Layered Cooperative Source and Channel Coding

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Abstract— **Cooperative techniques form a new wireless communication paradigm in which terminals help each other in relaying information to combat the random fading and to provide diversity in radio channels. Past work has focused on improving channel reliability through cooperation. We propose to jointly allocate bits among source coding, channel coding and cooperation to minimize the expected source distortion. Recognizing that not all source bits are equal, we further propose to protect the more important bits through user cooperation. To evaluate the gain of layered cooperation, we simulate four modes of communications that differ in their error protection strategy (equal vs. layered, with vs. without cooperation) with a practical channel coder, and show that, for i.i.d. Gaussian sources, layered cooperation can achieve significant performance gains over non-layered/non-cooperative communication. We also carry out an information theoretic analysis illustrating fundamental benefits of layered cooperation.**

Keywords- cooperative diversity; unequal error protection; layered compression; source and channel coding

I. INTRODUCTION

The wireless channel suffers from fading and multi-path distortion. It is now widely agreed that multiple transmit and receive antennas at both the mobile nodes and the base station/access can provide robustness against channel variations and substantially improve performance. However, the size of mobile devices limits the number of antennas that can be deployed, and it is essential to utilize other methods to obtain diversity. Cooperative communication techniques provide spatial diversity through the use of antennas that belong to different terminals. In a wireless environment, the signal transmitted by the source node can be "overheard" by other nodes, which we call partners. Cooperation is achieved through partners processing and re-transmitting the signals they receive. The destination combines the signals coming from the source and the partners, thereby creating spatial diversity.

Sendonaris et. al [1,2] were among the first to realize that even though the terminals are connected by noisy fading links, cooperation can provide benefits similar to those of multiple input-multiple output (MIMO) systems. It is shown that cooperation of wireless nodes not only results in robustness to channel variations, but also higher throughput, extended battery life for nodes and extended coverage [1,2]. Laneman et. al. [3] proposed various cooperative protocols and studied the outage probability to illustrate potential diversity gains. Motivated by the above information theoretic gains, research on the physical layer of cooperation explored design of cooperative channel codes. Hunter and Nosratinia [4] proposed a scheme involving rate compatible punctured convolutional (RCPC) codes and cyclic redundancy check (CRC) for error detection. Stefanov and Erkip [5] presented analytical and simulation results studying the diversity gains and suggested guidelines on how good cooperative codes can be designed.

Past work as reviewed above has mainly focused on how to improve channel reliability (or equivalently throughput for lossless data transmission) through cooperation. For transmission of multimedia signals, a more relevant performance measure is the end-to-end distortion caused by source compression and channel errors. In cooperative communication, for a given total channel usage, this distortion depends on both source and channel coding techniques, and the bit allocation among source coding, channel coding, and cooperation. We propose to jointly design source and channel coders, and to optimally allocate bits to minimize the end-to-end distortion. We further code the source into layers of different importance and apply unequal error protection through assigning different number of channel and cooperation bits to different layers. Our goal is to study the net effect of scalable coding and cooperative diversity on the overall source distortion. We also investigate information theoretic limits of source and channel cooperation and illustrate benefits of layered cooperative coding when ideal source and channel coders are utilized.

In the remainder of this paper, we first describe the four modes of communications that differ in their error protection strategy (equal vs. layered, with vs. without cooperation) and formulate the optimal bit allocation problems of these modes under a popular wireless channel model. We then compare the performance of the four communication modes, each under optimal bit allocation, for the i.i.d. Gaussian source. Section 4 reports the results obtained through simulation using the RCPC channel codes. Section 5 presents analysis results using an information theoretic approach. Section 6 summarizes this work.

II. FOUR MODES OF COOPERATION

We consider two active source terminals denoted by T1 and T2. In a cellular or wireless LAN system, these terminals cooperatively communicate with a common destination, the base station or the access point. In an ad-hoc network, the destinations for T1 and T2 could be different. Even though cooperative source and channel coding principles apply equally well to both scenarios and for larger number of terminals, for ease of exposure in this paper we consider two terminals communicating with a common destination and ignore interference from other wireless devices. This is illustrated in Figure 1 for a cellular system where the common destination is the base station (BS). We use S1 and S2 to denote the source information to be sent by terminals T1 and T2, respectively.

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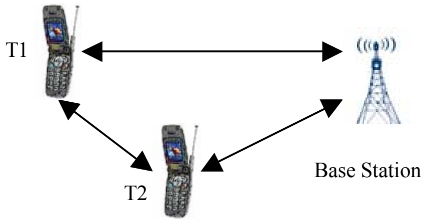

Figure 1: Illustration of user cooperation. Here, T1 and T2 cooperate to both send information to the base station..

