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Velocity spectra of superfluid turbulence

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Abstract. – We study numerically statistics of superfluid turbulence. We generate a quantized superfluid vortex tangle driven by a realistic model of normal-fluid turbulence whose energy spectrum obeys Kolmogorov's classical $k^{-5/3}$ law, where k is the wave number. We find that the resulting superfluid velocity spectrum has approximately a k^{-1}-dependence for wave numbers of the order of $1/\delta$ and larger, where δ is the average intervortex spacing. This result is similar to what happens in a pure superflow. We also find that the spectrum of the total velocity field follows the classical $k^{-5/3}$ law, even at temperatures low enough that the normal-fluid mass is only 5% of the total helium mass. We discuss our assumptions and results in view of recent experiments.

In the two-fluid model of superfluidity the motion of the fluid is described by two superimposed velocity fields, the normal-fluid velocity $\boldsymbol{V}_{\mathrm{n}}$ and the superfluid velocity $\boldsymbol{V}_{\mathrm{s}}$. The superfluid component flows with zero viscosity while the normal fluid component moves with a small but non-zero viscosity. This description of superfluidity as two interpenetrating components was originally put forward to explain some apparently paradoxical experiments in which helium II flows at fairly low velocity, but it has also described well the high-velocity counter-flow turbulence that occurs when the mean superfluid and normal-fluid velocities are in opposite directions [1]. Theoretical and experimental research on helium-II hydrodynamics is now turning to applying the two-fluid model to the co-flow turbulence of helium II, where the mean velocity of the two components is in the same direction, or is zero. In current experiments turbulence in helium II is produced using standard techniques of classical fluid dynamics, such as towing a grid [2, 3], rotating propellers [4] or moving a sphere [5].

In isolation, the normal-fluid component would have the Navier-Stokes turbulence expected of a fluid with non-zero viscosity. And, again in isolation, the superfluid component would have the turbulence behaviour of the inviscid Euler equation, with the extra constraint that the vorticity in the flow must be confined to vortex filaments with quantized circulation. But, except at absolute zero temperature, the two components are not isolated from one another, and the flow behaviours of both fluids, including turbulence, may be strongly affected by the coupling between the two fluid components [6]. In the interpretation of results of experiments in helium-II turbulence two assumptions are often made. The first assumption is that the turbulence in the normal fluid is still Navier-Stokes turbulence (at least approximately) despite the addition of the mutual friction force. The second assumption is that this normal-fluid turbulence drives the superfluid also into a classical turbulence behaviour. Vinen [7] suggested that the superfluid and normal-fluid velocity fields may lock together and become identical at length scales much larger than the average separation δ between the quantized vortices. If that happens, $\boldsymbol{V}_\mathrm{s}$ and $\boldsymbol{V}_\mathrm{n}$ will have the same spectrum at wave numbers $k \ll 1/\delta$.

The aim of this letter is to start an investigation of the spectrum of the superfluid component when the flow is driven by a normal-fluid velocity field with Kolmogorov's classical 5/3 power law kinetic-energy spectrum.

Our numerical calculation is based on the approach of Schwarz [8] in which a quantized vortex filament is represented as a space curve $\boldsymbol{s} = \boldsymbol{s}(\xi, t)$, where ξ is arclength and t is time, which moves with velocity $\mathrm{d}\boldsymbol{s}/\mathrm{d}t$ given by

$$\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} = \boldsymbol{V}_\mathrm{s} + \alpha \boldsymbol{s}' \times (\boldsymbol{V}_\mathrm{n} - \boldsymbol{V}_\mathrm{s}) + \alpha' \boldsymbol{s}' \times \big(\boldsymbol{s}' \times (\boldsymbol{V}_\mathrm{n} - \boldsymbol{V}_\mathrm{s})\big), \tag{1}$$

where $\boldsymbol{s}' = \mathrm{d}\boldsymbol{s}/\mathrm{d}\xi$, α and α' are known mutual friction coefficient, $\boldsymbol{V}_\mathrm{s}$ is the self-induced velocity at the point $\boldsymbol{s}$ given by the Biot-Savart integral

