



# **Broadband forcing of turbulence**

**Bernard J. Geurts, Arek K. Kuczaj**

**Multiscale Modeling and Simulation (Twente)  
Anisotropic Turbulence (Eindhoven)**

**IMS Turbulence Workshop  
London, February 18-19, 2008**

# Underwater canopies



# Urban dispersion



**DAPPLE:** Dispersion of Air Pollution and its Penetration into the Local Environment

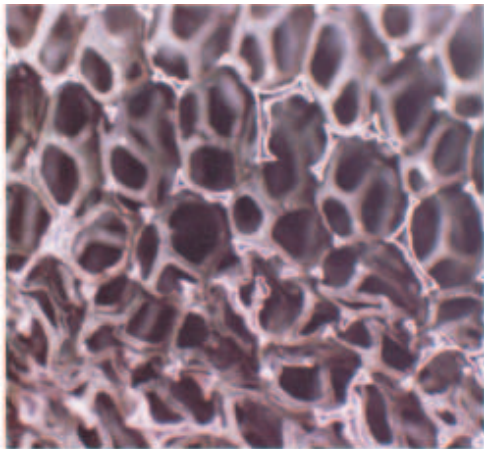
# Urban canopy



# Rural dispersion - water management

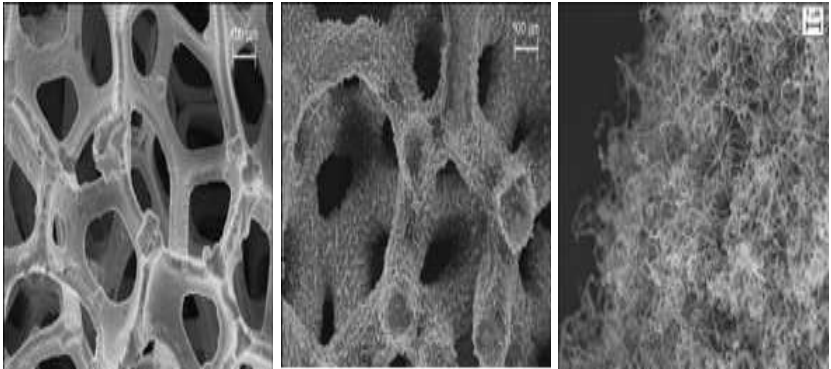


# Compact heat- and mass-transfer



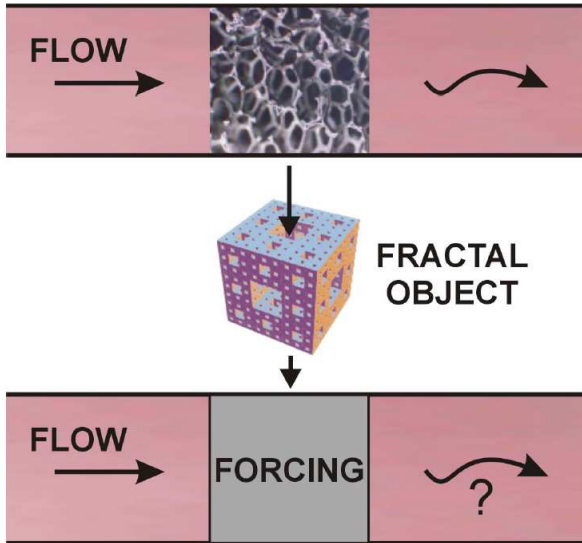
Nickel foam - heat-pump applications

# Compact heat- and mass-transfer



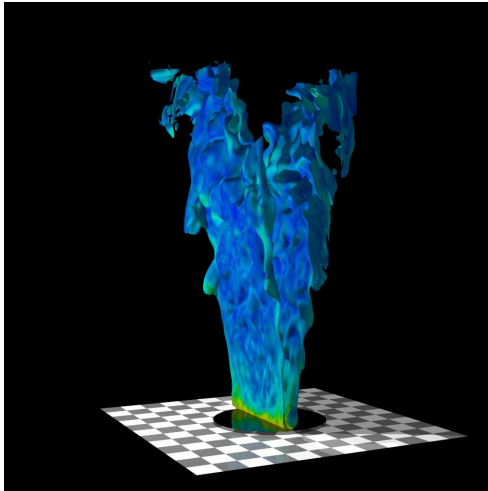
Coating with Carbon Nano Fibers - catalyst applications

# Fractal modeling of complex objects?



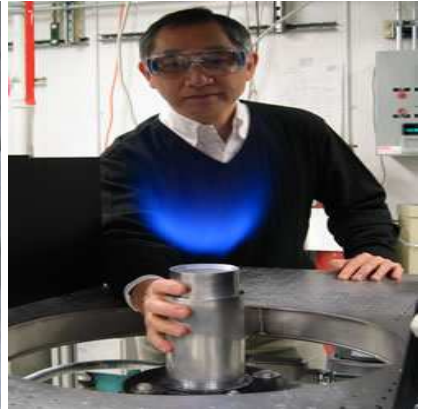
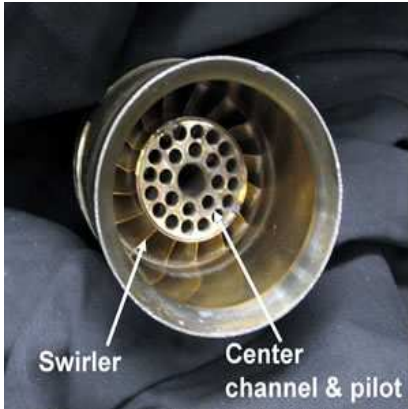


