25/01/13 STT (1

Last time: (5+1) theories in 10 and 11 d $\begin{cases} \text{11 d supergravity} \\ \text{Type II A, B, I} \\ \text{Heterotic, } SO(32),\ E_8 \times E_8 \end{cases}$

Note that only with 16 supercharges we can have non-Abelian gauge symmetries (not with 32)

Now start looking at the physics of the different forms (ie antisymmetric representations)

Review what know from electrostatics, go to (3+1)d and take A (1-form).

- Take A 1-form $\quad A = A_\mu dx^\mu$
- Construct field-strength $F = dA$
- for a pointlike electric source introduce $\delta^{(3)} = \underbrace{\delta(\vec{x}=0)\, dx_1 \wedge dx_2 \wedge dx_3}_{\text{volume form}}$
- write equations $d^*F = 0$ in absence of source

$d^* F = \delta^{(3)} Q$ in presence of electric source

Now suppose have magnetic sources $\quad dF = 0$ in absence of source

$dF = Q_M \delta^{(3)}$ in presence of magnetic source

What about Gauge-invariance? Given a gauge field A, can perform variation $\delta A = d\alpha$. If $F = dA$ then $\delta F = \delta dA = d\delta A = d^2\alpha = 0$

Write an effective action $\quad S = \int F \wedge * F$

Now generalize electrostatics to any form in any dimension

in D dimensions have form $C^{(p)}$ can write $C^{(p)} = C_{\mu_1 \dots \mu_p} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$

construct field-strength $G^{(p+1)} = dC^{(p)}$

equations of motion become $\quad d^* G^{(p+1)} = \delta^{(D-p)}$

(ie $D-p \to D-(p+1)+1$) eg $D=4$ $p=1$ for pointlike source in (3+1)d

so $d^* G^{(p+1)} = \delta^{(D-p)}$ electric

with magnetic sources

$dG^{(p+1)} = \delta^{(p+2)}$ eg in (3+1)d $p=1$ (ie magnetic sources care about codimension)

(d adds one dimension, $*$ does D-dimension)

Gauge invariance on $C^{(p)}$, $\delta C^{(p)} = dA^{(p-1)}$

$G^{(p+1)} = dC^{(p)}$ so $\delta G^{(p+1)} = 0$ by complete analogy

. Write an action $S = \int G^{(p+1)} \wedge * G^{(p+1)}$

...

Now apply it to the theories we know: <u>Brane spectroscopy</u>

• <u>(10+1)d supergravity multiplet</u> $[2000]_9 \oplus [1001]_9 \oplus \underset{C^{(3)}}{[0010]_9}$

Use the 3-form to ~~so~~ write $G^{(4)} = dC^{(3)}$

analogously $d * G^{(4)} = \delta^{(8)}$ ie source pointlike in 8 dimensions. electric source is a <u>membrane</u>

M2-brane.

$dG^{(4)} = \delta^{(5)}$ magnetic source is M5-brane

so in (10+1)d <u>2 branes</u> and <u>no strings</u> (2-form). Branes are <u>dual</u> with each other.

Mn means M theory in n space dimensions. ie electric, object pointlike in 8 dimensions, the 3 left are 1 time, 2 space ie M2. There are objects localized in more than 1 time dimension but their physics is more complicated.

• <u>Type IIA</u> $[2000]_8 \oplus \underbrace{[0100]_8} \oplus [0000]_8 \oplus [1010]_8 \oplus [1001]_8 \oplus [0010]_8 \oplus [0001]_8 \oplus \underbrace{[0011]_8} \oplus \underbrace{[1000]_8}$

Have 1-form, 2-form, 3-form $A^{(1)}, B^{(2)}, C^{(3)}$ (with $*$ in 10d) $[1000]_8, [0100]_8, [0011]_8$

from $A^{(1)}$ construct $F^{(2)} = dA^{(1)}$ and equation for electric source is $d * F^{(2)} = \delta^{(9)}$ 0-brane D

$\downarrow$

$[1000]_8$ for magnetic source is $dF^{(2)} = \delta^{(3)}$ 6-brane D

from $B^{(2)}$ construct $H^{(3)} = dB^{(2)}$ and equation for electric source is $d * H^{(3)} = \delta^{(8)}$, F1 electric

for magnetic source is $dH^{(3)} = \delta^{(4)}$ NS5-brane

⊛ F1 electric is a <u>string</u>

from $C^{(3)}$ construct $G^{(4)} = dC^{(3)}$ and equation for electric source is $d * G^{(4)} = \delta^{(7)}$ D2-brane

for magnetic source is $dG^{(4)} = \delta^{(5)}$ D4-brane

so found 4 different D-branes, one NS5-brane and one string

All theories in $(9+1)d$ ALWAYS have gravity, 2-form and scalar field SO from 2-form will always find a string which is called fundamental string. This is a character of the tensor product of 2 vectors. ~~[illegible]~~ NS5-brane always appears as well. These 2 are the electric object and magnetic object.

