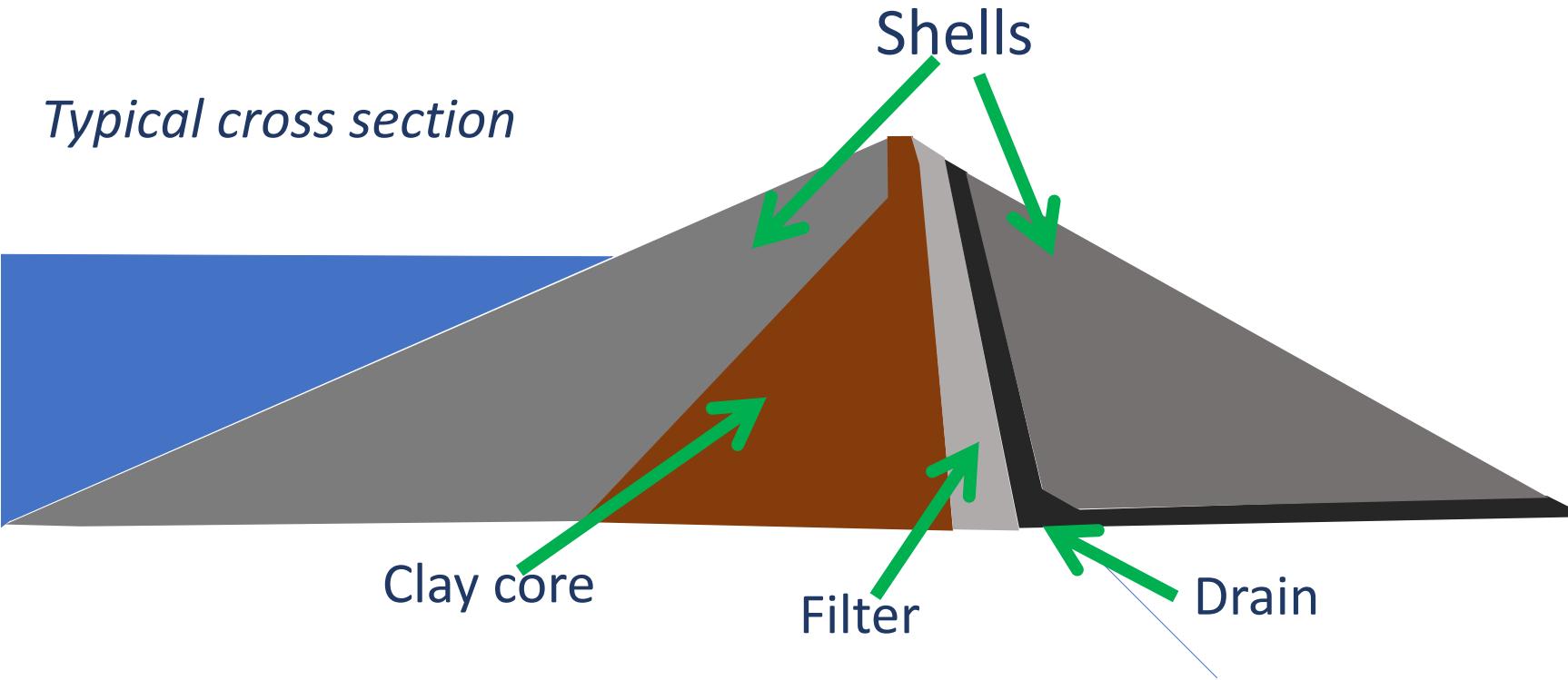


Pore network models in geotechnical engineering

Catherine O'Sullivan

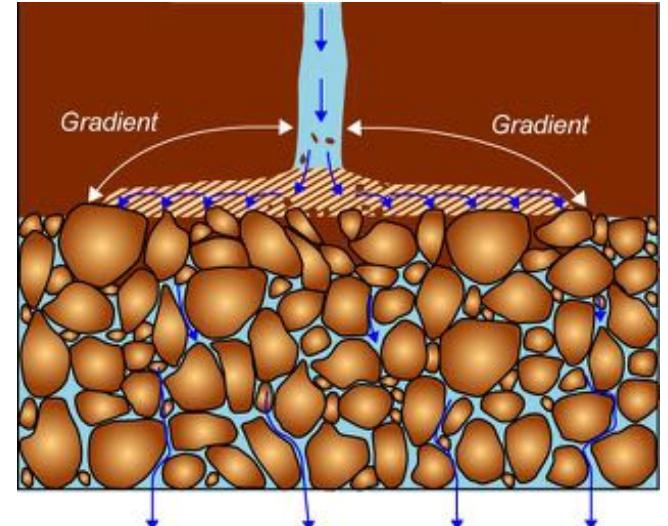
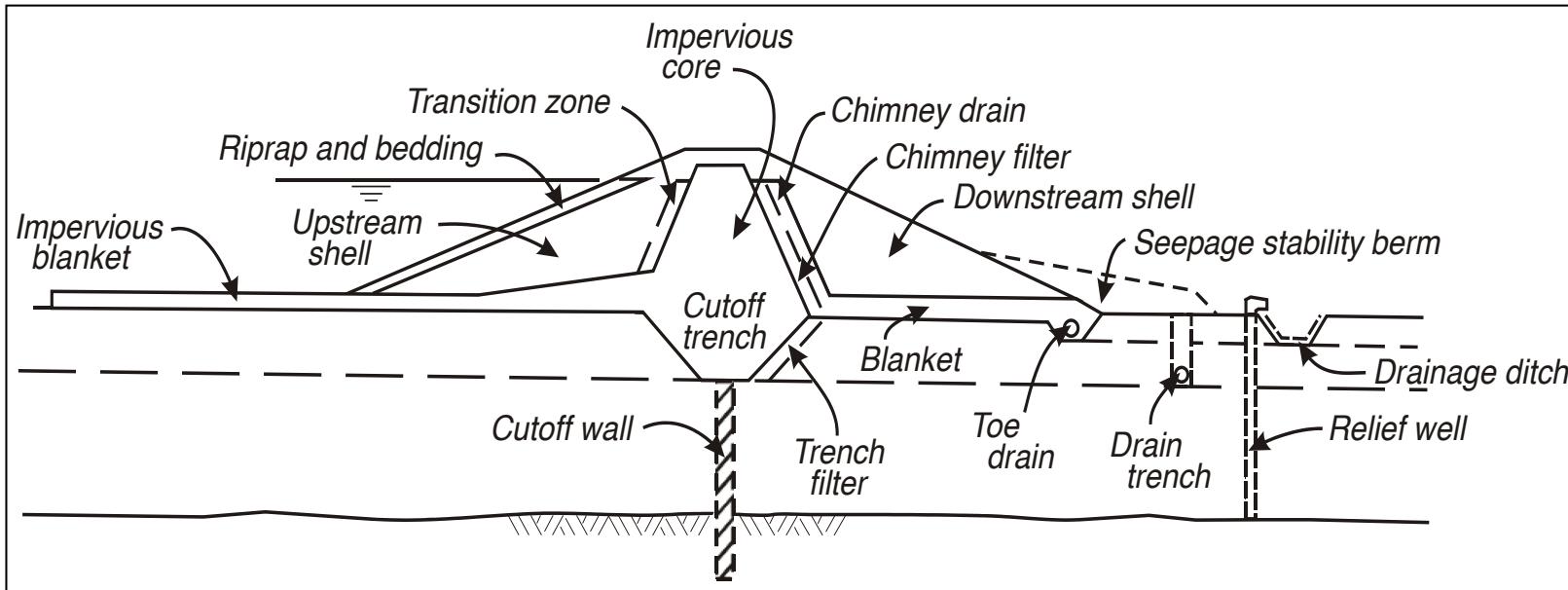
*Lis Bowman, Chris Knight, Tokio Morimoto, Thomas Shire, Nicoletta Sanvitale,
Adnan Sufian, David Taborda, Berend van Wachem, and Budi Zhao*

Application: Embankment dams



- Dams can be over 100 m high
- Water seeps through dam continuously
- Seeping water can preferentially erode fines
- In the UK about 2,500 dams retain reservoirs exceeding 25,000 m³
- In the US there are about 90,580 dams

Application: Embankment dams



(FEMA, 2011)



Paraperios dam - May 26 2010

Core

Sandwich filter

Maintaining dam integrity is crucial



WAC Bennett Dam

- High as a 60-storey building and two kilometres wide
- Holds back 360 kilometres of Williston Lake, the largest reservoir in North America



1996 Sinkhole at WAC Bennett Dam
(BC Hydro as cited by Muir Wood, 2007)

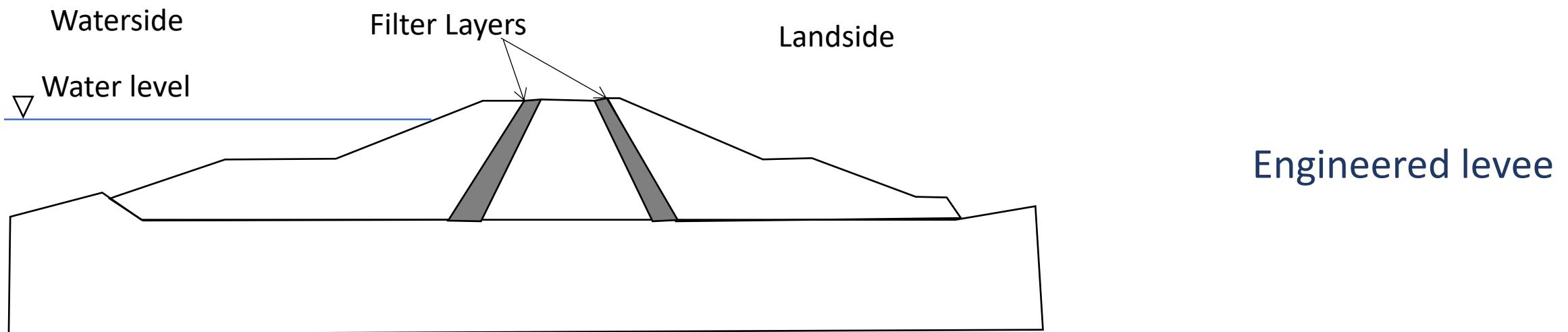
Application: Flood embankments (Levees)

- Levees – transient water levels, but can be very long.
- Concerned about seepage through embankment and foundations.
- 30,000 documented miles of levee in US
- 7,500 km of flood embankments in the UK

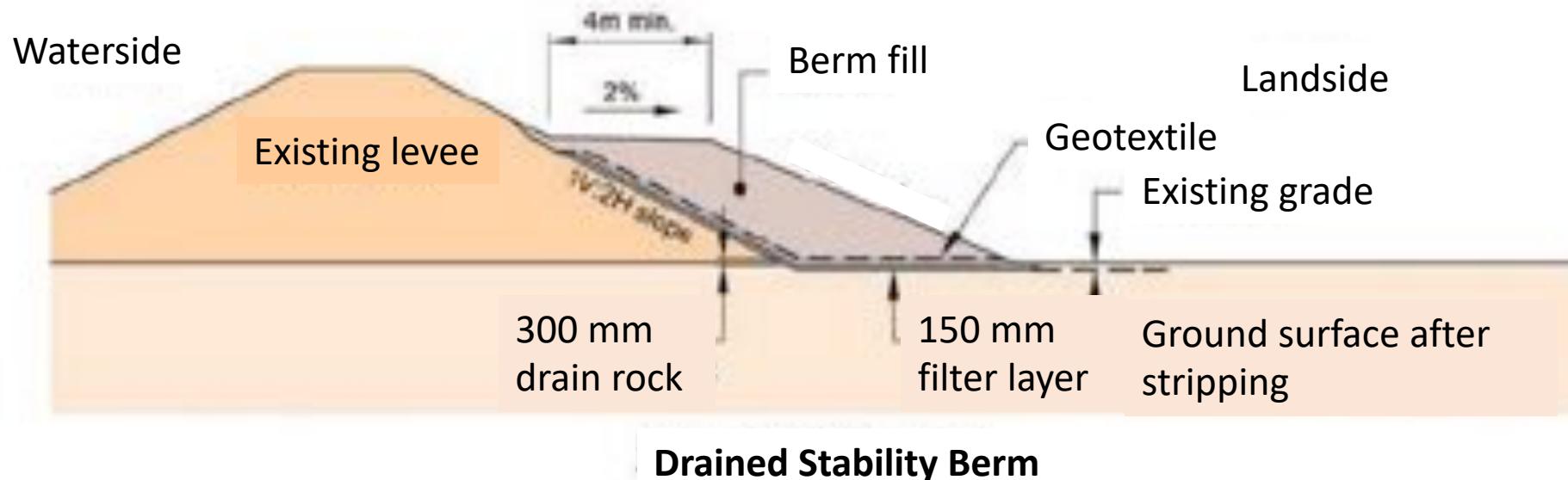


American River Levees California

Application: Flood embankments (Levees)



Engineered levee



Retrofit of
existing levee

(International Levee
Handbook, 2013)

Role of filters

Filters are designed and constructed to achieve specific goals such as preventing internal soil movement and controlling drainage (FEMA, 2011)

Five filter functions govern the capability of providing control for internal erosion.

1. Retention.
2. Self Filtration or stability.
3. No cohesion
4. Drainage.
5. Strength. ICOLD (2015)

Filter layers help to promote filtration, by preventing soil from migrating especially from the impervious core. (International Levee Handbook, 2013)



Filters for Embankment Dams
Best Practices for Design and Construction
October 2011



INTERNAL EROSION OF EXISTING DAMS, LEVEES AND DIKES, AND THEIR FOUNDATIONS

BULLETIN 164

Volume 1: INTERNAL EROSION PROCESSES AND ENGINEERING ASSESSMENT



The International Levee Handbook



Role of filters

Filters are designed and constructed to achieve specific goals such as preventing internal soil movement and controlling drainage (FEMA, 2011)

Five filter functions govern the capability of providing control for internal erosion.

1. Retention.
2. Self Filtration or stability.
3. No cohesion
4. Drainage (*permeability*).
5. Strength. ICOLD (2015)

Filter layers help to promote filtration, by preventing soil from migrating especially from the impervious core. (International Levee Handbook, 2013)



Filters for Embankment Dams
Best Practices for Design and Construction
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INTERNAL EROSION OF EXISTING DAMS, LEVEES AND DIKES, AND THEIR FOUNDATIONS

BULLETIN 164

Volume 1: INTERNAL EROSION PROCESSES AND ENGINEERING ASSESSMENT

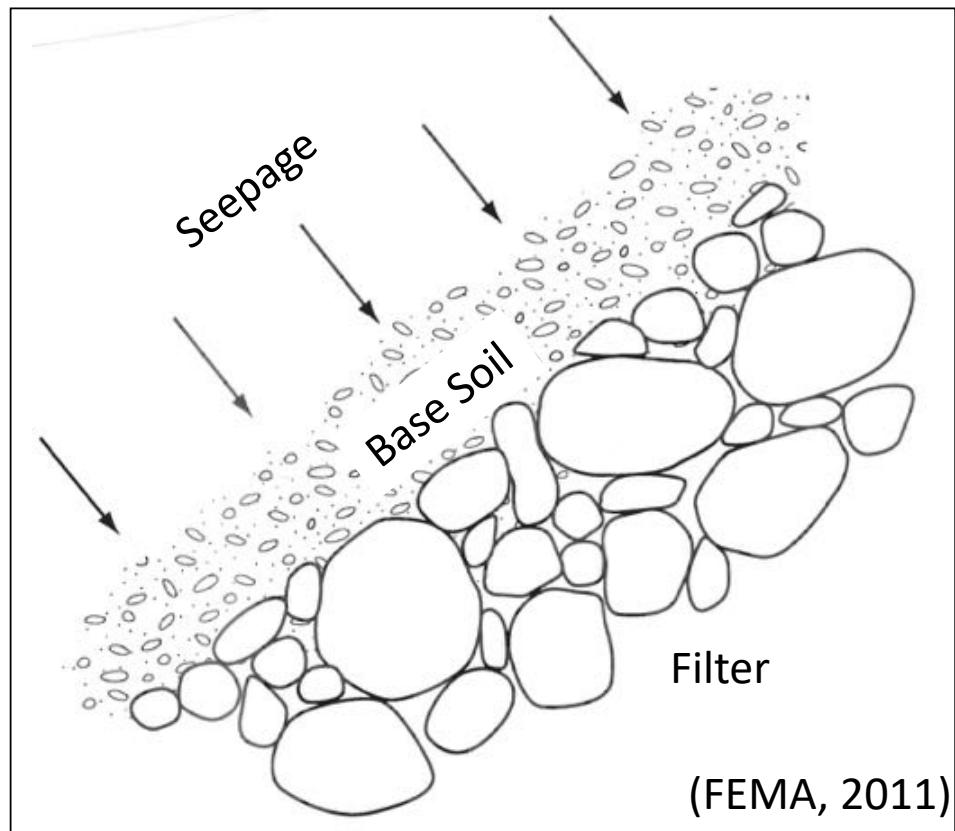


The International Levee Handbook



Retention

The voids in the filter should be sufficiently small to prevent erosion of the base soil (ICOLD,2015)



Geometric criteria:

- Consider particle size distribution
- Terzaghi's filter rule / Sherard & Dunnigan (1989)
 - D_{15F} of filter
 - D_{85B} of base
 - For retention $D_{15F} < 4 D_{85B}$
- Controlling constriction size

(ICOLD,2015)

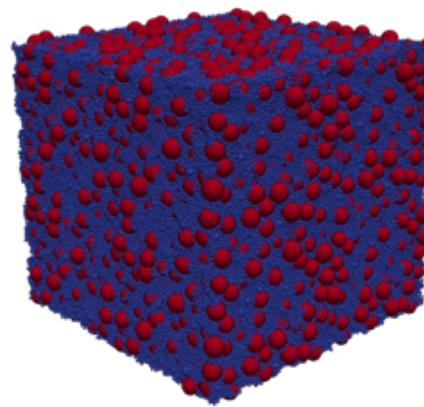
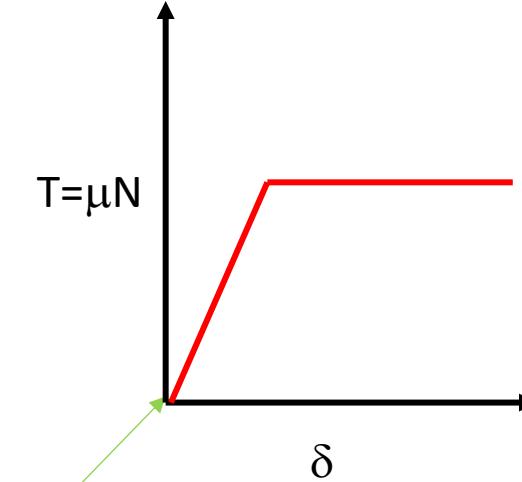
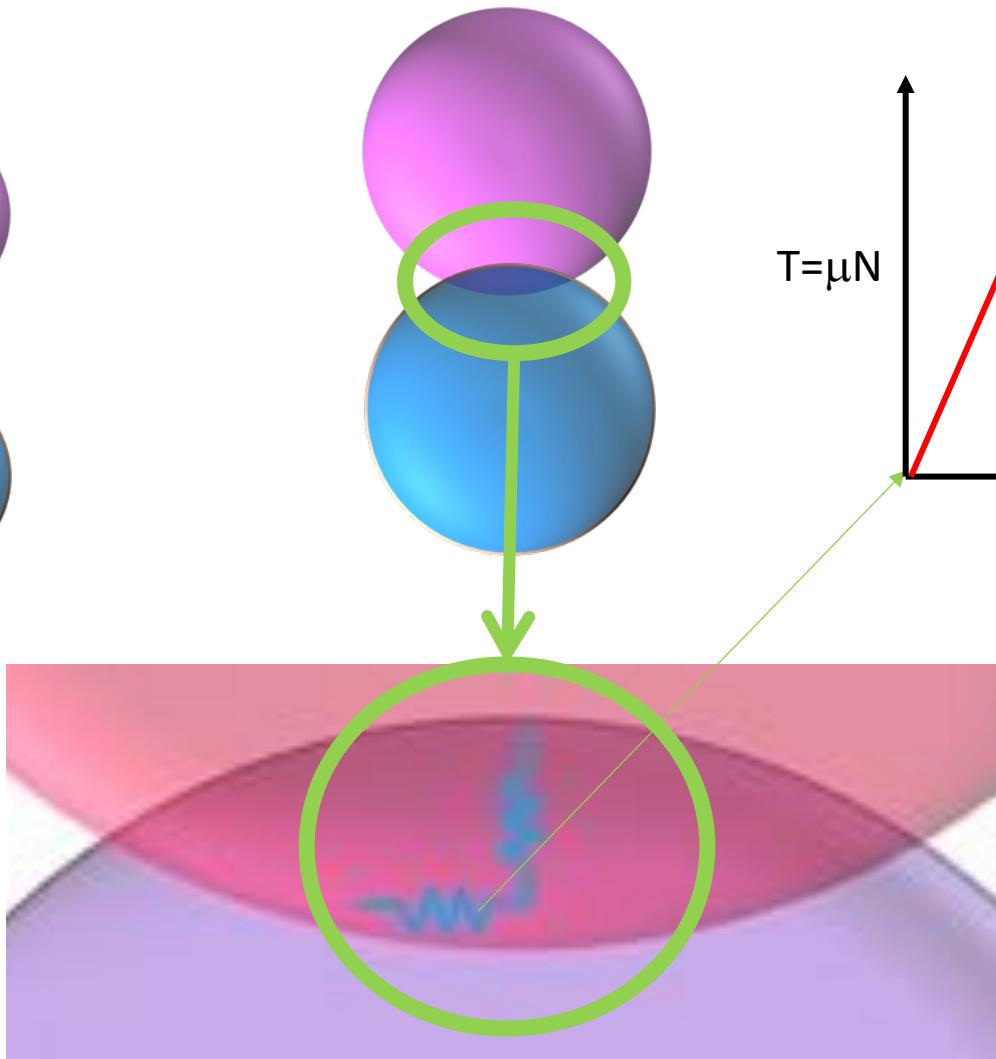
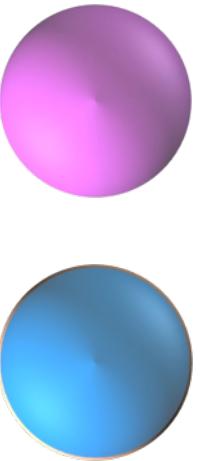
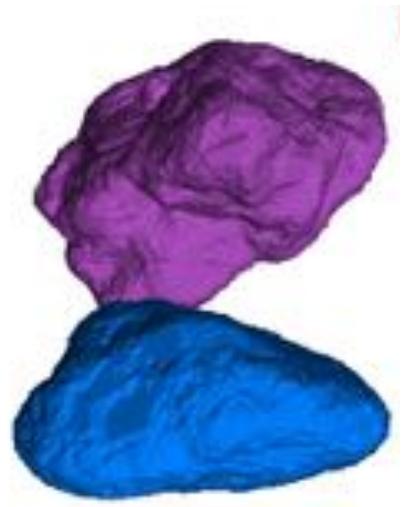
Research questions:

- **Retention:**
Does pore network modelling support use of the ratio D_{15F}/D_{85B} in design?
- **Internal Stability:**
Can we predict the fluid particle interaction forces with pore network modelling?

Research questions:

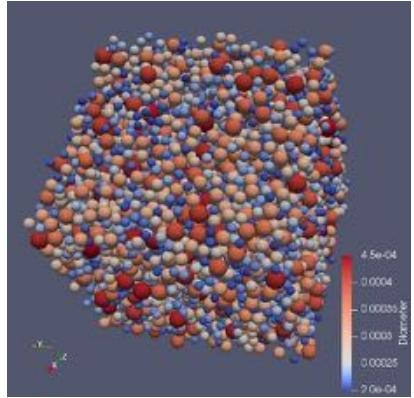
- **Retention:**
Does pore network modelling support use of the ratio D_{15F}/D_{85B} in design?
- **Internal Stability:**
Can we predict the fluid particle interaction forces with pore network modelling?

Discrete Element Method (DEM)

