Resolving cyclic events by used of dynamical systems tools

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Highly-resolved direct and large eddy simulations provide an enormous amount of spatial and temporal information, spanning several orders of magnitude in length and time scales, on intricate turbulence processes, often in very complex strain fields and geometries. Once the data have been generated and stored, often amounting to several Terabytes, the challenge is to find ways and means of visualising and analysing the data so as to derive insight into the flows and the interaction between turbulence and other relevant features – e.g. the geometry, boundary conditions, mean strain and thermal forcing.

There are a number of standard tools used routinely to analyse turbulence and turbulent flows. The simplest are visualisations and images of primitive fields, such as velocity, pressure and pressure fluctuations, and derived quantities, such as vorticity, enstrophy and strain. At a somewhat more elaborate level, there are tools to bring out vertical features, and these include the so-called "Q" and "λ_2" criteria, the latter being the second eigenvector of the former. Two examples are shown in Fig. 1 (see Leschziner et al): flows around a single cylinder and two cylinders in tandem, both visualised by means of contour surfaces of $Q = 0.5(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$.

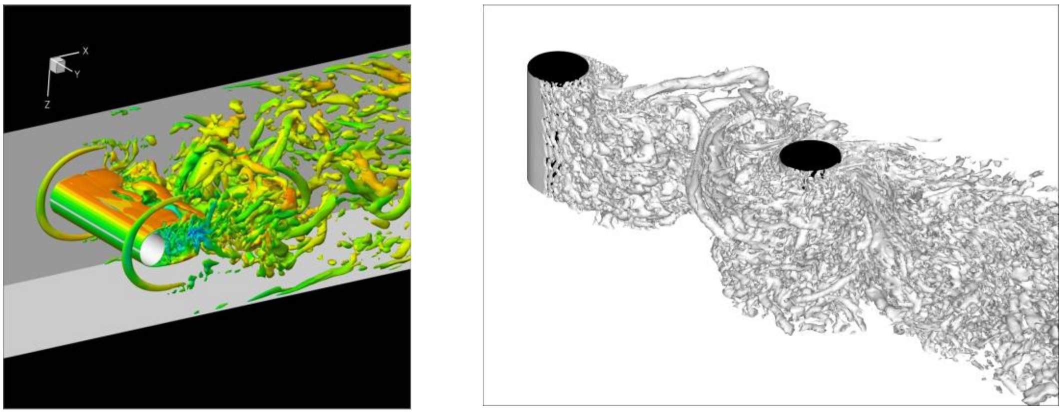


Fig. 1: Flow around a single finite-length cylinder and two cylinders in tandem, visualised with the "Q" criterion.

Yet more elaborate tools, especially targeting temporal features and the identification of exchange of energy among scales, include Fourier analyses of time series, energy spectra and wavelet analysis. At the most complex end are tools that are the subject of this summary. These target the resolution of cyclic (recurrent, but not necessarily periodic) events that are very difficult to resolve by means of time-series analysis.

The approach summarised herein has two key components: *Proper Orthogonal Decomposition* (POD) and *Recurrence Plots*, the latter well-known in analysing "chaotic systems" outside fluid mechanics. POD is based on Lumley's proposal that the velocity field can be decomposed as:

$$u_i(\vec{x},t) = \sum_{n=1}^{N} a_n(t)\phi_i^n(\vec{x})$$

where N is the number of modes used for the representation $a_n(t)$ are temporal modes and $\phi_i^n(\vec{x})$ represent basis functions that need to be selected to comply with a specific criterion upon which the representation of the flow is desired. A key feature of this decomposition is that the modes are organised according to decreasing kinetic-energy content, mode n=1 having the highest energy. Focusing on a small number of low-order modes then allows the most energetic motions of the flow to be examined, without the obscuring influence of higher-order modes. An example is shown in Fig. 2. This flow, examined by Avdis and Leschziner (2008), relates to the control of separation by the use of synthetic jets – i.e. jets that consist of periodic (say, sinusoidal) ejection and suction of fluid through an orifice. This type of jet involves no net mass flow, but there is a net ejection of momentum and vorticity into the flow, resulting in a reduction in the extent of separation.

