STRING THEORY

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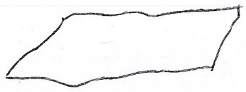
· 3 perspectives: - world sheet  2d, QFT, CFT

focus on this 2 [- space-time: D dimensions, QFT, renormalization, massless sector, classical equations
- branes extended objects, p+1 dimensions, p-branes (horrible joke) $-1 \leq p \leq 9$

bosonic string (S-T) 26D

We will rather focus on <u>superstrings</u> ie <u>supersymmetry</u> (gives exact results, solves the hierarchy problem) (*)

[<u>aside</u>: what is the hierarchy problem? It is: why are the natural scales in nature so separated?
[Take a look: VEV of Higgs $\langle\phi\rangle \sim 100\,GeV$, $m_L \sim g_L \langle\phi\rangle$ $g_L \to 10^{-6}$ $m_? \to 10^{15}\,GeV$ (check this better)

(*) bosonic strings have tachyons ie $m^2 < 0$
superstring first mass level is 0 : can compute the spectra using nonperturbative methods

. . .

SUSY in $(3+1)d$ $N = 1; 2, 4; 8$ (amount of supersymmetry)
gravity
chiral non-chiral

Extend Poincaré algebra to <u>superalgebra</u> with Q supercharges, Q_α^I

Index I basically tells me how many supercharges I have. In $(3+1)d$ it is $4 \cdot N$. <u>Why exactly 4</u>? Because we have 2 inequivalent spinors carrying an index running on 1,2 ~~[illegible]~~, ie dimension of spinor representation is $2^{rank} = 4$! Think of $[1;0]_{so(4)}$ Q_α^I and $[0;1]_{so(4)}$ $Q_{\dot\alpha}^I$

so α is the spacetime index

Actually, do we want higher-spin theories? Well, no! A theorem from Coleman-Mandula states that, when computing S-matrix elements, which is what counts, spin-higher-than-2-theories are free!

Last time: 3 perspectives (world sheet, spacetime, branes)

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Start looking at spacetime

We saw in $(3+1)d$ ie $SO(4)$ $4N$ supercharges $N=1,2,4,8$

And in $(10+1)d$? Focus on $N=1$, simplest supermultiplet, for simplicity. 2 Questions:

a) HOW MANY SUPERCHARGES? $(N=1)$ — completely analogous as $SO(4)$ $2+2$

Well, $SO(11) \cong B_5$, 1 type of spinor, dimension $2^5 = 32 \Longrightarrow$ 32 supercharges

b) HOW DO WE COUNT DEGREES OF FREEDOM? simple! From Wigner's little group

We are dealing with massless particles ie $p^2=0$ so can always boost to frame where $p^\mu = (|p|, 0, \dots, 0, p)$

The group that preserves momentum is then clearly $SO(D-2)$.

now characterize particles: d.o.f. $\rightarrow$ dimension of irreducible representation of $\underset{\text{massless}}{SO(D-2)} / \underset{\text{massive}}{SO(D-1)}$

as for massive $p^2 = m^2$, can boost to frame where $p^\mu = (m, 0, \dots, 0)$

SO in D dimensions only $D-2$ propagating d.o.f.

NOW take the graviton $g_{\mu\nu}$ 2nd rank traceless symmetric tensor in D dimensions

its representation is $[2,0,\dots,0]_{SO(D-2)}$. Dimension? $\binom{(D-2)+2-1}{2} - 1 = \frac{(D-1)(D-2)}{2} - 1 = \boxed{\frac{D(D-3)}{2}}$

$\dim(\mathrm{Sym}^k(n)) = \binom{n+k-1}{k}$ traceless

Some consequences: – in 3d gravity has 0 d.o.f. (moreover #d.o.f. gauge field = #d.o.f. scalar field scalar $\xrightarrow{\text{dual}}$ gauge field)

– in $D=2$ gravity is a constraint and have to add matter fields to give new degrees of freedom (massless gauge field has dimension 0 so no hope!)

$\rightarrow$ possible explanation of why we live in 4 dimensions ?!?

...

