Opportunistic Cooperation and Power Control Strategies for Delay-Limited Capacity

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Abstract —

In this paper, we study the delay-limited capacity of a cooperative relay network where the terminals are constrained by half-duplex assumption and average total network power. We show that, contrary to a single source-single destination case, where both terminals have only one antenna, a non-zero delay-limited capacity is achievable. Furthermore, we introduce simple transmission protocols which utilize the relay depending on the network channel state, and considerably improve the delay-limited capacity of the system compared to 'always' cooperate type protocols. This emphasizes the importance of feedback for cooperative systems that have delay sensitive applications.

I. Introduction

User cooperation has emerged as a spatial diversity technique to provide robustness against channel fluctuations in wireless environments [1], [2]. Cooperative diversity systems have been extensively studied in terms of different techniques and different performance metrics.

The main setting subject to analysis is composed of a single source-single destination pair and a relay terminal possibly assisting the communication among them as shown in Fig. 1. The channels among the terminals are modeled as independent quasi-static fading channels. Then the suitable performance metric is the outage probability which is shown to be the lower bound for the frame error rate of a coded system.

Most of the research on cooperation is based on the assumption of no channel state information at the transmitters (CSIT), where only the channel statistics is known, while receivers have the perfect channel state information (CSIR). Different from the main body of the research work on the subject, here we will follow the approach in [3], [4], [5] and assume the existence of perfect CSIT and CSIR. We will analyze the delay-limited capacity of the system. The availability of the CSIT allows the terminals to adapt their power levels and transmission times depending on the channel states, thus achieve a significant improvement in the performance. We will also offer an opportunistic cooperation strategy where the source-destination pair utilizes the available relay resources only from time to time depending on the channel states.

The analysis of the delay-limited capacity of a slowly fading communication channel is given in [6]. Delay limited capacity corresponds to the maximum achievable rate obtained by keeping the instantaneous mutual information constant with the help of power control strategies. This approach is justified by the delay sensitive applications such as voice and video communications. Here the channel state information is essential, since otherwise for any pre-assigned rate and power values, there is a non-zero probability of outage. In [6] the delay-limited capacity region of the multiaccess fading channel and the associated optimal power control strategy is characterized.

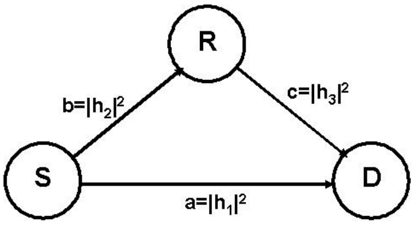

Figure 1: Illustration of the cooperative system model.

In [7] different simple protocols are offered to attain a lower outage probability through cooperation. Other than the incremental amplify-and-forward protocol which utilizes a single bit feedback, all of the other protocols offered in [7] or most of the subsequent work assume no CSIT, and thus no signal or power adaptation. In [3] authors assume perfect CSIT and study outage probability minimization for different protocols. Their analysis is mainly based on the full duplex assumption, i.e., the capability of simultaneous transmission and reception at the relay. The study of non-simultaneous relay transmission and reception is through amplify and forward protocol where they assume a fixed cooperation strategy, i.e, source and relay always cooperate regardless of the channel states. The authors show the importance of feedback for achieving minimum outage probability and further in [4] they show that even limited feedback improves the performance. In [5] effect of CSIT on ergodic capacity is studied.

In this work we consider a half-duplex system where the relay cannot transmit and receive at the same time, and analyze a decode and forward type strategy from the delay limited capacity perspective. In our scenario only the amplitudes of the channel states are available to the source and the relay. They either do not have, or do not utilize the phase information, thus the coherent combination of the source and the relay signals is not possible. Hence, unlike [9, 10] there is no benefit of the source and the relay transmitting at the same time. Channel state information is only utilized for power adapta-

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[1] This work was supported in part by NSF Grant No. 0430885.