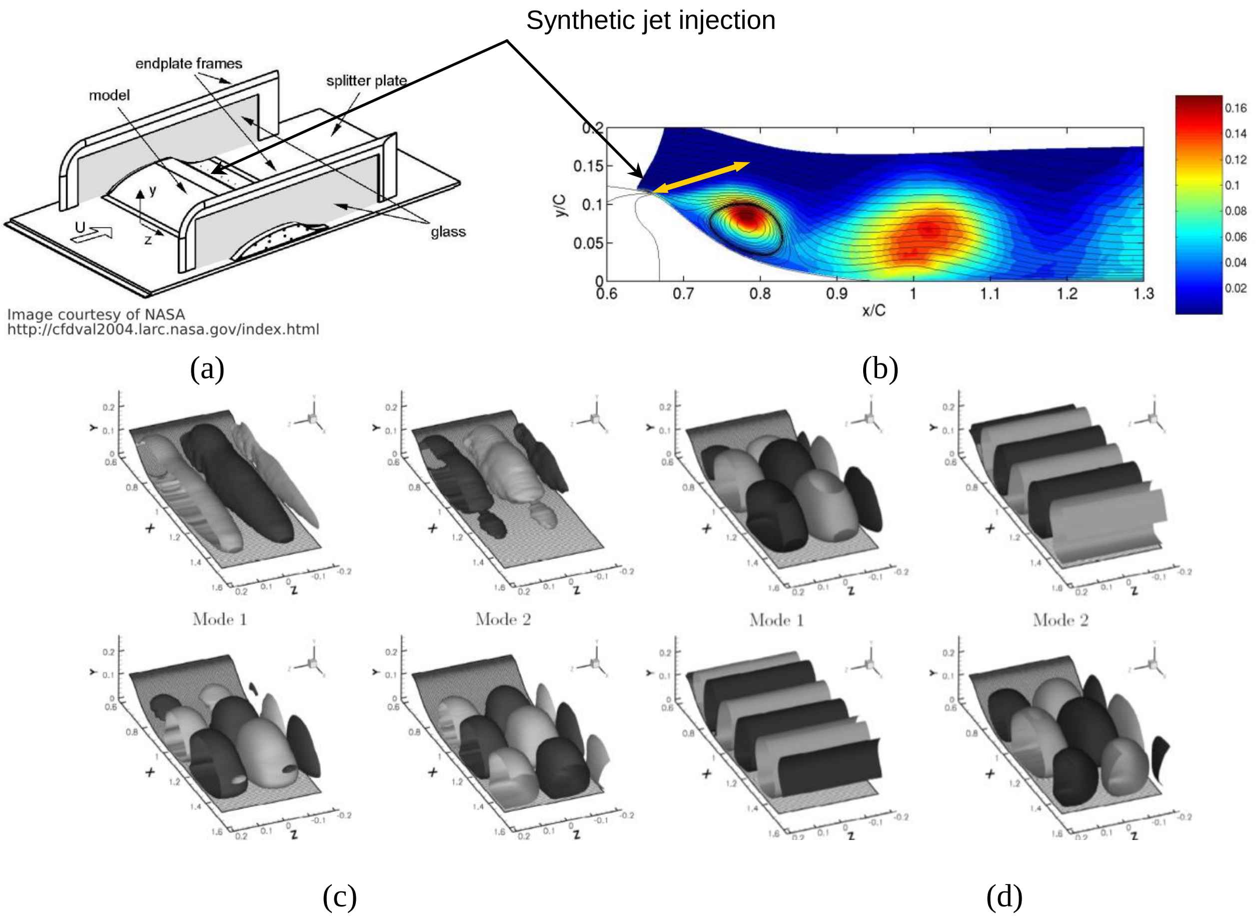


Fig. 2: POD analysis of synthetic-jet injection into a separated flow over a hump in a channel (a) Flow configuration, (b) phase-average visualisation showing the injection location (c)-(d)first 4 modes $\phi_i^n(\vec{x})$ without and with injection respectively.

