Comparison of turbulence profiles in high Reynolds number turbulent boundary layers and validation of a predictive model.

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The modified Townsend-Perry attached eddy model of Vassilicos et al (2015) combines the outer peak/plateau behaviour of rms streamwise turbulence velocity profiles and the Townsend-Perry log-decay of these profiles at higher distances from the wall. This model was validated by these authors for high Reynolds number turbulent pipe flow data and is shown here to describe equally well and with about the same parameter values turbulent boundary layer flow data from four different facilities and a wide range of Reynolds numbers. The model has predictive value as, when extrapolated to the extremely high Reynolds numbers of the SLTEST data obtained at the Great Salt Lake Desert atmospheric test facility, it matches these data quite well.

1. Introduction

The structure of zero pressure gradient turbulence boundary layer (TBL) flows has been a subject of both fundamental and applied research for many decades. Townsend (1976), Perry & Abel (1979) and Perry et al (1986) developed the well-known attached-eddy model to predict the profile of the turbulent kinetic energy with distance from the wall. This model is operative in an intermediate range far from the viscous layer and predicts that the turbulent kinetic energy scales with the square of the wall friction velocity u_τ and decreases logarithmically with distance to the wall. The model leads to a logarithmic decay of the mean square streamwise and spanwise fluctuating velocity as function of the wall distance y:

$$\frac{\overline{u'^2}}{u_\tau^2} = B_1 - A_1 log\left(\frac{y}{\delta}\right) \tag{1.1}$$

where A_1 is the Townsend-Perry constant, B_1 is the additive constant for the variance and δ is the pipe radius, channel half width of turbulent boundary layer thickness. The logarithmic decay of the mean square streamwise velocity has been characterised for several high Reynolds turbulent boundary layer flows (see e.g. Marusic *et al.* (2013); Vincenti *et al.* (2013)). Vallikivi *et al.* (2015) compared the behavior of $\overline{u'^2}$ in turbulent boundary layers and in turbulent pipe flows and concluded as Marusic *et al.* (2013) that the constant of the logarithm decay of Eq. (1.1) are very similar in both flows ($A_1 = 1.24$, $B_1 = 1.48$).

Various measurements of turbulent boundary layers and turbulent pipe flows over the last 20 years show that an outer peak or plateau appears in the mean square fluctuating streamwise velocity above the buffer region at distances which depend on the Reynolds number. This outer peak is known to be the consequence of very large scale motions which develop at high Reynolds numbers (see Hultmark *et al.* (2012, 2013), Vallikivi *et al.* (2015), Vincenti *et al.* (2013)). Despite several experimental studies of turbulent boundary layers flows at fairly large Reynolds numbers ($Re_\tau > 7\,000$), there is consensus neither on the slope of the log decay of streamwise normal Reynolds stresses nor on the increase of an outer peak and its wall normal distance for very large Reynolds number. This outer peak or plateau cannot be explained by the Townsend-Perry attached eddy model. This model leads to no more than (1.1) throughout the mean flow's logarithm range. Vassilicos *et al.* (2015) proposed a new model for the streamwise velocity spectra including a k^{-m} slope (with $0 < m < 1$) in a range of scales larger than those responsible for the k^{-1} slope of the attached eddy model of Perry *et al.* (1986). This model, which can fit the Princeton pipe flow data over a range of Reynolds numbers up to the maximum available $Re_\tau = 98\,000$, does not only predict the amplitude and position of the outer peak of the mean square fluctuating streamwise velocity profiles but also supports a more realistic variation of the integral length scale with distance to the wall than that predicted by the Townsend-Perry model. If most studies agree on the logarithmic decay of $\overline{u'^2}$ the scaling of the outer peak is less consensual. The advantage of the model proposed by Vassilicos *et al.* (2015) is to combine both the outer peak behavior and the logarithm decaying range in a single model. In this paper we apply this model of Vassilicos et al (2015) to high Reynolds number turbulent boundary layer data from various independent facilities around the world and establish that the model fits such data too without much variation in fitting parameters.