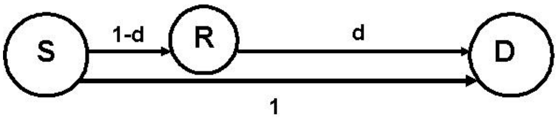

Figure 2: The model for the source, the relay and the destination locations.

tion and optimization of the proportion of the time that the relay receives information from the source.

The results obtained in this paper show that feedback, on top of cooperation will help the mobile terminals attain improved battery life and transmission rate by simple power allocation algorithms. Furthermore, it is shown that an opportunistic approach to cooperation, or 'asking for help when you really need', improves the performance of the overall communication in the delay-limited capacity sense, compared to the previously analyzed fixed strategies.

The outline of the paper is as follows: In Section II, the network model that is subject to our analysis will be introduced. In Section III, we will briefly make an analysis of the delay-limited capacity for the fixed decode-and-forward protocol. In Section IV, we will explain the opportunistic cooperation strategy. In Section V, an upper bound is found for the performance of the system. Section VI is devoted to the analysis of the numerical results and their discussions. Section VII includes the conclusion and the future work.

II. System Model

Our system consists of a single source(S), single destination(D) pair and an available relay(R) as shown in Fig. 1, where each terminal has one antenna. The links among the terminals are modelled as having quasi-static Rayleigh fading that are independent. The fading coefficients are denoted as h_i, $i \in \{1, 2, 3\}$ and they are circularly symmetric Gaussian random variables with zero mean. There is also additive white Gaussian noise with unit variance at each receiver.

Amplitude squares of the channel coefficients, which are exponentially distributed with λ_a, λ_b, and λ_c will be denoted by a,b and c as shown in Figure 1, i.e., $a = |h_1|^2$, $b = |h_2|^2$, and $c = |h_3|^2$. The parameters for the exponential distributions capture the effect of pathloss across the corresponding link. To consider the effect of the relay location on the performance of the network, we will follow the model in Fig. 2 and assume that the relay is located on the line joining the source and the destination. We will normalize the distance between the source and the destination and denote the relay-destination distance as d and the source-relay distance as $1 - d$, where $0 < d < 1$. Then the overall network channel state, $\mathbf{s} = (a, b, c)$ becomes a 3-tuple of independent exponential random variables with means $\lambda_a = 1$, $\lambda_b = \frac{1}{(1-d)^\alpha}$, and $\lambda_c = \frac{1}{d^\alpha}$, respectively, where α is the pathloss exponent. We will consider $\alpha = 1.5$ throughout the paper.

We assume that all the channel states a, b, and c are known at the source, the relay and at the destination, while the phase information is only available at the corresponding receivers. Furthermore, we assume that there is an average power limitation, P_{avg} on the total average power used by the network.

We constrain the terminals to employ half-duplex transmission, i.e., they are not allowed to transmit and receive simultaneously. The protocol for cooperation that we will suggest and analyze is based on the decode-and-forward (DF) protocol of [7]. Basically, the time slot or the channel frame of the source terminal, which corresponds to one fading block of the channel, is divided into two. In the first half of the time slot, the source transmits to both the relay and the destination, and in the second half if the relay decodes the message, it forwards the same message to the destination. The destination, receiving two copies of the same message from two independent fading channels combines them.

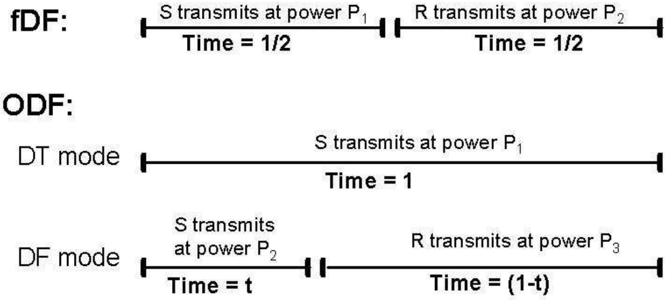

Figure 3: Cooperation protocols, fixed decode-and-forward (fDF) and opportunistic decode-and-forward (ODF).

