



JVSM: A Metric for Assessing Small Signal Voltage Stiffness

Ambuj Gupta¹, Balarko Chaudhuri¹, Mark O'Malley¹
¹ Control & Power Group, Imperial College London

I. GFM Definition & Compliance in Time-Domain

- GFM IBRS are expected to exhibit **“voltage source behind impedance (VSBI)”** characteristics in the (sub)-transient time frame.
- ISOs:** Assess active/reactive power response to voltage magnitude (MJ) & phase (PJ) jump.
- Test Setup: GFM connected to the Thevenin grid equivalent with SCR=3, X/R ratio = 6.

Table 1: GFM Voltage Source Requirements

ISO	Peak Value	Rise Time	Decay Time
AEMO/MISO	$P \geq 0.2 \text{ p.u.}/10^\circ \text{ PJ}$	$\leq 15 \text{ ms}$	-
ERCOT	$P \geq 0.2 \text{ p.u.}/10^\circ \text{ PJ}$	$\leq 16.6 \text{ ms}$	$\Delta P \geq 0 \forall t \leq 50 \text{ ms}$
	$Q \geq 0.03 \text{ p.u.}/3\% \text{ MJ}$		$\Delta Q \geq 0 \forall t \leq 100 \text{ ms}$
ENTSO-E	$I_p = 50\text{--}70\% I_{\text{RMS}}$	$\leq 10 \text{ ms}$	$t_{\text{decay}} = 0.016 \text{ s} \pm 33\%$

Assessing VSBI characteristics in the time-domain has some major drawbacks:

- Dependent on testing setup (e.g. SCR, X/R).
- Dependent on measurement device's characteristics, as requirements are at few-ms.
- Can not quantify or compare VSBI strength.

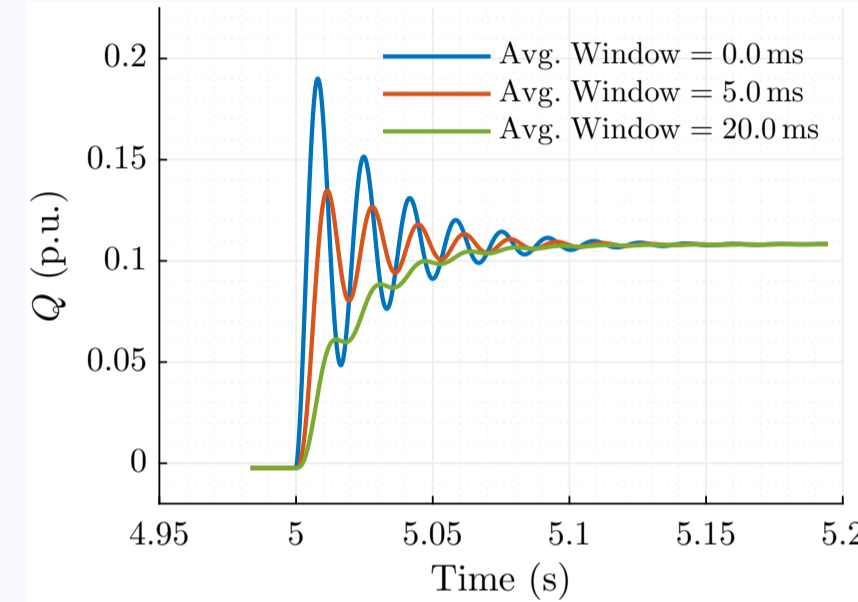


Figure 1: Effect of measurement filter averaging time window on reactive power response to a voltage jump at POI of GFM.

II. GFM Compliance in Frequency-Domain

- $Y_{qd}/Y_{dq} \rightarrow$ Reactive/Active power response to a voltage magnitude/phase perturbation.

$$-\begin{bmatrix} I_d(s) \\ I_q(s) \end{bmatrix} = \begin{bmatrix} Y_{dd}(s) & Y_{dq}(s) \\ Y_{qd}(s) & Y_{qq}(s) \end{bmatrix} \begin{bmatrix} V_d(s) \\ V_q(s) \end{bmatrix}$$

- Jacobian terms $P-\theta/Q-V$ analogous to Y_{dq}/Y_{qd}

$$\begin{bmatrix} P(s) \\ Q(s) \end{bmatrix} = \begin{bmatrix} \frac{P_0}{V_1} - \frac{3}{2} V_1 Y_{dd}(s) & -Q_0 - \frac{3}{2} V_1^2 Y_{dq}(s) \\ \frac{Q_0}{V_1} + \frac{3}{2} V_1 Y_{qd}(s) & P_0 + \frac{3}{2} V_1^2 Y_{qq}(s) \end{bmatrix} \begin{bmatrix} V_m(s) \\ \theta(s) \end{bmatrix}$$

GFM VSBI expectation [ESIG]:

- Nearly constant and $\geq 0.5 \text{ p.u./p.u.}$ magnitude of $P-\theta/Q-V$ spectra between 4 - 40 Hz.
- Phase of $P-\theta/Q-V$ spectra between 4 - 40 Hz should within $\pm 60^\circ$ of $\pm 180^\circ$.