# Controlling scales in flames



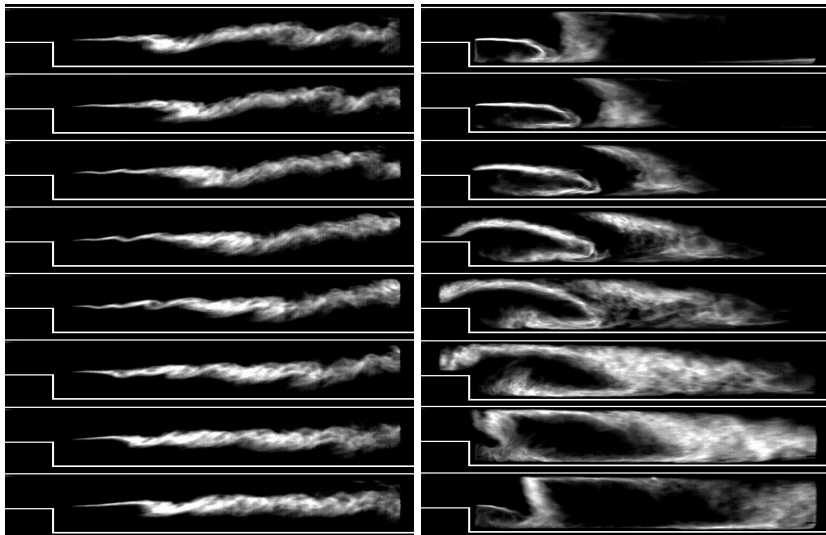
Effect of an upstream rod in flame

# Swirl control of lean combustion



Adding swirl stabilizes flame but hinders mixing

# Enhanced syngas combustion



Intensified combustion following upstream flow instability

## Will

- present broadband forcing methodology
- obtain controlled non-Kolmogorov turbulence
- consider effects on mixing rate
- investigate responsiveness to time-dependent forcing
- present problem of relating forcing to actual (fractal) object

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# Outline

- 1 Forcing at various scales
- 2 Mixing in manipulated turbulence
- 3 Optimal forcing?
- 4 Connections to real objects
- 5 Concluding remarks

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## Forcing incompressible flow

Physical space:  $\nabla \cdot \mathbf{v} = 0$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{f}$$

Spectral space: put  $\mathbf{F} = \mathcal{F}(\mathbf{f})$  and assume  $\mathbf{k} \cdot \mathbf{F} = 0$ . Then

$$\mathbf{v}(\mathbf{x}, t) = \sum_{\mathbf{k}} \mathbf{u}(\mathbf{k}, t) e^{i\mathbf{k} \cdot \mathbf{x}}$$

with

$$\left( \frac{\partial}{\partial t} + \nu k^2 \right) \mathbf{u}(\mathbf{k}, t) = \mathbf{D} \mathbf{W}(\mathbf{k}, t) + \mathbf{F}(\mathbf{k}, t)$$

where

$$D_{\alpha\beta} = \delta_{\alpha\beta} - \frac{k_{\alpha} k_{\beta}}{k^2} \quad ; \quad \mathbf{W}(\mathbf{k}, t) = \mathcal{F}(\mathbf{v}(\mathbf{x}, t) \times \boldsymbol{\omega}(\mathbf{x}, t))$$

Pseudo-spectral treatment, FFTW, de-aliased, ...

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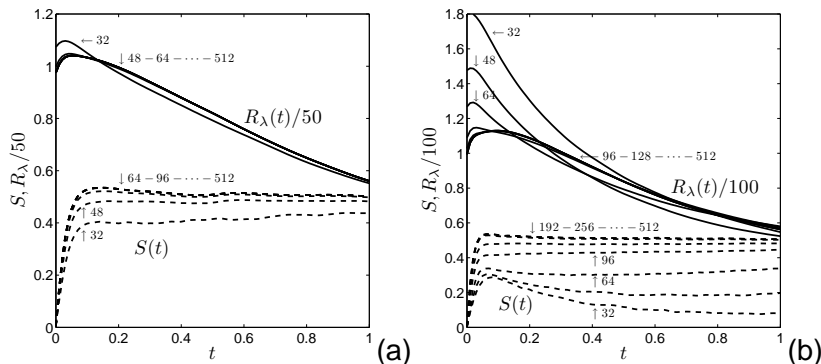
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# Convergence for decaying turbulence

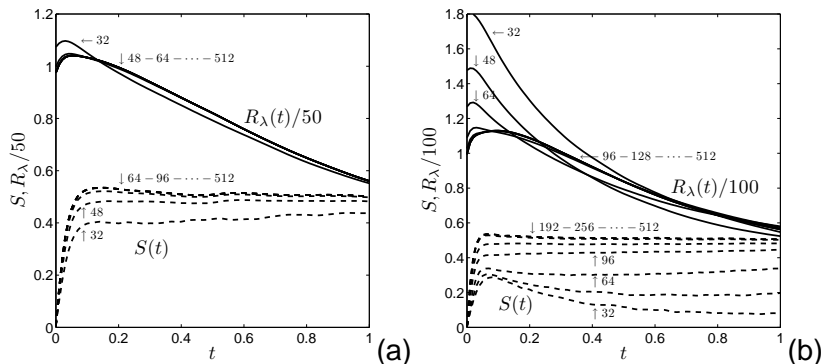


$R_\lambda$  and skewness  $S$  at initial  $R_\lambda = 50$  (a) and  $R_\lambda = 100$  (b)

$R_\lambda/N^3$	$32^3$	$48^3$	$64^3$	$96^3$	$128^3$	$192^3$	$256^3$	$384^3$	$512^3$
50	0.56	0.83	<u>1.11</u>	1.67	2.22	3.34	4.45	6.67	8.90
100	0.20	0.29	0.39	0.59	0.79	<u>1.18</u>	1.57	2.36	3.15

$k_{\max} \eta$  at different  $N$

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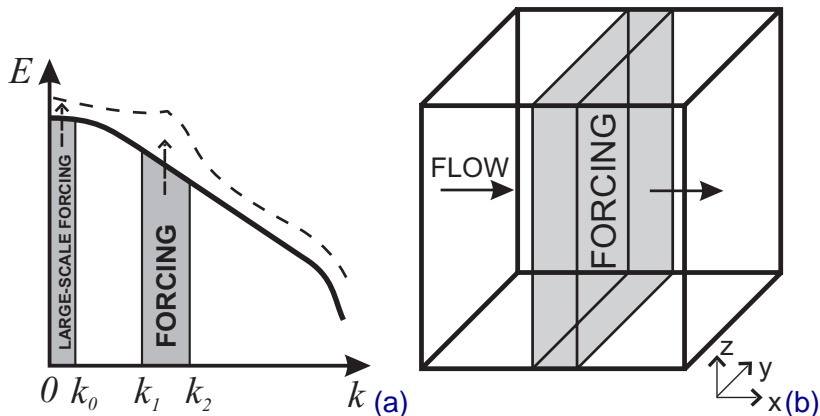


