OUTAGE MINIMIZATION BY OPPORTUNISTIC COOPERATION

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ABSTRACT

We consider a wireless relay network where communication is constrained by delay and average power limitations. We assume that partial channel state information is available at the transmitters while the receivers have the perfect channel state information, and consider the system performance in terms of outage probability. We propose an opportunistic user cooperation protocol that utilizes the channel state information to decide when and how to cooperate. We show that compared to fixed type cooperation schemes, our protocol improves the outage performance significantly and performs very close to the half-duplex lower bound while reducing the resources used by the relay.

1. INTRODUCTION

User cooperation is a spatial diversity technique that provides robustness against fading by forming a virtual antenna array [1, 2]. The model of a cooperative system builds upon the relay channel (see Fig. 1). The channels among the terminals are modelled as independent quasi-static fading. In applications that are not delay tolerant, the suitable performance metric is the outage probability which is shown to be the lower bound for the frame error rate of a coded system.

Although it is known that even limited feedback improves the system performance significantly in MIMO systems [4], most of the research on cooperative relaying is based on the assumption of no channel state information at the transmitters (CSIT), where only the channel statistics is known, while receivers have the perfect channel state information (CSIR). In particular, in [3] the authors introduced different simple cooperation protocols and proved that these protocols attain a lower outage probability than direct transmission of the message when perfect CSIR but no CSIT is present. In this paper, we will follow the approach in [4], [6], [7], [8], [9] and assume the existence of partial CSIT and perfect CSIR. We will show that with the help of feedback about the channel state, in this case the fading amplitudes, dynamic power and time allocation among the terminals is possible and the system can potentially achieve a considerable improvement in outage performance.

This work was supported in part by NSF Grant No. 0430885.

In [5] authors assume perfect CSIT/CSIR and study outage probability minimization for different protocols. Their analysis is mostly based on the full-duplex assumption, i.e., the simultaneous transmission and reception at the relay. They include the analysis of amplify-and-forward protocol with half-duplex, but due to the nature of the protocol dynamic time allocation is not possible, and thus the gains are limited. Further in [6] they show that even limited feedback improves the performance of amplify-and-forward protocol. In [8] and [9] effect of CSIT on ergodic capacity is studied, with total and separate power constraints on the source and the relay, respectively. In our recent work [7] we studied the delay-limited capacity of a cooperation system under partial CSIT and perfect CSIR assumption.

In this work we consider a relay that cannot transmit and receive at the same time and analyze decode-and-forward type strategies. In our scenario only the amplitudes of the channel states are available at the source and the relay. They either do not have, or do not utilize the phase information, thus the coherent combination of the source and the relay signals is not possible. Hence, the source and the relay do not benefit from transmitting at the same time and do not need to be symbol synchronized. Channel state information is only utilized for power and transmission time adaptation.

We consider a system that cannot tolerate large delays. Also, we assume the system power is limited so that zero outage cannot be guaranteed for a specified rate, i.e., the required transmission rate might be above the maximum delay-limited capacity that is achievable with the available average power. In this case, the system aims to minimize the outage probability by dynamically allocating power and time among the terminals over varying channel states.

The results obtained here prove the importance of feedback regarding channel state information and the considerable increase in the performance shows that feedback, on top of cooperation will help the mobile terminals attain improved battery life. Furthermore, we show that the dynamic nature of the proposed cooperation scheme, i.e., to cooperate when it is advantageous, and the ability to decide the

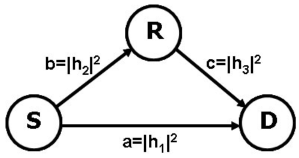

Fig. 1. Illustration of the cooperative relay system model.

amount of cooperation, improves the overall performance compared to the non-cooperative or the fixed type cooperative strategies.

The outline of the paper is as follows: In Section 2, the network model that is subject to our analysis is introduced. In Section 3, we analyze the minimum outage probability performance of the direct transmission and fixed decode-and-forward schemes. In Section 4, we explain the opportunistic cooperation strategy and analyze its outage probability. In Section 5, a lower bound to the outage probability is found. Section 6 is devoted to the analysis of the numerical results. Then the Conclusion and the Appendix follow.