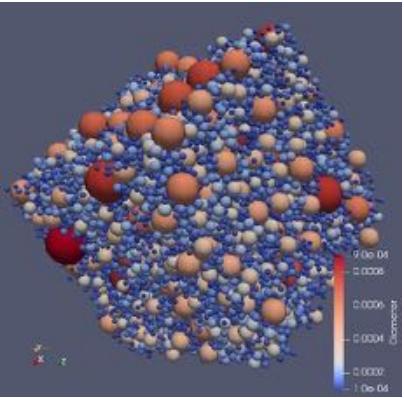


Samples considered

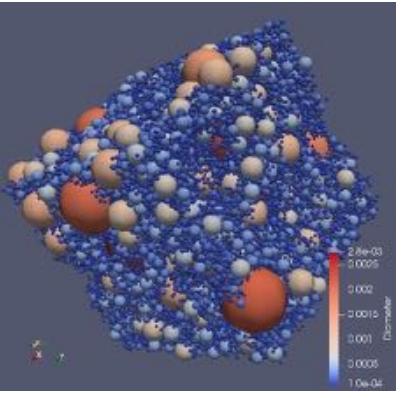
DEM Simulations



Spheres
 $C_u=1.2$

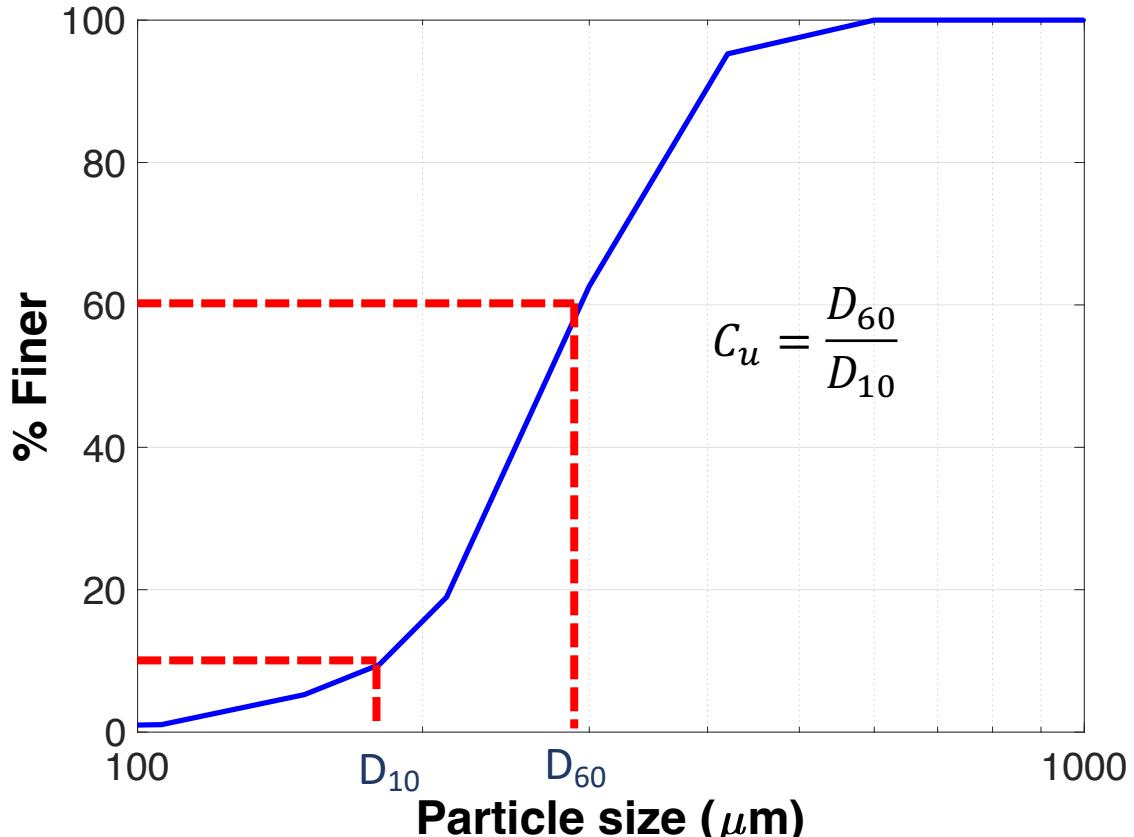


Spheres
 $C_u=3.0$



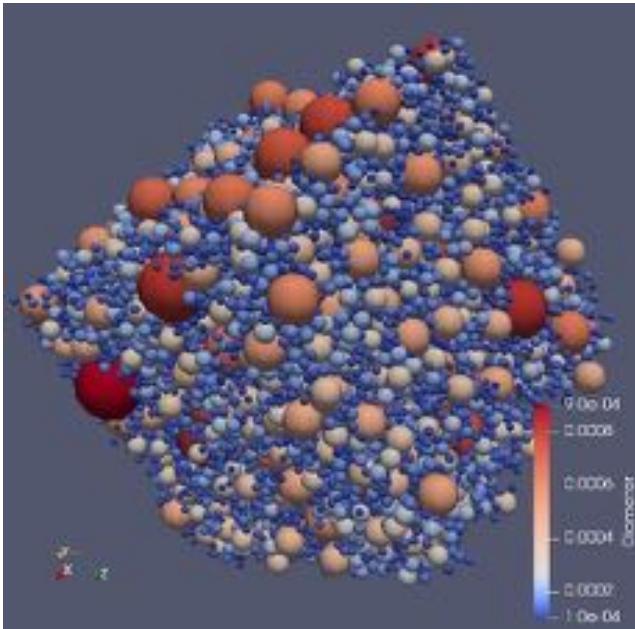
Spheres
 $C_u=6.0$

Do not consider possibility of filters containing fines

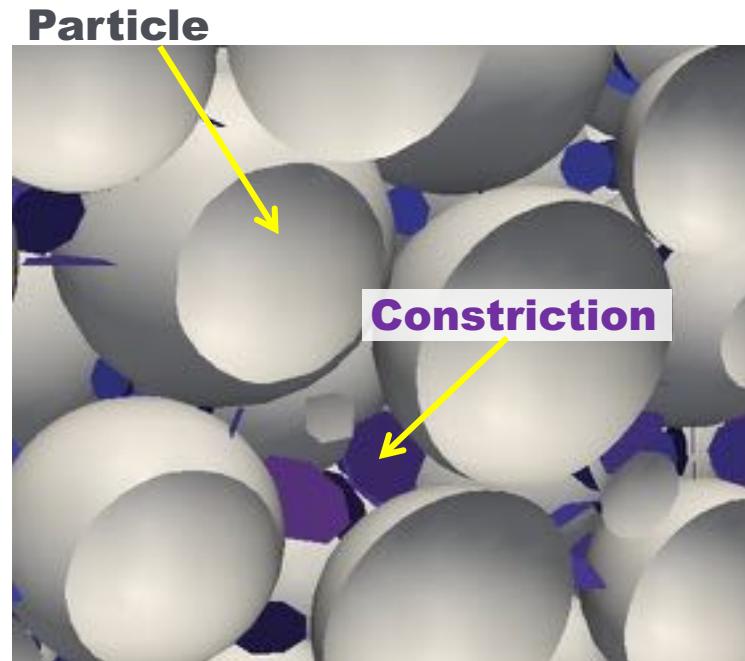


Determining constriction size distribution

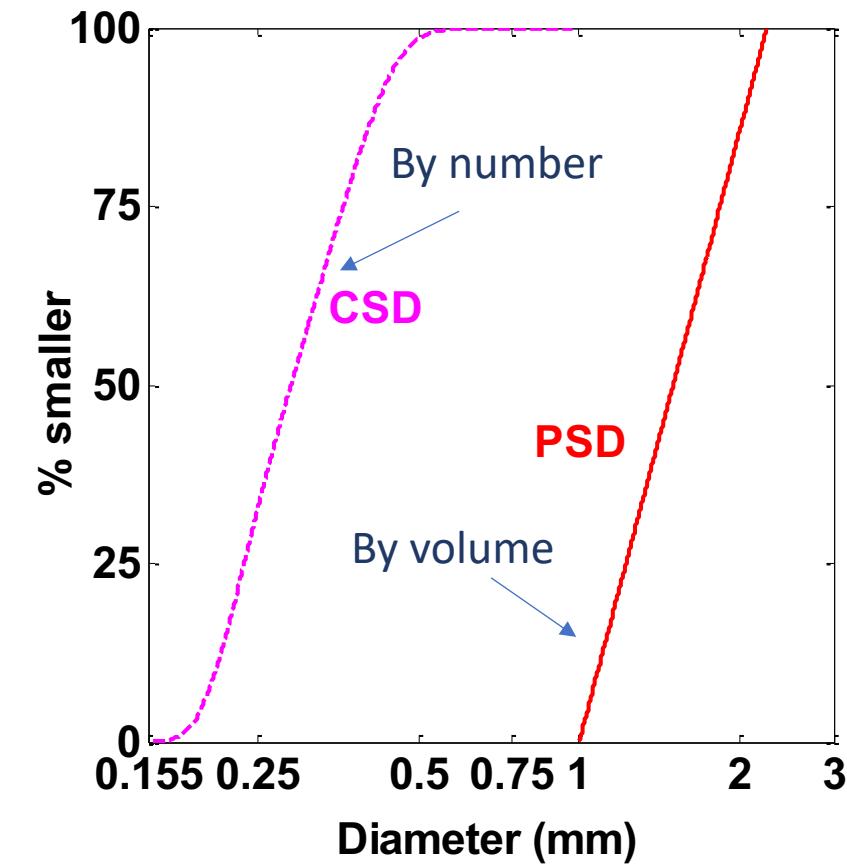
Generate particle scale data



Apply void partitioning algorithm

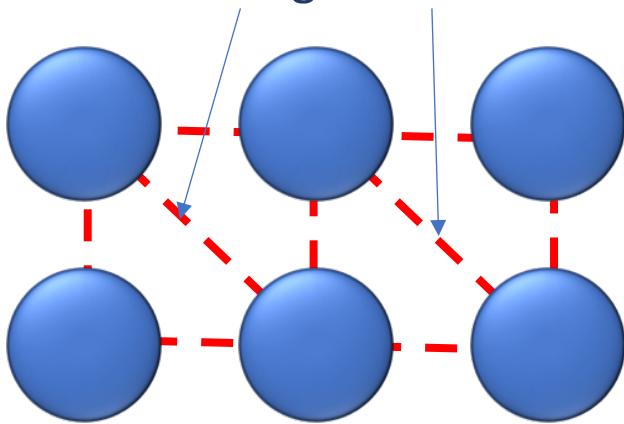


Calculate Constriction Size Distribution



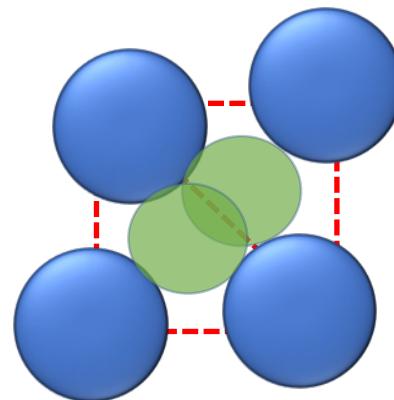
DEM constrictions: Triangulation method

False identification of constrictions due to over-segmentation

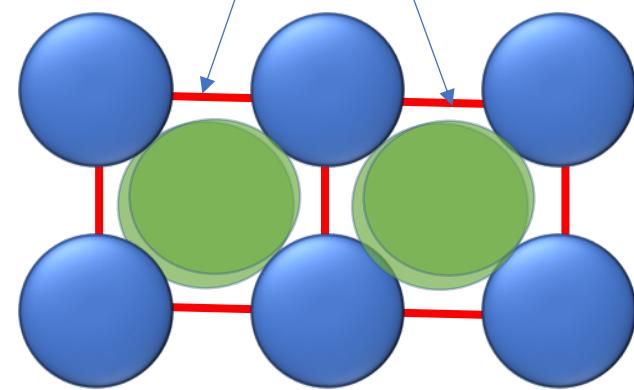


Delaunay triangulation based on particle centroids

Identify spheres tangent to particles forming Delaunay cell



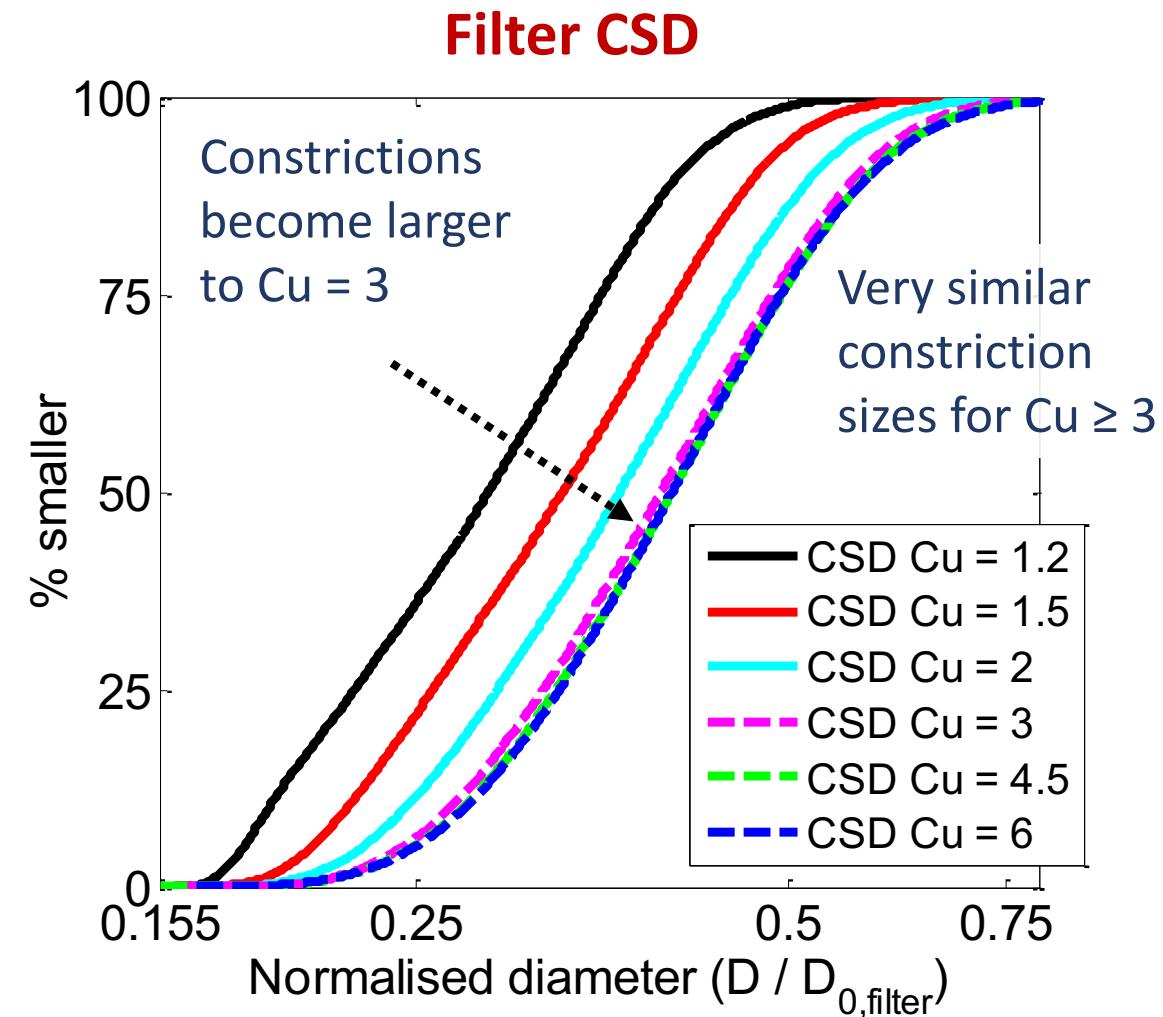
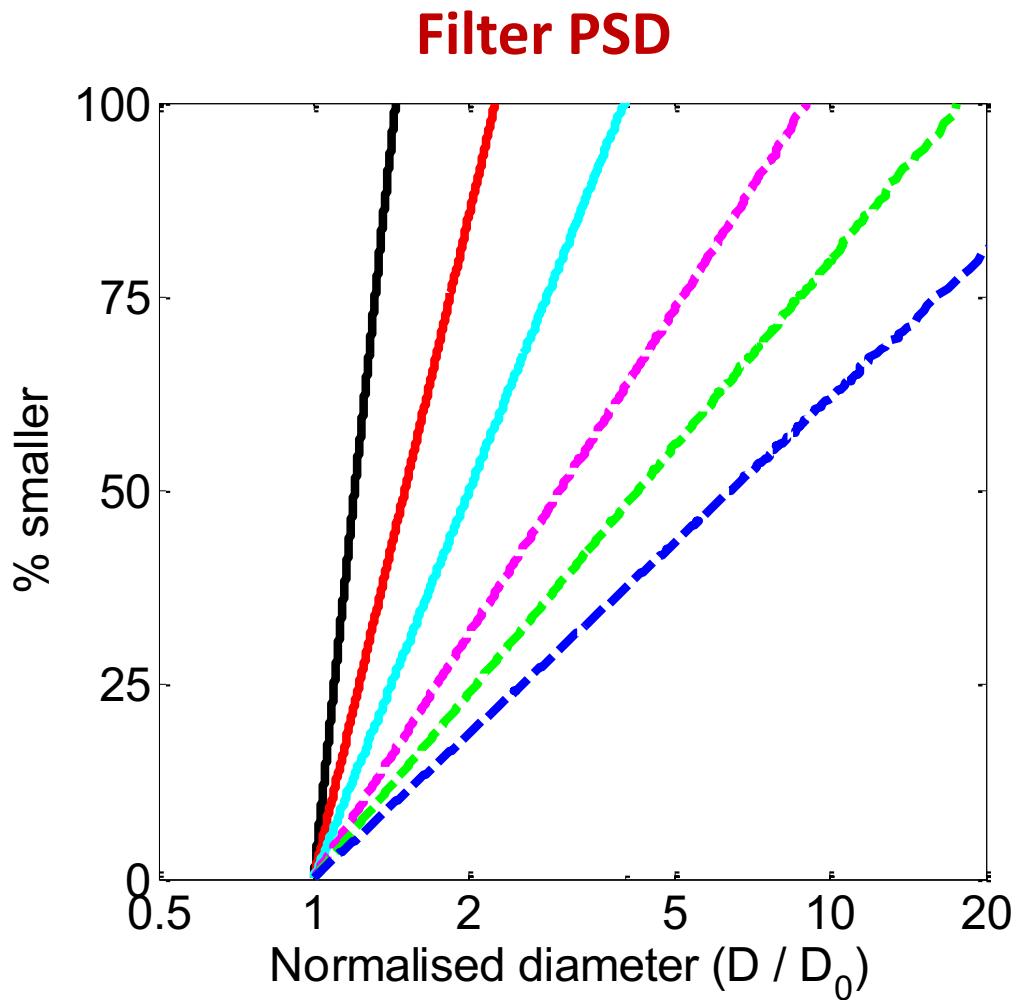
Valid constrictions



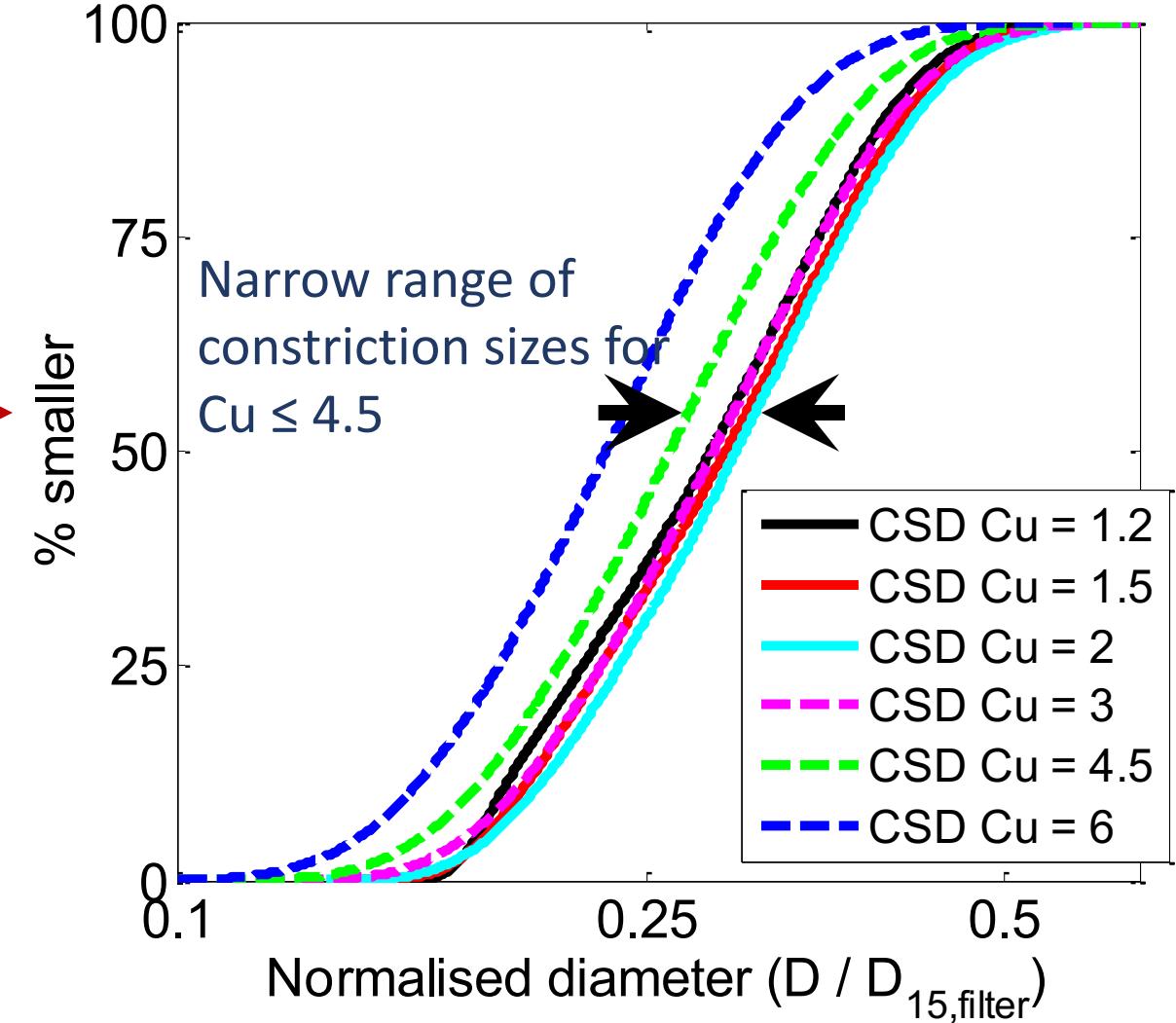
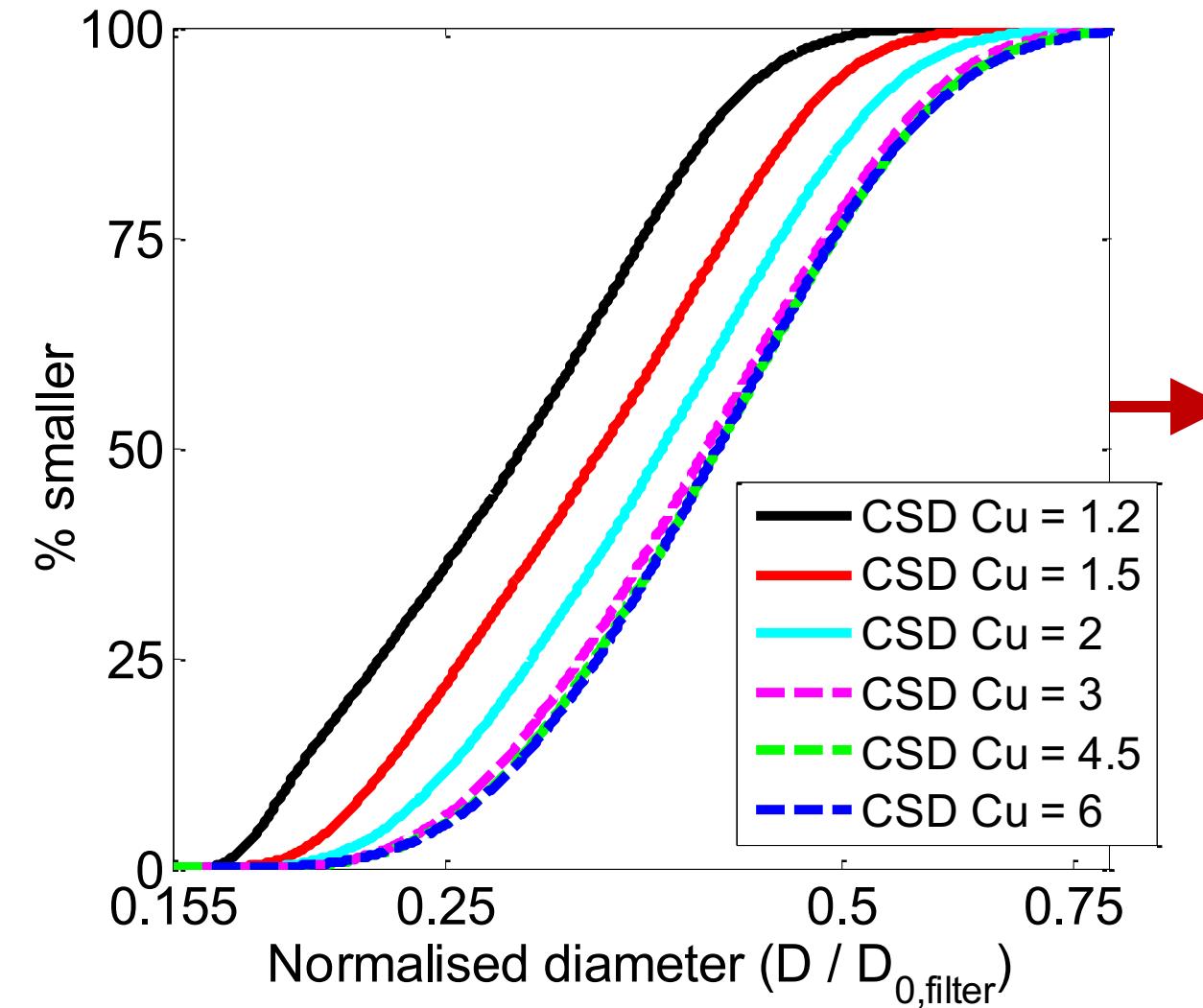
Where tangent spheres overlap Delaunay cells are merged

User decides magnitude of overlap

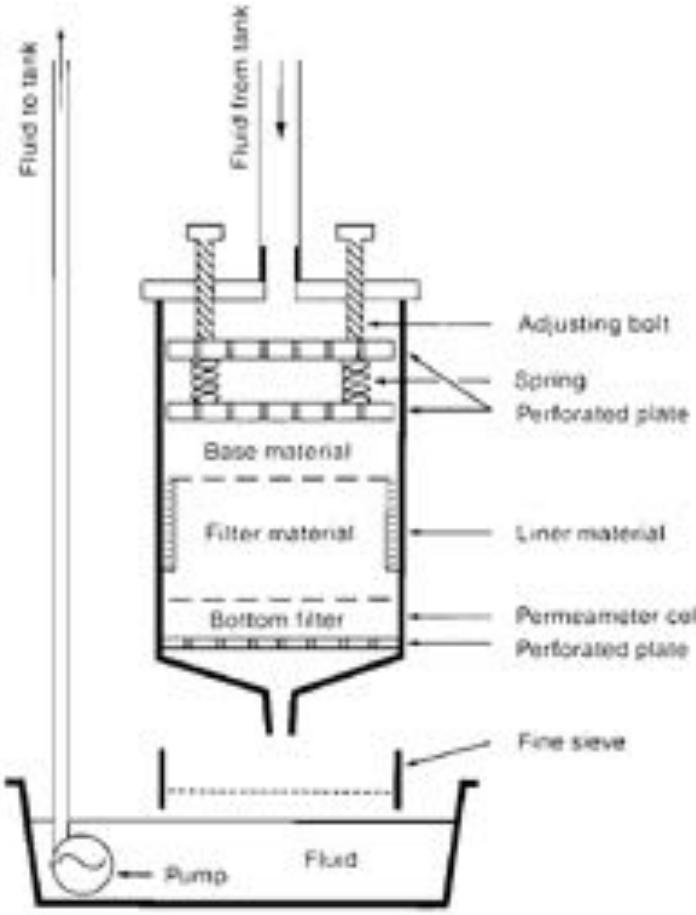
Constriction Size Distributions (DEM)



Constriction Size Distributions (DEM)

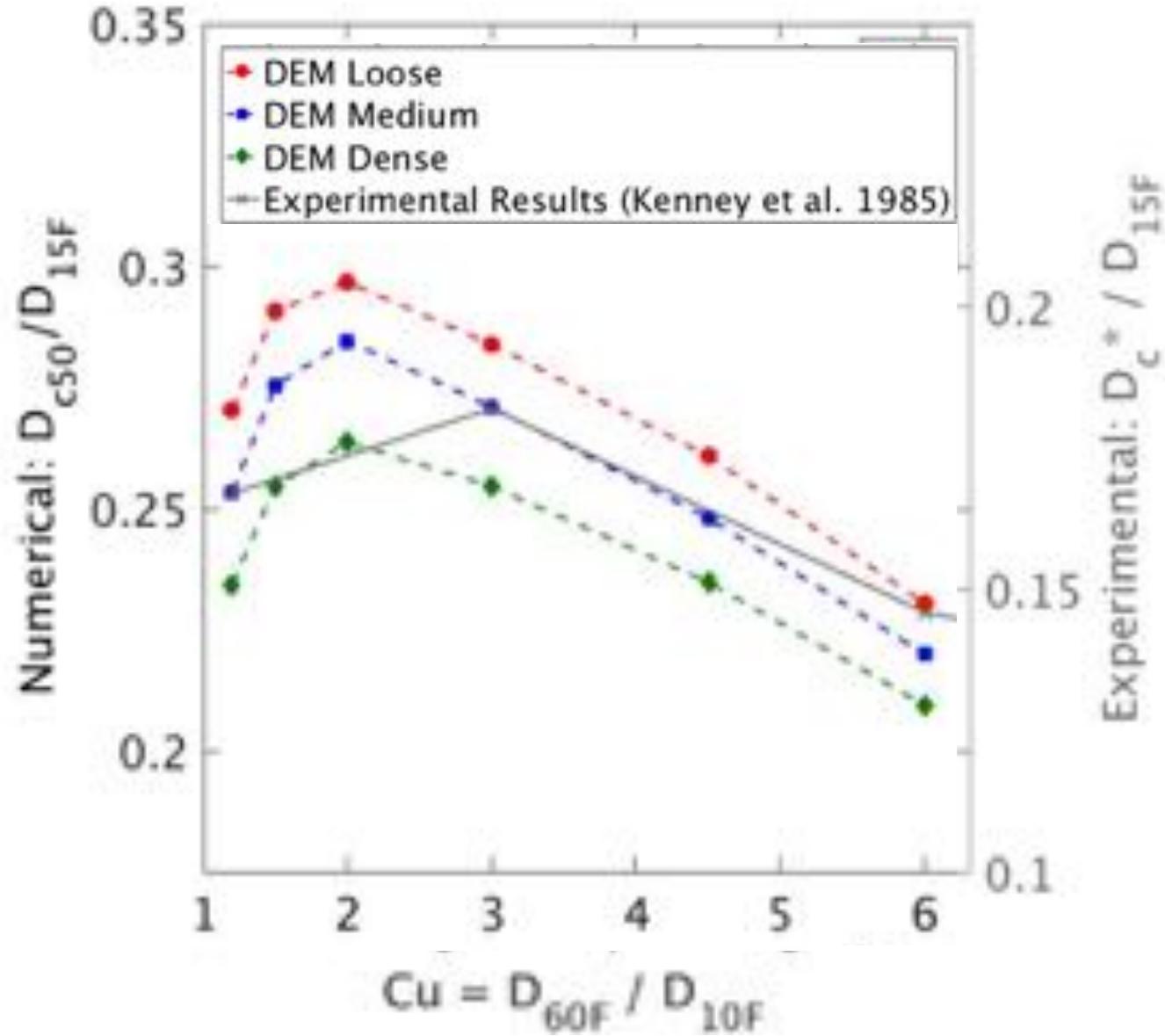


Controlling constriction size



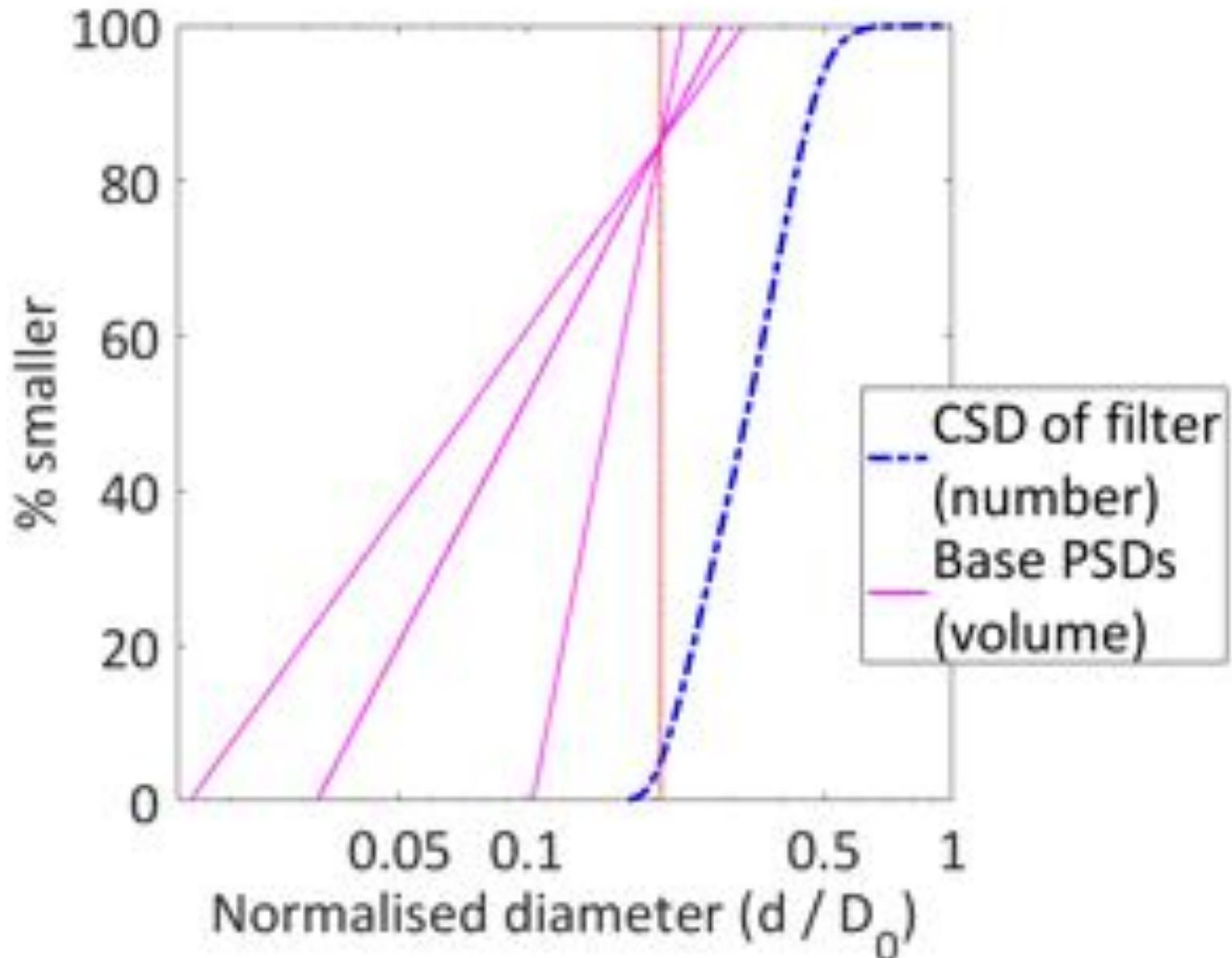
Kenney et al (1985): Base- Filter tests: Base-filer tests