- Type IIB $\underbrace{[2000]_8 \oplus [0100]_8}_{\text{2-form}} \oplus [0000]_8 \oplus 2[1001]_8 \oplus 2[0010]_8 \oplus \underbrace{[0002]_8}_{\substack{\text{self-dual} \\ \text{4-form}}} \oplus \underbrace{[0100]_8}_{\text{2-form}} \oplus \underbrace{[0000]_8}_{\substack{\text{scalar field} \\ \text{axion}}}$

actually one of the scalar fields is compact ⊛ (has branes associated) ie 0-form Gauge field, for the other one which is non-compact, no. So compact scalar is 0-form gauge field. Dilaton is non-compact.

So have $B^{(2)}, C^{(2)}, C^{(4)+}, C^{(0)}$

For $B^{(2)}$ have F1, NS5 $\Big\}$ same dimensions
For $C^{(2)}$ have D1, D5

For $C^{(4)+}$ field strength $G^{(5)} = dC^{(4)+}$ and by self-duality $*G^{(5)} = G^{(5)}$ (that's the meaning of +)

the brane is only one and it is both electric and magnetic source D3

For $C^{(0)}$ have D(-1), D7

What is a (-1)-brane? $C^{(0)}$ has field strength $G^{(1)} = dC^{(0)}$. $d*G^{(1)} = \delta^{(10)}$ ie object localized in all space and time dimension. D(-1) or D instanton because it happens in an instant.

⊛ comes from R-R sector. Combine Type II theories, find that Type II theories have Dp-branes, p is even for Type IIA, p is odd for Type IIB. So forms of Type IIA are odd, forms of Type IIB are ~~odd~~ even. (referring to spinor x spinor part).

Last time: brane spectroscopy

- M-theory: M2, M5 deriving from $C^{(3)}$ — M2 electric, M5 magnetic
- Type II for both type II there is $B^{(2)}$ — F1 electric, NS5 magnetic

D_p, p odd $\longrightarrow$ (II B): $C^{(0)}$ — D(-1), D7; $C^{(2)}$ — D1, D5; $C^{(4)+}$ — D3 <u>dyon</u>

D_p, p even $\longrightarrow$ (II A): $C^{(1)}$ — D0, D6; $C^{(3)}$ — D2, D4

<u>Notice</u>: 2 missing numbers ie $p = 8, 9$. These branes also exist but not so much from the point of view of spacetime. From the fundamental string point of view. We don't see them in spacetime because the number is too high and they generate a back-reaction. So theoretically could have D8-brane in type II A, D9-brane in type II B. However, in this case the back-reaction is so strong that type II B becomes type I. Moreover all these branes are supersymmetric and, in general, adding one brane "breaks" (ie reduces) the supersymmetry by half.

<u>SO</u> start with 32 supercharges, can't have more than 5 branes (think: add one, become 16 supercharges, etc.) <u>UNLESS</u> either:

- special orientation which doesn't break supersymmetry
- special combination which breaks all the supersymmetry in 1 go (eg brane-antibrane

... }

Now we want to look at <u>scalar fields</u>. Each scalar field has a vacuum expectation value which in general is different from 0. $\langle \phi \rangle \neq 0$. They are a measure of the simplest properties of the background. Look at their properties case by case (NEXT PAGE)

- $\boxed{\text{M-THEORY}}$: no massless scalar fields, no VEV $\Longrightarrow$ unique vacuum