$$\boldsymbol{V}_\mathrm{s} = \frac{\Gamma}{4\pi} \int \frac{(\boldsymbol{r} - \boldsymbol{s}) \times \mathrm{d}\boldsymbol{r}}{|\boldsymbol{r} - \boldsymbol{s}|^3}, \tag{2}$$

and $\Gamma = 9.97 \times 10^{-4}\ \mathrm{cm}^2/\mathrm{s}$ is the quantum of circulation. We make the standard assumption that the quantized vortex filaments reconnect when they cross one another. The mutual friction force on the superfluid vortices is proportional to the local difference in the velocity fields of the two fluid components, $\boldsymbol{V}_\mathrm{n} - \boldsymbol{V}_\mathrm{s}$. The mutual friction force exchanges energy between the two fluids, potentially in either direction, so the normal fluid may act as an energy source for the superfluid, driving the turbulence, or as an energy sink and thus a form of dissipation [9]. Ideally, one would solve the hydrodynamic equations for both fluid components simultaneously. But that is computationally very expensive and has been attempted only for very simple geometries [10]; the reasonable approximation which we follow is to determine the response of the superfluid to an imposed normal-fluid flow, neglecting the back reaction of the superfluid vortices on the normal fluid. Despite its limitation, the kinematic approach presented here represents a significant improvement on previous kinematic calculations in which the imposed normal fluid was laminar (uniform flow, Poiseuille flow, Gaussian vortex, ABC flow). Here we adopt a more realistic Lagrangian Kinematic Simulation model of turbulence (referred to as the KS model [11]). In this model of a turbulent flow the normal-fluid velocity field is given by $\boldsymbol{V}_\mathrm{n} = \boldsymbol{U}_\mathrm{n}^0 + \boldsymbol{v}_\mathrm{n}'$, where $\boldsymbol{U}_\mathrm{n}^0$ is a uniform flow and $\boldsymbol{v}_\mathrm{n}'$ represents a fluctuating turbulent part given by

$$\boldsymbol{v}_\mathrm{n}' = \sum_{m=1}^{M} [\boldsymbol{A}_m \times \hat{\boldsymbol{k}}_m \cos(\boldsymbol{k}_m \cdot \boldsymbol{x} + \omega_m t) + \boldsymbol{B}_m \times \hat{\boldsymbol{k}}_m \sin(\boldsymbol{k}_m \cdot \boldsymbol{x} + \omega_m t)]. \tag{3}$$

Here $M = 64$ is the number of modes, $\hat{\boldsymbol{k}}_m$ is a random unit vector with $\boldsymbol{k}_m = k_m \hat{\boldsymbol{k}}_m$, and the vectors $\boldsymbol{A}_m$ and $\boldsymbol{B}_m$ are chosen randomly but such that $|\boldsymbol{A}_m|^2 = |\boldsymbol{B}_m|^2 = (2/3) E_\mathrm{n}(k_m) \Delta k_m$ where the energy spectrum of the normal fluid, $E_\mathrm{n}(k_m)$, satisfies Kolmogorov's $k^{-5/3}$ law

$$E_\mathrm{n}(k_m) \sim \epsilon^{2/3} k_m^{-5/3} \tag{4}$$

and the energy dissipation rate per unit mass, ϵ, is a constant which depends on input parameters. Note that $\nabla \cdot \boldsymbol{v}_\mathrm{n}' = 0$ by construction. The frequency ω_m of the m-th mode is chosen to be equal to the eddy turnover time of that mode, so $\omega_m = \sqrt{k_m^3 E_\mathrm{n}(k_m)}$. Since in