2. SYSTEM MODEL

Our system consists of a single source(S), single destination(D) pair and an available relay(R) as shown in Fig. 1. The links among the terminals are modelled as having quasi-static Rayleigh fading that are independent. The fading coefficients denoted as $h_i,\ i \in \{1, 2, 3\}$ are circularly symmetric Gaussian with zero mean. There is also additive white Gaussian noise with unit variance at each receiver. Amplitude squares of the channel coefficients, denoted as $a = |h_1|^2$, $b = |h_2|^2$, and $c = |h_3|^2$ as shown in Fig. 1, are exponentially distributed with λ_a, λ_b, and λ_c. The parameters for the exponential distributions capture the effect of pathloss across the corresponding link. To consider the effect of the relay location on the performance of the network, we follow the model in Fig. 2. We normalize the distance between the source and the destination, and assume that the relay is located on the line connecting them. We denote the relay-destination distance as d and the source-relay distance as $1 - d$, where $0 < d < 1$. Then the overall network channel state, $\mathbf{s} = (a, b, c)$ becomes a 3-tuple of independent exponential random variables with means $\lambda_a = 1$, $\lambda_b = \frac{1}{(1-d)^\alpha}$, and $\lambda_c = \frac{1}{d^\alpha}$, respectively, where α is the pathloss exponent. We will consider $\alpha = 1.5$ in our numerical analysis. We assume that all the channel states a, b, and c are known at the source, the relay and the destination, while the phase information is only available at the corresponding receivers. Furthermore, we assume that there is an average transmit power limitation, P_{avg} on the total average power used by the network.

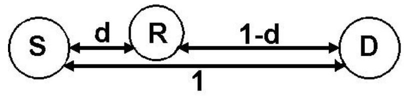

Fig. 2. The model for the terminal locations.

We constrain the terminals to employ half-duplex transmission, i.e., they are not allowed to transmit and receive simultaneously. The protocol for cooperation is based on the decode-and-forward (DF) protocol of [3], in which the time slot of the source terminal, which observes only one fading level towards the destination is divided into two. In the first half, the source transmits to both the relay and the destination, and in the second half if the relay decodes the message, it forwards the message to the destination. The destination, receiving two copies of the same message from two independent fading channels combines them. In the DF protocol defined in [3] the relay remains silent if it cannot decode at the end of the first half. However in our system, due to the availability of the channel state information, when the source decides to utilize the relay, it can transmit at a power level that guarantees decoding at the relay.

In this paper, we offer a cooperation strategy that dynamically adjusts the time that the relay listens and the power allocation among the source and the relay, subject to the average total power constraint P_{avg}, based on instantaneous channel gains to minimize the outage probability. We will allow the source to transmit its message directly to the destination throughout its whole time slot depending on the channel state information. Naturally, this is preferable in some channel states as we have a total power constraint for the source and the relay, hence the relay power cannot be utilized without cost.

Since the channel state information is limited to the amplitudes of the channel states and the phases of the fading coefficients are not known at the transmitters, the source and the relay do not need to transmit simultaneously to the destination after the relay listens to the source as in [12, 13]. No channel phase information at the transmitters means that coherent combination of the source and the relay signals (or beamforming) is not possible, thus simultaneous transmission leads to performance loss for total fixed transmit power.

3. OUTAGE MINIMIZATION FOR DIRECT TRANSMISSION AND FIXED DECODE-AND-FORWARD

It is known that, when each terminal has a single antenna, direct transmission (DT) can not achieve zero outage probability with a finite average power limitation [10]. However, if we allow the system to be in outage in case of deep fading, which requires high power to achieve a nonzero instantaneous channel capacity, then it is possible to achieve a nonzero constant transmission rate with finite average power [11]. In this section we will first find the minimum outage probability achieved in the case of DT to introduce the basic concepts and the ideas and then we will focus on the fixed decode-and-forward (fDF) cooperative protocol.

