

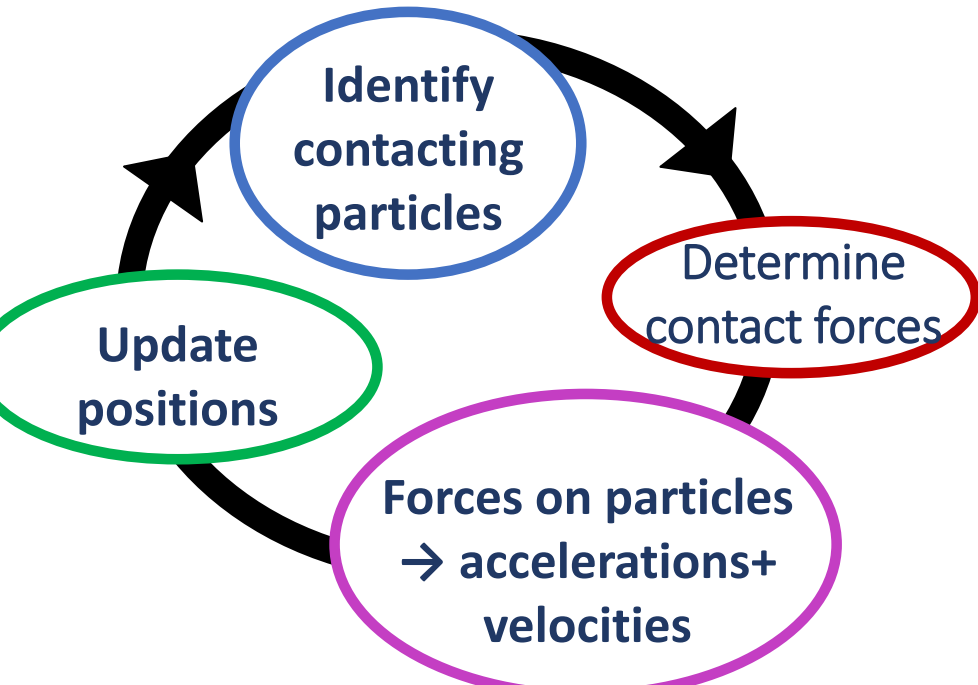
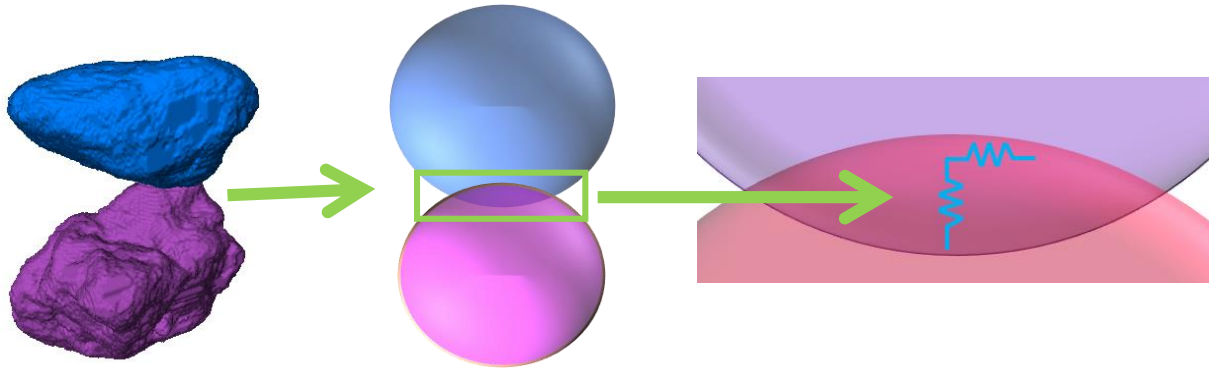
Particle scale insights into the load:deformation behaviour of sand

Catherine O'Sullivan

Kevin Hanley, Xin Huang, Masahide Otsubo

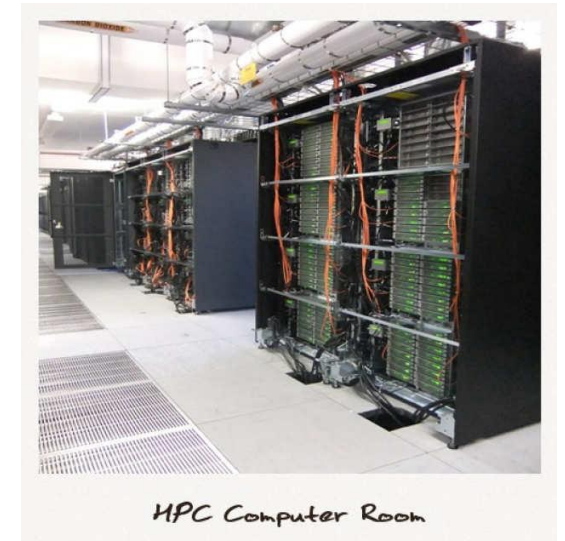
Discrete Element Method

Idealizes particle shape and contact behaviour

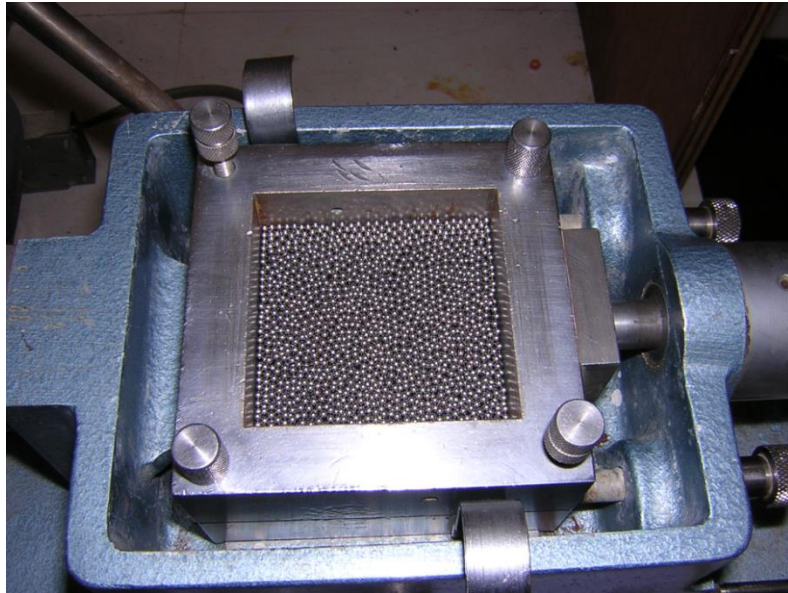


Conceptually simple algorithm – transient analysis – step forward in time

High performance computers enable larger samples to be simulated

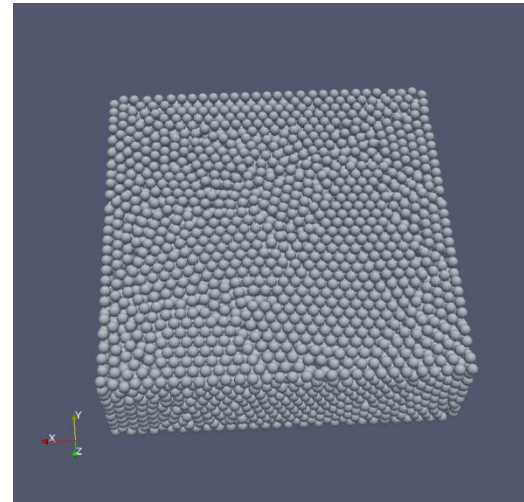


Direct Shear Test – illustration of benefits of DEM



Grade 25 steel precision balls
Diameter 0.992 mm

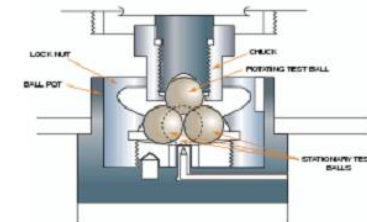
Cui and O'Sullivan (2006)
Géotechnique



DEM model: 11700 Spheres

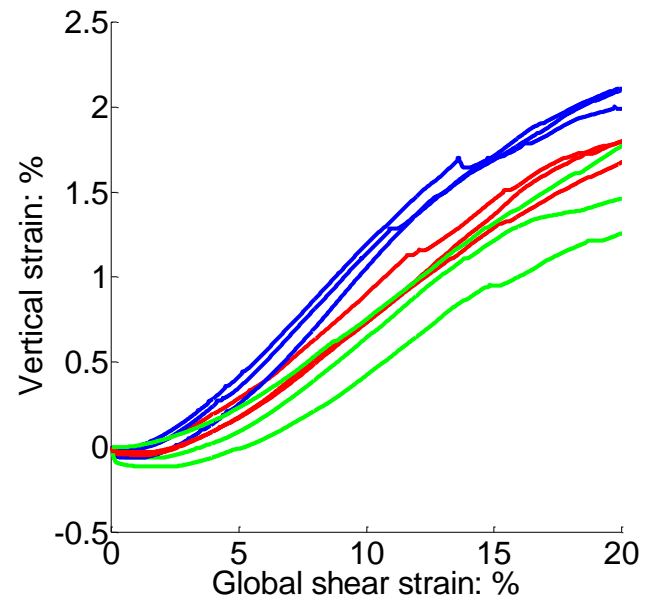
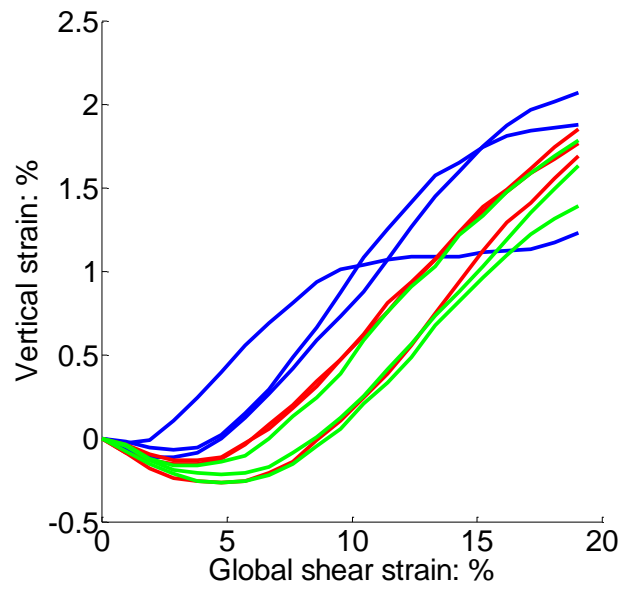
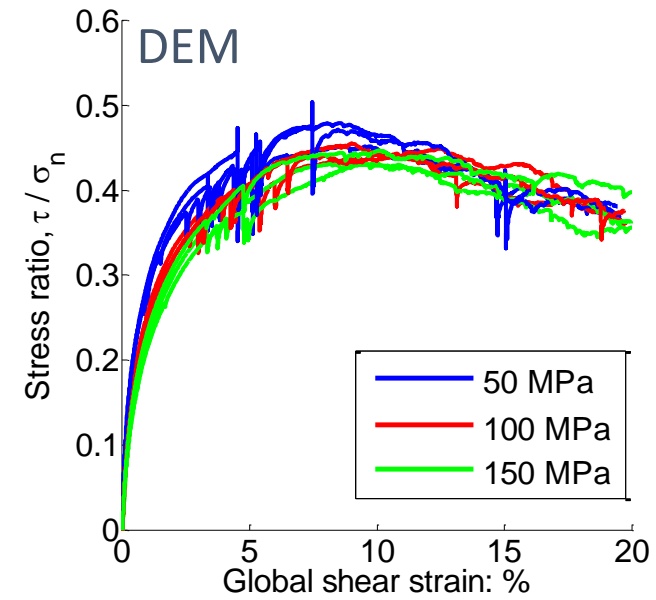
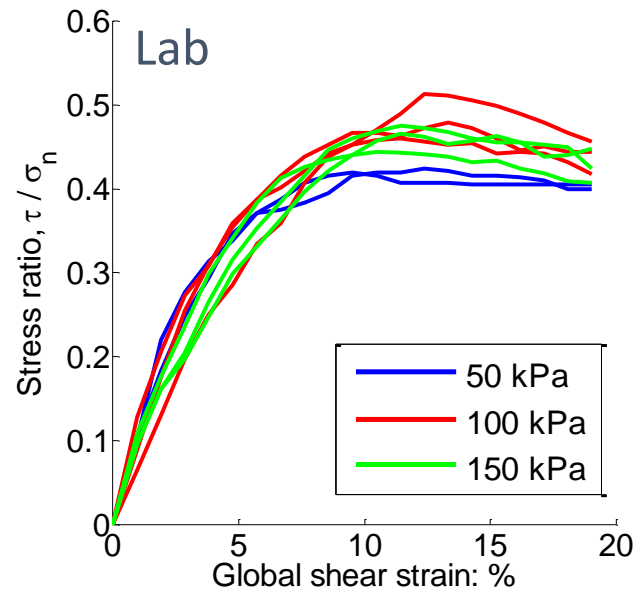


