QFT Exam Summer 2016
Post Exam Comments for Students[1]

General Comments on QFT course.

This course was given for the first time by Tim Evans in the Autumn of 2014. The current course dropped the previous emphasis on fermions and on the Lorentz group. This course focusses on the basics of QFT by using scalar fields alone, taking students through to the Feynman diagrams needed for the cross section calculation in a simple scattering process. Classical fermions and the Dirac equation were discussed in one lecture only but the material was not examinable for this course.

The ideas in QFT are challenging as many concepts are hard to visualise and are out of our everyday experience e.g. what is a quantum field. The algebra is often at the limit of our 4th year MSci students' experience, e.g. operators, contour integration, though they are all well within the students' capabilities. So this course provides them with the chance to gain experience with such techniques. Still students find the wealth of tricks and nomenclature quite confusing and the undergraduates get no chance to use these ideas at other times in their final year. The one exception is the use of classical scalar fields in the Unification course but there the emphasis is on symmetry and that aspect is hardly discussed in this course.

The course is also taken by a wide range of MSc students but my aim is to ensure that this examination satisfies the requirements of a typical fourth year MSci exam.

I tried to keep the questions in this exam fairly close to the lecture and problem sheet material. Length of question should prove the main challenge as a good understanding will be needed to work through the algebra quickly.

1. This material was covered in the lectures but with several details skipped over. The use of equations of motion to derive the classical form of the conserved current for a single complex scalar was given in the lectures. The rest of the question was part of PS4 Q4 though it was not covered in a rapid feedback nor marked as important. It is largely an exercise in manipulation of annihilation and creation operators which also formed the bulk of PS1 and PS4.

(i) I expected students to derive the expression for the current from the Lagrangian density given. However I did accept solutions which started from some general expression for current in terms of derivatives of a Lagrangian density which students had memorised. The simplest approach here was to insert the given expression for J^μ into the conserved current equation $\partial_\mu J^\mu = 0$ to show it is true, which still needs the use of the equations of motion.

Students were generally bad at handling the f in the Lagrangian density though most of time students ignored the problems they caused to get the right answer anyway. The function f is usually a polynomial but in any case it is a normal function and you should just use f' to denote its derivative. So for the usual form for a complex field we would have $f(\Phi^\dagger\Phi) = m^2\Phi^\dagger\Phi + (\lambda/4)(\Phi^\dagger\Phi)^2$ which means that f is nothing but $f(x) = m^2x + (\lambda/4)x^2$. This was only meant to save students from writing down two

[1] 24/5/2016, LaTeX'd May 24, 2016

terms when one with f' would do but clearly this hindered rather than helped students. You need to realise that

$$\frac{\partial}{\partial\Phi} f(\Phi^*\Phi) = \frac{\partial(\Phi^*\Phi)}{\partial\Phi} \left.\frac{df(z)}{dz}\right|_{z=\Phi^*\Phi} \tag{1}$$

$$= \Phi^* \left.\frac{df(z)}{dz}\right|_{z=\Phi^*\Phi} \tag{2}$$

$$= \Phi^* f'(\Phi^*\Phi) \tag{3}$$

(ii) The expression for the field was given at $t = 0$ while this question requires the time derivative of the field. This should be standard knowledge with the equation for the field only present to define the notation which it does successfully. The biggest problem here is that many students could not combine the two terms in J^0, the $-i(\partial_t\hat{\Phi}^\dagger(\boldsymbol{x}))\hat{\Phi}(\boldsymbol{x})$ and its hermitian conjugate (exploiting that property saves time). The $\hat{b}^\dagger\hat{b}$ and $\hat{c}^\dagger\hat{c}$ terms should add together while the $\hat{c}\hat{b}$ and $\hat{b}^\dagger\hat{c}^\dagger$ terms should cancel. With the wrong sign you get a commutators $[\hat{b}^\dagger, \hat{b}]$ and $[\hat{c}^\dagger, \hat{c}]$ but the $\hat{c}\hat{b}$ and $\hat{b}^\dagger\hat{c}^\dagger$ terms now no longer cancel, a clear indicator of a problem. I suspect some students expected a commutator and then 'made' the annoying terms go away.