D_c^* = controlling constriction diameter = largest particle that can pass through filter



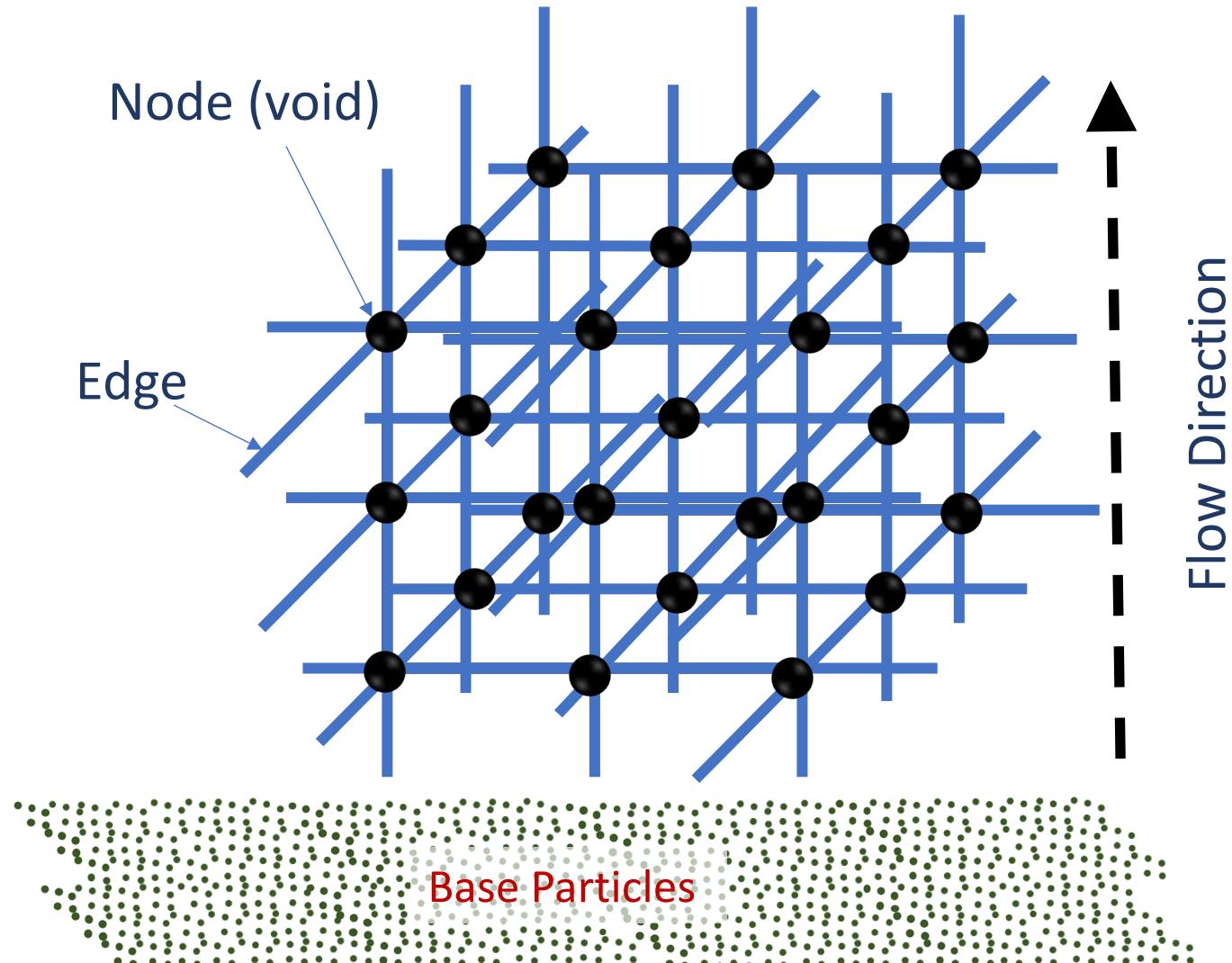
Retention – Network model

Can't judge a filter's effectiveness simply by visual comparison of the CSD of the filter and the PSD of the base material to be retained



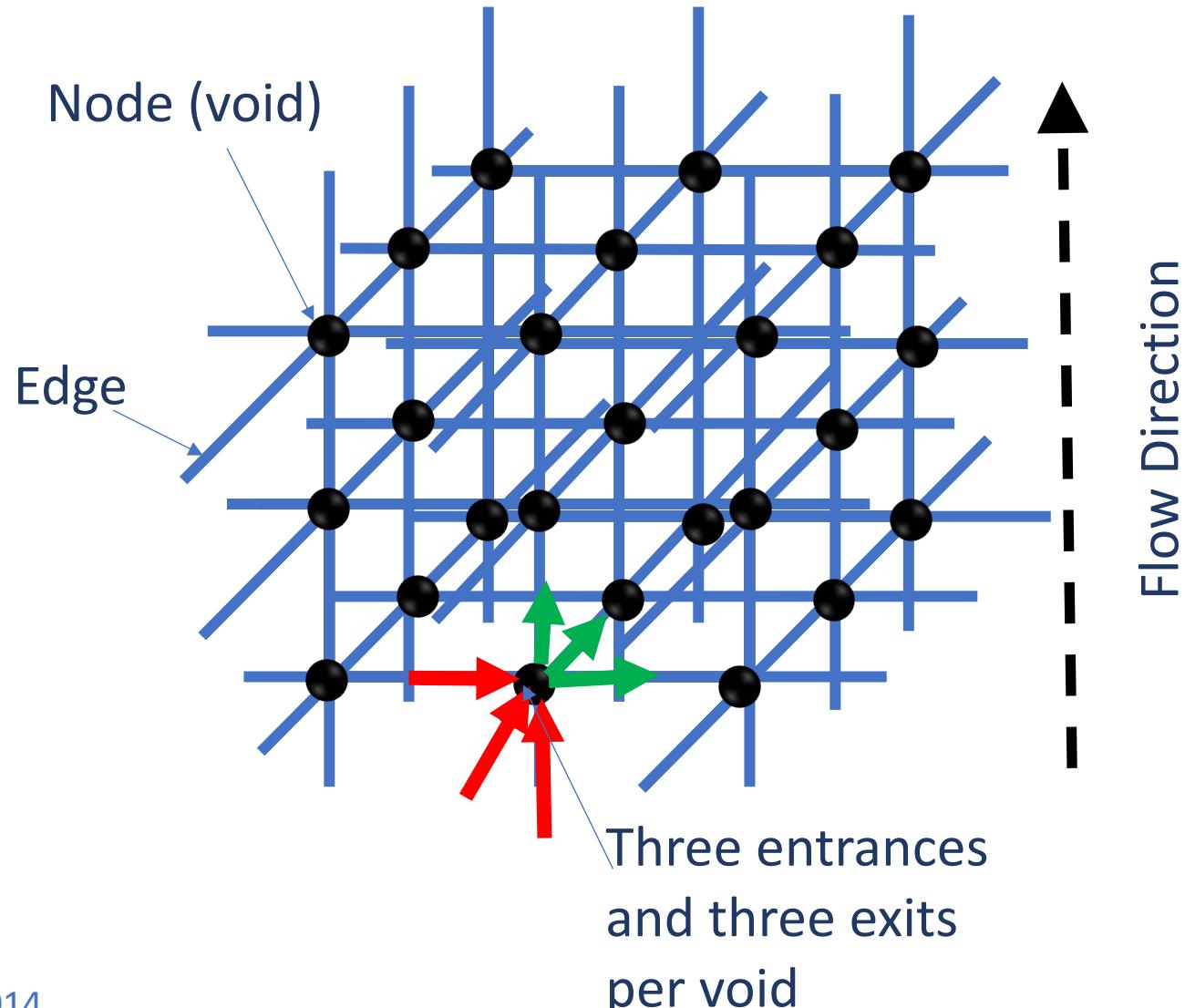
Retention – Network model

- Simulates migration of finer base particles through network
- Fluid flow not explicitly considered
- $40 \times 40 \times 60$ nodes (total 96,000 nodes), with 60 nodes in direction of flow
- Simple algorithm means up to 400 million base particles could be considered on a desktop pc

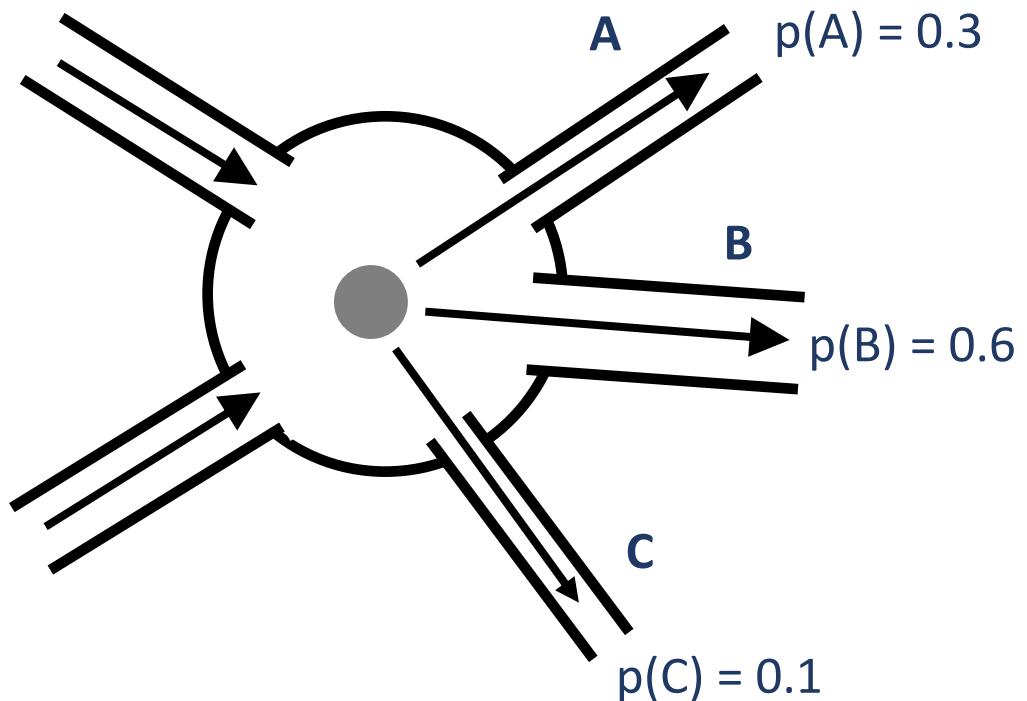


Retention – Network model

- Network model – lattice topology
- Nodes = individual voids
- Edges = inter void connections
- Edge diameters = constriction diameters – taken from CSDs

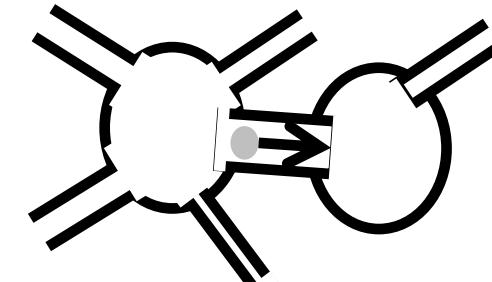


Area based random walk

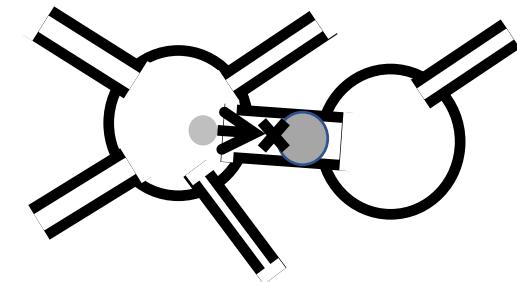


Likelihood of selecting a target edge to move through depends on constriction area

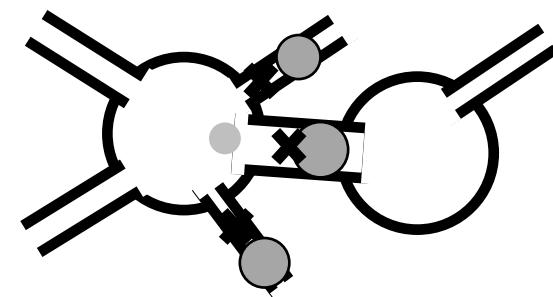
Base particle moves through constriction



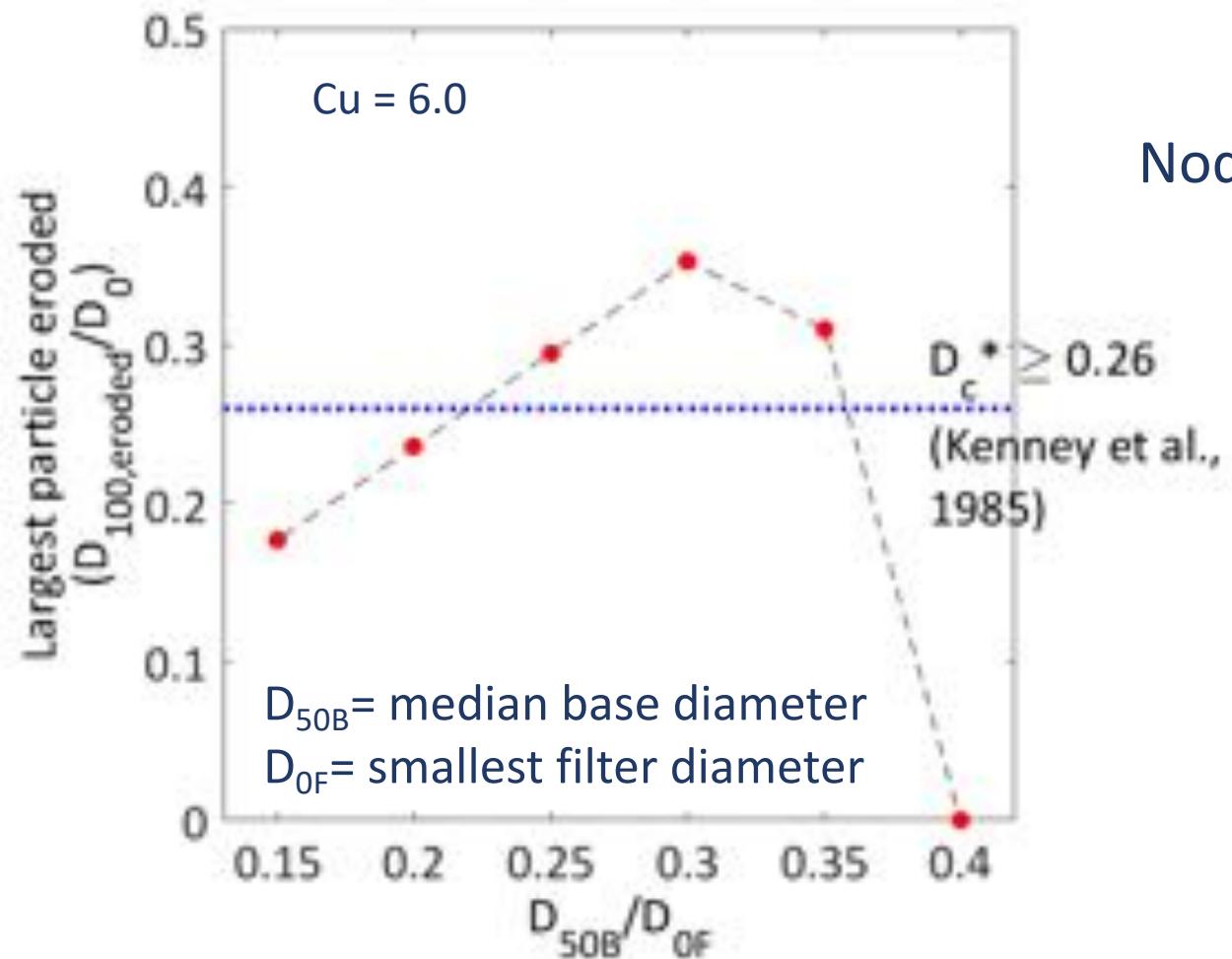
Base particle retained + constriction blocked



Base particle retained in void

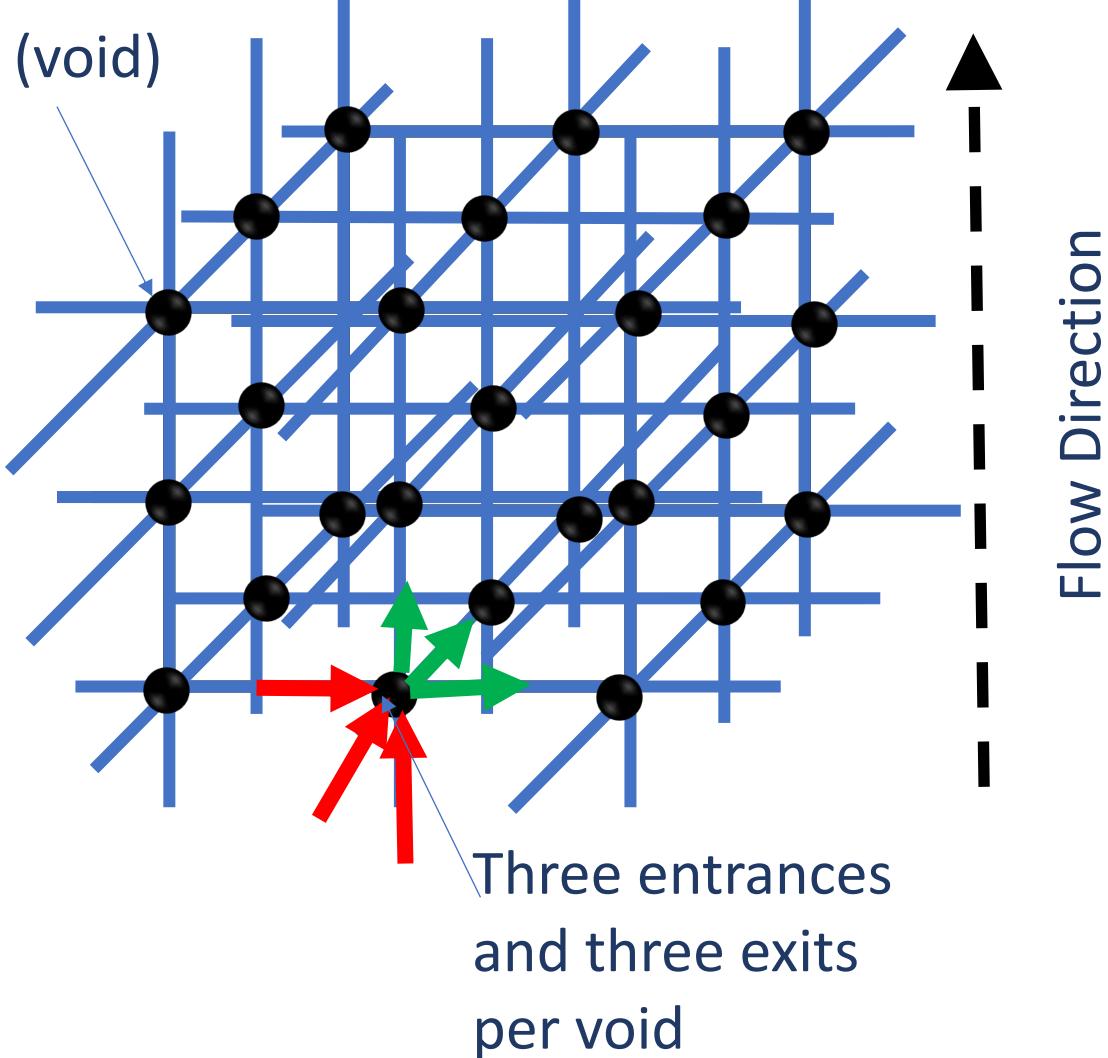


Retention – Network model

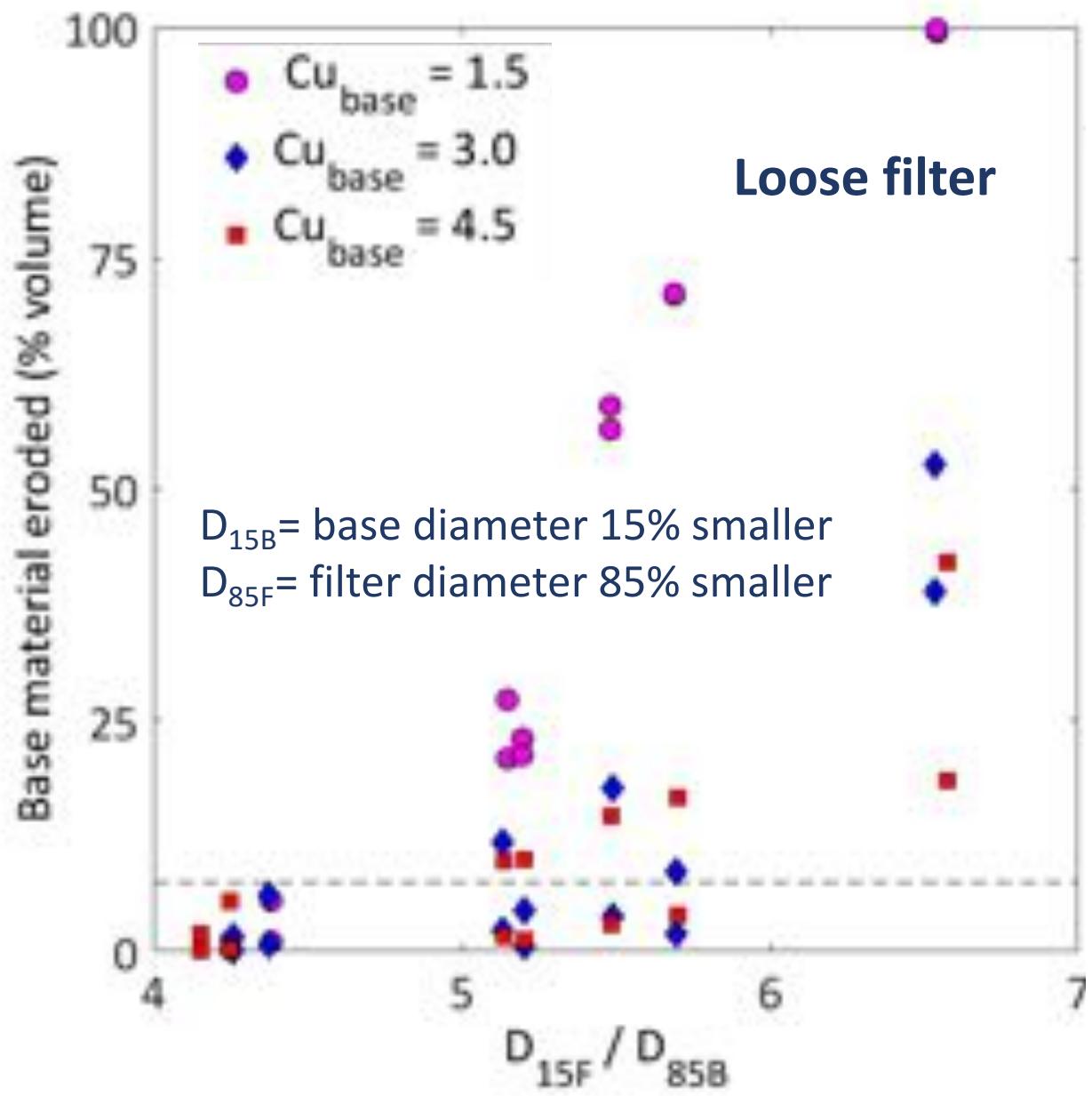


Filter Cu = 1.2, 3, 6, largest base particle eroded agrees with experimental data

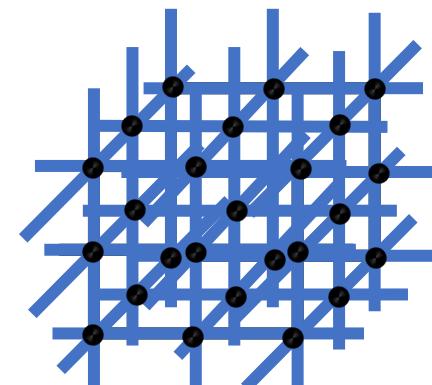
Node (void)



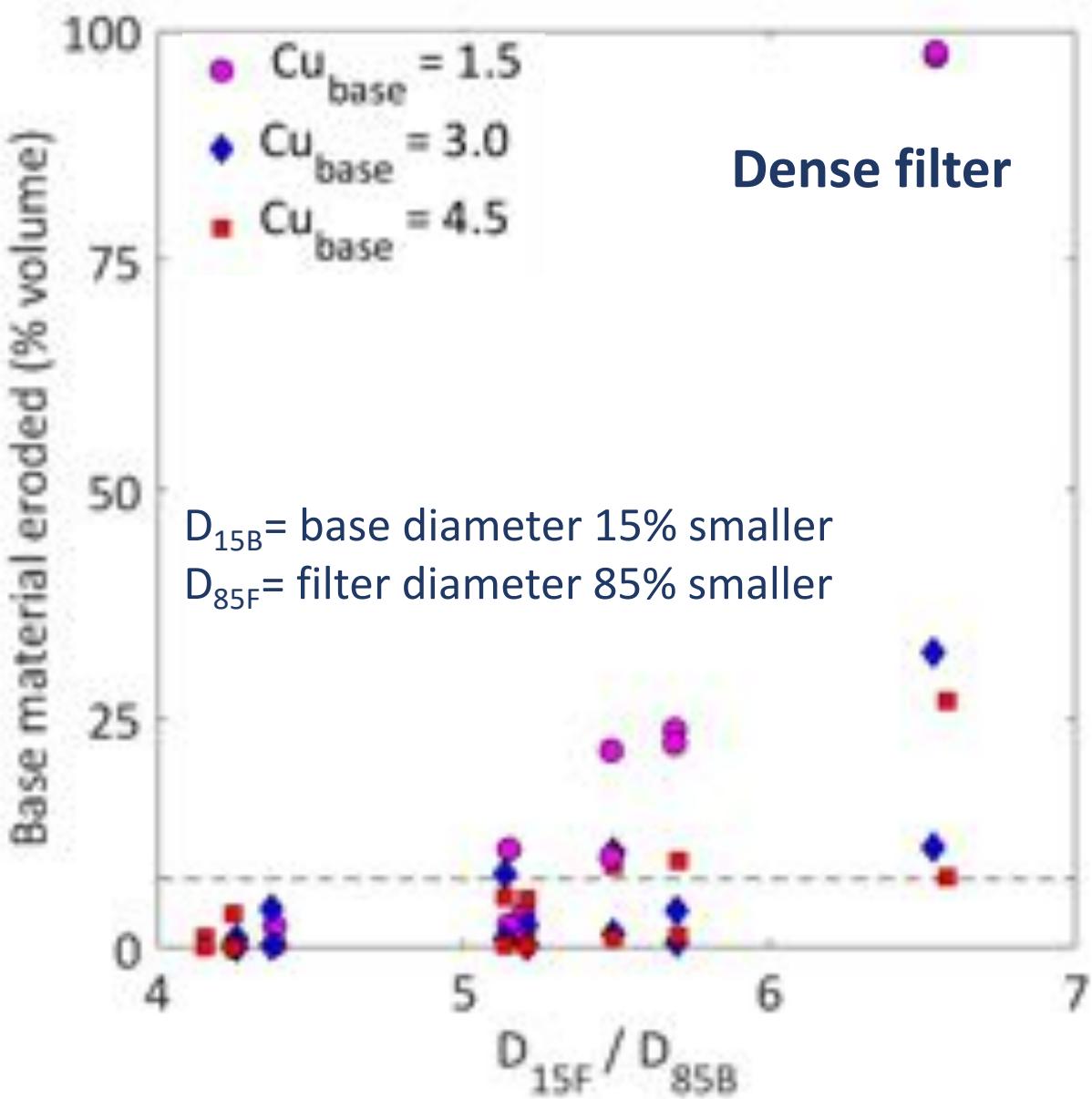
Retention – Network model



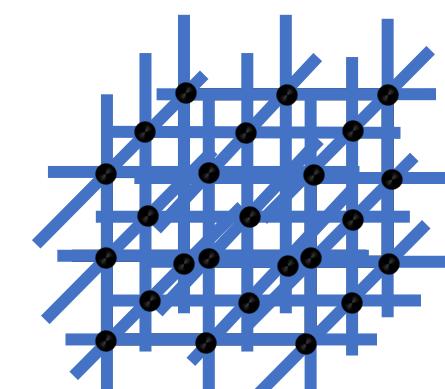
- Cu Filter = 1.5 and 3.0
- Network model that considers only constriction sizes and not full void space topology confirms experimental observation that filter characteristic diameter (D_{15F}) controls filtration



Retention – Network model



- Cu Filter = 1.5 and 3.0
- Network model that considers only constriction sizes and not full void space topology confirms experimental observation that filter characteristic diameter (D_{15F}) controls filtration



Research questions:

- **Retention:**
Does pore network modelling support use of the ratio D_{15F}/D_{85B} in design?
- **Internal Stability:**
Can we predict the fluid particle interaction forces with pore network modelling?