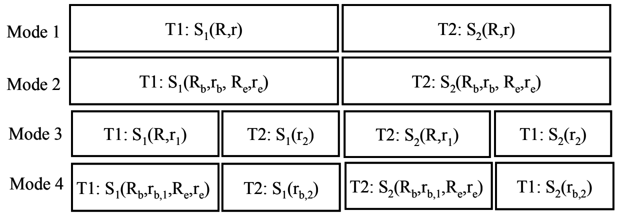

Figure 2: Four modes of communication.

Because of random fading, at any time, the channel between T1 and the BS may be very bad, whereas the channel between T1 and T2 and that between T2 and the BS may be good. In this case, it may be beneficial for T2 to listen to T1, and send S1 to the BS for T1. Similarly, T1 may help T2 in a symmetric manner. Assuming that T1 and T2 use time division multiplexing to share the channel, we consider the following modes of cooperation, illustrated in Figure 2.

For Sections 3 and 4 we assume that the modulation technique is fixed. We transmit B_0 bits in each channel use and each time slot includes N channel uses, allowing a maximum of B_0 N bits in each time slot. For Section 5, we consider an information theoretic approach which uses the best modulation scheme. We also assume within each time slot, M source samples are sent, with a maximum of $R_t = NB_0/M$ bits/sample. Generally, the B_0 N bits in a slot can be distributed among source coding, channel coding (the parity bits sent by the user itself), and cooperation (the parity bits sent by the partner). The four modes below differ in how the total number of bits is distributed among source coding, channel coding and cooperation. Although the actual bit allocation is done over all M samples, the description below is in terms of bits/sample.

Mode 1 (no cooperation or direct transmission): In this case, each terminal sends its own information using alternating time slots. T1 codes S1 using R source bits, applies a rate $R/(R+r)$ channel code, and transmits the resulting $R+r$ bits directly to the destination. The constraint is $R+r<=R_t$. Then T2 uses R source coding bits and r channel coding bits for S2.[1]

Mode 2 (scalable coding without cooperation): This is the conventional scalable coding with unequal error protection (UEP) through channel coding. In this case, S1 is coded into two layers, with R_b bits for the base-layer and R_e bits for the enhancement layer. We assume the base-layer is channel-coded with a rate $R_b/(R_b+r_b)$ code, and the enhancement layer is channel-coded with a rate $R_e/(R_e+r_e)$ code. Terminal T1 sends all the $R_b+r_b+R_e+r_e$ bits directly to the destination. The bit allocation must satisfy $R_b+r_b+ R_e+r_e<=R_t$.

Mode 3 (cooperative coding): Communication in this mode corresponds to cooperative coding of [4,5] but allows flexible bit allocation between source coding, channel coding, and cooperation. We assume that we have a rate $R/(R+r_1+r_2)$ channel code that can be punctured to rate $R/(R+r_1)$. Terminal 1 applies the $R/(R+r_1)$ channel code to R bits of S1, and sends a total of $R+r_1$ source and channel coding bits. If T2 can correctly decode the R source bits, which will be indicated by an error detection mechanism such as cyclic redundancy check (CRC) code, it will apply the $R/(R+r_1+r_2)$ channel code and send the remaining r_2 bits (called "*cooperation bits*") for T1. Otherwise, T2 will send a "frame error" signal to T1, and T1 will continue to send the r_2 channel bits itself.[2] In the next time slot, T2 sends S2 using $R+r_1$ source and channel coding bits, then T1 sends S2 using additional r_2 cooperation bits. The bit allocation must satisfy $R+r_1+r_2<=R_t$.

Mode 4 (layered cooperation): We propose this mode to allow for unequal error protection of source bits through both channel coding and cooperation. We assume the source is coded with R_b bits for the base-layer and R_e bits for the enhancement layer, and we apply cooperation only to the base-layer. Hence the partner only tries to decode the R_b source bits. The base-layer is protected by $r_{b,1}$ parity bits sent by the original terminal and $r_{b,2}$ cooperative parity bits sent by the partner. The enhancement-layer is protected by r_e parity bits. The bit allocation must satisfy $R_b + r_{b,1} + r_{b,2} + R_e + r_e <= R_t$. Note that mode 4 reduces to mode 3 when $R_e = r_e =0,$ to mode 2 when $r_{b,2} =0,$ and to mode 1 when $r_{b,2} =R_e = r_e =0.$

III. OPTIMAL BIT ALLOCATION PROBLEM

For each of the proceeding modes, there is an optimal allocation of source coding, channel coding and cooperation bits that will minimize the expected distortion (ED) at the receiver, given the source and channel characteristics. Because modes 1, 2 and 3 are special cases of mode 4, we only formulate the bit allocation problems for mode 4 below.