In $(3+1)d$ gravity not very interesting, our goal is $(10+1)d$

1) How many d.o.f.? $\frac{11 \cdot 8}{2} = 44$ d.o.f. (different polarization modes)

2) little group? $SO(9)$, tipical representation $[n_1, n_2, n_3, n_4]_{SO(9)}$

Recall representations of $SO(9)$ $\circ - \circ - \circ \Rightarrow \circ$ $[n_1, n_2, n_3, n_4]_{SO(9)}$

4 basic representations

$[1,0,0,0]$ <u>vector</u> of dimension 9 $[0,0,1,0]$ <u>3rd rank antisymmetric</u> of dimension $\binom{9}{3} = 84$

$[0,1,0,0]$ <u>adjoint</u> of dimension $\binom{9}{2} = 36$ $[0,0,0,1]$ <u>spinor</u> of dimension $2^{\text{rank}} = 2^4 = 16$

There is only a solution for the supergravity multiplet in $(10+1)d$

look at superalgebra sth like

32 Q's $\{Q, Q\} = \ldots P \ldots$ $\{Q_\alpha, Q_\beta\} = \underset{\uparrow\ \Gamma \text{ matrices}}{\Gamma^\mu_{\alpha\beta}} P_\mu + \ldots$ <u>but not important!</u>

let's construct representation like for highest weight:

result of supersymmetry, take for granted: - 16 Q's annihilate $|\,\rangle$ $(\forall\ |\,\rangle)$

- other 16 split in: 8 <u>creation</u>

8 <u>annihilation</u>

<u>BUT</u> supercharges are fermionic so single Q satisfies $Q^2 = 0$ so act by picking lowest weight state $|0\rangle$

act with $Q_\alpha Q_\beta |0\rangle$, 2^8 states so 256 polarization modes

Since we want supersymmetry, 128 fermions, 128 bosons

<u>Note</u> $84 + 44 = 128$, $84 + 44 + 128 = 256$

What do we know? <u>graviton</u> 44, <u>3rd rank antisymmetric</u> 84. We need sth of dimension 128.

<u>SO</u> ~~grav~~ ~~super~~ <u>supergravity multiplet</u>

$[2,0,0,0]_{SO(9)}$ $g_{\mu\nu}$ GRAVITON 44

$[0,0,1,0]_{SO(9)}$ $C^{(3)}$ 3RD RANK ANTISYMMETRIC (3-FORM) 84

$[1,0,0,1]_{SO(9)}$ $\psi_\mu{}^\alpha$ GRAVITINO 128

Note: as can see from Dynkin labels, 1 vector index

1 spinor index

Last time: 10+1 dimensional supergravity, multiplet of 3 different quantum fields

- use representations of little group to characterize

- $g_{\mu\nu} = g_{\nu\mu}$ so in $(10+1)d$ $\binom{12}{2} = 65$ components, 44 physical, 21 nonphysical

- Lagrangian for gravity can be written as

$$\frac{1}{G_N}\int dx^{10+1}\sqrt{-g}\,R,\ R \text{ Ricci scalar made of } g_{\mu\nu}$$

...

Dimensional reduction

We want to write all gravity supermultiplets with 32 supercharges in any dimension.

Consider a manifold of dimension d, M^d. Would like to take it to M'^{d-1}.

$M^d \longrightarrow M'^{d-1}$ by $x^M = (\overbrace{x^i}^{x'}, x^{d-1})$, $i=0,\dots,d-2$ ie forget the dependence on the last coordinate and use x'

eg $\phi(x) \longrightarrow \phi'(x')$ for a scalar field

And for a gauge field?

$A_\mu(x) \longrightarrow A_i(x'), A_d(x')$

A_i → vector index; A_d → singlet. SO gauge field decomposes in lower dimensional gauge field and scalar field

Now, specific case, from $(10+1)d$ to $(9+1)d$. Little group $SO(9) \supset SO(8)$.

$[1000]_9 = [1000]_8 + [0000]_8$ ie vector $SO(9) \longrightarrow$ vector $SO(8)$ + scalar (singlet $SO(8)$)

$9 = 8 + 1$

Compactification

Imagine a dimension is compactified, eg to S^1_R

Every field has a natural mode expansion $\phi(x) = \sum_n e^{in\vartheta}\phi_n'(x')$ $\vartheta \in S^1_R$

$\Big\}$ what does this mean?

because space is compact then momenta is quantized, can represent physics with x or x'

when compute dispersion relation $p^2 = m^2 = p'^2 + \left(\frac{n}{R}\right)^2$

at $R \to 0$ all modes decouple and left with 0 mode

Dimensional reduction is the end of IR flow of compactification ⟵ ?!?

. .

Now we want to decompose gravity. What happens to the 3 fields of the supergravity multiplet?