In the DF protocol defined in [7] the relay remains silent if it cannot decode the information after listening to the source. However in our system, due to the availability of the channel state information, when the source decides to utilize the relay, it transmits at a power level that guarantees decoding at the relay.

Our cooperation strategy optimizes the power allocation among the source and the relay, subject to the total power constraint P_{avg}, as well as the time that the source transmits based on the the instantaneous channel gains to maximize the delay-limited capacity. This is illustrated in Fig. 3 as the ODF protocol whose details will be discussed in Section IV. We will allow the source to transmit its message directly to the destination throughout its whole time slot depending on the channel state information. Naturally, this is preferable in some channel states as we have a total power constraint for the source and the relay which means that the relay power cannot be utilized without cost.

Note that since the phases of the fading coefficients are not known at the transmitters, there is no coherent combination of the source and the relay signals and they do not need to transmit simultaneously to the destination after the relay listens to the source as in [9, 10]. Since we consider delay-limited capacity and require the relay to decode the message for cooperation, simultaneous transmission would result in a performance loss.

III. Delay-Limited Capacity of Fixed Decode-and-Forward

We know that the single-user delay-limited power control strategy is 'channel inversion', i.e., the power level corresponding to fading amplitude a is $P(a) = 1/a$. However, for Rayleigh fading, the exponentially distributed fading amplitude has a probability density $f_a = (1/\lambda_a)exp(-a/\lambda_a)$ for which $\int_0^\infty f(a)a^{-1}da = \infty$. Thus the delay-limited capacity is zero for a finite expected power limitation, P_{avg} [6].

However, it is known that a non-zero delay-limited capacity can be achieved with multiple antennas in case of Rayleigh fading [8], where the channels corresponding to different antennas are uncorrelated. We will show that cooperation also provides a positive delay-limited capacity due to its increased diversity.

First we consider a simple non-opportunistic fixed decode-and-forward (fDF) strategy based on a minor modification of the DF protocol of [7]. We utilize the channel state information in the simplest way to achieve a non-zero delay-limited capacity. Here, independent of the channel conditions, the time slot is equally divided into two. In the first half, the source transmits at a power level that guarantees the decoding of the message either at the relay, or at the destination depending on the overall channel conditions.

We should note that, in the DF protocol of [7], the message is transmitted over the relay independent of the channel states. However, in the delay-limited capacity case, this would require a power level which would guarantee the relay decoding at the end of the first half. This would end up in a delay-limited capacity of zero as in the direct transmission case. Similarly, the decision to choose between direct transmission or decode-and-forward should also consider the state of the relay-destination channel. Otherwise, the delay-limited capacity would again be zero as it will be limited by the relay-destination channel.

In the case of message going through the relay (when the source-relay and relay-destination channels are both better than the source-destination channel), the relay retransmits the message with an independent Gaussian codebook and at a power level that is enough for the destination to decode the message by combining the signals from the source and the relay(Fig. 3). As argued in [9, 11], independent codebook usage increases the maximum mutual information, thus the rate at the receiver compared to repeating the message using the same codebook.

Our goal is to achieve maximum constant transmission rate over varying fading coefficients with zero outage probability by using a power adaptation strategy with average total power below P_{avg}. Let P_1 be the source transmit power during the first half of the time slot, and P_2 be the relay transmit power in the second half of the time slot. Both P_1 and P_2 are allowed to depend on the channel amplitude vector $\mathbf{s}$. Then for the delay limited capacity of the fixed decode-and-forward (fDF) strategy, in case of cooperation, we have:

$$C_d^{fDF} = \frac{1}{2}\log(1+P_1 b) = \frac{1}{2}\log(1+P_1 a)+\frac{1}{2}\log(1+P_2 c), \quad (1)$$

where 1/2 is due to the time-sharing.

