

Want to study theories with 32 supercharges. Consider toroidal compactification of  $\pi$ -theory  $(10+1)d$  in Minkowski spacetime  $\times T^n$  i.e.  $\mathbb{R}^{1,10-n} \times T^n$ ,  $T^n$   $n$ -dimensional torus. As the  $n$ -torus is compact it will have Fourier modes in every direction BUT dimensional reduction will tell us to restrict only to 0 modes i.e. forget about dependence on torus directions and focus only Minkowski space. Consider different cases for  $n$ :

-  $n=0$  i.e.  $(10+1)d$ : have graviton, gravitino, 3-form  $[2000]_5$ ,  $[1001]_5$ ,  $[0010]_5$ ,

-  $n=1$  i.e.  $(9+1)d$ : have type IIA and in general 1 graviton,  $p$ -forms, gravitinos, gravifermions, scalar fields

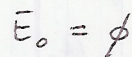
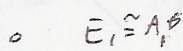
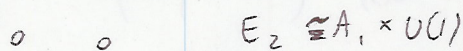
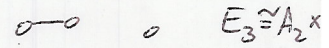
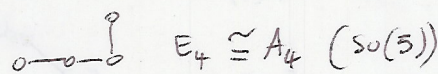
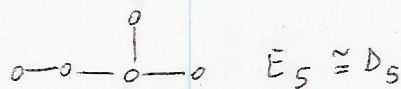
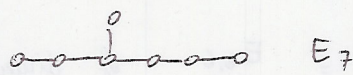
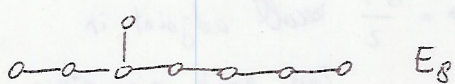
$p$ -forms give electric and magnetic sources, scalar fields have  $\langle \phi \rangle$  that parametrizes manifold

A relevant result is that ~~the~~ scalar manifold is of the form  $\frac{G_n}{H_n}$ .  $G_n$  is the maximally non-compact group with algebra  $E_n$ .  $H_n$  maximal compact subgroup of  $G_n$ .

It follows that the number of scalar particles is  $\dim \frac{G_n}{H_n}$ .

...

Let's review some  $E_n$  algebras



Now, for  $A_n$  the maximally non-compact ~~algebra~~ whatever is  $SL(n+1)$   
 for  $D_n$  " " " (?) is  $SO(n, n)$   
 for  $E_n$  " " " is  $E_{n, n}$

We don't care about  $E_3$  and over because not finite dimensional representation.

So  $H$  will be  $SO(n+1)$  for  $A_n$  and  $SO(n) \times SO(n)$  for  $D_n$



study  $\frac{G_n}{H_n}$  for  $n=1, \dots, 8$ . Before doing this recall  $H_n$  is a symmetry group. All massless fields transform in irreducible representations of  $H_n$ . So once we compute the massless fields we can fit them into the relevant representations.

=1 type IIA  $\tilde{E}_1 = SO(1,1) = \mathbb{R}^+$   $\dim(\mathbb{R}^+) = 1$  ie 1 scalar field non-compact

=1' type IIB  $E_1 = A_1$   $\frac{G_0}{H_1} = \frac{SL(2, \mathbb{R})}{SO(2)} = \mathbb{T}$   $\dim \mathbb{T} = 2$  ie 2 scalar fields   
 / 1 compact   
 \ non-compact

=2  $E_2 = A_1 \times U(1)$   $\frac{G_2}{H_2} = \frac{SL(2, \mathbb{R})}{SO(2)} \times \mathbb{R}^+$  So in (8+1)d have 3 scalars   
 / 2 non-compact   
 \ 1 compact   
 $3 = (3-1) + 1$

=3  $E_3 = A_2 \times A_1$   $\frac{G_3}{H_3} = \frac{SL(3, \mathbb{R})}{SO(3)} \times \frac{SL(2, \mathbb{R})}{SO(2)}$   $\dim \frac{G_3}{H_3} = (8-3) + (3-1) = 7$  scalars in (7+1)d.