- $\boxed{\text{Type IIA}}$: 1 scalar field ϕ called the dilaton. Take $\langle\phi\rangle \neq 0$ and ask, can it change the vacuum? Well, by supersymmetry the potential has to be $V = V(\phi) \equiv 0$. That because our single supermultiplet contains gravity $g_{\mu\nu}$, which has to appear in a derivative term (WHY?) BUT terms like $\partial\partial g$ would translate to terms like $\partial\partial\phi$ and the potential for a scalar field can't contain derivatives. Therefore, any value of $\langle\phi\rangle$ is a "good" vacuum, so have space of vacua parametrized by $\langle\phi\rangle$. Actually it is more useful to define e^{ϕ} with $\langle e^{\phi}\rangle \equiv g_s \neq 0$. g_s is the string coupling and we use it to parametrize the space of vacua. It measures the strength of the string perturbation theory. In addition introduce concept of moduli space of vacua $\equiv$ "Space of solutions for $V(\phi)=0$". In type IIA case $V(\phi)\equiv 0$ so any ϕ is a vacuum ie moduli space of type IIA is $\mathbb{R}^+$, parametrized by g_s

- $\boxed{\text{Type II B}}$: 2 scalar fields $\begin{cases} \phi & \text{dilaton} \\ C^{(0)} & \text{axion (0-form gauge field)} \end{cases}$

ϕ ~~compact~~ non compact, $C^{(0)}$ compact (ie can identify $C^{(0)}(x) \cong C^{(0)}(x) + 2\pi \quad \forall x$)

It looks like moduli space is $\mathbb{R}^+ \times S^1$? Actually no, it is the coset $\dfrac{SL(2,\mathbb{R})}{U(1)}$ which is equal to $\dfrac{SO(2,1)}{SO(2)}$ of dimension 2. The "2" counts the number of scalar fields. As a comparison write $\mathbb{R}^+$ as $SO(1,1)$ of dimension 1, parametrized by a single parameter ie type IIA has a single scalar field.

Convenient to parametrize space of vacua of type IIB by $\tau_{IIB} = \dfrac{i}{g_s} + \dfrac{C^{(0)}}{2\pi}$

- $\boxed{\text{Type I}}$: 1 scalar field ϕ. Moduli space is $\mathbb{R}^+$.

- $\boxed{\text{Heterotic theories}}$: 1 scalar field ϕ. Moduli space is $\mathbb{R}^+$.

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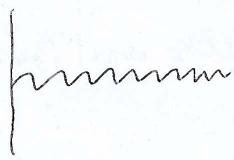
Branes ending on branes

Take a D_p brane, p even/odd for Type IIA / IIB.

From a worldsheet perspective a definition of a D_p-brane is the collection of points on which an F1 string can end

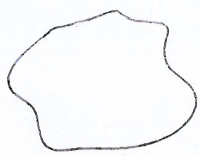


F_1 so in presence of branes can have open strings

D_p

Recall: BEFORE brane was solution to equation of motion, NOW points where strings end.

Strings can be either open or closed



closed string

open string

For open strings have to impose boundary conditions $\begin{cases} \text{Dirichlet ⊛} \\ \text{Neumann} \end{cases}$

⊛ D corresponds to Dirichlet.

Regarding the conditions, they tell whether the string is fixed or free to move.

Along the direction of the brane Neumann conditions, in transverse direction Dirichlet conditions.

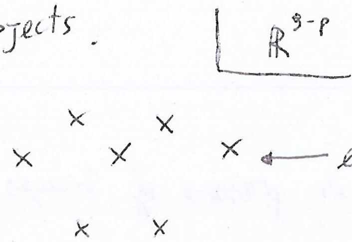
aside: in EM together with electron expect photon, in String Theory together with string expect 2-form B.

. . .

last time: • branes in string and M-theory. D_p-branes $p=-1,\dots,9$; p even type IIA
p odd type IIB

• fundamental strings F1, branes NS5, M2, M5. The world volume theory on each brane has 16 supercharges

Question: take a brane in $p+1$ dimensions embedded into $(9+1)d$. Pointlike in $9-p$ dimensions. Take $\mathbb{R}^{9-p}$
In $\mathbb{R}^{9-p}$ have collection of pointlike objects.