2. The modified Townsend-Perry model

The modified model proposed by Vassilicos *et al.* (2015) for the energy spectrum $E_{11}(k_1, y)$ in the region $\nu/u_\tau \ll y \ll \delta$ is defined in four wavenumber ranges. The Kolmogorov range $1/y < k_1$ is identical to the Townsend-Perry model following the form $E_{11}(k_1, y) \sim \epsilon^{2/3} k_1^{-5/3} g_K(k_1 y, k_1 \eta)$. However, what is referred to as the "attached eddy" range where $E_{11}(k_1) \approx C_0 u_\tau^2 k_1^{-1}$ in the Towsend-Perry model is restricted to the range $1/\delta_* < k_1 < 1/y$ (instead of the Towsend-Perry model's original range $1/\delta < k_1 < 1/y$) and a new scaling $E_{11}(k_1) \approx C_1 u_\tau^2 \delta (k_1 \delta)^{-m}$ with $0 < m < 1$ is proposed in the range $1/\delta_\infty < k_1 < 1/\delta_*$ where δ_* and δ_∞ are two large length-scales such that $\delta_* \leqslant \delta_\infty$. The spectra at very large scales $k_1 < 1/\delta_\infty$ are kinematically constrained to be independent of k_1 and therefore of the form $E_{11}(k_1) \approx C_\infty u_\tau^2 \delta$. This new model predicts that the new range $1/\delta_\infty < k_1 < 1/\delta_*$ where $E_{11}(k_1) \approx (k_1 \delta)^{-m}$ is present only for y smaller than a wall distance y_* above which the original Townsend-Perry model is valid without alterations.

By integration of $E_{11}(k_1)$ we obtain the modified model for the mean square streamwise fluctuating velocity in the range $\nu/u_\tau \ll y \leqslant y_* = \delta A^{1/p} Re_\tau^{-q/p}$:

$$\frac{1}{2}\overline{u'^2}(y)/u_\tau^2 \approx C_{s0} - C_{s1} \ln(\delta/y) - C_{s2} (y/\delta)^{p(1-m)} Re_\tau^{q(1-m)} \tag{2.1}$$

with

$$C_{s0} = \frac{C_0}{1-m} + C_0 \ln B + C_0 \alpha \frac{q}{p} \ln Re_\tau \tag{2.2}$$

$$C_{s1} = C_0(\alpha - 1) \tag{2.3}$$

$$C_{s2} = \frac{m C_0 A^{m-1}}{1-m}. \tag{2.4}$$

and where α, B, p and q are the parameters used in the power law formulation of the two scales δ_* and δ_∞ as a function of wall distance and Reynolds number:

$$\delta_*/\delta = B\,(y/\delta)^\alpha Re_\tau^\beta \tag{2.5}$$

and

$$\delta_\infty/\delta_* = A\,(y/\delta)^{-p} Re_\tau^{-q} \tag{2.6}$$

where $\beta = \alpha\, q/p$ and $B = A^{-\alpha/p}$. This model predicts that the location y_{peak} of the outer peak or plateau scales as $\delta\, Re_\tau^{-q/p}$, i.e. $y^+_{peak} \sim Re_\tau^{1-q/p}$. In the region were $y_* < y$ equation (1.1) holds and completes the model.

The parameters of this new model (2.1)-(2.4) have been evaluated against the Princeton superpipe energy spectra for a large Reynolds number range ($1\,985 < Re_\tau < 98\,160$) by Vassilicos *et al.* (2015). It was shown that the new model is able to reproduce the correct scaling of the outer peak with a single set of parameters for all Reynolds numbers. It is important to determine whether this new model can also account for a variety of high Reynolds number turbulence boundary layer data and whether it can do it without much variation in its defining parameters.

3. Fits of high Reynolds number TBL data

In order to test the universality of the model (1.1, 2.1-2.4), statistics from several large Reynolds number turbulent boundary layer experiments have been collected. As already shown by Hultmark *et al.* (2012), the mean square streamwise fluctuating velocity exhibits a clear outer peak for extremely large Reynolds number only. As very few experimental results are available for $Re_\tau > 40\,000$ and as the model is designed and able to capture the outer peak or plateau region even in the absence of a clear peak, the model is also fitted here on data from experiments with smaller Reynolds numbers where the statistics of $\overline{u'^2}$ do not display an outer peak but just a tendency towards a plateau. The minimum Reynolds number required to fit the parameters of the model should be such that there is at least a short range of wall distances where the log decay of $\overline{u'^2}$ is visible. In the present contribution, data sets from four turbulent boundary layer experiments are investigated and compared, covering a range of Reynolds numbers from $Re_\tau \simeq 3\,200$ to $Re_\tau \simeq 72\,000$. The models parameters are also compared with the original values from the fits of the Princeton superpipe data by Vassilicos *et al.* (2015).