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# Forcing: spectral and physical localization



a: Two-band forcing in spectral space

b: Forcing in a slab in physical space - spectral convolution



## Energy and forcing

### > Evolution of Fourier coefficients

$$\left( \frac{\partial}{\partial t} + \nu k^2 \right) u_\alpha(\mathbf{k}, t) = \Psi_\alpha(\mathbf{k}, t) + F_\alpha(\mathbf{k}, t)$$

where  $\Psi_\alpha(\mathbf{k}, t) = D_{\alpha\beta} W_\beta(\mathbf{k}, t)$

### > Energy evolution: $E(\mathbf{k}, t) = \frac{1}{2} |\mathbf{u}(\mathbf{k}, t)|^2$

$$\frac{\partial E(\mathbf{k}, t)}{\partial t} = -\varepsilon(\mathbf{k}, t) + T(\mathbf{k}, t) + T_F(\mathbf{k}, t)$$

- dissipation  $\varepsilon(\mathbf{k}, t) = 2\nu k^2 E(\mathbf{k}, t)$
- transfer  $T(\mathbf{k}, t) = u_\alpha^*(\mathbf{k}, t) \Psi_\alpha(\mathbf{k}, t)$
- forcing  $T_F(\mathbf{k}, t) = u_\alpha^*(\mathbf{k}, t) F_\alpha(\mathbf{k}, t)$

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## Basic forcing of type 'A'

Choose to have  $\partial_t u_\alpha = 0$ , i.e.,  $\partial_t E(\mathbf{k}, t) = 0$  for **forced** modes

Obtain forcing:

$$A1 : F_\alpha(\mathbf{k}, t) = \nu k^2 u_\alpha(\mathbf{k}, t) - \Psi_\alpha(\mathbf{k}, t)$$

Extensions keeping  $|\mathbf{u}(\mathbf{k}, t)|$  constant (Chasnov)

$$F_\alpha(\mathbf{k}, t) = (\varepsilon(\mathbf{k}, t) - T(\mathbf{k}, t)) \frac{u_\alpha(\mathbf{k}, t)}{2E(\mathbf{k}, t)}$$

or shell-averaged version (Kerr)

Or average over all modes: (and assign to  $P$  forced modes)

$$A2 : F_\alpha(\mathbf{k}, t) = \frac{\widehat{\varepsilon}(t)}{P} \frac{u_\alpha(\mathbf{k}, t)}{2E(\mathbf{k}, t)}$$

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Energy input rate  $\varepsilon_w$  fixed per forced mode:

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Multiscale stirrer: (Mazzi, Vassilicos)

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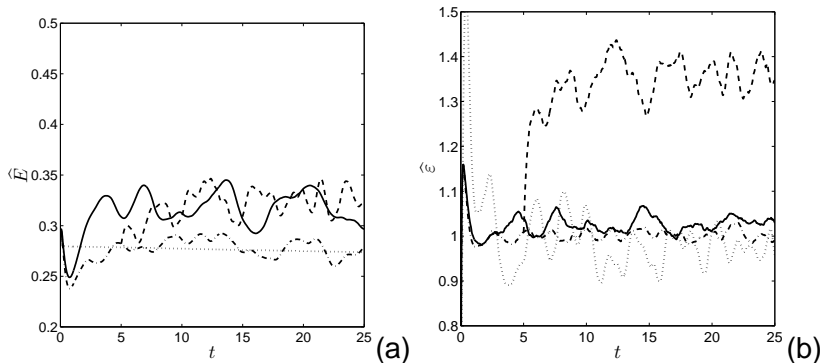
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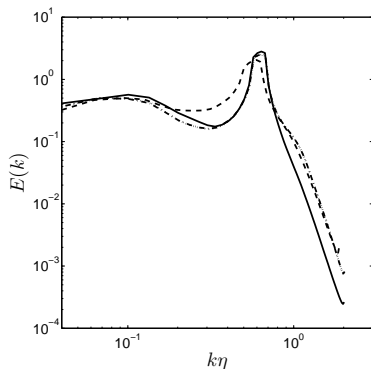
## Two-band forcing



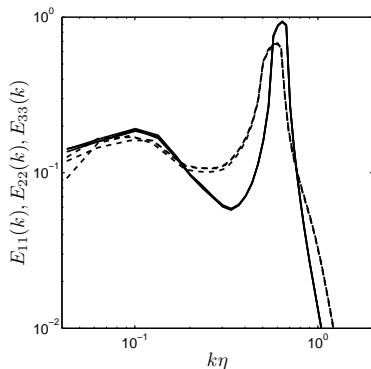
Kinetic energy  $\hat{E}$  (a) and energy-dissipation-rate  $\hat{\varepsilon}$  (b)

A1 (dashed), A2 (dotted), B1 (dash-dotted), B2 (solid)

## Two-band forcing: compensated spectra



(a)



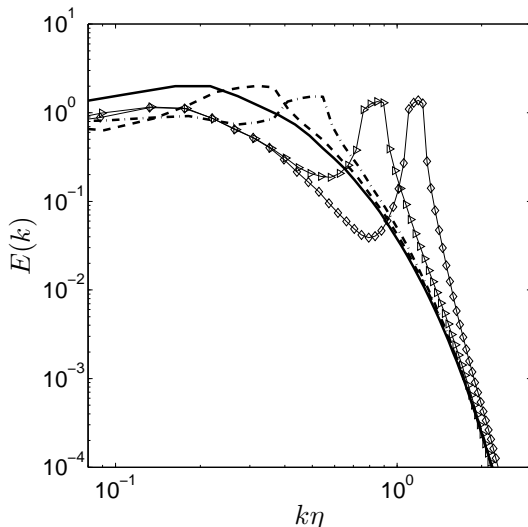
(b)

two-band forced turbulence:  $k \leq 3\pi$  and  $33\pi < k \leq 41\pi$

(a) A1 (dashed), A2 (dotted), B1 (dash-dotted), B2 (solid)