$[2000]_9 = [2000]_8 + [1000]_8 + [0000]$ ⟵ $\boxed{\text{GRAVITON}}$

$\;\;44 \qquad\qquad 35$

can think as $g_{\mu\nu} \to g_{ij},\, g_{i,10},\, g_{10,10}$

and the gravitino? [aside notice that $\underset{16}{[0001]_9} = \underset{8}{[0010]_8} + [0001]_8$]

so $\underset{128}{[1001]} = \underset{56}{[1001]_8} + \underset{56}{[1010]_8} + \underset{8}{[0010]_8} + [0001]_8$ ⟵ $\boxed{\text{GRAVITINO}}$

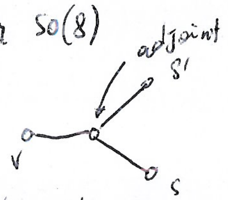
vector $[1000]$ and decomposition of spinor part as in aside

compute dimension of first 2 as gravitino $(D-3)\cdot \text{dim spinor} =$

$= (10-3)\cdot 8 = 56$

OR use dimension formula for $SO(8)$

for $SO(8)$



positive roots = 12

$\dim[n_1, n_2, n_3, n_4] = (n_1+1)(n_2+1)(n_3+1)(n_1+n_2+2)(n_2+n_3+2)(n_2+n_4+2)(n_1+n_2+n_3+3)\cdot$
$\cdot(n_2+n_3+n_4+3)(n_1+n_2+n_4+3)(n_1+n_2+n_3+n_4+4)(n_1+2n_2+n_3+n_4+5)\cdot \frac{1}{2^3 3^2 \cdot 4 \cdot 5}$

(n_1: vector, n_2: adjoint, n_3: spinor, n_4: spinor')

decomposed graviton and gravitino, what about the $\boxed{\text{3-form?}}$

$\underset{84}{[0010]} = \underset{56}{[0011]_8} + \underset{28}{[0100]_8}$

84 → 3rd rank antisymmetric; 56 → 3rd rank antisymmetric; 28 → adjoint

$[0011]_8$ has dimension 56 because:
1) DO IT! (formula)
2) $\binom{8}{3} = 56$
3) S_3 symmetry ⟶ compare to $[1001]_8$ and $[1010]_8$, you exchange the right Dynkin labels

SO what did we obtain?

	$(10+1)\,d$	$(9+1)\,d$	
GRAVITON	$[2000]_g$	$[2000]+[1000]+[0000]$	$g_{ij} + A_i + \phi$ — graviton, gauge field, scalar field $\to$ dilaton
+		+	
GRAVITINO	$[1001]_g$	$[1001]+[1010]+[0010]+[0001]$	$\underbrace{\psi_i^{\alpha} + \psi_i^{\dot\alpha}}_{\text{gravitino gravitino}'} + \underbrace{\psi^{\alpha} + \psi^{\dot\alpha}}_{\text{opposite chirality gravifermions}}$
+		+	
3-FORM	$[0010]_g$	$[0011]+[0100]$	$C^{(3)} + B^{(2)}$ — 3-form gauge field, 2-form gauge field

NOTE: 1 vector index, 1 spinor index

Type IIA supergravity multiplet $3 \to 9$

we got the supergravity multiplet in 10 dimensions

Now we need to understand the physics behind this field, ALL massless spectra

Compute all supergravity multiplets with 32 supersymmetry charges in any dimension

What information do we lose when we ~~do~~ perform a dimensional reduction?

THIS QUESTION DID NOT GET ANSWERED

...

Sth which looks "unrelated"

- think of Kaluza-Klein compactification — AND SO?
- dimensions are Einstein's attempt to unify gravity with electromagnetism — NICE TRY

sth (WHAT?) ensures Gauge invariance in lower dimension (IS IT RELATED TO THE HIGGS EFFECT

Last time: · supergravity multiplet in $(10+1)d$: $[2000]_9+[1001]_9+[0010]_9$

· dimensional reduction

· obtained type IIA supergravity multiplet

...

Now take a look again at type IIA gravity supermultiplet.

Identify 3 sectors

$$\underbrace{[2000]_8+[0100]_8+[0000]_8}_{\text{NS-NS sector}}+\underbrace{[1010]_8+[1001]_8+[0001]_8+[0010]_8}_{\text{R-NS / NS-R sector}}+\underbrace{[1000]_8+[0011]_8}_{\text{R-R sector}}$$

What if we could obtain it as a tensor product of "vector-and-spinor" representation?

