REPORT ON THE IMS/SIG 35/COST 20/ERCOFTAC WORKSHOP:

Interscale energy transfer in various turbulent flows Institute for Mathematical Sciences (IMS), Imperial College, London, UK, March 26-28, 2007

Claude Cambon[1], Arkadi Tsinober[2,3], Christos Vassilicos[2,3]
[1] Laboratoire de Mécanique des Fluides et d'Acoustique, UMR 5509,
Ecole Centrale de Lyon/Université Claude Bernard Lyon/INSA/CNRS, 69134 Ecully Cedex, France.
[2] Institute for Mathematical Sciences, Imperial College, London
[3] Department of Aeronautics, Imperial College, London

Context and objectives of the workshop

The SIG (ERCOFTAC Special Interest Group) 35, entitled *Multipoint Turbulence Structure and Modelling*, includes a large spectrum of topics, from fundamental studies to applications. Interscale energy transfer is a key concept for a better understanding of turbulent mixing, for improving the parametrisation of small scales in LES, etc.

Not to mention artificial forcings (narrow-band in Fourier space, white noise in time), physically relevant effects, reflected in linear operators added to Navier-Stokes equations, can inject energy and/or anisotropy at various scales, forming a domain of turbulence with interactions : interactions with body forces (Coriolis, buoyancy, Lorentz), with large-scale gradients (shear, strain). These effects can dramatically alter the classical scheme of inertial range with constant flux slaved to the dissipation rate, which is inherited from Kolmogorov. Initialisation and/or forcing by fractal objects yield even more surprizing results in this context.

The workshop was attended by about 50 participants, senior scientists as well as students. The participants represented several different technical fields, from Applied Mathematics to engineering, and 8 different countries (EU, USA, Israel and Japan). Among the participants, 3 students were supported by ERCOFTAC scholarship.

Nineteen talks were given. Each talk was 40 mn. long, including questions, except the last one, added later, and a general discussion took place at the end of the meeting. The talks can be roughly dispatched in 6 groups as follows, keeping in mind, of course, their large overlaping.

A) Fundamentals. Alignment properties

Darryl Holm (London) *averaged vorticity aligment dynamics* : a large survey of Lagrangian and Eulerian exact equations, derived from Euler equations for flow ranging from strictly incompressible to baroclinic, was offered, with reminders of conservation properties (e.g. Ertel theorem). As for all the talks in this group, aligment properties concerned the vorticity within the eigenframe (principal axes) of the symmetric velocity gradient tensor. New developments in progress call into play quaternions.

Koji Okhitani (Sheffield) *Survey and numerical simulation of Lagrangian stability of ideal fluid motion* : a modern version of the Arnold's theory was revisited, with emphasis of a sectional approach to Lagrangian stability. As a very promising perspective, the solenoidal constraint (divergence-free velocity field) is being incorporated in the fully Lagrangian dynamical description.

Alain Pumir (Nice) *Geometry and Statistics in Turbulence* : Lagrangian dynamics of a transported tetrahedron yields predictions for the alignment and Q-R maps (Q and R being the second and third invariants of the velocity gradient tensor). A stochastic model, with related Fokker-Planck equation was derived, using possibly a pure spherical form for the pressure Hessian (assumption coined as "restricted Euler equations").

Beat Luethi (Risoe, Denmark, and Switzerland) *Energy flux to subgrid scales as obtained from particle tracking* : the experimental method, based on PTV, gives access to various Eulerian and Lagrangian statistics. Among them, emphasis was placed on accurate evaluation of the velocity gradient tensor. New results, very relevant for LES applications, delt with the aligment of the subgrid scale tensor (or Reynolds stresses from the smallest scales) within the principal axes of the coarse symmetric velocity gradient tensor.

B) Balance and unbalance of enstrophy production. Improved closures

Robert Rubinstein (Hampton, VA, USA) *Unbalanced vortex stretching in non-stationary turbulence* : an historical survey was given for the study of the balance, and unbalance, of the nonlinear vortex stretching term (enstrophy production) with its "dissipative" counterpart (palinstrophy or enstrophy destruction). Models by Lumley and by Bernard & Speziale were contrasted. Finally, an isotropic spectral model with time-periodic injection of energy at large scale (as in modulated turbulence) was used to investigate the response of smaller structures, directly linked to the unbalance under consideration.