Source Model: The distortion at the receiver depends on both the distortion introduced by the source encoder as well as the channel characteristics. We use $D(R)$ to represent the distortion per source sample incurred by the source encoder when the source rate is R bits/sample. This distortion-rate function can be obtained theoretically or can be experimentally determined given a particular source model and source coder. Assuming the source is successively refinable (which is true for i.i.d. Gaussian sources), the distortion is $D(R_b+R_e)$ if both the base-layer and enhancements bits are correctly received, is $D(R_b)$ if only the base-layer bits are decodable, and is $D(0)$ if the base-

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[1] With a systematic channel code, the first R bits are the original source bits, the following r bits are parity bits. The proposed system can work with non-systematic channel codes as well. In either case, we call the additional r bits added by the channel coder as the "channel coding bits".

[2] In a simpler realization, T2 may choose not to transmit a frame error signal and the reserved channel slot for cooperation is not utilized.

layer bits are not decodable, regardless whether the enhancement-layer bits are decodable.

Channel model: We assume a wireless environment with flat, quasi-static fading and represent the fading level between terminal i and the destination by h_i. This link will be called channel i. Quasi-static fading suggests that h_i remains constant within each time slot. We consider Rayleigh fading which is independent for each terminal. The instantaneous fading levels are assumed to be accurately measured at the receivers, but not at the transmitters. Transmitters only know the statistics of the fading. The terminals can "overhear" each other's transmissions through another independently fading channel with quasi-static Rayleigh fading h_{12}. This overheard signal enables them to use each other's antennas in a cooperative fashion to obtain spatial diversity. Typically h_{12} and h_{21} will have the same statistics, enabling the terminals to help each other at the same rate.

To calculate the expected distortion for S1 (similarly for S2) at the receiver in mode 4, we define the following frame error rates (FERs). Throughout, SNR_i denotes average received signal to noise ratio in channel i (averaged over all fading values h_i) and $E_{hi,hj}$ denotes expectation with respect to fading levels h_i and h_j

- $P_{in} = P_1(R_b, r_b; SNR_{12})$ is the probability that the block of $R_b + r_b$ bits transmitted through the inter-partner channel are undecodable by the partner, averaged over inter–partner fading h_{12}.
- $P^1_{b|h_1} = P_1(R_b, r_{b,1}, r_{b,2}; SNR_1 \mid h_1)$ is the probability that the block of $R_b + r_{b,1} + r_{b,2}$ bits transmitted through channel 1 are undecodable by the destination, given a channel realization h_1.
- $P^2_{b|h_1,h_2} = P_1(R_b, r_{b,1}, r_{b,2}; SNR_1, SNR_2 \mid h_1, h_2)$ is the probability that the block of $R_b + r_{b,1} + r_{b,2}$ bits, with $R_b + r_{b,1}$ bits transmitted through channel 1, and $r_{b,2}$ bits through channel 2, are undecodable by the destination given channel realizations h_1, h_2.
- $P_{e|h_1} = P_1(R_e, r_e; SNR_1 \mid h_1)$ is the probability that the block of $R_e + r_e$ bits transmitted through channel 1 are undecodable by the destination, given channel realization h_1.

Note that the proceeding FER expressions depend on the bit allocation among source coding, channel coding and cooperation as well as the channel SNR. Then we have

$$\begin{aligned} ED = {} & \{(1-P_{in})E_{h_1,h_2}[(1-P^2_{b|h_1,h_2})(1-P_{e|h_1})] \\ & + P_{in}E_{h_1}[(1-P^1_{b|h_1})(1-P_{e|h_1})]\}D(R_b+R_e) \\ & + \{(1-P_{in})E_{h_1,h_2}[(1-P^2_{b|h_1,h_2})P_{e|h_1}] \\ & + P_{in}E_{h_1}[(1-P^1_{b|h_1})P_{e|h_1}]\}D(R_b) \\ & + \{(1-P_{in})E_{h_1,h_2}[P^2_{b|h_1,h_2}] + P_{in}E_{h_1}[P^1_{b|h_1}]\}D(0) \end{aligned} \qquad (1)$$

The bit allocation problem is to minimize the expected distortion in (1), subject to the respective rate constraints for different modes. Solving this problem analytically for real-world signal sources and source and channel coders are extremely challenging due to the lack of good models for $D(R)$ and various FER probabilities involved. Nevertheless it is possible to perform simulations to yield models for these functions, and then conduct numerical optimization to determine the optimal bit allocation.