Coarse-grid DEM CFD coupling

Combination of DEM and CFD

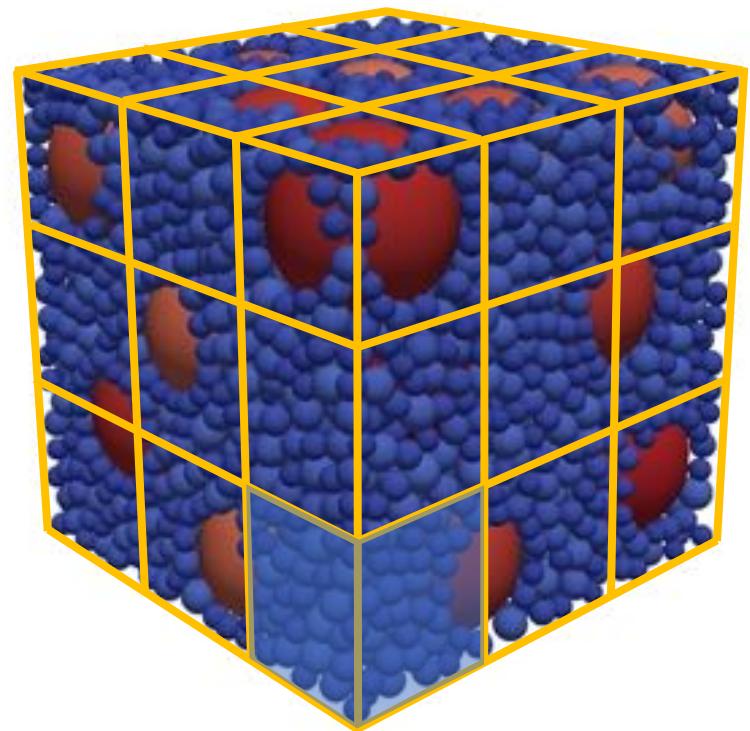
- DEM for soil particles
- CFD for water seepage

Data exchange



- | | |
|-------------|--------------------------|
| -Porosity | -Fluid velocity |
| -Drag force | -Fluid pressure gradient |

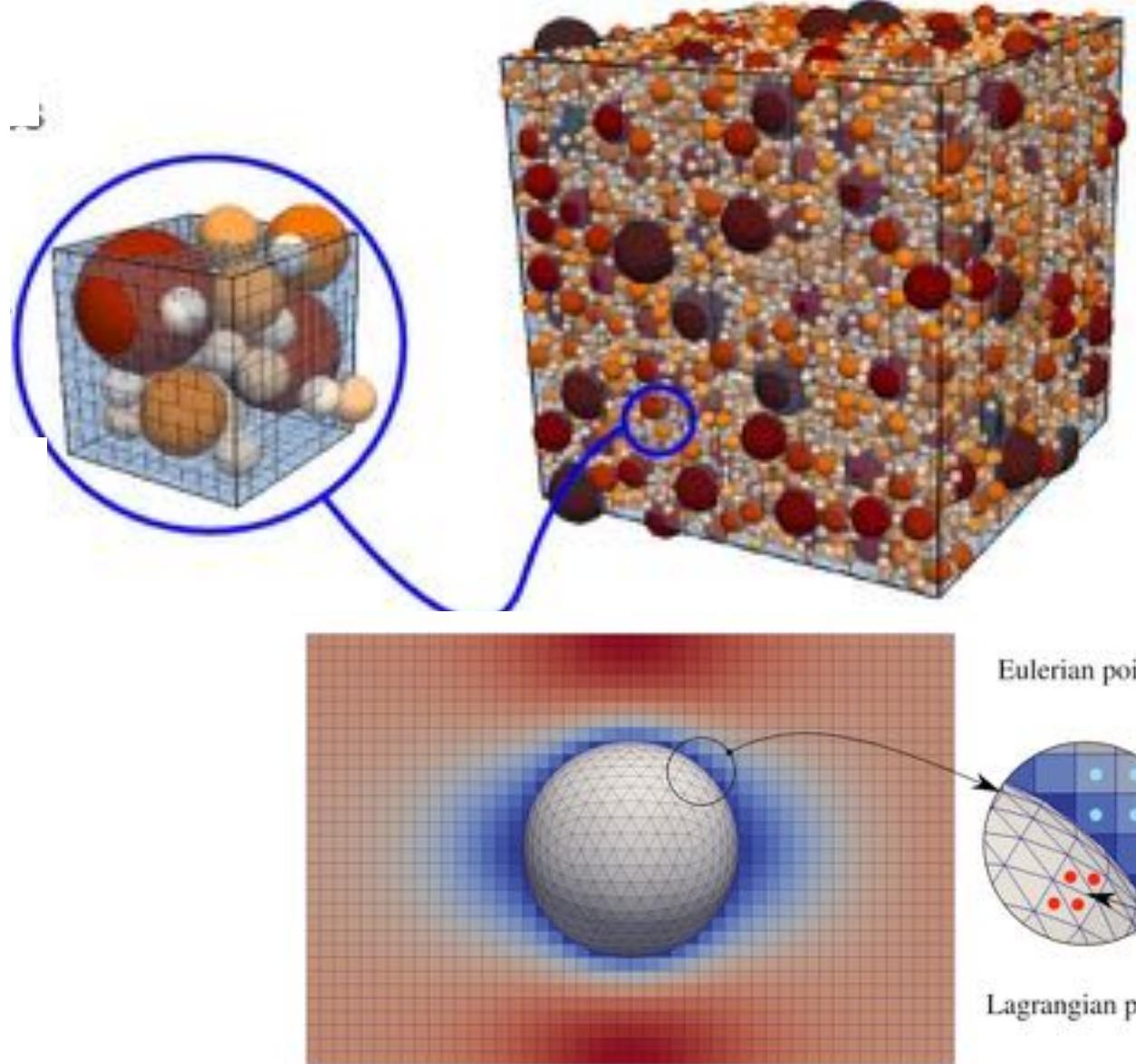
Coarse grid method proposed
by Tsuji



(Tsuji et al., 1993, Xu and Yu, 1997)

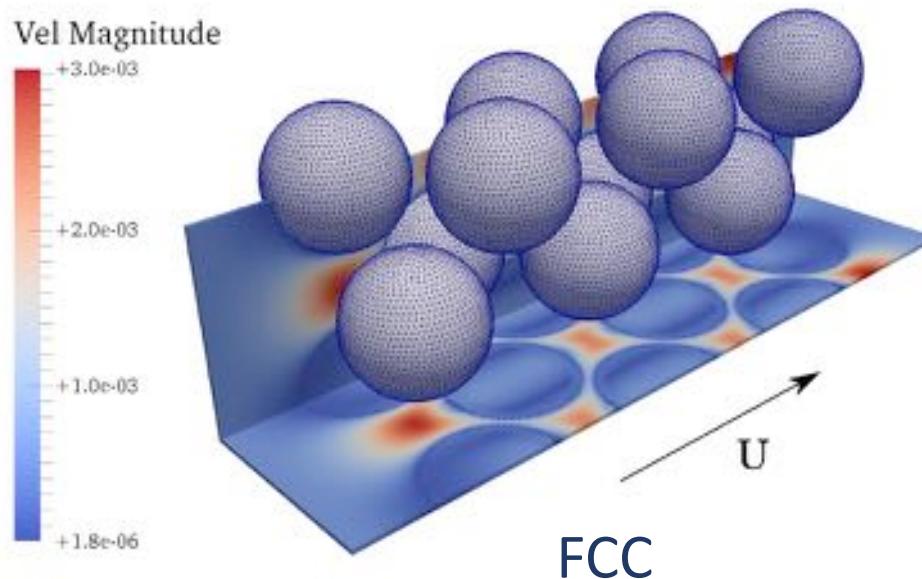
Simulation of flow in void space

- Immersed boundary Method (IBM)
- Code developed by Prof. Berend van Wachem, Mech. Eng. Imperial / Universität Magdeburg, Germany
- Additional developments by PhD student Chris Knight
- Enables direct calculation of fluid-particle interaction force + subparticle scale simulation of fluid flow



Verification simulations

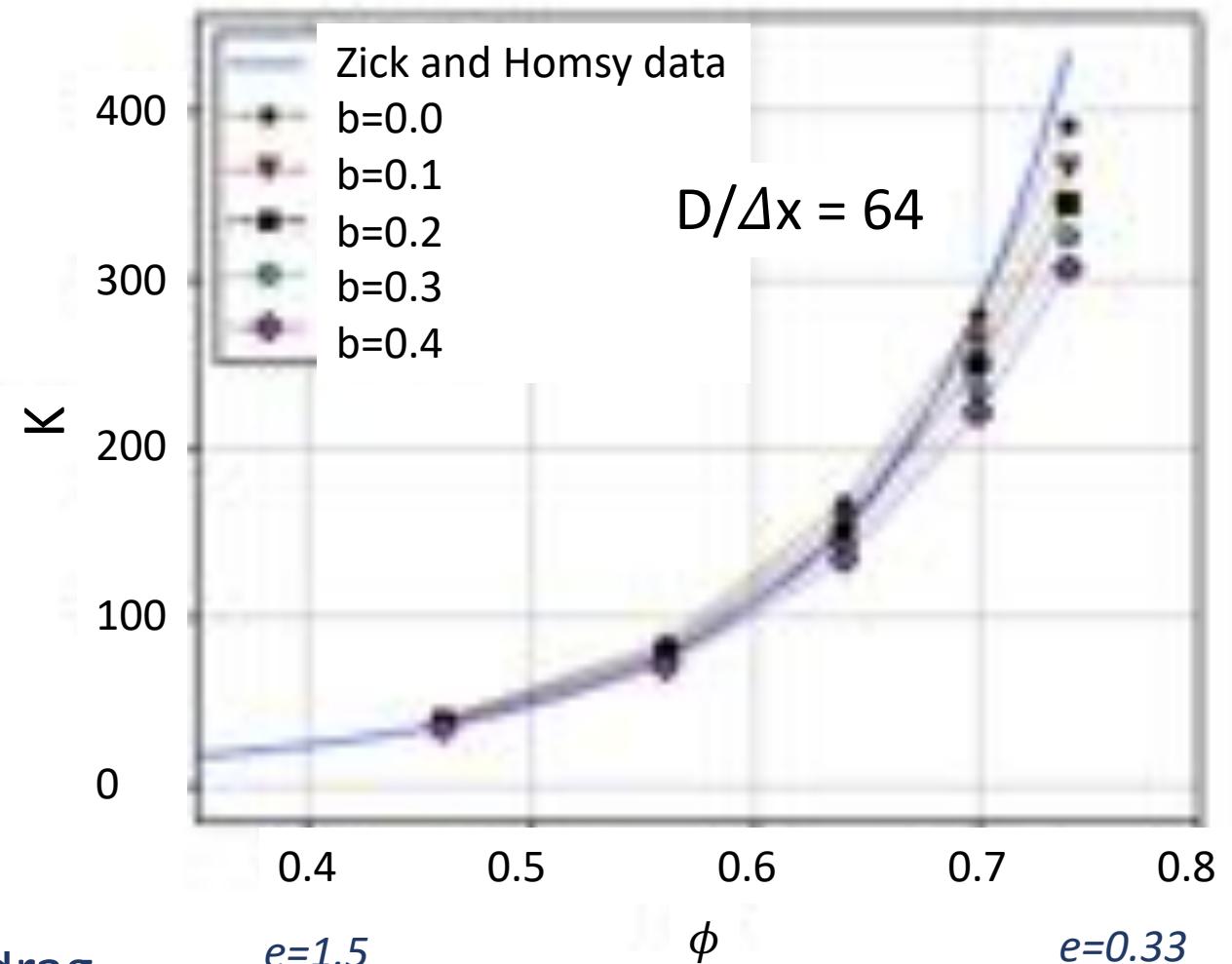
Zick and Homsy – boundary integral method data



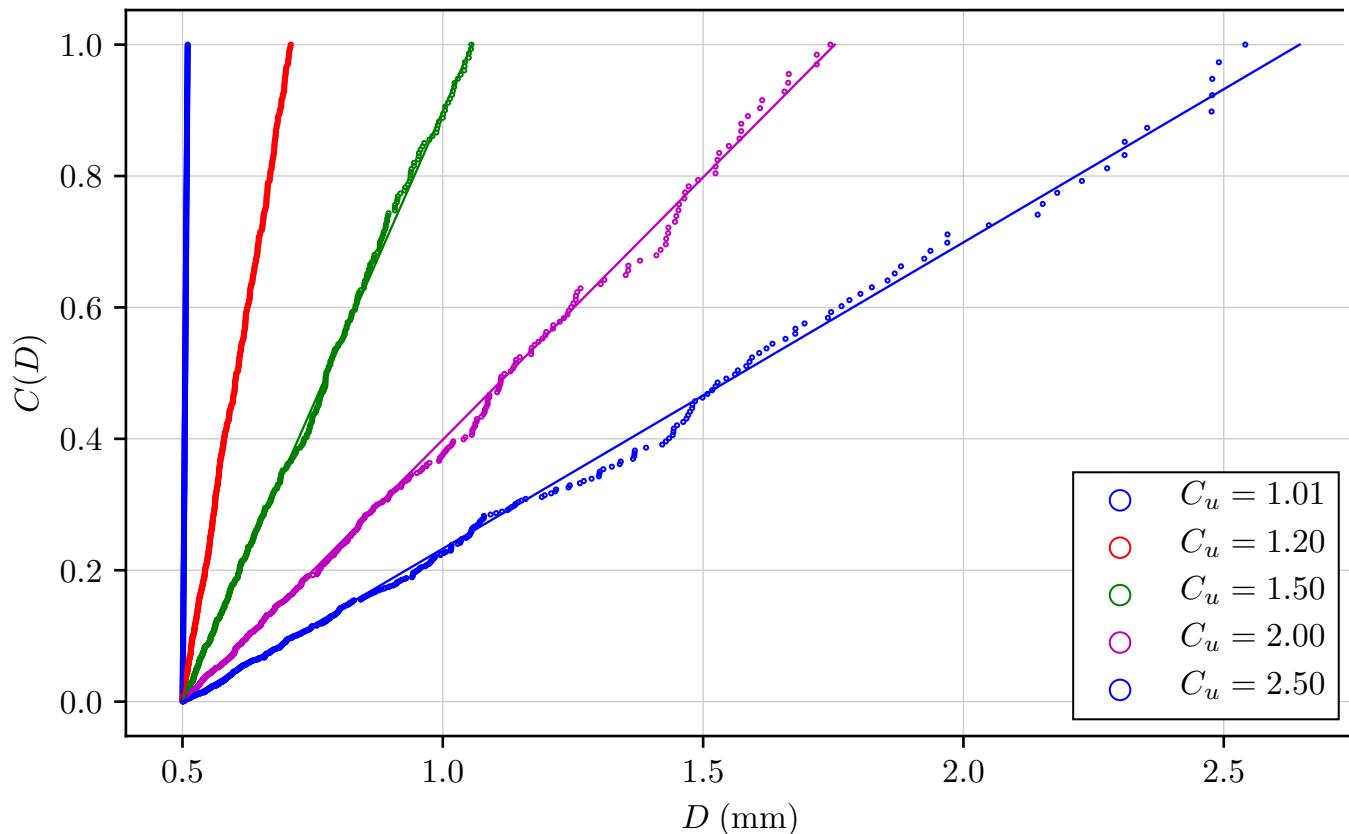
ϕ = solids fraction = $V_{\text{solids}} / V_{\text{total}}$

b = radius retraction parameter
(retraction = $b \Delta x$)

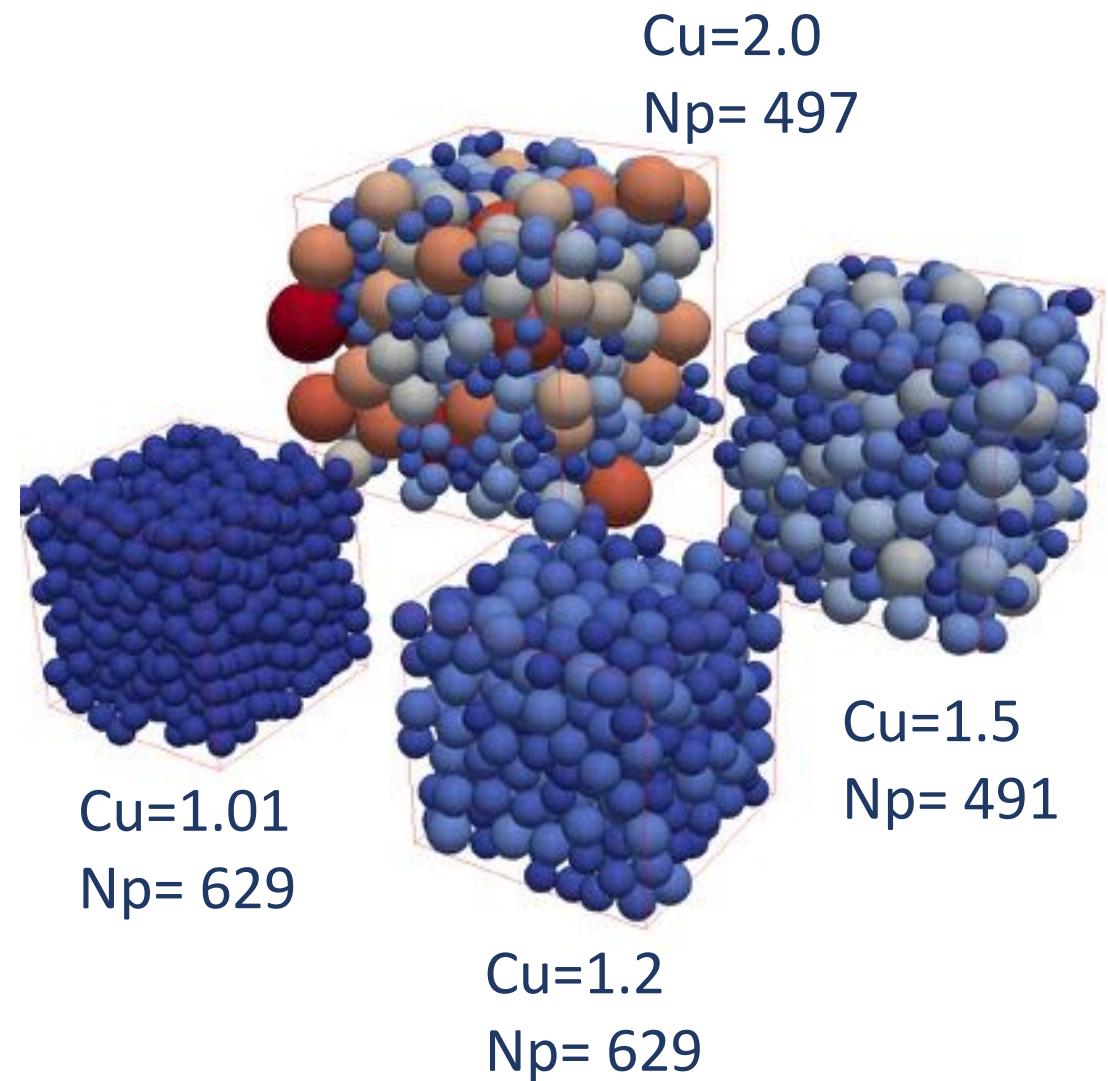
K = fluid particle interaction force / Stokes drag



Samples considered



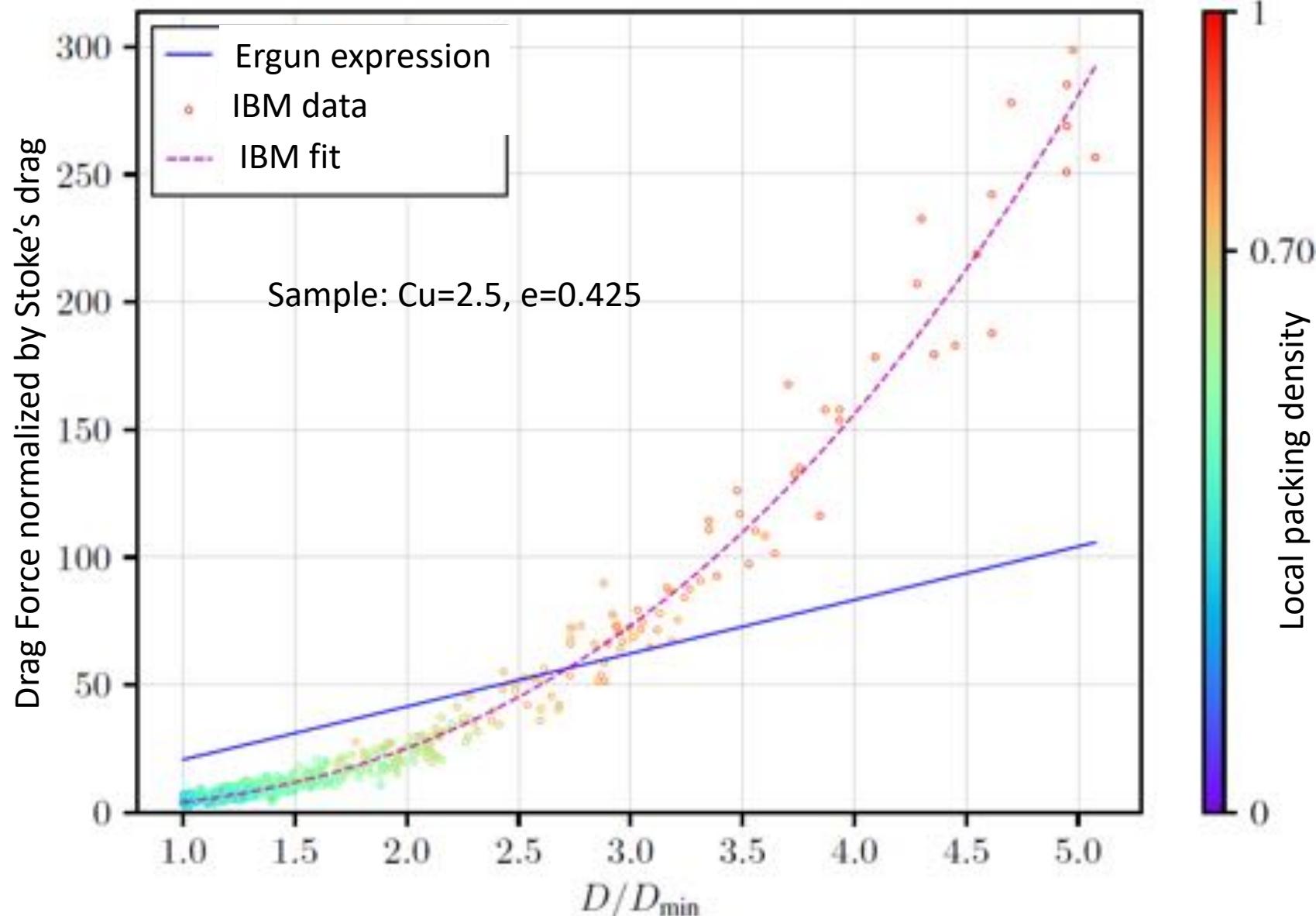
Computational cost restricted to small samples and small Cu values (< 2.5)



Fluid particle interaction force ($F_{f \rightarrow s}$)

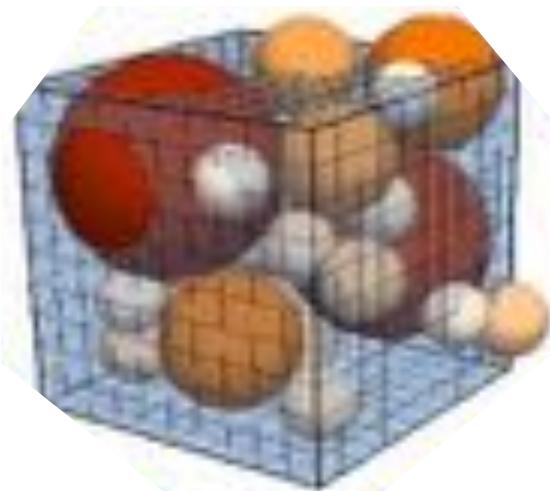
Drag coefficients used in conventional DEM-CFD were developed for mono-disperse samples

IBM data indicate that they give poor match to fluid-particle interaction forces ($F_{f \rightarrow s}$) in poly-disperse and gap-graded samples



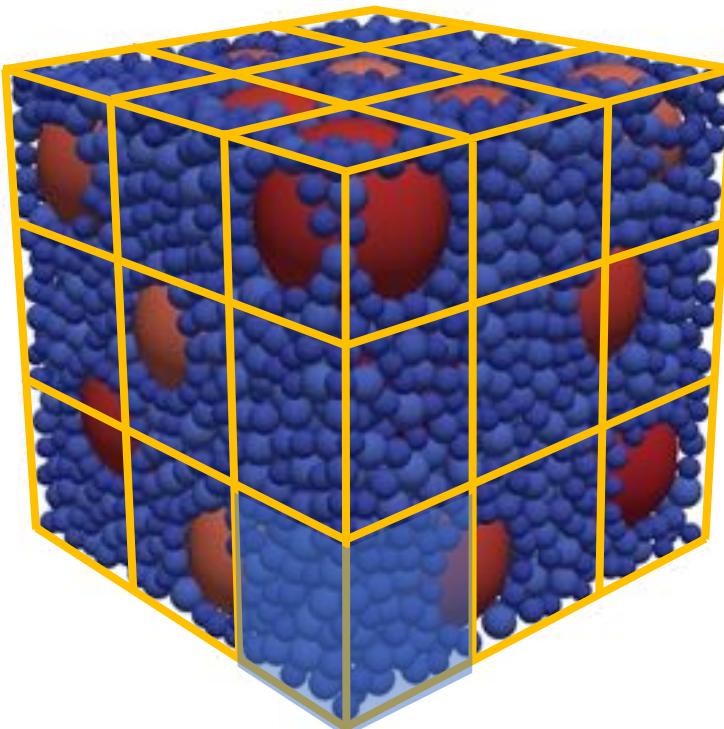
PNM DEM CFD coupling

Fully resolved flow



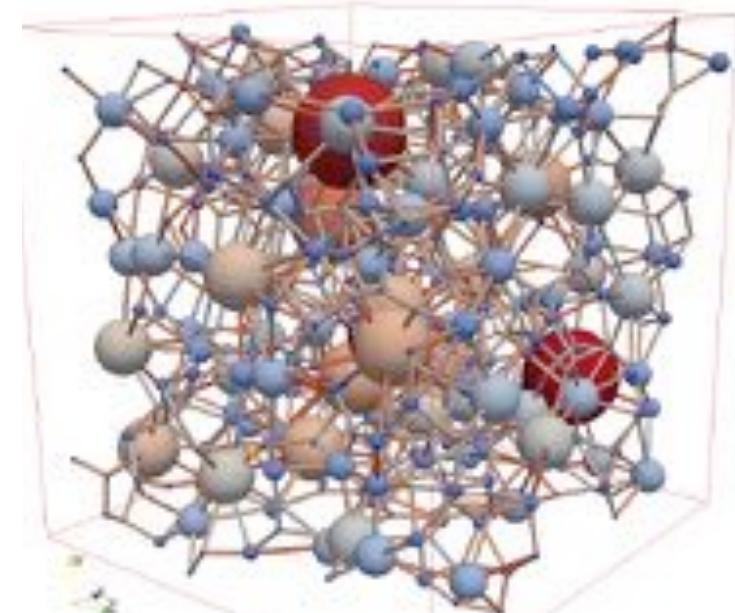
Very computationally expensive
Accuracy can be excellent