Arkady Tsinober (London, and Israel) *On balance of enstrophy production and its dissipation* : even if the scaling by Lumley is relevant, this bal-

ance remains a fundamental question in turbulence. The enstrophy production term can be also compared to the cubic invariant of the symmetric part of the velocity gradient tensor, which is a priori more relevant for the budget of dissipation rate. The order of magnitude of these different terms was estimated from data in a bounded channel flow. Finally, results from a new pseudo-spectral DNS with various large-scale (e.g. ABC) forcings, was shown to support the idea of a balance, not only in average, but at every time-step.

K. Kikani (with David Mc Comb, Edinburgh) *Markovianized single-time local energy transfer : a K41 compatible Eulerian spectral closure for isotropic turbulence with no tuning parameters* : a general introduction of renormalized perturbation theories, e.g. including Kraichnan's DIA and TFM, was given. Interscale energy transfer is evaluated by the spectral transfer function, which is the counterpart of the third-order structure function in the Karman-Howarth equation, and its closure calls into play both the second-order spectral tensor and the response function, which are essentially two-point two-time correlation functions. Using the LET concept, an abridged single-time markovianized version is derived with no need for a tuning parameter (as in EDQNM), and (apparently) no mixed Lagrangian/Eulerian information, as requested in LHDIA, LRA or LDIA.

C) New large scale interactions in experiments

Shigeo Kida (Kyoto) *Flow structure in a sphere of precession* : this study is experimental, numerical and theoretical. In addition to the dominant angular velocity, a second one, normal to it, characterizes the precession. As in the geophysical case, the ratio of the main angular velocity to the precession one (Poincaré number), even very weak, is a key parameter. Preliminar flow visualizations (experiment) and pseudo-spectral DNS results in terms of toroidal/poloidal potentials showed a very multiform flow organisation, from chaotic to turbulent. Low Reynolds steady states were analytically calculated by a perturbative method and are recovered by DNS.

Herman J. H. Clercx (Einhoven) *Turbulent rotating Rayleigh-Bénard convection : DNS and SPIV measurements* : In contrast with the following group, instable thermal convection is studied, both experimentally and numerically. A nice stereo-PIV procedure was used for the measurements. The results showed that the Bolgiano-Obukhov scaling works without rotation, but that rotation modifies this scaling.

D) Turbulent flows in a stably stratified fluid with and without rotation

Franck Nicolleau (Sheffield) *Kinematic Simulation and Rapid Distortion Theory of turbulence in stably stratified and rotating flow* : KS means building synthetic realizations of an incompressible velocity field, by means of superposed spatial Fourier modes, in order to compute trajectories and to derive Lagrangan statistics. In the basic version of the model, temporal evolution of the velocity field was possibly mimiced by means of empirical temporal frequency, only affecting the phase of the Fourier modes. Incorporating the linear dynamics of inertia-gravity waves is much more informative in the presence of rotation and sta-
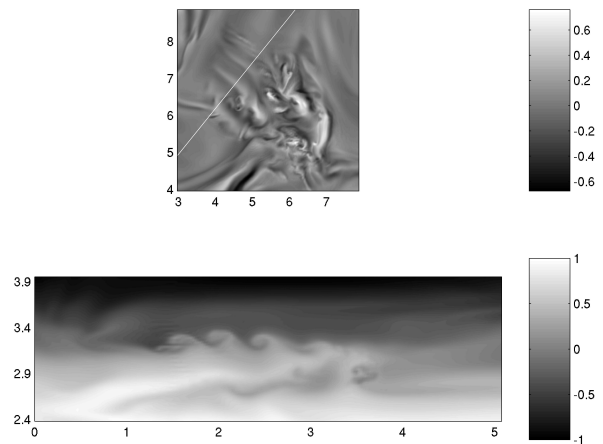

