Spatial Rigid/Flexible Dynamic Model of Biopsy and Brachytherapy Needles Under a General Force Field

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Abstract — Computer-aided simulation of percutaneous needle insertion, as a training tool for junior surgeons, is expected to significantly increase targeting accuracy during minimally invasive operations. An essential requirement for the development of these simulation solutions, is the complete and accurate characterisation of the underlying dynamics of needle insertion. In this regard, this work presents a novel mathematical model, based on the theory of flexible multibody dynamics, that captures the spatial dynamics of needles used for brachytherapy and local anaesthetic transperineal prostate (LATP) biopsy, under a general three-dimensional force field. Due to its accuracy and computational efficiency, the proposed model is expected to constitute a valuable tool for both real-time visual/haptic simulation and control of percutaneous needle insertion.

Index Terms — Medical Robotics, Needle Steering, Flexible Multibody Dynamics, Finite Element Method, Rayleigh-Ritz Method

I. INTRODUCTION

Minimally invasive surgery (MIS) and localised therapy have become integral parts of modern medical practices as they are characterised by decreased recovery time, reduced patient discomfort and lower risk of infection when compared to open surgery [1]. Percutaneous needle insertion constitutes one of the main practices for performing MIS, including a plethora of diagnostic and therapeutic applications, such as tissue biopsy, brachytherapy, neurosurgery, and deep brain stimulation [2].

The success rate of these operations heavily relies on the accuracy of needle placement, while imprecise targeting can often lead to severe complications, such as false negatives in biopsy or ablation of healthy tissue [3]. At the same time, accurate percutaneous needle placement is a highly challenging task. The limited visual feedback during the operation, combined with the tissue anisotropy and the variability in anatomical structures among different patients, significantly complicates navigation through the tissue and thus decreases the operation’s overall accuracy [3].

In recent years, the biomechanics and robotics research communities have developed different solutions that have the potential to significantly increase the accuracy of needle placement procedures. These mainly focus on the development of robotic systems that allow autonomous or semi-autonomous navigation and accurate needle placement to targeted locations inside soft tissue. Another approach, is the development of high-fidelity visual/haptic medical simulators for training junior doctors in a variety of surgical scenarios or for the pre-operative planning of complex procedures by experienced surgeons [2].

This work focuses on the formulation of a mathematical model that captures the spatial dynamics of brachytherapy and local anaesthetic transperineal prostate (LATP) biopsy needles under a general 3D force field. The proposed model considers: a) the needle’s base as a rigid body that follows any arbitrary spatial trajectory, which corresponds to the surgeon’s hand movements during percutaneous needle insertion, and b) the needle’s shaft as a flexible body that deforms elastically under a general state/time dependent field of forces, which corresponds to the interactions between the needle and the surrounding tissue (distributed load $p(x,t,\dot{q},\ddot{q})$ and point load $F_B(t,\dot{q},\ddot{q})$). By employing this multibody rigid/flexible modelling approach, this work captures both the dynamics of the needle’s deflection and the reaction forces that act on the surgeon’s hand, while accounting for the inertial forces caused by the overall motion of the system. The accuracy and the computational efficiency of the proposed model, allow its application to both real-time visual/haptic simulation and control of percutaneous needle insertion.

II. METHODS

The development of the spatial dynamic model of the needle is based on the theory of flexible multibody dynamics.

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Fig. 1. Model of rigid/flexible brachytherapy/LATP needle.
Different approaches have been proposed in the literature for the analysis of such systems, such as the component mode synthesis [4], the rigid finite element (RFE) method, the absolute nodal coordinate (ANC) formulation and the floating frame of reference (FFR) formulation [5]. This work is based on the FFR formulation as it allows the description of arbitrarily large rigid-body displacements and rotations and a straightforward addition of complex force functions and constraint equations [5].