Coarse grid method



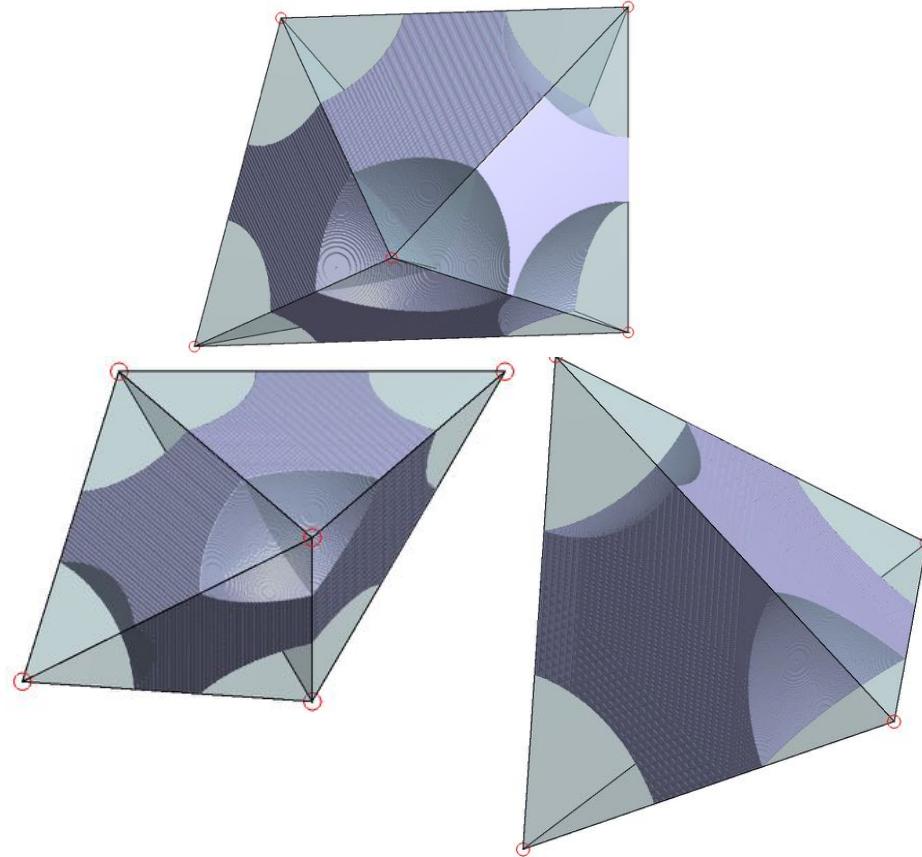
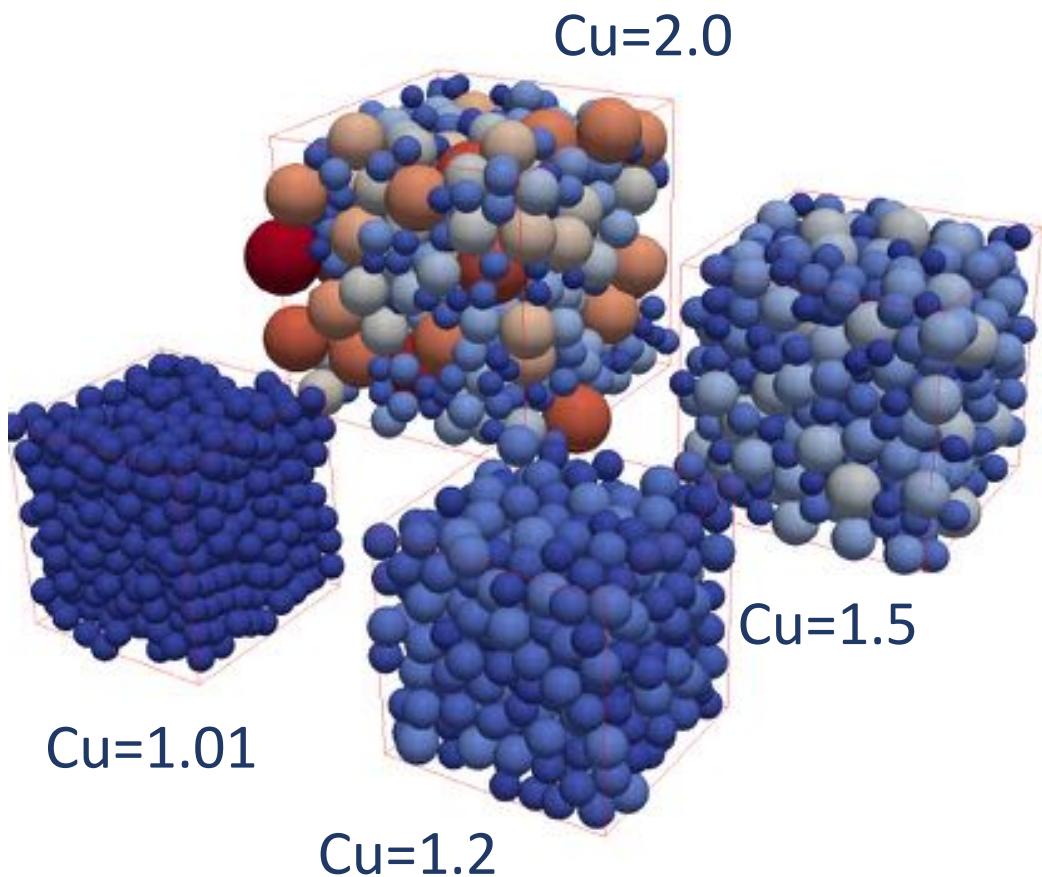
Computationally expensive
Accuracy limited

Pore Network Model (PNM)



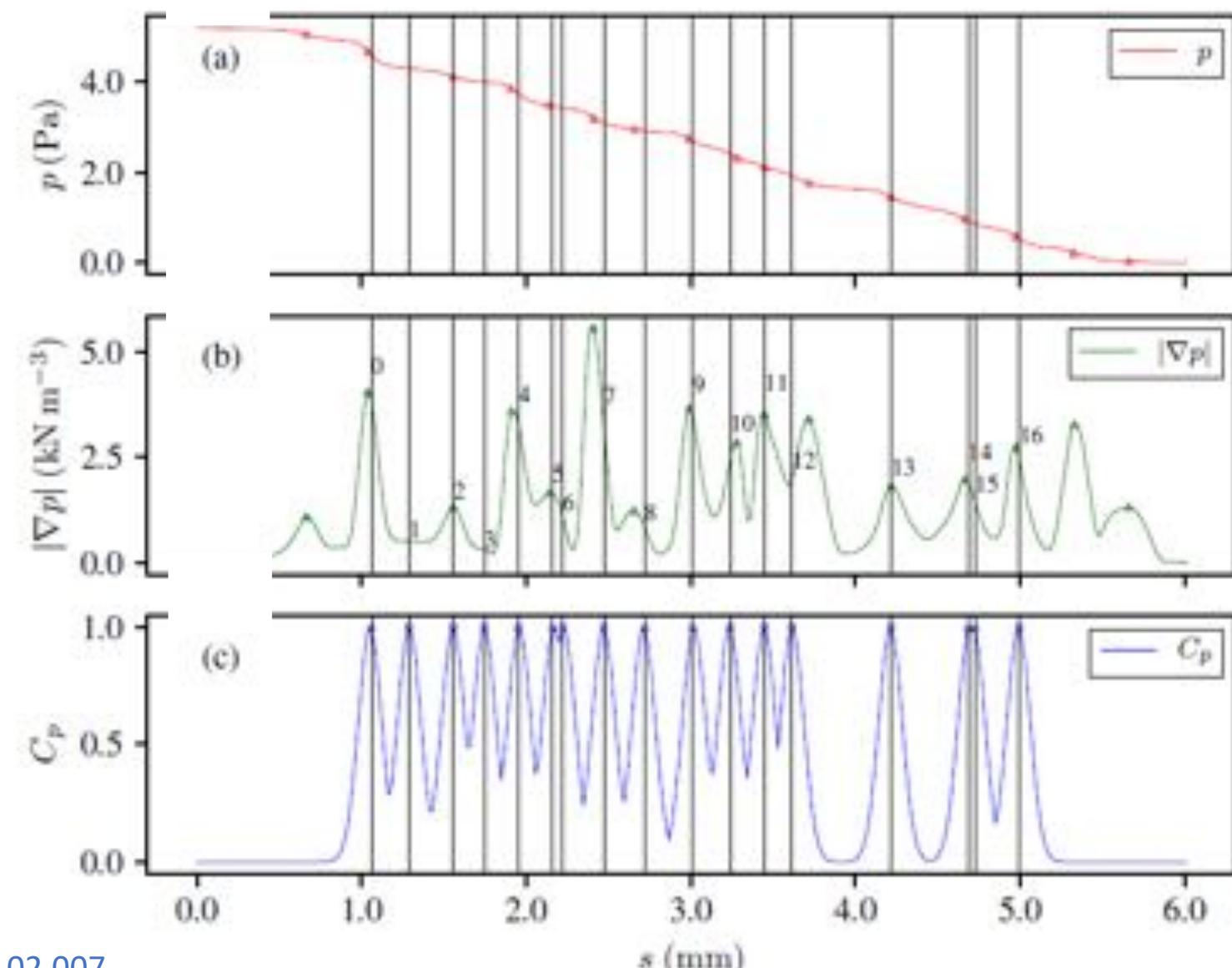
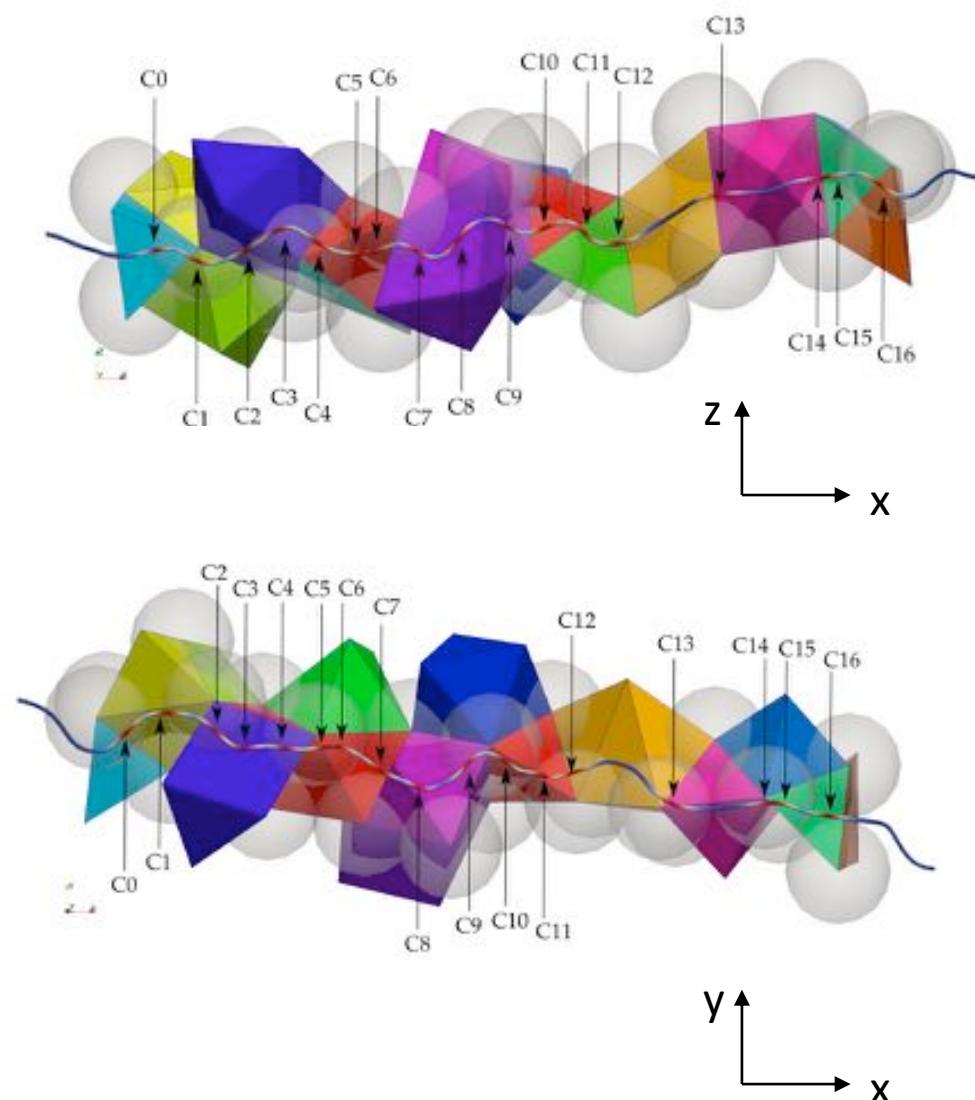
Very computationally efficient
Accuracy not completely understood

Identifying individual pores

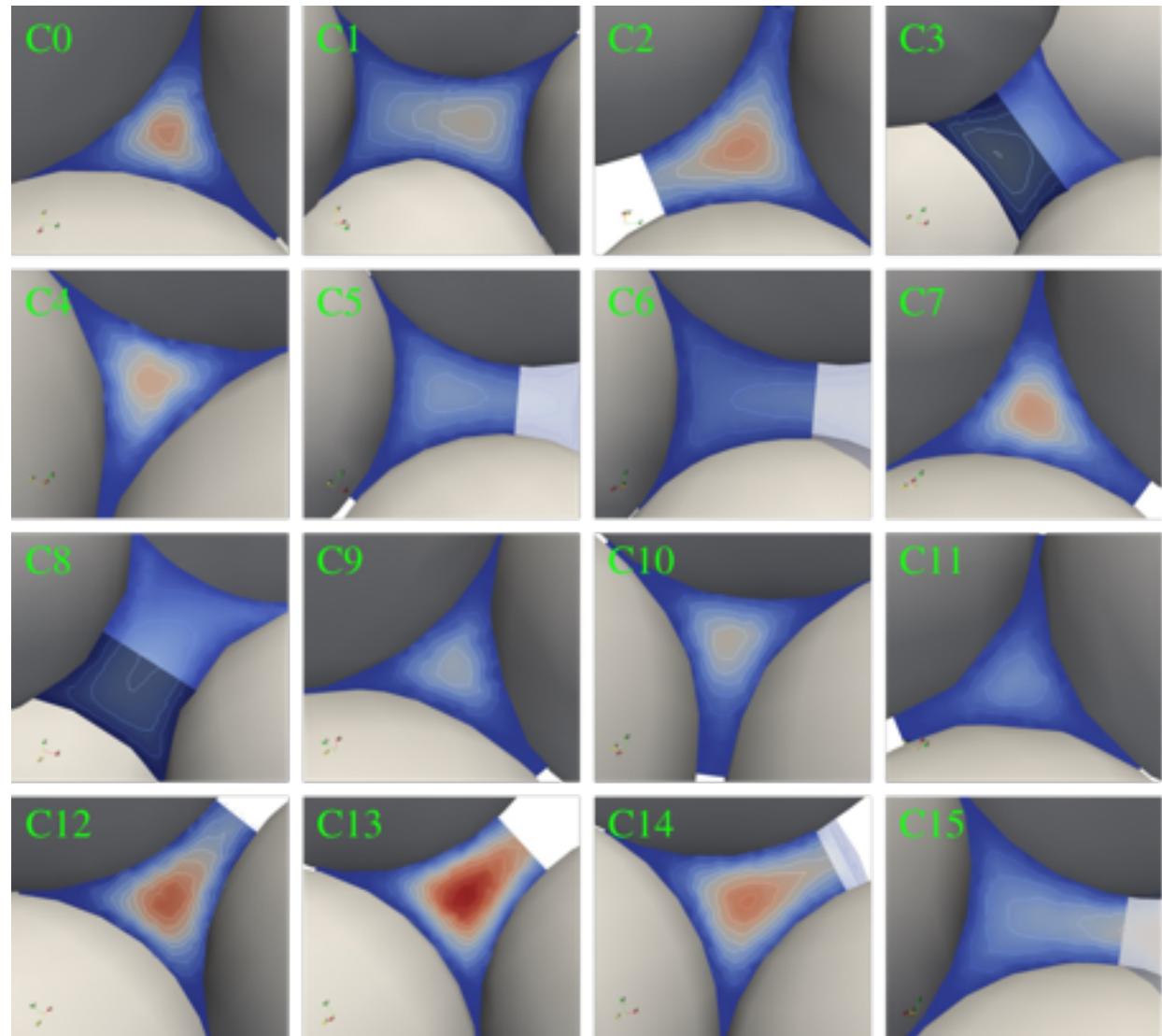
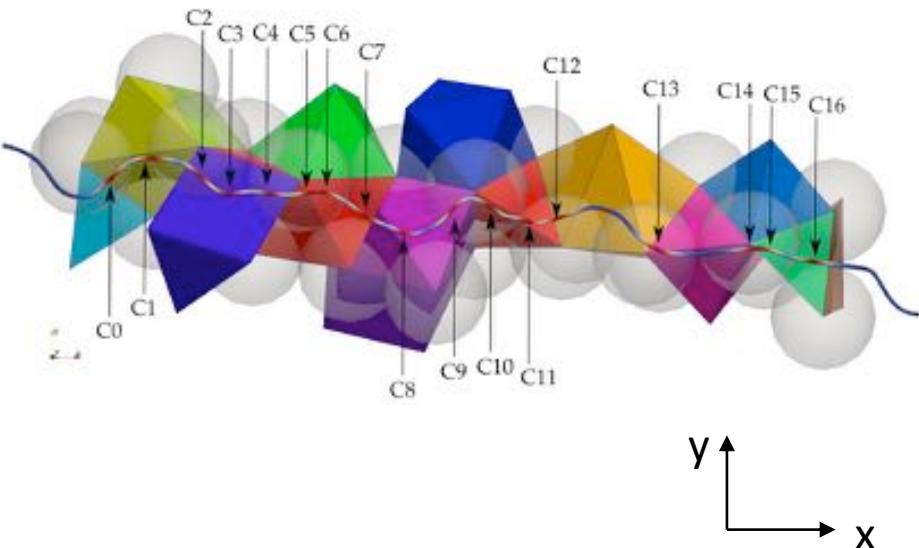
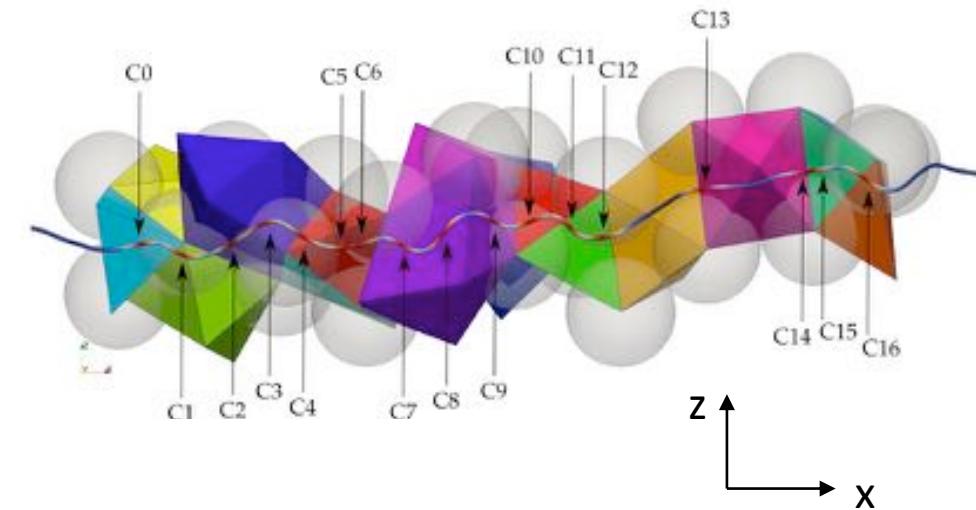


Triangulation + optimized merging to ensure inscribed sphere contained in pore

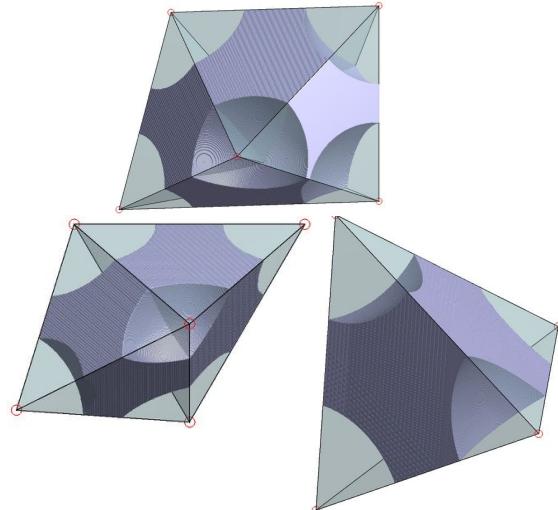
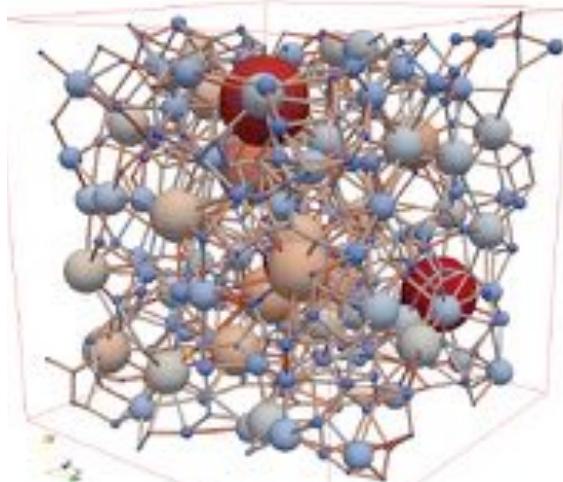
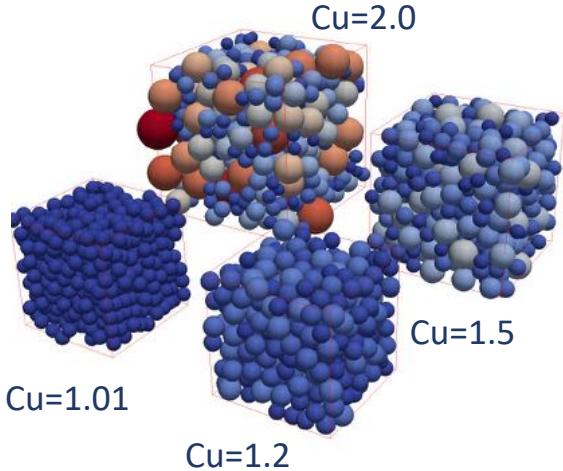
Tracing Flow along a Streamline



Tracing Flow along a Streamline



IBM and pore network model



Flow from pore a to pore b

$$q_{ab} = g_{ab}(p_a - p_b)$$

$$g_{ab} = \frac{\alpha A_t R_h^2}{8\mu l_c}$$

g_{ab} = conductance

p_a, p_b = pressure at nodes

$$\alpha = 0.5$$

} Shape factor

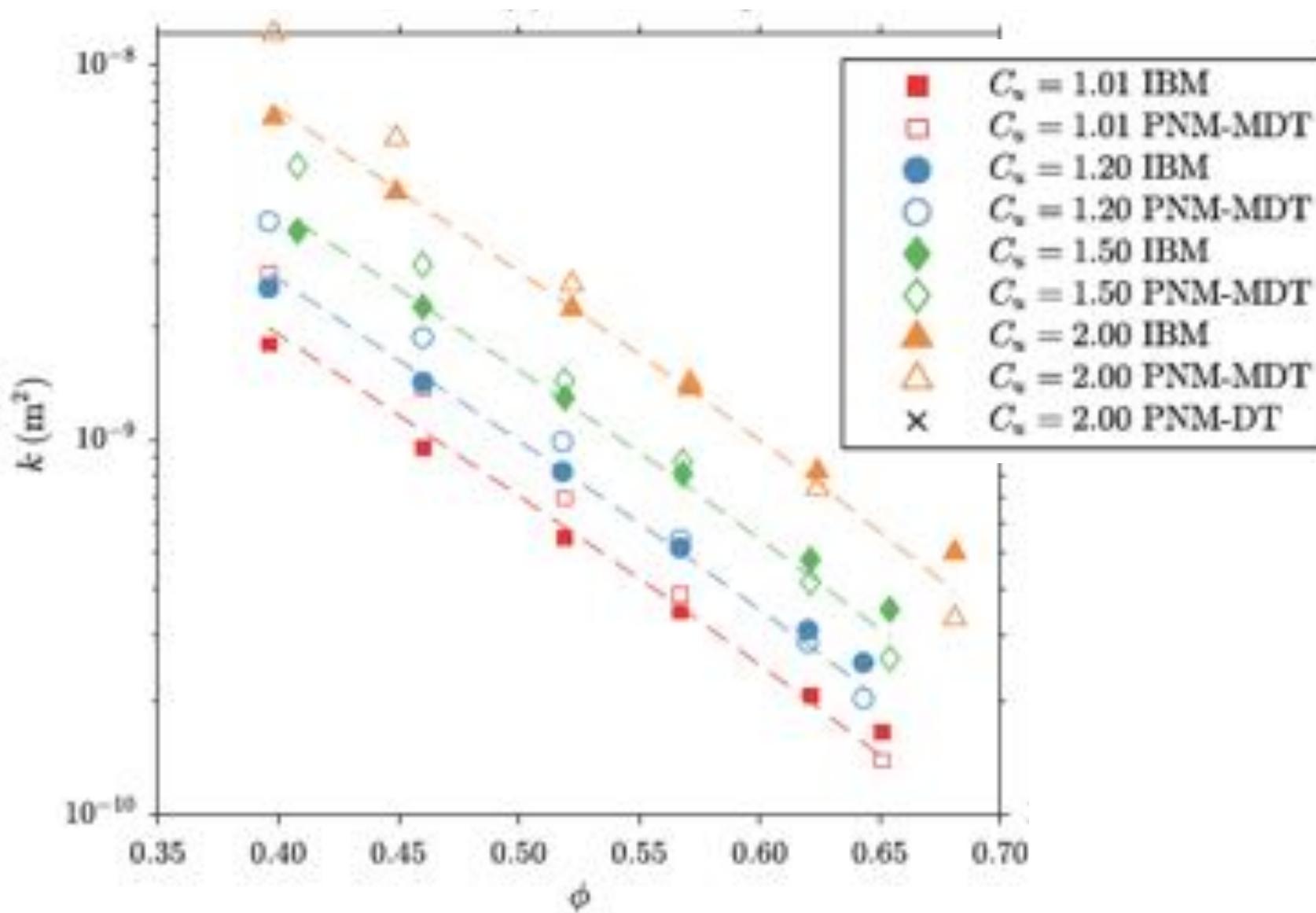
A_t = throat area

R_h = hydraulic radius
(ratio of volume to surface area)

l_c = centre to centre distance

} Assumed pipe geometry

IBM and pore network model permeabilities



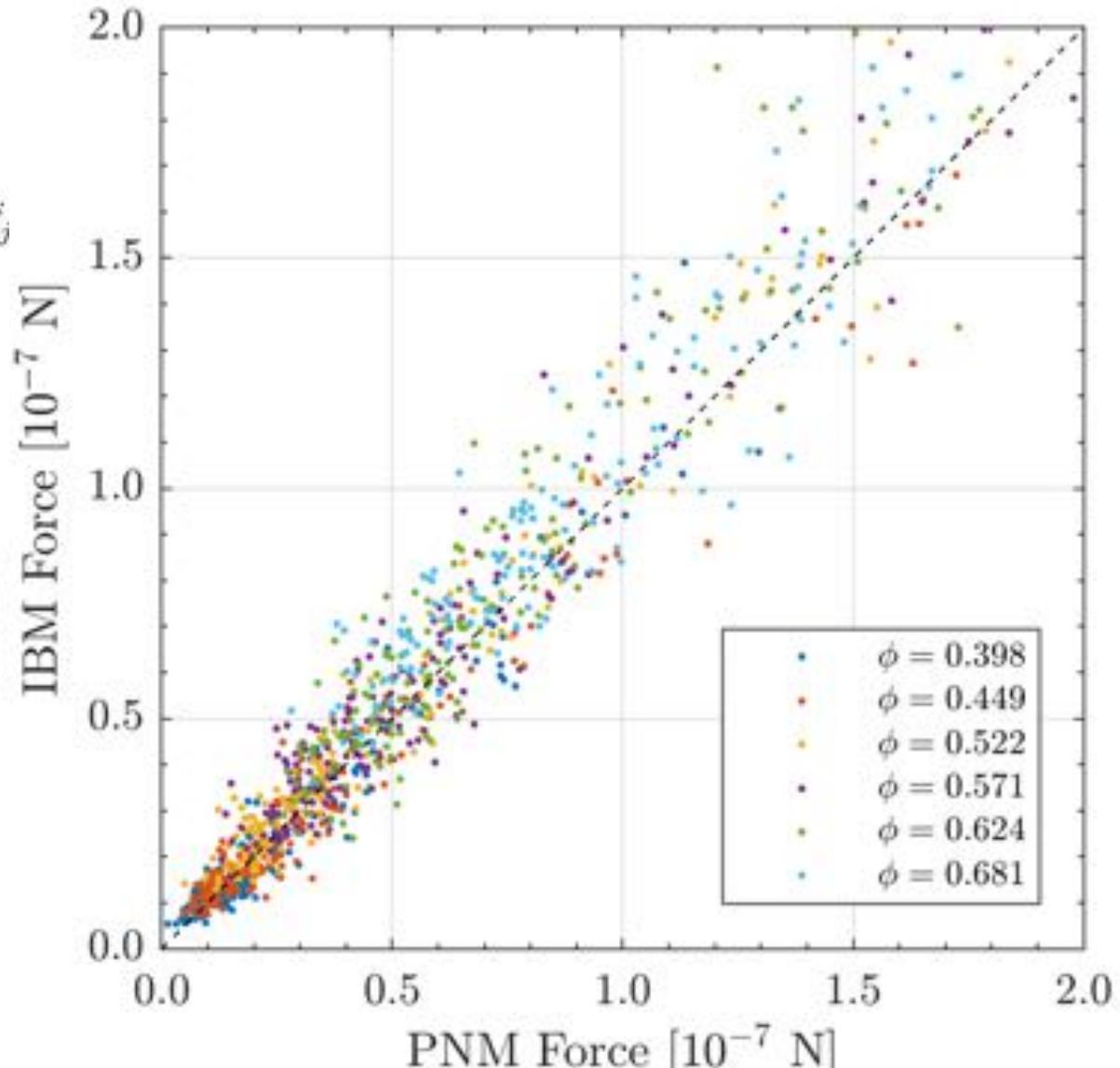
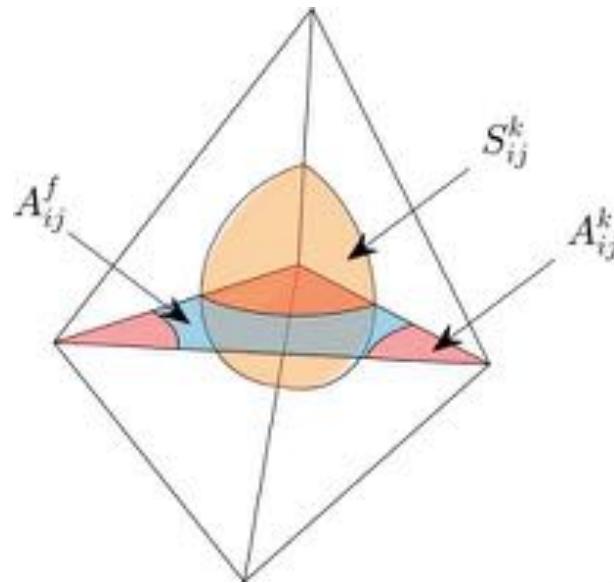
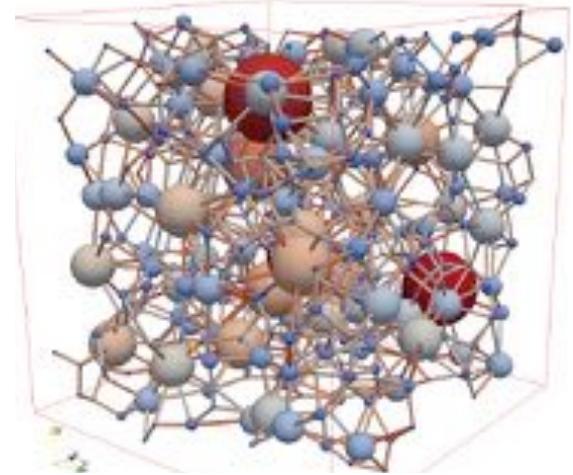
PNM = pore network model