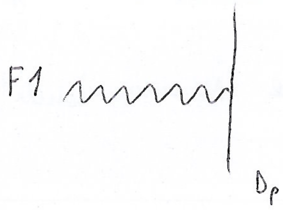


← each is a pointlike object (brane)

If I choose to place a brane somewhere I break translational invariance in $9-p$ dimensions.
It is spontaneous breaking of translation invariance. By Goldstone's theorem get massless fields.
So get $9-p$ massless scalar fields.
For D_p-brane get $9-p$ massless scalars, eg for F1 get 8 massless scalars
for NS5 get 4 (Type IIB)
for M5 get 5
for M2 get 8

Can also talk about massless fermions because we also have spontaneous SUSY breaking as we go from 32 supercharges to 16. Goldstone's theorem for SUSY gives 8 massless fermions.
We need 8 bosons too. Look at a D_p-brane

If I'm observer on D_p-brane, the end of F1 gives a point on D_p like a charged particle in $(p+1)d$. By Gauss' law



$$\int_{S^2} \bar{E}\cdot d\bar{S} = Q$$

Write Gauss' Law in differential form: $\int_{S^2} *F = Q$

Check dimensions: $*F$ is a 2-form in $(3+1)d$, S^2 is a 2-form, so integrating $*F$ over S^2 we get a scalar. Now generalize to $(p+1)d$, integrate over S^{p-1}

$$\int_{S^{p-1}} *F = Q \longrightarrow = \text{charge carried by the end of the fundamental string}$$

Therefore by complete analogy I conclude the existence of a massless Gauge field (analogous of photon) on D_p-brane.

Now recall the V-plet in $(9+1)d$ with 16 supercharges $([1000]_8 \oplus [0001]_8)$ gives 8 massless fermions and 8 massless bosons. D_9-brane will have a Gauge field A_μ on it, 0 massless scalars and 1 massless fermion: Gauge field and gaugino.

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Now perform dimensional reduction $SO(8) \longrightarrow SO(7)$

$[1000]_8 \longrightarrow [100]_7 \oplus [000]_7$, $[0001]_8 \longrightarrow [001]_7$ so V-plet becomes $\boxed{\underset{7}{[100]_7} \oplus \underset{8}{[001]_7} \oplus \underset{1}{[000]_7}}$

As we now have 1 massless scalar, ~~which~~ $9-p=1$ ie this representation is on the world volume of D8

...

Let's do further reduction : $SO(8) \supset SO(6) \times SO(2)$. For $SO(2)$ charge introduce fugacity q.

$[1000]_8 \longrightarrow [100]_6 \oplus [000]_6 (q^2+q^{-2})$ $\qquad [0001]_8 \longrightarrow [010]_6 q \oplus [001]_6 q^{-1}$

The vector $\rightarrow$ vector + 2 SO(2)-charged scalars, spinor $\rightarrow$ 2 inequivalent SO(2)-charged spinors

So now V-plet is $[100]_6 \oplus [010]_6 q \oplus [001]_6 q^{-1} \oplus [000]_6 (q^2+q^{-2})$

2 massless scalars, $9-p=2$, so lives on D7-brane.

Summarizing, on D_p-brane have always gauge field A_μ, 8 fermions, $9-p$ ϕ's.

Count degrees of freedom: - massless A_μ in $(p+1)d$: $p-1$

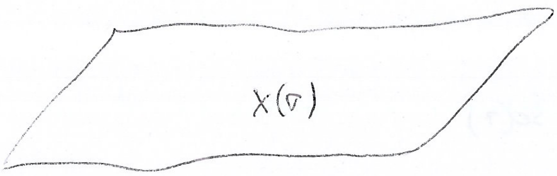
- 8 massless fermions $\quad 8 \quad +$
- $9-p$ massless ϕ's $\quad 9-p \quad +$

massless V-plet, $\Longrightarrow$ $8+8$

16 supercharges

So now we know the massless content of D_p-brane. Now we have to write effective action. Learn that there is very natural way to construct Gauge theories as strings end on branes, and therefore the point where they end is charged (due to the coupling of strength g_s).

F1 string, try to embed it into spacetime. Use 2 coordinates σ_1, σ_2 ie $X^\mu(\sigma_1, \sigma_2)$ $\mu = 0, \dots, D$



Simplest action: consider X as a scalar field ie $\int d^2\sigma\, \partial_i X^\mu \partial_i X_\mu$; ∂_i: derivatives on the world sheet with respect to σ_1 and σ_2

It looks like we have $D+1$ $(\mu = 0, \dots, D)$ massless fields. Take $D=9$, ie 10 massless fields. Did we not say they were 8? Therefore this formulation encorporates 2 non-physical ~~dofs~~ degrees of freedom and it is related to the fact that Goldstone's theorem applies to longitudinal modes (?) ~~check~~ or to the fact that 2 coordinates are identifying the string.