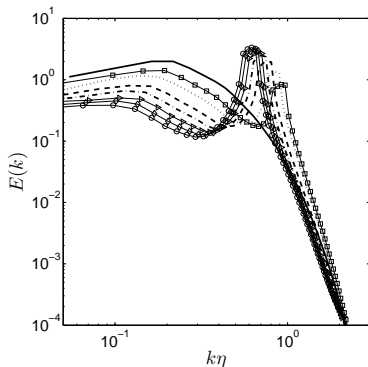
(b) Co-spectra  $E_{11}$ ,  $E_{22}$ ,  $E_{33}$  for A1 (dashed) and B2 (solid)

## Peak where you want: B2

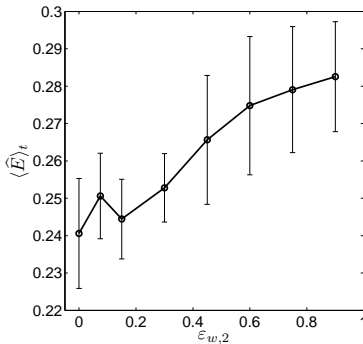


Equal forcing per band:  $\varepsilon_{w,1} = \varepsilon_{w,2} = 0.15$

## Peak with desired strength: B2



(a)



(b)

- (a)** large-scale forcing  $\varepsilon_{w,1} = 0.15$  in  $k \leq 3\pi$ : second band  $33\pi < k \leq 41\pi$ :  $\varepsilon_{w,2} = 0.075, 0.15, 0.30, \dots, 0.90$
- (b)** Corresponding time-averaged total kinetic energy

Forcing removes energy from large scales - nonlocality

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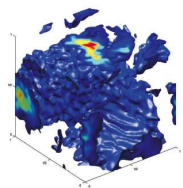
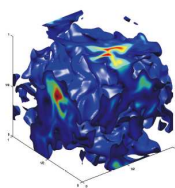
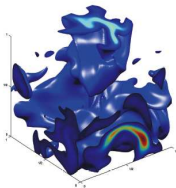
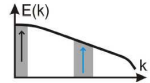
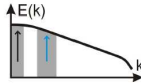
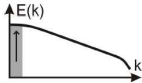
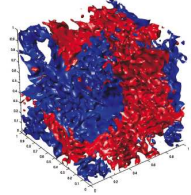
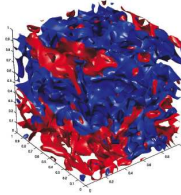
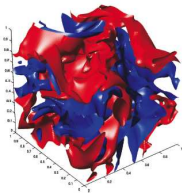
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# Fractal stirring at various scales

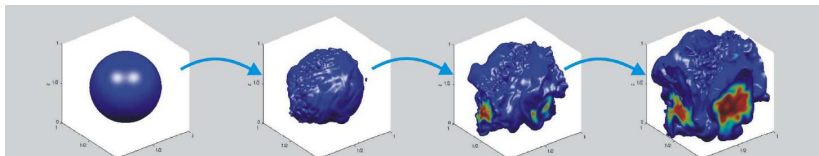




# Controlled mixing in two-band forcing



# Scalar mixing: area and wrinkling



Geometric properties level-set: where  $c(\mathbf{x}, t) = a$

$$I_g(a, t) = \int_{S(a,t)} dA g(\mathbf{x}, t)$$

- $g(\mathbf{x}, t) = 1$ : area  $A$
- $g(\mathbf{x}, t) = |\nabla \cdot \mathbf{n}(\mathbf{x}, t)|$ : wrinkling  $W$

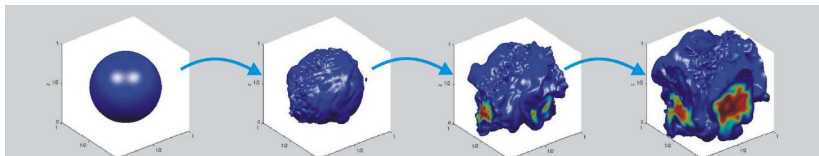
Instantaneous and cumulative:

$$\vartheta_A(a, t) = A(a, t)/A(a, 0) \quad ; \quad \vartheta_W(a, t) = W(a, t)/W(a, 0)$$

$$\zeta_A(a, t) = \int_0^t \vartheta_A(a, \tau) d\tau \quad ; \quad \zeta_W(a, t) = \int_0^t \vartheta_W(a, \tau) d\tau$$

Distinguish: rate and maxima ( $\vartheta$ ) and total effect over time ( $\zeta$ )

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$$I_g(a, t) = \int_{S(a,t)} dA g(\mathbf{x}, t)$$

- $g(\mathbf{x}, t) = 1$ : area  $A$
- $g(\mathbf{x}, t) = |\nabla \cdot \mathbf{n}(\mathbf{x}, t)|$ : wrinkling  $W$

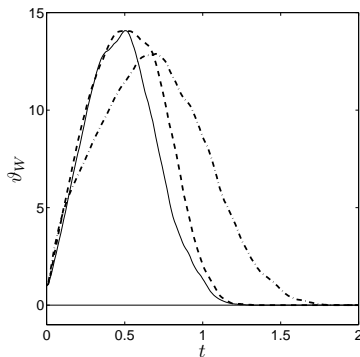
Instantaneous and cumulative:

$$\vartheta_A(a, t) = A(a, t)/A(a, 0) \quad ; \quad \vartheta_W(a, t) = W(a, t)/W(a, 0)$$

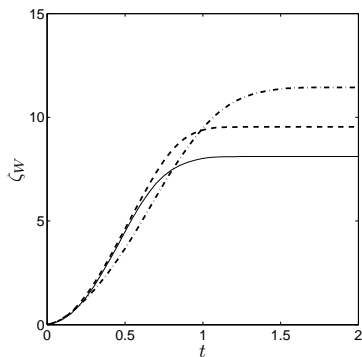
$$\zeta_A(a, t) = \int_0^t \vartheta_A(a, \tau) d\tau \quad ; \quad \zeta_W(a, t) = \int_0^t \vartheta_W(a, \tau) d\tau$$