That leads us to the concept of $\boxed{\text{factorization}}$

Consider the following tensor products

$$\underset{\text{vector}}{[1000]^2}=\underset{\substack{\text{2nd rank}\\\text{traceless}\\\text{symmetric}}}{[2000]}+\underset{\substack{\text{2nd}\\\text{rank}\\\text{antisymmetric}}}{[0100]}+\underset{\text{trace}}{[0000]}$$ is exactly the NS-NS sector

· $[1000]\otimes[0001]=[1001]+[0010]$ is part of the second sector. The other part is obtained by $[1000]\otimes[0010]=[1010]+[0001]$

· $[0010]\otimes[0001]=[0011]+[1000]$ is the R-R sector. To compute it easily use triality transformation of $[1000]\otimes[0001]$ ie move the "1" 2 places to the right

SO what did we obtain?

type IIA ~~gravity~~ supergravity multiplet can be decomposed as $\underbrace{([1000]+[0010])}_{L}\underbrace{([1000]+[0001])}_{R}$

ie as tensor product of 2 sectors of opposite chirality

Recall type IIA is a massless sector $\qquad \mathrm{IIA}=([1000]\oplus[0010])\otimes([1000]\oplus[0001])$

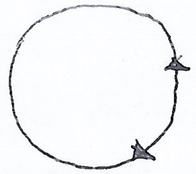
NOW change perspective and switch to worldsheet: consider embedding of a string into $(9+1)d$ and characterize it by a collection of massless scalars and fermions (massless in 2 dimensional sense). In 2d masslessness has curios features, to see it take a scalar field X and write action (only kinetic term as no mass term)

$$\int d^2\sigma\, \partial_i X \partial^i X \qquad X(\sigma_1, \sigma_2)$$ and switch to $z = \sigma_1 + i\sigma_2$, $\bar{z} = \sigma_1 - i\sigma_2$

rewrite the action as $\int dz \int d\bar{z}\, \partial X \bar{\partial} X$ and Euler-Lagrange equations of motion $\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}$ are $\partial \bar{\partial} X = 0$. What does it mean? Propagation can be composed out of functions of z or $\bar{z}$ only

THINK: $X = X(z) + \bar{X}(\bar{z})$ solves $\partial\bar{\partial} X = 0$

What does it mean physically? Imagine to take a worldsheet which is ~~a [illegible]~~ S^1. Then we have 2 types of propagation, left-handed or right-handed, which decouple and do not influence each other



That's because there is no interaction as there is no mass term in the Lagrangian (remember, all massless modes)

Consequence is that excitations are either left moving OR right moving and one can decompose the Hilbert space into $\mathcal{H}_L \oplus \mathcal{H}_R$ which factorize. Why is masslessness important From a high scale of energy all the dynamics can be inferred from the massless modes. (effective theo

Think of 2d example as analogous of 4d case $I = \int d^4x\, \partial_\mu \phi \partial^\mu \phi$, $\Box\phi = 0$, $p^2 = 0$ so

$\phi(x) = \int d^4p\, \delta(p^2 - m^2) \left(e^{ipx} a(p) + c.c. \right)$ analogous to $X = X(z) + \bar{X}(\bar{z})$

...

RETURN to spacetime perspective. Obtained something analogous, ie factorization of type IIA into $L \otimes R$, each one with 1 vector and 1 spinor. Call the vector "1" and the spinor "2", then the contributions are $[2000] + [0100] + [0000] : 1_L \otimes 1_R$, $[1010] + [0001] : 2_L \otimes 1_R$,

$[1001] + [0010] : 1_L \otimes 2_R$, $[1000] + [0011] : 2_R \otimes 2_R$

So massless sector comes from factorization principle

WHAT IF I had done the factorization in another way?

Think: L sector 8 bosons, 8 fermions
R sector 8 bosons, 8 fermions } BUT fermions have opposite chirality, SO what if I take fermions which transform under the same representation?