=4  $E_4 = A_4$   $\frac{G_4}{H_4} = \frac{SL(5, \mathbb{R})}{SO(5)}$  of dimension 14

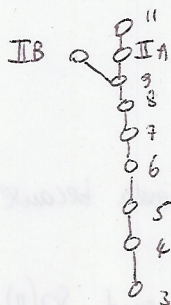
5  $E_5 = D_5$   $\frac{G_5}{H_5} = \frac{SO(5,5)}{SO(5) \times SO(5)}$  of dimension 25

=6  $E_6$   $\frac{G_6}{H_6} = \frac{E_{6,6}}{Sp(4)}$  of dimension  $42 = 78 - 36$    
 $\rightarrow = \frac{8 \cdot 9}{2}$  recall adjoint in 2nd rank symmetric

Side think of  $SU(3) \supset SO(3)$   $[1,1]_{SU(3)} = [2]_{SO(3)} + 2[1]_{SO(3)} + [0]_{SO(3)}$    
 (reducing by adjoint)   
 (Diagram: A horizontal line with a vertical line crossing it in the middle, representing the adjoint representation of SU(3).)

=7  $E_7$   $\frac{G_7}{H_7} = \frac{E_{7,7}}{SU(8)}$  of dimension  $133 - 63 = 70$  scalars

8  $E_8$   $\frac{G_8}{H_8} = \frac{E_{8,8}}{SO(16)}$  of dimension  $248 - 128 = 120$  scalars





Last time: theories with 32 supercharges

Some common theories are:  $N=8$  supergravity,  $AdS_7 \times S^4$ ,  $AdS_4 \times S^7$ ,  $AdS_5 \times S^5$

eg  $AdS_7 \times S^4$  massless content is that for  $n=4$  in the previous discussion

let's take it further:  $E_4 \cong A_4 = SU(5)$

$M = \frac{SL(5, \mathbb{R})}{SO(5)}$ , little group is  $SO(5)$

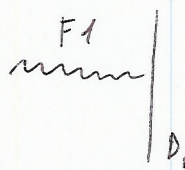
$\Rightarrow$  massless states are in irreducible representations of  $SO(5) \times SO(5)$ :

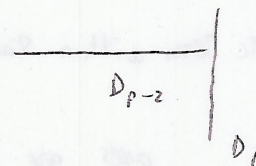
- bosons  $([20;00] \oplus [02;10] \oplus [10;02] \oplus [00;20])$ , gravitino  $[11;01]$ , gravifermion  $[01;11]$

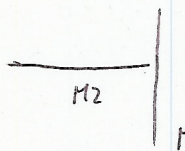
|          |        |        |        |    |    |
|----------|--------|--------|--------|----|----|
| graviton | 2-form | 1-form | scalar |    |    |
| 14       | 50     | 50     | 14     | 64 | 64 |

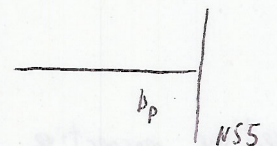
### Branes ending on branes

We saw some cases:

$F1$    $-1 \leq p \leq 9$  and magnetic dual

  $2 \leq p \leq 8$  (both in type II)

$M2$   (in M theory)

  $0 \leq p \leq 6$

let's write equations for the 1st case

$d *_{g_{p+1}} H = \delta^{(p)} Q - \delta^{(p-1)} *_{g_{p+1}} F$ , can integrate from an action. Note now we are on SPACETIME

perspective. So construct a low energy effective action, at most quadratic in  $B$  (derivatives).

