Examination Paper

M.Sc. in Quantum Fields and Fundamental Forces

TP.6 – Supersymmetry

14.00 - 17.00 Tuesday, 18th. May, 1999

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Answer **THREE** out of the following four questions

Use a separate booklet for each question. Make sure that each booklet carries your name, the course title, and the number of the question attempted.

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SUPERSYMMETRY, May 1999

Answer three of the following four questions.

1.

(a) Define the $N = 1$ super-Poincare algebra. Verify that the super Jacobi identities are satisfied for the case of three fermionic generators.

(b) Express the momentum operator in terms of anticommutator of fermionic operators and show positivity of the energy.

(c) Consider a massless representation of $N = 1$ supersymmetry algebra. Relate the algebra of fermionic generators to a Clifford algebra and thus explain the way of constructing other states starting with a "Clifford vacuum" state. Describe content of the multiplet with lowest helicity 3/2.

2.

(a) Express the anticommutator part of supersymmetry algebra in terms of commutators using grassmann parameters.

(b) Using the formula $e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\cdots}$ where dots stand for other commutator terms show that if $U(x,\theta,\bar\theta) \equiv \exp(\theta Q + \bar\theta\bar Q - ix^\mu P_\mu)$ then

$$U(a^\mu,\xi,\bar\xi)U(x,\theta,\bar\theta) = U(x^\mu + a^\mu + i\xi\sigma^\mu\bar\theta - i\theta\sigma^\mu\bar\xi, \theta+\xi, \bar\theta+\bar\xi) \ .$$

(c) Write down the kinetic part of the action of the Wess-Zumino model (ignore the auxiliary field). Use dimensional considerations to fix the structure of the supersymmetry transformations. Determine the relation between the coefficients in the transformation rules from the condition of the invariance of the action.

3.

(a) For generic superfield $\Phi(x,\theta,\bar\theta)$ transforming under supersymmetry as

$$\delta_\xi\Phi = (\xi Q + \bar\xi\bar Q), \quad Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \quad \bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu.$$

explain which derivatives of Φ transform as superfields. Given the derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu, \quad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu,$$

show that they anticommute with $Q, \bar Q$ and compute their anticommutator.

(b) Formulate the conditions of spontaneous supersymmetry breaking. Show that there is no supersymmetry breaking in the Wess-Zumino model for one chiral superfield with the superpotential containing quadratic and cubic terms.

(c) Let V be a real scalar superfield. Write down the superfield analog of the Maxwell action for the superfield $W_\alpha = \bar{D}^2 D_\alpha V$ and derive the corresponding superfield equation for V.

4.

(a) Solve the constraint $\bar{D}_{\dot{\alpha}}\Phi = 0$ and thus show that a chiral superfield can be represented as

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \quad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}\ .$$

(b) Starting with the chiral superfield action

$$\int d^4x d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi + [\int d^4x d^2\theta \mathcal{W}(\Phi) + h.c.]$$

and introducing the unconstrained superfield X such that $\Phi = \bar{D}^2 X$ find the field equation for Φ.

(c) Consider the system of 3 chiral superfields with the superpotential

$$\mathcal{W}(\Phi) = \lambda\Phi_1 + m\Phi_2\Phi_3 + g\Phi_1\Phi_2^2.$$

Eliminate the auxiliary fields from the corresponding action to determine the scalar potential and show that supersymmetry is broken at its minimum.