Compute $([1000]\oplus[0001])^2 = \underbrace{[2000]\oplus[0100]\oplus[0000]}_{\text{vector}\otimes\text{vector}}\oplus\underbrace{2[1001]\oplus2[0010]}_{\text{vector}\otimes\text{spinor}}+\underbrace{[0002]\oplus[0100]\oplus[000}_{\text{spinor}\otimes\text{spinor}}$

to compute the (spinor $\otimes$ spinor) sector, one might think of it as the triality transformation of $[1000]\otimes[1000]$ ~~...~~

[as an aside in D_n $[0\ldots11]$ is $(n-1)$th rank antisymmetric tensor
$[0\ldots20]$ is nth rank antisymmetric tensor } self-dual
$[0\ldots02]$ is nth rank antisymmetric tensor } ~~dual~~ antiself-dual

about ~~...~~ self-dual/antiself-dual, think of EM, $F_{\mu\nu}$, $*F_{\mu\nu}=\varepsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma}$ and
$F_{\pm}=F_{\mu\nu}\pm\varepsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma}$ ⊕ combination self-dual, under contraction with ε goes back to itself
⊖ combination antiself-dual, under contraction with ε goes to minus itself

NOW let's compute dimensions of this new supermultiplet

$[2000]$ $35=\binom{9}{2}-1$ as 2nd rank symmetric traceless

$[0100]$ $28=\binom{8}{2}$

$[0000]$ 1

$[1001]$ $56=(10-3)\cdot2^{4-1}$ as for gravitino $(D-3)\cdot$dim spinor and for D_n dim spinor $=2^{\text{rank}-1}$ 2^{n-1}

$[0010]$ $8=2^{4-1}$ spinor

$[0002]$ 35 by triality (4th rank self-dual antisymmetric tensor)

$[0100]$ $28=\binom{8}{2}$ as before

$[0000]$ 1 as before

We call this new multiplet <u>type IIB</u> supergravity multiplet, also massless.

(lower mass contribute more to dynamics than high mass).

It also ~~cases~~ contains 2 gravitinos, <u>BUT</u> of opposite chirality. <u>So</u> type IIA supergravity multiplet (obtained from dimensional reduction) is <u>non-chiral</u>, whilst type IIB supergravity multiplet is <u>chiral</u> (obtained from analogous factorization).

...

However, note that $[1000]\oplus[0010]$ can be a supermultiplet on its own right, composed of 8 bosons and 8 fermions, 16 supercharges

- Of 16 supercharges 8 Q's s.t. $Q|\rangle = 0$
- other 8 supercharges $\begin{cases} 4 \text{ creation} \\ 4 \text{ annihilation} \end{cases}$, left with 2^4 states $\begin{cases} 8 \text{ bosons} \\ 8 \text{ fermions} \end{cases}$

<u>So</u> minimal supermultiplet in $(9+1)d$ is $[1000]\oplus[0001]$ Vector multiplet (V-plet)

<u>NOW</u> want to make a gravity multiplet (G-plet). We can either:

- tensor product of 2 V-plets (32 SUSY) that explains the saying "Gravity is the square of Gauge theories"
- take V-plet ($[1000]\oplus[0001]$) and do the minimal action (ie tensor product) to obtain $[2000]$ ie graviton out of that. Just multiply by a vector!

$$([1000]\oplus[0001])\otimes[1000] = [2000]\oplus[0100]\oplus[0000]\oplus[1001]\oplus[1010]$$

↳ due to this operation, still equal number of bosons and fermions

$[2000]\oplus[0100]\oplus[0000]\oplus[1001]\oplus[1010]$ is $(9+1)d$ gravity supermultiplet with 16 supercharg

Up to now $\Rightarrow$ we found 3 differents multiplets with 32 supercharges, massless

by performing dimensional reduction

- $[2000]_9 \oplus [1001]_9 \oplus [0010]_9$ (10+1)d supergravity, massless limit of M-theory (spectrum unique can be derived by supersymmetry)
- $([1000]_8 \oplus [0001]_8)([1000]_8 \oplus [0010]_8)$ type IIA supergravity, massless limit of type IIA String Theory in (9+1)d. String with 2 different modes by factorization

analogous factorization

- $([1000]_8 \oplus [0001]_8)^2$ type IIB supergravity, massless limit of type IIB String Theory.

aside note that dimensional reduction of type IIA supergravity is the same as dimensional reduction of type IIB Supergravity. Physically that's because I can't distinguish between one type of spinor and another. Also, $([1000]_8 \oplus [0010]_8)^2$ gives an equivalent Type IIB supergravity multiplet.

Moreover, massive sector of type IIA and type IIB coincide. In the notation, "II" counts the number of gravitinos. Can use other notation that takes into account chirality of gravitinos ie type IIA is called (1,1) SUSY in (9+1)d (gravitinos with opposite chirality), type II B is also called (2,0) SUSY in (9+1)d.