$$\int_{\text{spacetime}} H \wedge *_{g_{p+1}} H + \int_{F1} B + \int_{\text{linear } B} B \wedge *_{g_{p+1}} F$$

quadratic  $B$       linear  $B$       linear  $B$

quadratic derivatives



let's consider particle. Terms in action are:

old line  $Q \int A$  linear response to gauge field  $= \int A_\mu(x) \frac{dx^\mu}{dt} dt$   
 $\uparrow$   
 via pullback  $\underbrace{\quad}_{\text{current}}$

or  $Q \int_{F1} B = Q \int_{F1} B_{\mu\nu}(X) \partial_i X^\mu \partial_j X^\nu \epsilon^{ij} d^2\sigma = Q \int_{F1} B_{\mu\nu} \underbrace{J^{\mu\nu}}_{\text{current}}$  (B antisymmetric, so  $\epsilon$  antisymmetric too)  
 $\uparrow$   
 2 parameters

charge that string carries with respect to 2-form

(P11) in complete analogy term that tells us that the brane is electrically charged with respect to the form.

or let's look at the WORLD SHEET perspective of F1:

B is coupling for kinetic term with a scalar field BUT mixing 2 scalar fields.

$\underbrace{\int G_{\mu\nu}(x) \partial_i X^\mu \partial_j X^\nu d^2\sigma}_{\text{couplings}}$  axes are scalars,  $G_{\mu\nu}$  is coupling in worldsheet perspective and metric in spacetime perspective.

The same term in spacetime perspective is how fundamental string couples to 2-form, for worldsheet perspective just coupling for kinetic term.

call vector  $\times$  vector =  $\text{Sym}^2(\text{vector}) + \Lambda^2(\text{vector}) + \text{trace}$  (ie graviton, 2-form, dilaton)

can write the trace  $\int d^2\sigma \phi(x) \partial_i X^\mu \partial_i X_\mu$

the 3 fields  $(g_{\mu\nu}, B, \phi)$  always appear as couplings of the kinetic term

upon quantization have renormalization group flow,  $\beta$ -functions equal to 0  $\iff$  field equations in spacetime

for  $\text{Sym}^2$  get Einstein's equations, for B the others, similarly for dilaton.

$\uparrow$   
 $G_{\mu\nu}$



Now try to understand the other term. Recall Gauge invariance of B  $\delta B = d\Lambda^{(1)}$ ,  $H = dB \Rightarrow \delta H = 0$

do terms 1,2,3 on worksheet give 1,2,3 in spacetime?

$$\oint_{F1} B = \int_{F1} dA_1 = 0$$

Stokes' theorem

But Stokes' theorem holds in absence of a boundary so the story is unfinished.  $F1$  has a boundary (pointlike) and its end is source for 1-form. In presence of boundaries  $\int_{\partial F1} A$  says that the end of the string is charged under the 1-form.

$$\delta \left( \int_{F1} B - \int_{\partial F1} A \right) = \int_{F1} d\Lambda^{(1)} - \int_{\partial F1} \Lambda^{(1)} = 0 \text{ as } \delta B = d\Lambda^{(1)} \quad \delta A = \Lambda^{(1)} \text{ so have to add this term to the action}$$

look at  $\delta B = d\Lambda^{(1)}$ ,  $\delta A = \Lambda^{(1)}$ ,  $F = dA \Rightarrow \delta F = \Lambda^{(1)} d\Lambda^{(1)}$

B is not Gauge-invariant  
A is not Gauge-invariant  
F is not Gauge-invariant

However  $\delta(F-B) = 0$  so  $\{F-B\}$  is Gauge invariant

Now can write effective action for gauge field on brane  $\int_{D_p} (F-B) \wedge *_{p+1} (F-B)$

If there was no 2-form as source it was  $F \wedge * F$

Now because of Gauge invariance when expand find  $B \wedge *_{p+1} F$  but also  $B \wedge * B$



recall from last time 3 interaction terms in the action

$$\int_{\Sigma} d^2\sigma \sqrt{\eta} \eta^{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} G_{\mu\nu}(x) \quad \text{interaction term for the metric on worldsheet}$$

gives rise to Einstein's equations

$$\int_{\Sigma} d^2\sigma \epsilon^{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} B_{\mu\nu}(x) = \int_{\Sigma} B \quad \text{interaction term for the 2-form, pull-back of 2-form}$$

into world sheet

↳ B first depends only on spacetime coordinate, then depends on parametrization of worldsheet

~~the~~ relevant equations from  $\beta$ -functions is  $d *_{g_{11}} H = \delta^{(8)} \mathcal{G} - \delta^{(9-p)} *_{p+1} F$ , from  $\int H \wedge * H + \int_{\Sigma} B + \int_{D_p} B \wedge *_{p+1} F$

$$\int_{\Sigma} d^2\sigma \sqrt{\eta} \eta^{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\mu} \phi(x)$$

...