Therefore have $\underline{D-1}$ massless fields. In $(25+1)d$ have 24 massless bosons.

In $(9+1)d$ have 8 massless bosons.

aside: why do we have 2 non-physical degrees of freedom in the last example?

Because of Goldstone's theorem!

Think of particle in D dim (easier example). Write action with $X^\mu(\tau)$ $\mu = 0, \dots, D-1$

important: just used 1 coordinate τ.

$I = \int d\tau \, \dot{X}^\mu \dot{X}_\mu + \dots$, looks like we have D massless fields. But do we really?

By Goldstone's theorem, simply by placing a particle in spacetime I spontaneously break D-1 symmetries, i.e D - (# of coordinates identifying the particle), D-1 space translations. So have D-1 massless fields.

Now extend to string, ~~the~~ embedding function uses 2 coordinates, so in completely analogous way lose D-2 generators, have D-2 massless fields. $I = \int d^2\sigma \, \partial_i \vec{X}^\mu \partial_i X_\mu$

[Polyakov action is $I = \int \sqrt{\eta} \, \partial_i X^\mu \partial_j X^\nu \eta^{ij} \underbrace{G_{\mu\nu}(x)}_{\text{not trivial metric}}$]

..

Now ask, where are the scalars? How do I compute in general the massless content of a D_p brane? Take as example D8.

have V-plet, little group is $SO(7)$: $\underset{A_\mu}{[100]_7} \oplus \underset{\psi}{[001]_7} \oplus \underset{\phi}{[000]_7}$ $\mu = 0, \dots, 8$

Write effective action $I = \int d^9x \left(F^2 + \bar{\psi} \partial\!\!\!/ \psi + (\partial_\mu \phi)^2 \right)$, $\langle \phi \rangle \neq 0$ in general. In 9 directions the brane is a point, but where? * * * The where is fixed by $\langle \phi \rangle$

So $\langle \phi \rangle$ characterizes position of the brane on transverse coordinate.

Now look for a general method to compute massless spectra. Take general D_p - brane.

Recall : • world volume is $(p+1)$-d

• little group is $SO(p-1)$

• already computed we get $(9-p)$ ϕ's (scalars)

So can try to decompose $SO(8)$ representations ~~of it~~ into <u>little group</u> and <u>R-symmetry group</u>, which rotates the scalars among themselves.

$$SO(8) \supset \underbrace{SO(p-1)}_{\text{little group}} \times \underbrace{SO(9-p)}_{\text{R symmetry group}}$$

We already performed this decomposition for $p = 8, 7$. Now decompose for $p = 6$.

$SO(8) \supset SO(5) \times SO(3)$

$$[1000]_8 = [10;0] \oplus [00;2]$$

$[10;0]$: vector $SO(5)$, singlet $SO(3)$; $[00;2]$: singlet $SO(5)$, vector $SO(3)$

think of $8 \left\{ \begin{pmatrix} 5 \\ \hline 3 \end{pmatrix} \right.$ just dividing into 2 vectors separately

$$[0001]_8 = [01;1]$$

So V-plet in $(6+1)$d has A_μ, $\Psi_\alpha^{\ a}$ (α: spinor index of $SO(5)$, a: spinor index of $SO(3)$), $\phi^{ab} = \phi^{ba}$ scalar. Got massless content of D_6 - brane.

Now try to get massless content for D_5 brane.

$SO(8) \supset SO(4) \times SO(4)$ ie 4 copies of $SU(2)$.

$$[1000]_8 = [1;1;0;0] \oplus [0;0;1;1] \qquad A_\mu + 4\ \phi^{ab} \text{ scalars}$$

$$[0001]_8 = [1;0;1;0] \oplus [0;1;0;1] \qquad 2 \text{ vectors, 2 spinors opposite chirality}$$

this is called $(1;1)$ SUSY in $(5+1)$d.

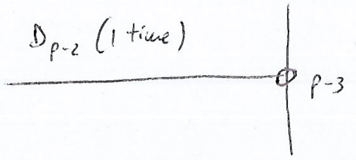
So now we know the massless field content in any D_p brane.

