Imperial College London

MSc EXAMINATION May 2017

This paper is also taken for the relevant Examination for the Associateship

STRING THEORY

For Students in Quantum Fields and Fundamental Forces

Wednesday, 10 May 2017: 14:30 to 17:30

Please answer all 3 questions.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 3 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 3 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

1. Branes ending on Branes.

Write down equations for a (p, q) string ending on a (p, q) 5 brane as follows:

(i) First find the little group on the world volume of the (p, q) 5 brane, and the corresponding R symmetry group. What is the type of supersymmetry? [3 marks]

(ii) Find the massless spectrum on the (p, q) 5 brane and identify the representation of the massless fields under the little group and the R symmetry group. Use a highest weight notation for the representations. [16 marks]

(iii) Match each representation with a corresponding field in the field theory on the world volume of the brane. For fermions specify the type (Dirac, Weyl, Majorana, Symplectic). [12 marks]

(iv) Write the space time source equation for a (p, q) string ending on a (p, q) 5 brane. [16 marks]

(v) Deduce the (p, q) 5 brane world volume source equation for the end of the (p, q) string. [8 marks]

(vi) Identify the terms in the source equation of (iv) that can be derived from an action, and write each contribution to the action. [9 marks]

(vii) Find the gauge variations of the form that couples to the (p, q) string, including gauge variations of all fields involved in the action from the previous question. [14 marks]

(viii) Write down an action that includes the terms in (vi) and is gauge invariant under these gauge variations. [7 marks]

[Total 85 marks]

2. String theory in 6+1 dimensions with 32 supercharges.

(i) What is the little group L for massless states in 6+1 dimensions? [1 mark]

(ii) Specify all of its irreducible representations and their dimension formula. [6 marks]

(iii) What is the U duality group for this amount of supersymmetry, its non-compact form, its dimension, and the corresponding maximally compact subgroup, H. How many non-compact generators it has? How many compact generators? [9 marks]

(iv) Find the spectrum of the supergravity multiplet and fit the states into representations of the little group L as well as into representations of H. Specify the names of the corresponding fields, and their corresponding dimensions. [46 marks]

(v) Check that the number of bosonic and fermionic degrees of freedom in this theory match. [14 marks]

(vi) What kind of symmetry does this spectrum exhibit? [5 marks]

(vii) Write down the moduli space as a coset space, $\mathcal{M} = G/H$, with G the maximally non-compact version of the E_n algebra and H is its maximally compact subgroup. Verify it has the right dimension. [10 marks]

[Total 91 marks]

3. Brane Tensions.

(i) State the Moduli space of supergravity theories with 32 supercharges in 8+1 dimensions. Find the dimension of this moduli space. [4 marks]

(ii) Which compactifications of string and M theories have this 8+1 supergravity theory as low energy limit? State 3 different examples. [6 marks]

(iii) Write down the dimensionless moduli which parametrize this moduli space. [2 marks]

(iv) For each background identify these moduli. [9 marks]

(v) Describe the branes that one can find in this theory. In particular, give the dimension of their worldvolume and state how many types arise in each case. [16 marks]

(vi) Using tension formulas of branes find the duality relations between the 3 different theories. [9 marks]

[Total 46 marks]