Mean μ_{surface}
5.5°

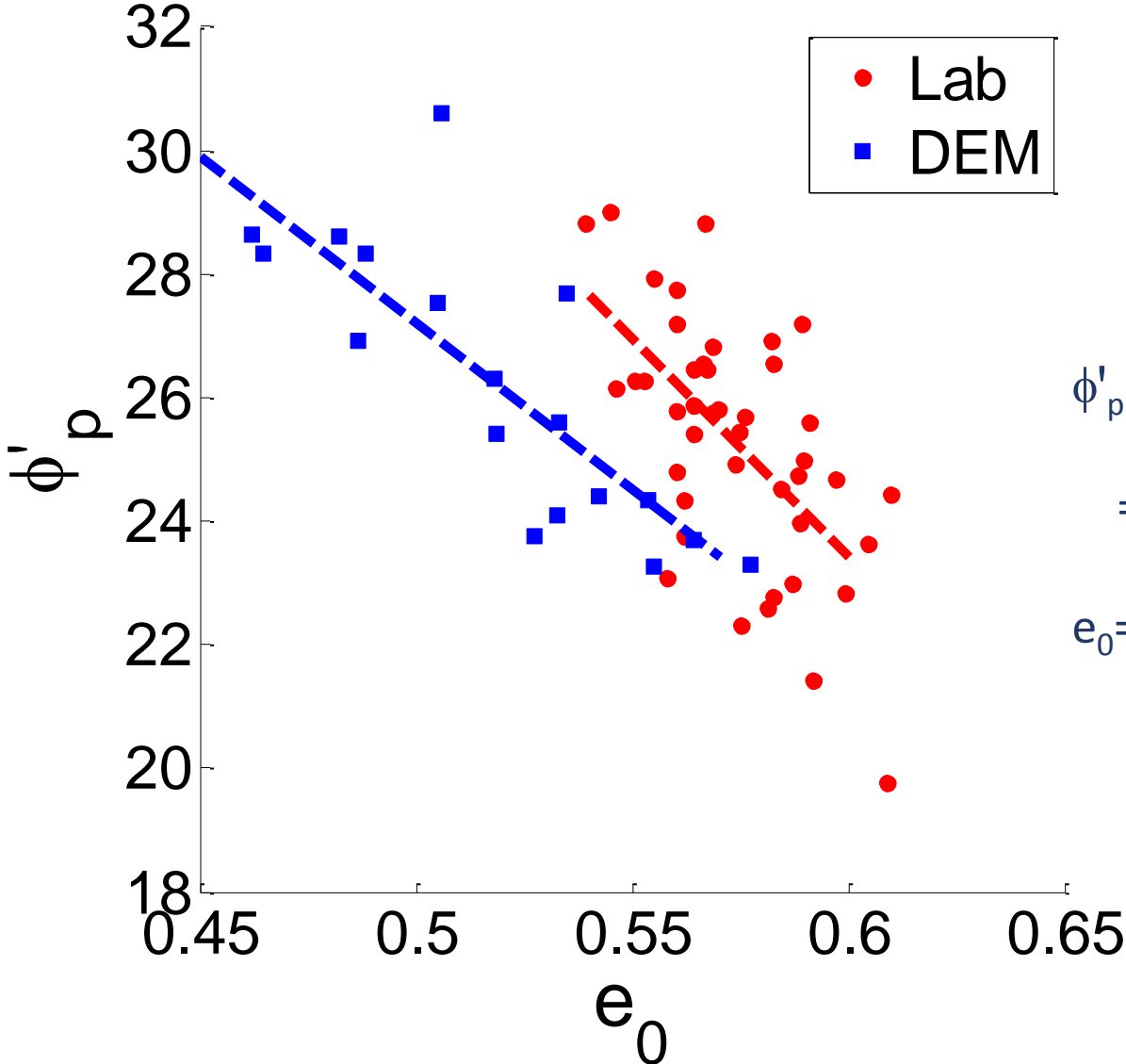


Mean μ_{surface}
5.7°

Direct Shear Test

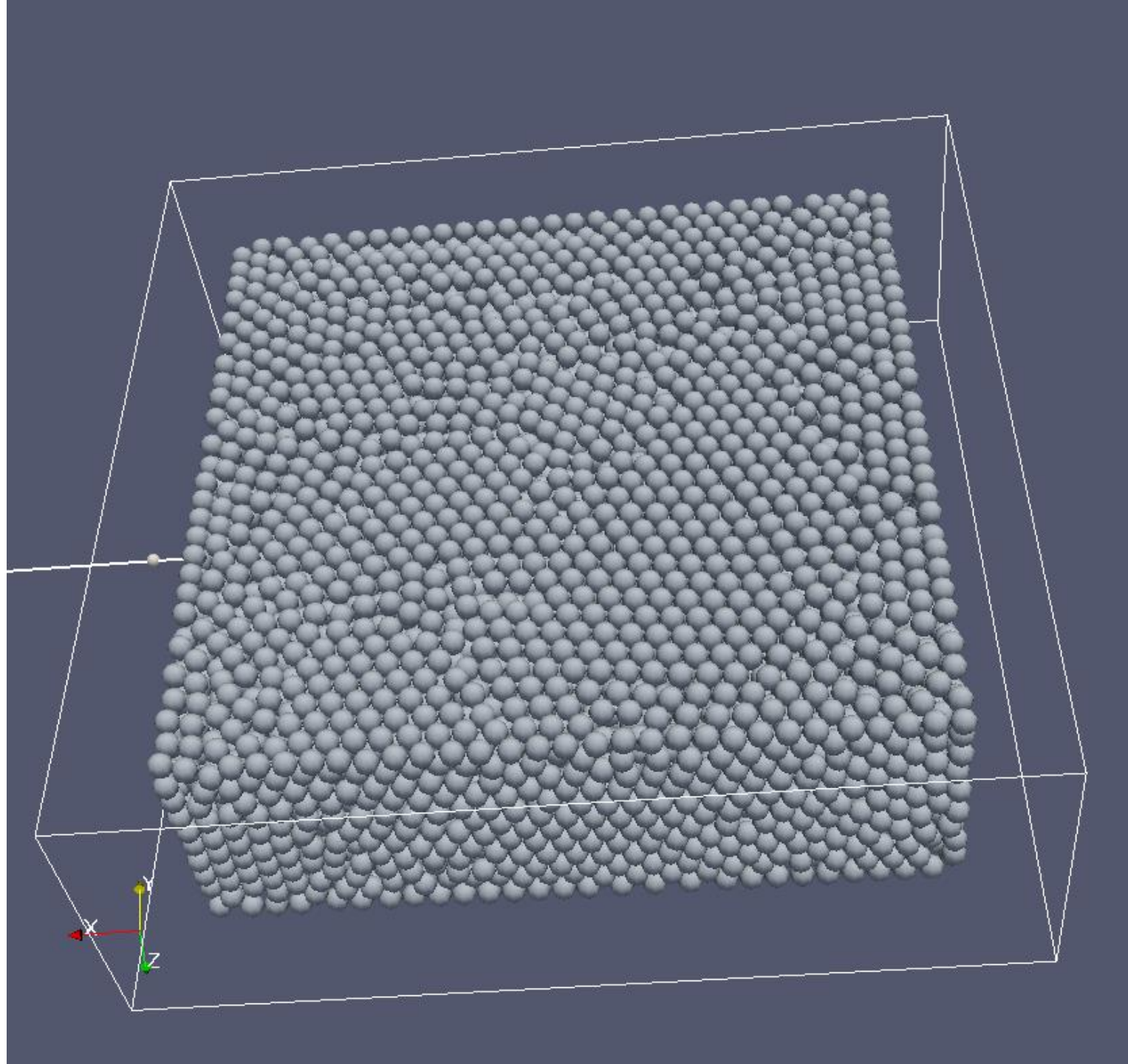


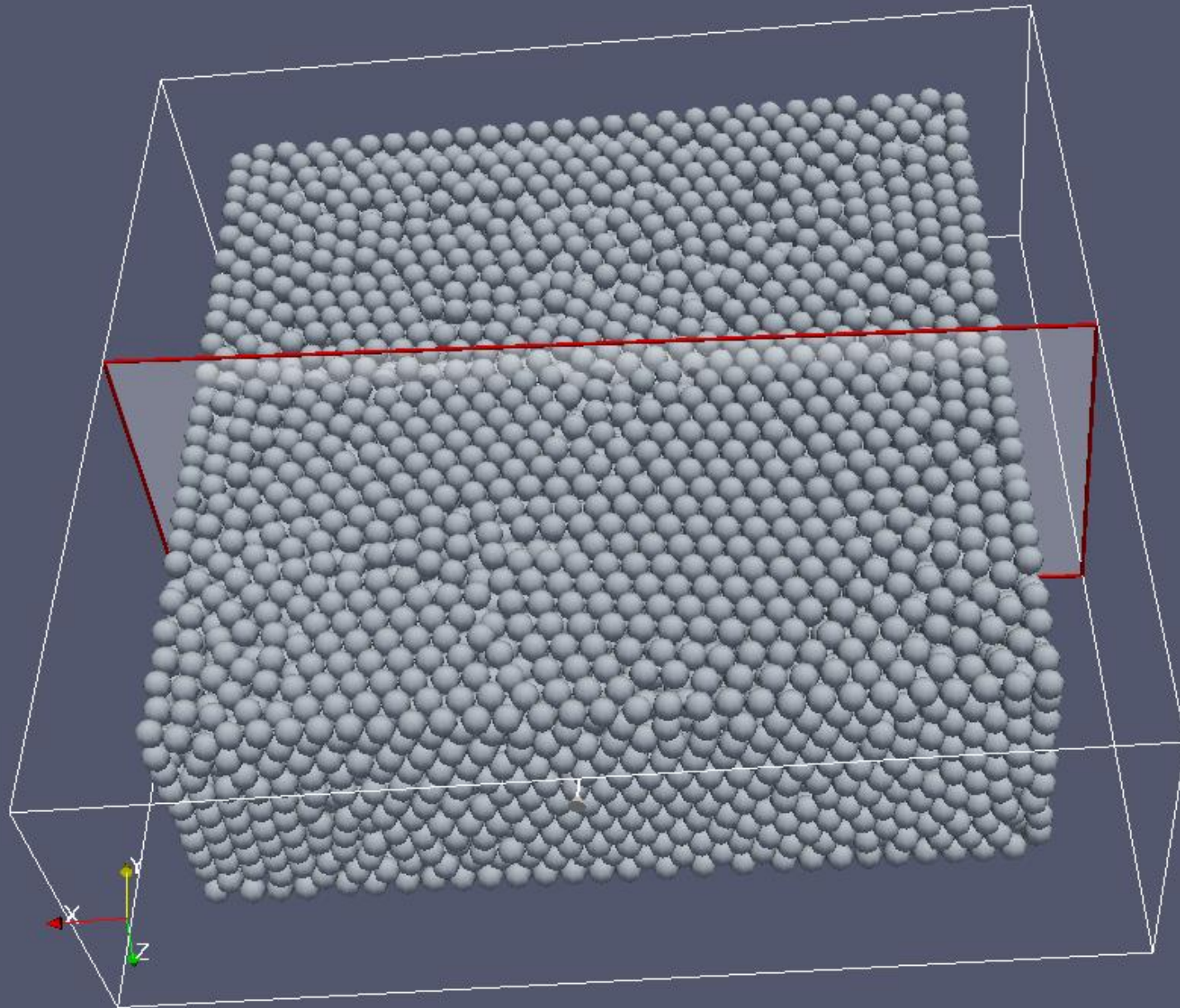
Direct Shear Test

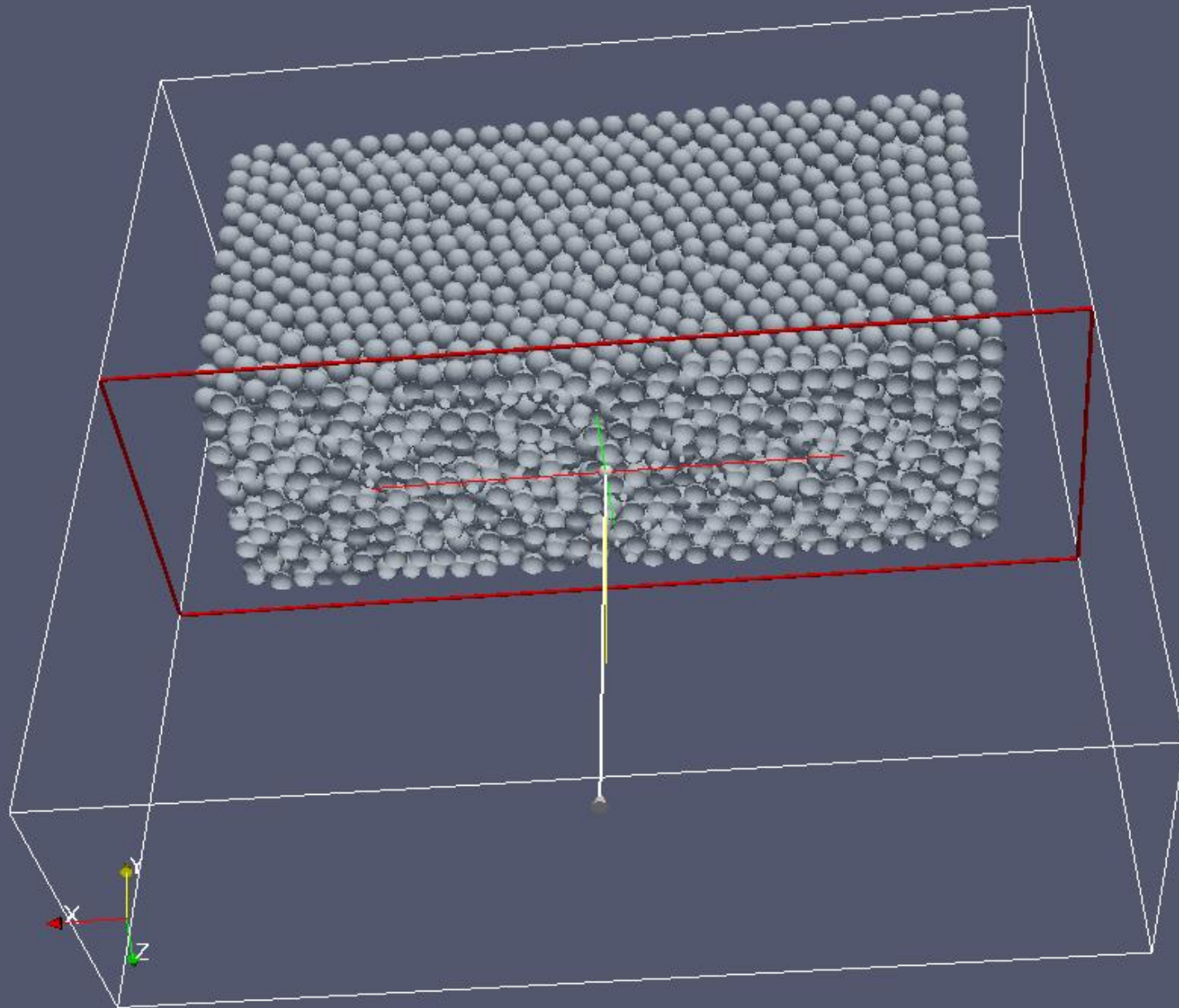


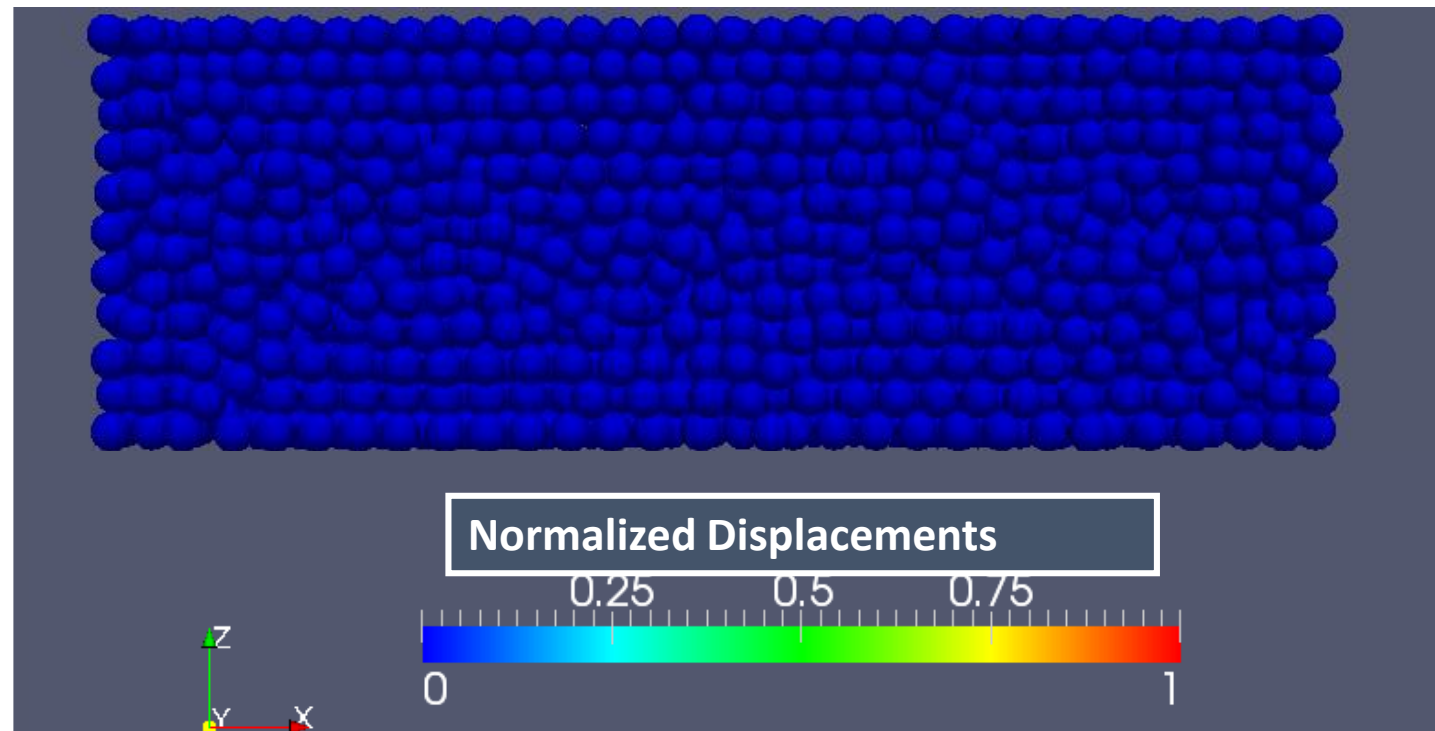
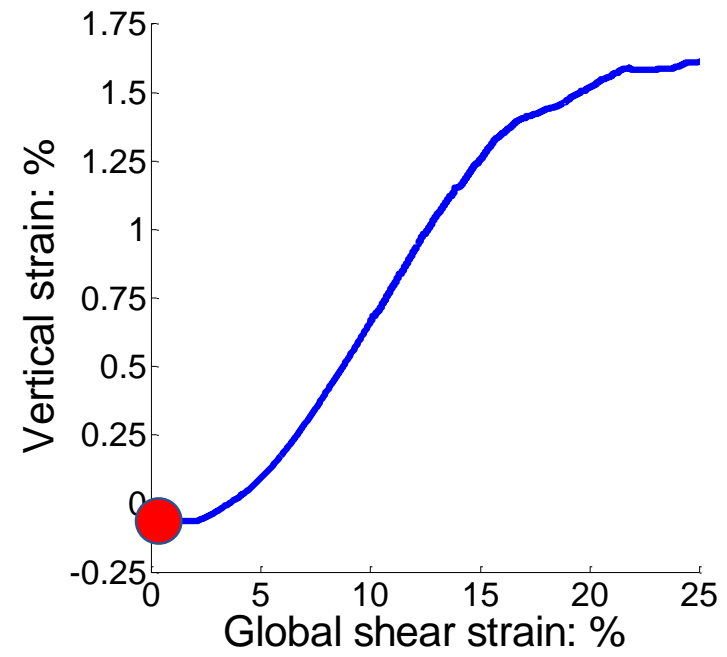
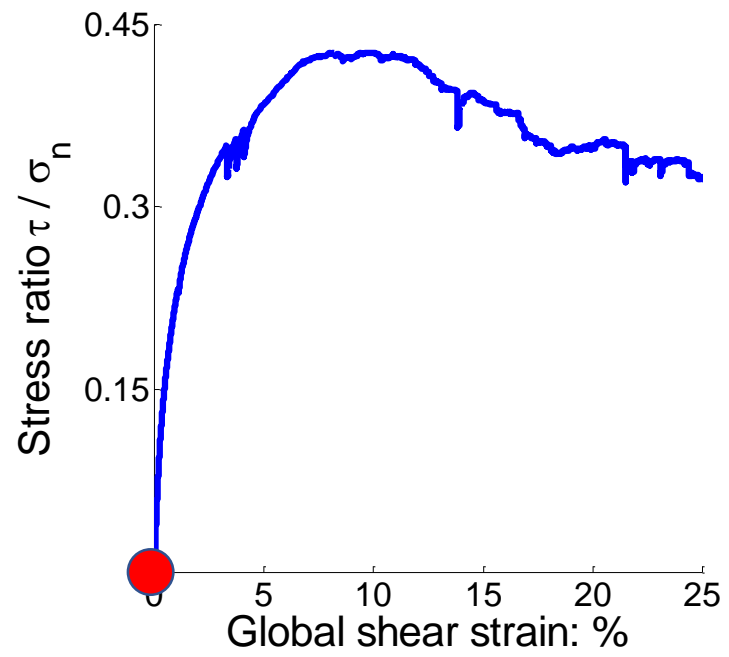
ϕ'_p = peak angle of shearing resistance
= peak "friction" angle

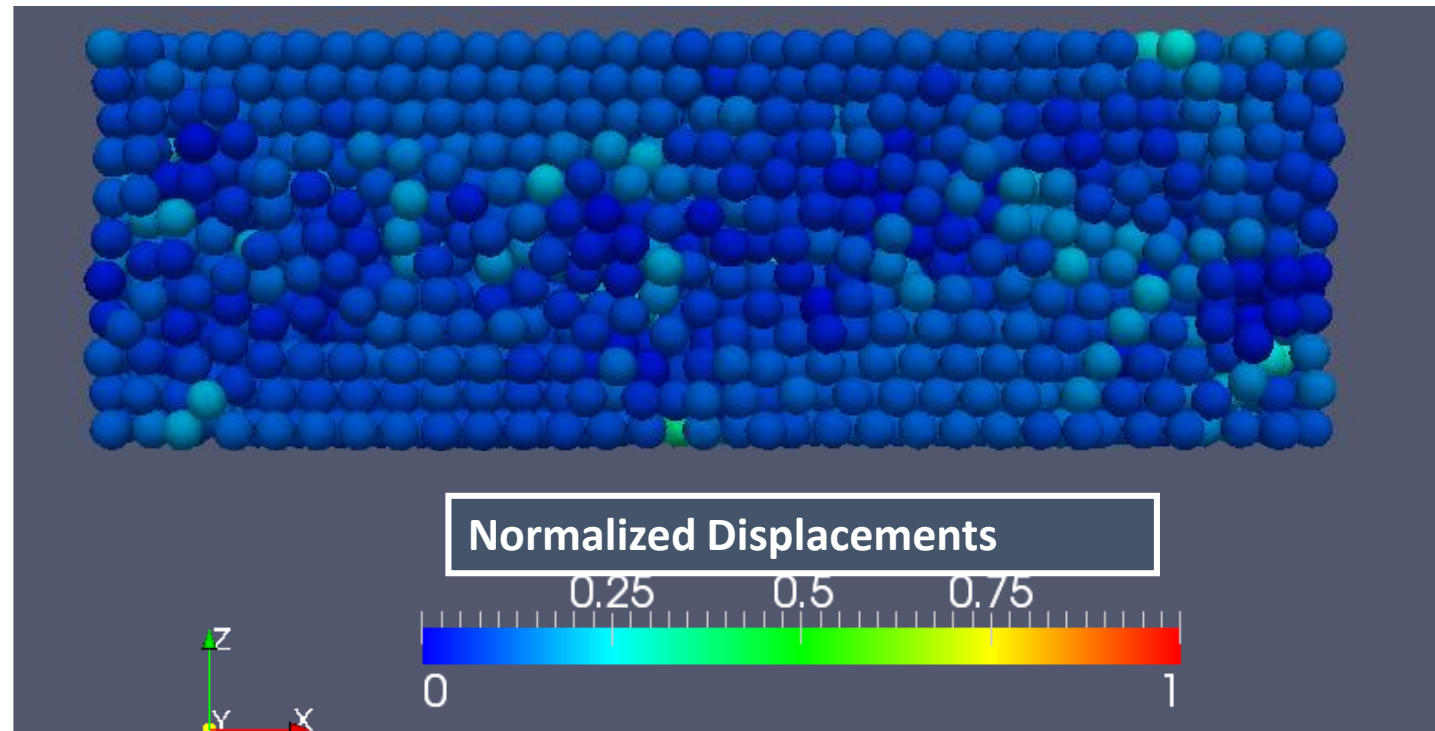
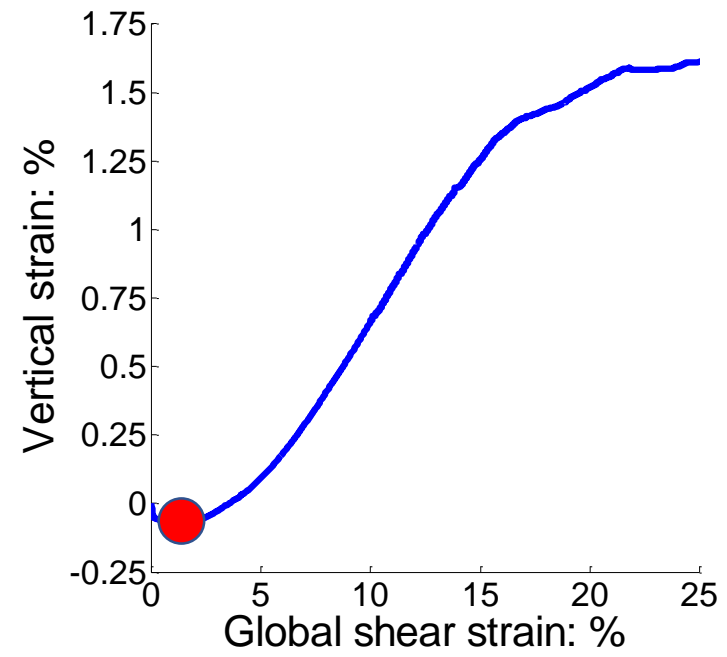
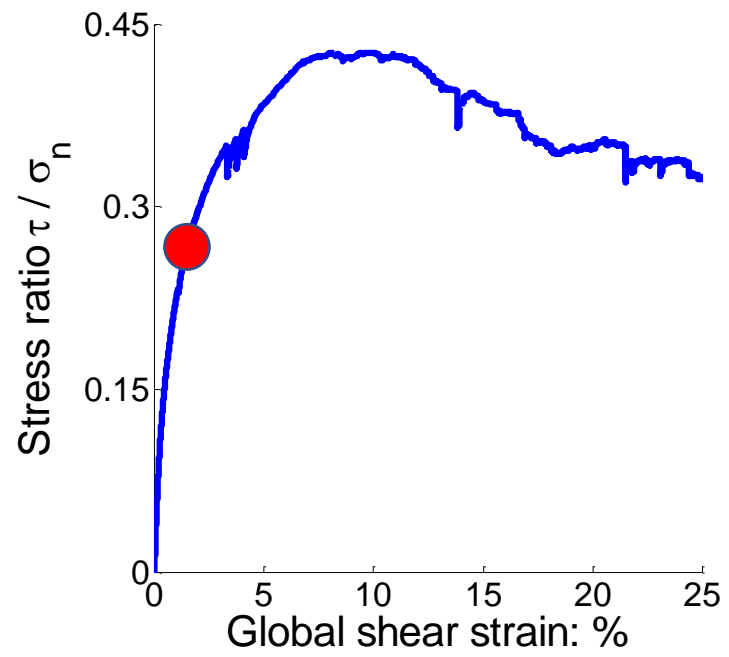
e_0 = initial void ratio

