Notes on 2017 QFT Test

Tim Evans
(Friday, January 13, 2017 07:37)

Overview of Course

This course was given for the first time by Tim Evans in the Autumn of 2014. The focus was on the basics of QFT by using scalar fields alone, taking students to the Feynman diagrams needed for the cross section calculation in a simple scattering process.
The course was supported by a series of five 1 hour problem classes in which a postgraduate presented answers to two (usually) key questions from six problem sheets. The same PG also gave written feedback on the student answers. No typed notes provided but the course followed very closely the format and notation used in a set of free notes by David Tong in Cambridge from his Part III lectures.
The course is taken by fourth year MSci physics students. The ideas are challenging as many concepts are hard to visualise and are out of our everyday experience e.g. what is a quantum field. The algebra is often at the limit of these students experience, e.g. operators, contour integration, though they are all well within the students capabilities. So this course provides them with the chance to gain experience with such techniques.
The course is also taken by a wide range of MSc students but my aim to focus on examination requirements of a fourth year MSci student. Many of these MSc students do see the ideas of this QFT course applied in several of their MSc level courses.

Question 1

This material was covered in the lectures but with several details skipped over. Most parts of this question were then covered in detail as Problem Sheet 2 Q7, a question which was in the rapid feedback sessions. The form of the Hamiltonian in terms of normal modes, part (ii), was only done for the classical case in PS2 Q6.

The last part on the continuum limit of the 1+1 dimensional model of masses on springs (a typical model of phonons from condensed matter physics) was very poorly done. The continuum limit in the final part was only covered in lectures but very few people seemed to know how to proceed. The clue was in the dimensions. All you needed to remember was that you want the continuum field $\phi(t,x)$ to represent the deviation from equilibrium position of the masses $u_n(t)$ so that $\phi(t, x = na) \sim u_n(t)$. Then all you have to do in your answer is make the dimensions match here and the easiest way to do this is to make both sides dimensionless. Clearly we need $u_n(t)/a$ as u is a distance, the deviation of a mass from equilibrium position and we are given that $\phi(t,x)/\sqrt{ma}$

is dimensionless. Setting these equal then you can proceed. The rest of the calculation was better remembered e.g. students recognised how to turn the $(u_{n+1} - u_n)$ term into a spatial derivative. Several students tried to start from our usual interaction picture for for a free field but that has $c = 1$ and gives no relationship to the u_n functions.

Question 2

Very similar to Q2 from summer 2016 exam. Wick's theorem is a core part of the course. It is covered in lectures, problems sheets and in a rapid feedback class. This question is easier than that covered in the lectures as we immediately simplify to the standard case for normal ordering, as relevant for vacuum expectation values. Part (i) and (ii) are special cases of PS5, Q3, (iii) is also one example from PS5, Q3 while (iv) is part of PS5 Q4.

The exam in 2015 also had, in Q3 part (iii), a request to state Wick's theorem. The 2015 question went on to apply this to the two-point Green function in $\lambda\phi^4$ to order $O(\lambda)$. The term in that 2015 question is essentially the same as here in the last part but the context is very different.

Definitions of Normal ordering, Time ordering and Wick's theorem were too often given in a loose or imprecise language. While I may say things "time ordering moves later times to the left" when discussing other topics, it is a loose shorthand when I am focussing on other things e.g. matrix element Green function relationship. When you are asked explicitly for a definition in an exam question worth a couple of marks you must give a clear and precise definition not a sloppy shorthand phrase. So don't forget normal ordering leaves the order of ϕ_i^+ fields unchanged and separately the order of ϕ_i^- fields unchanged. There is a term with pure ϕ^- factors which does not annihilate the ket vacuum state so you need to discuss the annihilation of the bra vacuum state in this case. Time ordering does not "move latest fields to the left" (there is only one latest time, later than what, left of what, etc). WHy not write out one example of each of the first few terms of Wick's theorem e.g. one with zero contractions, one with one contraction and one with two contractions to illustrate clearly your definition.

Question 3

This is essentially the reveerve of questions about $\phi \to \psi\bar{\psi}$ decay. The diagrams are the same but we link the external legs of diagrams for the Green funtion to initial and final states in the matric element in a different way than we do for decay. This means this was almost identical to the 2016 summer exam, Q4 and the question on ϕ decay in Problem Sheet 6. The PS6 question was not a heavily recommended question and we only looked at the tree level g^1 calculation in lectures. We have, however, used the same model and various $\psi\psi$ and $\psi\bar{\psi}$ scattering processes as the primary example in this course so many diagrammatic elements ought to be familiar and the Feynman rules should be very familiar.

I am not sure why students use the formula from topology to calculate loops. Most of them are obvious. Ignore the arrows and the types of line in our diagrams as this is equivalent to counting loops in the momentum variables (the same diagrams can be interpreted in momentum space though that is not was was requested here. If you can see a topological loop in the diagram it is a loop in the sense here and it will be associated with a free 'loop' momentum integration when you do the Feynman diagrams with momentum space rules. I'd only use the formula when loops overlap e.g. one of the vacuum diagrams.

Another common mistake (appearing in this question in particular but seen in other places too) is for students to talk about the ψ field being "the particle field" or "creating particles" or similar types of language with any relativistic field. The ψ field contains both $\hat{b}$ and $\hat{c}^\dagger$ operators so has both particle and anti-particles parts built into its definition. Only when you link the field to an initial or final state, as we do in part (i) here, might you begin to tie a relativistic field to a particle or anti-particle.