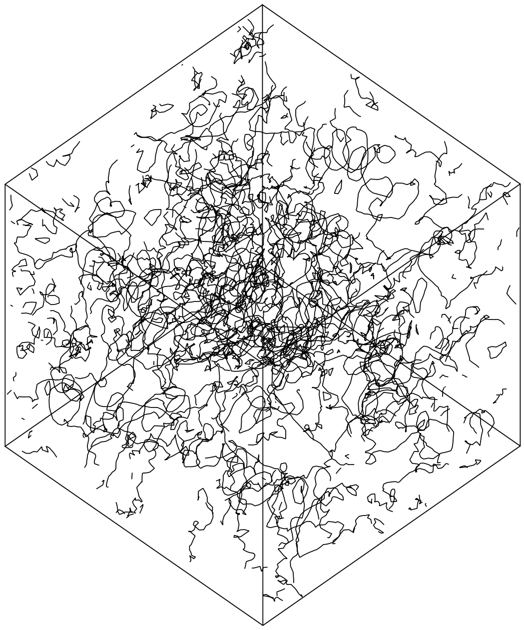

Fig. 1 – Vortex tangle at $t = 0.093$ s.

our calculation there is no uniform superflow which is applied, $\boldsymbol{v}_{\rm s}$ consists only of fluctuations caused by the superfluid vortices and we call $\boldsymbol{v}'_{\rm s} = \boldsymbol{v}_{\rm s}$.

The time-dependent velocity field generated by the KS model is a qualitatively realistic representation of the type of eddying, straining and streaming flows observed in turbulence. Our aim is to calculate the response of superfluid vortex filaments to the mutual friction force generated by this turbulent flow.

We perform our numerical calculation in a cubic periodic box of size $h = 0.1$ cm at temperature $T = 1.3$ K. The mean normal flow is $V_{\rm n}^0 = 2.36$ cm/s and the rms value $V_{\rm rms}$ of the turbulent fluctuations $\boldsymbol{v}'_{\rm n}$ is $V_{\rm rms} = 11.93$ cm/s. The Kolmogorov length is $\eta = \ell R^{-3/4} = 0.00078$ cm, where $\ell = h/2$ is the integral scale, $R = V_{\rm rms}\ell/\nu_{\rm n} = 255$ is the Reynolds number, $\nu_{\rm n} = \mu/\rho_{\rm n} = 2.34 \times 10^{-3}$ cm^2/s is the kinematic viscosity of the normal fluid, $\rho_{\rm n} = 0.00652$ g/cm^3 is the normal-fluid density and μ is the viscosity of helium II (note that we define the Reynolds number using $\nu_{\rm n} = \mu/\rho_{\rm n}$ rather than $\nu = \mu/\rho$). The calculation starts with 20 superfluid vortex rings of random size and orientation (the result is independent of the initial condition). The initial superfluid filaments interact with each other and with the normal fluid and soon a tangle of filaments is formed. We stop the calculation at time $t = 0.093$ s (see fig. 1) at which point the total length of the tangle of filaments is $\Lambda = 14.18$ cm, hence the vortex line density is $L = \Lambda/h^3 = 14180$ cm^{-2} and the average intervortex spacing is $\delta \approx L^{-1/2} = 0.008$ cm. Calculations with larger values of Λ are not practical because the time required to evaluate the Biot-Savart integral is proportional to the square of the number of the discretization points.

To analyse the result we define the velocity spectra of the two fluid components:

$$E_{\rm n}(k) = 4\pi k^2 |\widetilde{\boldsymbol{v}}'_{\rm n}|^2, \tag{5}$$

$$E_{\rm s}(k) = 4\pi k^2 |\widetilde{\boldsymbol{v}}'_{\rm s}|^2, \tag{6}$$

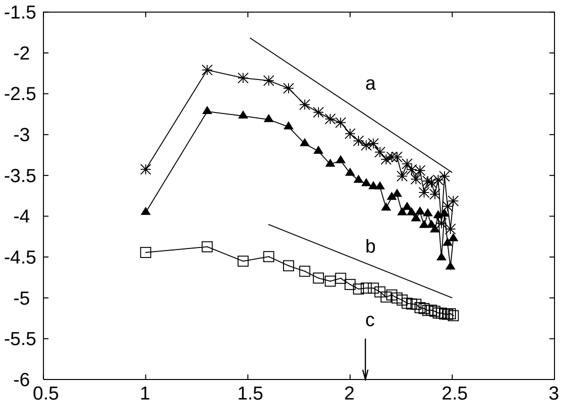

Fig. 2 – Velocity spectra plotted *vs.* wave number $\log_{10} k$. The units of k are cm^{-1}. Asterisks: normalized normal-fluid velocity spectrum $\log_{10}\big((\rho_\mathrm{n}^2/\rho^2)E_\mathrm{n}(k)\big)$; dark triangles: total-velocity spectrum $\log_{10}\big(E_\mathrm{t}(k)\big) - 0.5$; the downward shift of 0.5 units is introduced to distinguish the curve of $(\rho_\mathrm{n}^2/\rho^2)E_\mathrm{n}(k)$ from the curve of $E_\mathrm{t}(k)$ which would otherwise overlap. Open squares: normalized superfluid velocity spectrum $\log_{10}\big((\rho_\mathrm{s}^2/\rho^2)E_\mathrm{s}(k)\big)$. The line labelled "a" illustrates the slope of Kolmogorov's $k^{-5/3}$ law; the line labelled "b" illustrates the k^{-1} slope. The arrow labelled "c" marks $\log_{10}(k_\delta)$, where $k_\delta = 1/\delta = 119\ \mathrm{cm}^{-1}$ corresponds to the intervortex spacing δ.

where $\widetilde{\boldsymbol{v}}'_\mathrm{n}$ and $\widetilde{\boldsymbol{v}}'_\mathrm{s}$ are the Fourier transforms of $\boldsymbol{v}'_\mathrm{n}$ and $\boldsymbol{v}'_\mathrm{s}$, respectively. Figure 2 shows the resulting (normalized) normal-fluid and superfluid velocity spectra $E_\mathrm{n}(k)$ and $E_\mathrm{s}(k)$. The velocity spectrum of the normal fluid, $E_\mathrm{n}(k)$, follows Kolmogorov's $k^{-5/3}$ power law, as it should be by construction. The velocity spectrum of the superfluid, $E_\mathrm{s}(k)$, is evidently different. We conclude that, at least in this example, the superfluid vorticity tangle that develops from the mutual friction force induced by the normal fluid does not share the turbulence statistics of the driving normal-fluid flow.

We know from our previous calculation [12] at $T = 0$ that, in the absence of the normal fluid, the velocity spectrum of superfluid vortices has $E_\mathrm{s}(k) \sim k^{-1}$-dependence which arises from a Kelvin wave cascade [7, 12, 13], a result which has recently been confirmed by Araki *et al.* [14]. The finite-temperature spectrum presented in fig. 2 shows that $E_\mathrm{s}(k)$ is much shallower than the driving $k^{-5/3}$ normal-fluid dependence, the power law of $E_\mathrm{s}(k)$ being close to k^{-1}. It is interesting to note that the k^{-1}-dependence of $E_\mathrm{s}(k)$ actually extends into the region of wave numbers $k < k_\delta = 119\ \mathrm{cm}^{-1}$, where $k_\delta = 1/\delta$ corresponds to the average intervortex spacing.

Ideally one would like to explore the spectrum in the region $k \ll 1/\delta$ and test Vinen's idea that $\boldsymbol{V}_\mathrm{s} \approx \boldsymbol{V}_\mathrm{n}$ at length scales much larger than the vortex separation δ. Unfortunately, it is difficult to generate a larger separation of scales, because the large L required to reduce δ makes the evaluation of the necessary Biot-Savart integrals computationally prohibitive. The resolution of more length scales would also allow direct comparison with the experiments by Stalp *et al.* [3] who measured the decay of superfluid vorticity created by a towed grid. They modelled their results by assuming that the superfluid obeys Kolomogorov's spectrum, but there is no conflict between their model ($E_\mathrm{s}(k) \sim k^{-5/3}$) and our result ($E_\mathrm{s}(k) \sim k^{-1}$) because the two investigations are likely to refer to different parts of the spectrum ($k \ll 1/\delta$ and $k \geq 1/\delta$, respectively); this would certainly be the case if the two fluids lock together in a classical behaviour at large length scales, as suggested by Vinen [7].