Let P be the power allocation when the source-destination channel has amplitude squared a, $P = P(a)$. Then for this power allocation, the maximum instantaneous mutual information is $I(a, P) = \log(1 + aP)$, which can be achieved by Gaussian codebooks. The outage probability for an attempted transmission rate R becomes

$$P_{out}^{DT}(R, P) = Pr(I(a, P) < R).$$

Then we can formulate the outage minimization problem as

$$\begin{aligned} \min \quad & P_{out}^{DT}(R, P) = Pr\left(log(1 + aP) < R\right), \\ \text{s.t.} \quad & E_a[P] \leq P_{avg}. \end{aligned} \tag{1}$$

Intuitively, to achieve minimum probability of outage within the average power limitation, one should transmit during the better channel states and not transmit at all when the channel is in deep fade. Using the results outlined in Appendix, one can see that the optimal power allocation function should be of the form

$$P = \begin{cases} (2^R - 1)/a & \text{if} \quad a \geq a^*, \\ 0 & \text{if} \quad a < a^*. \end{cases} \tag{2}$$

We can rewrite the outage probability and the average power constraint in terms of a^* as

$$Pr(I(a, P) < R) = Pr(a < a^*), \tag{3}$$

$$E[P] = (2^R - 1)E\left[\frac{1}{a}|a \geq a^*\right]. \tag{4}$$

We observe that the outage probability is an increasing function of a^* while the average power is a decreasing function of it. We conclude that the minimum outage probability can be obtained with the power allocation function that satisfies the average power constraint with equality. Since a has a continuous distribution, there always exists a^* such that $E[a^{-1}|a \geq a^*] = P_{avg}/(2^R - 1)$. The outage probability corresponding to a^* obtained from this equation is the solution to the optimization problem of (1). The minimum outage probability vs. average power (P_{avg}) of direct transmission for transmission rate $R = 1$ is shown in Fig. 4.

If there is a relay terminal available to help the source, we can achieve a smaller probability of outage by user cooperation. First we consider a simple non-opportunistic fixed decode-and-forward (fDF) strategy. Here, independent of the channel conditions, the relay first decodes the message and if successful, retransmits (see Fig. 3).

For fDF, the time slot is divided into two equal portions and in the first half, the source transmits either to the destination or to the relay depending on the channel states. If the source-destination channel is better than the source-relay channel, then the source transmits at a power level that is enough for decoding at the destination and the relay is not utilized. Otherwise, it aims the relay to decode. In this case, the relay decodes and retransmits the message in the second half using an independent Gaussian codebook. The destination then combines the signals coming from the source and the relay. Then the instantaneous mutual information $R^{fDF}(\mathbf{s}, \mathbf{P})$, where $\mathbf{P} = (P_1(\mathbf{s}), P_2(\mathbf{s}))$ is

$$R^{fDF} = \max(R_{DT}^{fDF}, R_{DF}^{fDF}),$$

where

$$\begin{aligned} R_{DT}^{fDF} &= \tfrac{1}{2}\log(1 + aP_1), \\ R_{DF}^{fDF} &= \min\Big(\tfrac{1}{2}\log(1 + bP_1), \tfrac{1}{2}\log(1 + aP_1) + \\ & \tfrac{1}{2}\log(1 + cP_2)\Big) \end{aligned}$$

and

$$P_{out}^{fDF} = Pr(R^{fDF}(\mathbf{s}, P) < R).$$

Then we can state the optimization problem for fDF as:

$$\begin{aligned} \min \quad & P_{out}^{fDF} = Pr(R^{fDF}(\mathbf{s}, P) < R), \\ \text{s.t.} \quad & E\left[\mathbf{P}\right] \leq P_{avg}, \end{aligned} \tag{5}$$

where

$$E[\mathbf{P}] = E\left[\frac{P_1 + P_2}{2}\right].$$

We define $P_{req}(R, \mathbf{s})$ as the minimum required total power for successful transmission at channel state $\mathbf{s}$,

$$P_{req}(R, \mathbf{s}) = \min_{P_1, P_2}\left(\frac{P_1 + P_2}{2} : R^{fDF}(\mathbf{s}, \mathbf{P}) \geq R\right). \tag{6}$$

Again, using the results in Appendix, the optimization of (5) can be reduced to searching among the power allocations that result in total required power $P_{req}(R, \mathbf{s})$ below a threshold value. The probability of channel states that require total power more than this threshold is the outage probability. The required values for P_1 and P_2 for successful transmission in the fixed decode-and-forward protocol are

$$P_{req}^1 = \frac{2^{2R} - 1}{\max(a, b)} \tag{7}$$

$$P_{req}^2 = \begin{cases} 0 & \text{if} \quad a \geq b; \\ \frac{2^{2R}/(1 + aP_1) - 1}{c} & \text{if} \quad a < b. \end{cases} \tag{8}$$

Then the minimum total required power is $\frac{P_{req}^1 + P_{req}^2}{2}$.