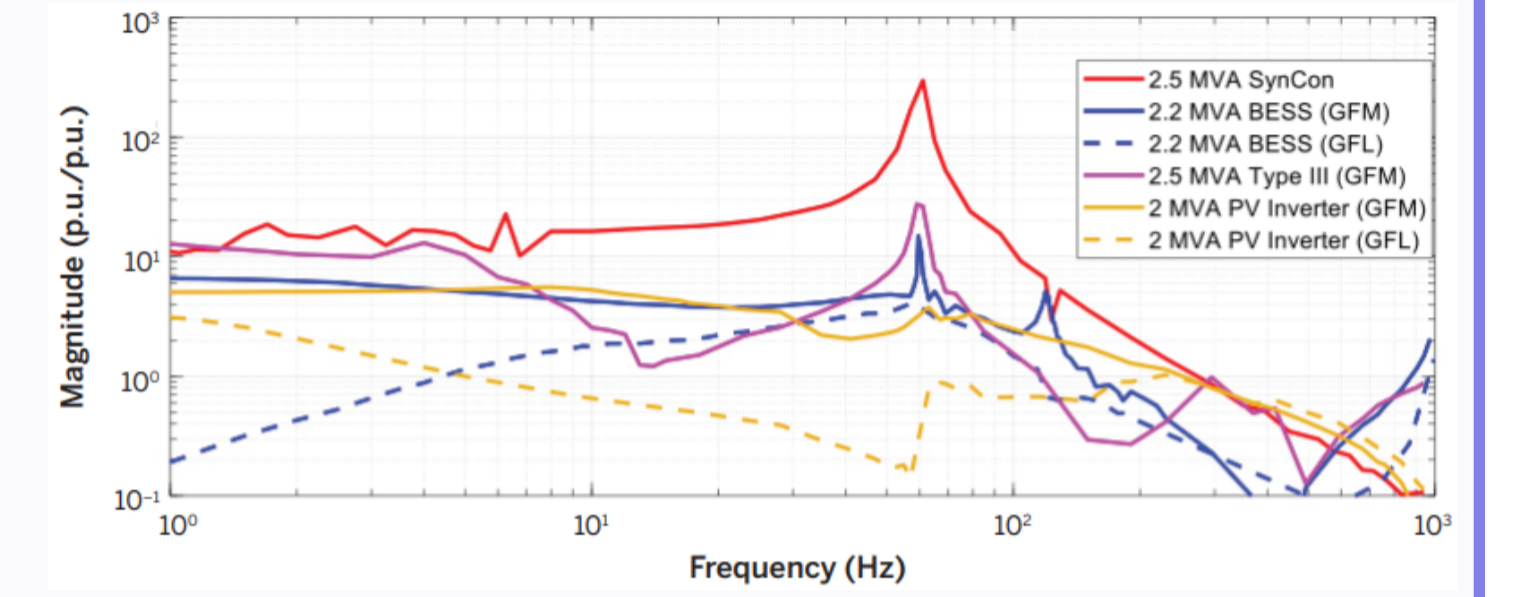


Figure 2: Magnitude of Q/V Frequency Spectra for different resources. Source - ESIG (Testing the Performance of Grid-Forming Resources).

Drawbacks with the current approach:

- No quantification of “nearly constant”.
- A min. $P-\theta/Q-V$ mag. at low frequencies may exclude GFMs with good VSBI strength.
- Can not quantify or compare VSBI strength.