IBM = Immersed boundary method

Dashed lines – Kozeny Carmen

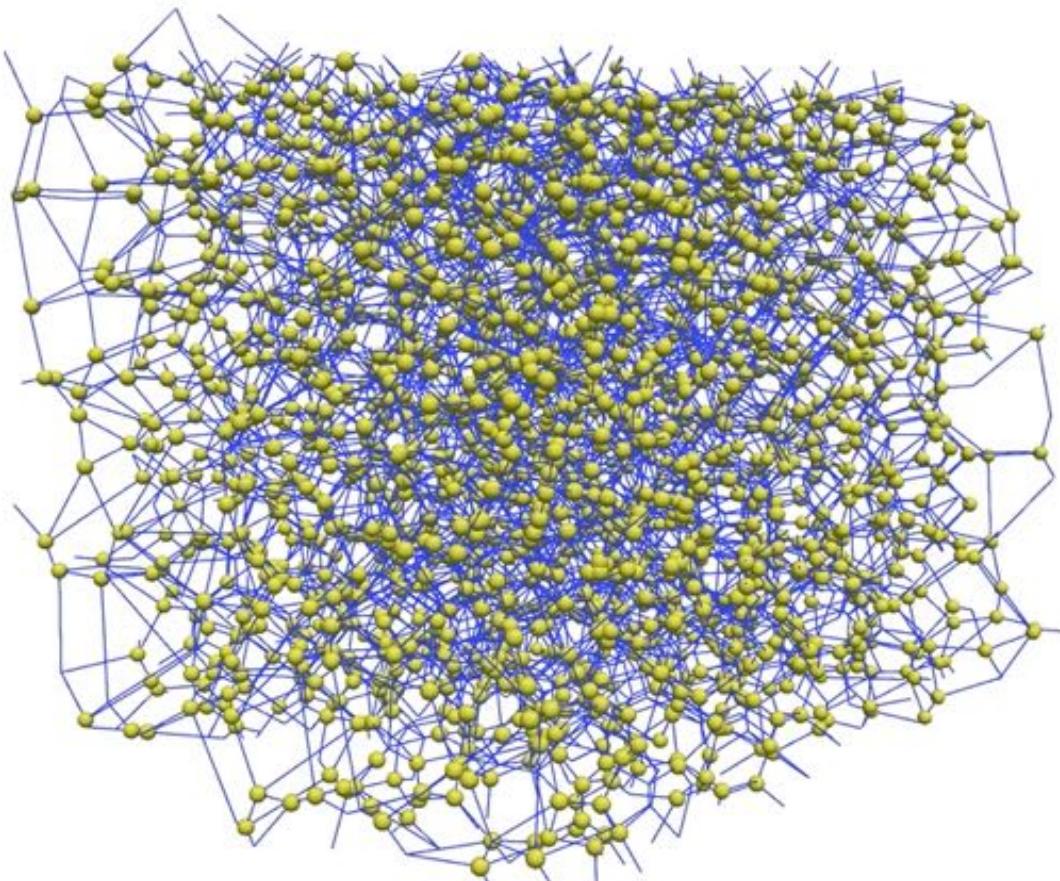
Pore throat series model

Determining $F_{f \rightarrow s}$

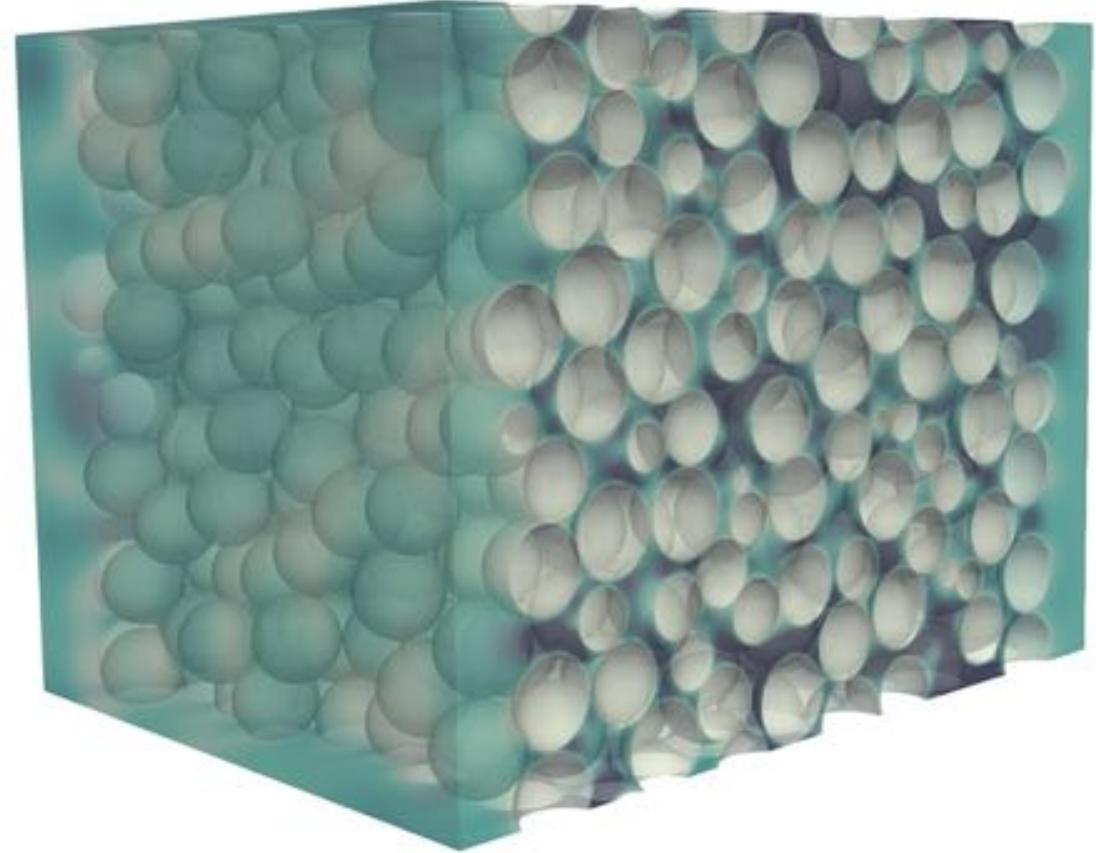


- Chareyre et al. approach to determine $F_{f \rightarrow s}$
- Considers pressure drop, area of intersection of particle and constriction and surface area of particle along edge

Towards a more efficient and accurate PNM

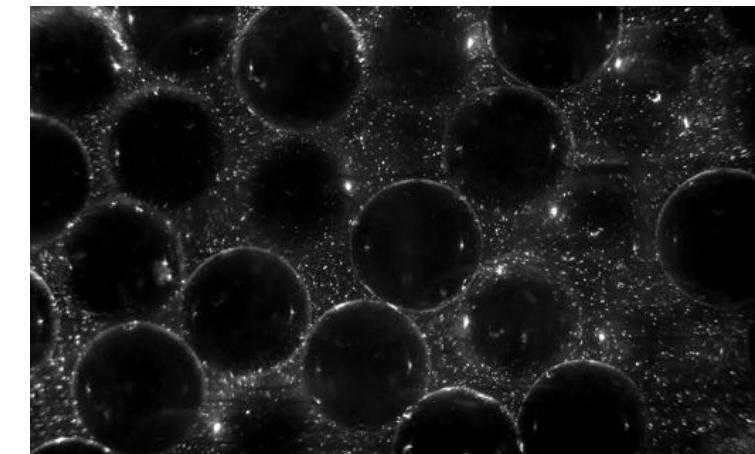
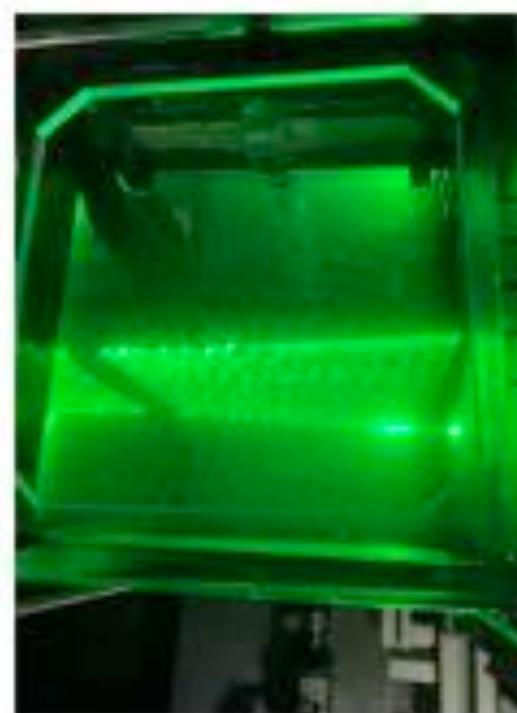
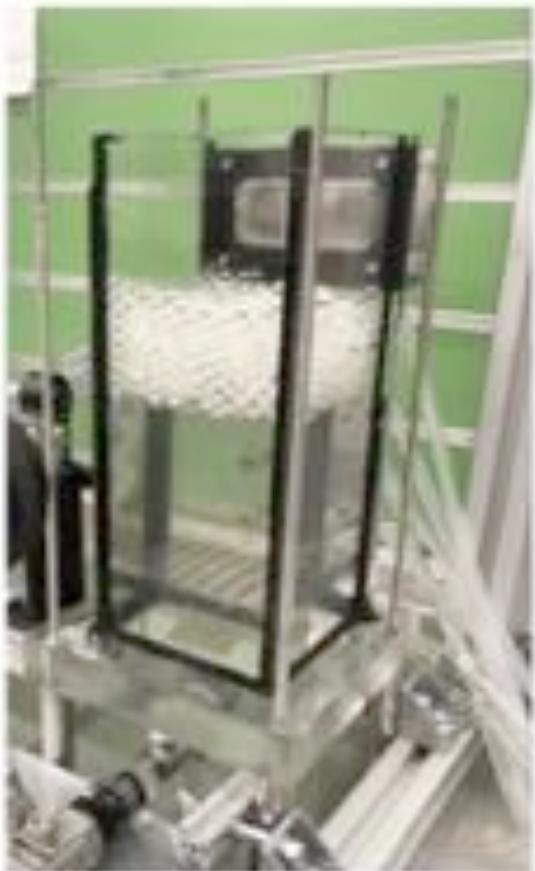


No merging of tetrahedra – focus on PNM formulation to improve accuracy

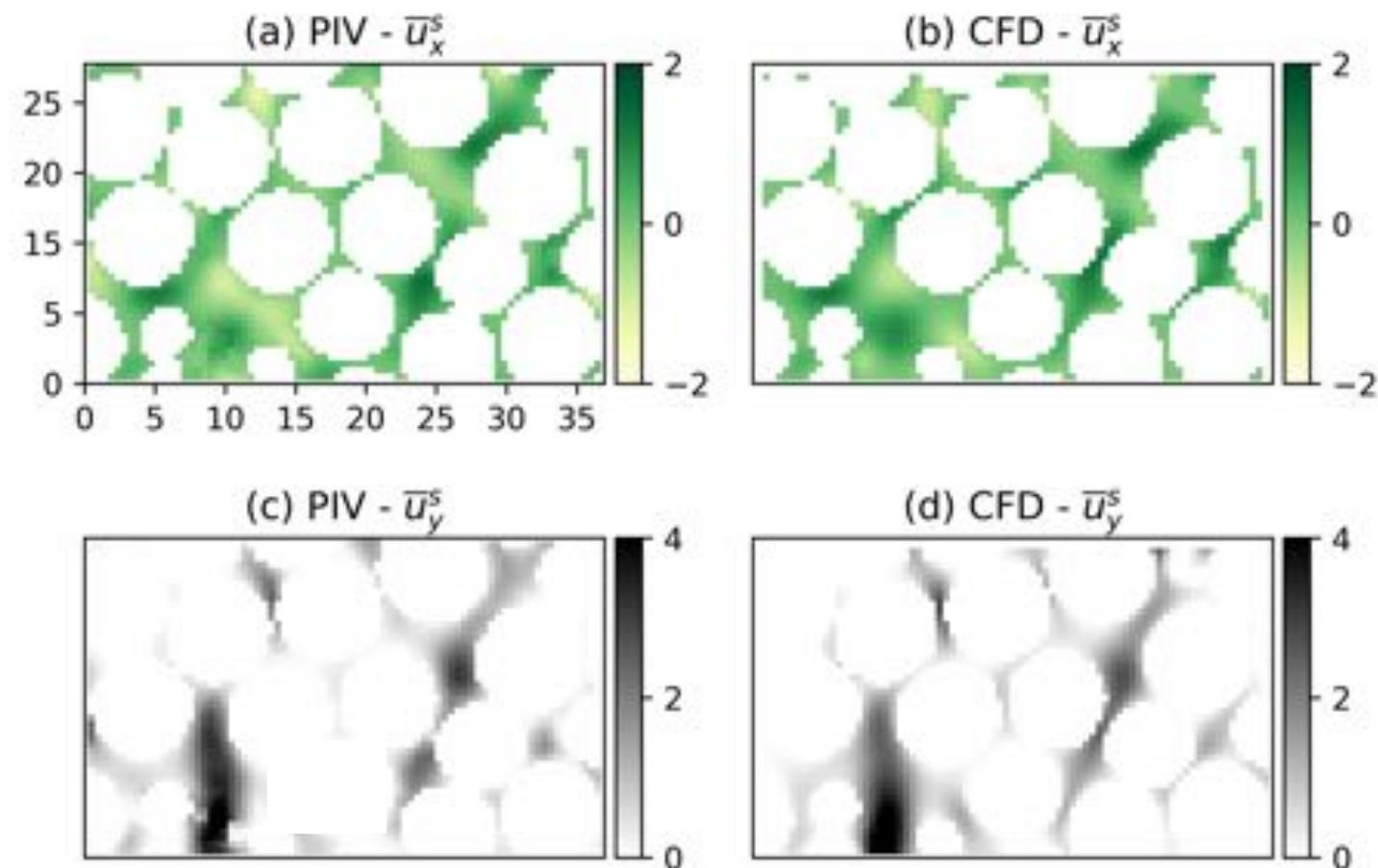
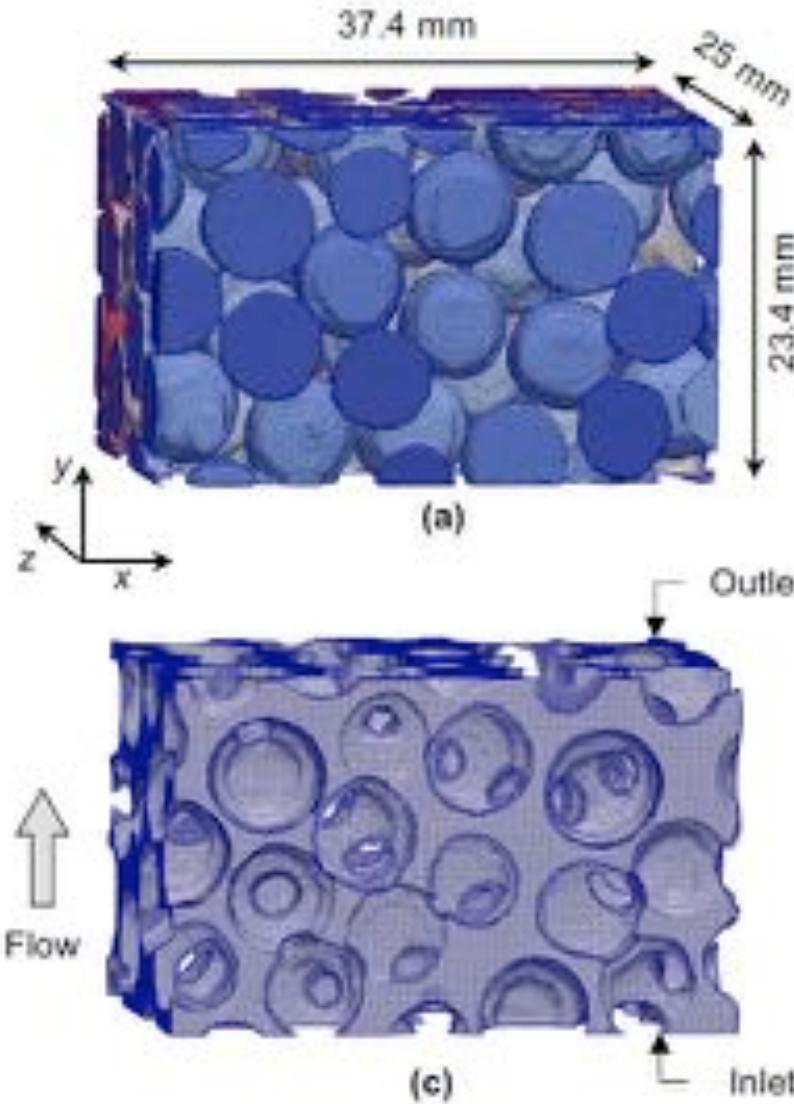


Calibration of empirical parameters using CFD simulations in OpenFOAM

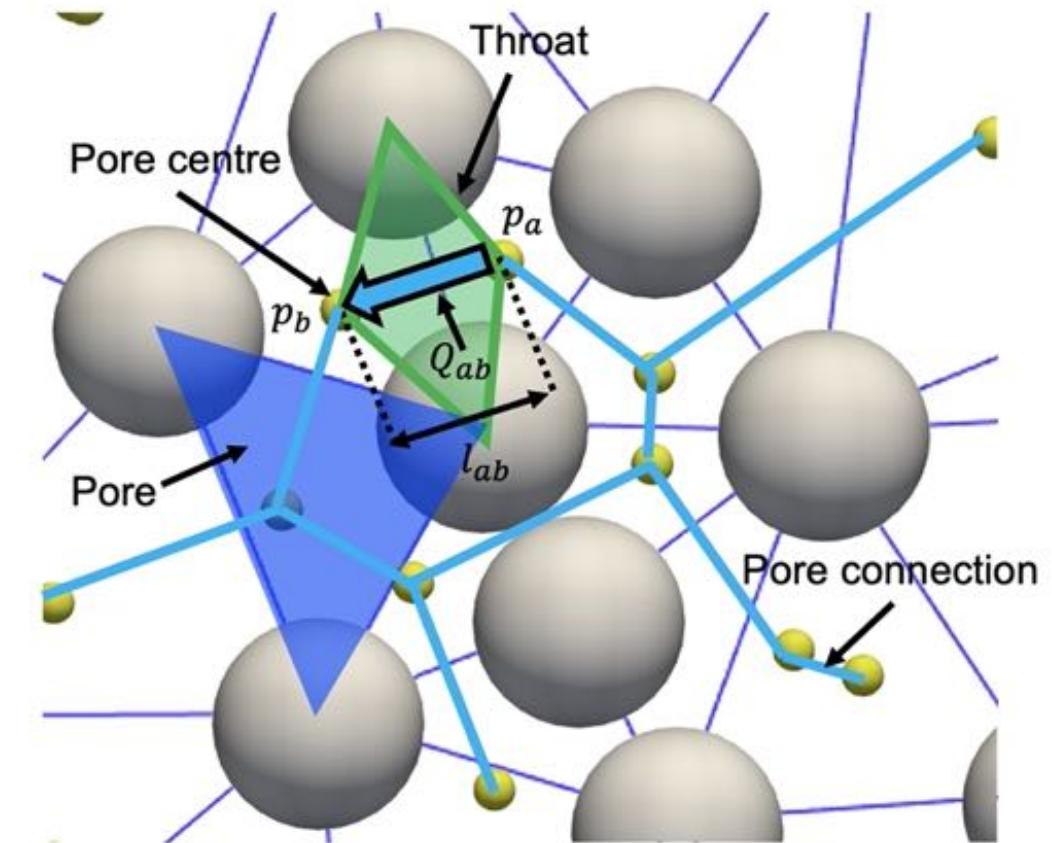
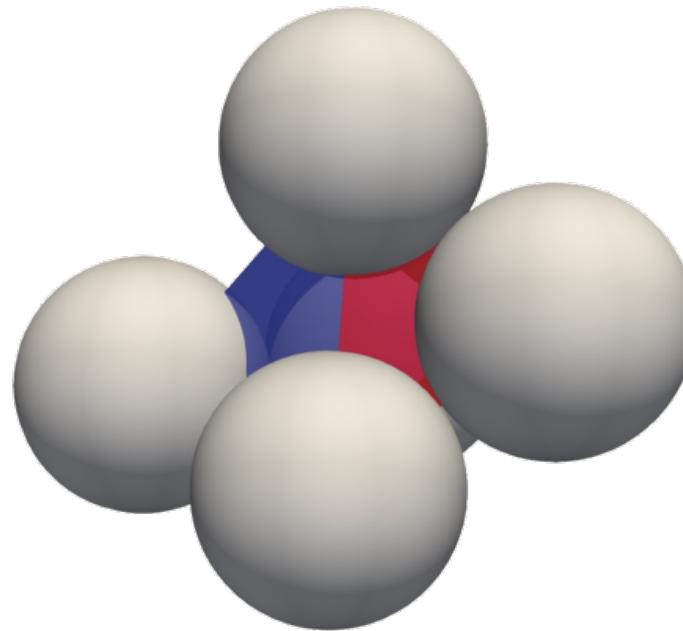
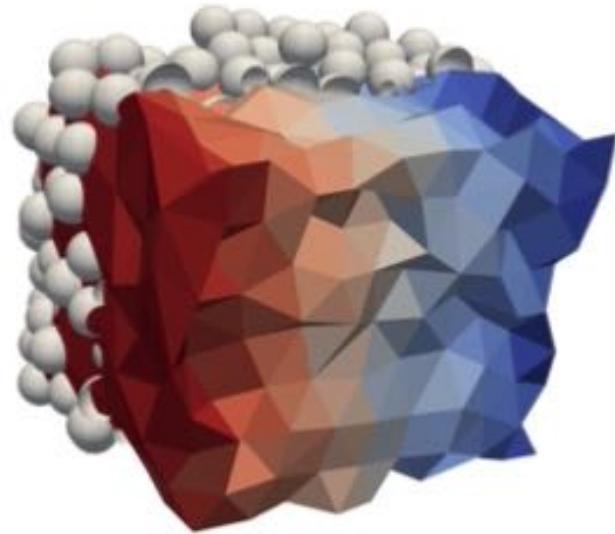
Validating OpenFOAM approach



Validating OpenFOAM approach

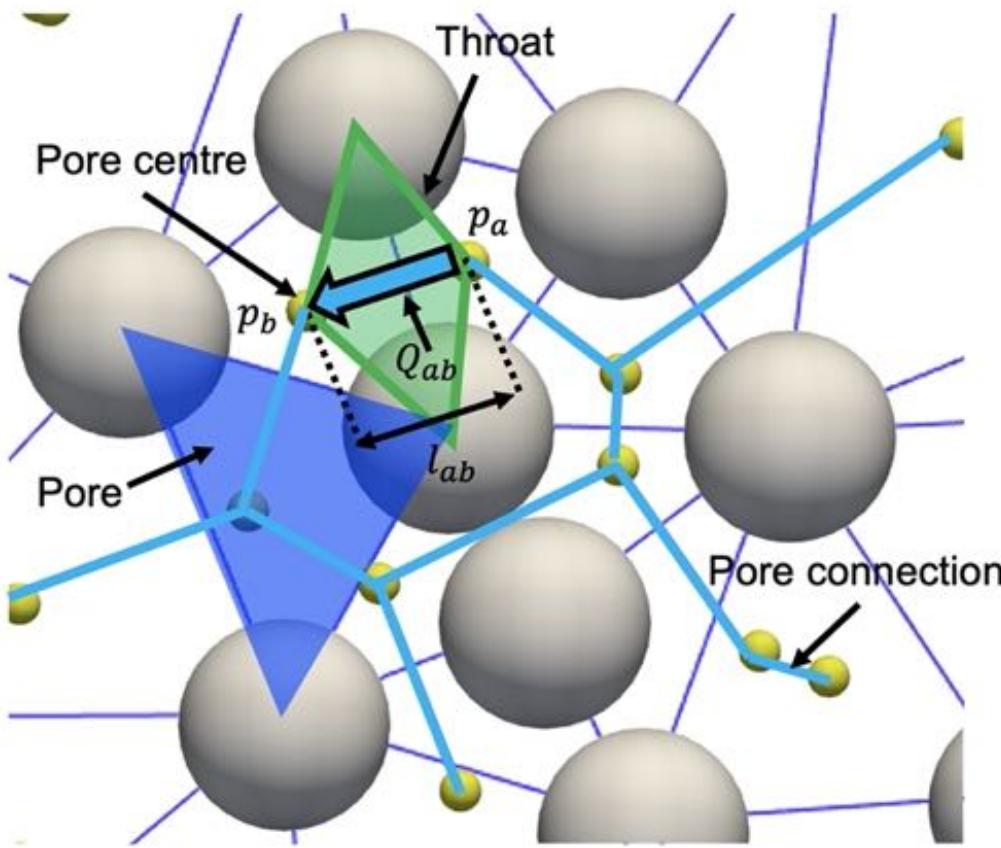


Developing the PNM

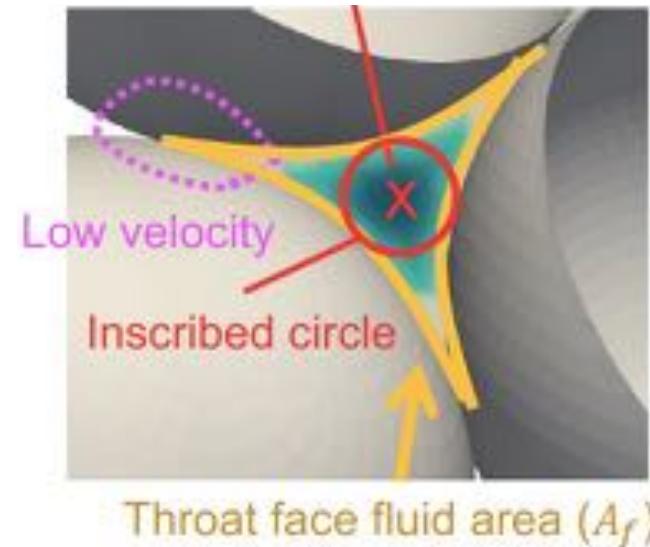


Developing the PNM

Flow from pore a to pore b $q_{ab} = g_{ab}(p_a - p_b)$

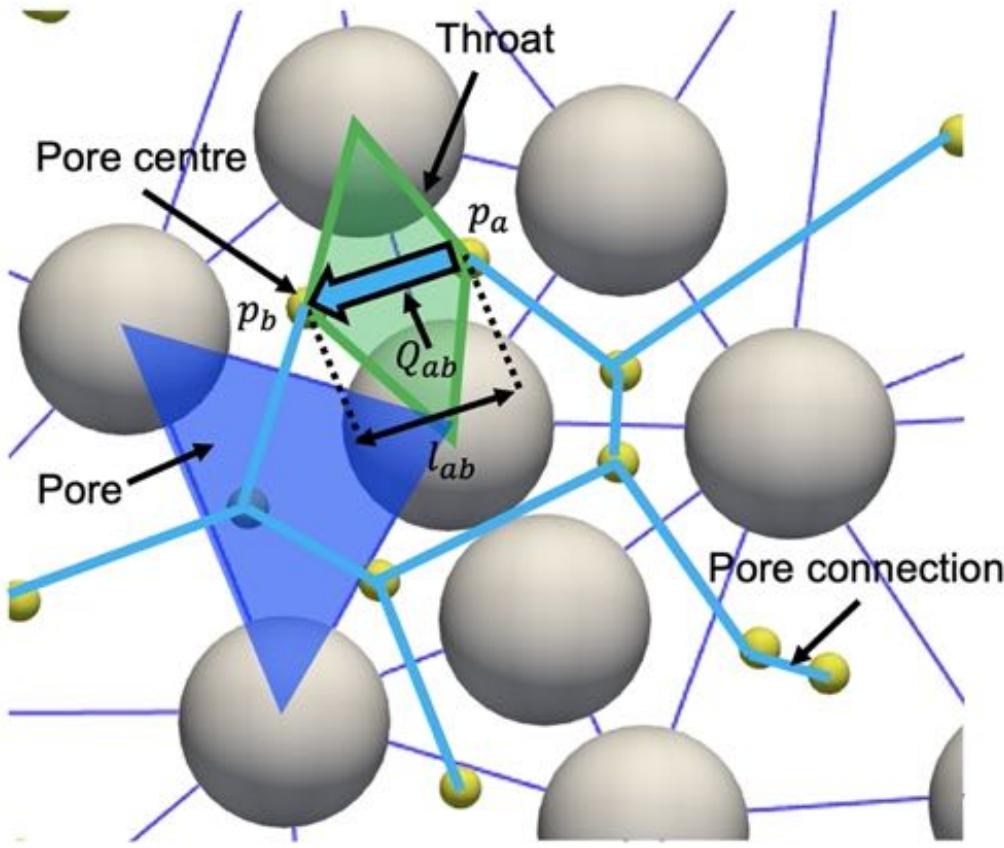


ER: $g_{ab} = \frac{\alpha_{ER} \pi r_{eff}^4}{\mu l_{ab}}$; $r_{eff} = f(\text{throat incircle}, A_f)$



