QFT Exam Summer 2017
Post Exam Comments for Students

7/6/2017

General Comments on QFT course.

This course was given for the first time by Tim Evans in the Autumn of 2014. The focus of this course was on the basics of QFT. This was achieved by using scalar fields alone, taking them from classical free fields to the Feynman diagrams needed for a cross-section calculation in a simple scattering process.
The course is taken by fourth year MSci physics students. The ideas are challenging as many concepts are hard to visualise and are out of our everyday experience e.g. what is the nature of a quantum field? The algebra is often at the limit of these students experience, e.g. operators, contour integration, though they are all well within the students' capabilities. So this course provides them with the chance to gain experience with such techniques. Still students find the wealth of tricks and nomenclature quite confusing and the undergraduates get no chance to use these ideas at other times in their final year. The one exception is the use of classical scalar fields in the Unification course but there the emphasis is on symmetry and that aspect is hardly discussed in this course.
As a result I have tried to keep the questions fairly close to the lecture and problem sheet material. Length of question will prove the main challenge as a good understanding will be needed to work through the algebra quickly.
The course is also taken by a wide range of MSc students. This year I had 105 students take the exam of which 56 (53%) are UG (almost all 4th year MSci students), 4 are visitors (typically Erasmus students) and the remaining 45 (43%) are MSc students (including 27 (26%) QFFF students). My aim is to produce an exam which satisfies the examination requirements of a fourth year MSci student. Many of these MSc students do see the ideas of this QFT course applied in several of their MSc level courses.

1. This material was covered in the lectures but with several details skipped over. Most parts of this question were then covered in detail as Problem Sheet 2 Q2, a question which was in the rapid feedback sessions. The conservation of current is PS2 Q8. The 1+1 dimensional model of masses in a ring and the continuum limit of the classical theory is part of the lectures and worked in detail in PS2 and in an associated rapid feedback class. However we have not done the two-dimensional version posed here (I mentioned this verbally in a lecture). The challenge for students is for them to realise that the algebra in this last part is almost identical to the one spatial dimension case discussed in the lectures. However this two-spatial dimension case requires me to phrase the model slightly differently and to use a slightly different notation which will catch out those not fully confident with this model.

This question was popular but was not well done.

(i) As $(\phi_1)^2 + (\phi_2)^2 = 2|\Phi|^2$ don't forget to square the factor of 2 when finding the $\lambda|\Phi|^4$ from the $g((\phi_1)^2 + (\phi_2)^2)^2$ term, many students did.

(ii) Many students used a general derivation learnt from other courses but this was a lengthy approach and in that case they needed to explain what the symmetry was and so state that a symmetry was needed (even if not proved to exist in this case).

I expected people to calculate $\partial^\mu J_\mu$ directly from the given expression. You find factors of $\Phi(\partial_\mu \partial^\mu \Phi^*)$ and its complex conjugate but the $(\partial_\mu \partial^\mu \Phi)$ factors appear in the explicit form of the equation of motion found in part (i). The answer then comes out in a few lines.

(iii) This part was poorly done. I tended to get some version of the one-dimensional example we did in the lectures. The point of this last section is to see if students had picked up a key point which I made at the end of the discussion of the model in the lectures. That it is the *deviation* of the masses from their equilibrium fields which is proportional to the value of the field. The location of the field is given by the equilibrium position of the mass. The deviations for the n-th mass were clearly defined in the question to be $x_n(t)$ in the x direction and $y_n(t)$ in the y direction. The location of the n-th mass in equilibrium is $z = na$ (you can ignore the equilibrium position in the other two directions as it doesn't change from mass to mass, always zero). That is the deviations are given by a two-dimensional vector $(x_n(t), y_n(t), 0)$ for the n-th mass and this will be matched to $(\phi_1(t, z = na), \phi_2(t, z = na), 0)$, i.e. we need two independent fields as each mass has two degrees of freedom, two independent dimensions it can move in.

To find the exact scaling relation don't guess (I think some students may have just remembered the form, many got it wrong). Try the ansatz that $x_n(t) = k\phi_1(t, z = na)$ and focus on the quadratic terms for x_n. You will find the result you need comes most easily from the kinetic term. The quadratic terms for y_n work exactly the same as there is a symmetry here so without any calculation you can state the same factor is needed for $y_n(t) = k\phi_2(t, z = na)$. You should find from the spatial derivatives that you get something of the form $c^2(\partial\phi_i/\partial z)^2$ where c is a velocity, the speed of the wave, to get the units right. The existence of a non-zero c needed to be noted. Note the question carefully stated we were not using natural units.

As an aside, also note that in most systems these masses may move in a third direction, the z direction, In that case we would need a third field. That is if the position of the n-th mass is given by $(x_n(t), y_n(t), z_n(t) + na)$ we will set this equal to match this with $(k_1\phi_1(t, z = na), k_2\phi_2(t, z = na), k_3\phi_3(t, z = na))$ where k_i are constants to be determined. In the example in the lecture we focussed on masses only moving longitudinally, with a $z_n(t)$ deviation and we were studying just the $\phi_3(t, z)$ field. So in fact the relationships between discrete model and continuous model parameters are exactly as in the example in the lecture.