IV. PERFORMANCE FOR I.I.D. GAUSSIAN SOURCES WITH RCPC CHANNEL CODES

We simulated the four modes assuming S1 and S2 are two independent i.i.d. unit variance Gaussian sequences. We use the well-known successively refinable Gaussian rate-distortion function [6] $D(R) = 2^{-2R}$ to characterize the source encoder.

The effect of the channel is simulated using the model described in Sec. 3. For channel coding, the rate 1/4 RCPC code [7] is used, which can be truncated to rate 1, 2/3, ½, 1/3. For a given channel realization and assumed bit allocation among source, channel, and cooperation, we use the Viterbi algorithm to decode the source bits from all received bits assuming the channel fading levels are known at the receivers. This allows us to determine the various FER's for channels 1 and 2 through simulations.

Figure 3 shows the minimal distortions achievable by different modes, for varying channel SNR between T1 or T2 and the destination. We assume the symmetric case in which the two user-to-destination channels have the same average received SNR. For the inter-partner channel, we assume a fixed average signal to noise ratio indicated by SNR_{12} and study the effect of varying SNR_{12} on the various modes of cooperation. For each mode and a given SNR, the optimal bit allocation is determined by searching through a chosen set of source coding rates, and channel code rates (including 1, 2/3, 1/2, 1/3, 1/4,), with the total rate constraint $R_t = 4$ ($N = 192$, $B_0 = 1$, $M = 48$).

Figure 3(a) shows the results when the inter-partner channel has an average received SNR of 30dB. First of all, we see that mode 2 consistently outperforms mode 1, which reveals the importance of unequal error protection (UEP). Secondly, we see that mode 3 and mode 4 have substantial gain over mode 1 and mode 2, respectively, by applying additional protection to all or important bits through cooperation. At high channel SNRs (14 dB and beyond), mode 3 does not perform as well as mode 2. This shows that at high channel SNR, applying cooperation equally to all bits is not as effective as unequal protection without cooperation. Mode 4 has a consistent gain over mode 3 except when SNR is very low (0dB) (at which point mode 4 operates with R_e=0 and thus reduces to mode 3). For the same distortion, mode 4 requires a channel SNR that is 1-3 dB lower than the best of the other three modes. For the same channel SNR, mode 4 yields a distortion that is 1-2 dB lower than the other modes.

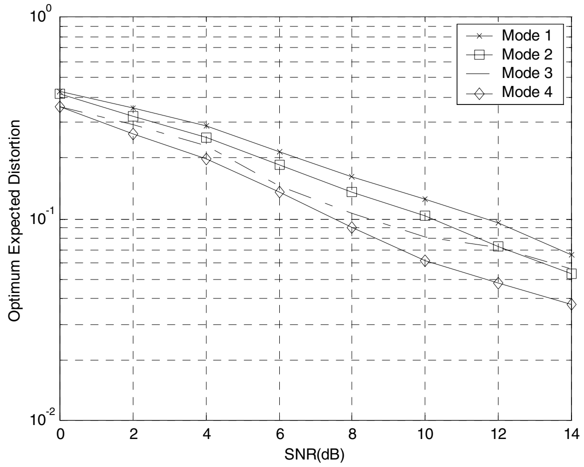
Figure 3(b) shows the comparison when the inter-partner channel is very poor with an average SNR of 0 dB. As expected, the gains of mode 3 and mode 4 over mode 1 and mode 2, respectively, are reduced compared to Figure 3(a). In fact, mode 2 is better than mode 3 for channel SNR>=2dB. This is because when the channel SNR is not too bad, it is more effective to protect important source bits through channel coding, than to protect all source bits equally through channel coding/cooperation, for bad inter-partner channel. It is encouraging; however, that mode 4 still outperforms all other modes over a large range of SNR.

The optimal bit allocations within each channel frame (total bits=192) used by different communication modes for selected channel SNR points are provided in Table 1. Comparing the results in Table 1(a) and Table 1(b), we see that, for mode 3 and mode 4, more (sometimes equal) cooperation bits are allocated when the inter-partner channel is better. For the same inter-partner channel, when the user channel gets better, fewer bits are used for channel-coding/cooperation for the base-layer, and more bits are shifted to the enhancement layer, so that a greater number of source bits are transmitted.