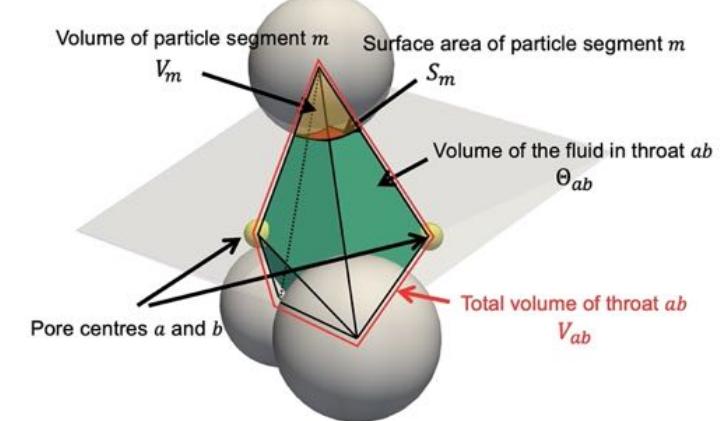
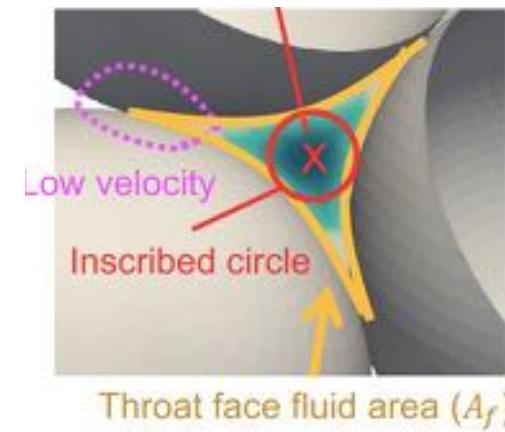
Developing the PNM

Flow from pore a to pore b $q_{ab} = g_{ab}(p_a - p_b)$



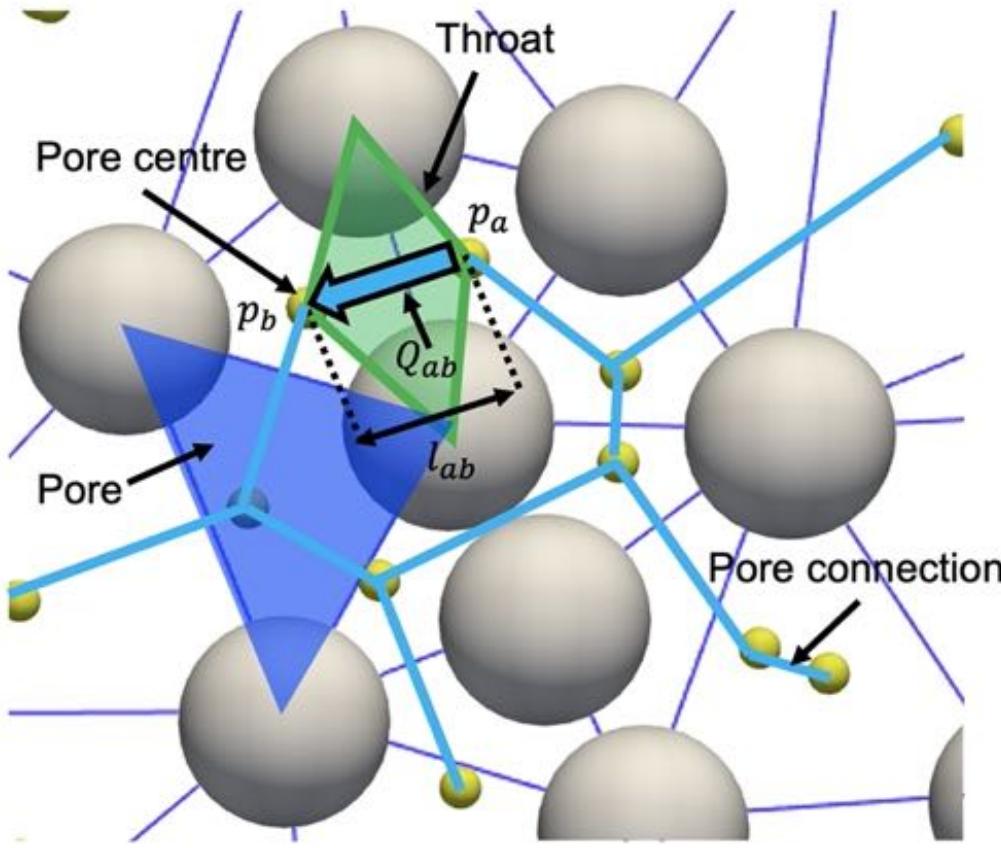
ER: $g_{ab} = \frac{\alpha_{ER}\pi r_{eff}^4}{\mu l_{ab}}$; $r_{eff} = f(\text{throat incircle}, A_f)$

HR: $g_{ab} = \frac{4\alpha_{HR}A_f r_h^2}{\mu l_{ab}}$; $r_h = f(\Theta_{ab}, S_m)$



Developing the PNM

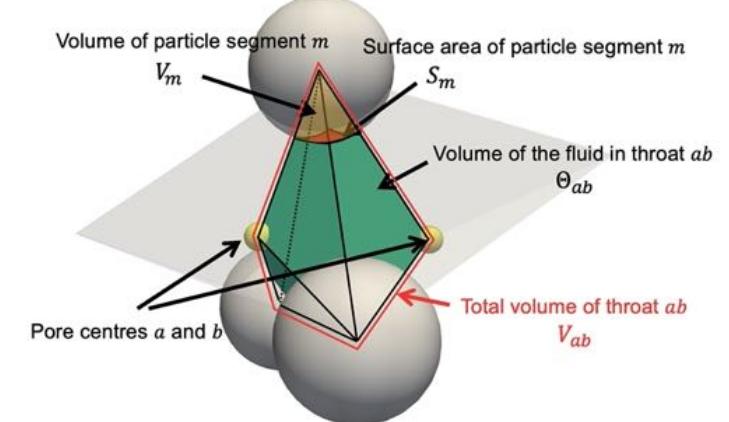
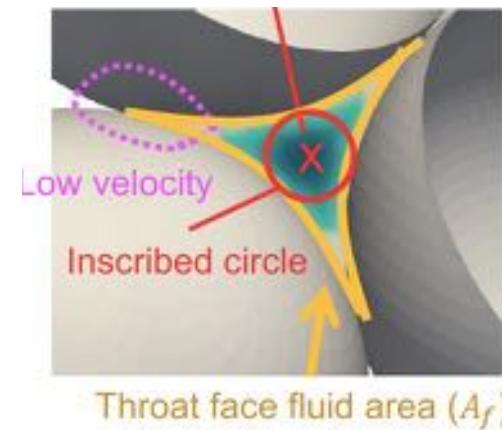
Flow from pore a to pore b $q_{ab} = g_{ab}(p_a - p_b)$



$$\text{ER: } g_{ab} = \frac{\alpha_{ER}\pi r_{eff}^4}{\mu l_{ab}}; r_{eff} = f(\text{throat incircle, } A_f)$$

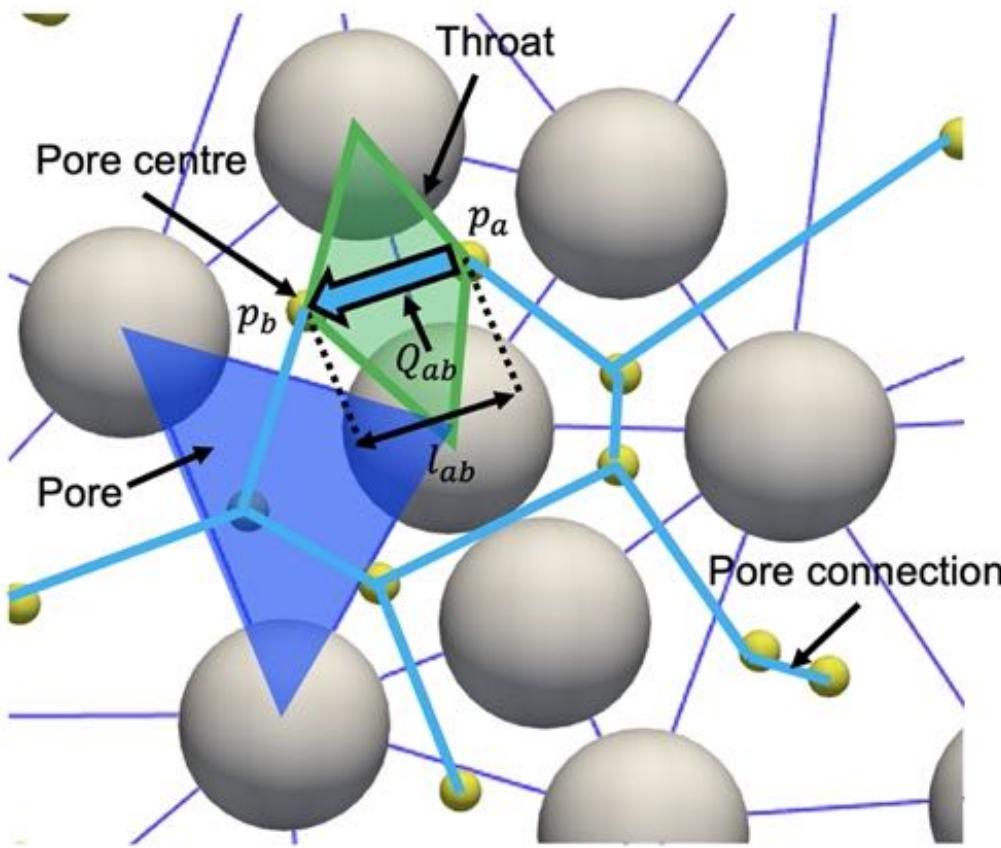
$$\text{HR: } g_{ab} = \frac{4\alpha_{HR}A_f r_h^2}{\mu l_{ab}}; r_h = f(\Theta_{ab}, S_m)$$

$$\text{HR4: } g_{ab} = \frac{4\alpha_{HR4}r_h^4}{\mu l_{ab}}; r_h = f(\Theta_{ab}, S_m)$$



Developing the PNM

Flow from pore a to pore b $q_{ab} = g_{ab}(p_a - p_b)$



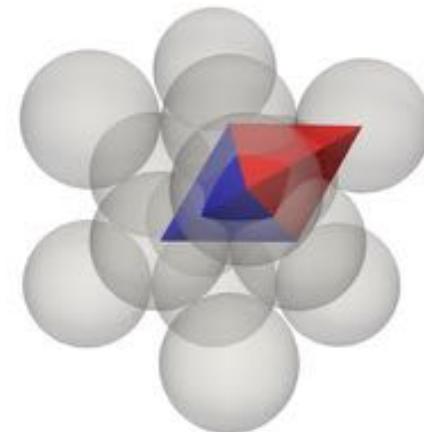
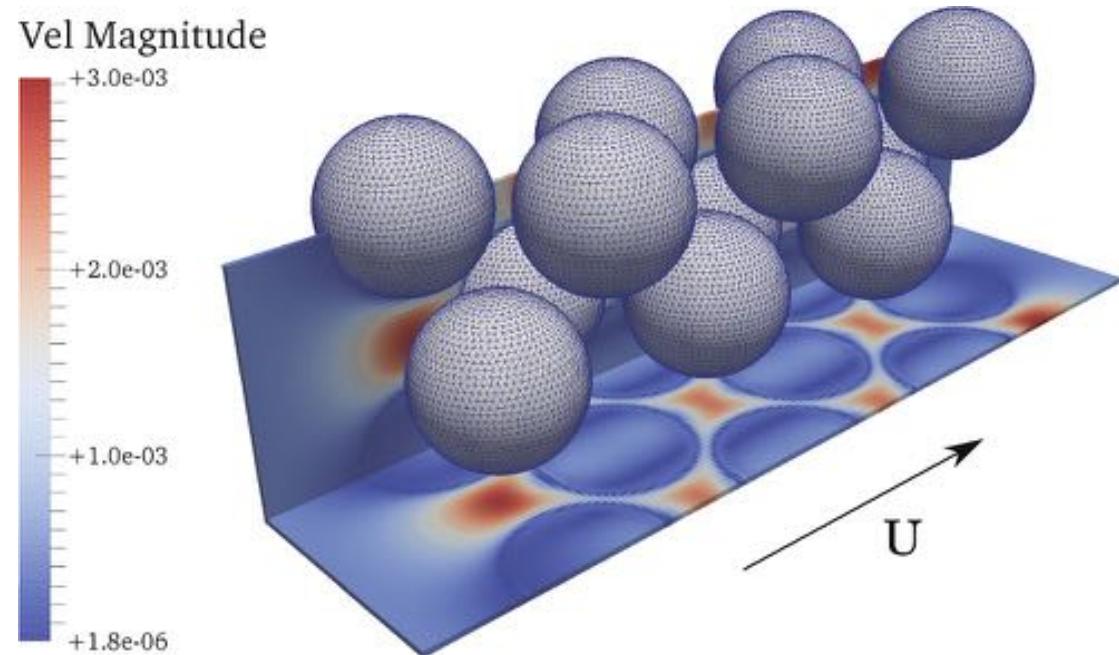
ER: $g_{ab} = \frac{\alpha_{ER} \pi r_{eff}^4}{\mu l_{ab}}$; $r_{eff} = f(\text{throat incircle, } A_f)$

HR: $g_{ab} = \frac{4\alpha_{HR} A_f r_h^2}{\mu l_{ab}}$; $r_h = f(\Theta_{ab}, S_m)$

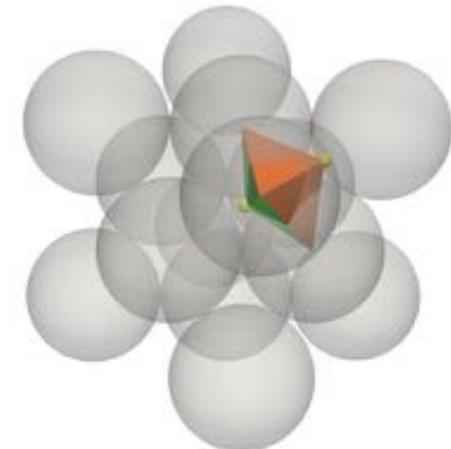
HR4: $g_{ab} = \frac{4\alpha_{HR4} r_h^4}{\mu l_{ab}}$

ER-HR: $g_{ab} = \frac{4\alpha_{ER_HR} r_{eff}^2 r_h^2}{\mu l_{ab}}$

Using analytical study to back-calculate conductances

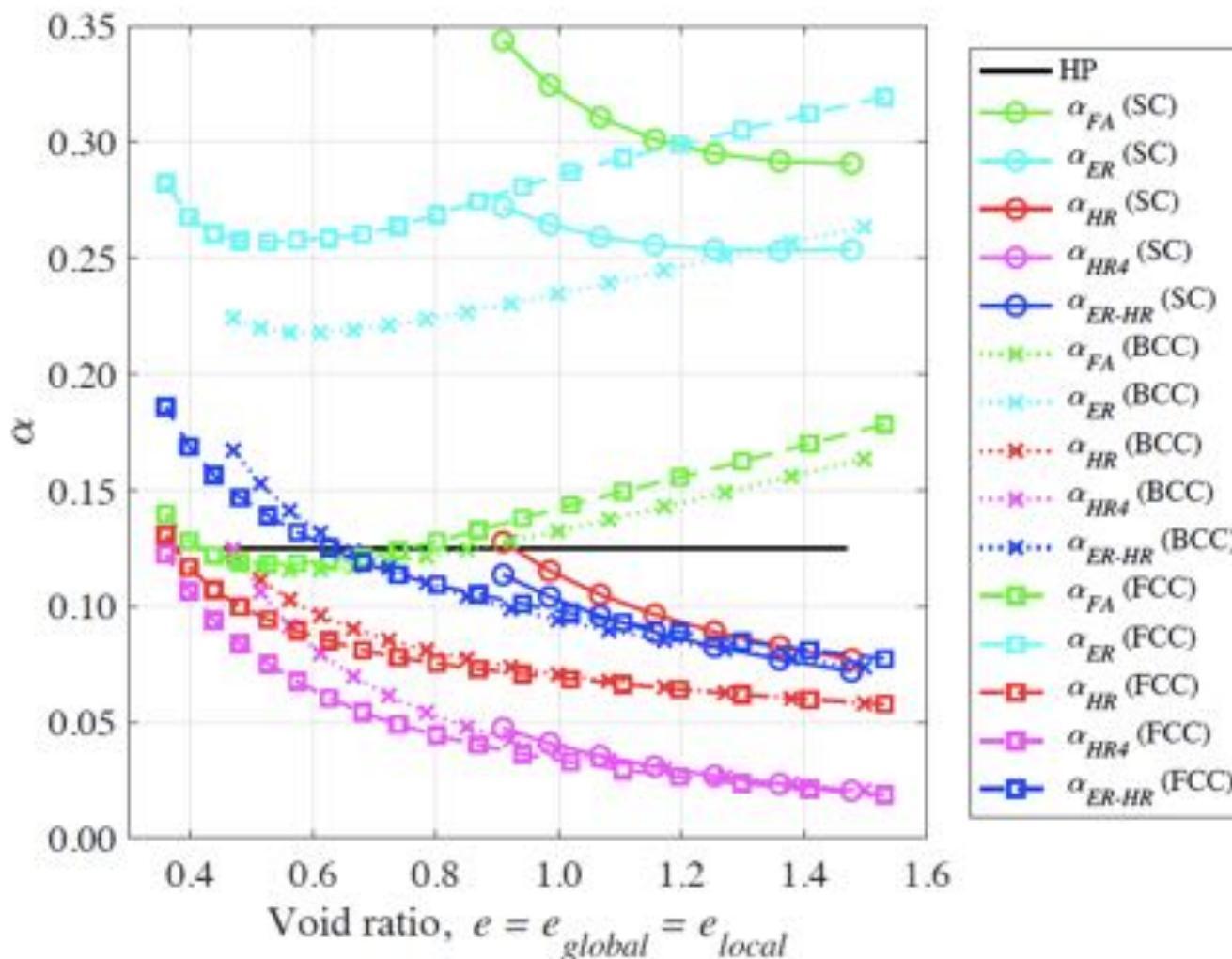


Pore - FCC



Using analytical study to back-calculate conductances

Flow from pore a to pore b $q_{ab} = g_{ab}(p_a - p_b)$



$$\text{ER: } g_{ab} = \frac{\alpha_{ER} \pi r_{eff}^4}{\mu L}; r_{eff} = f(\text{throat incircle, } A_f)$$

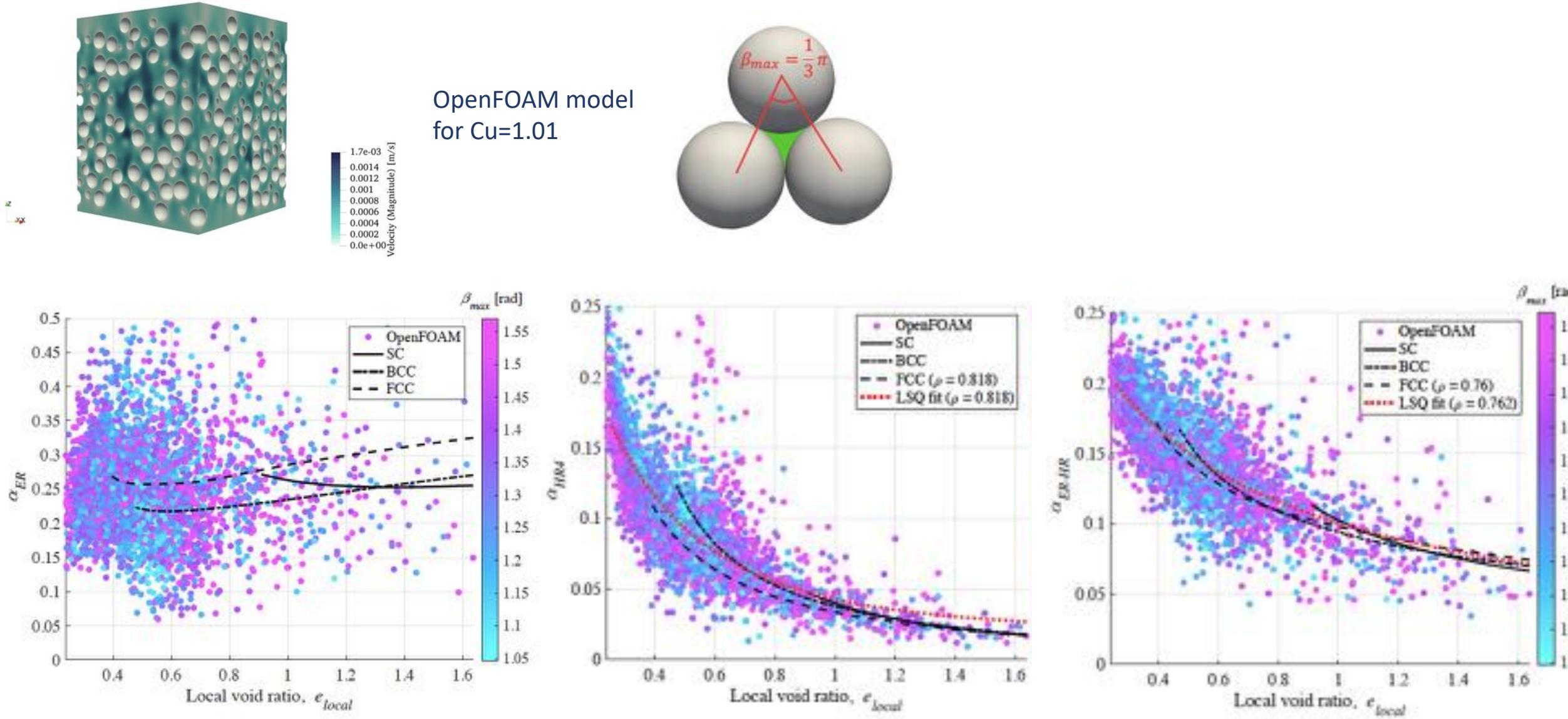
$$\text{HR: } g_{ab} = \frac{4\alpha_{HR} A_f r_h^2}{\mu L}; r_h = f(\Theta_{ab}, S_m)$$

$$\text{HR4: } g_{ab} = \frac{4\alpha_{HR4} r_h^4}{\mu L}$$

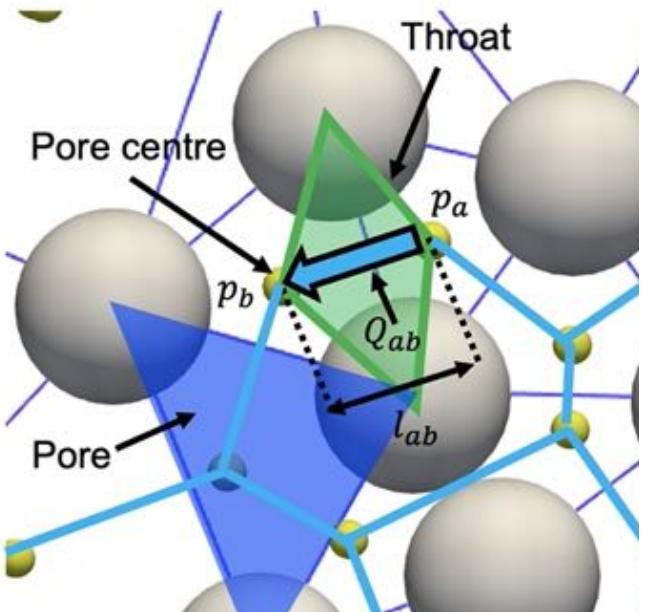
$$\text{ER-HR: } g_{ab} = \frac{4\alpha_{ER_HR} r_{eff}^2 r_h^2}{\mu L}$$

Data used to develop expressions for $\alpha(e)$ for each packing

Accuracy of conductance models

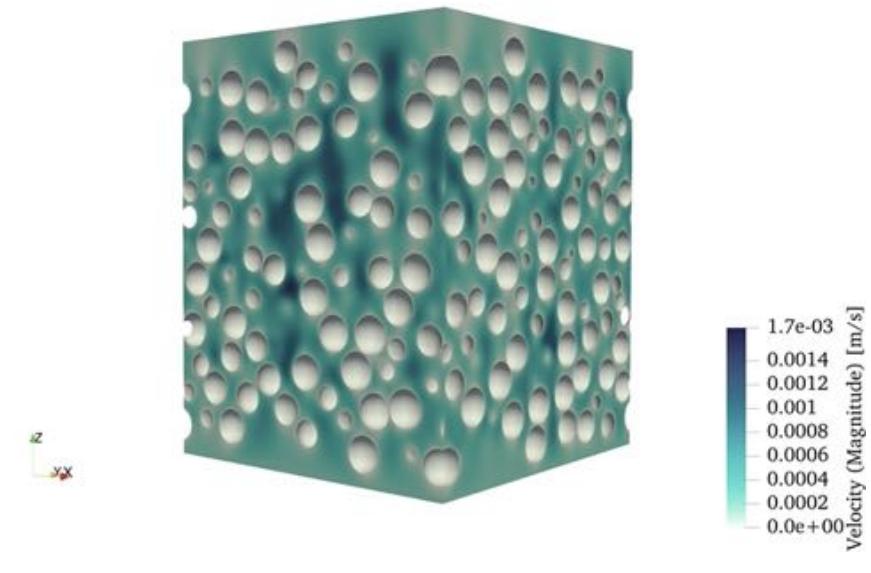


Importance of defining edges and nodes

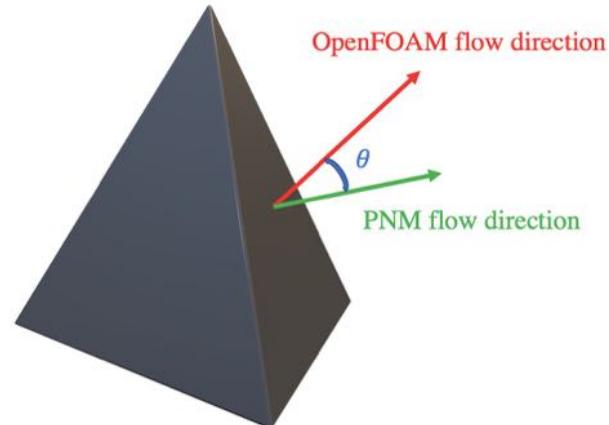


V-V model – pore
nodes at Voronoi
nodes – flow
orthogonal to throat
face

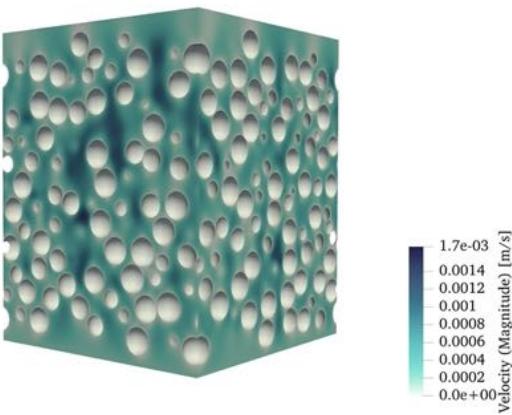
C-C model – pore
notes at centroids of
pore cells – flow not
orthogonal to throat
face