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Now observe that one of the factors in the Type II A/B is already a V-plet of 16 supercharges (ie $[1000] \oplus [0001]$), more in detail

$$[1000]_8 \oplus [0001]$$

A_μ $\quad \psi_\alpha \longrightarrow$ gaugino, superpartner of Gauge field

$\hookrightarrow$ Gauge field

Can construct other supermultiplets in 2 ways: a) multiply this V-plet by another vector to obtain gravity (as we already did)

b) Yang-Mills extension (see more later)

$\Longleftarrow$ $\boxed{\text{IN BOTH CASES STILL SAME NUMBER OF BOSONS AND FERMIONS}}$

a) We already did it

$$\left([1000]_8 \oplus [0001]_8\right) \otimes [1000]_8 = \underset{35}{[2000]} \oplus \underset{28}{[0100]} \oplus \underset{1}{[0000]} \oplus \underset{56}{[1001]} \oplus \underset{8}{[0001]} \xrightarrow[\text{dimensions}]{\text{observe by}} \begin{cases} 64 \text{ bosonic} \\ 64 \text{ fermionic} \end{cases}$$

b) Yang-Mills extension is performed by multiplying by a Gauge field in the adjoint representation of a group

$$\left([1000]_8 \oplus [0001]_8\right) \otimes \mathrm{Adj}_G$$

<u>aside</u> one usually says there are 5 superstring theories in $(9+1)d$, 6 if we consider the M-theory in $(10+1)d$ as well. The first 3 are the M-theory, the type IIA and the type IIB. What about the other 3? They are obtained from Yang-Mills extensions, so, while the other 3 have 32 supercharges, these have 16 supercharges. The three theories are: (and use the adjoint of the following groups)

- <u>Heterotic theories</u>, Gauge field in adjoint of $E_8 \times E_8$ (behave differently under left-right exchange
- <u>a theory</u> in which Gauge field in adjoint of $SO(32)$
- <u>Type I theory</u>, Gauge field again in adjoint of $SO(32)$.

...

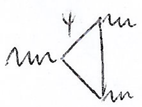
An important concept is that of <u>anomalies</u> ie classical symmetries which are violated in quantum theor

Take Dirac fermion in $(3+1)d$, ~~the~~ axial current $J_\mu{}^A = \bar\psi \gamma_\mu \gamma_5 \psi$

$$\partial_\mu J^{\mu A} = O(m) + c\,\varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma} \qquad d^* J^A = c\, F \wedge F$$

↳ as contains field strength, only happens when couple to Gauge theory.

how do we calculate c? It gets various contributions depending on the field content. In diagrams:

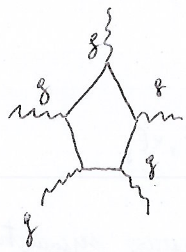


anomaly field theory $\sum_i c_i = 0$

How does this generalize to $(9+1)d$? Instead of 3-point function get 5-point function!

Have to consider all couplings to gravity ie via graviton.

g: graviton



all fields which couple to gravity
each time the 2 spinor labels differ, $c \neq 0$
exchange of 2 spinor labels $c \to -c$

let's compute the anomaly for <u>type IIA supergravity</u>. Spinor labels to [1001], [1010] cancel as $c \to -c$

THINK: [0010], [0001] cancel, [0011] has identically $c=0$ ← Same for 2 gravifermions, and 3-form is identically

So type IIA is <u>chiral</u> as there is no anomaly (can always find opposite chirality spinor)

Next, compute for type IIB. Relevant spinor representations are 2[1001], 2[0010] and [0002]. It's not trivial and not obvious, but miracolously $\sum_i c_i = 0$!

So type IIB is anomaly free.

In general we want physical theories to be anomaly free, so <u>gravity anomaly</u> in (9+1)d restricts the number of vector multiplets, 496! All the other are anomalous. So look for a group whose adjoint has dimension 496. Note that $\dim SO(32) = 496$ and $\dim(E_8 \times E_8) = 2 \cdot \dim(E_8) = 2 \cdot 248 = 496$.

So adjoint of this group makes sure theories are anomaly-free.

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SUMMARY 6 STRING THEORIES

32 SUPERCHARGES	16 SUPERCHARGES
(10+1) d Supergravity, M-theory	Heterotic, gauge field in adjoint of $E_8 \times E_8$
Type IIA String Theory	" " $SO(32)$
Type IIB String Theory	Type I " " $SO(32)$

The last 3 are contructed by Yang-Mills extension. In principle they might be anomalous, but Green-Schwarz mechanism makes them no more anomalous.