III. Jacobian Voltage Stiffness Metric (JVSM)

- Voltage Stiffness \rightarrow Speed and Magnitude of P (or Q) response to θ (or V) perturbation.
- $P-\theta/Q-V$ spectra for IDVS & SM: $\omega_{res} = \omega_1 = 50/60 \text{ Hz}$. Thus, the same speed of response.
- SCR as a metric \rightarrow Measure of magnitude.
- A GFM can have a slower response.
- How to quantify both magnitude & speed of response in one metric?**

$$JVSM = \sqrt{\frac{JVSM_\theta^2 + JVSM_V^2}{2}}$$

- JVSM - Voltage Phase ($JVSM_\theta$):**

$$JVSM_\theta = \frac{\int_{f_{min}}^{f_{max}} \left| \frac{P(s)}{\theta(s)} \right| \log_{10}(f) df}{\int_{f_{min}}^{f_{max}} \log_{10}(f) df}$$

- JVSM - Voltage Magnitude ($JVSM_V$):**

$$JVSM_V = \frac{\int_{f_{min}}^{f_{max}} \left| \frac{Q(s)}{V(s)} \right| \log_{10}(f) df}{\int_{f_{min}}^{f_{max}} \log_{10}(f) df}$$

- $[f_{min}, f_{max}] = [5, 100] \text{ Hz}$.
- $f_{min} \rightarrow$ end of transient time & ignores slower dynamics like droop.
- $f_{max} \rightarrow$ ignores faster dynamics like network dynamics / current control loop.
- $\log_{10}(f) \rightarrow$ Emphasizes response at higher frequencies, thus captures speed of response.
- $\left| \frac{Q(s)}{V(s)} \right| / \left| \frac{P(s)}{\theta(s)} \right| \rightarrow$ captures magnitude of response, in p.u. rather than Bode Gain (dB).

IV. Case Studies

JVSM & SCR for Ideal Voltage Source (IDVS):

- IDVS \rightarrow varying only response magnitude.

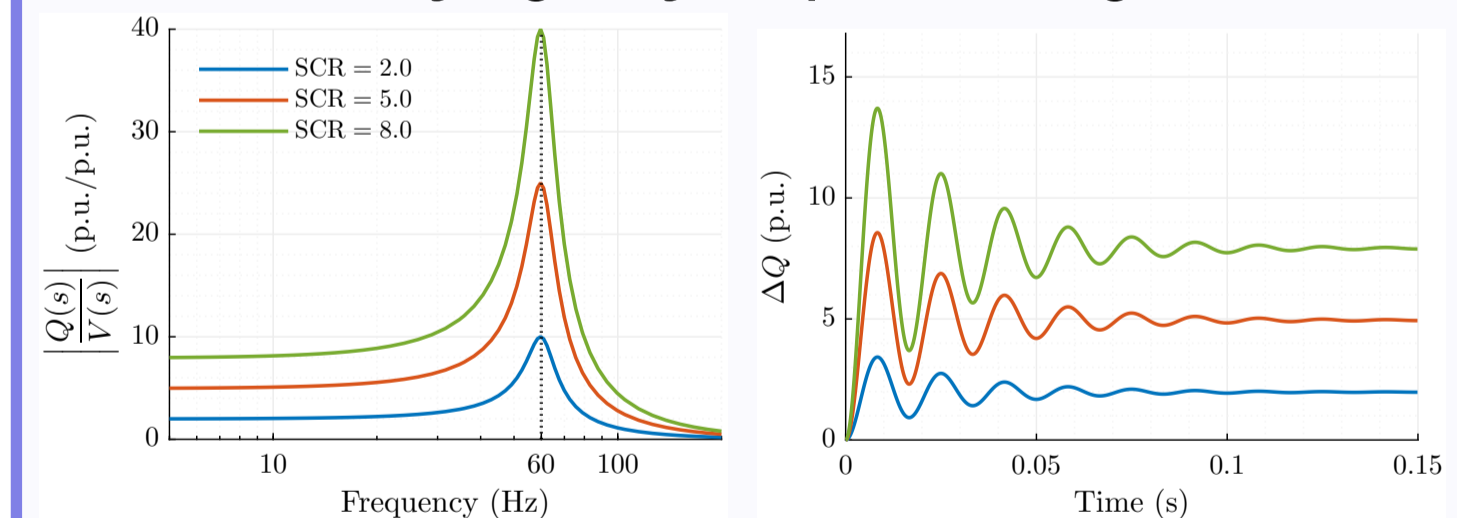


Figure 3: Magnitude of Q/V Spectra of an IDVS; X/R = 10. Figure 4: Time-domain response of an IDVS; X/R = 10.