Side: how to perform dimensional reduction from 5d to 4d ie  $so(3) \supset so(2)$

think of  $so(2)$  as the Cartan generator inside  $so(3)$

So, we have 5 cases; in every case let weight of  $so(3)$  become  $2 \cdot h$  in  $so(2)$  (twice the helicity)

• graviton  $[4] = x^4 + x^2 + 1 + x^{-2} + x^{-4}$  weights  $4, 2, 0, -2, -4 \Rightarrow$  helicities  $0, \pm 1, \pm 2$

as always  $\text{graviton}_5 \rightarrow \text{graviton}_4, 1\text{-form}_4, \text{scalar}_4$   
helicity:  $\pm 2, \pm 1, 0$

• gravitino  $[3] = x^3 + x + x^{-1} + x^{-3}$  weights  $\pm 3 \Rightarrow$  helicities  $\pm \frac{3}{2}$ : gravitinos  
weights  $\pm 1 \Rightarrow$  helicities  $\pm \frac{1}{2}$ : gravifermions

so  $\text{gravitino}_5 \rightarrow \text{gravitino}_4, \text{gravifermions}_4$

• 1-form  $[2] = x^2 + 1 + x^{-2}$

analogously as before  $1\text{-form}_5 \rightarrow 1\text{-form}_4, \text{scalar}_4$



• fermion  $[1] = x + x^{-1}$

fermion<sub>5</sub> → fermion<sub>4</sub>

• scalar  $[0] = 1$

scalar<sub>5</sub> → scalar<sub>4</sub>

Note fields always come in pairs for CPT invariance

...

So now we know how to compute all supergravity multiplets in any dimension with 32 supercharges

We see there is always a single graviton, collection of gravitinos, p-form fields, fermions, scalars

We realised that p-form fields lead to branes (generalized electromagnetism)

$$\begin{array}{ll} D_p & -1 \leq p \leq 9 \\ F1 & \\ NS5 & \\ M5 & \\ M2 & \end{array}$$

massless fields: • D<sub>p</sub> → V-plet 16 supercharges

• F1 → scalars and fermions (no gauge fields)  $x^M, \psi^M$   
10 10

• M5 → tensor multiplet; little group  $SO(4)$  so 2 Dynkin labels  $SO(4) \propto SU(2) \times SU(2)$

R-symmetry  $Sp(2)$ . Represent as  $[SO(4); Sp(2)]$

tensor multiplet is  $[2, 0; 0, 0] \oplus [10; 10] \oplus [0, 0; 0, 1]$

self-dual 2-form      4 fermions      5 scalars; recall 2nd rank traceless antisymmetric represent of  $Sp(2)$

• NS5 of type IIB already did

R-symmetry  $SO(4) \cong Sp(1) \times Sp(1)$

$D=11$  | NS5 (conclude that world volume theory contains gauge field)

(use this representation because in 6 dimension supersymmetry fermions are pseudoreal)

$[11; 0, 0] \oplus [0, 1; 10] \oplus [10; 0, 1] \oplus [0, 0; 11]$ ; always compute by dimensional reduction of V-plet on D9-brane. Note is same massless content as for D5-brane.

vectors are  
scalars of  
R-symmetry

fermions

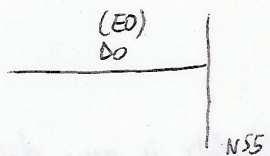
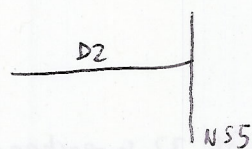


NS5 of type IIA. We face a puzzle: We don't have a D1-brane in

type IIA (recall  $p$  even for type IIA, odd for type IIB)  $\min \Big|_{NS5}$

at best D2-brane. So there will be a string on the world-volume of NS5.

