Imperial College London

MSc EXAMINATION May 2012

This paper is also taken for the relevant Examination for the Associateship

STRING THEORY SPRING 2012

For Students in Quantum Fields and Fundamental Forces

Monday, 9 May 2012: 14:00 to 17:00

Answer 3 out of the following 5 questions.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 3 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 3 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

1. Branes ending on Branes.

Write down the equations for branes ending on branes for the case of a E0 brane ending on a NS5 brane. Please include:

(i) The source equation. [10 marks]

(ii) The term in the action which describes the interaction between the bulk gauge field and the field strength which is localized on the brane. [5 marks]

(iii) The corresponding Gauss Law, either in differential form or in integral form.

[5 marks]

[Total 20 marks]

2. String theory in 5+1 dimensions with 32 supercharges.

(i) Give the little group for the massless states in 5+1 dimensions. Identify fundamental representations of the little group both in terms of their dimension and highest weights.

[2 marks]

(ii) Find the R symmetry for this amount of supersymmetry and its basic representations.

[3 marks]

(iii) Find the field content of the supergravity multiplet in 5+1 dimensions with 32 supercharges. Specify the different fields and their transformation laws under the little group and the R symmetry.

[9 marks]

(iv) Check that the number of bosonic and fermionic degrees of freedom in this theory match.

[2 marks]

(v) What is the dimension of the moduli space of vacua?

[2 marks]

(vi) Write down the coset space, G/H which is the scalar manifold of the theory, with G the maximally non-compact version of the E_n algebra and H is its maximal compact subgroup.

[2 marks]

[Total 20 marks]

3. Supergravity multiplets with 16 supercharges in various dimensions.

(i) Find the shortest massless multiplet in 6+1 dimensions with 16 supercharges. Write the different massless fields in terms of highest weights for irreducible representations of the little group, as well as the R symmetry group. [3 marks]

(ii) Using this multiplet find another massless supermultiplet in 6+1 dimensions with this amount of supersymmetry. [3 marks]

(iii) As a check, verify that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom. [1 mark]

(iv) What are the two supersymmetric theories with 16 supercharges in 5+1 dimension. What is the R symmetry in each case? On which branes can one find these supersymmetries. [3 marks]

(v) Find the shortest supermultiplet in each case. [5 marks]

(vi) For each case, construct another massless supermultiplet. [5 marks]

[Total 20 marks]

4. Brane Tensions.

(i) State the Moduli space of supergravity theories with 32 supercharges in 8+1 dimensions. Find the dimension of this moduli space. [2 marks]

(ii) Which compactifications of string and M theories have this 8+1 supergravity theory as low energy limit? State 3 different exmples. [3 marks]

(iii) Write down the dimensionless moduli which parametrize this moduli space. [2 marks]

(iv) For each background identify these moduli.

(v) Describe the branes that one can find in this theory. In particular, give the dimension of their worldvolume and state how many types arise in each case. [4 marks]

(vi) Using tension formulas of branes find the duality relations between the 3 different theories. [9 marks]

[Total 20 marks]

5. Quantum Field Theory on the Brane.

(i) State the massless bosonic fields that live on the world volume of a single Dp brane.
[2 marks]

(ii) Count the bosonic polarization modes and show the dependence on p. [2 marks]

(iii) Find the R symmetry group and determine the transformation laws of the bosonic fields under this group. [2 marks]

(iv) Write down the little group in $9+1$ dimensions and show how it breaks in the presence of the Dp brane. [2 marks]

(v) For the special case of $p = 9$ find the massless fermion on the world volume of the brane and write how it transforms under the little group. [2 marks]

(vi) For the special case of $p = 4$ find the massless fermions on the world volume of the brane and write down their transformation laws under the little group and the R symmetry group. [2 marks]

(vii) Find the gauge theory living on the world volume of N Dp branes. Specify the bosonic content of this theory. How many supercharges are there for this theory? [2 marks]

(viii) Find the moduli space of vacua for the system of these N Dp branes. [2 marks]

(ix) What is the gauge group on a generic point in the moduli space? Find the special points on the moduli space. What happens on these special points? [2 marks]

(x) Find the most general gauge symmetry on the moduli space. [2 marks]

[Total 20 marks]