- JVSM captures the variation in magnitude:

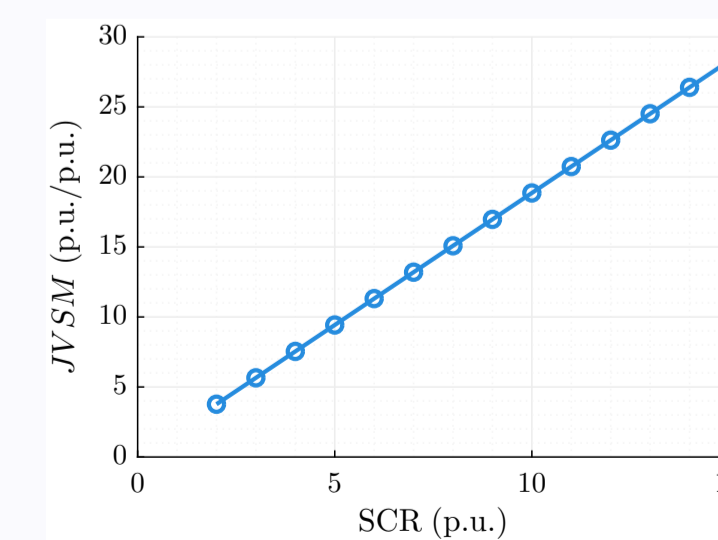


Figure 5: JVSM vs SCR for an IDVS with X/R = 10.

GFM with varying voltage control BW:

- GFM \rightarrow varying response magnitude & speed.

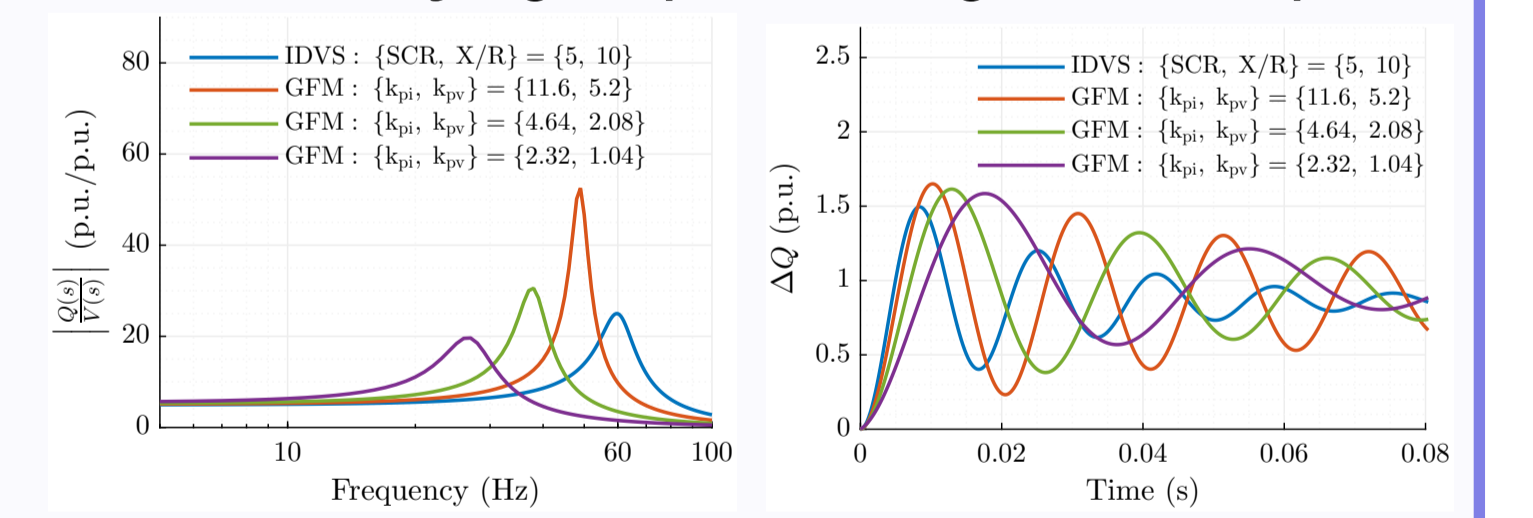


Figure 6: Magnitude of Q/V Spectra of an IDVS; X/R = 10. Figure 7: Time-domain response of an IDVS; X/R = 10.

- JVSM captures both magnitude & speed:**

Table 2: Effect of changing voltage control loop bandwidth on JVSM of GFM

Device	Parameters (p.u.)	JVSM (p.u./p.u.)
IDVS	$\{SCR, X/R\} = \{5, 10\}$	9.378
GFM	$\{k_{iv}, k_{pv}\} = \{11.60, 5.20\}$	9.799
GFM	$\{k_{iv}, k_{pv}\} = \{4.64, 2.08\}$	6.516
GFM	$\{k_{iv}, k_{pv}\} = \{2.32, 1.04\}$	4.057

- JVSM is an indicator of small-signal grid strength & GFM compliance ($JVSM_{min} = 5.6$; equivalent to IDVS - SCR = 3, X/R = 10).**