The mathematical description of the needle’s deformation field is based on the assumed modes method. For this, two approaches are examined; namely the Rayleigh-Ritz and the finite element method. The first approach estimates the needle’s deformation field as a linear combination of a finite series of shape functions that satisfy both the smoothness and the geometric requirements of the structure’s boundary conditions (admissible functions). In the proposed model, the admissible shape functions are the orthonormal mode shapes of the cantilever beam, as this ensures that both requirements are satisfied. In the second approach, the beam is discretised by a finite number of beam elements, each of which is deformed based on local admissible shape functions, and characterised by local mass, damping and stiffness properties. In both of the aforementioned approaches, the position of an arbitrary point \( P \) on the flexible needle, with respect to the FFR \( f \) (Fig. 1), is approximated as

\[
\ell_{AP/f} = \ell_{AP_0/f} + \Phi(x) \, q_f
\]

where, \( \ell_{AP_0/f} \) is the position of the point before deformation, \( \Phi(x) \) is the matrix of shape functions and \( q_f \) is the vector of generalised elastic coordinates. Note that both \( \Phi(x) \) and \( q_f \) differ between the two approaches.

Having defined the position of an arbitrary point on the flexible body and by applying D’Alembert’s form of the principle of virtual work for the flexible needle of Fig. 1, subject to generalised contraint forces from the rigid body, we obtain the set of differential equations

\[
M_f \ddot{q}_c + C_f \dot{q}_c + K_f q_c = \int_{\nu_f} F_c + Q_f + Q_c
\]

where, \( q = \left( \ell_{DCA/F}^T \, \theta_f^T \, \nu_f^T \right)^T \) is the state vector that describes both the rigid-body motion of the needle and its elastic deformation. The highly nonlinear mass matrix \( M_f \) is both dependent on the system’s state and the selection of the shape functions, the stiffness matrix \( K_f \) is constant, while the damping matrix \( C_f \) is defined based on the classical Rayleigh damping as \( C_f = \mu M_f + \kappa K_f \). The vectors \( \ell_{\nu_f}, Q_f, \) and \( Q_c \) represent the coriolis/centrifugal forces, the generalised external forces and the generalised constraint forces, respectively.

Given that the position and the orientation of the rigid body are known functions, with continuous second time derivatives, equation (2) can be integrated numerically to obtain the time evolution of the elastic coordinates \( q_f \) and, thus, the vibrational behaviour of the needle. Furthermore, formulating the equations of motion of the rigid-body, we can easily evaluate the reaction forces and moments \( F_c^F \) and \( M_c^F \) acting on the needle’s handle (haptic feedback).

### III. RESULTS

To illustrate the performance of the proposed model, a simple test case of step force loading is considered. Let \( F_{B,1}^f = [0 \quad 0 \quad F_B \quad \mathcal{H}(t-1)]^T \), an external force vector applied at the needle’s tip as shown in Fig. 1, where \( F_B \) is the applied force magnitude with an arbitrary value of 0.5 N and \( \mathcal{H}(t) \) is the Heaviside function. Next, the response of the system is considered for two distinct cases. In the first case the surgeon’s hand remains fixed with respect to the inertial frame \( F \), while in the second case its trajectory (position and orientation) is described as \( \ell_{DCA/F}^P = \left[ 0 \ 0 \ a_z \sin(2\pi f_{dz} t) \right]^T \) and \( \theta_f = 0 \), where \( a_z \) is the displacement magnitude with an arbitrary value of 0.2 m and \( f_{dz} \) the displacement frequency chosen as 2.0 Hz.

![Fig. 2. System response under step force loading.](image)

(a) Reaction force in z direction for fixed and moving base.  
(b) Reaction moment in y direction for fixed and moving base.

As shown in Fig. 2(a and 2(b), the proposed model can capture the dynamics of the needle’s vibration while also accounting for the inertial forces caused by the overall rigid body motion. It should be noted that, the selection of the appropriate number of terms in the Rayleigh-Ritz method and the appropriate number of elements in the FEM leads to equal eigenvalues (natural frequencies) for the two systems and thus to identical system responses. Thus, no explicit mention of the employed technique is given. In this regard, the selection of the appropriate approximation method is solely dependent on the application’s requirements, such as the required accuracy and computational efficiency. Finally, even though this example examines only a simple 2D case, the proposed model can be used for any arbitrary spatial trajectory of the needle’s base and under any 3D loading conditions that might be present during percutaneous needle insertion operations.

### REFERENCES