Despite this limitation, our result that $E_\mathrm{s}(k) \sim k^{-1}$ at wave numbers of the order of magnitude of k_δ is significant because *a priori* we would expect this power law only in the region $k \gg 1/\delta$ (isolated vortex line). Furthermore, our result generalizes what is known in the $T=0$ case.

We can also define a velocity field $\boldsymbol{v}_\mathrm{t}$ based on the total mass flux

$$\rho \boldsymbol{v}_\mathrm{t} = \rho_\mathrm{s} \boldsymbol{v}'_\mathrm{s} + \rho_\mathrm{n} \boldsymbol{v}'_\mathrm{n}, \tag{7}$$

where $\rho_\mathrm{s} = 0.1386\ \mathrm{g/cm^3}$ is the superfluid density at $T = 1.3\ \mathrm{K}$ and $\rho = \rho_\mathrm{s}+\rho_\mathrm{n} = 0.1451\ \mathrm{g/cm^3}$ is the total density of helium II. The corresponding total velocity spectrum is

$$E_\mathrm{t}(k) = 4\pi k^2 |\widetilde{\boldsymbol{v}}_\mathrm{t}|^2. \tag{8}$$

In strong contrast to the non-classical statistics of the superfluid velocity spectrum $E_\mathrm{s}(k)$, we find that the spectrum of the total velocity, $E_\mathrm{t}(k)$, obeys the classical $k^{-5/3}$-dependence —see fig. 2. To understand this result we square eq. (9), Fourier-transform it and obtain

$$E_\mathrm{t}(k) = \left(\frac{\rho_\mathrm{s}}{\rho}\right)^2 E_\mathrm{s}(k) + \left(\frac{\rho_\mathrm{n}}{\rho}\right)^2 E_\mathrm{n}(k) + 2\left(\frac{\rho_\mathrm{s}\rho_\mathrm{n}}{\rho^2}\right) E_\mathrm{c}(k), \tag{9}$$

where we have introduced the cross-term spectrum $E_\mathrm{c}(k) = 4\pi k^2 \widetilde{\boldsymbol{v}}'_\mathrm{s} \cdot \widetilde{\boldsymbol{v}}'_\mathrm{n}$ which measures the alignment of $\boldsymbol{v}'_\mathrm{s}$ and $\boldsymbol{v}'_\mathrm{n}$ in Fourier space (not in real space). We observe that $E_\mathrm{c}(k)$ is two to three orders of magnitude smaller than $E_\mathrm{n}(k)$ and $E_\mathrm{s}(k)$, indicating that there is very little alignment of the normal fluid and superfluid velocities in Fourier space. Note that fig. 2 shows *normalized* velocity spectra to show directly the relative size of the quantities $E_\mathrm{t}(k)$, $(\rho_\mathrm{n}^2/\rho^2)E_\mathrm{n}(k)$ and $(\rho_\mathrm{s}^2/\rho^2)E_\mathrm{s}(k)$ which appear in eq. (9). Figure 2 clearly shows that in our calculation the turbulent energy of the superfluid is very small compared with that of the normal fluid on the length scales concerned.