We understood the low-energy physics of Dp-brane, characterized by dynamics of massless V-plet in (p+1)d. Now need to understand for M5, M2, F1, NS5. But let's still review about D_p-brane.

Recall existence of Gauge field due to string that ends on brane. So have 2-form $F_{\mu\nu}$.

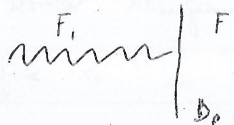
relative equation is $d *_{p+1} F = \delta^{(p)}$ $\quad (1 + (p+1) - 2) = p$

Have magnetic excitations as well, $dF = \delta^{(3)}$. It means that not only a string can end on a Dp-brane but also another object localized in 3 space dimensions



So by electric-magnetic duality D_{p-2} brane ends on D_p brane. End of D_{p-2} brane is a magnetic source for A_μ that lives on the D_p brane.

Let's write some equations. For



have 2-form B, field strength H

$d * H = \delta^{(8)} Q$
$\quad \llcorner$ charge of F1. But as F1 extends only in part of spacetime use θ-function.

$d * H = \delta^{(8)} Q\, \theta(x)$
$\quad \hookrightarrow$ selects where string is localized

But ~~??~~ wait! If I take d on both sides, on 1 side $d(d*H) = 0$ but $d(\delta^{(8)} Q \theta(x)) = Q\delta^{(9)}$ as $d\theta = \delta$. So the equation is wrong unless $Q = 0$. To solve this add another term.

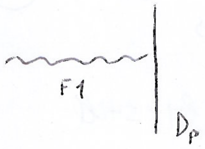
As string ends on Dp brane, add term $-\delta^{(9-p)}$ to take into account world volume of the brane.

$$\boxed{d * H = \delta^{(8)} \theta(x) Q - \delta^{(9-p)} *_{p+1} F_2} \Rightarrow d(d*H) = 0 = Q\delta^{(9)} - \delta^{(9-p)} d *_{p+1} F_2 \Rightarrow \boxed{d *_{p+1} F = \delta^{(p)} Q}$$

We recovered the electric source equation. Check dimensions of $d*H$ equation:

$1 + (10-3) = 8 = 9 - p + (p+1) - 2$ ok!

Notice that Q on F1 induces charge of particle A_μ.



relevant fields: B coupled to F1, $C^{(p+1)}$ coupled to D_p, A_μ which lives on the world volume of the brane.

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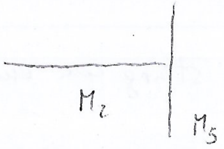
Repeat this story for M5 brane



Only M2 brane can end on M5 brane

$C^{(3)}$ is the one that couples both to M_2 and M_5

end of M_2 brane is a closed string which propagates on M_5 brane so have 2-form B

Now write equations

· field strength $G^{(4)}$ from $C^{(3)}$

· $d*_{11}G^{(4)} = \delta^{(8)}\theta - \delta^{(5)}H^{(3)}$, by taking exterior derivative find $0 = \delta^{(5)} - \delta^{(5)}dH^{(3)} \Rightarrow \boxed{dH^{(3)} = \delta^{(4)}}$

ie the end of the membrane is a magnetic source for the 2-form that lives on the 5-brane.

It is an electric source as well as $C^{(3)}$ is self-dual.

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Now recall on M_5 brane expect 5 massless scalar fields so that massless supermultiplet on M_5 brane is $\underset{\substack{\uparrow \\ \text{self-dual} \\ \text{2-form}}}{[2,0]} + 4\underset{\substack{\uparrow \\ \text{fermions}}}{[1,0]} + 5\underset{\substack{\uparrow \\ \text{massless} \\ \text{scalar fields}}}{[0,0]}$ in $SO(4)$

As scalar fields are completely analogous to each other R-symmetry groups which rotates them is $SO(5)$. So the massless supermultiplet is <u>tensor multiplet</u>:

$\underset{\substack{\uparrow \\ \text{little group} \\ SO(4)}}{[2,0;}\underset{\substack{\uparrow \\ \text{R-symmetry} \\ \text{transformation} \\ \text{group}}}{0\,0]} \oplus [1\,0;0\,1] \oplus [0\,0;1\,0]$ ie

· 2-form in <u>singlet</u> of $SO(5)$

· fermions in <u>spinor</u> of $SO(5)$

· scalars in <u>fundamental</u> of $SO(5)$

This multiplet contains chiral fields so is subject to <u>anomalies</u>

It has 16 supercharges but as it is chiral it is different from SUSY on D5-brane.