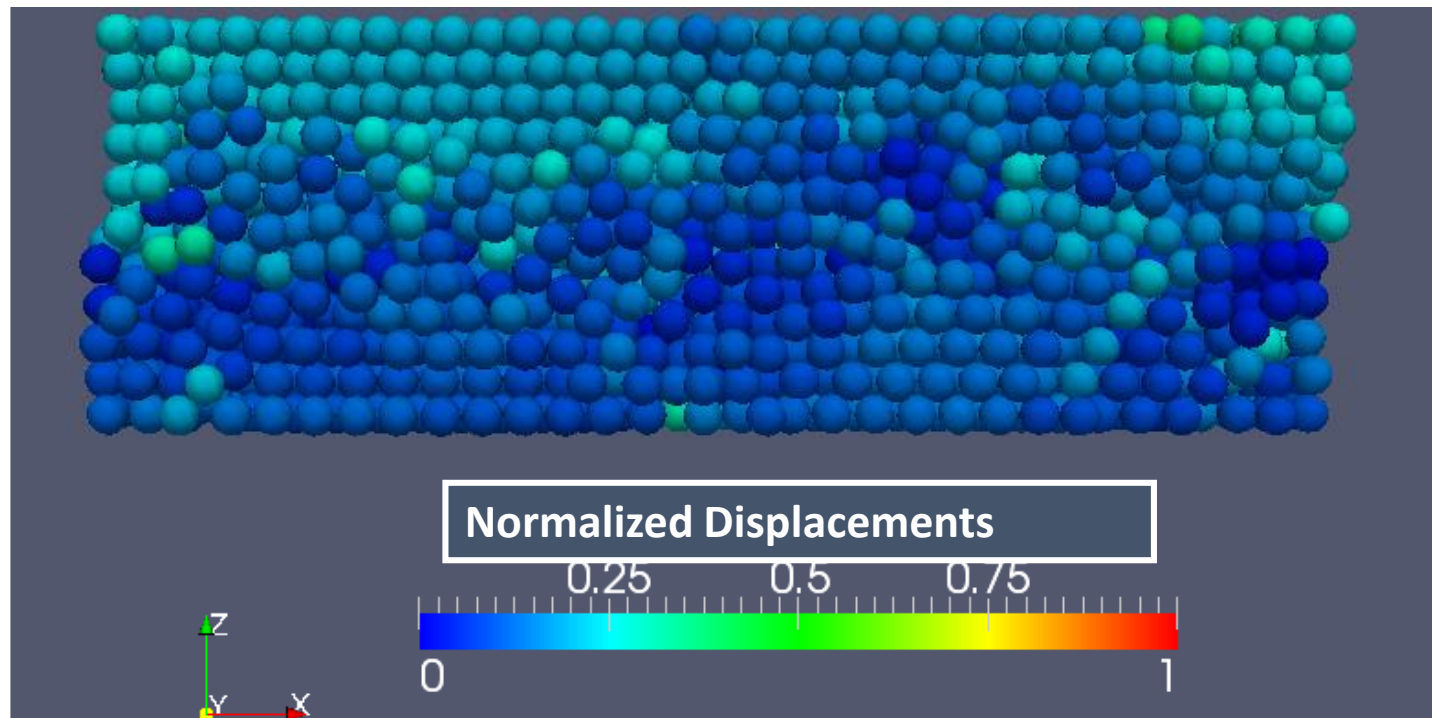
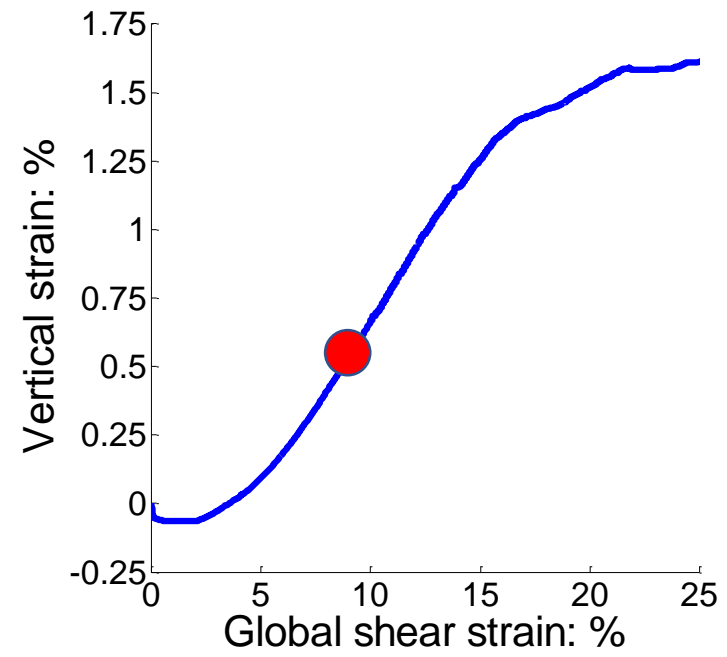
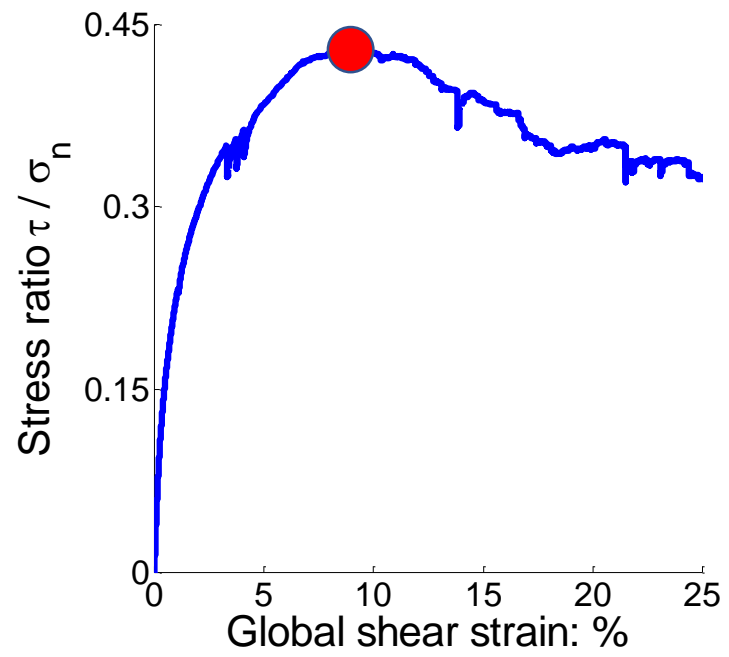


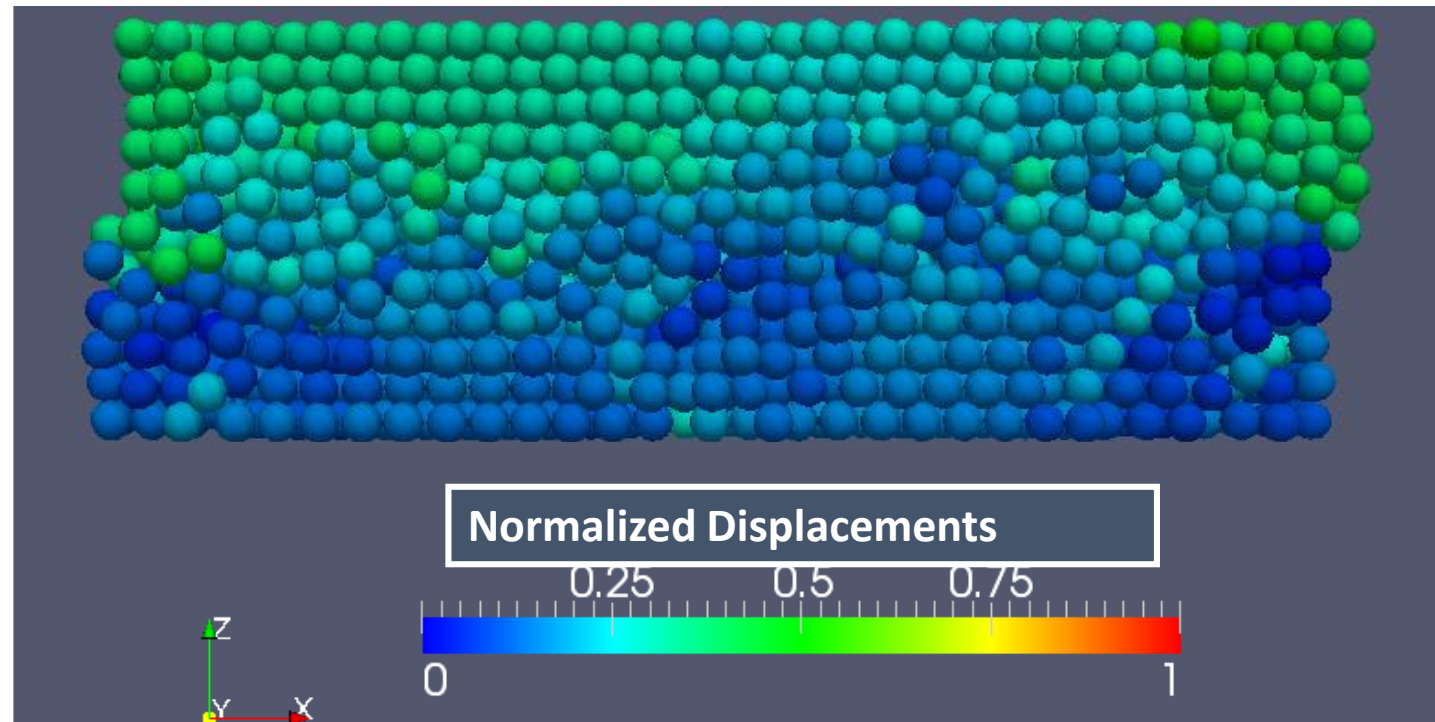
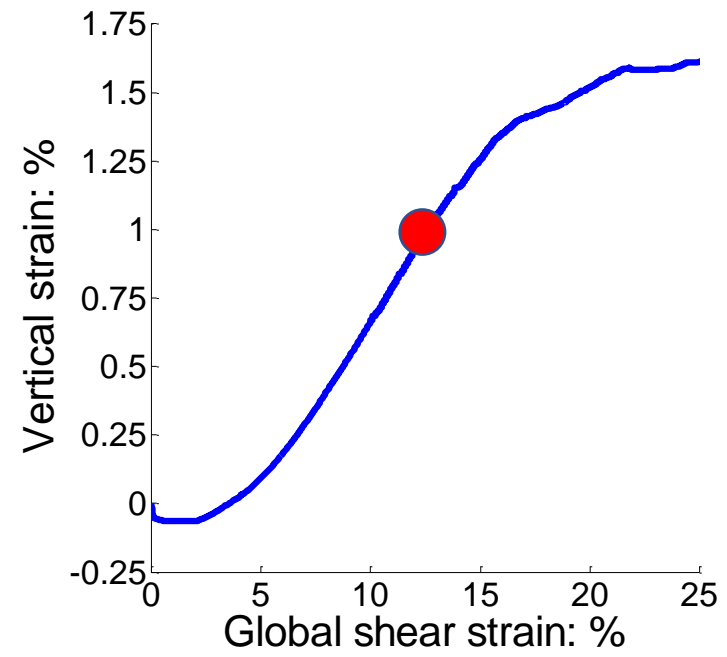
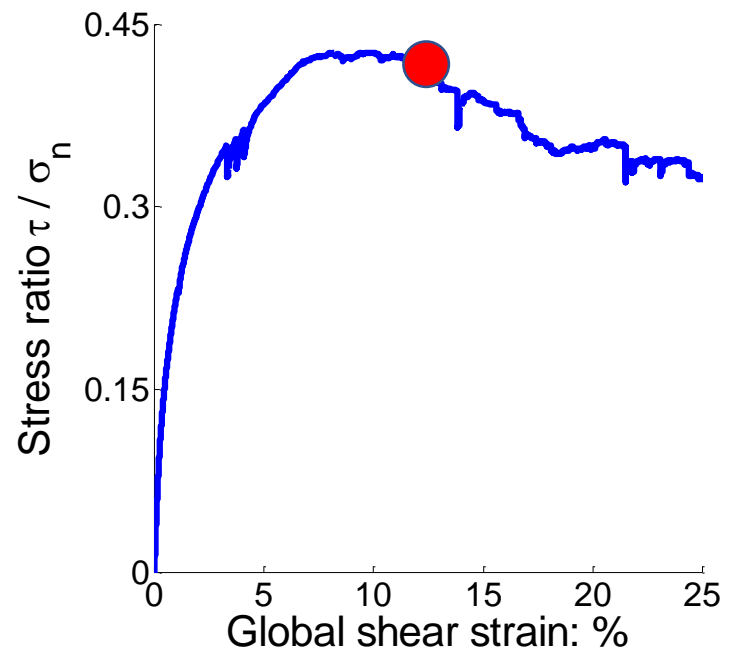


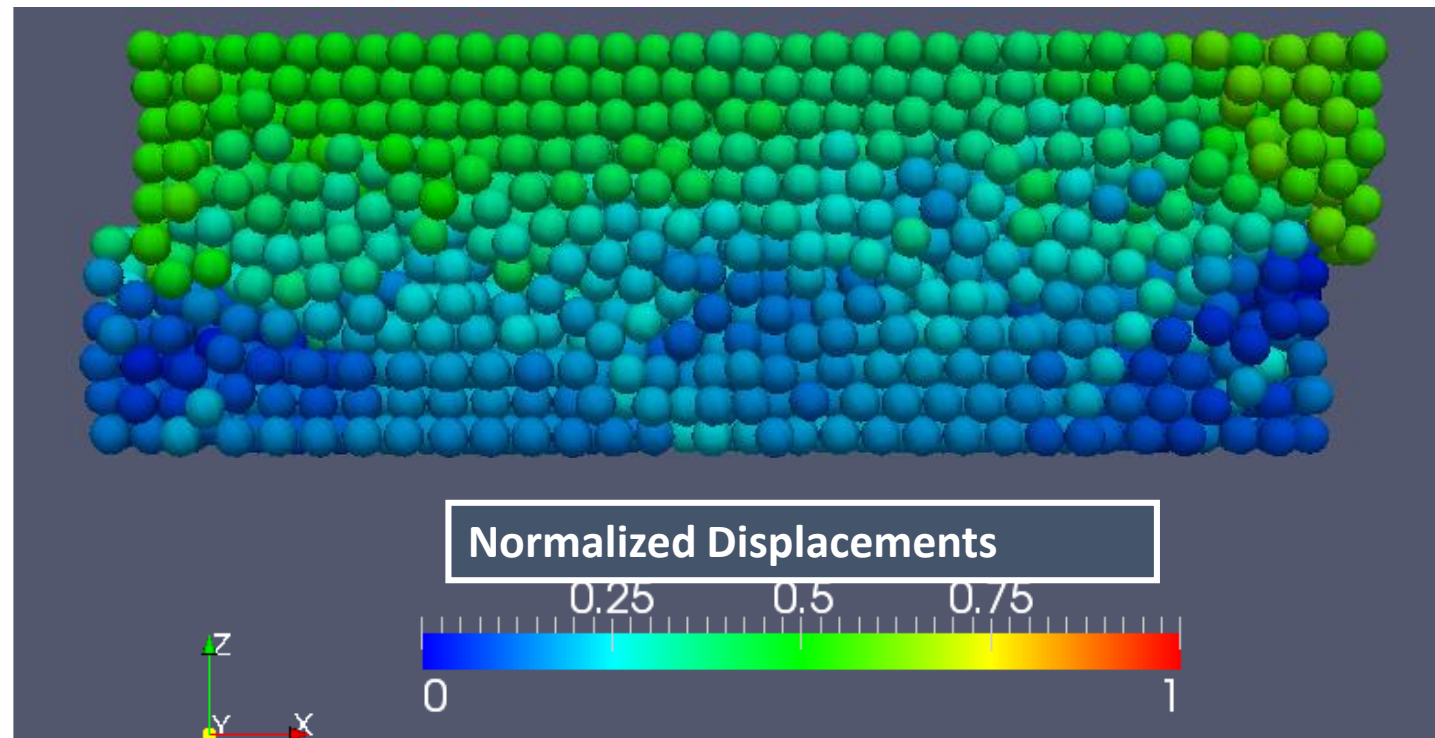
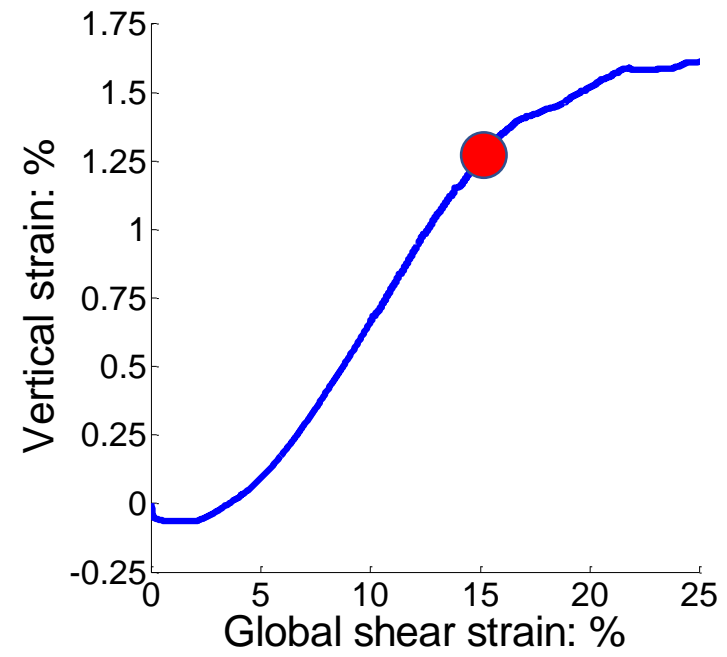
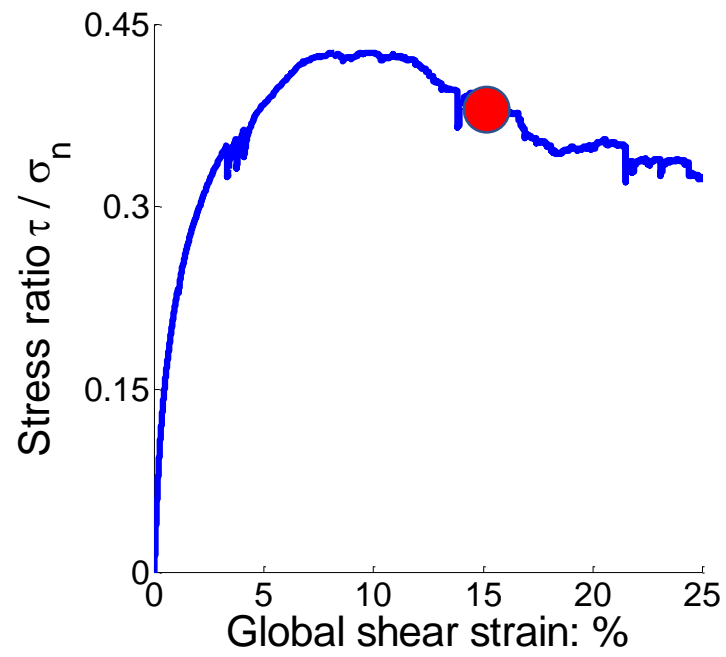