We should note that $P_2 = 0$ when $a \geq b$ or $a \geq c$. In this case

$$C_d^{fDF} = \frac{1}{2}\log(1+P_1 a). \quad (2)$$

Then we can summarize the optimization problem for fDF as:

$$\begin{array}{ll} \max C_d^{fDF}, & \\ \text{subject to} \quad E\left[\frac{P_1+P_2}{2}\right] \leq P_{avg}, & \end{array} \quad (3)$$

where P_1 and P_2 satisfy equations (1) and (2).

The numerical results for the delay-limited capacity of fDF and its comparison with the ODF protocol that will be introduced in the next section can be found in Section VI.

IV. DELAY-LIMITED CAPACITY OF OPPORTUNISTIC DECODE-AND-FORWARD

In the fDF protocol, we force the source to send its message through the relay. However this may result in a degradation of the performance depending on the channel gains. In opportunistic decode-and-forward (ODF) we let the terminals to operate in two different modes, direct transmission(DT) mode or decode and forward(DF) mode. In DT mode, the source transmits directly to the destination throughout the whole time slot and the relay neither tries to decode the message nor transmits at any portion of this time slot. In DF mode, however, the source first transmits its message to the relay, and the relay decodes and retransmits this message using an independent Gaussian codebook. Let P_1 be the source power in DT mode, and P_2 and P_3 be the source and the relay powers in DF mode. Note that all the powers are functions of the channel state vector $\mathbf{s}$.

In ODF, the source and the relay divide the time slot into two parts which are not necessarily equal. We will introduce the parameter t $(0 \leq t \leq 1)$ for optimization of the time allocation, where the time slot is normalized as 1. This is illustrated in 3. Note that t also depends on $\mathbf{s}$. Let $\mathbf{A}$ be the set of network states that DT is used and $\mathbf{A^c}$ be the set of states that DF is used.

The instantaneous capacity, C_d for each mode can be written as

$$\begin{aligned} C_d^{DT} &= \log(1+P_1 a), && (4)\\ C_d^{DF} &= t\log(1+P_2 b), && (5)\\ &= t\log(1+P_2 a)+(1-t)\log(1+P_3 c). && (6) \end{aligned}$$

Since we want to achieve a constant rate, say R, for all channel states with zero outage probability, we want the instantaneous capacity achieved at DT mode (Eqn. 4) to be equal to the instantaneous capacity achieved at DF mode (Eqn. 5, 6), both of them being R. Then the delay-limited capacity maximization problem can be stated as

$$\begin{array}{lll} \max & R = C_d^{DT} = C_d^{DF}, & \\ \text{s.t.} & E[P_1|\mathbf{s}\in\mathbf{A}]P(\mathbf{s}\in\mathbf{A}) & +E[tP_2+(1-t)P_3|\mathbf{s}\in\mathbf{A^c}]\cdot \\ & \cdot P(\mathbf{s}\in\mathbf{A^c}) \leq P_{avg}. & \end{array} \quad (7)$$

Calculating the expected transmit power, $E[P]$ for each mode, we get the following:

$$E[P_1|\mathbf{s}\in\mathbf{A}] = (2^R-1)E\left[\frac{1}{a}|\mathbf{s}\in\mathbf{A}\right], \quad (8)$$

$$E[P_2|\mathbf{s}\in\mathbf{A^c}] = E[\frac{(2^{R/t}-1)}{b}|\mathbf{s}\in\mathbf{A^c}], \quad (9)$$

$$\begin{aligned} &E[P_3|\mathbf{s}\in\mathbf{A^c}] = \\ &E\left[\frac{1}{c}\left\{2^{R/(1-t)}\left[1+\frac{a}{b}\left(2^{R/t}-1)\right)\right]^{t/(t-1)}-1\right\}|\mathbf{s}\in\mathbf{A^c}\right]. \end{aligned} \quad (10)$$