The first set of data has been recorded in the turbulent boundary layer wind tunnel of Lille which has a test section length of $20.6\,m$ in the stream-wise direction (x-direction) and a cross-section 2 m wide and 1 m high. Four free-stream velocities were investigated from $U = 3\,m/s$ to $U = 10\,m/s$ leading to a Reynolds number range from $Re_\tau = 3\,200$ to $Re_\tau = 7\,000$. The statistics are detailed in Carlier & Stanislas (2005). The hot wire data were recorded at 19.6 m from the entrance of the test section and the friction velocities were measured using macro-PIV by Foucaut *et al.* (2006). The second set of data is from hot-wire measurements in the large boundary layer wind tunnel at University of Melbourne. It has a $27\,m$ long test section of $2 \times 2\,m^2$. The statistics of the two lower Reynolds numbers are presented in Marusic *et al.* (2015) whereas the data for $Re_\tau = 19\,000$ were extracted from Marusic *et al.* (2010). The third set of data is from the turbulent boundary layer flow physics facility (FPF) at the University of New Hampshire (UNH) which is able to reach similar Reynolds numbers with a much longer test section ($72\,m$) and a lower free stream velocity ($U = 13.75\,m/s$). The facility and the hot wire data are described in Vincenti *et al.* (2013). The facility is an open circuit suction tunnel that draws from, and discharges to, the atmosphere. The streamwise free-stream turbulence intensity for $U > 7\,m/s$ is less than about 0.3%. The fourth and last set of data investigated here are the turbulent boundary layer and the turbulent pipe data from the Princeton superpipe (see Vallikivi *et al.* (2015)). A comparison of spectra from both superpipe experiments over the same range of Reynolds numbers ($3\,000 < Re_\tau < 70\,000$) was conducted by Vallikivi *et al.* (2014) leading to 5 scaling regions for the turbulent pipe and the turbulent boundary layer. The data acquired with the Nano Scale Thermal Anemometry Probe (NSTAP) are known to be affected by insufficient probe resolution (see Table 1). As in most studies of the authors of the NSTAP data, the present analysis is conducted with the data corrected for spatial filtering (see Smits *et al.* (2011)). The correction affects mainly the statistics of the two highest Reynolds numbers ($Re_\tau > 40\,000$) for $y^+ < 300$. However the present analysis was also conducted on the raw data to check that the quantitative conclusions of the present paper do not change significantly.

The data of each experiment are affected by different statistical convergence levels and/or slightly different experimental conditions. Some important characteristics of the four TBL experiments are summarized in Table 1. For instance, the LML facility produces a slight favorable pressure gradient (-0.5 Pa/m at 10 m/s) which is not the case for the New Hampshire and the Melbourne facilities. Hot wire acquisition time are of the order of $12\,000\ U_\infty/\delta$ for the Melbourne and LML facilities and 2 to 4 times smaller for the New Hampshire facility depending on the Reynolds number. This can explain apparent different levels of convergence of some statistics. There are also different tripping conditions for the TBL experiments with respect to the size of the boundary layer thickness (see Table 1) and in the way that scaling parameters such as friction velocity (u_τ) and boundary layer thickness (δ) have been determined by different techniques. For instance the boundary layer thickness is determined as δ_{99} except for the Melbourne data where δ is determined from a modified Coles law of the wall/wake fit to the mean velocity profile. This method can overestimate δ by up to 25% with respect to δ_{99} depending on the Reynolds number. The friction velocity u_τ is either determined by Preston tube or Clauser plot. The accuracy of both methods is known to be of the order of few percent. The values of u_τ for Lille experiments are validated by micro-PIV.

In figure 1 we plot, for comparison, mean square streamwise fluctuating velocity profiles from different experiments at two similar Reynolds numbers. As both the boundary layer thickness and the length of the hot wires are different, the comparison of the results in the region of the first peak ($y^+ < 30$) is not significant, especially at the highest Reynolds number. Due to a different definition of the boundary layer thickness, the Reynolds number for the Melbourne data may be over-estimated which may explain the slightly lower values of mean square streamwise fluctuating velocity profiles. However, taking into account the slightly different Reynolds number values and experimental conditions in each plot of figure 1 and the uncertainty in the friction velocity evaluations, it can be said that the four experiments exhibit comparable overall behaviours. The values of $\overline{u'^2}$ in the New Hampshire TBL are however slightly higher than the average of the three other

TABLE 1. Parameters of the four turbulent boundary layer experiments. The boundary layer thickness δ corresponds to δ_{99} except for Melbourne TBL where δ is determined from a modified Coles law of the wall/wake fit to the mean velocity profile. ℓ^+ is the sensor length in wall unit.