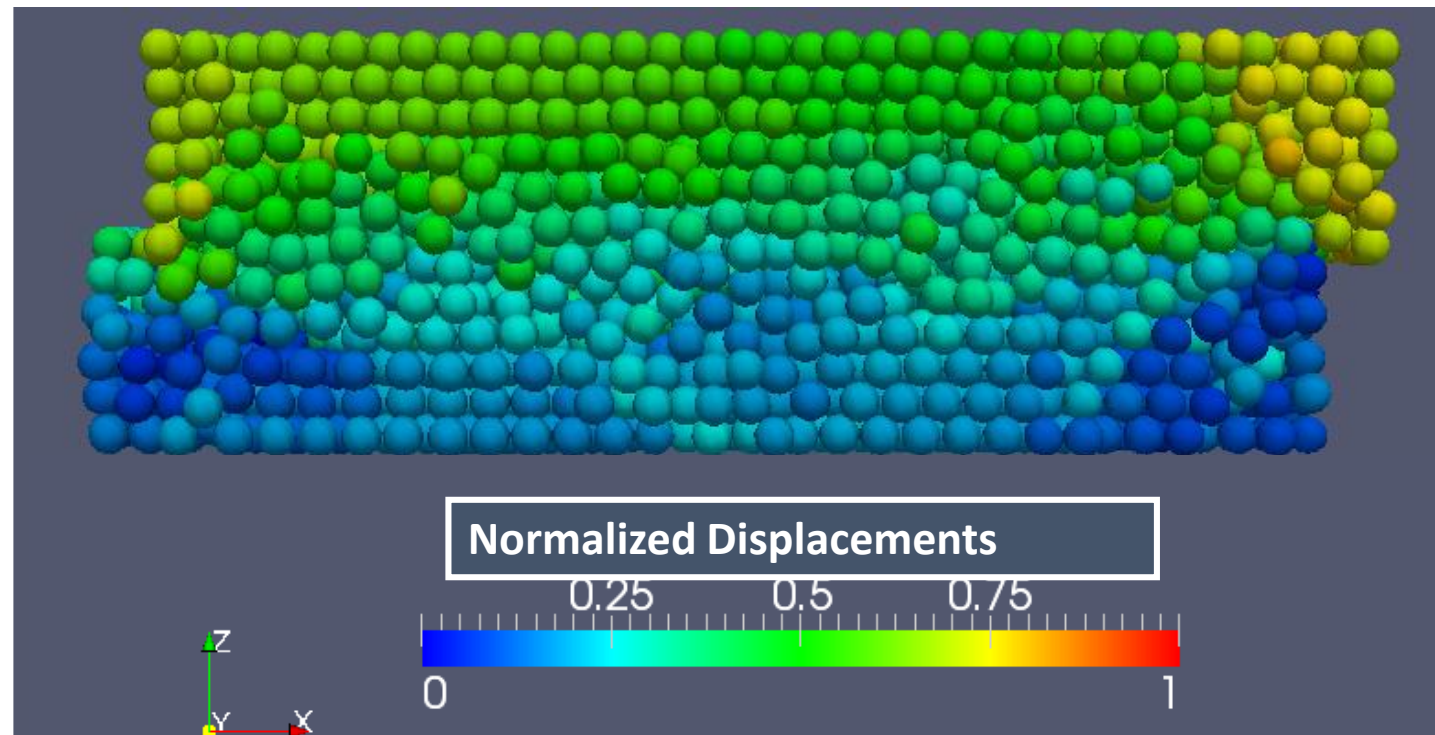
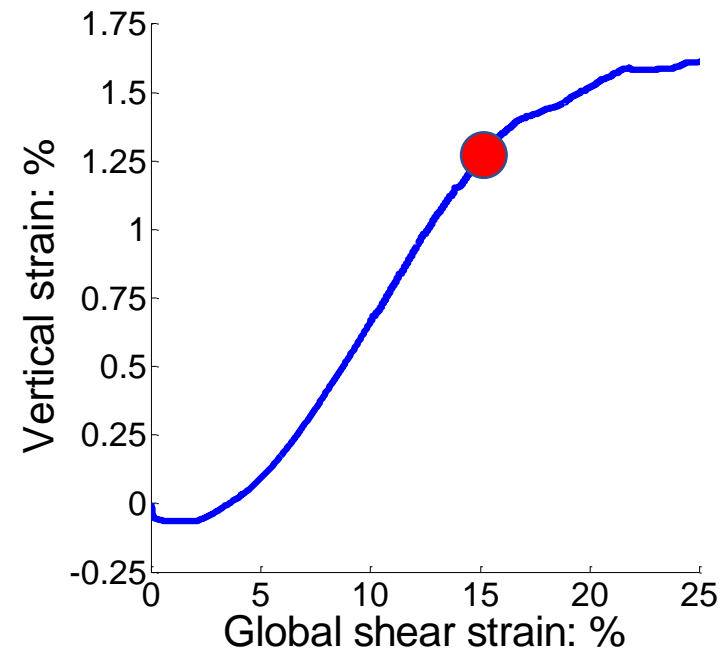
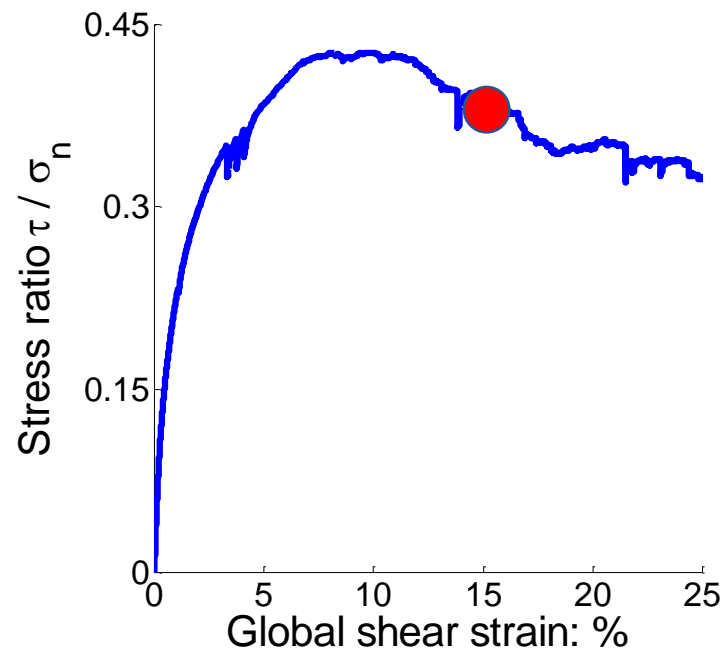


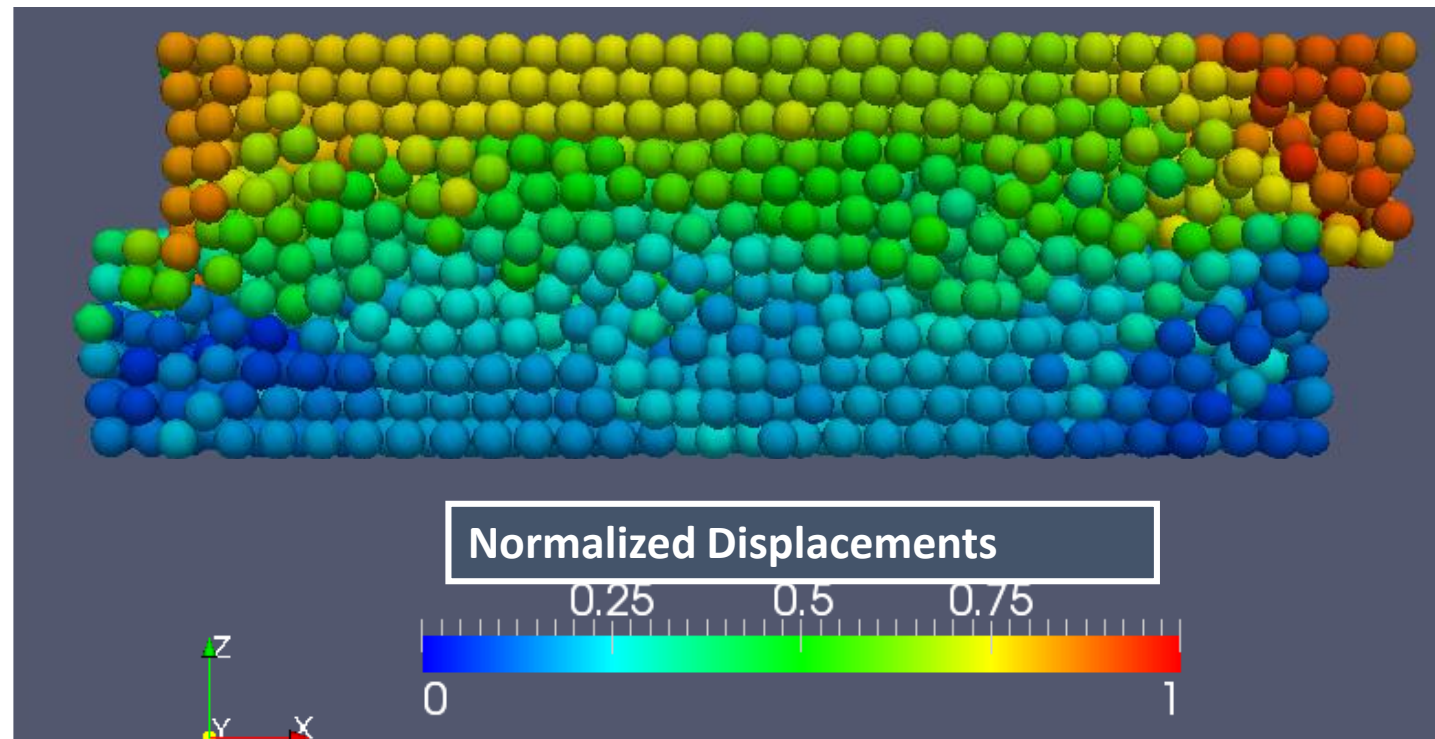
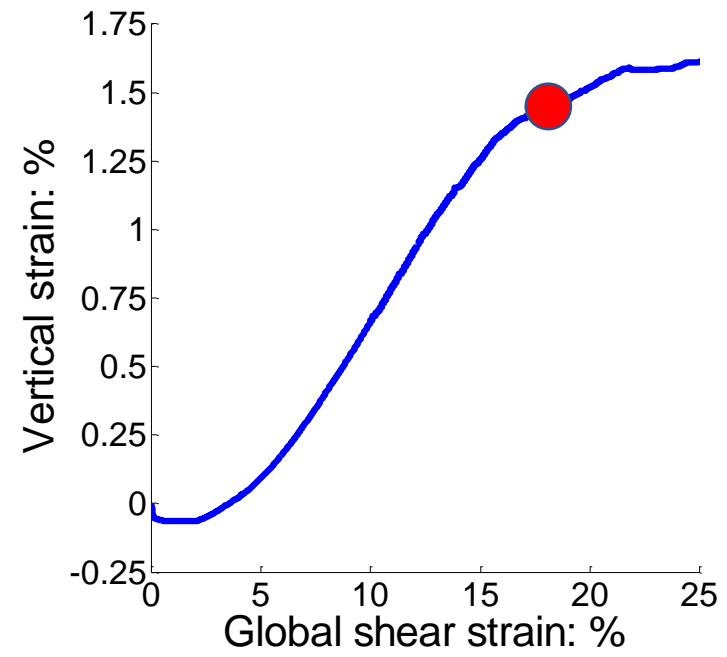
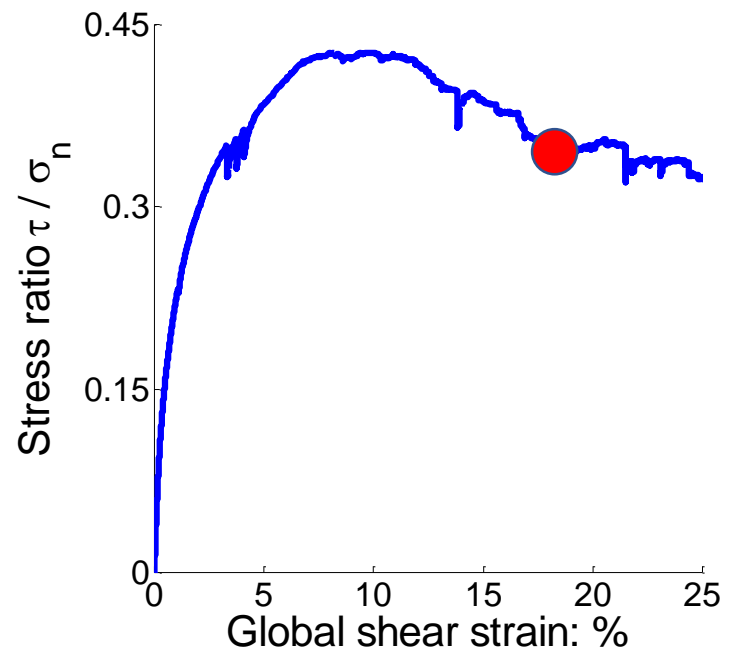


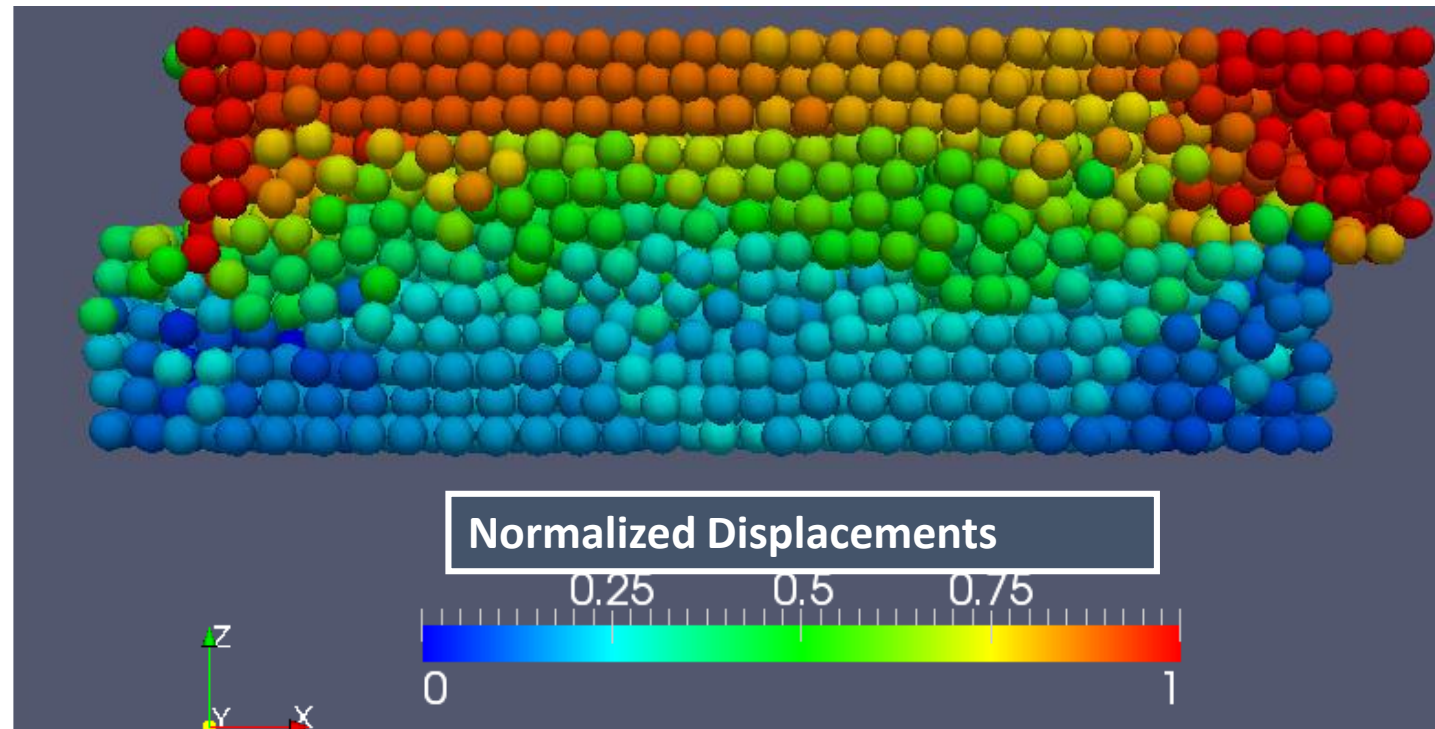
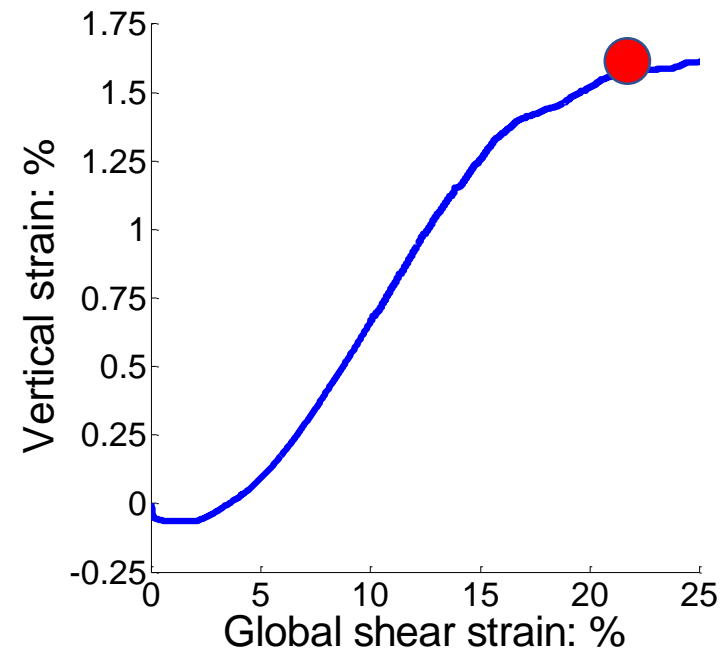
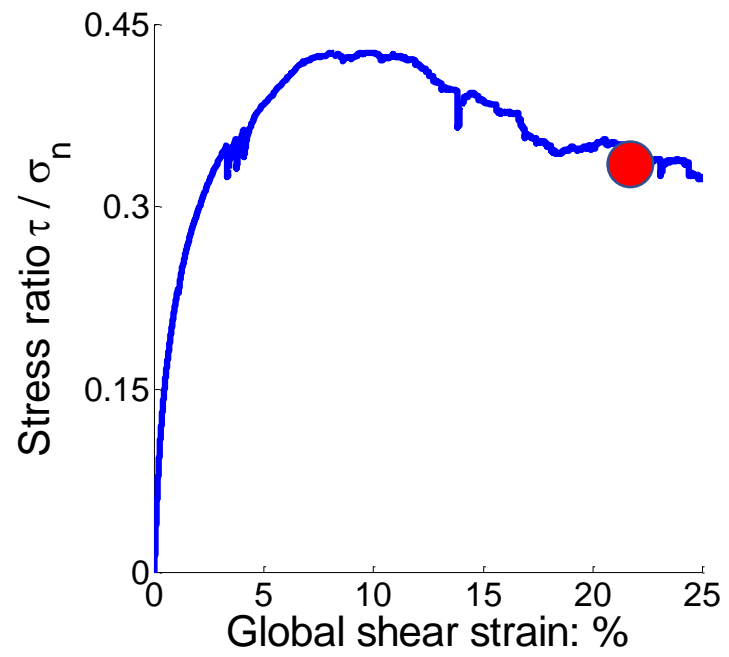


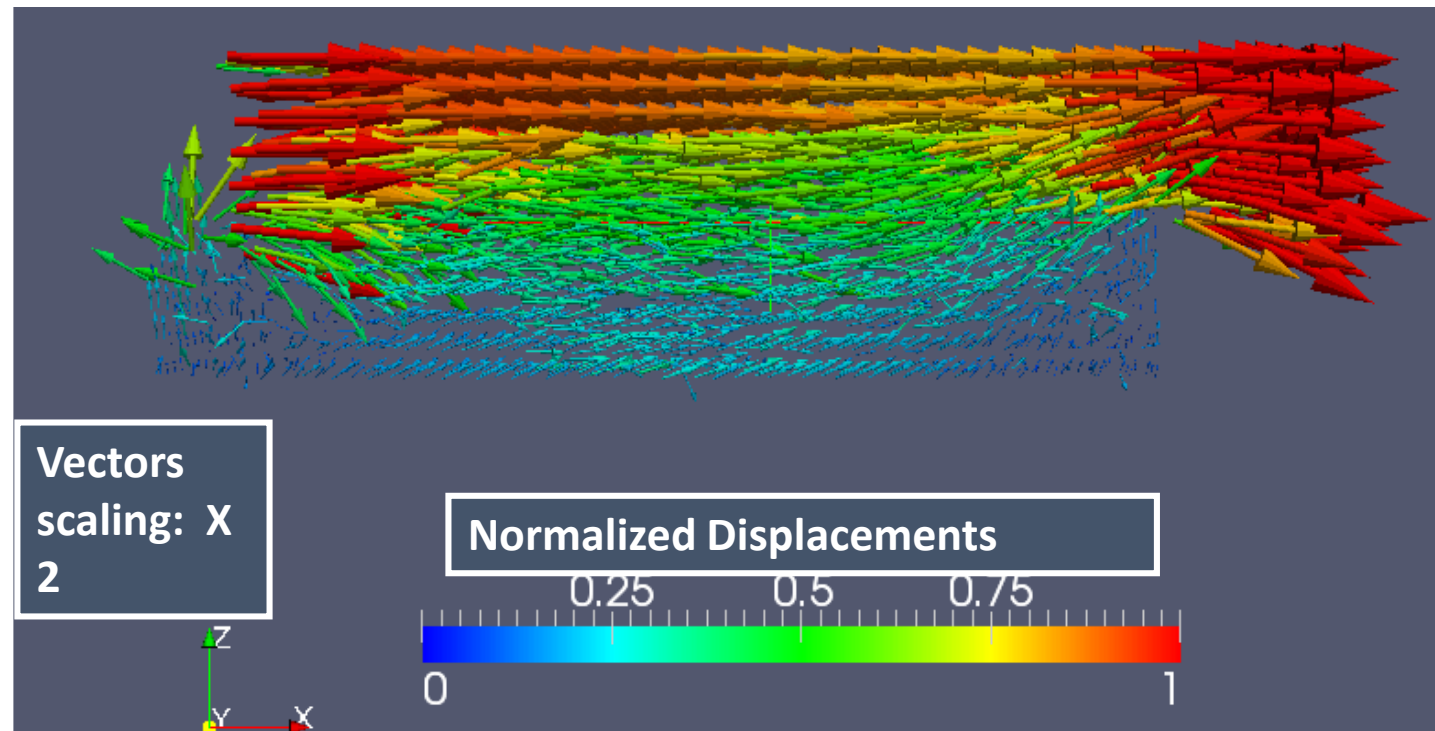
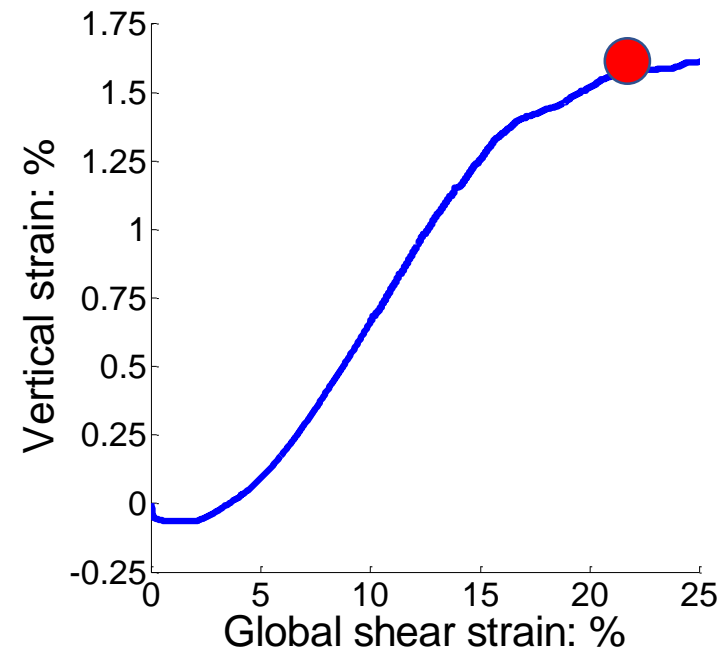
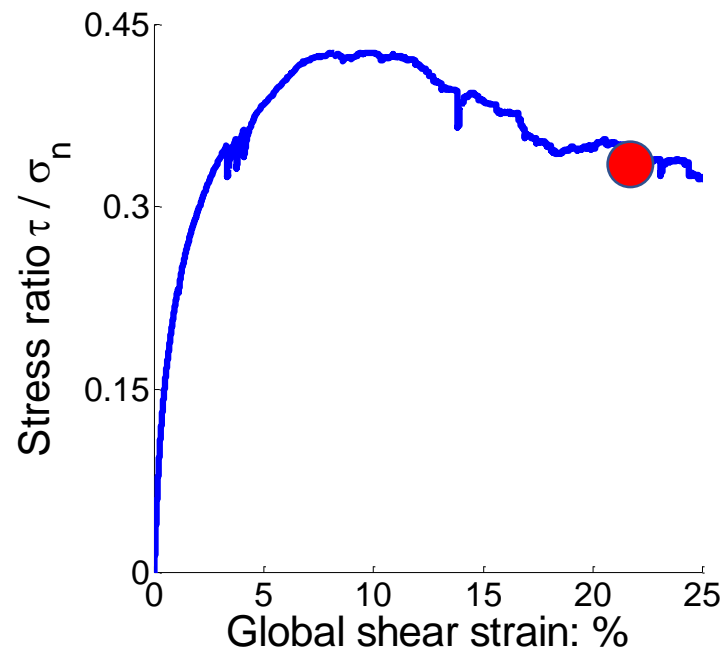