Obviously DT is chosen when we have $a \geq b$ or $a \geq c$, because there is no need to ask for the relay's help when the source-destination channel is better than the source-relay or the relay-destination channels. If $a < b$ and $a < c$, then the source prefers to transmit the message over the relay if the

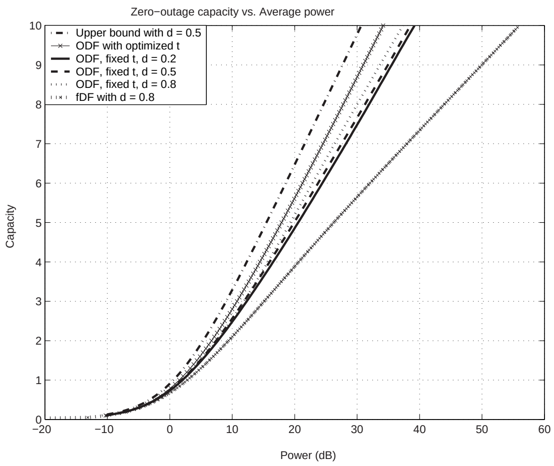

Figure 4: Delay-limited capacity vs. average sum power.

required power for DT is more than the required power for DF for that specific channel realization. Hence

$$\mathbf{s} \in \mathbf{A} \iff a \geq b \text{ or } a \geq c \text{ or } P_1 < tP_2 + (1-t)P_3 \text{ for } \forall t \in (0,1), \tag{11}$$

ODF is a more advanced protocol compared to fDF in three aspects: i) dynamic time allocation, ii) more advanced decision rule among DT and DF modes, iii) more efficient usage of the channel in case of DT. However, it still keeps the simple nature of the decode-and-forward protocol in the coding sense.

The analytic computation of the delay limited capacity based on this opportunistic cooperation, where an optimal power and time allocation strategy is used, is hard to obtain. Thus, our analysis will be based on the numerical results. To simplify our numerical calculations, instead of solving (7), we will solve the equivalent optimization problem of minimizing the average total power to achieve a target delay-limited capacity. Numerical results show that significant performance improvement is provided by ODF.

V. UPPER BOUND

In this section, we want to find out how far we are from the optimal gain by using a simple protocol like ODF. Although the upper bound we will introduce is loose, we will see that ODF performs close to this bound for moderate power levels.

We consider two different upper bounds for the delay-limited capacity corresponding to each relay location. For the first upper bound, we will consider the case where the source message is assumed to exist at the relay a priori. Thus, this is equivalent to a MISO channel with two transmit antennas where coherent combination of the messages from these antennas is not allowed. Basically, only one of the terminals with the best instantaneous channel state transmits during each time slot.

For the second upper bound, we assume that the relay has a direct connection to the destination, but joint decoding is not allowed. This is equivalent to the source trying to send its message to either of the relay or the destination whichever has the the best channel quality at the specific time slot.

Since both of these case are idealizations of our relaying scenario, they both upper bound the performance of any protocol and the smaller one for each channel realization will be the tighter one.

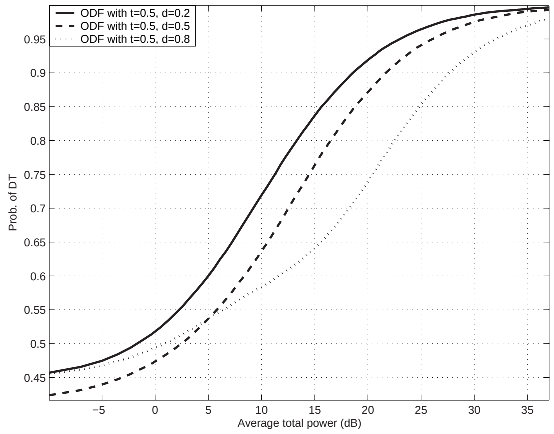

Figure 5: Probability of using direct transmission vs. average sum power constraint for ODF with fixed time allocation of t=1/2.