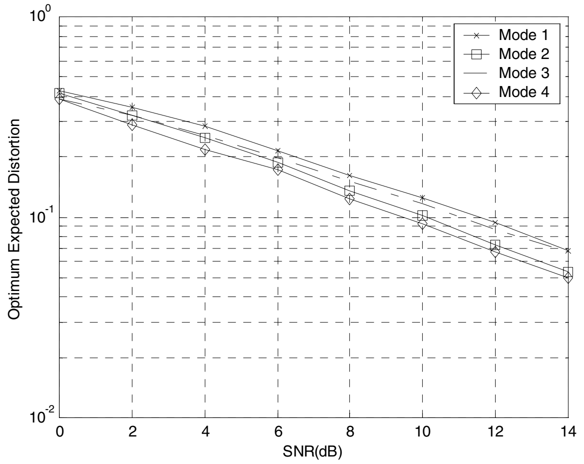
V. INFORMATION THEORETIC APPROACH

The performance gains reported in Sec. 4 are for a particular family of channel coders. We also examine the performance of different modes with "ideal" channel coding and modulation that can realize Shannon channel capacity, and characterize the behavior of various modes when the channel SNR is large. The effect of the fading and channel noise for this analysis is expressed by the outage probability, the probability that for a given SNR a particular source rate is greater than the instantaneous capacity of the channel, and hence cannot be reliably transmitted. In this case, rather than determine the optimal bit allocation among source coding, channel coding, and cooperation, we need to determine the total number of information bits to be sent per source sample, the partition between base layer and enhancement layer in the layered case, and the proportion of bits to be sent directly vs. through partner in the cooperation case.

Assume that with best modulation and channel coding, we send R_s information bits/channel use. In the special case of M=N with mode 1, we have R_s information bits/source sample. The total number of channels uses, N, is assumed to be large, and the cooperating terminals share these N channel uses. As in [3], we assume that repetition coding is used for cooperation. The error rate is now determined by the outage probability. Let's take mode 4 for example, we assume that T1 uses αN channel uses to send the base layer, and $(1-2\alpha) N$ channel uses to send the enhancement layer, both at the same channel rate R_s. Hence the base layer corresponds to αR_s bits/sample, the enhancement layer $(1-2\alpha)R_s$ bits/sample. If T2 can correctly decode the base layer (that is the inter-user channel is not in outage), it will forward the base layer bits using the remaining αN channel uses. Otherwise, this part of the channel slot is wasted. By replacing the FERs in (1) with the outage probabilities of various links, which now depend on R_s and α, we can determine optimal α and R_s, for given channel SNRs for all the modes.



(a) inter–partner channel SNR=30dB



(b) inter–partner channel SNR = 0dB

Figure 3. Comparison of different communication modes for the i.i.d. Gaussian source with RCPC codes. The horizontal axis is the common SNR between T1/T2 to the destination.

We first analytically examine the expected distortion for various modes when SNR is large. We assume the average SNR in all the links are the same including the inter-partner channel. We study mode 1 (α=1) closely to illustrate the trade-offs in such a system. For high SNRs, we consider a logarithmic grow of rate as a function of SNR that is $R_s = m \log(SNR)$ where the best SNR exponent m (or the multiplexing gain) will be determined[3]. Approximating the outage probability with $(2^{R_s}-1)/SNR=SNR^{m-1}$ for large SNR, and using the Gaussian rate distortion function, we get $ED(R_s,SNR)\sim SNR^{-2m}+SNR^{m-1}$. For optimal performance we need to maximize $min(2m,1-m)$ which results in m=1/3. Defining the *average distortion exponent* as [8]

$$\Delta = \lim_{SNR\to\infty} \frac{-\log(ED(R_s, SNR))}{\log(SNR)}$$

and using the optimal multiplexing gain m=1/3 we get Δ=2/3 for mode 1.

This distortion exponent Δ can be interpreted as the equivalent of diversity for source-channel coding systems: For large SNR we have $ED(R_s,SNR)\sim SNR^{-\Delta}$. Obviously, the mode that can yield the highest Δ is most desirable.

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[3] In order to change the exponent m and hence the channel rate Rs, we assume we can change the modulation as well as channel coding as a function of SNR.