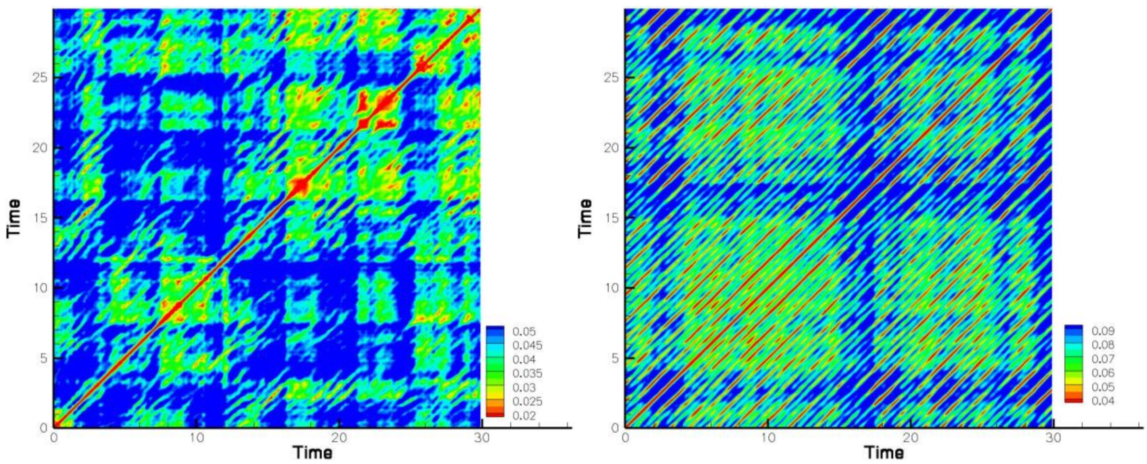
Here, the separation occurs from a hump placed in a duct. The synthetic jet is injected along a slot orifice placed close to the separation line along the hump (in spanwise direction). The periodic injection results in strong flapping of the separated shear layer and the formation of large-scale structures in the recirculation zone downstream of the hump, as visualized by one phase-averaged streamline plot given in Fig. 2(b). Figs. 2(c)-(d) show the first 4 modes resulting from a POD analysis of the streamwise-velocity field. Modes 2 and 3 (Fig. 2d), in particular, reveal the agitation of the flow by the periodic jet, but the most energetic mode 1 reflects the predominance of large three-dimensional vortices which are 'inherited' from the jet-free baseline flow (Fig. 2c) and are 'modulated' by the periodic injection.

While the above POD analysis is well-established, this is but a precursor step to the main, novel part of the analysis summarised below, namely the use of *Recurrence Plots*. A Recurrence Plot (RP) a two-dimensional map with both axes being time. If the state of any dynamical system being examined is characterized at time t_i by the *n*-dimensional vector $\vec{x}_i$ and at time t_j by $\vec{x}_j$, a 'distance' norm between the two states at the two different times may be computed. This can be done for any time value t_i, associated with one axis of the RP, and any other time value t_j, associated with the other axis. When this distance is below a given threshold ε, the system (the flow) may be said to be close to the state on a previous orbit (recurrence). Mathematically, this can be expressed as:

$$R_{ij} = \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|)$$

where Θ is the Heaviside function. A value of 1 is then attributed to events at a distance less than ε, and a value 0 identifies events at a distance greater than ε. The value 1 is then entered as a black mark on the RP, while locations corresponding to the value 0 remain white. For non-stochastic processes, the RP features characteristic patterns to which specific meanings can be attributed, as explained below. The proposal made herein is that the phase-space identifying the dynamical system should be described by the temporal POD coefficients $a_n(t)$, so that the vector $\vec{x}_i$ is defined as $\vec{x}_i = [a_1(t_i), ..., a_N(t_i)]^T$.

Several applications of the method to "toy" systems and real turbulent flows are in Lardeau et al (2008). Fig. 3(b) shows the RP that arises for the flow in Fig. 2. This is compared to the RP that arises for the same case, but without injection (Fig. 3a).



(a) (b)

Fig. 3: Recurrence Plots corresponding to Fig. 2, without (a) and with (B) synthetic-jet injection

The plot for the baseline flow shows little evidence of periodicity – i.e. there are very few lines that are parallel to "Line of Identity" – the diagonal in the square plot. On the other hand, there is fairly clear evidence of cyclicity with a time scale 10 time units. In other words, the RP suggests that the system undergoes recurrent (cyclic) changes in state - perhaps reflecting a spanwise meandering, in time, of streamwise oriented structure seen in the POD representation for this case. The Recurrence plot for the actuated case (with jet) is richer and even more interesting. First, it provides clear evidence, of the periodic injection process, by way of the streaks parallel to the Line of Identity. Second, it appears that the cyclic behaviour identified in baseline flow remains, essentially, unaffected by the injection. Hence, while the injection destabilizes elongated structures of the baseline case (identified in mode 1 and 2, baseline case), the convective behaviour of the lumpy structures of higher modes remain unaffected, and the system continues to undergo major changes in state at a time scale more than one order of magnitude larger than that of the injection process. This case thus illustrates well that RPs can provide useful information which is at best very difficult to derive from conventional analytical tools.

References

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