**Distinguish:** rate and maxima ( $\vartheta$ ) and total effect over time ( $\zeta$ )

# Evolution of wrinkling $\vartheta_W$ and $\zeta_W$



(a)

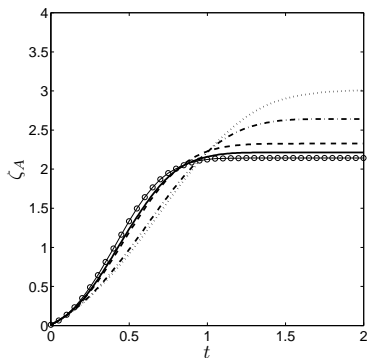


(b)

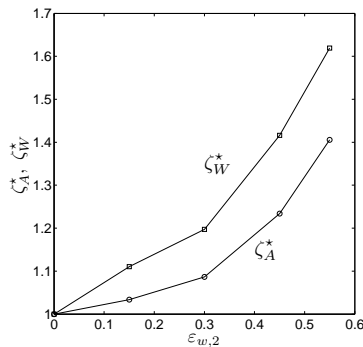
## Comparing: forcing

- $\mathbb{K}_{1,1}$  at  $\epsilon_W = 0.60$  (solid)
- $\mathbb{K}_{1,1}$  at  $\epsilon_W = 0.15$  and  $\mathbb{K}_{5,8}$  at  $\epsilon_{W,2} = 0.45$  (dashed)
- $\mathbb{K}_{1,1}$  at  $\epsilon_W = 0.15$  and  $\mathbb{K}_{13,16}$  at  $\epsilon_{W,2} = 0.45$  (dash-dotted)

## Mixing: value for money



(a)



(b)

- Forcing  $\mathbb{K}_{1,1} - \mathbb{K}_{13,16}$  at (0.60 – 0.00) ( $\circ$ ), (0.45 – 0.15) (solid), (0.30 – 0.30) (dash), (0.15 – 0.45) (dot-dash), (0.05 – 0.55) (dot)
- surface-area (a) and wrinkling  $\zeta_W^*$  at  $t = 2$  (b)

# Outline

- 1 Forcing at various scales
- 2 Mixing in manipulated turbulence
- 3 Optimal forcing?**
- 4 Connections to real objects
- 5 Concluding remarks

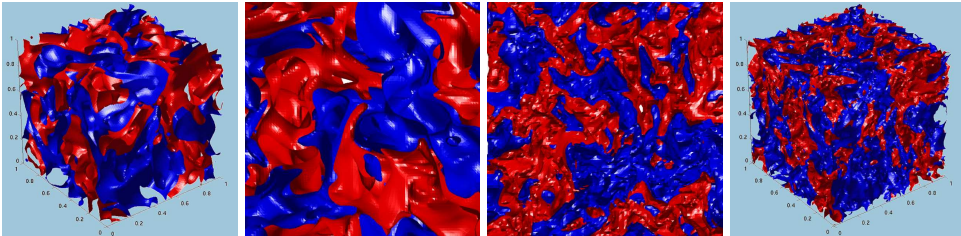
# Preferred frequency for turbulence agitation?

So far: forcing at various spatial scales

What about time-modulation of forcing?

Consider forcing at:

- (1) large-scales only
- (2) various scales simultaneously



# Modulated Forcing

Time-modulation of forcing (B1):

$$F_{\alpha}(\mathbf{k}, t) = \left[ \frac{\varepsilon_w}{P} \frac{u_{\alpha}(\mathbf{k}, t)}{2E(\mathbf{k}, t)} \right] \left( 1 + A \sin(\omega t) \right)$$

## Expect:

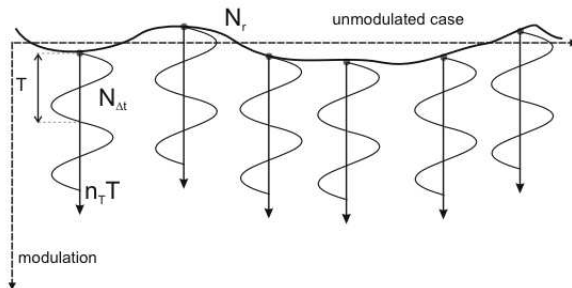
- $\omega \gg 1$ : modulation too rapid: no/small effect
- $\omega \ll 1$ : modulation quasi-stationary: no/small effect

**Q1:** optimal modulation frequency/frequencies?

**Q2:** increased turbulence/transport/mixing?



# Ensemble of forced simulations



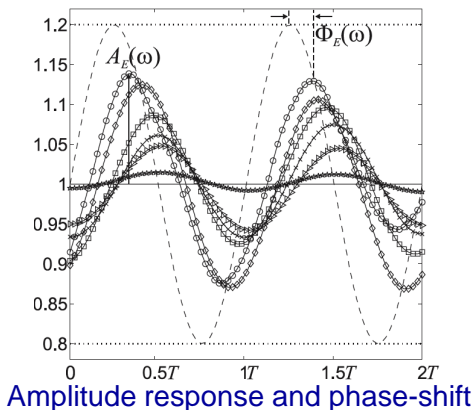
## Registration total kinetic energy:

- start:  $j$ -th initial condition,  $N_r$  realizations
- forced - no modulation:  $E_j^{(0)}(t)$
- forced - modulation:  $E_j(t, \omega)$

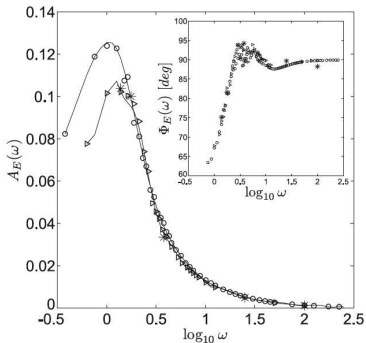
# Extract Amplitude and Phase

Averaging over  $N_r$  realizations:

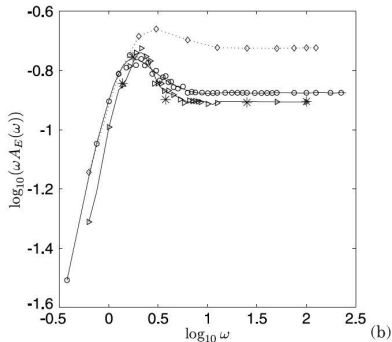
$$\langle E(t, \omega) \rangle = \frac{1}{N_r} \sum_{i=1}^{N_r} E_j(t, \omega) = a(\omega) + A(\omega) \sin(\hat{\omega}(t + \Phi(\omega)))$$



# Response maxima: energy



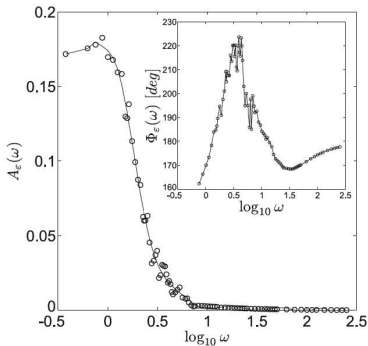
(a)



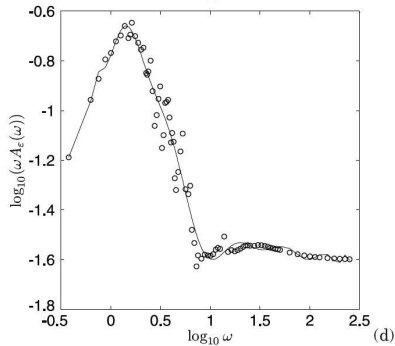
(b)

- $R_\lambda = 50$ :  $\circ$ , and  $R_\lambda = 100$ :  $\triangle$
- Phase-shift:  $\omega \gg 1$  then  $\rightarrow$  90-degrees
- Compensated spectrum:  $\omega^{-1}$  decay

# Response maxima: dissipation



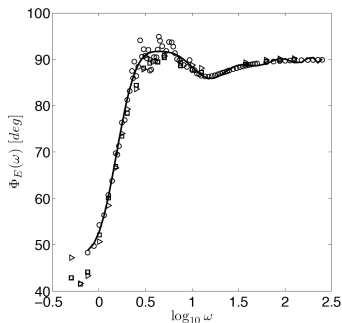
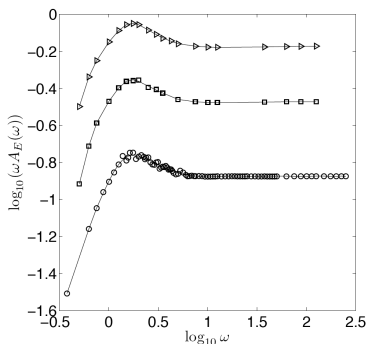
(c)



(d)

- Phase-shift:  $\omega \gg 1$  then  $\rightarrow$  180-degrees

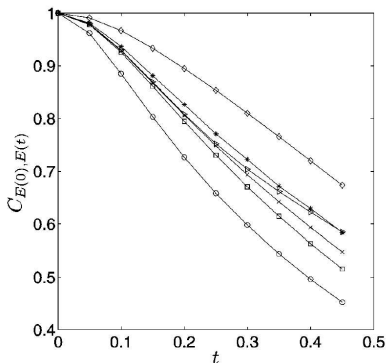
# Effect of amplitude of modulation



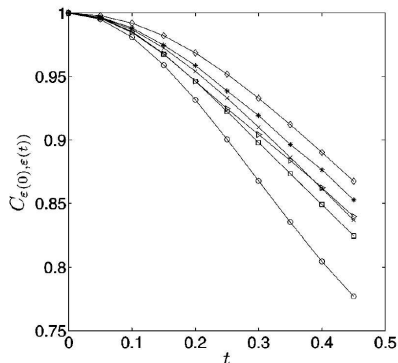
Modulation depth:  $A = 1/5$  ( $\circ$ ),  $A = 1/2$  ( $\square$ ),  $A = 1$  ( $\triangle$ )

# Response maxima and correlations?

## Kinetic energy

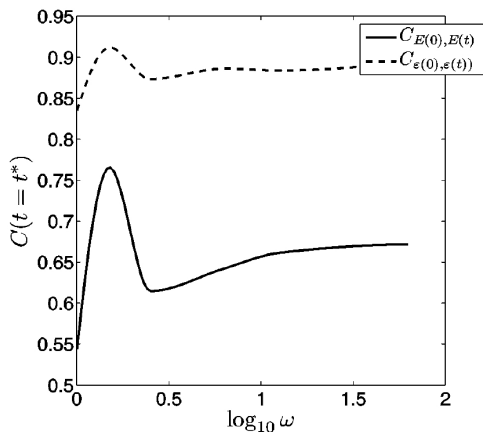


## Energy-dissipation rate



$$C_{E(0),E(t)}(\omega) = \frac{\langle E(0)E(t) \rangle}{\langle E(0)^2 \rangle^{1/2} \langle E(t)^2 \rangle^{1/2}}$$

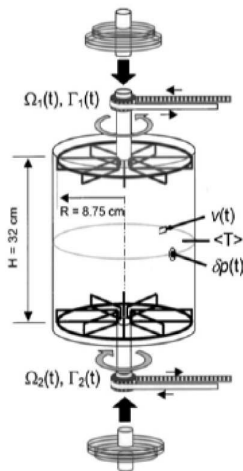
# Response maxima and correlations?