SNR	Mode	R_b	$r_{b,1}$	$r_{b,2}$	R_e	r_e	ED
0 dB	1	48	144	0	0	0	0.43
	2	56	112	0	16	8	0.42
	3	48	48	96	0	0	0.36
	4	48	48	96	0	0	0.36
2 dB	1	64	128	0	0	0	0.35
	2	48	96	0	32	16	0.32
	3	64	32	96	0	0	0.29
	4	48	48	48	32	16	0.27
6 dB	1	96	96	0	0	0	0.22
	2	56	56	0	53	27	0.19
	3	96	0	96	0	0	0.15
	4	80	0	80	32	0	0.14
10dB	1	96	96	0	0	0	0.13
	2	48	48	0	64	32	0.10
	3	96	0	96	0	0	0.08
	4	72	0	72	48	0	0.06
14dB	1	128	64	0	0	0	0.07
	2	56	56	0	80	0	0.05
	3	128	0	64	0	0	0.06
	4	56	0	56	80	0	0.04

(a) Inter-partner channel SNR=30 dB

SNR	Mode	R_b	$r_{b,1}$	$r_{b,2}$	R_e	r_e	ED
0dB	3	48	48	96	0	0	0.39
	4	48	48	96	0	0	0.39
2dB	3	64	64	64	0	0	0.32
	4	48	48	48	32	16	0.29
6dB	3	96	48	48	0	0	0.20
	4	48	24	24	64	32	0.17
10dB	3	96	0	96	0	0	0.12
	4	64	32	32	64	0	0.09
14dB	3	128	0	64	0	0	0.07
	4	72	0	36	84	0	0.05

(b) Inter-partner channel SNR=0 dB. Bit allocations for modes 1 and 2 are the same as in Table 1(a).

Table 1: Optimal bit allocations for different modes for i.i.d. Gaussian source with RCPC codes.

By minimizing *ED*, we have found the optimal operating parameters and the corresponding distortion exponents to be, for mode 3 (in the special case of repetition coding with $\alpha=0.5$): $m=2/3$, $\Delta=2/3$, and for mode 4: $m=0.6$, $\alpha=1/3$, $\Delta=0.8$. It is interesting to see that simple application of cooperative coding (mode 3), when partners share a time slot equally does not improve the distortion exponent over direct transmission (mode 1). This is further illustrated by Figure 4, which shows numerical calculations of optimum expected distortions as a function of channel SNR for all the modes studied. We observe that mode 3 can be worse than mode 1. Through this simple cooperation, one obtains spatial diversity, which is highly desirable from the perspective of channel coding, but in return has to give up a significant portion of its spectral efficiency. Layered cooperation (mode 4) allows one to achieve proper trade-off between spectral efficiency and diversity, by offering diversity only to the important source bits. Overall we find our observations are consistent with Figure 3.

VI. CONCLUSIONS

Layered coding with UEP is a well-established technique for wireless multimedia transmission, where UEP is typically realized by applying more channel coding redundancy to the more important source layer. In a cooperative network, where we get spatial diversity benefits, we can also vary the level of cooperation by changing the number of bits the partner sends. We showed that applying cooperation to the base layer (possibly on top of FEC) is more effective than direct transmission with UEP or applying FEC alone in a cooperative system. Simulation results with Gaussian sources and RCPC channel codes indicate that the proposed layered cooperation scheme can significantly reduce end-to-end distortion under the same total channel usage, compared to non-layered cooperation or layered coding without cooperation, over a large range of channel SNR and inter-partner channel conditions. Our information theoretic analysis illustrated that the mentioned gains of layered cooperation are of fundamental nature and continue to exist with optimal modulation and channel coding.

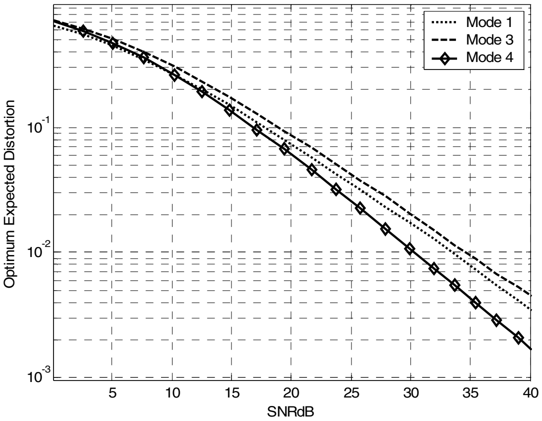

Figure 4: Comparison of different communication modes from an information theoretic perspective. The horizontal axis is the common SNR between T1/T2 to the destination, as well as the inter-user channel.

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