Mathematically, the upper bound becomes

$$\max \ \log\left[1 + P \min(\max(a,c), \max(a,b))\right], \tag{12}$$

$$\text{subject to } E[P] \leq P_{avg}. \tag{13}$$

VI. NUMERICAL RESULTS AND DISCUSSIONS

Fig. 4 illustrates the delay-limited capacity vs. the average total power constraint of the system for various communication scenarios. The topmost curve corresponds to the upper bound for a relay location of $d = 0.5$. Since this is the worst relay location for our upper bound computation, this curve also serves as an overall bound for any relay location. We did not include upper bounds corresponding to other relay locations for clarity of the figure.

The fDF curve shows that although its performance is inferior to the upper bound and the ODF protocol, it can still achieve a nonzero delay-limited capacity which is not possible without cooperation. This proves the importance of cooperation, even in the simplest form, on the performance of delay-limited systems.

The results for ODF protocol show that the improvement compared to the fDF protocol is considerable. We also include results for ODF protocol with different relay locations where time allocation is fixed with $t = 1/2$. We see that, optimization of time allocation improves the performance of ODF protocol compared to the case where the time allocation between the source and the relay is fixed in the DF mode. The curve corresponding to 'ODF with optimized t' is for a relay location of $d = 0.8$, however different relay locations result in almost the same performance for the average total power levels under considerations. This means that, in case of time allocation optimization, any relay will serve the source to obtain most of the highest possible capacity. This result is in contrast with [12], where it was shown that DF type strategies are suboptimal when the relay is close to the destination.

A very important characteristic of the ODF protocol is the fact that the relay is not utilized all the time. In fact the probability of the channel states that result in cooperation decreases with increasing available sum power. In Fig. 5 we plot the probability of operating in DT mode with respect to the sum power constraint when $t = 1/2$. We observe that

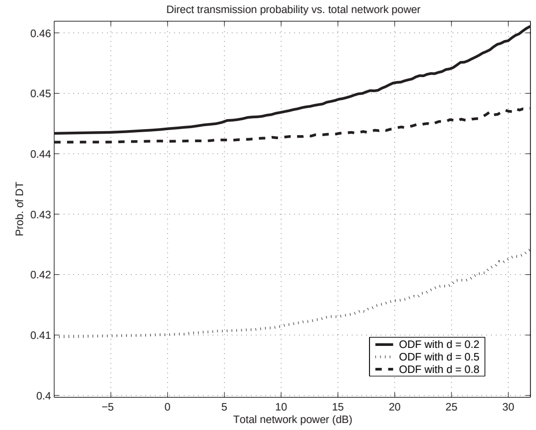

Figure 6: Probability of using direct transmission vs. average sum power constraint for ODF with time allocation optimization.

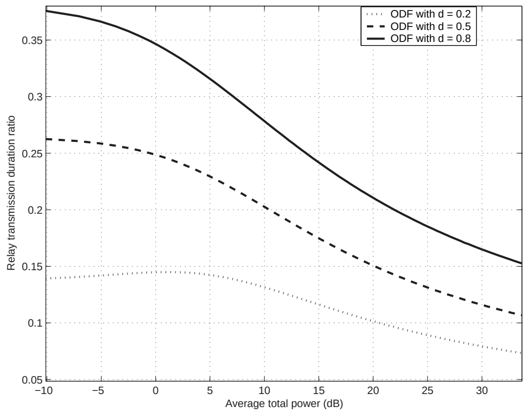

Figure 7: The ratio of relay transmission duration to the whole time slot (E[1-t]) vs. average sum power constraint for ODF with time allocation optimization.

the proportion of the states where the source utilizes the relay for cooperation depends on the relative channel qualities and decreases to zero with increasing sum power constraint. However, in Fig. 6, where the probability of direct transmission in case of ODF with optimized t is plotted, we see that cooperation is now more probable. This is due to the fact that, for some channel realizations, although cooperation with $t = 1/2$ may be worse than DT, it might be possible to improve the performance with some other time allocation. Fig. 7 shows the relay transmission duration ratio that is the expected value of $(1 - t)$, in case of ODF with time allocation optimization. Here we see that, for increasing total power, although cooperation occurs almost with probability 1/2 (Fig. 6), most of the cases result in a time allocation where the relay transmits only a small amount of time, i.e., the relay listens most of the time to transmit for a shorter time period.