Figure 1: Top panel shows part of a horizontal slice through the w field. The white dashed line gives the orientation of a vertical slice through the horizontal plane. the bottom panel shows ρ_T on that vertical slice. From J. J. Riley & S. M. de BruynKops (2003), "Dynamics of turbulence strongly influenced by buoyancy," Phys. Fluids, **15** (7), 2047–2049.

ble stratification : this amounts to incorporate the so called "inviscid Rapid Distortion Theory" entirely, in any realization, or to replace the empirical frequencies by the exact dispersion frequencies of waves. Nontrivial anisotropic time development is found for mean square displacements related to one and two particles. The most original result is a pseudo-Brownian behaviour, which results from inertial wave phase-mixing, whose underlying mechanism is completely different from the classical one (Taylor).

Jim Riley (Seattle) *Turbulence and energy transfer in strongly-stratified turbulence* : a very complete survey of observations, experiments and DNS/LES was offered. Scaling laws of strongly stratified turbulence were discussed, showing the importance of a $R_b \sim ReF_r^2$ number (buoyancy efficiency) in addition to more conventional Froude F_r and Reynolds Re numbers. DNS results with large-scale initialisation by 3D Taylor-Green modes were shown for the most interesting parameter range, with $R_b > 1$. Horizontal spectra exhibit a conventional $-5/3$ law, already found by Lindborg and coworkers with very different calculations using flatened boxes, 2D-2C empirical forcing, and hyperviscosity.

Claude Cambon (Lyon) *Toroidal energy cascade: anisotropic morphology and nonlinear dynamics of strongly-stratified turbulence*: Dominant rotation and dominant stratification were first considered face to face. In the first case, the role of inertial wave phase-mixing was shown to be an essential explanation, from the abovementioned Nicolleau's results to the preferential development of cyclonic vorticity. Similar effects only affect the poloidal part of the flow in the presence of strong stratification, so that strong turbulence governs the toroidal part (toroidal/poloidal is essentially the same as the Riley's vortex/wave *without rotation*). A toy-model of pure toroidal cascade is consistent with anisotropic

morphology (horizontal layering) and dynamics. In spite of detailed (triadic) conservation of both energy and vertical enstrophy, this dynamics is very far from the 2D one, with dominant direct cascade in terms of spherical shells of different size, but a significant part of inverse cascade in terms of cylinders. This suggests revisiting the Quasi-Geostrophic case using exactly the same tools, when rotation is reintroduced.

Joel Sommeria (Grenoble) *Decaying grid turbulence in a rotating stratified fluid*: the experimental study was carried out using PIV in the large rotating tank (13 meters diameter "Coriolis platform" in Grenoble) for a large range of f/N (system vorticity to Brunt-Waisala frequency) ratios, the turbulence being generated by a moving rake. Conventional $-5/3$ slopes were found for horizontal spectra when stratification is dominant. At the larger rotations rate, until $f/N \sim 1.2$, the results lend support to the quasi-geostrophic Charney's model, with inverse energy cascade and direct cascade of potential vorticity, the aspect ratio of elongated columnar structures remaining close to f/N.

Jose Redondo (Barcelone) *Hyperdiffusivity in rotating stratified flows*: among a lot of results, mostly experimental, shown, a diagram in terms of three nondimensional numbers, Richardson, Reynolds and Rossby, was shown to collect various cases with turbulence and instabilities. A new experiment of stratification with a vertical oscillating grid could be considered as the counterpart of a well-known experiment by Hopfinger and coworkers in the rotating flow case. Finally several results illustrate the crucial importance of $f/N \sim 1$ for such flows.

E) Fractal initialisation or forcing on turbulence

Christopher Keylocks (Leeds) *Properties of the wake structure behind fences with a fractal arrangement of horizontal strut*: Wavelet, fractal and structure functions analyses were carried out of turbulent velocity signals in the lee of fractal fences in a wind tunnel. Questions were asked relating to potential modifications caused by the fences's fractal nature in the shape of structure functions and in the mix of chirp, spiral-like (local) and global (fractal-like) singularities in the turbulence signal. It was claimed that the turbulence generated by fractal fences may have less pronounced chirp singularities. This is a very difficult but promising direction of investigation which has only recently started in a collaboration between Leeds and Japan.