Moreover, how many transverse scalars? 4! That's different from the 5 we found on the M5-brane. There is a missing scalar. Let's see why. Consider:



end of 0-brane is pointlike in all directions (instanton), carries charge as usual so expect 0-form Gauge field propagating on world-volume of brane

But remember  $p=0$  Gauge field is periodic (compact) scalar field

So we also should expect periodic scalars

Finally: 4 transverse scalars + 1 compact scalar  $\longrightarrow$  5 scalars !!!

So we solved the problem of having in principle 5 scalars but only observing 4

or let's talk about duality of type IIA and M-theory

remember: M-theory  $[2000]_9 \oplus [1001]_9 \oplus [0010]_9$  massless multiplet

think of M-theory compactified on  $S^1_R$ . Write action in 11d

$\int \sqrt{g} R d^{10+1}x$ . Count dimensions and observe there is scale.  $[R]=2$   
 $[g]=0$   
 $[x]=-11$  (space)

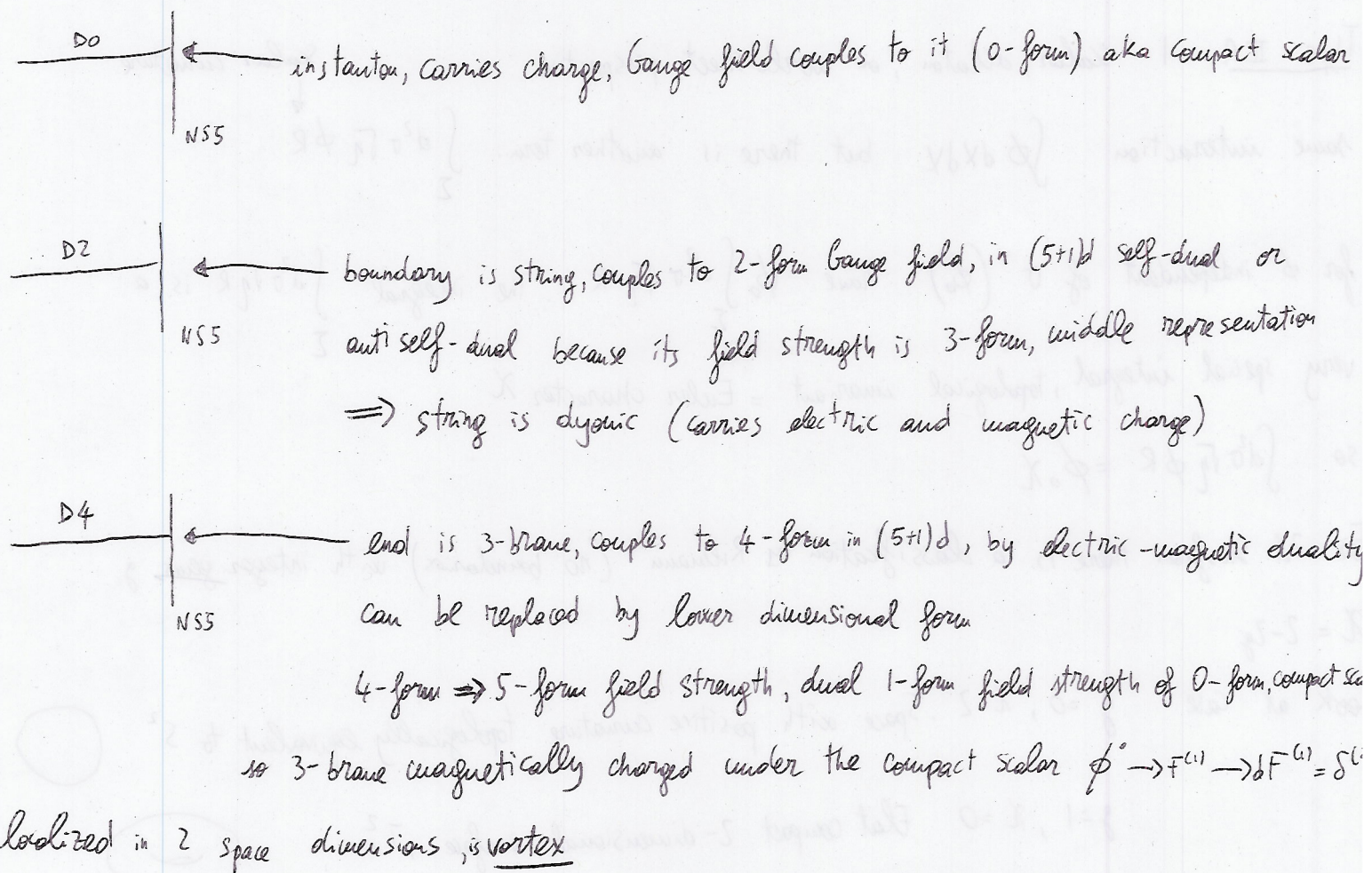
So introduce coefficient to make action dimensionless