The outage probability vs. average total power for fDF protocol is included in Fig. 4. The discussion of the results is left to Section VI.

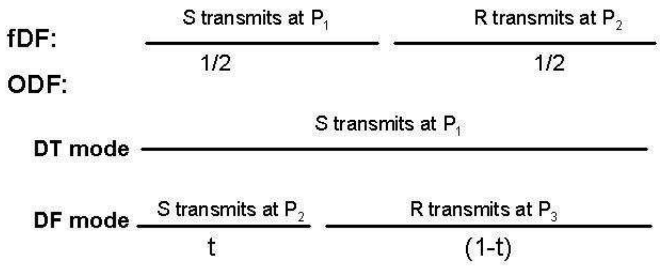

Fig. 3. The allocation of the time slot among the terminals.

4. OUTAGE MINIMIZATION FOR OPPORTUNISTIC DECODE-AND-FORWARD

In the fDF protocol, discussed in Section III, the source is constrained to transmit only during the first half and the selection among direct transmission and decode-and-forward is made depending only on the relative values of the channel states a and b. In opportunistic decode-and-forward (ODF), similar to fDF, we let the terminals decide when to cooperate by operating in two different modes, direct transmission(DT) mode and decode-and-forward(DF) mode. However, in DT mode the source transmits directly to the destination throughout the whole time slot and the relay neither tries to decode the message nor transmits at any portion of this time slot. In DF mode, however, the source first transmits its message to the relay, the relay decodes and retransmits this message using an independent Gaussian codebook. Let P_1 be the source power in DT mode, and P_2 and P_3 be the source and the relay power allocations in DF mode with $\mathbf{P} = (P_1, P_2, P_3)$. Note that all the powers are functions of the channel state vector $\mathbf{s}$. At each channel state, the system operates in either one of the modes, so we either have $P_1 > 0$ and $P_2 = P_3 = 0$, or $P_1 = 0$ and $P_2 > 0$, $P_3 > 0$ corresponding to DT and DF modes, respectively.

We introduce another degree of freedom in the performance optimization of the ODF protocol. In ODF, the source and the relay divide the time slot into two parts that are not necessarily equal. Thus it will be possible to optimize the performance over time allocation t, the portion of the time slot that the relay listens $(0 \leq t \leq 1)$. Here the total time slot is normalized as 1. This is illustrated in Fig. 3.

For ODF protocol, we define the instantaneous capacities for each mode separately

$$\begin{aligned} R_{DT}^{ODF}(\mathbf{s},\mathbf{P}) &= \log(1 + aP_1), \\ R_{DF}^{ODF}(\mathbf{s},\mathbf{P}) &= \min\big(t\log(1 + bP_2), \\ &\quad t\log(1 + aP_2) + (1-t)\log(1 + cP_3)\big). \end{aligned} \tag{9}$$

Then the instantaneous capacity corresponding to power $\mathbf{P}$ and time allocation t is

$$R^{ODF}(\mathbf{s}, P, t) = \max(R_{DT}^{ODF}, R_{DF}^{ODF}), \tag{10}$$

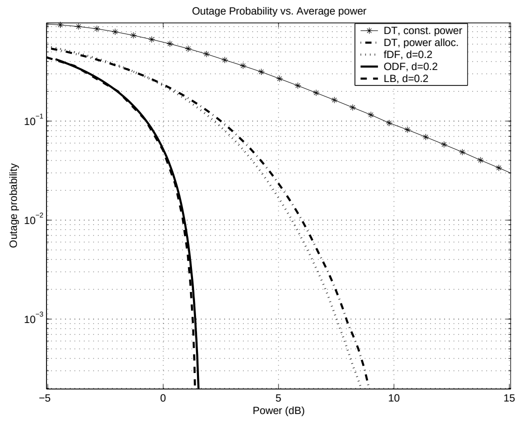

Fig. 4. The outage probability vs. average total power (Rate=1).