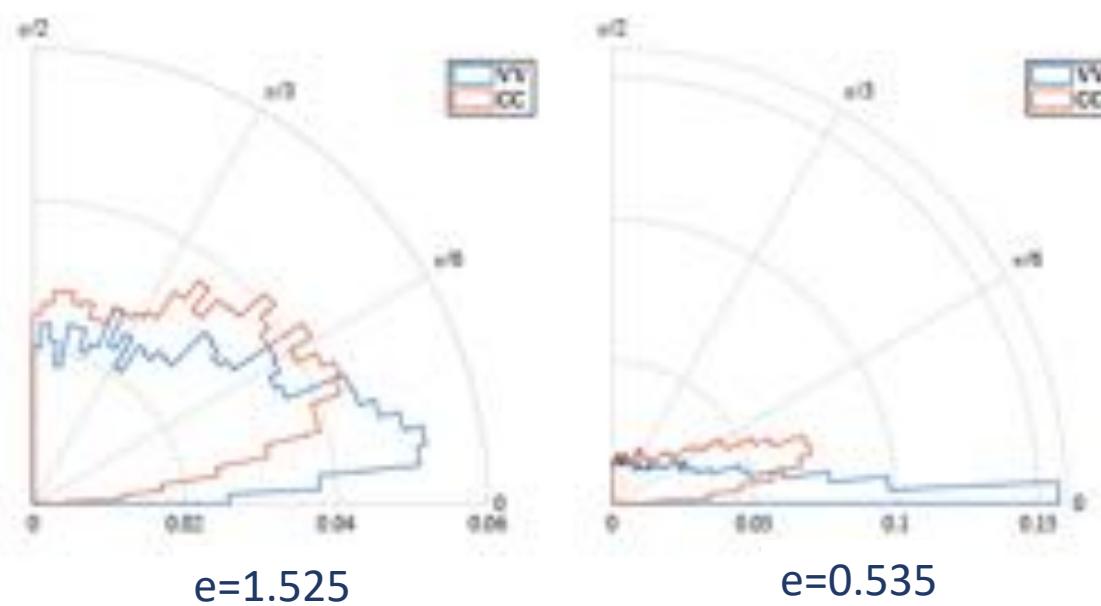
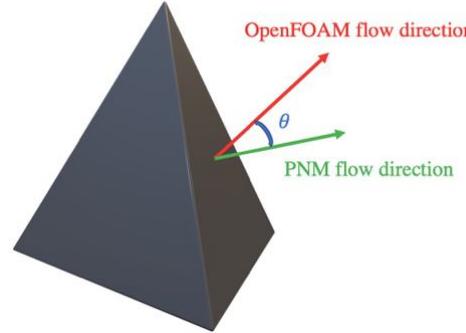
OpenFOAM model for Cu=1.01



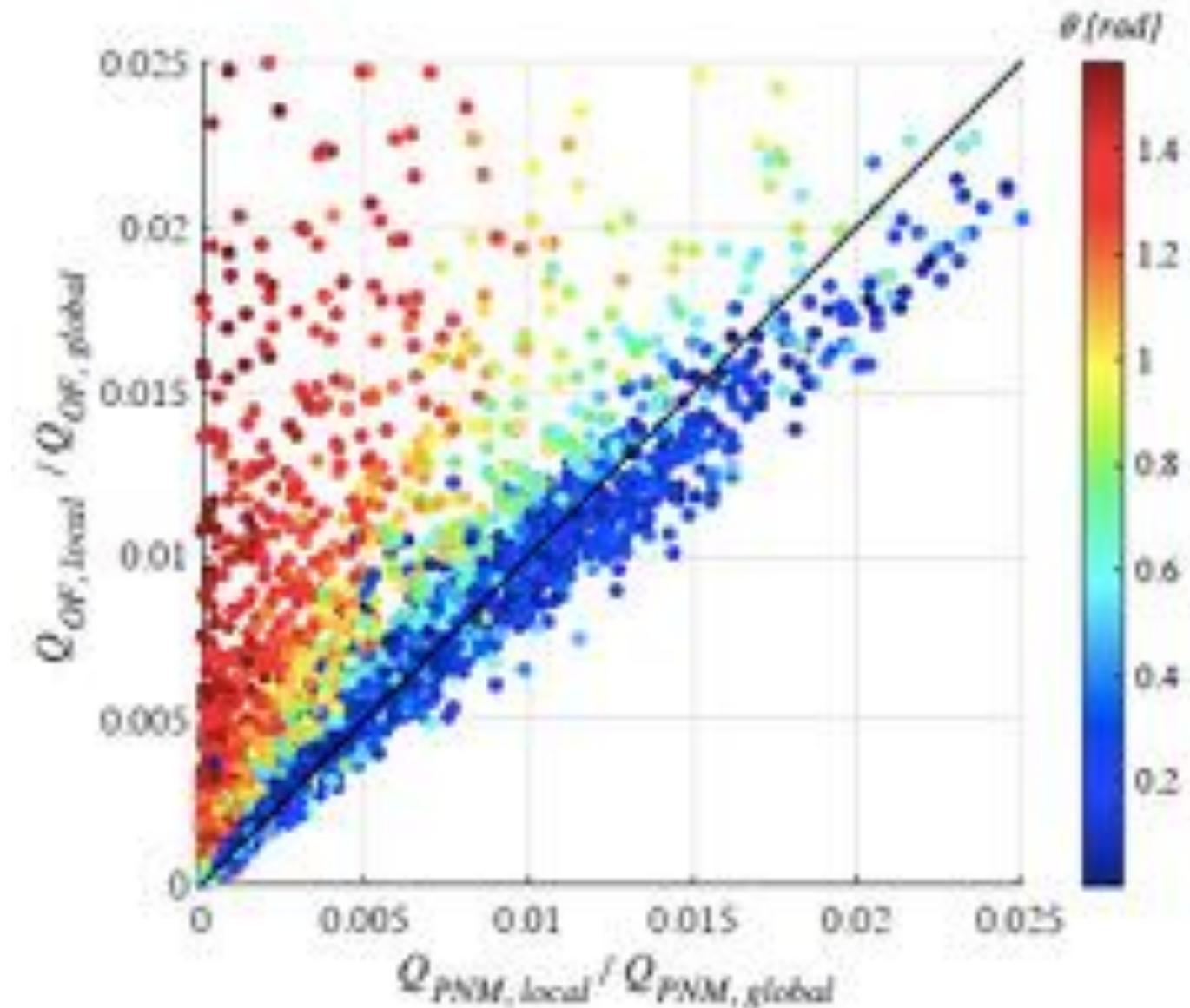
Importance of capturing flow direction



OpenFOAM model for Cu=1.01

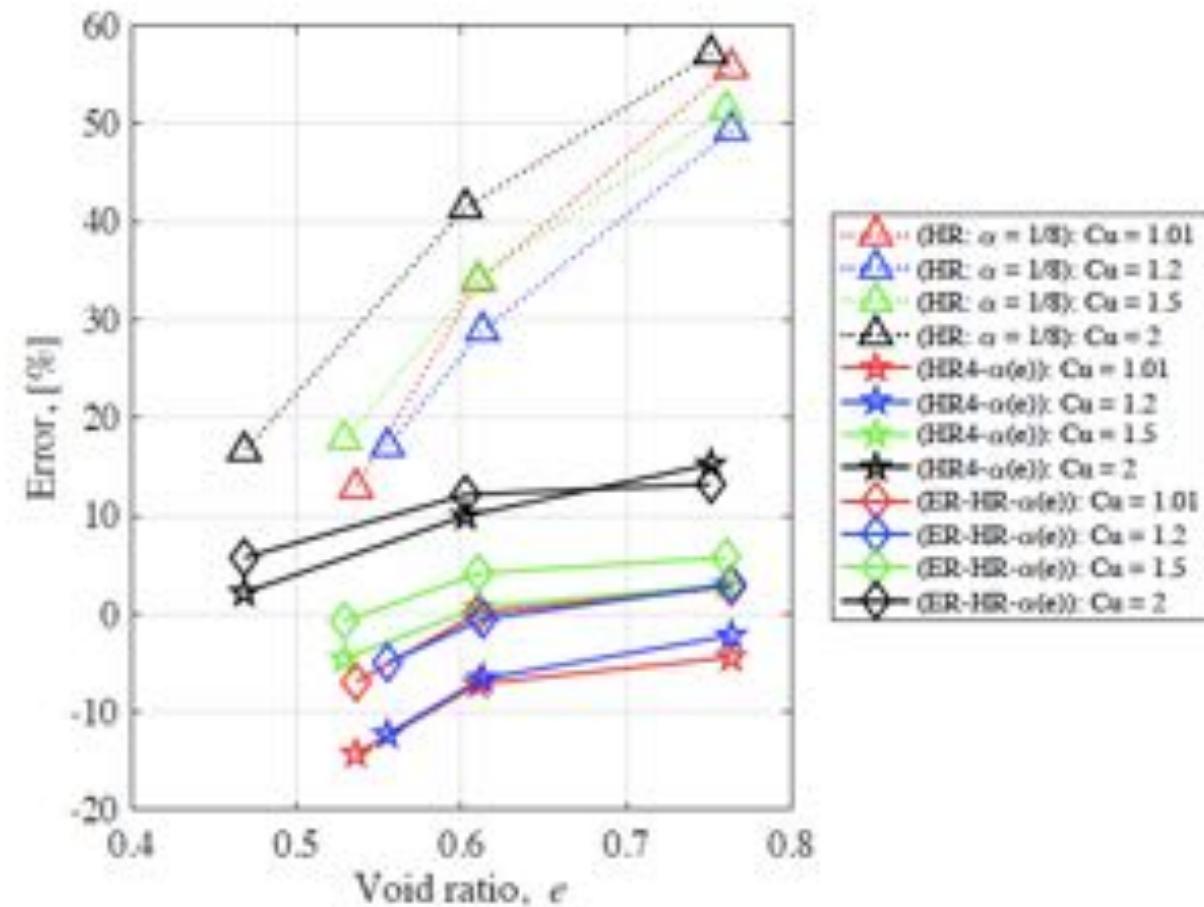
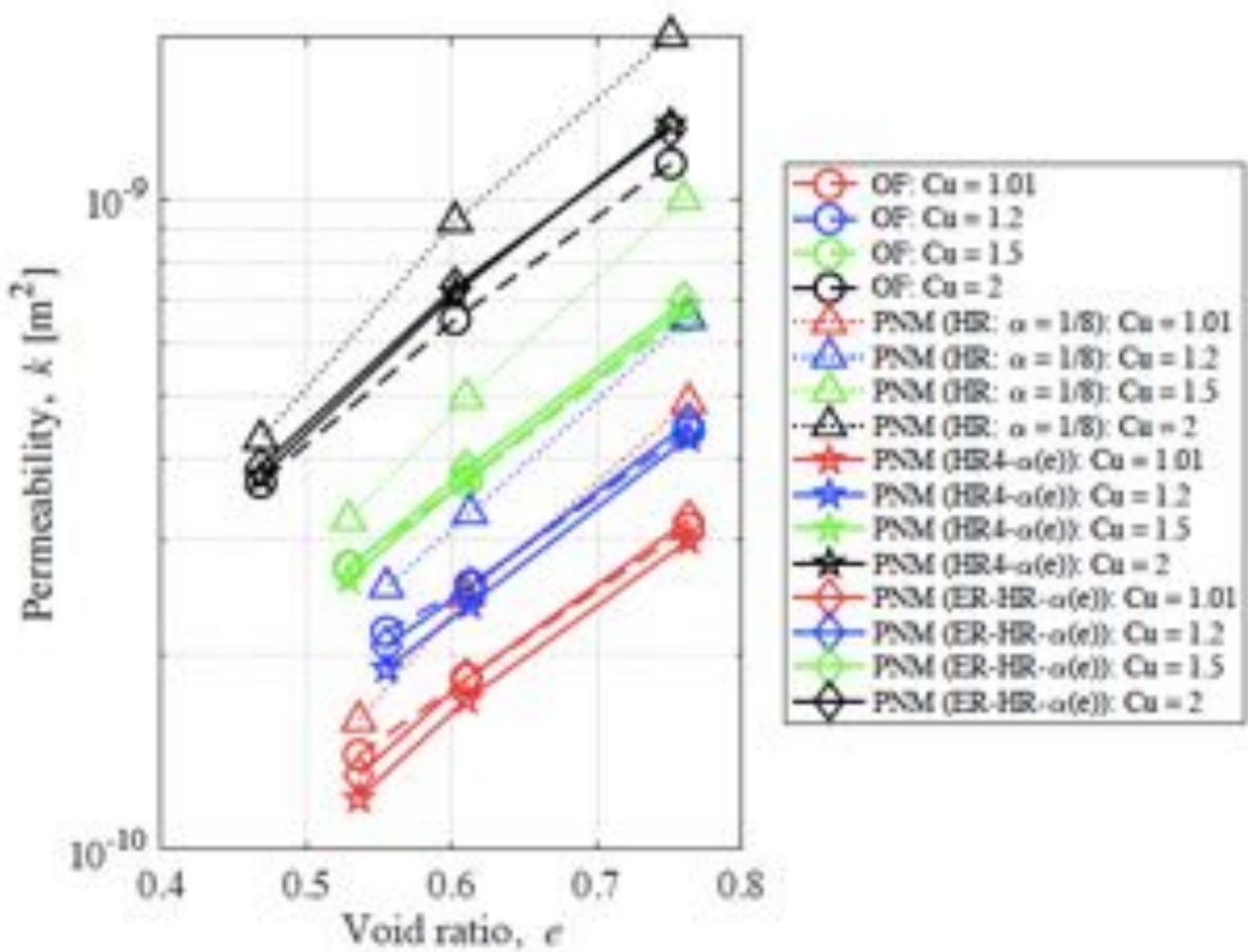


Cu=1.01

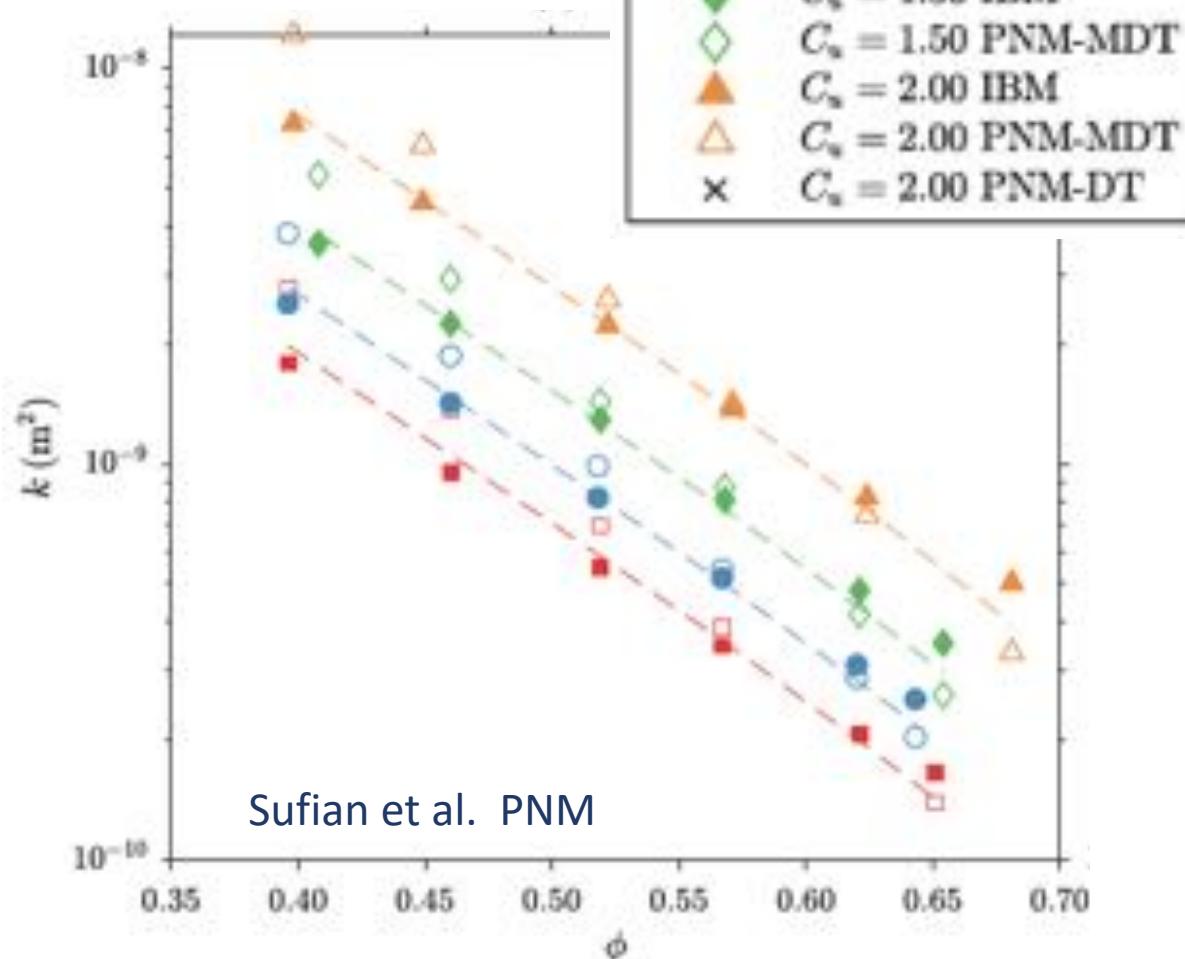
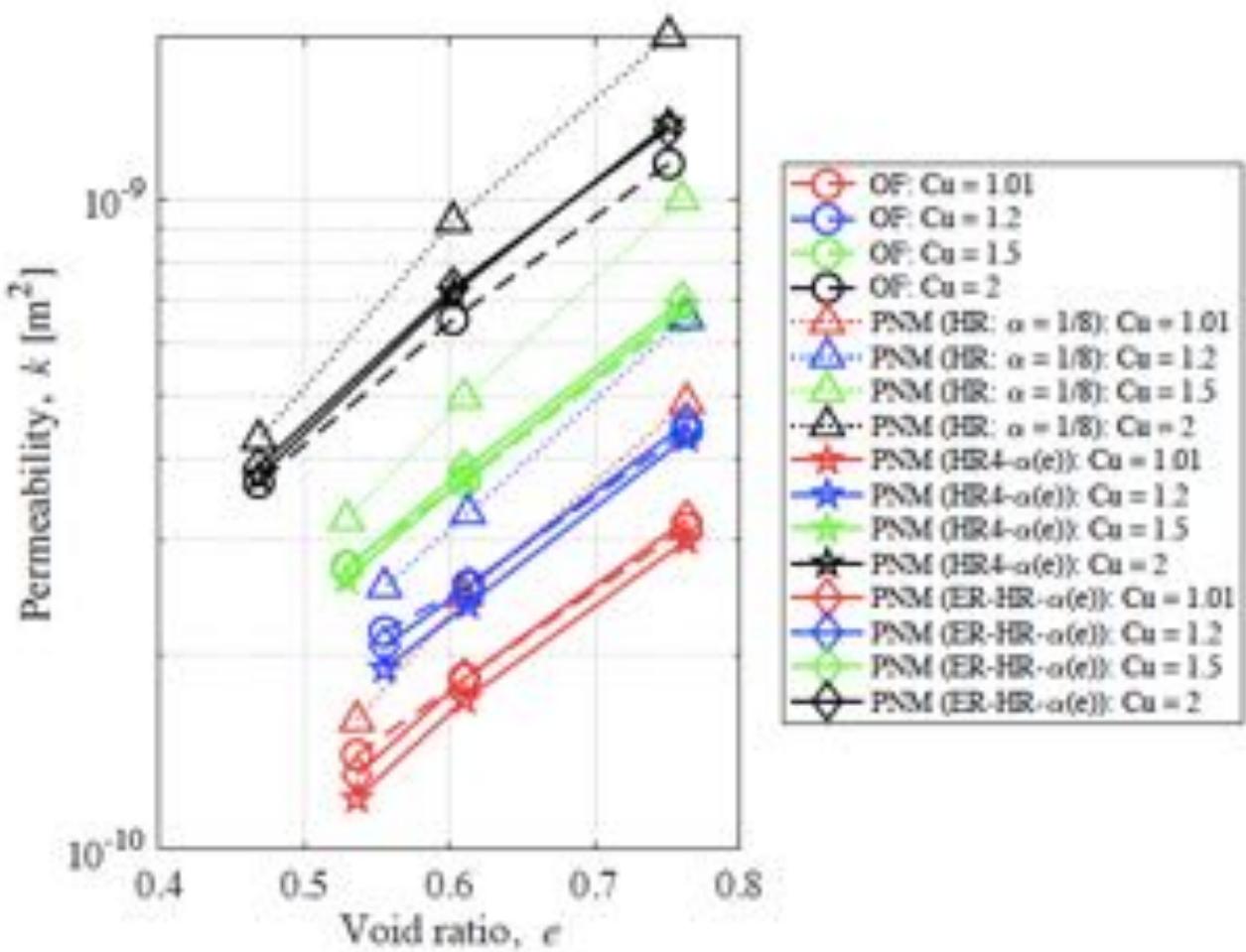


Morimoto et al. (2022) <https://doi.org/10.1016/j.compgeo.2022.104900>

Accuracy of permeability data

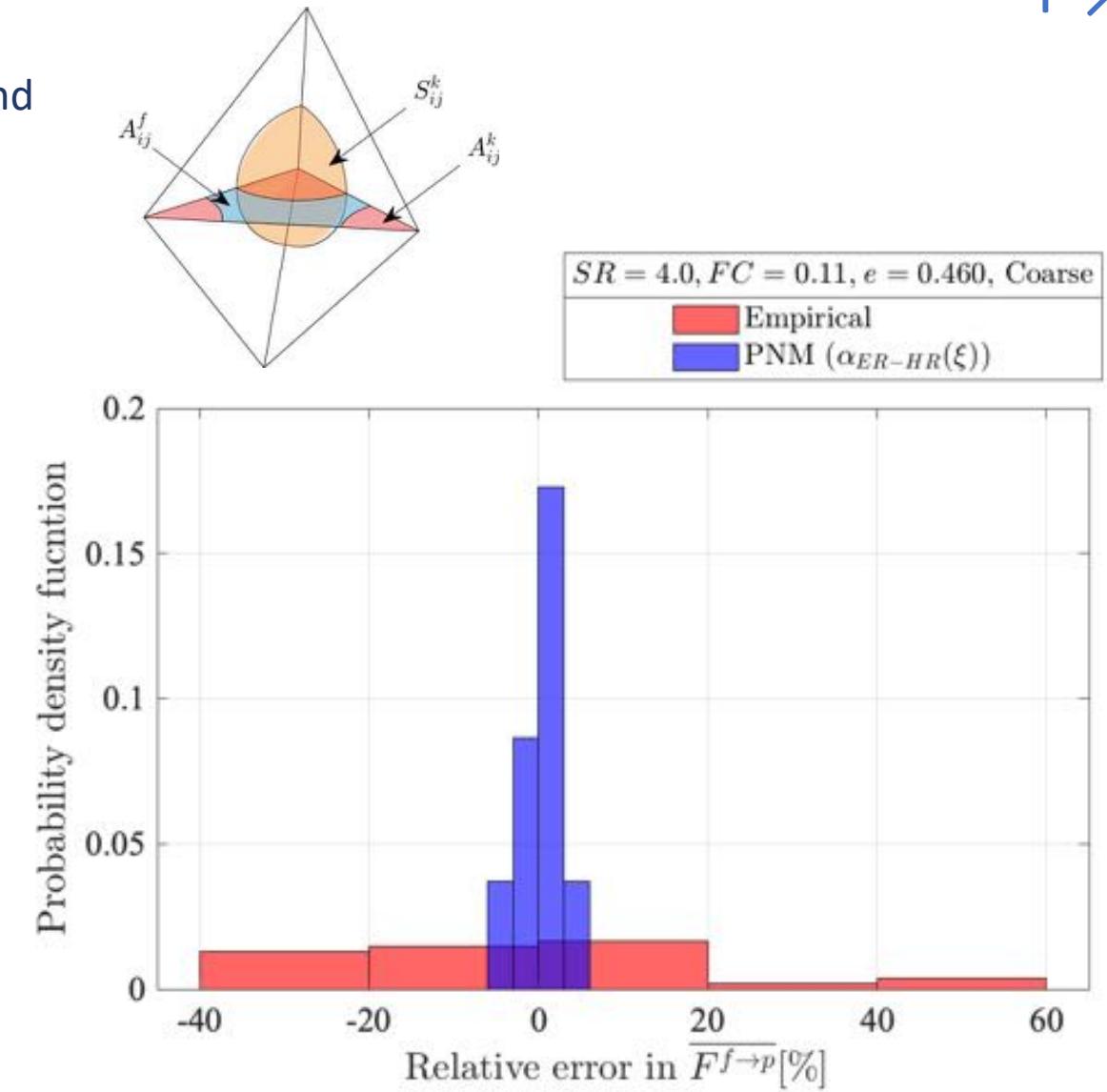
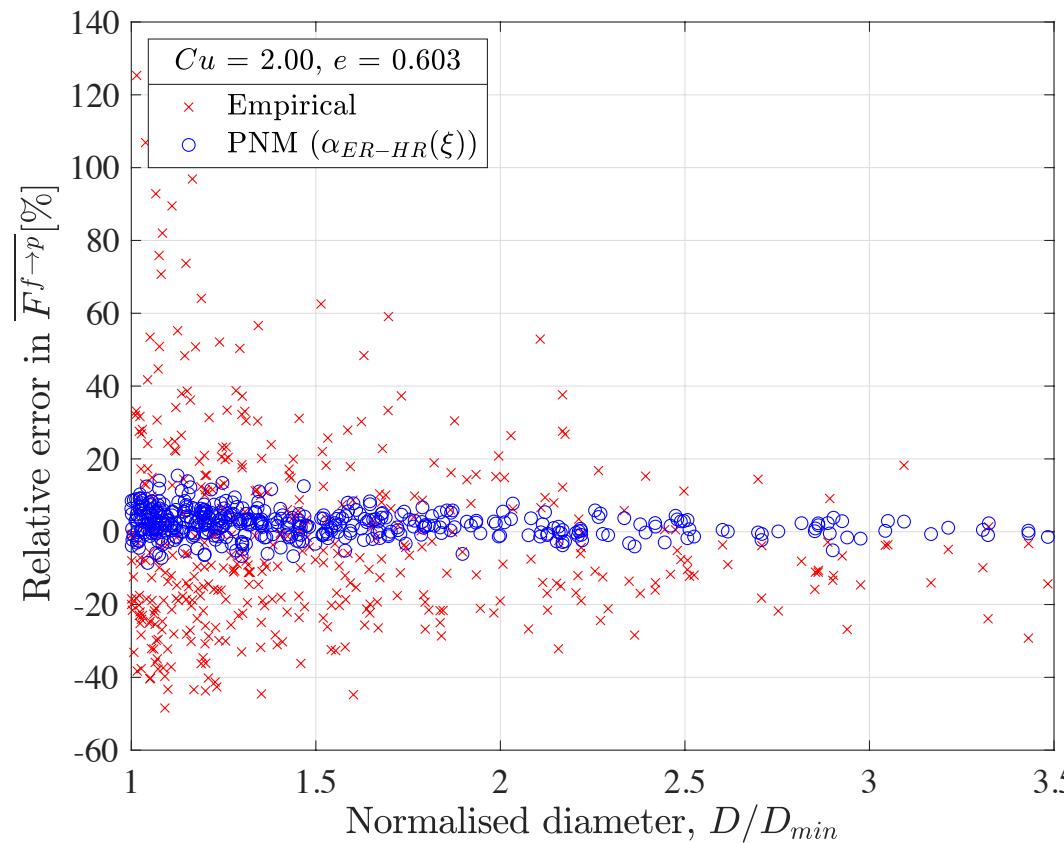


Accuracy of permeability data



Improved prediction of fluid particle interaction force $F_{f \rightarrow s}$

$F_{f \rightarrow s}$ Considers pressure drop, area of intersection of particle and constriction and surface area of particle along edge



Research questions:

- **Retention:**
Does pore network modelling support use of the ratio D_{15F}/D_{85B} in design?
- **Internal Stability:**
Can we predict the fluid particle interaction forces with pore network modelling?

Conclusions

- Filters in dams and flood embankments must retain a base material, provide adequate drainage, and be internally stable.
- Simple pore network models that do not account for flow can be used to explore empirical correlations that consider only constriction size.
- Accuracy of Pore Network Models is improved if the conductance formulations accounts for local porosity.
- Pore network models cannot accurately capture flow in very loose packings and are unlikely to be effective to simulate e.g. fluidized beds.
- Once an appropriate conductance model is found, pore network models present a very attractive option to for fluid-coupled DEM simulations.

Acknowledgements

- Engineering and Physical Sciences Research Council:
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- Horizon 2020 Marie Curie ITN Mathegram
- Research Computing Services at Imperial College London