Experiments	Re_τ	δ	ℓ^+	Tripping conditions
Lille TBL Carlier & Stanislas (2005)	3196 5006 7022	0.319 0.298 0.304	6.1 8.4 11.5	Grid 5 mm thickness and 10 cm spacing on 2 m at the entrance of the test section
Melbourne TBL Marusic *et al.* (2010) Marusic *et al.* (2015)	10000 13600 19000	0.226 0.315 0.303	23.9 22 22	Grit P40 Sandpaper 154 mm long located at the entrance of test section (Re_τ=10000)
New Hampshire TBL Vincenti *et al.* (2013)	10770 15480 19670	0.736 0.717 0.688	14.6 22 28.6	6mm threaded rod 1 mm above the surface at 1.4 m from the test section entrance
Princeton TBL Vallikivi *et al.* (2014)	8261 25062 40053 72526	0.0283 0.0257 0.0257 0.0291	17 29 47 75	1mm square wire 76mm from leading edge 1.82 m upstream the measurement location.

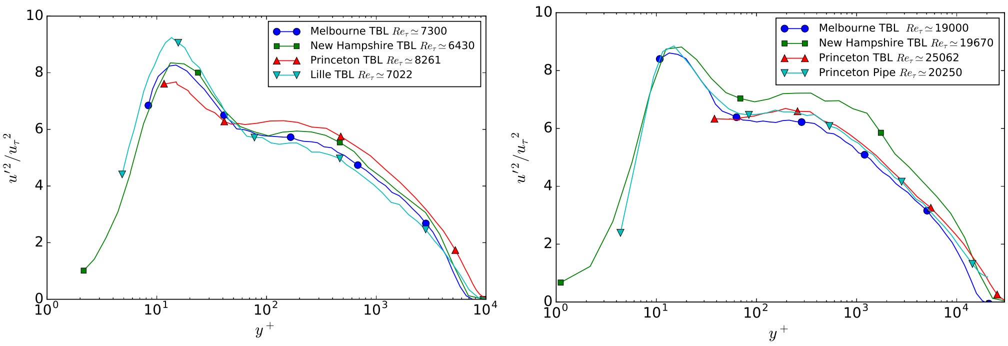

FIGURE 1. Comparison of $\overline{u'^2}/u_\tau^2$ versus y^+ for several turbulent boundary layer and pipe flow experiments at two Reynolds numbers: a) $Re_\tau \simeq 7\,000$ b) $Re_\tau \simeq 20\,000$

experiments for the higher of the two Reynolds numbers (figure 1b). The log decay (1.1) is clearly defined only at the highest Reynolds number ($Re_\tau \simeq 20\,000$) except for the New Hampshire TBL data for which the outer peak is more pronounced and extends to higher y^+.

As a first test, the model (1.1), (2.1)-(2.4) was fitted to each Reynolds number data of each dataset independently. This results in a set of optimal parameters defined in table 2. The two regions $y < y_*$ and $y > y_*$ are fitted at the same time but as the parameters A_1 and B_1 involved in the log decay region $y > y_*$ are fitted independently of C_0, the two fitted parts do not perfectly match. The extent of the region with the logarithmic decay (1.1) is not a priori certain. In the present case a conservative but widely accepted upper bound $y < 0.15\delta$ was used. This choice leads to a rather short logarithmic region for the lower Reynolds numbers investigated here and consequently a substantial uncertainty in the determination of the two constants B_1 and A_1. The two parts of the model as well as the wall distance y_* where the two models merge are fitted simultaneously using a

TABLE 2. Best values of the model parameters (1.1, 2.1-2.4) for the fit of the four turbulent boundary layer datasets.