$k \int d^{10+1}x \sqrt{g} R$   $[k]=9$

write  $k = l_p^{-9}$   $l_p$  is Planck's length.  $R$  radius of  $S^1_R$  measured in units of  $l_p$



Last time: compute the massless fields on the world volume of type IIA and NS5-brane.



as always have 4 massless scalars, Goldstone bosons

Now we have to discuss why does this extra scalar (fifth) appears

Discuss Type IIA - M theory duality on a circle

In M theory there is 1 scale: Planck scale. Take 1 direction, compactify on a circle,

have vacuum expectation value of a scalar:  $R$

$$[2000]_9 = [2000]_8 \oplus [1000]_8 \oplus [0000]_8$$

think of  $g_{10,10}(x^{10}) = \sum a_n e^{i n x^{10}}$  Fourier decomposition, take 0-mode, associated to scalar

moreover write invariant length  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{10,10} (dx^{10})^2 + \dots$

$\uparrow$   
coefficient interpreted as radius



Moduli space parametrized by value of  $R: \mathbb{R}^+$

Type IIA: 1 scalar, dilaton, on worldsheet perspective

same interaction  $\oint dX dX$  but there is another term:  $\int_{\Sigma} d^2\sigma \sqrt{\eta} \phi R$

scalar curvature  
↓

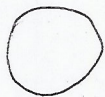
for  $\phi$  independent of  $\sigma$  ( $\phi_0$ ) have  $\phi_0 \int_{\Sigma} d^2\sigma \sqrt{\eta} R$ . The integral  $\int_{\Sigma} d^2\sigma \sqrt{\eta} R$  is a very special integral, topological invariant = Euler character  $\chi$

so  $\int d^2\sigma \sqrt{\eta} \phi R = \phi_0 \chi$

For 2d surfaces there is a classification by Riemann (no boundaries) with integer genus  $g$

$\chi = 2 - 2g$

look at case:  $g=0, \chi=2$ . Space with positive curvature topologically equivalent to  $S^2$



$g=1, \chi=0$  Flat compact 2-dimensional surface,  $T^2$



$g=2, \chi=-2$

$g, \chi = 2 - 2g$



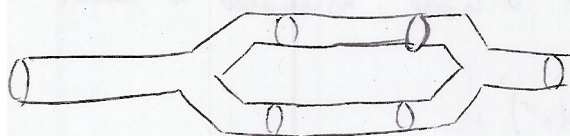
$g$  counts number of holes

low think of string perturbation theory and take these surfaces in their profile

$S^2$ : closed string creates itself, propagates as closed string and then disappears

$T^2$ : no string, then 1 string decomposes into 2 strings which propagate, annihilate, then nothing again

all these are processes where strings interact. Holes: where strings generate and interact.



