GENERALISED EM AND BRANES

Suppose we have a k-form potential $C^{(k)}$

Then the field strength will be $G^{(k+1)} = dC + \ldots$ and the dual $(*G)^{(D-k-1)}$

Electric localised source involves $(d*G)^{D-k} = \delta^{(D-k)} Q_E$

Magnetic localised source involves $(dG)^{k+2} = \delta^{(k+2)} Q_M$

$$\Rightarrow \quad Q_E = \int_{S^{D-k-1}} *G \quad , \quad Q_M = \int_{S^{k+1}} G$$

Note: $\delta^{(m)}$ means that the source is localised in m spatial dimensions.

Therefore, it is allowed to extend in the remaining $D-1-m$ spatial dimensions

We label the brane by the spatial dimension $D-1-m$ it extends to

$$\Rightarrow \boxed{\begin{array}{l} \text{A k-form is electrically coupled to a } D-1-(D-k) = k-1 \text{ brane} \\ \text{A k-form is magnetically coupled to a } D-1-(k+2) = D-k-3 \text{ brane} \end{array}}$$

Denote by: $C^{(k)} \begin{cases} \nearrow k-1 \\ \searrow D-k-3 \end{cases}$

- Example: $C^{(3)}$ in $D=11$: $C^{(3)} \begin{cases} \nearrow 2 \Rightarrow M2 \\ \searrow 5 \Rightarrow M5 \end{cases}$

- Example: $C^{(2)}$ and $C^{(3)}$ in TYPE IIA : $C^{(2)} \begin{cases} \nearrow 1 \Rightarrow F1 \\ \searrow 5 \Rightarrow NS5 \end{cases}$, $C^{(3)} \begin{cases} \nearrow 2 \Rightarrow D2 \\ \searrow 4 \Rightarrow D4 \end{cases}$

- Example: $C^{(2)}$, $B^{(2)}$ and $C^{(4)+}$ in TYPE IIB : $C^{(2)} \begin{cases} \nearrow F1 \\ \searrow NS5 \end{cases}$, $B^{(2)} \begin{cases} \nearrow D1 \\ \searrow D5 \end{cases}$, $C^{(4)+} \left.\begin{matrix} \nearrow 3 \\ \searrow 3 \end{matrix}\right\} \Rightarrow D3$

- Example: $C^{(2)}$ and $C^{(3)}$ in $D=9$: $C^{(2)} \begin{cases} \nearrow 1 \Rightarrow D1 \\ \searrow 4 \Rightarrow D4 \end{cases}$, $C^{(3)} \begin{cases} \nearrow 2 \Rightarrow D2 \\ \searrow 3 \Rightarrow D3 \end{cases}$

Aside: Electromagnetic duality corresponds to the fact that a k-form potential has the same degrees of freedom as a $(D-k-2)$-form potential since under $SO(D-2)$:

$$\binom{D-2}{k} = \binom{D-2}{D-2-k}$$

Therefore working with this new form will correspond to having:

$$(d*G')^{D-(D-2-k)} = (d*G')^{k+2} \quad \text{and} \quad (dG')^{(D-2-k)+2} = (dG')^{D-k}$$

Comparing this to the original discussion we see that this amounts to:

$$d*G \longrightarrow dG' \quad \text{and} \quad dG \longrightarrow d*G'$$