Experiments	Re_τ	m	p	q	A	α	C_0	B_1	A_1
Lille TBL Carlier & Stanislas (2005)	3193	0.38	2.13	0.77	1.12	1.26	1.23	1.70	1.15
	5006	0.38	2.14	0.76	1.10	1.33	1.28	1.70	1.18
	7022	0.38	2.16	0.77	1.50	1.03	1.27	1.69	1.18
Melbourne TBL Marusic *et al.* (2010) Marusic *et al.* (2015)	7172	0.38	2.20	0.77	1.57	1.09	1.35	1.70	1.29
	10000	0.38	2.13	0.76	1.15	1.20	1.36	1.70	1.30
	13600	0.35	2.30	0.79	0.92	1.13	1.32	1.50	1.30
	19000	0.38	2.11	0.79	1.04	1.22	1.28	1.59	1.23
New Hampshire TBL Vincenti *et al.* (2013)	10770	0.35	2.30	0.75	1.80	1.10	1.56	1.40	1.65
	15740	0.38	2.12	0.78	1.06	1.32	1.47	1.70	1.59
	19670	0.38	2.12	0.78	1.03	1.33	1.51	1.70	1.64
Princeton TBL Vallikivi *et al.* (2014)	8261	0.38	2.10	0.81	0.99	1.28	1.37	1.70	1.43
	25062	0.38	2.09	0.81	0.99	1.18	1.30	1.43	1.29
	40053	0.38	2.05	0.84	0.95	1.13	1.21	1.24	1.24
	72526	0.39	2.00	0.89	0.90	1.20	1.17	1.13	1.16

L-BFGS-B algorithm developed by Byrd *et al.* (1995) for the solution of the optimisation problem.

The first conclusion is that the model is able to fit the complete dataset with parameter values which do not vary significantly with Reynolds number and from one experiment to the other. The two exponents p and q control the wall distance of the outer peak or plateau as well as the related Reynolds number and wall-distance dependencies of the new spectral range extent δ_∞/δ_*. The values obtained for p and q appear relatively constant across Reynolds numbers and across TBL experiments. They appear also fairly close to those obtained by Vassilicos *et al.* (2015) for the Princeton high Reynolds number turbulent pipe flow. These values of q and p suggest an outer peak position scaling like $y^+_{peak} \sim Re_\tau^{2/3}$ (following section 2, $1 - q/p \simeq 0.61$, which is close to 2/3) and the fact that α is always larger than 1 suggests that an outer peak does indeed exist (see Vassilicos *et al.* (2015) for an explanation of this point). However the ratio q/p seems to decrease slightly for the Princeton TBL with increasing Reynolds number. The parameter m which, in the model, controls the slope of the k_1^{-m} spectrum in the new range $1/\delta_\infty < k_1 < 1\delta_*$ is extremely stable as compared, for instance, to α. However, the two values appear to be correlated so that a small variation of the former can be compensated by the later. The parameter C_0 which controls the amplitude of the outer peak does not vary too much and remains close to a mean value of 1.33. The larger values of C_0 for the New Hampshire data are linked to the more pronounced outer peak as already noticed in Fig. 1. The parameters A_1 and B_1 of the log decay model (1.1) are only significant at the highest Reynolds numbers investigated where this log decay region is clearly defined. They are also dependent on the lower bound on y chosen to fit the model (1.1). In our model, the log decay region starts at the wall distance y_* which was used to fit the two parameters A_1 and B_1. The average values of these two parameters keeping only the 7 cases with $Re_\tau > 12\,000$ are $A_1 \simeq 1.35$ and $B_1 \simeq 1.45$. The average slope $A1$ is close to the value reported by Marusic *et al.* (2013) with a different definition of the fitting lower bound and taking into account their 95% confidence estimated error bars.

As a second test of robustness of the model, the data of the four TBL experiments were fitted with the model whilst keeping a constant value for four parmeters ($m = 0.38$,

TABLE 3. Best values of the model parameters (1.1, 2.1-2.4) for the fit of the four turbulent boundary layer datasets and the Princeton superpipe data. The fit was performed fixing four of the parameters ($m = 0.38$, $p = 2.1$, $A = 1.0$ and $\alpha = 1.2$).

Experiments	Re_τ	q	C_0	B_1	A_1
Lille TBL Carlier & Stanislas (2005)	3193	0.71	1.26	1.70	1.17
	5006	0.70	1.29	1.70	1.19
	7022	0.78	1.23	1.70	1.15
Melbourne TBL Marusic *et al.* (2010) Marusic *et al.* (2015)	7172	0.78	1.28	1.70	1.25
	10000	0.74	1.36	1.70	1.30
	13600	0.76	1.31	1.58	1.27
	19000	0.78	1.28	1.59	1.23
New Hampshire TBL Vincenti *et al.* (2013)	10770	0.70	1.48	1.70	1.50
	15740	0.70	1.53	1.70	1.63
	19670	0.70	1.58	1.70	1.67
Princeton TBL Vallikivi *et al.* (2014)	8261	0.76	1.40	1.70	1.44
	25062	0.81	1.30	1.44	1.28
	40053	0.90	1.20	1.34	1.20
	72526	0.95	1.17	1.13	1.16
Princeton Pipe Hultmark *et al.* (2012)	10480	0.78	1.31	1.70	1.28
	20250	0.79	1.33	1.69	1.26
	37690	0.82	1.30	1.34	1.29
	68160	0.83	1.27	1.43	1.22