Christos Vassilicos (London) *Fractal-generated turbulence*: Wind tunnel measurements of fractal-generated homogeneous isotropic turbulence were investigated, with various pattern of fractal grids. Results suggest that it is possible to turn off the dissipation anomaly in such a turbulence and make the kinetic energy dissipation rate decrease with Reynolds number as its inverse power. In spite of a strong alteration of decay laws and dissipation rate due to the fractal structure of the grid, and related anomalous initialisation of turbulence, energy spectra with $-5/3$ power-law dependence on wavenumber is recovered.

F) Reduction of degrees of freedom

Edriss Titi (Israel and USA) *Mathematical study of certain analytical models of turbulence*: After an overview of various issues, including rapidly rotating turbulence, the author gave a nice presentation of the alpha model making the point clear that it allows regularisation of the equations without loss of energy conservation.

Sergei I. Chernyshenko (with Maxym N. Bondarenko, Southampton) *Master mode set for 3D turbulent channel flow* : a general procedure was presented to extract a minimal number of master modes, irrespectively of the nature of the modal decomposition (Fourier-Chebychev, pod, wavelets ...). It was shown that turbulence velocity fields can be represented quite accurately with only 1% of modes (Fourier-Chebychev here) in a channel flow.

The two talks below could be also related to "stably stratified and/or rotating turbulence" theme.

Bernard Geurts (Twente) *Regularization modelling of compressible rotating turbulence*: incompressible rotating turbulence was first addressed, using DNS and LES with the regularization method applied to small scales. Two issues were particularly discussed: the alteration of the decay law for the turbulent kinetic energy and the asymmetry of the distribution in terms of cyclonic and anticyclonic vorticity. The compressible case finally considered was a rotating mixing layer. In contrast with the conventional case without rotation, compressibility effects were shown to be significant even at moderate Mach number.

Kai Schneider (Marseille) *Quantifying anisotropy in stratified and rotating turbulence using orthogonal wavelets*: previous (Lichtenstein et al. 2005) and new DNS results were analyzed using both toroidal/poloidal decomposition and wavelets analysis. It was shown that the nonlinear dynamical method CVE (Coherent Vortex Extraction), successfully applied to isotropic turbulence, yielded to an even better (smaller) coherent/incoherent rate for wavelet coefficients in the strongly anisotropic case altered by dispersive waves. Some results were somewhat surprizing and not consistent with too simple ideas, such as the fact that the poloidal part (preferentially affected by waves) of the flow appeared more intermittent than its toroidal counterpart.

Conclusions, open issues

The first issue which created much interest and debate during the conference was the Tennekes-Lumley balance between enstrophy production and dissipation. It was shown that this balance is valid at every time step of a Direct Numerical Simulation (DNS) of homogeneous turbulence with periodic boundary conditions forced at the large scales (A. Tsinober). The questions therefore also arose as to whether this balance might also hold during decay, for any boundary and initial conditions and for any forcing whether large scale or broad-band.

A related question which also arose from the talks during the discussions was the dependence of kinetic energy dissipation rate on Reynolds number and the time-dependence of the decreasing kinetic energy during turbulence decay. Fractal initialisation of turbulence yielded the most surprizing behaviour in this context (J. C. Vassilicos).

Laboratory experiments and DNS of rotating stratified turbulence were presented and vigorously discussed. In the limit where the rotation dominates

over stratification, they clearly show that the power-law time-dependence of the kinetic energy becomes shallower with decreasing (faster rotation) Rossby number. For instance, the fact that the exponent of the power law decay becomes close to half its value in the non-rotating case at largest Reynolds number and smallest Rossby number was recovered by B. Geurts, in complete agreement with several previous studies. A different régime where the decay would be completely blocked is not consistent with experimental and theoretical studies. Yet, the horizontal integral length-scale in such turbulence keeps the same increasing power-law time-dependence irrespective of Reynolds number. This is a surprising result worthy of further study, even if the evolution of integral length scales was the object of many experimental and theoretical approaches so far.