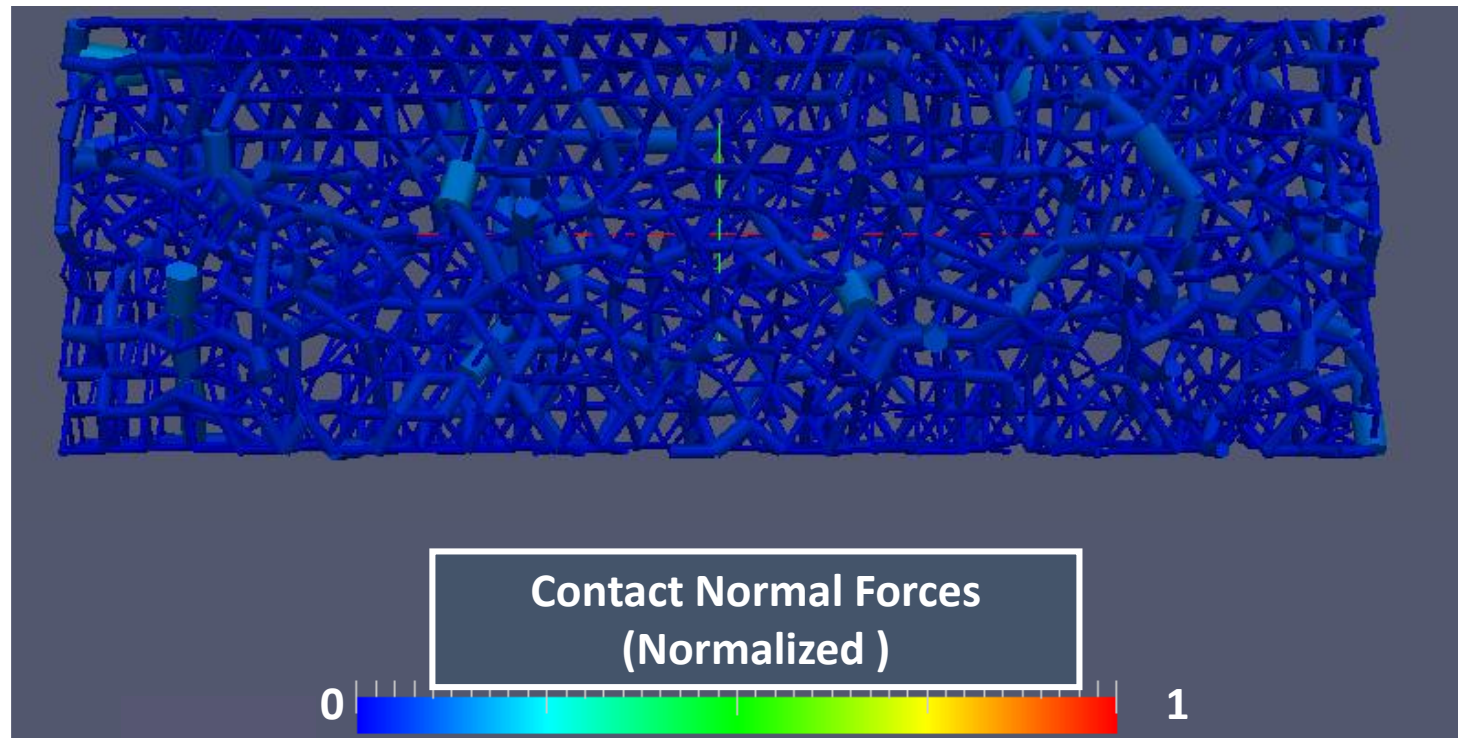
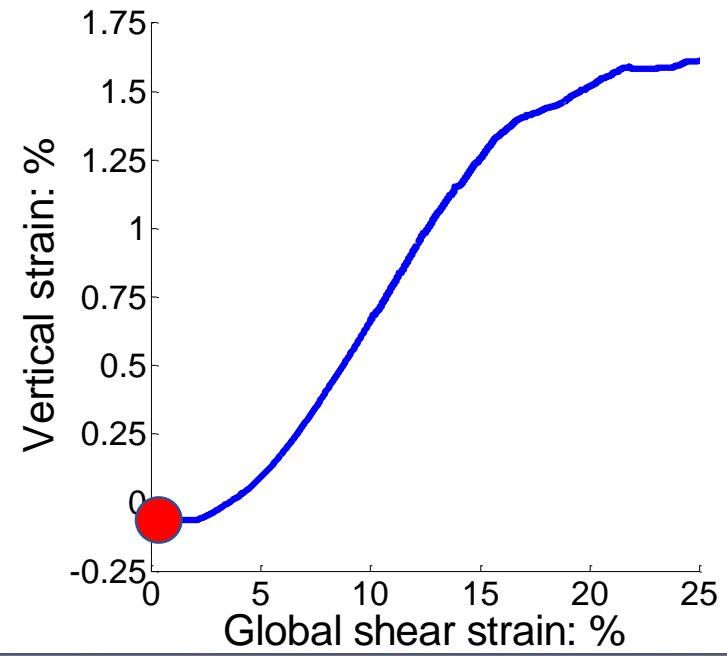
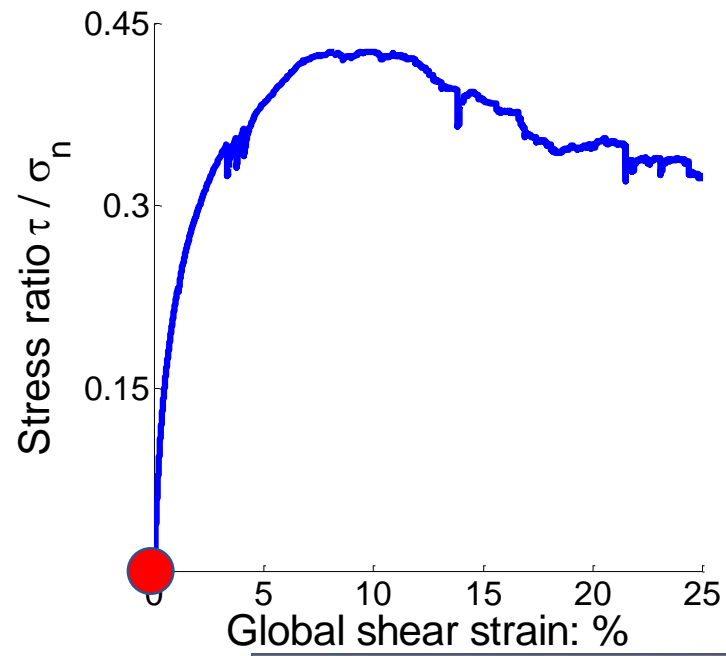


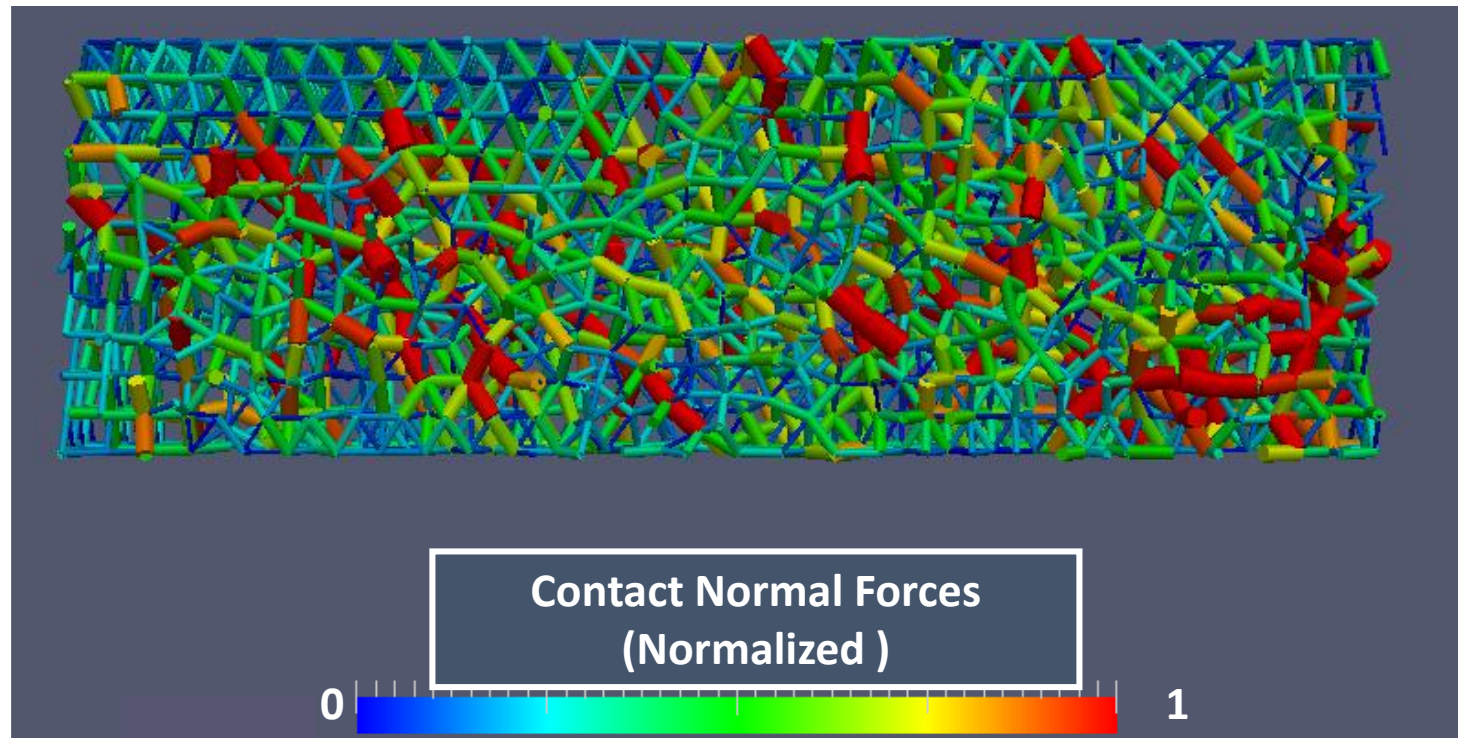
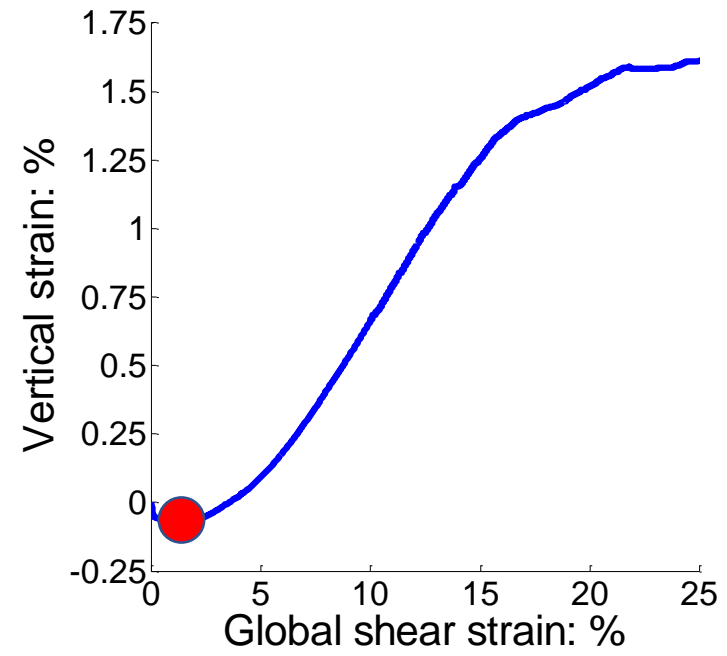
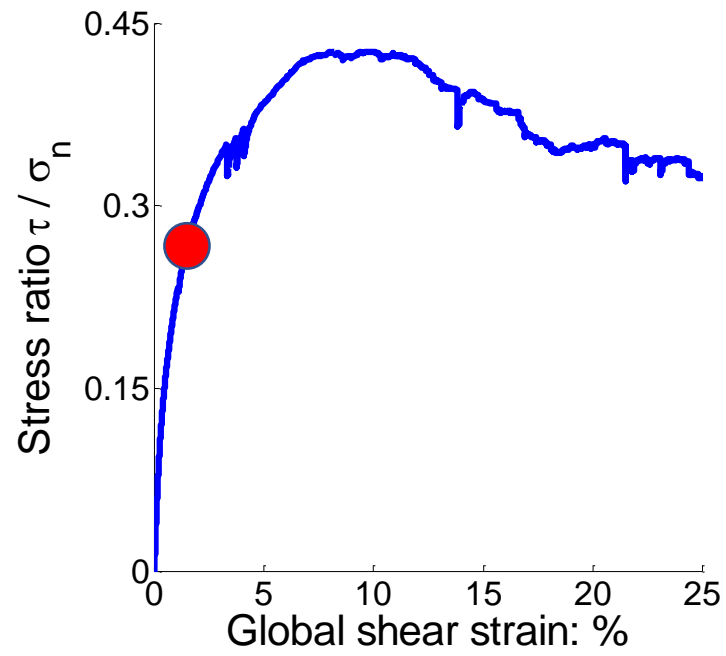


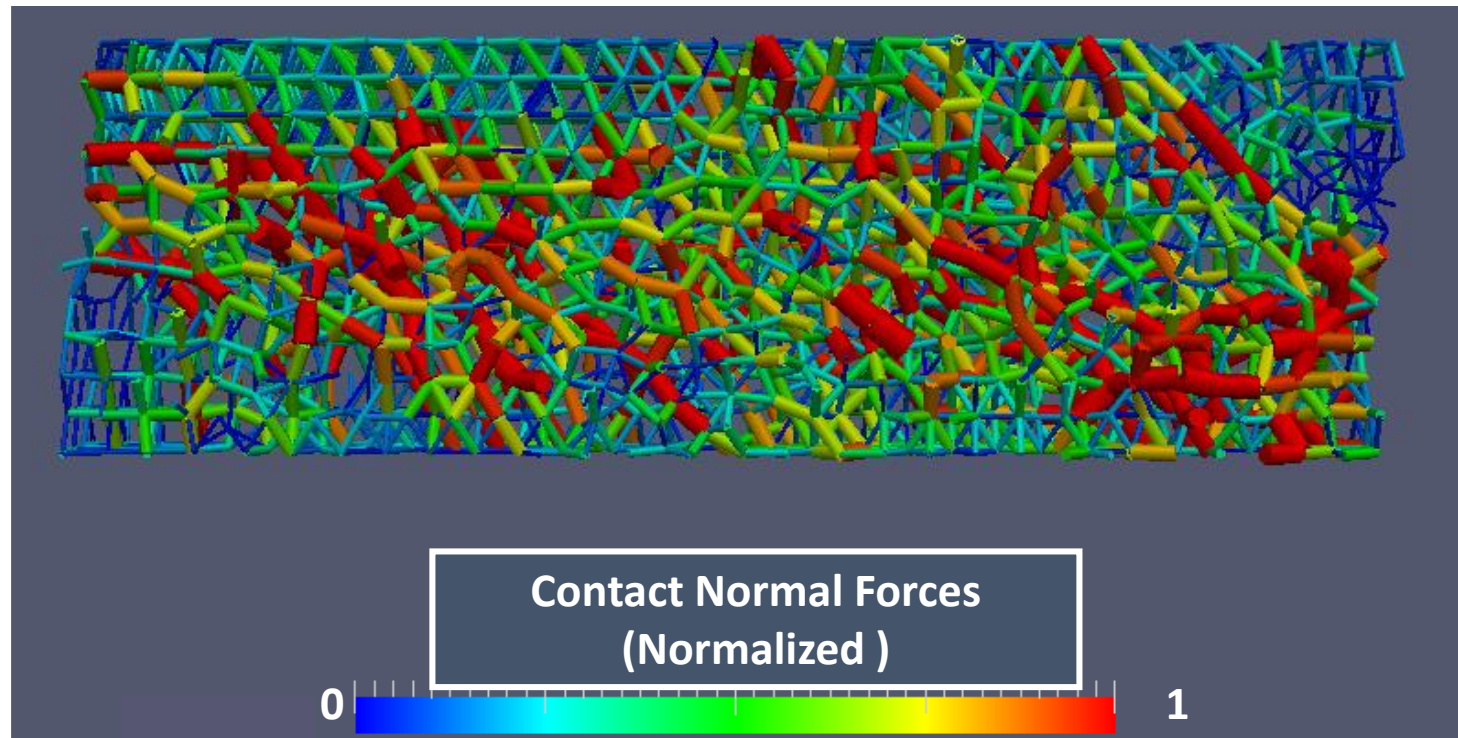
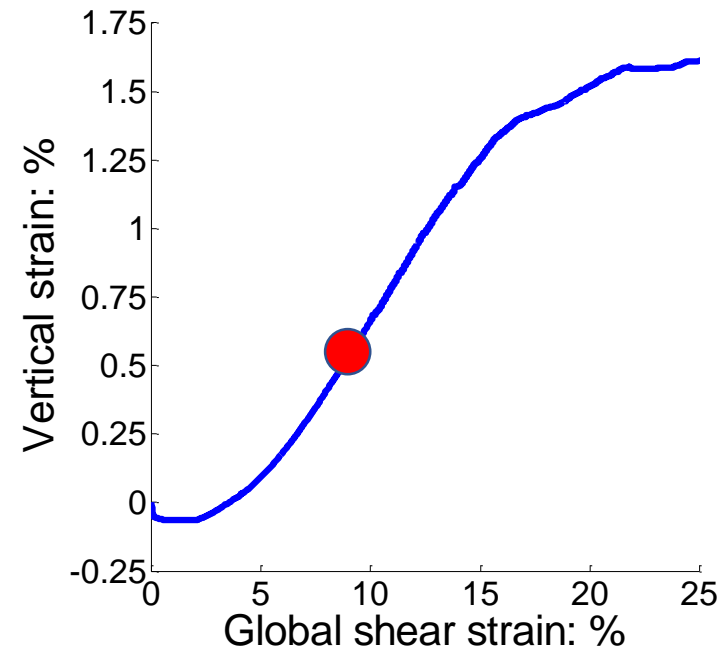
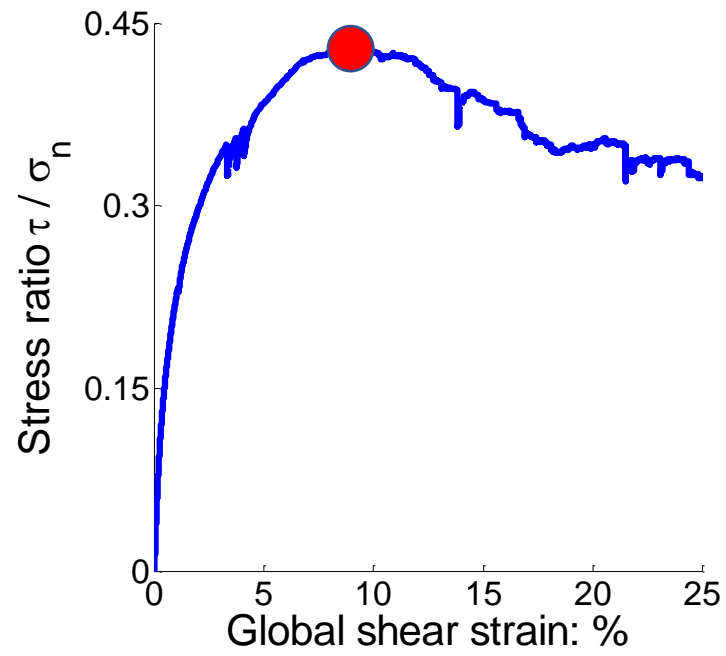