$p = 2.1$, $A = 1.0$ and $\alpha = 1.2$). The values of these four parameters are taken as the approximate average of the first estimation when all parameters are fitted. We choose to adjust the four remaining parameters q, C_0, B_1 and A_1 for two reasons. Firstly, the model is sensitive to the ratio p/q which controls the location of the outer peak and more importantly the wall distance y_* associated to the merging of the two parts of the model. Secondly, the other three parameters C_0, B_1 and A_1 are associated with the amplitude of the outer peak and the slope and level of the log-decay region. The values of A_1 and B_1 in (1.1) vary from one experiment to the other in particular because this region is well established only at extremely large Reynolds numbers but also because there is no clear consensus on the exact bounds on y where this log-decay region exists. The four fitted parameters are given in table 3 and the resulting fits for the four TBL data (and for the Princeton turbulent pipe data for comparison) are shown in Fig. 2. The first part of the model (i.e. equations (2.1)-(2.4) for $80\nu/u_\tau \lesssim y < y_*$) which results from the addition of the new spectral range $1/\delta_\infty < k_1 < 1/\delta_*$ is able to return good fits for all the experiments and all the Reynolds numbers investigated. The parameters C_0 and q which control the amplitude and location of the outer peak (or plateau for $7\,000 < Re_\tau < 20\,000$) take values similar to the previous fit (Table 2); the rms difference on these two parameters is only about 3% between Tables 2 and 3. The variations in the values of the log decay parameters B_1 and A_1 with respect to Table 2 are due to the slight changes of the lower bound y_*. Finally, the parameters q, C_0, A_1 and B_1 are also given for the Princeton turbulent pipe data at four Reynolds numbers. At similar Reynolds numbers, the differences in the values of these four parameters between the two types of flow (TBL and turbulent pipe flow) are comparable to the differences in their values from one TBL experiment to the other.

The evidence obtained from the present analysis points to a wide applicability of the model (2.1)-(2.4) and (1.1), with parameter values which do not vary much from one