corrections to propagator of closed string

Can think of genus as measuring strength of interaction in perturbation theory



Look at the relevant term in ~~the~~ the action:

~~$$S = \dots + \phi_0 \chi + \dots$$~~

$$S = \dots + \phi_0 \chi + \dots$$

$$\int D\chi e^{-S} = \int D\chi e^{-\phi_0(\chi - \chi_0) + \dots}$$

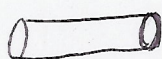
so  $g$  measures the strength of the interaction.  $e^{\phi_0} = g_s$  string coupling, one for each splitting

$g_s = e^{\langle \phi \rangle}$   $\phi$ : dilaton. All these surfaces have no boundaries.

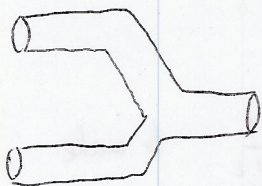
...  
What if the surfaces have boundaries?



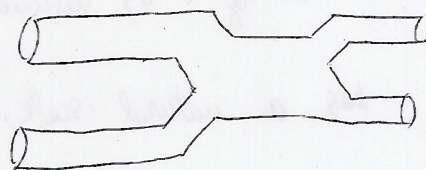
disc  
 $S^2$  with 1 boundary



cylinder  
 $S^2$  with 2 boundaries



$S^2$  with 3 boundaries



$S^2$  with 4 boundaries

think of scattering of 2 strings

In this case  $\chi = 2 - 2g - b$ , so each boundary adds 1 more power to  $g_s$

Type IIA has 1 scalar field, moduli space is  $\mathbb{R}^+$  parametrized by  $g_s$

$g_s$  doesn't have a length scale

effective term in action is  $\frac{1}{l_s^8} \int d^{9+1} \sqrt{G} R$ ;  $l_s$  string scale

Look at other effective term:  $\frac{1}{l_s^2} \int d^2 \sigma \partial X^\mu \partial X_\mu = T \int d^2 \sigma \partial X \partial X$  as  $X$  has scale

$T = \frac{1}{l_s^2}$  has dimension of tension

$T$  is interpreted as tension of F1



curiosities: why gravitational force  $\sim \frac{1}{r^2}$ ? Measures space dimensions,  $\frac{1}{r^{d-1}}$

To small scales all we can say is that it goes like  $\frac{1}{r^{(2+\epsilon)}}$ , but we didn't go under the scale of  $1 \mu\text{m}$ , maybe at lower scales there are hidden dimensions.

why only the gravitational force sees dimensions? Maybe because we live in a brane.

While electromagnetic sources are localized gravity is not localized on the brane.

1st time: M theory on  $S^1$ , and compare type IIA. By itself M theory has scale parameters, which measures all scales in  $(10+1)d: l_p$ . No scalar fields so that's the only scale BUT if we make one direction compact we introduce another parameter  $R$ , radius of the circle, think of it as vacuum expectation value of scalar.

Type IIA has a natural scale, the tension of the string  $T_{F1} = \frac{1}{l_s^2}$  to make

action dimensionless  $\frac{1}{l_s^2} \int_{F1} dx dx$ .  $T$ : energy per unit length.

There is charge for coupling to  $p$ -form field, what about mass? So brane has energy per unit volume! For particle (pointlike) have mass, and for string (1d) have energy per unit length, tension  $[T]=2$ . For  $p$ -brane have tension  $[T_p]=p+1$ .

we think of the different branes we've seen  $(F1, D_p, NS5, M5, M2)$ ,  $l_s^2 = \alpha'$

$T_{F1} = \frac{1}{l_s^2}$ ;  $D_p$ :  $T_{Dp} \sim \frac{1}{g_s l_s^{p+1}}$  why have  $\frac{1}{g_s}$ ? Remember  $g_s$  measures strength of

perturbative expansion. If  $g_s \ll 1$  then perturbation theory, processes with high genus are suppressed.