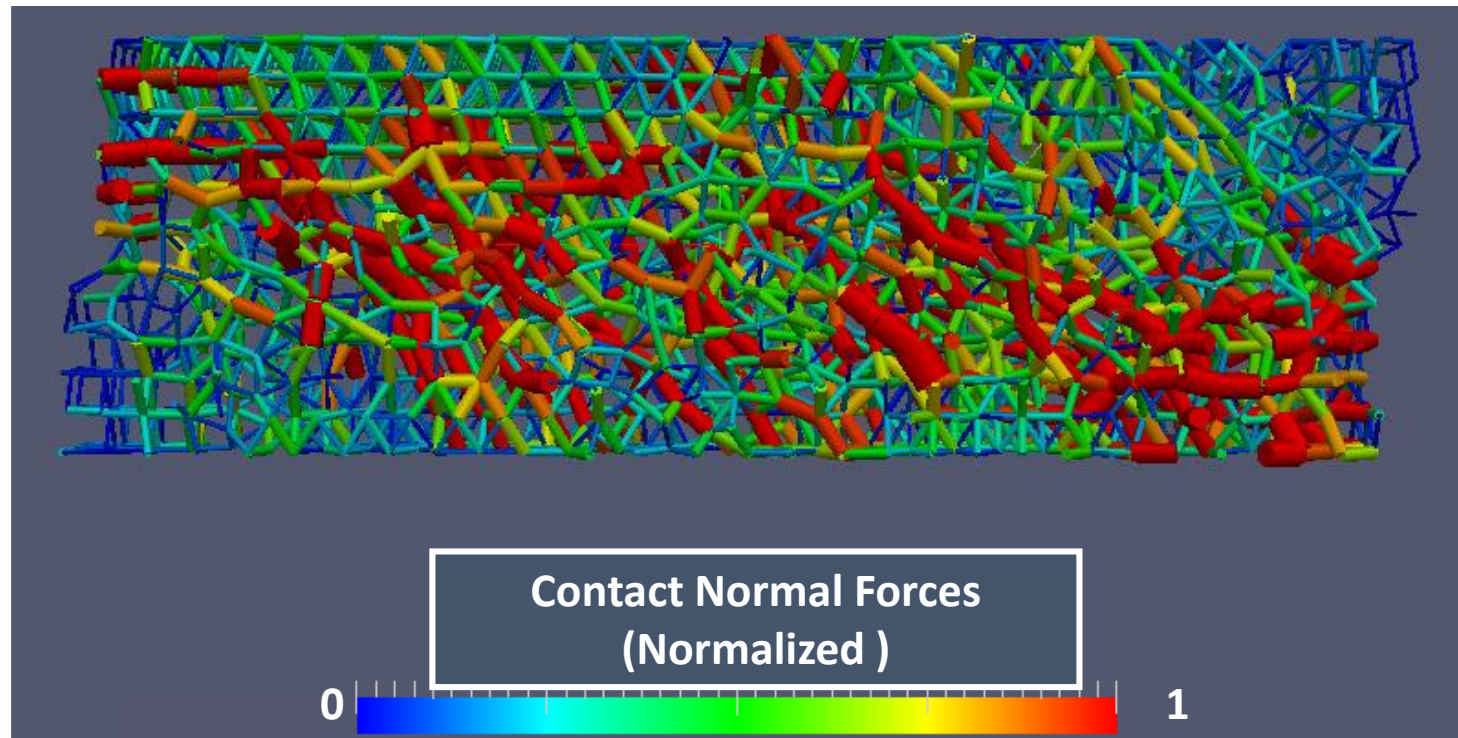
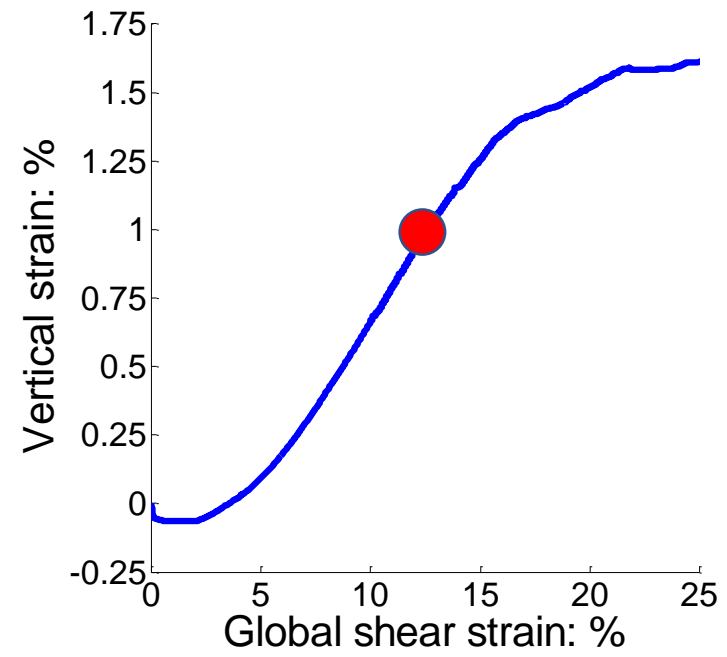
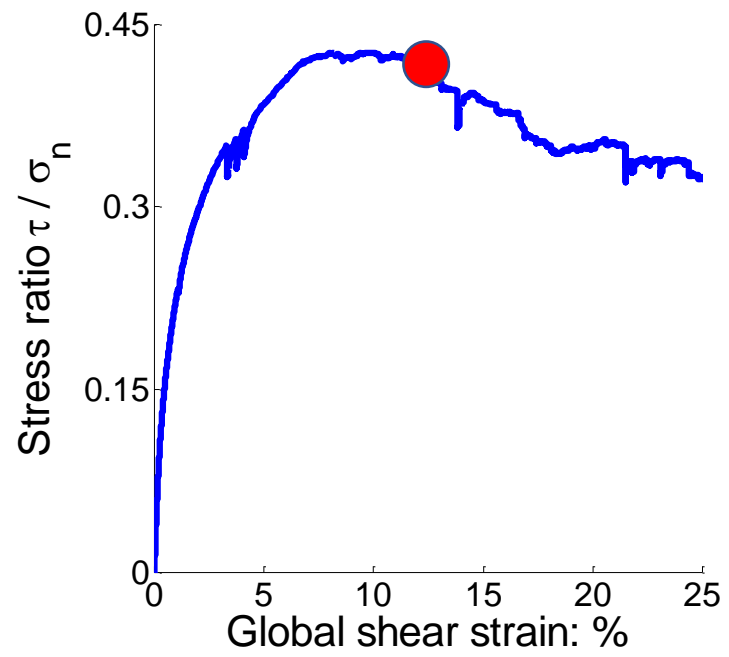


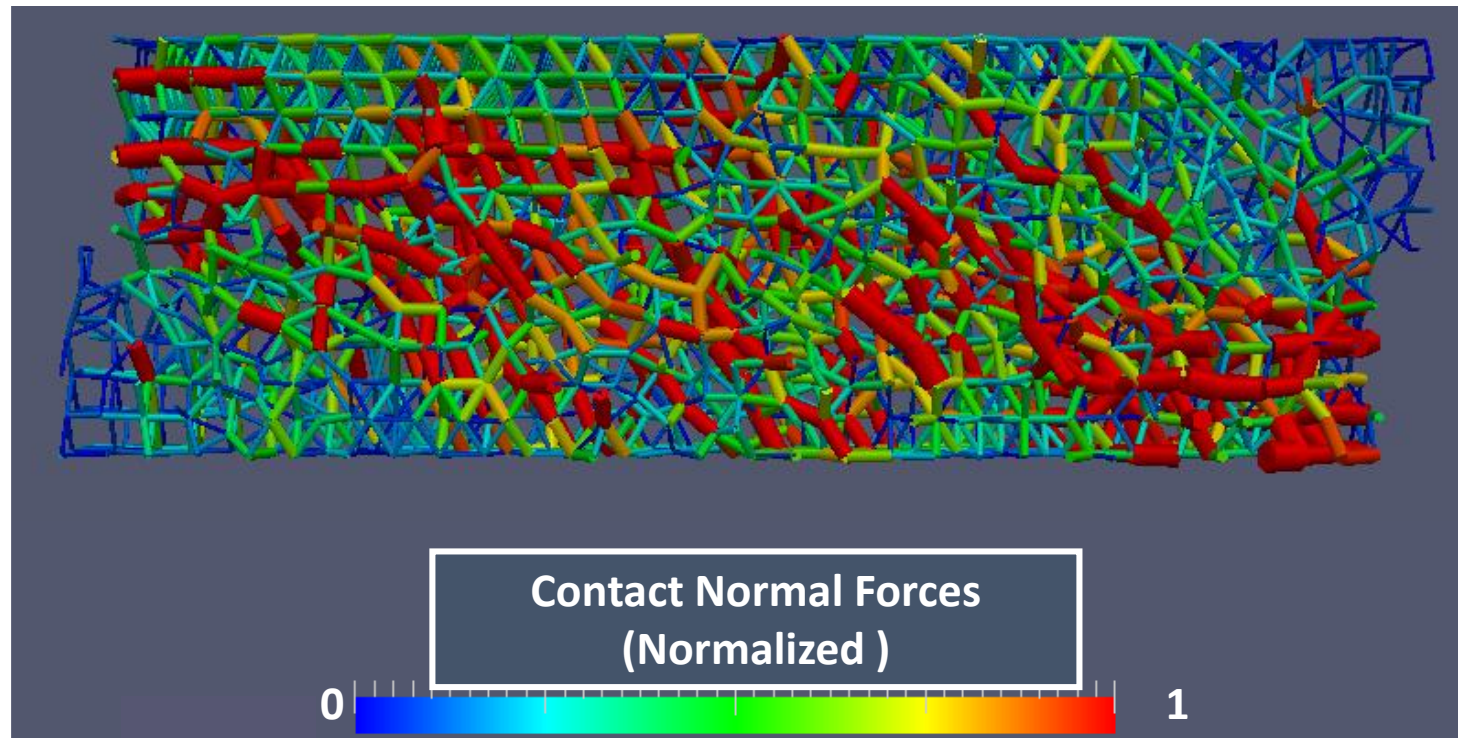
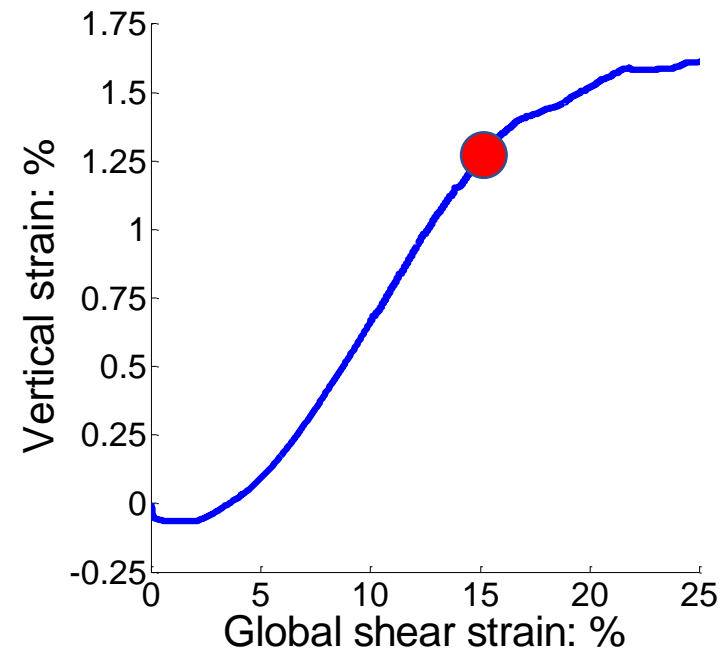
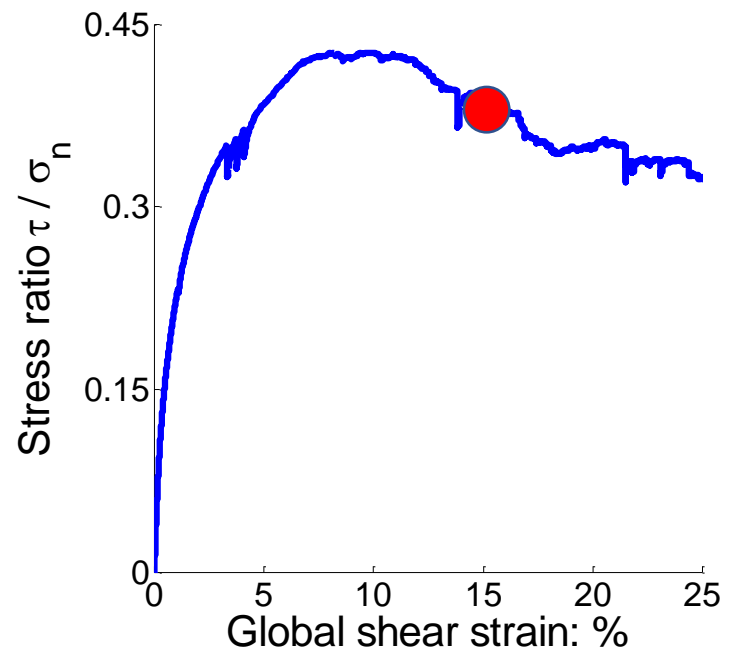


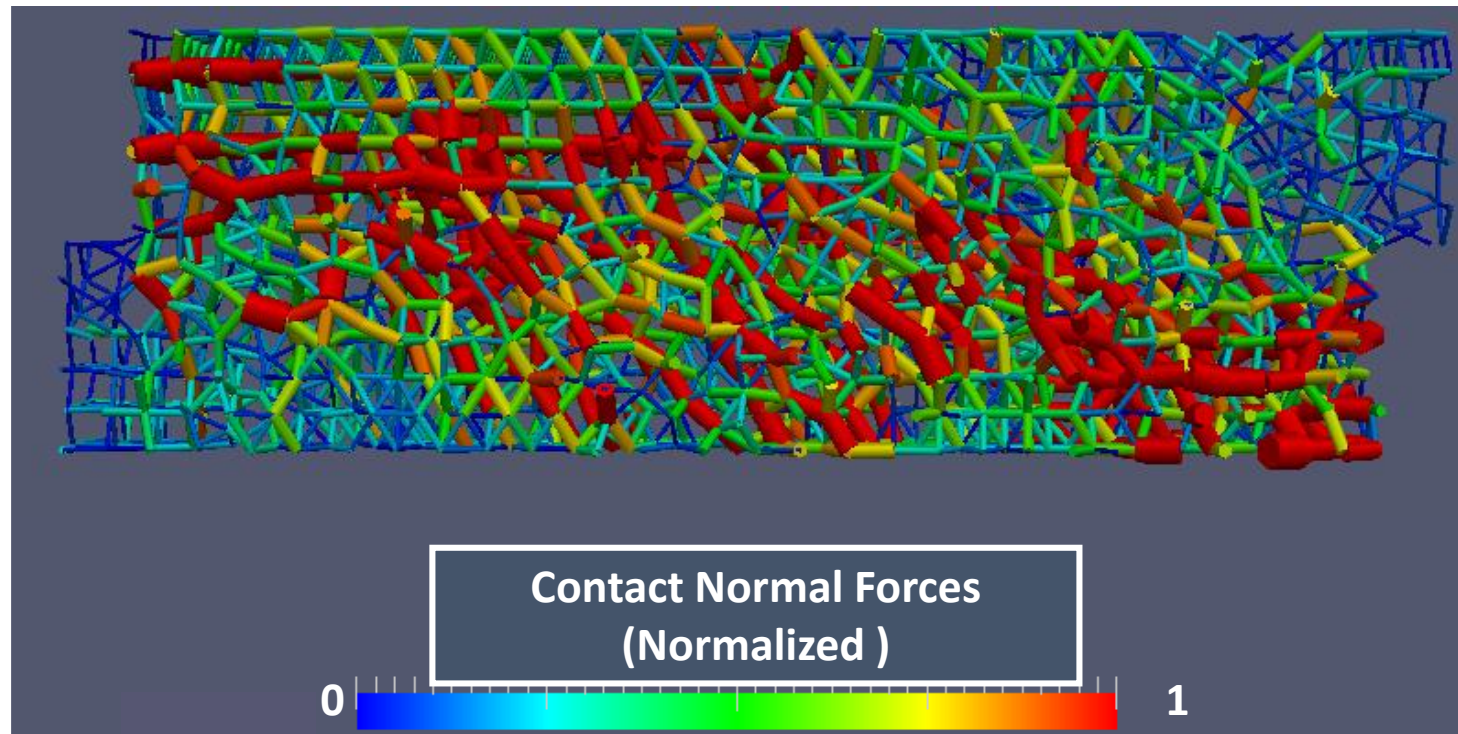
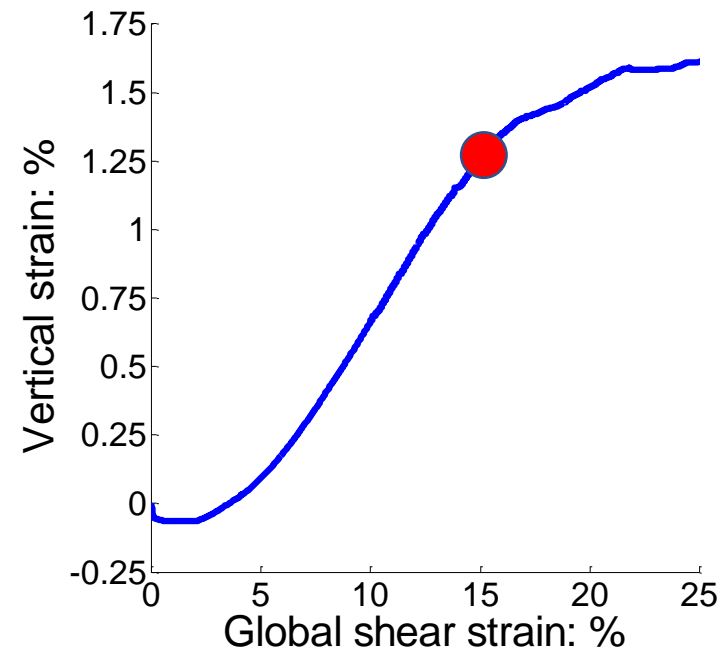
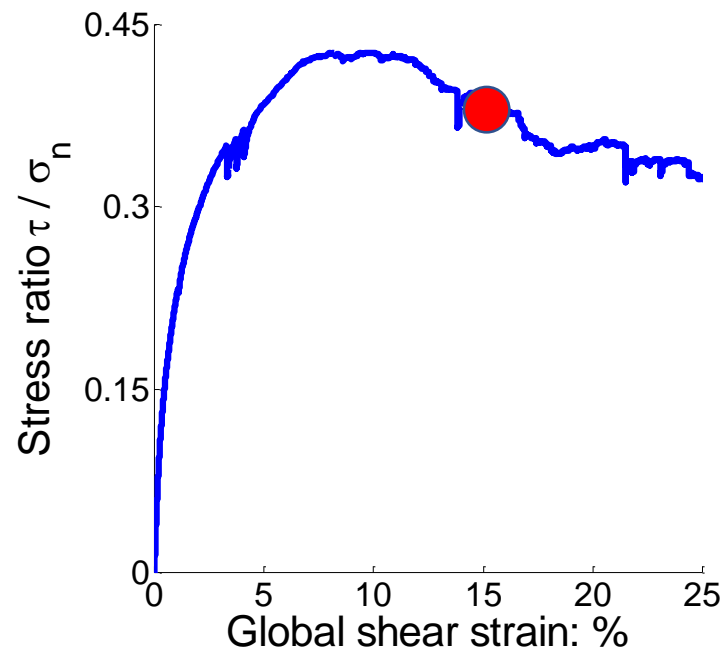


