance improvement can not be extended to more complex networks, and thus the applicability is doubtful.

In order to illustrate how the suggested source-channel coding techniques perform for arbitrary SNR values, we have plotted expected distortion vs. SNR for the usual direct transmission (LS, BS with 1 layer), LS and BS with 2 layers, UT and upper bound in Fig. 4 for $b = 2$. The results are obtained from an exhaustive search over all possible rate, channel and power allocations. The figure illustrates that the theoretical performance results that were found as a result of the high SNR analysis hold, in general, even for moderate SNR values. However we should note that the slopes for UT and the upper bound shown in the figure are smaller than the calculated $\triangle$ values. This is due to the slow convergence rate of ED as argued in Section V.

VIII. Generalization to Other Sources

Throughout this paper, we have used the Gaussian source assumption. This made it possible to use the known distortion-rate function for complex Gaussian source and also to utilize the successive refinable nature of this specific source. However, our results hold for any memoryless source with finite differential entropy and finite second moment. Recall that the distortion of a Gaussian source at a given rate is a tight upper bound for other sources with squared error distortion in the high resolution regime, that is when R is large [9]. Furthermore, although most sources are not successively refinable, it was proven in [10] that all sources are nearly successively refinable. Due to Corollary 1 in [10], for any $0 < D_M < ... < D_2 < D_1$, $(M \geq 2)$ and squared error distortion, there exists an achievable M-tuple with $L_k \leq 1/2$, $k \in \{1, ..., M\}$, where $L_k = R_k - R(D_k)$ is the rate loss at step k. This means that to achieve the distortion levels we used in our analysis, we need to compress the source at a rate that is at most $1/2$ bits/sample greater than the rates required for Gaussian source. Then the transmission rate over the channel for non-Gaussian source, R' satisfies $R' \leq R + \frac{1}{2b}$, where R is the rate used for the Gaussian source. However, since we scale the operating rate with increasing SNR as $r \log SNR$, the high SNR behavior of the outage probability does not change by relaxing the Gaussian assumption.

TABLE I

Distortion exponent ($\triangle$) in terms of bandwidth expansion (b) and the number of layers (n).

Strategy	Distortion Exponent
Layered Source with Progressive Transmission	$1 - 1/(\frac{b}{n}+1)^n$
Broadcast Strategy with Layered Source	$1 - 1/(1+b+...+b^n)$
Uncoded Transmission	$\begin{cases} 0 & \text{if } b < 1; \\ 1 & \text{if } b \geq 1 \end{cases}$
Upper Bound	$\begin{cases} b & \text{if } b < 1; \\ 1 & \text{if } b \geq 1 \end{cases}$

IX. Conclusion

We consider the expected distortion (ED) of a system which transmits a continuous amplitude source over a quasi-static fading channel. Due to non-ergodic nature of the channel, the optimal performance can be achieved by joint source-channel optimization. We use distortion exponent ($\triangle$) as the performance metric, which is the exponential decay rate of ED, and show how the optimal performance can be approached by joint source and channel coding strategies with optimized parameters. We are currently extending our results to a MIMO system with block fading and possible application to the cooperation scenario, where a third terminal is available for relaying the source information can be found in [2], [3].

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