If  $g_s \sim 1$  have to take into account all topologies, perturbation theory not very reliable.

If  $g_s \gg 1$  non perturbative effects.



Now, recall  $D_p$ -brane naturally associated to open string (worldsheet with a boundary).

What is the simplest worldsheet with a boundary? Disc! Euler character = 1, so power of  $\frac{1}{g_s}$

NS5 brane couples to NS2 mode (closed string), now topology is sphere so  $T_{NS} \sim \frac{1}{g_s^2 l_s^6}$

Now compare  $T_{D5}$  to  $T_{NS5}$   $T_{D5} \ll T_{NS5}$  for  $g_s \ll 1$ . So should think in perturbation theory, of NS5 brane as extremely heavy object compared to D5.

$T_{F1} \ll T_{D1}$  because  $T_{F1}$  has no  $g_s$  dependence  $\frac{1}{l_s^2} < \frac{1}{g_s l_s^2}$

So we are left to determine  $T_{M5}$  but that's easy because M-theory has no scale

$T_{M5} = \frac{1}{l_p^6}$   $T_{M2} = \frac{1}{l_p^3}$ . We are missing factors of 2 and  $\pi$ . To fix them just use

Dirac quantization. Note funny features like  $T_{M2} = \sqrt{T_{M5}}$ .

Now think of M-theory over  $S^1$ . Recall have 3-form  $C^{(3)}$   $C_{\mu_1 \mu_2 \mu_3}$ . Now reduction to type II

$$C_{\mu_1 \mu_2 \mu_3} \begin{cases} C_{ijk} & 3\text{-form} \\ B_{ij,10} & 2\text{-form} \end{cases}$$

We know M2 and M5 brane couple to  $C^{(3)}$ . But in

type IIA F1 and NS5 couple to 2-form and D2 and D4 couple to 3-form. So do (D2, D4, F1, NS5) ~~derive~~ descend from (M2, M5) in  $(10+1)d$ ?

Think M2 brane can wrap  $S^1$  (either transverse to the circle or along the circle). In limit of very small  $S^1$  it loses a dimension and becomes F1. If it does not wrap it stays 2d and becomes D2. Same for M5, if it wraps it loses a direction and becomes D4. If it does not wrap becomes NS5. So branes of type IIA can derive from branes of M-theory by either wrapping them or not.



$$\text{cell } T_{F1} = \frac{1}{l_s^2} = T_{M2} \cdot R$$

radius of circle, because when brane wraps a circular direction just need to multiply length by tension. There will be a  $2\pi$ .

$$\frac{1}{l_s^2} = T_{M2} R = \frac{R}{l_p^3} \longrightarrow R l_s^2 = l_p^3$$

$$T_{D2} = \frac{1}{g_s l_s^3} = T_{M2} = \frac{1}{l_p^3} \longrightarrow l_p^3 = g_s l_s^3$$

$$\boxed{g_s l_s = R}$$

$$T_{D4} = \frac{1}{g_s l_s^5} = T_{M5} R = \frac{R}{l_p^6} \longrightarrow \text{consistent previous relations}$$

$$|0| \text{ SELF CONSISTENT } \quad \cancel{g} T_{NS5} = \frac{1}{g_s^2 l_s^6} = T_{M5} = \frac{1}{l_p^6}$$

parameters 2 relations are enough (was obvious as formulated with  $\frac{1}{l_s^{p+1}}$ )

validity between M on  $S^1$  and type IIA

$$\boxed{\begin{aligned} g_s l_s &= R \\ g_s l_s^3 &= l_p^3 \end{aligned}}$$

Strong-Weak duality (depending on strength of ~~couple~~ the string coupling)

$g_s \ll 1 \longrightarrow R \ll 1$  (type IIA) perturbative regime

$g_s \gg 1$  type IIA not valid, no perturbation regime, BUT can do computations in M-theory

because are effectively in  $(R \text{ big})$  ~~in~~  $(10+1)d$  with explicit map parameters.