In both cases, either with optimized or fixed t, the improvement in the performance comes with a limited use of the relay terminal, and the relay utilization depends on the relay location. This feature becomes more important when considered in a network with more than two mobile units. Then using opportunistic strategies, it is possible that a relay helps multiple sources communicating to different destinations simultaneously by relaying one of the sources (the one with the worst direct channel) at each time slot, but still improving the delay limited capacity for all the sources. This scenario becomes more meaningful for ad hoc and/or sensor networks where simultaneous transmissions and relaying are essential. Thus, there is a tradeoff between less relay utilization by fixed time allocation, and higher delay-limited capacity by time allocation optimization. Again for such networks, keeping relay silent as much as possible, might improve the overall network performance as it will decrease the interference caused by the relay as well. If interference is our prior concern, then time allocation optimization becomes more attractive as it keeps the relay in the listening mode for most part of the time slot (Fig. 7).

Furthermore, in Fig. 8 we show the ratio of the average power spent by the relay to the total average network power with respect to the sum power constraint where time allocation is optimized. The ratio is always less than 1/2 and very small for a relay relatively close to the source. We also observe that the ratio goes to zero with increasing sum power constraint. One basic concern about cooperative protocols in general is the lack of incentives for the terminals to help each other. In cooperation protocols where CSIT is not available, terminals relay information independent from the channel states, which means that they spend half of their battery power for helping their partner. However, with opportunistic cooperation the amount of power dedicated to relaying is reduced to a minimal amount which makes it easier to promote cooperation. Again in a denser network scenario, where multiple candidates are available for relaying, it is possible to pick the node that requires the least relay power for achieving the same delay-limited capacity. This will significantly reduce the resources spent by the relay terminal.

VII. Conclusion

In this paper we assumed the availability of the perfect channel state information both at the transmitters and at the receivers and analyzed the delay-limited capacity of a relay system with an average sum network power constraint. Based on a network model consisting of one source, one destination and one relay, we showed that the channel state information at the transmitter makes it possible to achieve a nonzero delay-limited capacity for delay sensitive communications with the help of cooperation. Furthermore, we introduced the opportunistic decode-and-forward cooperation protocol (ODF) where the relay terminal is utilized depending on the channel states among all terminals. We showed that ODF brings a considerable improvement to the delay-limited capacity with a limited use of relay resources. Our future work will include the minimization of the outage probability using ODF in cases where the required transmission rate is above the delay-limited capacity. We will also explore the effects of limited feedback on the performance improvement achieved by the ODF protocol.

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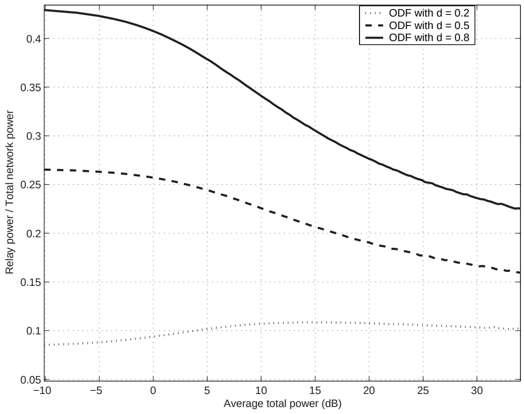

Figure 8: The ratio of the relay power to the total network power vs. sum power constraint for various relay locations.

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