since only the capacity corresponding to the chosen mode is non-zero. Then the outage minimization for ODF can be written as

$$\begin{aligned} \min \quad & P_{out}^{ODF} = Pr(R^{ODF}(\mathbf{s},\mathbf{P},t) < R), \\ \text{s.t.} \quad & E[\mathbf{P},t] \leq P_{avg}, \end{aligned} \tag{11}$$

where

$$E[\mathbf{P},t] = E[P_1] + E[tP_2 + (1-t)P_3]. \tag{12}$$

The minimum required total power for ODF at channel state $\mathbf{s}$ is

$$P_{req}(R,\mathbf{s}) = \min\left(P_{req}^{DT}(R,\mathbf{s}), P_{req}^{DF}(R,\mathbf{s})\right), \tag{13}$$

where

$$\begin{aligned} P_{req}^{DT}(R,\mathbf{s}) &= \min_{P_1}\{P_1 : R_{DT}^{ODF}(\mathbf{s},P) \geq R\}, \\ P_{req}^{DF}(R,\mathbf{s}) &= \min_{P_2,P_3,t}\{tP_2 + (1-t)P_3 : R_{DF}^{ODF}(\mathbf{s},P) \geq R\}, \end{aligned}$$

and let $(\mathbf{P},t)_{req}$ be the power and time allocation that results in minimum required total network power of $P_{req}(R,\mathbf{s})$.

Overall, there are three improvements in ODF compared to fDF. The first is the utilization of the whole time slot in case of direct transmission. The second is the dynamical time allocation among the source and the relay in case of decode-and-forward, and the third improvement is the advanced decision rule that is used to make a decision among the two modes. While in fDF decision is based only on the relative values of a and b, ODF operates in the mode that requires the least total network power. The solution to the optimization problem corresponding to ODF protocol is outlined in Appendix. Similar to the previous cases, this outage minimization problem is equivalent to finding the right threshold for the required total power. Since outage probability increases with increasing threshold, while the average power decreases; each threshold gives us an average power-minimum outage probability pair. This pair is achieved by a power and time allocation function $(\mathbf{P}, t)$, which assigns positive powers that add up to the minimum required total power for each channel state only if this minimum value is below the threshold.

In Fig. 4 we observe the performance of ODF protocol. The gain provided by the opportunistic nature of the protocol and the dynamic time allocation among the source and the relay is significant. In Section VI we will further discuss the effect of relay location on the outage performance and some other advantages of ODF.

5. LOWER BOUND TO THE OUTAGE PROBABILITY

In this section, we find a lower bound to the outage probability when channel amplitude information is available at the transmitters. We use the cut-set bounds for the 'cheap relay' that are introduced in [14] specialized to our scenario. Considering the fact that, in our scenario, beamforming is not possible as the channel phases are not known at the transmitters, only one of the terminals with the best instantaneous channel state transmits during each time slot. Then we can upper bound the instantaneous capacity for the half-duplex relay as

$$\begin{aligned} R^{UB} &= \underset{0 \leq t \leq 1}{sup} \min \Big(t \log(1 + (a+b)P_1), \\ & \quad t \log(1 + aP_1) + (1-t)\log(1 + cP_2) \Big). \end{aligned}$$

Here the first term in the minimization corresponds to the cut-set around the source during the transmission of the source and the second term corresponds to the cut-set around the destination. Then the outage probability of the system can be lower bounded by

$$P_{out}^{LB} = Pr(R^{UB}) < R). \tag{14}$$

The minimization problem for the lower bound is

$$\begin{aligned} \min \quad & P_{out}^{LB}(R, P_{avg}) \\ \text{s.t.} \quad & E_s[P] < P_{avg}, \end{aligned} \tag{15}$$

6. NUMERICAL RESULTS AND DISCUSSIONS

Fig. 4 illustrates the minimum outage probability vs. the average total power constraint of the system for various scenarios for a transmission rate of $R = 1$. The topmost curve corresponds to the case of DT where the source transmits with constant power. Comparison of the constant power curve to the DT with dynamic power allocation curve shows that a power reduction of almost 8 dB is possible at $P_{out} = 10^{-1}$ by optimal power allocation.