The resulting expression is

$$\widehat{Q} = \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left(\hat{c}^\dagger_{\boldsymbol{p}}\hat{c}_{\boldsymbol{p}} - \hat{b}^\dagger_{\boldsymbol{p}}\hat{b}_{\boldsymbol{p}}\right) + \text{infinite c-number}\,. \tag{4}$$

The real point of the question is that the two terms are number operators for the two types of scalar particle. These have to be interpreted as showing that the particles created by $\hat{c}^\dagger$ have the opposite charge of their anti-particles $\hat{b}^\dagger$ because of different sign in the expression.

(iii) Just basic practice manipulating free field expressions. I expected students to be familiar with the expression for $\widehat{Q}$ so they could have done this part from that knowledge.

The physics at the end is to emphasise that $\hat{\Phi}$ and $\hat{\Phi}^\dagger$ are eigenstates of charge. That is $\hat{\Phi}$ and $\hat{\Phi}^\dagger$ act on any state to increase or decrease the conserved charge by one. While this is like the effect of the particle and anti-particle annihilation and creation operators in the previous part, there is an *important difference*. The field $\hat{\Phi}$ is *not* "the particle field" and $\hat{\Phi}^\dagger$ is *not* "the anti-particle field". Both these fields contain both particle ($\hat{b}$,$\hat{b}^\dagger$) and anti-particle ($\hat{c}$,$\hat{c}^\dagger$) contributions. In fact the excitations created by fields are a superposition of pure particle and pure-antiparticle states in the usual quantum mechanical sense of a superposition. The reason these fields are useful is that the particle and anti-particle contributions appear in opposite senses. One field has terms which add one particle and other terms remove one anti-particle so the net change on a state when acted on by such a field is to add one unit of charge to the state — fields are eigenstates of charge. The other field, the hermitian conjugate, will do the reverse. Neither the number of particles nor the number of anti-particles is conserved when acting a field on a state.

By way of contrast we could use the real field representation $\hat{\phi}_1 = (\hat{\Phi} + \hat{\Phi}^\dagger)/\sqrt{2}$ and $\hat{\phi}_2 = -i(\hat{\Phi} - \hat{\Phi}^\dagger)/\sqrt{2}$ and that captures the physics perfectly well. However it is not the best basis for this work because $\hat{\phi}_1$ and $\hat{\phi}_2$ when acting on state do not change that state by a definite amount of the conserved charge. These real scalar fields are not eigenstates of charge and as such as not naturally reflecting the physics of problem. Using them will lead to more complicated calculations though you would eventually get the right answer. The real scalar fields are expressed in terms of $\hat{a}_{1\boldsymbol{p}} = (\hat{b}_{\boldsymbol{p}} + \hat{c}^\dagger_{-\boldsymbol{p}})/\sqrt{2}$ and $\hat{a}_{2\boldsymbol{p}} = -i(\hat{b}_{\boldsymbol{p}} - \hat{c}^\dagger_{-\boldsymbol{p}})/sqrt2$ which are legitimate annihilation operators but the conserved charge is not given in terms of the number of these quanta making these quanta a poor match to the physics but again you can do the calculations with them if you try.

2. Wick's theorem is a core part of the course. It is covered in lectures, problems sheets and in a rapid feedback class. Part (i) and (ii) are special cases of PS5, Q3. Part (iii) is also one example from PS5, Q3 while (iv) is part of PS5 Q4.

(i) When answering an exam question where the focus is on a *definition*, it is not enough to give loose definition. When you are busy answering a question on a different topic but a definition or theorem is used in passing, then a quick note about the definition rather than a lengthy full definition would be appropriate.

Here is clearly a case where a proper formal definition of normal ordering is needed. Say that normal ordering moves "ϕ^+ to the right and ϕ^- to the left" begs the question to the left/right of what or what about the order within the ϕ^+ themselves? Too many students failed to give a careful definition.

The normal ordered product of fields contains a term with just ϕ^- so it is not enough to say $\hat{\phi}^+|0\rangle \sim \hat{a}|0\rangle = 0$ when explaining why expectation values of normal ordered products are zero here.