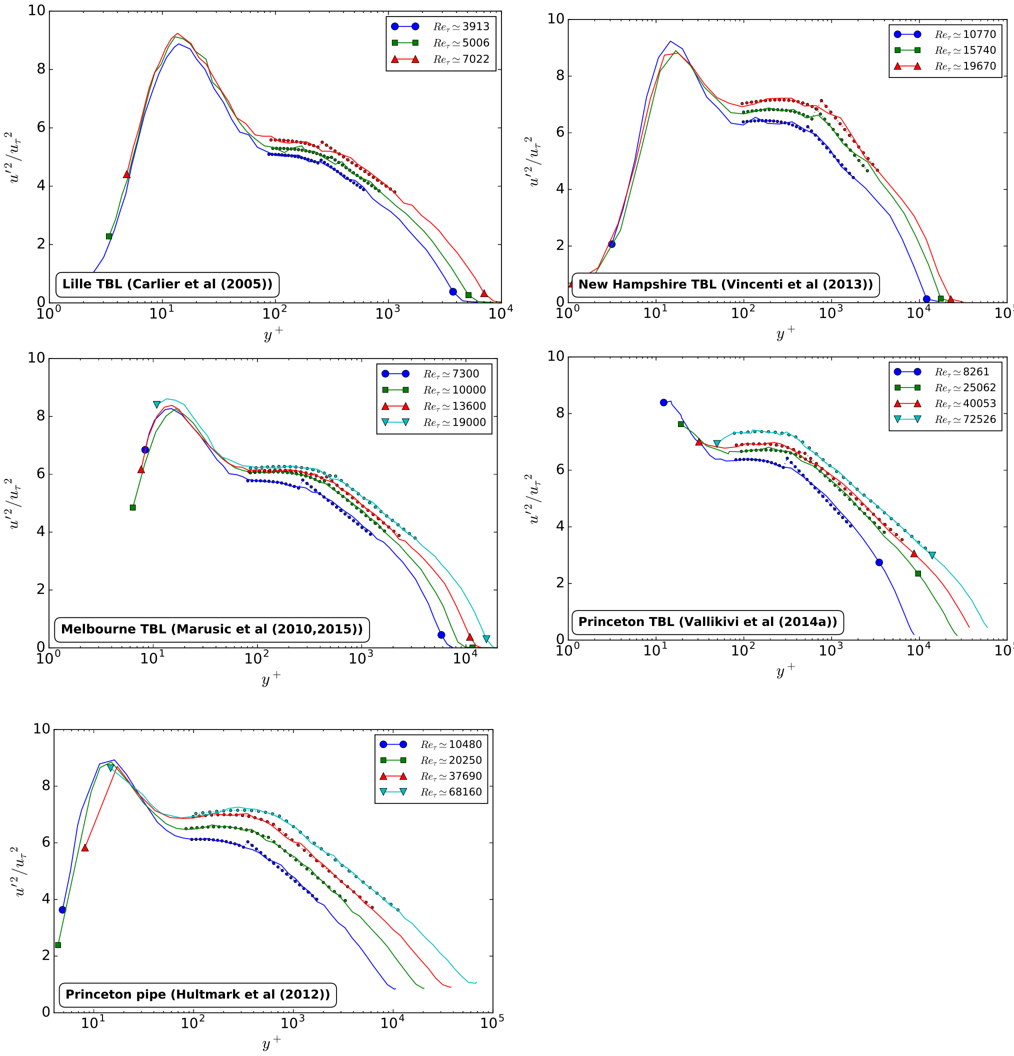

FIGURE 2. Fit of each experiment dataset with the model (1.1, 2.1-2.4) using 4 variable parameters (q, C_0, B_1, A_1) and the remaining ones fixed at $m = 0.38$, $p = 2.1$, $A = 1.0$ and $\alpha = 1.2$.

flow to the other (a wide range of Reynolds numbers Re_τ from 7 000 to 70 000, two types of wall-bounded turbulent flows and four different installations). This motivates us to propose a best set of values for all eight parameters of the model which can be used for predictive purposes, for example when extrapolating to near-wall turbulence at even higher Reynolds numbers. However very few data are available to test such extrapolation. An example of such rare data was obtained from highly documented measurements performed at SLTEST, an atmospheric facility in the Great Salt Lake Desert. Several sets of experiments were conducted in this facility but we focus only on the most reliable ones, the sonic anemometer data from Hutchins *et al.* (2012) and the hot wire and sonic anemometer data from Metzger *et al.* (2007). The first set covers only part of the log decay region while both hot wire and sonic anemometer data are available from the second set covering the two specific regions of the present model. As parameters vary a little

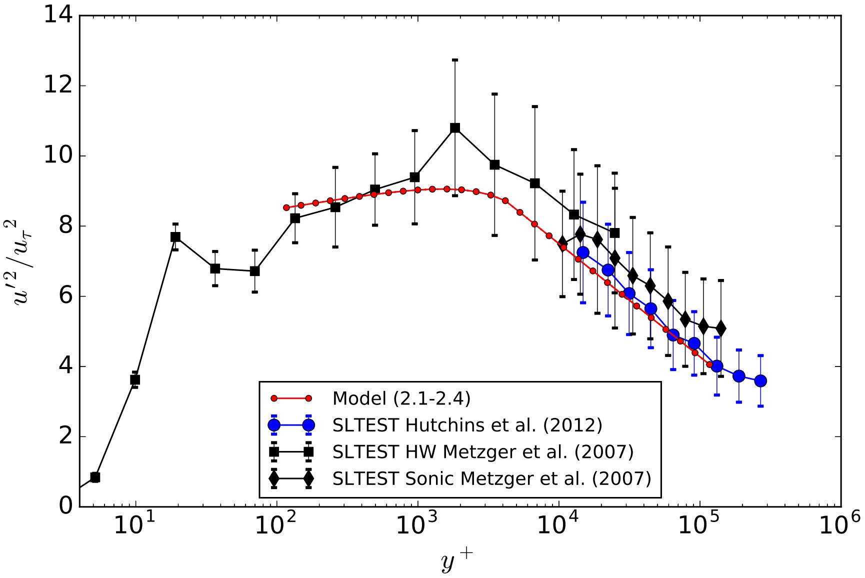

FIGURE 3. Comparison of $\overline{u'^2}/u_\tau^2$ for the measurements campaign of the SLTEST experiments with the model (1.1)-(2.1-2.3) using the averaged parameters obtained from previous fit for the highest Reynolds numbers ($m = 0.38$, $p = 2.1$, $q = 0.81$, $A = 1.0$, $\alpha = 1.21$, $C_0 = 1.4$, $B_1 = 1.4$, $A_1 = 1.4$). The Reynolds numbers based on boundary layer thickness were estimated to 60 millions and 80 millions of the Hutchins *et al.* (2012) and Metzger *et al.* (2007) corresponding to $Re_\tau \simeq 770\,000$ and $Re_\tau \simeq 780\,000$ respectively.

