QFT Exam Summer 2018[1]

Post-Exam Comments for Students

4/6/18

General Comments

The exam was taken by 133 students, of which 40% were UG (all but one were final year MSci students), 6% were Erasmus students and the remaining were MSc students: 36% QFFF MSc, 11% MSc Physics, 8% CQD MRes. Note that these number represents an increase of 13% over last years (105 students) and a 66% increase over the number taking this course when I first gave it (80 in the summer 2015 exam). The number of students taking the exam now exceeds the capacity of the lecture theatre used for this course.

1. The Harmonic Ring is discussed in lectures, problem sheets and has appeared in various guises in several exams. Only the last part is not straight book work though the results we have discussed using a slightly different approach, working in coordinate not momentum space.

This question was popular but poorly done, especially the last part. First just substitute in the Fourier transforms given to get fields $\hat{\phi}(t,k)$ as they are the natural match for the $\widehat{U}_k$ we started with. Then note that on dimensional grounds that

$$(\Delta k)\sum_k \quad \equiv \quad \int dk\,. \tag{1.1}$$

Many students remembered the version in space coordinates we used in lectures, where $(\Delta x)\sum_n \equiv \int dx$ where equilibrium positions are at $x_n - = na$ so its pretty obvious $\Delta x = a$. However on dimensional grounds k is wavenumber so k has dimensions of inverse length. That means you would guess that $\Delta k \propto 1/a$ where the precise constant of proportionality[2] only rescales the fields so M and c are unaffected.

The other point students missed is that they end up trying to compare the relativistic dispersion relation $E^2 = (\hbar k)^2 c^2 + M^2 c^4$ against the energy of a mode in terms of the dispersion relation for the Harmonic Ring i.e. $(\hbar\omega_k)^2$ where $\omega_k = |\sqrt{4\omega^2\sin^2(ka/2)+\Omega^2}|$. The only way to get these to match in terms of k dependence is if you expand ω_k in terms of small a and compare the low order terms.

2. This question was done exceedingly well and almost all students did this question. It is largely book work and the contour integration is the main example I used in the lectures. Note that this question contained a typo in equation (7) where a factor of ω should have been an p_0 but this did not seem to cause any problems in the answers I saw.

Many students applied the time derivative $\partial/\partial t$ to a commutator with x and y variables without making it clear what t was. In fact t should be chosen to be the time component of one of the four vectors, not both, so either $t = x_0$ or $t = y_0$ but not both. You take the equal time-limit later. You could do the question without this trick, and many did it that way round.

Note, as I continually said in the lectures, the ECTR consist of three commutators and all three must always be checked.

3. A challenging question in terms of time but it is straight book work so if you really know how this proof works, it should have been achievable, as some students demonstrated. A very small number of students did this question.

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[1] LaTeX'd July 2, 2018

[2] You can get this Δk precisely if you remember how wavelengths are quantised on a periodic lattice.

A more serious typo appeared in equation (12) in this question. The correct version of the identity is

$$[\widehat{A}_1 \, , \, \widehat{A}_2 \ldots \widehat{A}_m] = \sum_{i=2}^{m} \widehat{A}_2 \ldots \widehat{A}_{i-1} [\widehat{A}_1, \widehat{A}_i] \widehat{A}_{i+1} \ldots \widehat{A}_m \, . \tag{3.1}$$

Part of the explanation for this typo is that it was added at the last minute to after most of the detailed checks had been performed. I've noted that more care will be needed to check the changes made at such a late stage. I was responding to comments from the external examiner to make this question easier and they had suggested this identity could be given[3] Again this error seemed not to cause any problems. Students, like me, seemed to know the appropriate identity to use in the context of the proof, so this identity really served as a prompt or to indicate that no proof was needed for its use.

In proofs of Wick's theorem, students often forgot to link the commutator $[\phi_1^+, \phi_i]$ to the propagator defined in the first part of the question. They often forgot to finish the induction proof by noting where Wick's theorem is trivially true by definition, for either one or two fields.

4. This question was popular and also very well done.

Feynman rules were sometimes little careless. I asked for the rules for SYTh (Scalar Yukawa Theory) specifically so give me those rules. Too many students gave me generic rules without the specific SYTH example requested. Why not sketch the diagrammatic elements? A sketch is much quicker than words.

Students are often confused by a ratio of polynomials, each to a given order, then expanded consistently to the same order but this year's cohort seems to know this.

Useful Definitions

This extra page of identities used in several questions was suggested by an external examiner in 2017. It has proved a great success. For 2019 I may add a line on the 'slash' notation common in relativistic QFT for various factors of $(2\pi)^n$ to save students noting if they use it. I will continue to write out factors of $(2\pi)^n$ in the question for clarity.

[3] I also deleted another small part of the question in response to those comments.