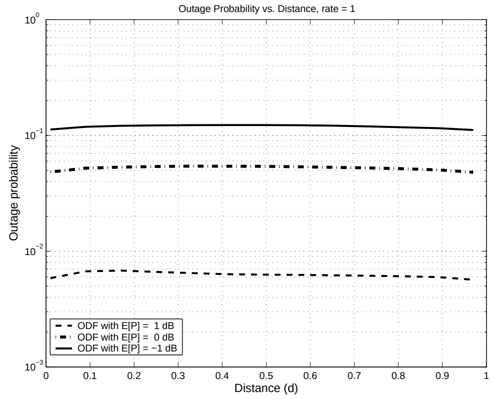

Fig. 5. The outage probability vs. the source-relay distance(d) for ODF protocol (Rate=1).

The third curve is the performance results of fDF with $d = 0.2$. We see that the system performance improves slightly with fixed cooperation protocol. The power savings compared to DT with power optimization is less than 1 dB. The next curve corresponds to ODF with distance $d = 0.2$. Now the performance improvement is substantial, about 7 dB for $P_{out} = 10^{-3}$ compared to fDF. We see that dynamic power and time allocation brings the performance very close to the lower bound for the relay location of $d = 0.2$, which is the lowest curve in the figure.

To see that effect of the relay location on the performance of the ODF protocol, in Fig. 2 we plot the minimum outage probability vs. the source-relay distance. Here we observe that, although the the minimum outage probability decreases with increasing average total power, outage probability is almost independent of the relay location for constant average power. This means that, when we have both power and time allocation optimization, any relay will serve the source to obtain most of the highest possible performance gain.

Another important characteristic of the ODF protocol, which results from the dynamic nature of the optimization, is that the relay is not utilized all the time. Since relaying is preferred only when it performs better than direct transmission, relay resources are used only in a limited fashion. The probability of the channel states that result in cooperation decreases with increasing available sum power. Naturally the probability of the states where the source utilizes the relay for cooperation depends on the relative channel qualities. In Fig. 6 we can see how the ratio of the average power spent by the relay to the total average network power changes with increasing available total average power. The ratio is always less than $1/2$ and very small for a relay

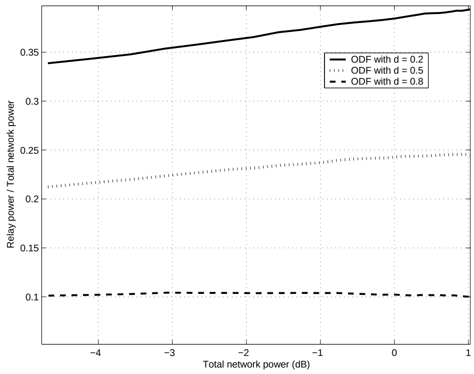

Fig. 6. The ratio of the relay power to the total network power vs. average total power constraint for varying relay location.

relatively close to the destination. Fig. 6 focuses on the $(-4dB, 1dB)$ range since the outage probability of ODF protocol practically vanishes for power values beyond 1 dB.

The fact that the relay power in ODF is spent in a limited amount is important in the cases where cooperation is not mutual. One basic concern about cooperative relaying protocols in general is the lack of incentives for the terminals to help each other. In cooperation protocols where CSIT is not available, terminals relay information independent of the channel states, which means that they spend half of their battery power for helping their partner. However, with opportunistic cooperation the amount of power dedicated to relaying is reduced to a minimal amount which makes it easier to promote cooperation. Again in a denser network scenario, where multiple candidates are available for relaying, it is possible to pick the node that requires the least relay power for achieving the same outage probability. This will significantly reduce the resources spent by the relay terminal. As we can observe from Fig. 2, ODF performance is uniform over the relay locations. Thus, it is possible to achieve a performance close to the lower bound by utilizing a negligible amount of relay power when there is an available relay close to the destination.

7. CONCLUSION

Current wireless communication systems are now designed to support transmission of delay intolerant applications, such as real-time multimedia, using battery-power limited terminals in fading environments. These applications might require constant transmission rates that are higher than the possible delay-limited (zero-outage) capacity of the system. In this paper, we develop cooperative transmission protocols that improve the system performance considerably in the minimum outage probability sense with the help of feedback which provides partial channel state information at the transmitters. We propose an opportunistic decode-and-forward (ODF) protocol where the relay terminal is utilized depending on the overall network state and power and time allocation are done dynamically. We show that ODF brings a considerable improvement with a limited use of relay resources.