with Reynolds number when the Reynolds number is not high enough, the parameters used for the extrapolation were computed as their average for TBL with $Re_\tau \geqslant 12\,000$ (Table 2) only. This leads to $A_1 \simeq 1.35$, $B_1 \simeq 1.45$, $A \simeq 1.0$, $\alpha \simeq 1.21$, $p \simeq 2.1$, $q \simeq 0.81$, $m \simeq 0.38$ and $C_0 \simeq 1.32$. However, as the continuity of the model at $y = y_*$ imposes $A_1 = B_1 = C_0$ (see Vassilicos *et al.* (2015) for full explanations) we choose a compromise value of $A_1 = B_1 = C_0 = 1.4$ keeping the other parameters to their average value.

These average values for p, m, α and A are also the values chosen for our second fit (Table 3). The two sets of SLTEST data as well as our model parameters and prediction are shown in Fig. 3. The values chosen for A_1 and B_1 lead to a prediction of the log decay region in general agreement with the two sets of data taking into account the error bars proposed by the authors. The model predicts an outer peak in global agreement with the hot wire data of Metzger *et al.* (2007) given the accuracy of the experimental results. In fact, the outer peak returned by the model looks more like a plateau with a weak slope. Putting aside the point of Metzger et al. at $y^+ = 2000$ which is potentially questionable, the global shape and level predicted by the extrapolation of our model to these extremely high Reynolds numbers agrees reasonably well with the SLTEST data down to $y^+ = 150$. It would be of interest to have more reliable measurements of the inner peak in such a flow to see how the inner and outer peaks connect.

4. Conclusions

Several studies of high Reynolds number turbulent pipe, channel and zero-pressure gradient boundary layer flows have shown that a region of high turbulence develops further away from the wall than the well known near-wall turbulence peak (see Hultmark *et al.* (2012), Morrisson *et al.* (2004),Marusic *et al.* (2010)). In a recent paper, Vassilicos *et al.* (2015) developed a model which, based on a new reading of turbulence spectra, combines this outer high turbulence region with the Townsend-Perry log-decay of the rms turbulence velocity profiles. The model is able to account for this new turbulence region and its Reynolds number dependence from a newly identified low wavenumber range of the energy spectrum with attached eddy physical significance. The model was validated by Vassilicos *et al.* (2015) on the exceptionally high Reynolds number data of the Princeton superpipe.

It is of course important to establish how widely this model holds and to see whether its validity can be expanded to turbulent boundary layers. For that purpose, we have collected data from the few facilities around the world which can provide turbulent boundary layer measurements at Reynolds numbers sufficiently high to be relevant. Thanks to the kindness of colleagues from Melbourne, New Hampsire and Princeton, a fairly comprehensive dataset could be assembled covering a range of Re_τ from approximately $3,000$ to $70,000$. After checking the coherence of the dataset, two different fits where tried: one where all the parameters were free to be fitted by the optimisation procedure and one where four of the parameters where chosen a priori on the basis of the first fit and four were fitted. Both approaches give a reasonable prediction of the entire dataset and values of the parameters which are in good agreement. Furthermore, these values are fairly close to the ones obtained by Vassilicos *et al.* (2015) for the Princeton superpipe data.

These encouraging results allow us to propose a set of parameters to be used to predict high Reynolds number near wall turbulence. We therefore tried the predictive power of our model on the very highest Reynolds number data available: those of the SLTEST facility in the Salt Lake desert in USA. Two datasets from two different teams (Hutchins et al 2012 and Metzger et al 2007) where selected as the most representative of this Salt Lake desert flow. The results show that the prediction of our model is in good agreement with the measurements within measurement accuracy. Our model can therefore be used to extrapolate the near wall turbulence intensity distribution to very high Reynolds numbers and may even provide a means to improve near-wall LES predictions until a better approach becomes available. It is interesting to note that at Reynolds numbers as high as those of the SLTEST facility ($Re_\tau = 780,000$) the model does not yield a very marked peak but rather a plateau with a weak slope.

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