2. The commutators for a complex field were considered in lectures but the full details were only considered in PS4, Q5 which was a starred question though not covered in rapid feedback. The real field equivalents were covered in detail in the lectures. The form of the Feynman propagator in four-momentum has been studied in lectures with similar manipulations for retarded or advanced propagators done on PS4, in rapid feedback class, in various previous exams and tests.

This question was not popular and it was not well done.

(i) The question stated that we had "a pair of non-interacting free real scalar fields $\hat{\phi}_1$ and $\hat{\phi}_2$". It is clear from the subscripts that these are not the field $\phi(x)$ given in equation 1 in this question. The subscripts also make it clear they are *distinct* fields. This requires that we generalise $\hat{a}_k$ to two distinct types of operator, say $\hat{a}_{1k}$ and $\hat{a}_{2k}$ with appropriate commutation relations as was discussed in the context of a complex field in the lectures.

These are not equal-time correlation functions. I always try to write an explicit t or $t = 0$ notation if I want to work in terms of time and three-space coordinates, the latter also being in bold font.

(ii) (No comments)

(iii) A lot of confusion over the position of the poles of the Feynman propagator, I saw all four possible combinations. Students need to interpret the Feynman propagators as follows

$$\frac{1}{p^2 - m^2 + i\epsilon} = \frac{1}{p_0^2 - \omega^2 + i\epsilon} = \frac{1}{p_0^2 - (\omega - i\epsilon')^2} \tag{1}$$

where $\epsilon' = 2\omega\epsilon + O(\epsilon^2)$ is also an infinitesimal positive quantity if $0 < \epsilon \ll 1$. Sometimes students just lost signs expanding expressions out too quickly. The poles are at $p_0 = +\omega - i\epsilon$ and $p_0 = -\omega + i\epsilon$.

The explicit form given here means the poles are not on the real axis with the curve distorted round them. This distorted contour approach is a legitimate alternative way to express this result but to answer this question properly requires the student to link the deformed path version to the form used here.

3. The basic definitions for this case of an arbitrary split of the fields were all covered in lectures. Wick's theorem for four bosonic fields is PS5, Q4, and included both the case of an arbitrary split (and in fact for arbitrary numbers of different scalar fields too) as well as the usual vacuum expectation value case. This question was also covered in the rapid feedback classes. So this question ought to be very familiar.

This question was very popular but was not well done.

(i) Some statements were too brief.

(ii) The only difference here from PS5, Q4 is that you have to remember that, as was discussed in the lectures, the *only* case where Wick's theorem is useful in an operational sense is when normal ordering is defined such that the relevant expectation values of any normal product are zero, i.e. $\langle N(\text{fields})\rangle = 0$. In PS5, Q4 you were told to assume that. In this exam setting, you had to state that you need to make this choice in the penultimate part of this subquestion (ii). This was then intended to lead students to apply the principle in the last part of this subquestion. Having the confidence in your understanding of Wick's theorem to make this assertion was part of the test here, though it carried few marks.

(iii) You can exploit the fact that the bra and ket states are eigenstates of particle number (zero number) and the annihilation and creation operators subrace or add one to the number of a ket respectively. Therefore you only need to consider combinations with two annihilation and two creation operators. Add in the fact that the vacuum ket (vacuum bra) is annihilated by the annihilation operator (creation operator) you quickly realise that only two terms (from the 16 formed by multiplying out the product of four fields) can contribute.

4. $\psi\psi \to \psi\psi$ scattering in Scalar Yukawa Theory is a core course problem, set in Problem Sheet 6 and covered in a rapid feedback session. This question is almost entirely book work. The question on deriving the link between Green function and matrix element is part of PS6 Q4. Being able to connect a matrix element, which appears in calculations of physical quantities such as cross-sections and decay rates, to Feynman diagrams represents the central goal of course and requires a confidence in all the course material.

This question was very popular and it was very well done.

(i) You *must* take times to $\pm\infty$ to connect the Green function coordinates with the matrix element coordinates time of the incoming and outgoing particles.

(ii) Feynman rules for matrix elements or Green functions, momentum space or coordinate space all accepted, though the context of the question was Green functions in coordinate space.

(iii) Diagrams in momentum space or coordinate space were all accepted even though I specifically asked for diagrams contributing to the coordinate space Green function. If you gave the diagram in terms of momentum then you should have stated the Fourier transform needed to reach the coordinate Green function.

You must label the legs (coordinates or momentum) to indicate how they are linked to the Green function arguments and because there are further diagrams of same shape but with different external leg labels which are technically different though they represent expressions which only differ by permutations of the Green function arguments. It was enough to note what these permutations were and then to state how many extra diagrams there were. Students often gave some of these cases, but it was common to miss some of these permutation cases.

Count loops and give symmetry factors for *all* diagrams, including $O(g^0)$ which are perfectly good diagrams.

The vacuum expectation value (vev) of a field is $\langle 0|\hat{\phi}S|\,0\rangle$. This one-point Green function has one external field and so it is given by diagrams with one external leg known as *tadpoles*. You just need to find parts of your diagrams of this shape and they occur in several places.