(ii) Again, this called for a precise definition of *Time Ordering*. Phrases such as "latest times to the left" were often seen. This phrase leaves open several questions: time of what (fields), move what to the left (fields of the latest times?), to the left of what (fields at medium and early times?), and how late does the time of a field have to be to be the latest? After that still have to tell the examiner how late and early time fields are ordered within themselves. Yes, all students writing brief things like that probably know what time ordering means but full marks could only be given for a proper definition. Yes writing a formal definition is a bit annoying and painful but this is the only time you have to do it, namely in explicit requests for definitions in a question like this.

(iii) Generally well done.

(iv) Generally well done.

3. The discussion of the U operator is in PS5, Q2 in great detail, and was also covered in a rapid feedback class. The question here, part (i), is simpler than in the PS as here the result is given. The main focus of this question is on the difference between the physical ($|\Omega\rangle$) and the free ($|0\rangle$) vacuum. The example given here, the full propagator in $\lambda\phi^4$, was studied in the last problem sheet (PS7, Q1) and covered in the last rapid feedback class at the start of the Spring term but this problem was not explicitly covered in lectures. The question in PS7 covered many aspects and the nature of vacuum diagram contributions discussed here was just one small part or the problem sheet.

(i) This part was poorly done. You need to start by arguing why the interaction Hamiltonian H_{int} in the S matrix can be replaced by the full Hamiltonian.

(ii) You just apply the result from part (i), you don't need to do some lengthy derivation.

(iii) Too many people forgot the lowest order term in the perturbative expansion of Z, namely 1, i.e. $Z = 1 + \lambda z_1 + O(\lambda^2)$.

(iv) In this part you only needed some simple expansions but the lowest order term of Z is essential. Many peoplemissed this and could not express the expansion of Π_c as series in λ.

4. This is about $\phi \to \psi\bar{\psi}$ decay in Scalar Yukawa Theory, taken from Problem Sheet 6. We only looked at the tree level g^1 calculation in lectures. We have, however, used the same model and various $\psi\psi$ and $\psi\bar{\psi}$ scattering processes as the primary example in this course. So similar results and diagrams appear in lectures, in problem sheet questions marked "important", in the New Year tests (made available to all students) and in the 2015 exam. So the Feynman rules should be very familiar and other parts mimic very closely what was asked in similar questions. This year was much better than last year in terms of the Feynman rules and diagrams.

(i) Book work. Don't forget that in the formula used in lectures the times of the in (out) fields in the Green function need to be taken to $-\infty$ ($+\infty$). If you used the LSZ formula then the question asked for a derivation so that needed to be given.

(ii) Don't forget the rules for external legs. That is for each field in the Green function there is an external vertex with that coordinate, the coordinate is not integrated over (as you do with internal vertices) but you do have one leg/line/edge coming out of that vertex corresponding to the appropriate explicit field in the Green function (not a field from the S matrix).

(iii) Generally well done.

(iv) Generally well done. There were six third order $O(g^3)$ diagrams that did not contain tadpoles (which, as stated in the question, were excluded but they did not lose marks if given). Of these the two that students most often forgot (one,or sometime both) were B and E in figure 1.

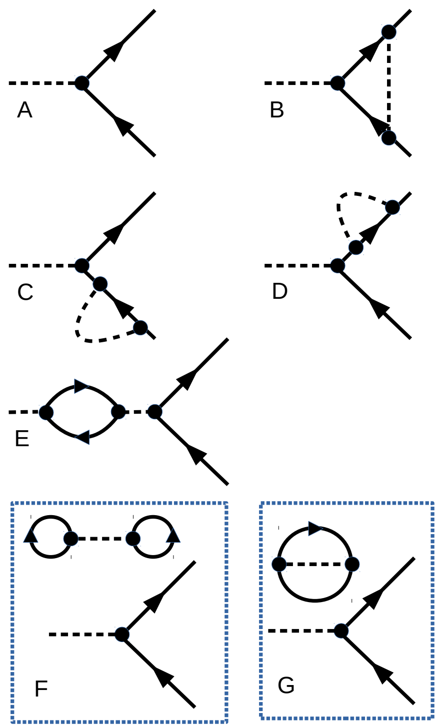

Figure 1: The Feynman diagrams for the full Green function describing the decay $\phi \to \phi\bar{\psi}$ in Scalar Yukawa theory showing all contributions up to $O(g^3)$.