8. APPENDIX

In this appendix we prove that the power and time allocations, $\mathbf{P}$ and t, respectively, that solve (11) are of the following form:

$$(\mathbf{P}, t) = \begin{cases} (\mathbf{P}, t)_{req}(R, \mathbf{s}) & \text{if} \quad P_{req}(R, \mathbf{s}) \leq P^*, \\ 0 & \text{if} \quad P_{req}(R, \mathbf{s}) > P^* \end{cases} \tag{16}$$

Recall that both $\mathbf{P}$ and t are functions of the network state $\mathbf{s}$ and $P_{req}(R, \mathbf{s})$ is the minimum total network power that is required to achieve a transmission rate of R at network state $\mathbf{s}$. $(\mathbf{P}, t)_{req}(R, \mathbf{s})$ corresponds to the power allocation vector, time allocation couple that results in this minimum required average network power. This form is valid for all the protocols mentioned in the paper while $P_{req}(R, \mathbf{s})$ is protocol dependent.

Given any of the protocols described in this work, we let the maximum rate that can be transmitted to the destination using this protocol with power and time allocation $(\mathbf{P}, t)$ as $R(\mathbf{P}, t, \mathbf{s})$ ($t = 1/2$ independent of $\mathbf{s}$ for fDF). Then the optimization problem we want to solve is

$$\begin{aligned} \min \quad & P_{out} = Pr(R(\mathbf{P}, t, \mathbf{s}) < R), \\ \text{s.t.} \quad & E[\mathbf{P}, t] \leq P_{avg}. \end{aligned} \tag{17}$$

where R is the attempted transmission rate which is given.

Let $\Gamma = \{(\mathbf{P}, t) : E[\mathbf{P}, t] \leq P_{avg}\}$ and $\gamma \in \Gamma$ be any power and time allocation pair that satisfies the average power constraint.

Now, consider

$$\gamma' = \begin{cases} \gamma & \text{if} \quad R(\mathbf{s}, \gamma) \geq R, \\ 0 & \text{if} \quad R(\mathbf{s}, \gamma) < R, \end{cases} \tag{18}$$

where $R(\mathbf{s}, \gamma)$ is the maximum transmission rate that can be achieved by γ.

It is easy to see that the outage probabilities corresponding to power allocations, γ and γ' are equal, i.e., $P_{out}^{\gamma'} = P_{out}^{\gamma} = Pr(R(\mathbf{s}, \gamma) < R)$ and $E[\gamma'] \leq E[\gamma]$. Therefore $\gamma' \in \Gamma$ as well and we can only concentrate on power allocation functions in the form of γ'.

Now, consider the following power-time allocation

$$\bar{\gamma} = \begin{cases} P_{req}(R, \mathbf{s}) & \text{if} \quad P_{req}(R, \mathbf{s}) \leq P^*, \\ 0 & \text{if} \quad P_{req}(R, \mathbf{s}) > P^* \end{cases} \tag{19}$$

where P^* is chosen to satisfy $P_{out}^{\bar{\gamma}} = P_{out}^{\gamma'}$. This is possible since channel states are continuous, and thus $P_{out}^{\bar{\gamma}}$ is a continuous function of P^*. Let $F(\mathbf{s})$ be the cumulative distribution function of the channel states and define $\mathcal{A} = \{\mathbf{s} : R(\mathbf{s}, \gamma) \geq R\}$, and $\mathcal{B} = \{\mathbf{s} : P_{req}(R, \mathbf{s}) \leq P^*\}$. Then we have

$$\begin{aligned}
E_s[\gamma'] &= \int_{\mathcal{A}} \gamma' dF(\mathbf{s}), \\
&= \int_{\mathcal{A}\cap\mathcal{B}^c} \gamma' dF(\mathbf{s}) + \int_{\mathcal{A}\cap\mathcal{B}} \gamma' dF(\mathbf{s}), \\
&\overset{(a)}{\geq} \int_{\mathcal{A}\cap\mathcal{B}^c} P^* dF(\mathbf{s}) + \int_{\mathcal{A}\cap\mathcal{B}} \gamma' dF(\mathbf{s}), \\
&= P^* Pr(\mathcal{A}\cap\mathcal{B}^c) + \int_{\mathcal{A}\cap\mathcal{B}} \gamma' dF(\mathbf{s}), \\
&\overset{(b)}{=} P^* Pr(\mathcal{A}^c\cap\mathcal{B}) + \int_{\mathcal{A}\cap\mathcal{B}} \gamma' dF(\mathbf{s}), \\
&\overset{(c)}{\geq} \int_{\mathcal{A}^c\cap\mathcal{B}} P_{req} dF(\mathbf{s}) + \int_{\mathcal{A}\cap\mathcal{B}} P_{req} dF(\mathbf{s}), \\
&= \int_{\mathcal{B}} P_{req} dF(\mathbf{s}), \\
&= \int_{\mathcal{B}} \bar{\gamma} dF(\mathbf{s}) \\
&= E_s[\bar{\gamma}].
\end{aligned}$$

