Imperial College London

MSc EXAMINATION May 2013

This paper is also taken for the relevant Examination for the Associateship

STRING THEORY SPRING 2013

For Students in Quantum Fields and Fundamental Forces

Friday, 10 May 2012: 14:00 to 17:00

Answer 4 out of the following 4 questions.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 4 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 4 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

Turn over for questions

1. Branes ending on Branes.

Write down the equations of M2 brane ending on M5 brane as follows:

(i) Massless sector on the M5 brane. First find the little group on the world volume of the M5 brane, and the corresponding R symmetry group. [2 marks]

(ii) Find the massless spectrum on the M5 brane and identify the representation of the massless fields under the little group and the R symmetry group. [3 marks]

(iii) Match each representation with a corresponding field in the field theory on the world volume of the brane. [3 marks]

(iv) Write the source equation for a M2 brane ending on a M5 brane. [3 marks]

(v) Write the source equation for the end of the M2 brane in the world volume of the M5 brane. [2 marks]

(vi) Derive the source equation in (iv) from an action. [3 marks]

(vii) Find the gauge variation of the form that couples to the M2 brane, including gauge variations of all fields involved in the action from the previous question. [4 marks]

(viii) Ignoring self duality issues, write down an action which is gauge invariant under these gauge variations. [5 marks]

[Total 25 marks]

2. String theory in 4+1 dimensions with 32 supercharges.

(i) What is the little group L for massless states in 4+1 dimensions? [1 mark]

(ii) Specify all of its irreducible representations and their dimension formula. [2 marks]

(iii) List the most common representations which are discussed in physics, and the names of the corresponding fields. [5 marks]

(iv) What is the U duality group for this amount of supersymmetry, its non-compact form, and the corresponding maximally compact subgroup, H. [3 marks]

(v) Specify the basic representations of H, compute their dimension, and write their names. [12 marks]

(vi) Find the field content of the supergravity multiplet in 4+1 dimensions with 32 supercharges. Specify the different fields and their transformation laws under $L \times H$. [15 marks]

(vii) Check that the number of bosonic and fermionic degrees of freedom in this theory match. [2 marks]

(viii) What is the dimension of the scalar manifold? [1 mark]

(ix) Write down the coset space, G/H which is the scalar manifold of the theory, with G the maximally non-compact version of the E_n algebra and H is its maximal compact subgroup. Verify it has the right dimension. [2 marks]

[Total 43 marks]

3. Supergravity multiplets with 16 supercharges in 5+1 dimensions.

(i) How many different super-algebras are there in 5+1 dimensions with 16 supercharges. Specify the corresponding R symmetry and the 5 brane on which these super-algebras are realized. [6 marks]

(ii) What is the little group for the massless states in 5+1 dimensions. Identify fundamental representations of the little group both in terms of their dimension and highest weights. [6 marks]

(iii) Find the shortest massless multiplet for the non-chiral theory in 5+1 dimensions with 16 supercharges. Write the different massless fields in terms of highest weights for irreducible representations of the little group, as well as the R symmetry group. Specify the dimensions and name the different fields. [3 marks]

(iv) Using this multiplet find another massless supermultiplet in 5+1 dimensions with this type of supersymmetry. Use highest weights, dimensions, and name each field. [27 marks]

(v) As a check, verify that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom. [1 mark]

[Total 43 marks]

4. Enhanced Gauge Groups in Type I′

The dilaton equation in Type I′ takes the form

$$\left(\frac{l_s}{g_s}\right)'' = -\sum_i Q_i \delta(x - x_i), \tag{1.1}$$

where x is a coordinate along the interval of size R, $'$ denotes a derivative with respect to x, i runs over the charged objects on the interval, Q_i are their charges and x_i their positions. Set the boundary conditions on the interval to be $g_s(0) = g_0; g_s(R) = g_R$.

(i) What is the condition on the charges Q_i coming from the Gauss Law. [1 mark]

(ii) Find the solution to Equation (1.1), and express it using the two boundary values above. [3 marks]

(iii) Find all enhanced simple gauge groups in Type I′ (simple here means the gauge group in the vector multiplet is simple). [3 marks]

(iv) Describe the position of the branes in each case and write down the different charges of each object, as well as the values of the string coupling, by writing the solution as found above. [8 marks]

[Total 15 marks]