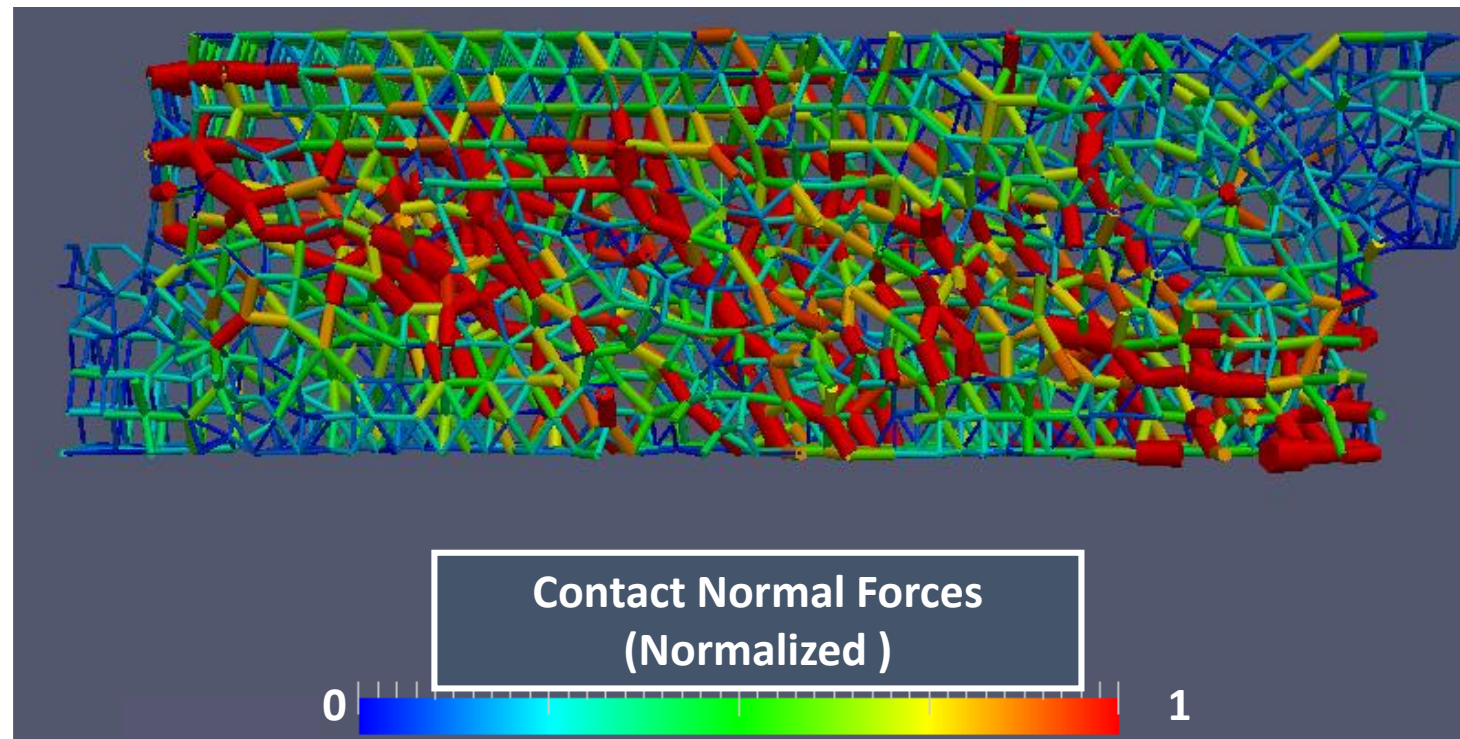
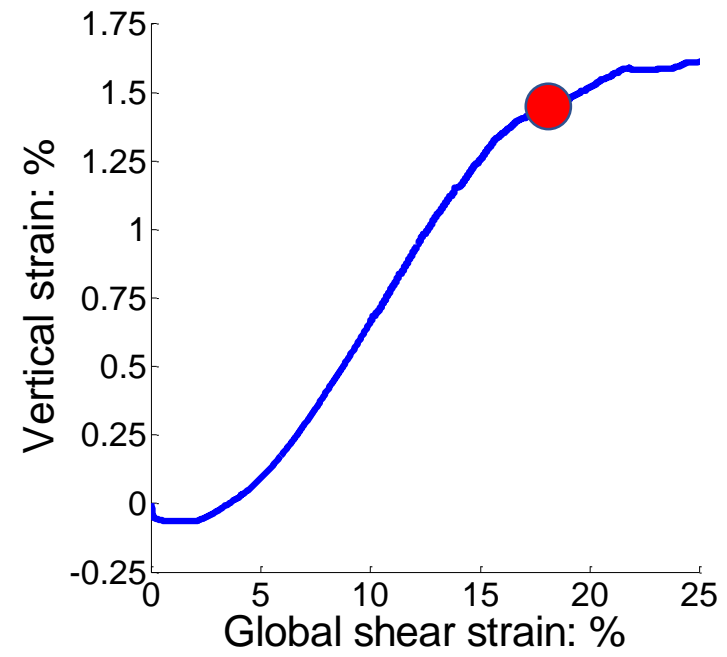
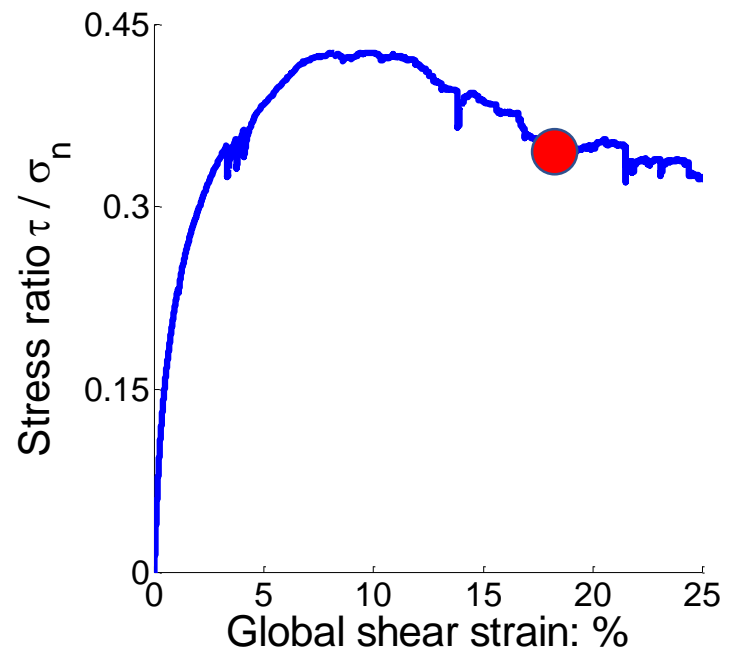


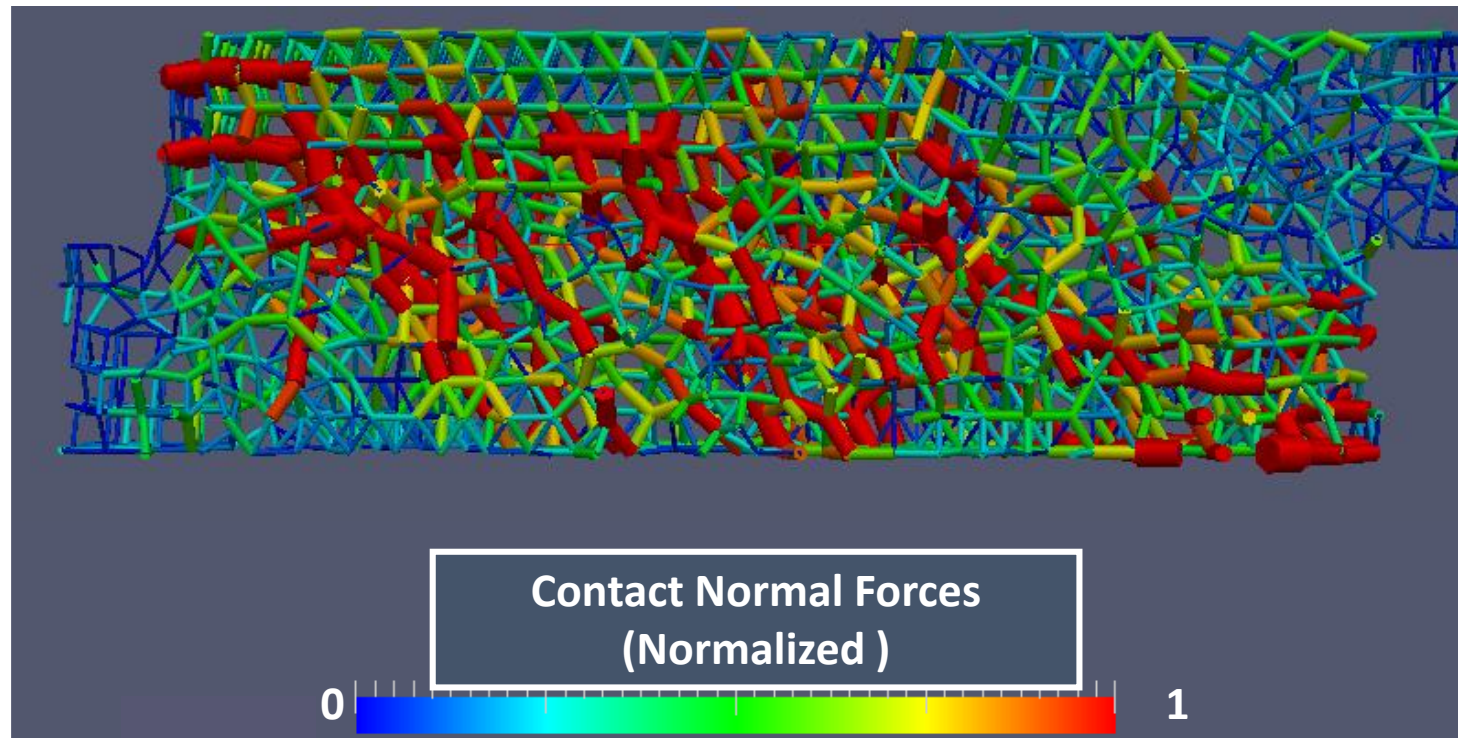
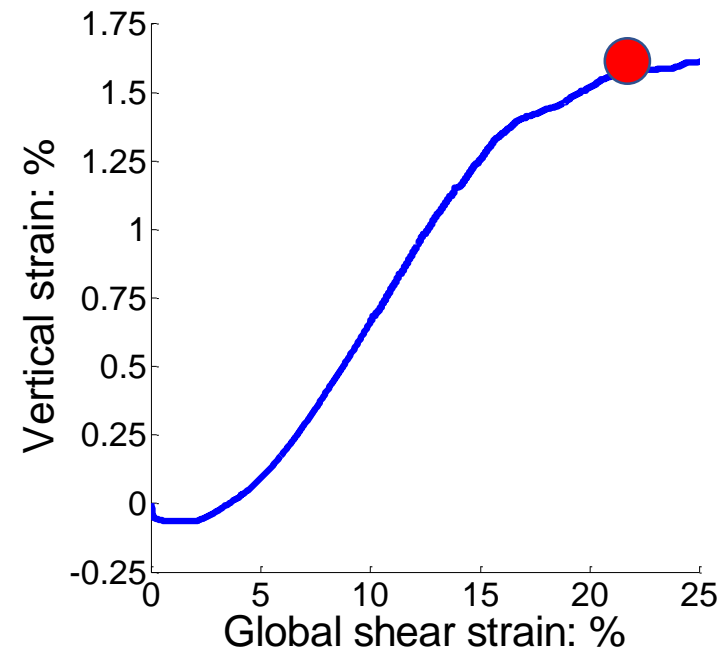
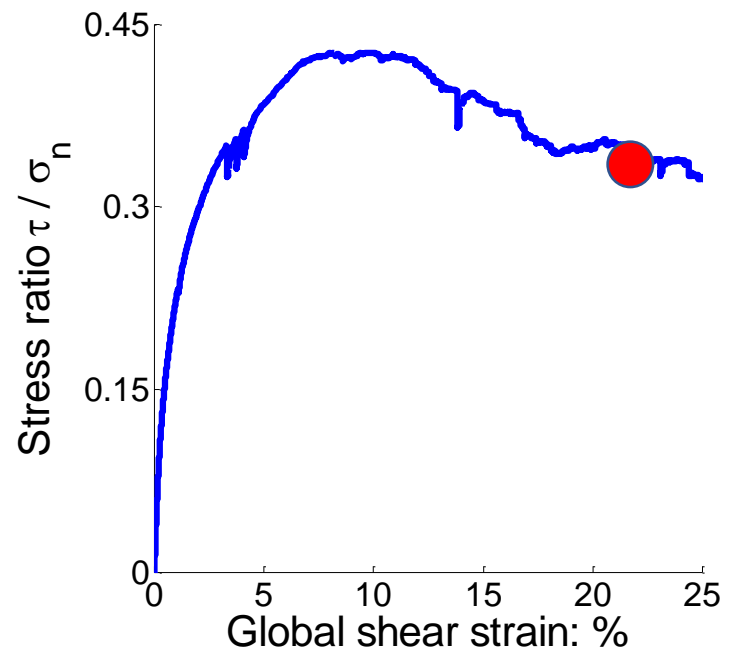


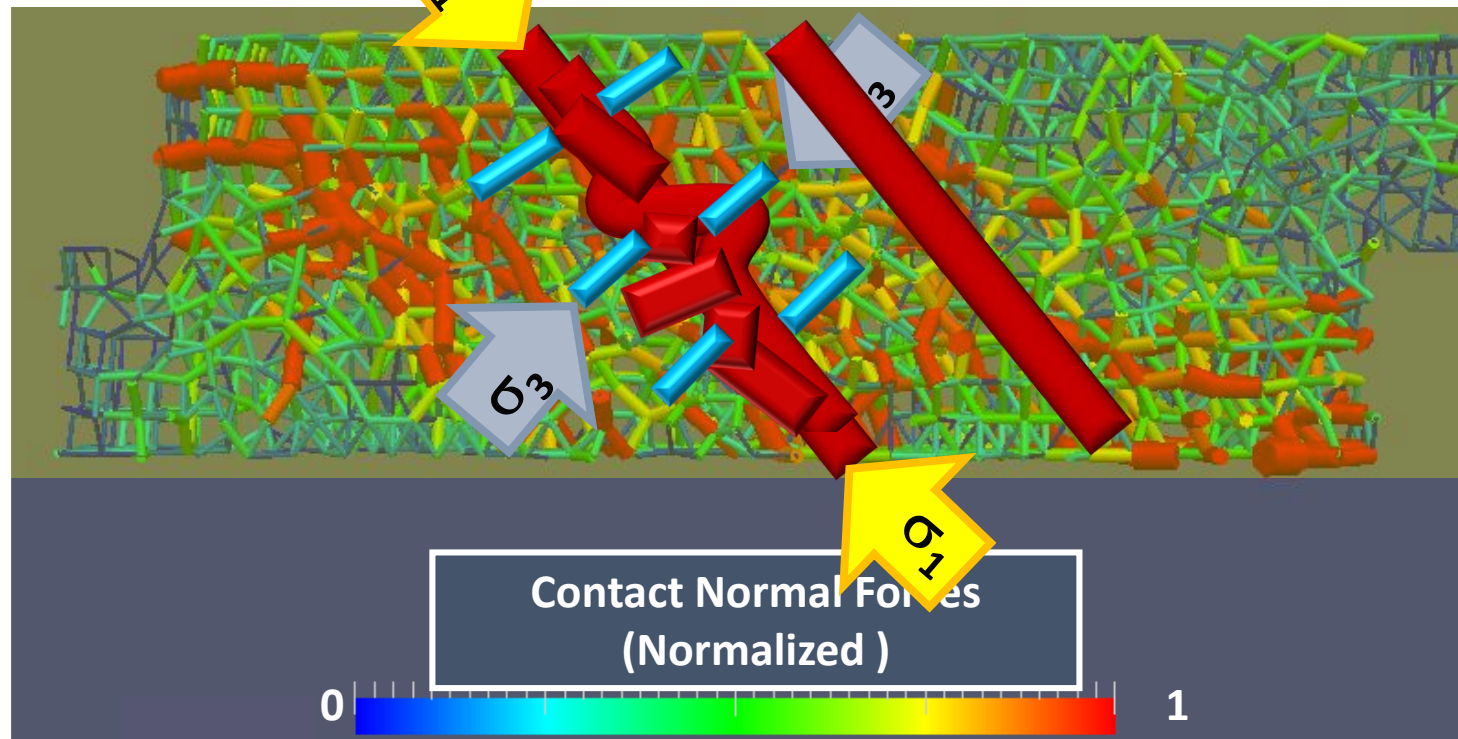
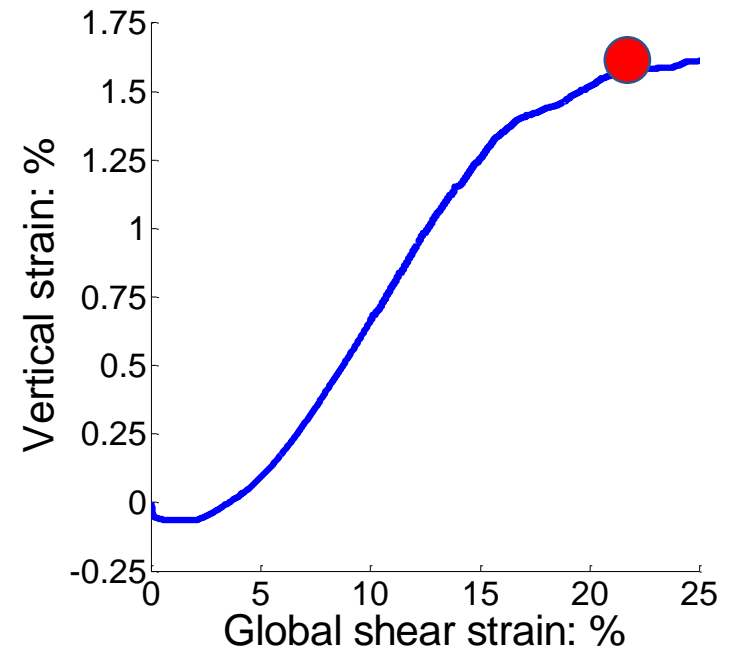
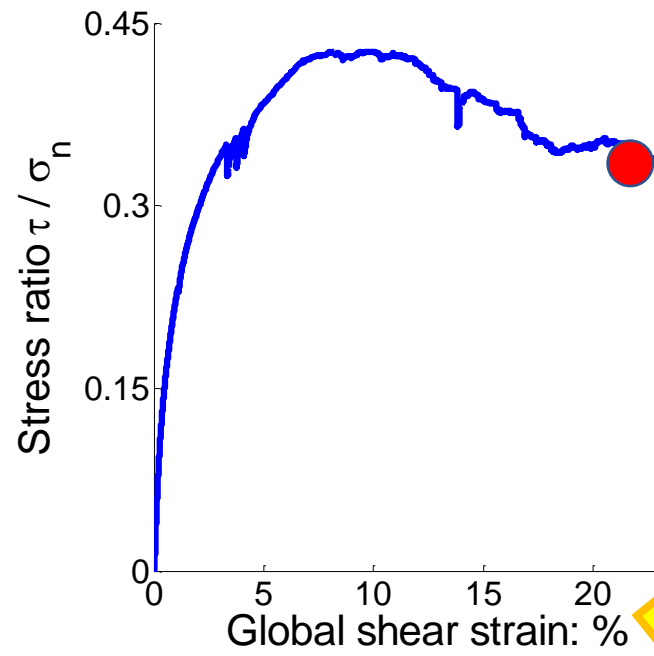