Maximal correlation at  $\omega$  at which response maximum

Here:  $t^* = 0.3$

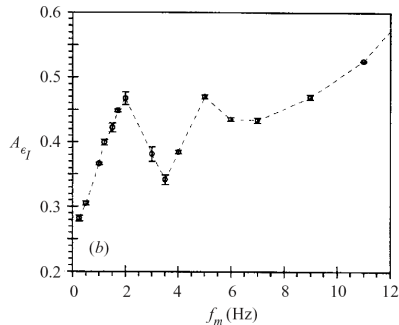
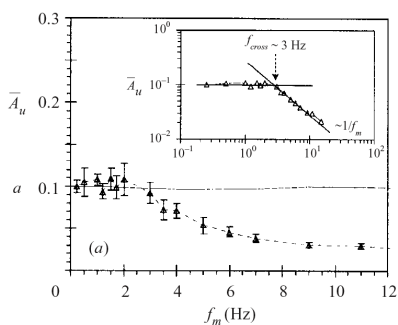
# Experimental 'similarities': washing machine



Cadot-Titon-Bonn (JFM 485, 2003)



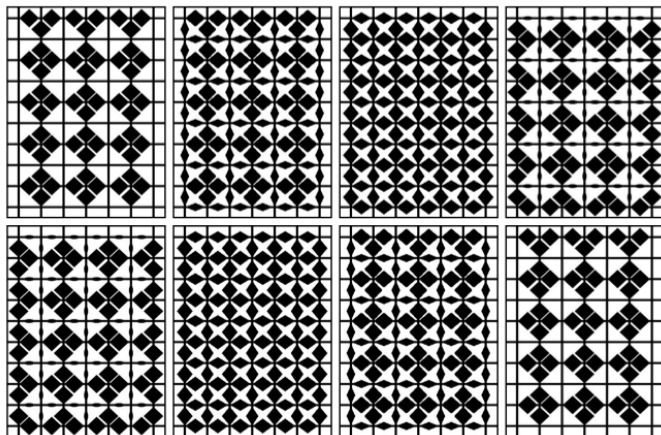
# Experimental 'similarities': washing machine



Velocity fluctuations (left) and power-input (right)

# Possible Connections with Experiments?

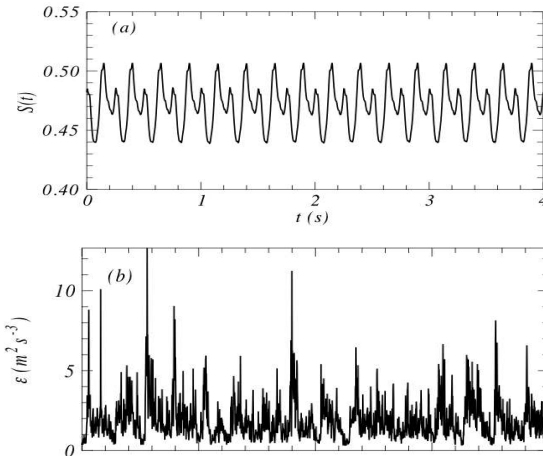
Periodic active grid mode: Tipton - van de Water



Grid can be cycled at different frequencies

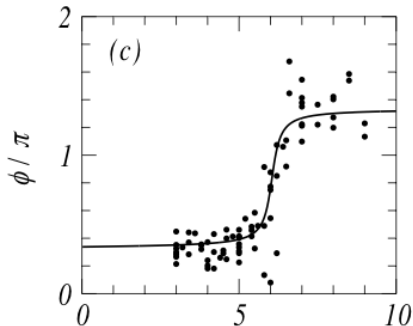
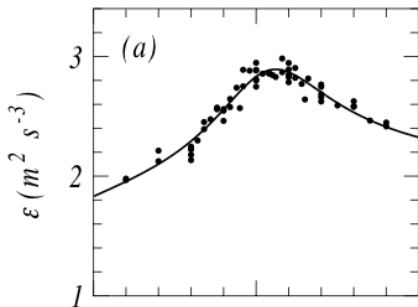
# Dissipation rate in modulated turbulence

Grid solidity and dissipation rate:



Low-pass filtering of the dissipation-rate

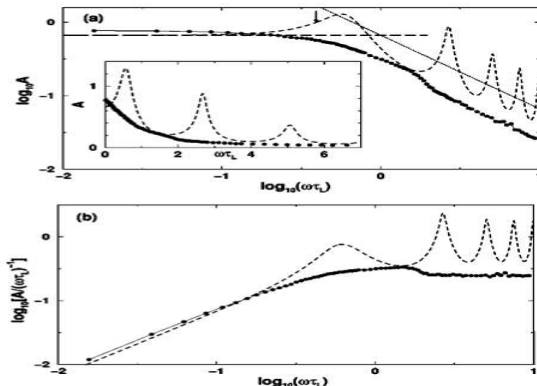
# Frequency dependence



Resonant dissipation - phase shift of 180 degrees

# Mean field and GOY?

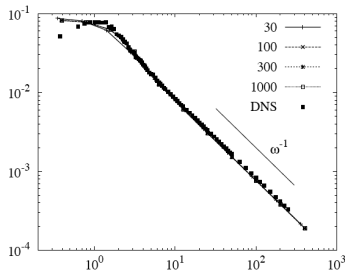
Heydt-Grossman-Lohse



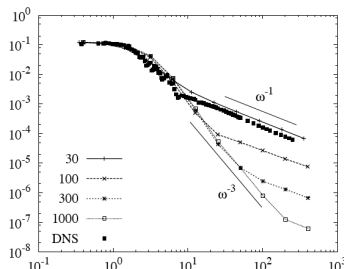
- Dashed: mean-field, Dots: GOY simulation
- GOY and REWA simulations show only small effect
- **dissimilar** to numerical NS experiments

# Two point closure

## Bos-Rubinstein



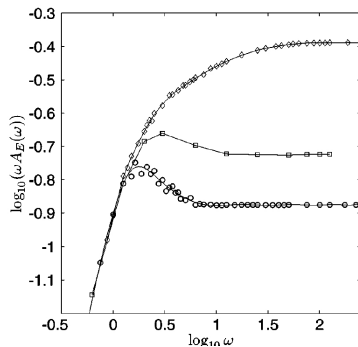
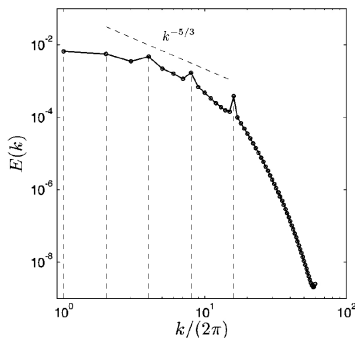
(a)



(b)

