Particle Symmetries Problem Set 3 (2012)

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Representations of Product Groups

1. Show that the algebra of $SO(4)$ is isomorphic to the algebra of the product group $SU(2) \times SU(2)$.

2. Denote a representation of $SO(4)$ by $[n_1; n_2]$, where n_i are the highest weights of each of the two $SU(2)$'s. Consider the simplest nontrivial representation of this product group, $[1; 1]$, and compute its dimension. What well known quantum field is associated with this representation?

3. Compute the following expressions for representations of $SO(4)$:

   (a) $\text{Sym}^2[1; 1]$

   (b) $\text{Sym}^3[1; 1]$

   (c) $\wedge^k[1; 1]$ for any positive k.

   (d) Show that it is enough to know $\text{Sym}^2[1; 1]$ in order to compute $\text{Sym}^k[1; 1]$ for any positive k.

4. Find a general expression for the Taylor expansion of $\text{PE}[[1; 1]t]$.

5. For a field $\phi_{\alpha,\alpha'}, \alpha = 1, 2; \alpha' = 1, 2$ transforming in the $[1; 1]$ representation of $SO(4)$ write down a composite field which transforms in the $[k; k]$ representation of $SO(4)$.

6. Next consider the representation $[2; 1]$ and find its dimension. Which quantum field does it represent.

7. Compute the following expressions for this representation of $SO(4)$:

   (a) $\text{Sym}^2[2; 1]$

   (b) $\text{Sym}^3[2; 1]$

   (c) $\text{Sym}^4[2; 1]$

   (d) $\wedge^k[2; 1]$ for any positive k.

8. Find a general expression for the Taylor expansion of $\text{PE}[[2; 1]t]$.