It is (0,2) SUSY, on world-volume of D5-brane was (1,1) SUSY.

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Recall on D5 R-symmetry SO(4) (not SO(5)!), little group is SO(4) as well

Supermultiplet is $[1,1;0,0] \oplus [1,0;1,0] \oplus [0,1;0,1] \oplus [0,0;1,1]$ for every chiral matter there is opposite chirality

little group ↑ ↑ R-Symmetry

The different SUSY's in (5+1)d are denoted based on number of gravitinos:

- 32 supercharges : (2,2) 2 copies of each type gravitino and R-Symmetry is $Sp(2) \times Sp(2)$
- 16 Supercharges : (1,1) on D5-brane, R-symmetry is $Sp(1) \times Sp(1)$
  (0,2) on M5-brane, R-symmetry is $Sp(2)$
- 8 supercharges : (0,1) R-symmetry is $Sp(1)$

So (m,n) notation denotes copies of gravitino OR rank of R-symmetry

(Think: could actually call $Sp(2)$ $SO(5)$)

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The fact that M2 ends on M5 tells us 2-form is self-dual and string is dyonic (both
electric and magnetic)

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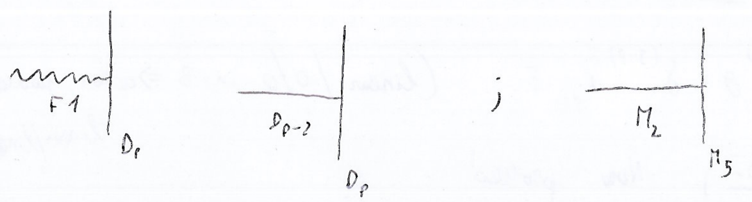
onsider other options for branes ending on branes. Use the principle that for every electric excitation
ook for magnetic excitation



hat happens on NS5 in Type IIB theory? (16 supercharges)

SUSY's, but either : . 4 transverse ϕ's

OR

. 5 transverse ϕ's

Type IIB we have 4 transverse ϕ's $\Longrightarrow$ (1,1) SUSY $\Longrightarrow$ there is 1-form A

? what is the Source for the Gauge field?

Source for A has to be pointlike, so look for something localized in 5 dimensions!

F1 already ends on D_p, so have D1 which ends on NS5.

Exercise: write analogous formulas and consider this is <u>electric source</u>.

For <u>magnetic source</u> there is D_3 brane because end of 3-brane is 2-brane with codimension 3.

As a result it's like a <u>magnetic monopole</u> (think in (3+1)d it's particle, in (4+1)d it's string, in (5+1)d it is sth 2-dimensional) A magnetic monopole is <u>always</u> a codimension-3 object.

$0 \leq p \leq 6$

Actually any D_p brane ends on NS5 brane

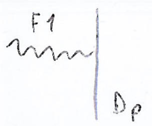
(but different for Type IIA, Type IIB)

What's difference between D1 and F1? On world volume of D1 there is Gauge field, but no Gauge theory on F1 (only fermions, etc.). Theory with Gauge field is difficult to quantize. This because on D_p brane F1 ends. F1 can end on D1 dividing its world volume into 2 different parts: <u>domain wall</u>

So recall 4 different cases:

...

<u>Now</u> want to derive these equations from an <u>action principle</u>



for this configuration $d *_{3,1} H = \delta^{(8)} g - \delta^{(8-p)} *_{p+1} F$ (linear / 0 / 0 in B $\Rightarrow$ action quadratic / linear / linear in

$\frac{1}{2}\int H \wedge * H$ for 1st term. (and $\frac{\delta I}{\delta B}$) Now proceed!

$$\frac{1}{2}\int H \wedge * H - \int_{F1} B + \int_{D_p} B \wedge *_{p+1} F$$

because of $\to$ F1 δ

Try to understand the last 2 terms. For $\int_{F1} B$ think of particle of charge Q, has coupling on its world volume like $Q\int \vec{A}\cdot d\vec{X} = Q\int A$

$\vec{X}(\tau) \to = Q\int A_\mu(x) \frac{dX^\mu}{d\tau} d\tau = Q\int A_\mu J^\mu \qquad J^\mu = Q\frac{dX^\mu}{d\tau}$