Here, (a) and (c) follow from the definition of the set $\mathcal{B}$, and (b) follows from the fact that $1 - P_{out}^{\bar{\gamma}} = P(\mathcal{B}) = 1 - P_{out}^{\gamma'} = P(\mathcal{A})$. Thus we conclude that $E_a[\bar{\gamma}] \leq E_a[\gamma']$, i.e., $\bar{\gamma} \in \Gamma$. This means that of all the functions in Γ, the minimum outage probability is achieved by a function of the form $\bar{\gamma}$. We further observe that the outage probability is an increasing function of the threshold P^* while the average power is a decreasing function. We conclude that the minimum outage probability can be obtained with the power allocation that satisfies the average power constraint with equality. Since $E_s[\bar{\gamma}]$ is a continuous function of P^*, there always exists P^* such that $E_s[\bar{\gamma}] = P_{avg}$. The outage probability corresponding to this P^* is the solution of our optimization problem.

9. REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity, Part I: System description," IEEE Trans. on Communications, vol. 51, pp. 1927-1938, November 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity, Part II: Implementation aspects and performance analysis," IEEE Trans. on Communications, vol. 51, pp. 1939-1948, November 2003.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," IEEE Trans. Inform. Theory, vol. 50, no. 12, Dec. 2004.

[4] K. K. Mukkavilli, Sabharwal, E. Erkip, B. Aazhang. "On beamforming with finite rate feedback in multiple antenna systems," Trans. on Info. Theory, vol. 49, no.10, pp. 2562-2579, October 2003.

[5] N. Ahmed, M.A. Khojastepour, B. Aazhang, "Outage Minimization and Optimal Power Control for the Fading Relay Channel," IEEE Information Theory Workshop, San Antonio, TX, October 24-29, 2004.

[6] N. Ahmed, M.A. Khojastepour, A. Sabharwal, B. Aazhang, "On Power Control with Finite Rate Feedback for Cooperative Relay Networks," The 2004 Intern. Symp. on Inform. Theory and App.,Parma, Italy, Oct. 10-13, 2004.

[7] D. Gunduz, E. Erkip. "Opportunistic cooperation and power control strategies for delay-limited capacity," In Proceedings of the 2005 Conference on Information Sciences and Systems, Baltimore, March 2005.

[8] A. Host-Madsen, J. Zhang, "Ergodic Capacity and Power Allocation in Wireless Relay Channels," In Proceedings of IEEE Globecom, Dallas, TX, 2004.

[9] Y. Liang, V.V. Veeravalli. "Resource allocation for wireless relay channels," Asilomar Conference on Signals, Systems and Computers, Nov. 2004.

[10] S. V. Hanly and D. N. C. Tse, "Multiaccess fading channels: part II: Delay-limited capacities," IEEE Transactions on Information Theory, vol. 44, no. 7, pp. 2816-2831, Nov. 1998.

[11] G. Caire, G. Taricco, and E. Biglieri,"Optimum power control over fading channels," IEEE Trans. on Info. Theory, vol. 45, pp. 1468-1489, July 1999.

[12] K. Azarian, H. El Gamal, and P. Schniter,"On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," submitted to the IEEE Transactions on Information Theory, July 2004.

[13] Rohit U. Nabar, Helmut Bolcskei, Felix W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," IEEE Journal on Selected Areas in Communications, vol. 22, no. 6, Aug 2004 pp. 1099-1109

[14] M.A. Khojastepour, A. Sabharwal, B. Aazhang, "Cut-set Theorems for Multi-state Networks," Submitted for publication in IEEE Trans. on Information Theory