Before discussing whether this energy balance is general, we show that the total velocity can be a useful concept to interpret the experimental technique of Maurer and Tabeling [4]. Maurer and Tabeling produced turbulence using two counter-rotating disks and observed Kolmogorov's classical $k^{-5/3}$ law for local pressure fluctuations obtained on a small head-tube. After making a number of reasonable assumptions they derived a two-fluid Bernoulli theorem which relates the instantaneous measured pressure $P_\mathrm{meas}(t)$ to the actual pressure $P(t)$ and the upstream superfluid and normal-fluid velocities $V_\mathrm{s}(t)$ and $V_\mathrm{n}(t)$: $P_\mathrm{meas}(t) = P(t)+\frac{1}{2}\rho_\mathrm{s}V_\mathrm{s}^2(t)+\frac{1}{2}\rho_\mathrm{n}V_\mathrm{n}^2(t)$. Then, after decomposing $V_\mathrm{s}(t)$, $V_\mathrm{n}(t)$, $P(t)$ into time-independent mean fields U_s^0, U_n^0, P^0 (which are related to the main axial swirls induced by the disks) and time-dependent fluctuations $v'_\mathrm{s}(t)$, $v'_\mathrm{n}(t)$, $p'(t)$, and after noticing that the fluctuations have zero mean, they concluded that what is measured is the spectrum of the quantity $P_\mathrm{meas}(t) = p'(t) + \rho_\mathrm{s}U_\mathrm{s}^0 v'_\mathrm{s}(t) + \rho_\mathrm{n}U_\mathrm{n}^0 v'_\mathrm{n}(t)$. Maurer and Tabeling noticed that, as in ordinary turbulence, the right-hand side is dominated by the dynamic term, hence by measuring the pressure fluctuations at the head of the tube one has direct access to the velocity fluctuations. At this point, by combining their analysis with Vinen's idea that the large-scale fields are the same ($U_\mathrm{n}^0 = U_\mathrm{s}^0 = U^0$), we conclude that what is actually measured is the total velocity, $P_\mathrm{meas}(t) = p'(t) + U^0\rho v'_\mathrm{t}(t)$, where $\rho v_\mathrm{t}(t) = \rho_\mathrm{n} v'_\mathrm{n}(t) + \rho_\mathrm{s} v'_\mathrm{s}(t)$.

To complete the discussion of Maurer and Tabeling's experiment we notice that their main result, that the spectrum of $P_\mathrm{meas}(t)$ obeys Kolmogorov's law independently of temperature, can be accounted for in two ways. The first explanation is that the normal-fluid term is always greater than the superfluid term, even at low temperature; if that is the case, although the magnitude of the spectrum depends on temperature via ρ_n, the power law remains the same, $k^{-5/3}$. The second explanation is based on Vinen's idea: if not only the mean flows ($U_{\rm n}^0$ and $U_{\rm s}^0$) but also the fluctuations ($v_{\rm n}'(t)$ and $v_{\rm s}'(t)$) are locked together, then, using the two-fluid Bernoulli theorem, since $\rho_{\rm s}+\rho_{\rm n}=\rho$ and $V_{\rm s}(t)=V_{\rm n}(t)=V(t)$, we conclude that $P_{\rm meas}(t)=P(t)+\rho V^2(t)$ is indeed independent of temperature. In the experimental conditions of Maurer and Tabeling the second explanation is also possible. Although we do not know δ (because L was not measured), it is fair to assume that, because of the relatively large diameter of the head-tube (0.1 cm), the flow region Δ which was probed must have been much larger than δ (given the large Reynolds number involved, $L \gg \Delta^{-2} = 100$ cm^{-2}).

We discuss now the choice of parameters used in the simulation. We chose the low-temperature end ($T = 1.3$ K) of the typical experimental range in order to minimize the contribution of the normal fluid to the total velocity spectrum. At this temperature, $\rho_{\rm n}/\rho = 0.045$ and $\rho_{\rm s}/\rho = 0.96$, so that the contribution of $E_{\rm n}(k)$ to $E_{\rm t}(k)$ is multiplied by a very small coefficient. Despite this, we find that $E_{\rm t}(k)$ is still dominated by $E_{\rm n}(k)$. At higher temperatures this dominance would only increase, given a superfluid vortex tangle of a similar line density.