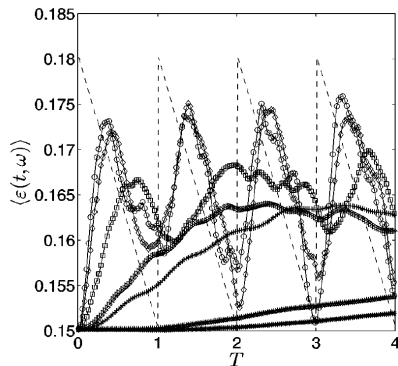
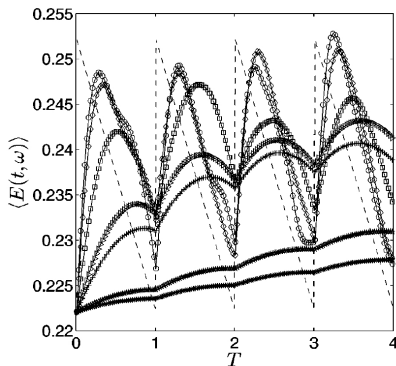
Energy (a) and dissipation (b): two-point closure approach compares closely to DNS

# Multiscale forcing



Response maxima pronounced when large scales forced  
More pronounced as  $Re$  lower

# Complex forcing strategies



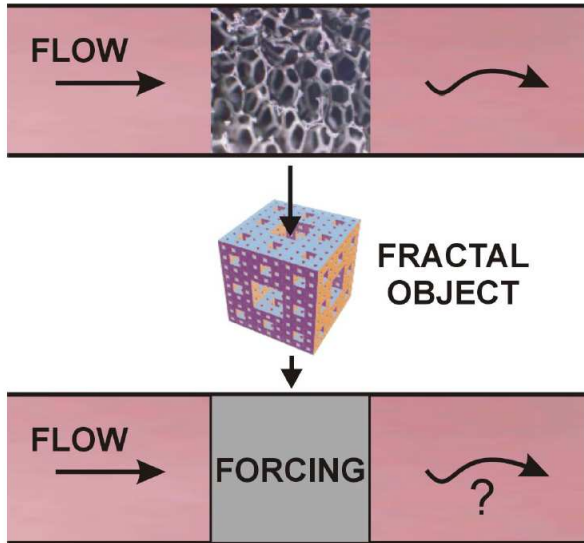
Response to saw-tooth forcing



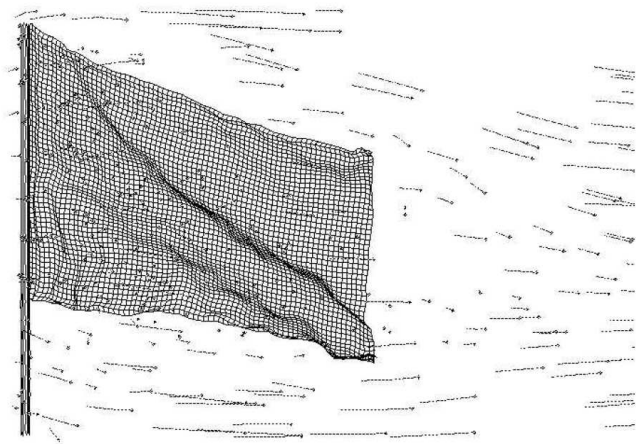
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# Fractal modeling of complex objects?



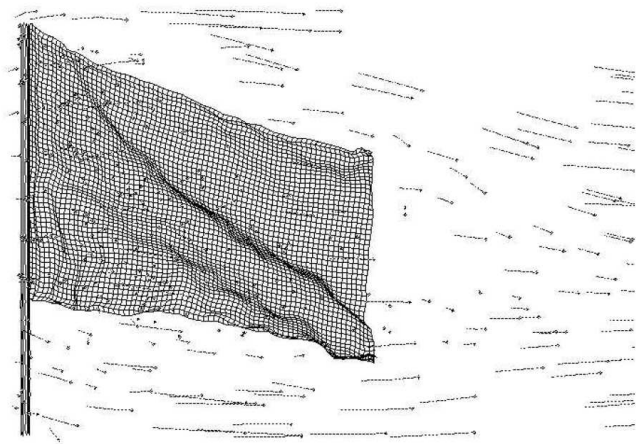
# IBM - basics



Peskin, c.s.

- Compute on simple grid - cut out object
- fast solvers - complex geometries

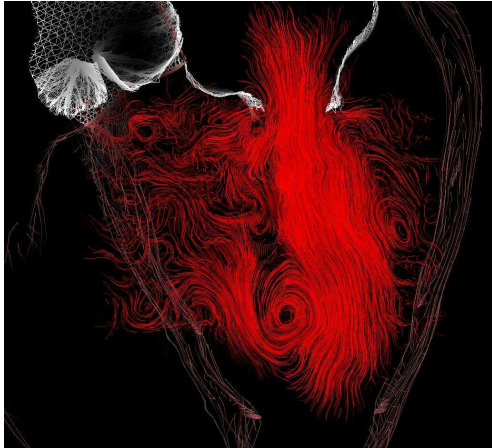
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# IBM in life-sciences



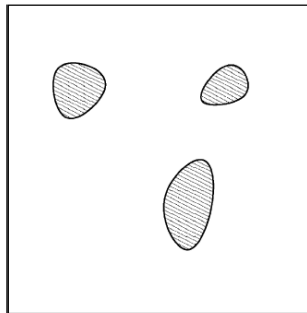
Famous application: flow in realistic heart

# IBM - volume penalization

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \frac{1}{Re} \Delta \mathbf{u} + \frac{1}{\epsilon} H \mathbf{u} = 0$$

Indicator function:

$$H = \begin{cases} 1 & \mathbf{x} \in \Omega_s \\ 0 & \mathbf{x} \in \Omega_f \end{cases}$$



# How to relate forcing to IBM?

## Case studies:

- **bottom-up:** optimize forcing to comply with simulated flow?
- **top-down:** relate 'objects' to 'local fractal dimension'?
- ...
- can lack of detailed resolution be 'modeled away' at all?
- how much geometric detail is needed?
- can two-point closure provide guidance?
- ...

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- quantitative and qualitative changes of cascading possible
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- 'receptivity' to agitation probed with time-modulated forcing
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- connect complex geometry to specific forcing?

Thanks: NCF

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