When it is the stratification which dominates over rotation, the power-law time dependence of the kinetic energy seems very similar to that of decaying isotropic turbulence, indicating a strong presence of 3D dynamics in this respect (Jim Riley, Joel Sommeria).

Energy spectra with $-5/3$ power-law dependence on wavenumber seem ubiquitous: for example they are found in strongly stratified turbulence in the horizontal, and in fractal-generated wind-tunnel turbulence. How little does this $-5/3$ exponent mean ? Is the generaly approved idea that it is a direct consequence of constant interscale flux valid even in fractal-generated turbulence where the kinetic energy dissipation rate does not seem to be independence of Reynolds number?

The role of coherent structures was also, though rather more briefly, the focus of some interest. This relative brevity was not caused by any perceived unimportance of the issue, but by its extreme difficulty! Their role was discussed in relation to transfers of energy, in particular angle to angle in stratified and/or rotating turbulence (C. Cambon, K. Schneider). Their role was however also discussed in relation to anisotropic turbulent diffusion (F. Nicolleau) as they do not seem to have much impact on one-particle statistics but may well have a much more determining impact on two-particle statistics (not to mention multi-particle and multi-point statistics). In fact, the concept of a coherent structure should perhaps be viewed quite broadly; they may be local well-defined structures which move by keeping their coherence intact, but they may also be dispersive inertia-gravity waves, as pointed out by Cambon. Both of these types of coherent structures can have significant effects on turbulent diffusion, but it is easier to visualize eddies than waves.

On the other hand, the role of spatio-temporal structures resulting from instabilities is really essential (more than neutral gravity waves or too specific zig-zag instabilities) for the nonlinear dynamics of stratified turbulence without rotation. For instance, Kelvin-Helmholtz (K-H) structures resulting from inter-layer vertical shearing are evidenced in the figure 1 in the regime $R_b > 1$ (Jim Riley). An important question is the possible consistency of full DNS including K-H instabilities with a strongly anisotropic statistical model for toroidal cascade (C. Cambon).

Different mixed Eulerian and Lagrangian approches were presented. How much of the Eulerian structure is needed for correct predictions of Lagrangian statistics ? It is this question that is at the heart of the philosophy behind Kinematic Simulations. Kinematic Simulations of various turbulent flows, in particular isotropic turbuence and stratified and/or rotating turbulence turned out to be of some interest to a large number of participants. Another pair of important related questions which were also touched upon in this Eulerian/Lagrangian context are energy transfer and geometrical alignments. Energy transfers are up- and down-scale in stratified turbulence but inhibited by rotation in rotating turbulence. It is a new question to know what they are in fractal-generated turbulence. The important role of the pressure hessian in any reduction on non-linearity was also brought up.

An issue of great importance, both fundamentally and for applications, is the reduction of the number of degrees of freedom. It was shown that turbulence velocity fields can be represented quite accurately with only 1% of all wavelet coefficients (Kai Schneider) or 1% of master modes (S. Chernychenko). In both cases, the two important questions, not yet answered, are: (i) how does this number increase with Reynolds number? As Reynolds number to the 9/4 power or less? (ii) How can one predict future velocity fields from a given 1% good representation of a velocity field at time zero? Clearly this last question is related to the famous predictability problem, an issue which was also discussed.

Finally, a very important question which is not raised often enough is: what is the ideal turbulence experiment? The problem of turbulence in a sphere is one which lends itself to accurate direct numerical simulation studies, as well as to various asymptotic studies and, crucially, to detailed and well-controlled experimental investigations. All aspects of the problem, theoretical, numerical and experimental, are dealt with the same boundary conditions and in the absence of a mean flow, a unique occurence! This research project is currently under way in Kyoto, Japan, and appears promising.

SPONSORSHIP/ Pilot centers and SIG involved