The natural question to ask is whether this situation (that the normal fluid dominates the total energy spectrum) is a special case resulting from our choice of input parameters (very large normal-fluid turbulent velocity and relatively weak vortex line density), or is this situation common? Although the spectra of fig. 2 do not correspond to a statistical steady state, it is likely that the situation described in our numerical calculation is typical of recent experiments in helium-II turbulence. First let us consider the normal fluid's Reynolds number (remember that we define R using the normal-fluid's density $\rho_{\rm n}$, not the total density ρ). Our values of mean flow and turbulent rms velocity are much smaller than some of the normal-fluid velocities that are quoted in the experiments. For example our Reynolds number ($R = 255$) is significantly smaller than values reported by Stalp *et al.* [3] in their table 1 (from $R = 10^3$ up to $R = 20 \times 10^4$ using grid velocities $v_{\rm g}$ from 5 up to 100 cm/s at $T = 1.5$ K). Our Reynolds number is also smaller than what is quoted by Maurer and Tabeling [4] ($R = 1.4 \times 10^5$). Secondly, let us consider the superfluid's vortex line density. As said before, there are practical computational constraints which prevent us from reaching the high values of L which we would like; our value $L = 14180$ cm^{-2} is smaller than than the initial value $L \approx 2 \times 10^5$ cm^{-2} shown by Stalp *et al.* in their fig. 2 for $v_{\rm g} = 5$ cm/s. Nevertheless, our L is of the same order of magnitude of their value of L at saturation when the observed classical decay begins (*e.g.*, $L \approx 18000$ cm^{-2} for $v_{\rm g} = 5$ cm/s). Moreover, this classical decay is observed down to vortex lines densities as low as $L \approx 10^3$ cm^{-2}, so our L is clearly in the range of the phenomena of interest. Notice that to boost the $E_{\rm s}(k)$ term in eq. (9) would require a much larger initial vortex line density for the same normal-fluid energy; since we expect that in a random vortex tangle the superfluid velocity scales like $\Gamma/\delta \approx \Gamma L^{1/2}$, we have $E_{\rm s}(k) \sim L$. Therefore, to make $E_{\rm s}(k)$ as big as $E_{\rm n}(k)$ in our fig. 2, we would need to increase L by more than two orders of magnitude; this would make L one order of magnitude bigger than the initial value $L = 2 \times 10^5$ cm/s reported by Stalp *et al.* for the lowest grid velocity. The situation described in our calculation is therefore not atypical and $E_{\rm n}(k)$ is likely to be the major contribution to the total energy spectrum $E_{\rm t}(k)$.

If the superfluid vortex tangle were not random, but were instead polarized with large amounts of superfluid vortex lines locally aligned, then the scale of the superfluid velocity would be larger than that of a random tangle. But there is no evidence from either experiments or numerical simulations for the formation of strong local organization in the superfluid vorticity in turbulent helium-II flow. Whether such polarization can take place at large length scale is an important issue which our simulation cannot address because we do not have resolution over a wide range of scales.

In conclusion, we calculated the superfluid velocity spectrum resulting from an imposed KS model of turbulence, possibly the most realistic description available of the turbulent normal fluid of current experiments. Our choice of parameters (Reynolds number and vortex line density) should be typical of experimental situations. We found that, while the driving normal-fluid velocity field has Kolmogorov's $k^{-5/3}$ energy spectrum, the superfluid velocity field that develops from the mutual friction forcing does not have a $k^{-5/3}$ energy spectrum, but instead obeys a shallower power law similar to the k^{-1} power law calculated at $T = 0$ when the normal fluid is absent. We also found that, even at $T = 1.3$ K, where the influence of the normal fluid would presumably be smallest due to the very low normal-fluid density (representing less than 5% of the total mass), the velocity spectrum of the total mass flow is still dominated by the contribution from the normal fluid and obeys the classical $k^{-5/3}$ law.

The scaling laws which we found apply to a limited region of the spectrum at wave numbers k of the order of $k_\delta \approx 1/\delta$ and $k > k_\delta$. It is hoped that further work will generate more intense vortex tangles in order to determine whether the scaling $E_{\rm s}(k) \sim k^{-1}$ extends into the large-scale flow region $k \ll k_\delta$, or whether in that region the superfluid velocity field is so strongly coupled to the normal-fluid velocity field that $E_{\rm n}(k) \sim E_{\rm s}(k)$. This is certainly the most important issue in the current study of helium-II turbulence. In the meantime we have explored some consequences of the assumption that the two fluids are similar at large scales in the interpretation of recent experiments. On the experimental side, it is apparent that progress would arise if one carried out simultaneous measurement of *two* quantities which involve $\boldsymbol{V}_{\rm s}$ and $\boldsymbol{V}_{\rm n}$; for example, the set-up of Maurer and Tabeling [4] should include the measurement of superfluid vortex line density, and the set-up of Stalp *et al.* [3] should include the measurement of pressure spectra.

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