Particle Symmetries Problem Set 4 (2012)

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Representation Lattices

1. For the simple roots, $\alpha_1 = (1,0), \alpha_2 = (-1,1)$ compute the Cartan matrix.
2. Identify the algebra which has this root system and find the fundamental weights.
3. Plot the weights of the representations $[1,0]$ and $[0,1]$ on a 2 dimensional lattice. What is the type of lattice which is generated by the fundamental weights?
4. What is the shape of the lattice for the representation $[n,0]$? What is the length measured by n? Compute the multiplicity of inner points on the lattice.
5. What is the shape of the lattice for the representation $[0,n]$? What is the length measured by n? Compute the multiplicity of inner points on the lattice.
6. What is the shape of the lattice for the representation $[n_1,n_2]$? What are the lengths measured by n_1, n_2? Compute the multiplicity of inner points on the lattice.