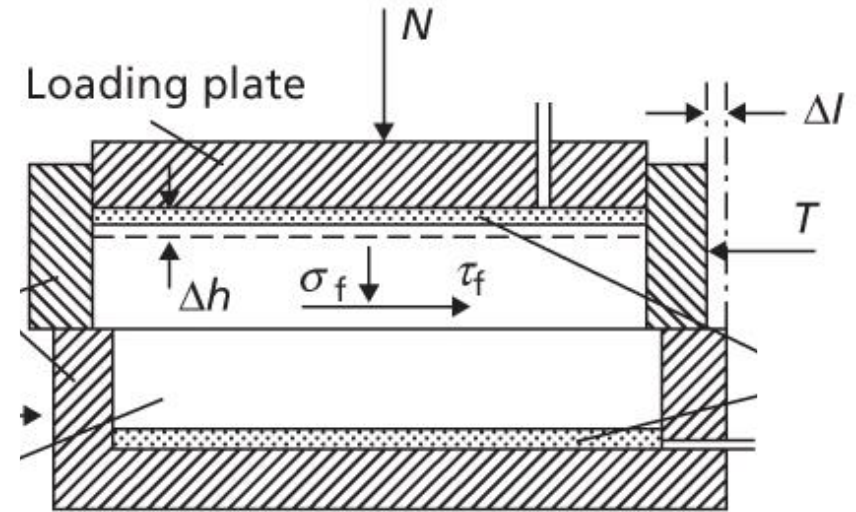
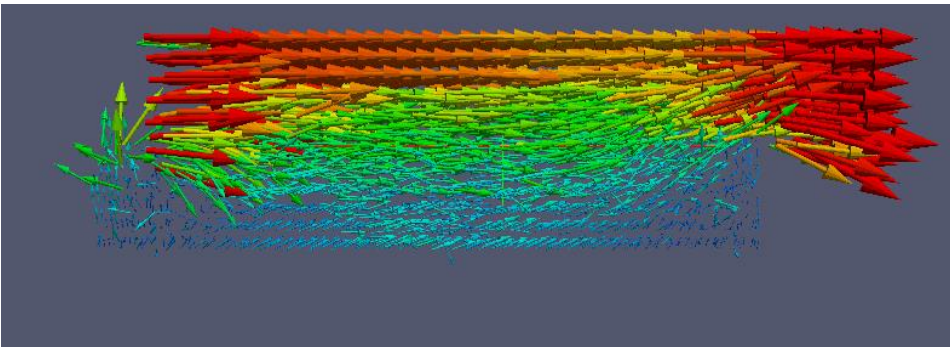
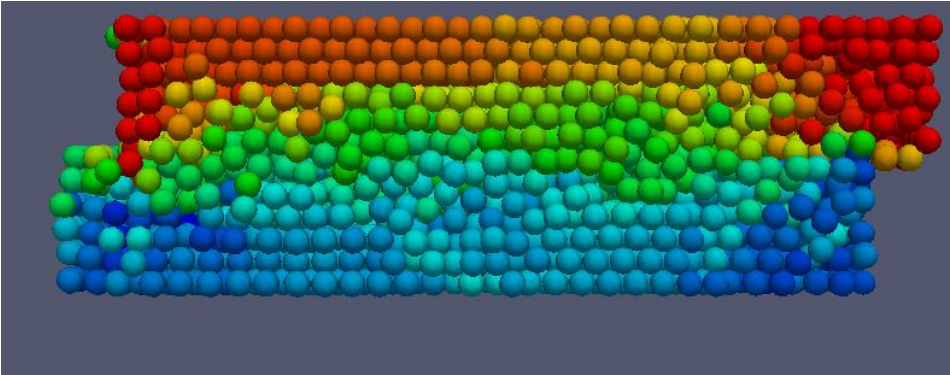
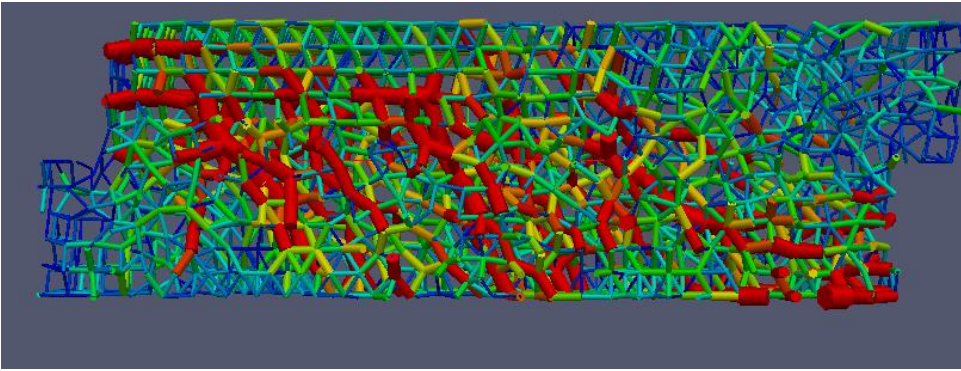
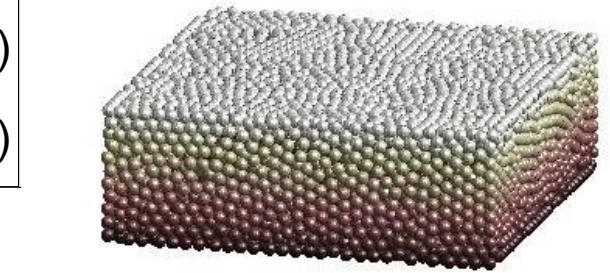
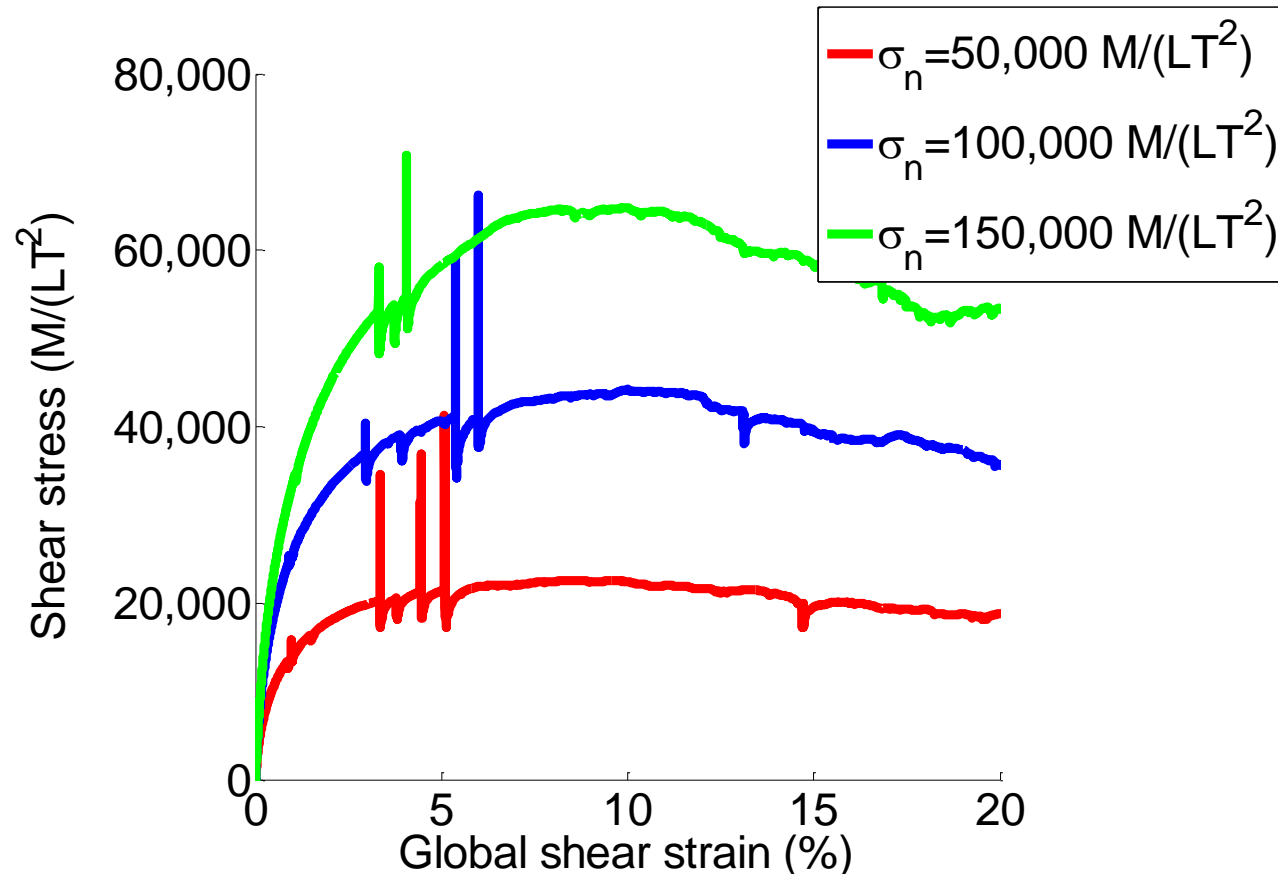


Diagram from
Craig's Soil Mechanics
Knappett and Craig(2012)

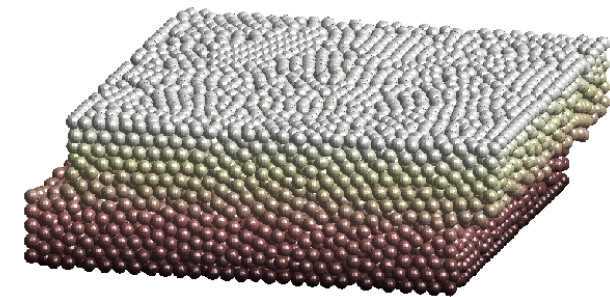
DEM and sand behaviour

- Even using spherical particles and simple contact models, DEM simulations can capture many of the complex mechanical response characteristics of sand
- The demonstrated ability to capture this behavior means that DEM can be used:
 - To advance understanding of the fundamental mechanisms that underlie the observed response in laboratory element tests
 - In parametric studies to better understand how various particle-scale parameters influence the overall load:deformation response
 - To complement experimental studies e.g. by considering stress and strain states that cannot be attained in experiments

DEM can capture stress-dependant/frictional strength



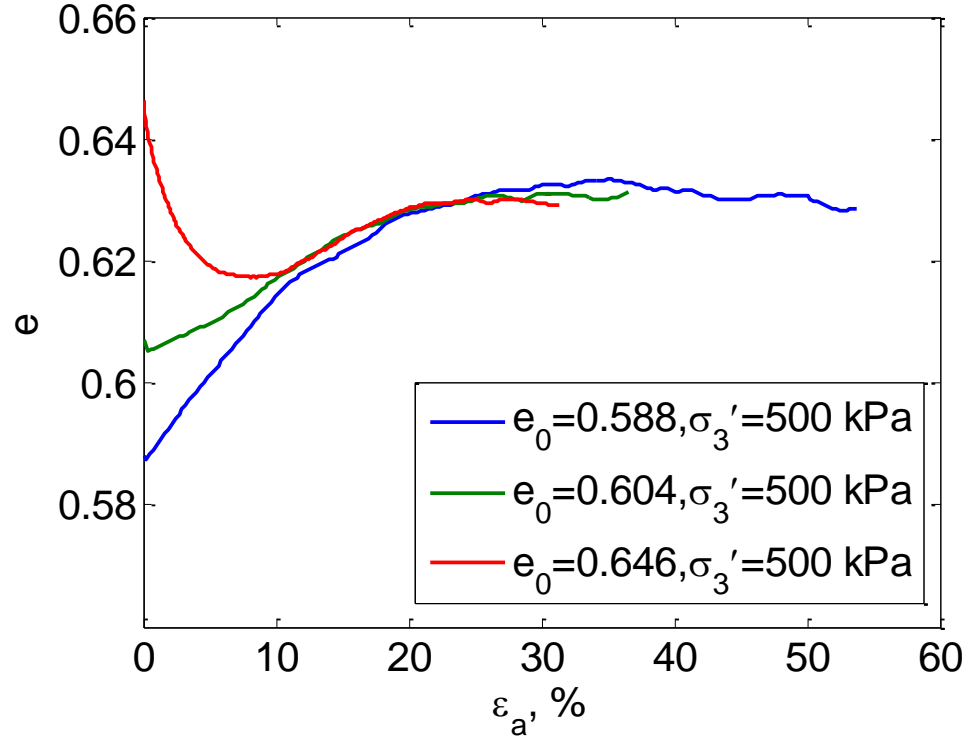
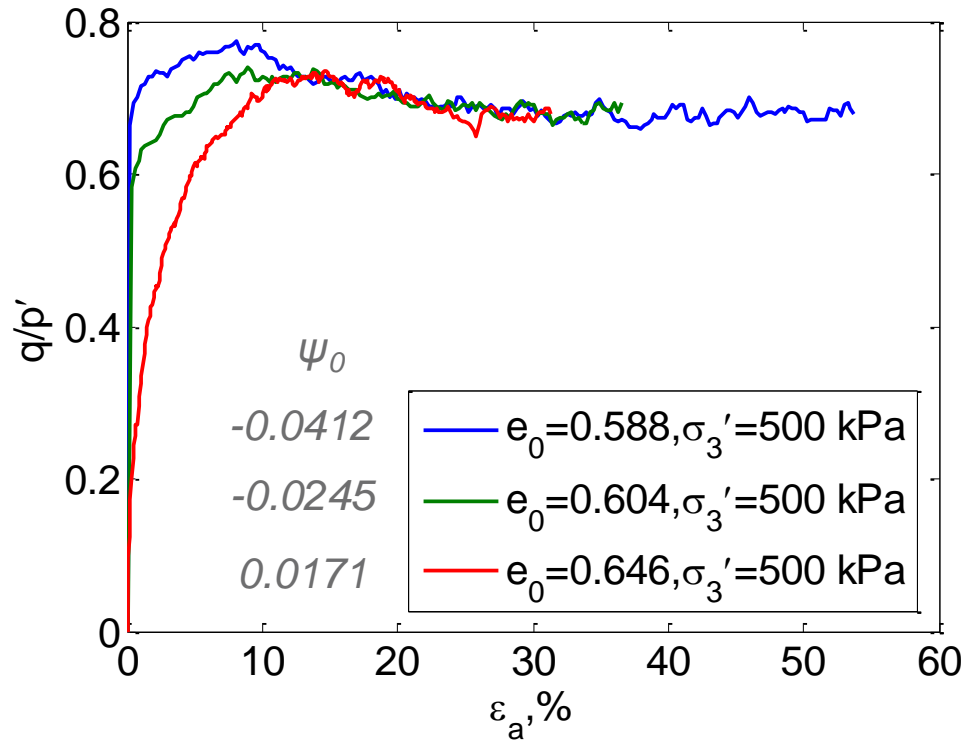
$\gamma = 0\%$



$\gamma = 15.3\%$

Direct shear test simulations
Modified version of Trubal
Dr.L. Cui

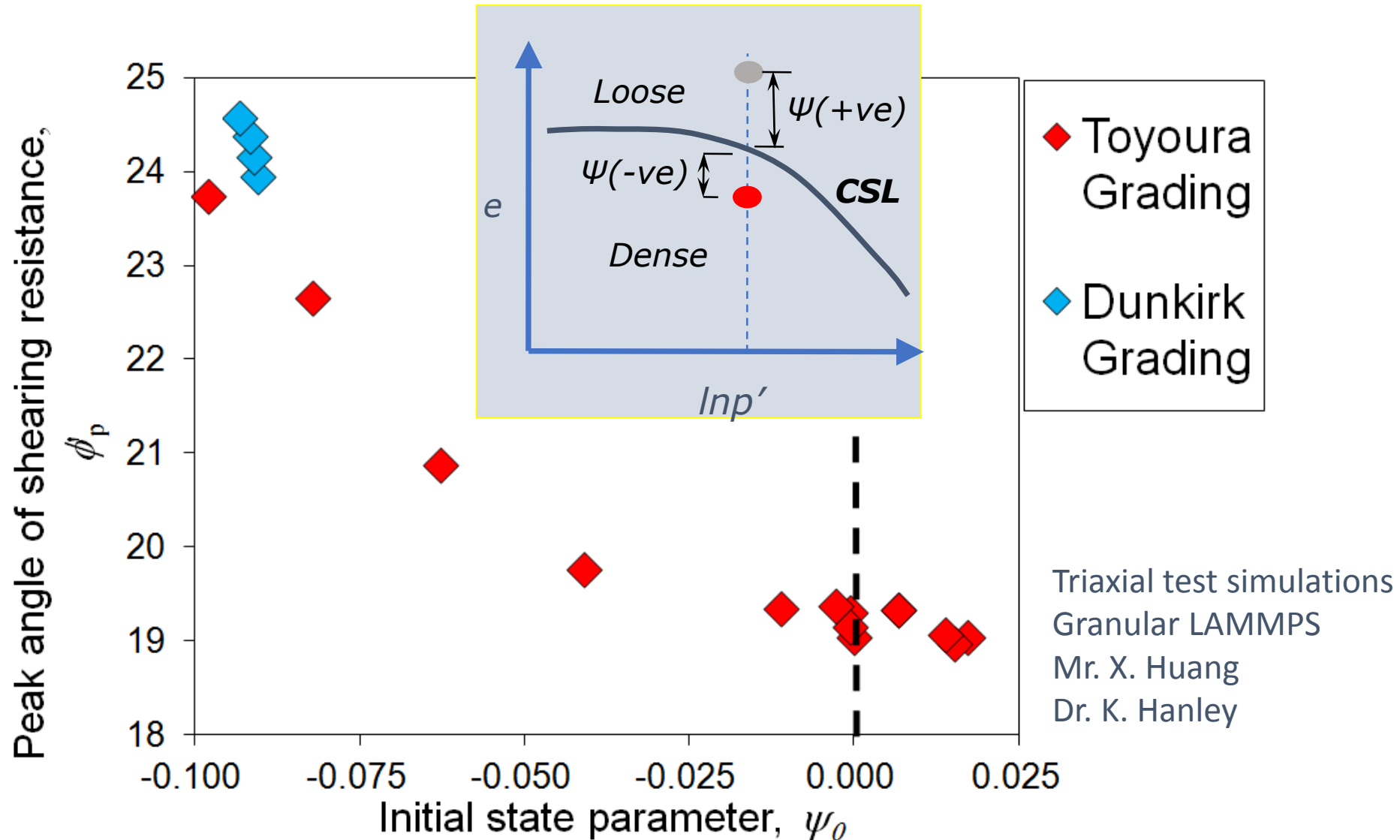
DEM can capture the existence of a critical state



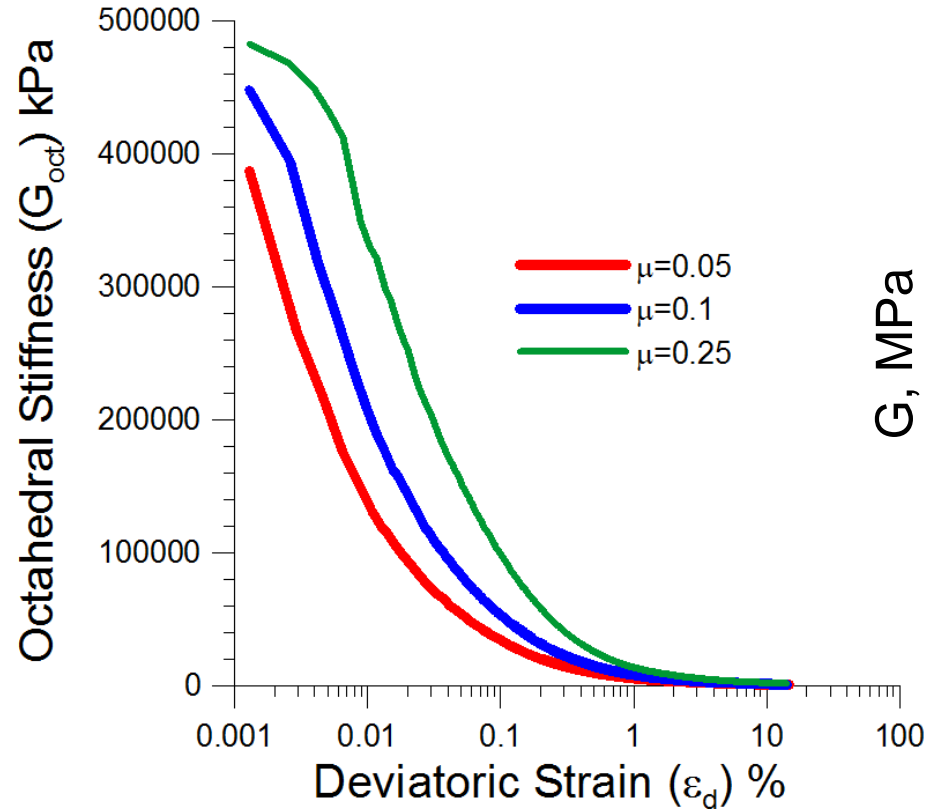
$\psi > 0$ loose (wet)
 $\psi = 0$ critical state
 $\psi < 0$ dense (dry)

Triaxial test simulations
Granular LAMMPS
Mr. X. Huang

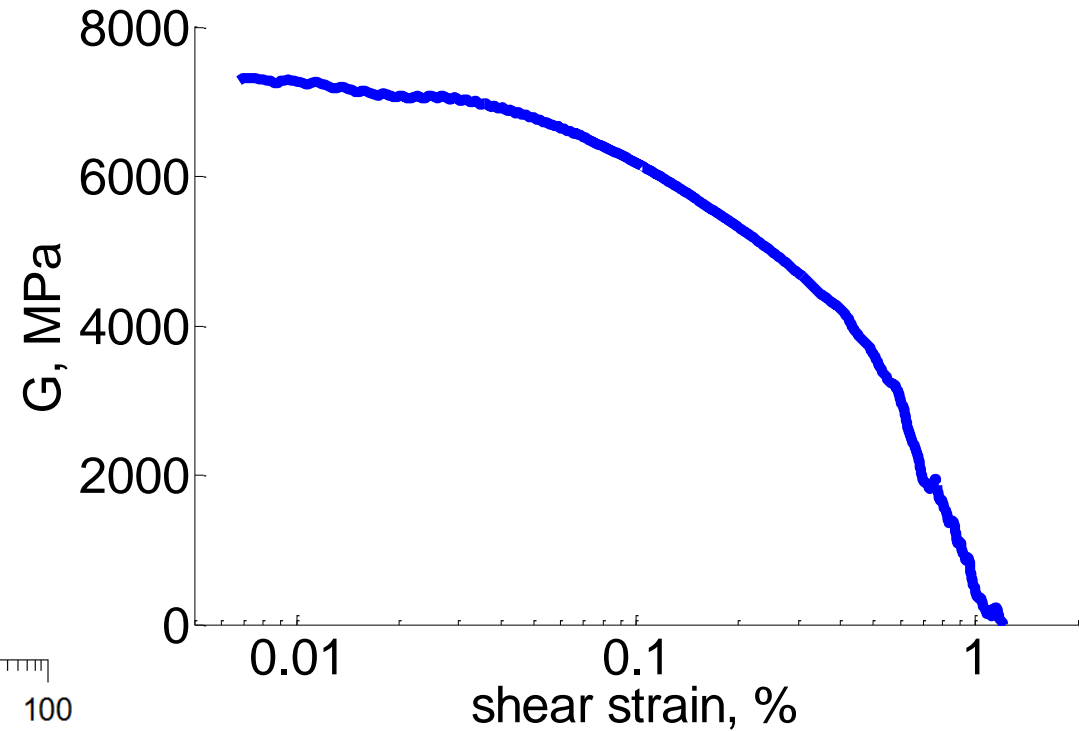
DEM can capture sensitivity of Φ_p to void ratio (state)



DEM can capture the non-linear nature of soil stiffness

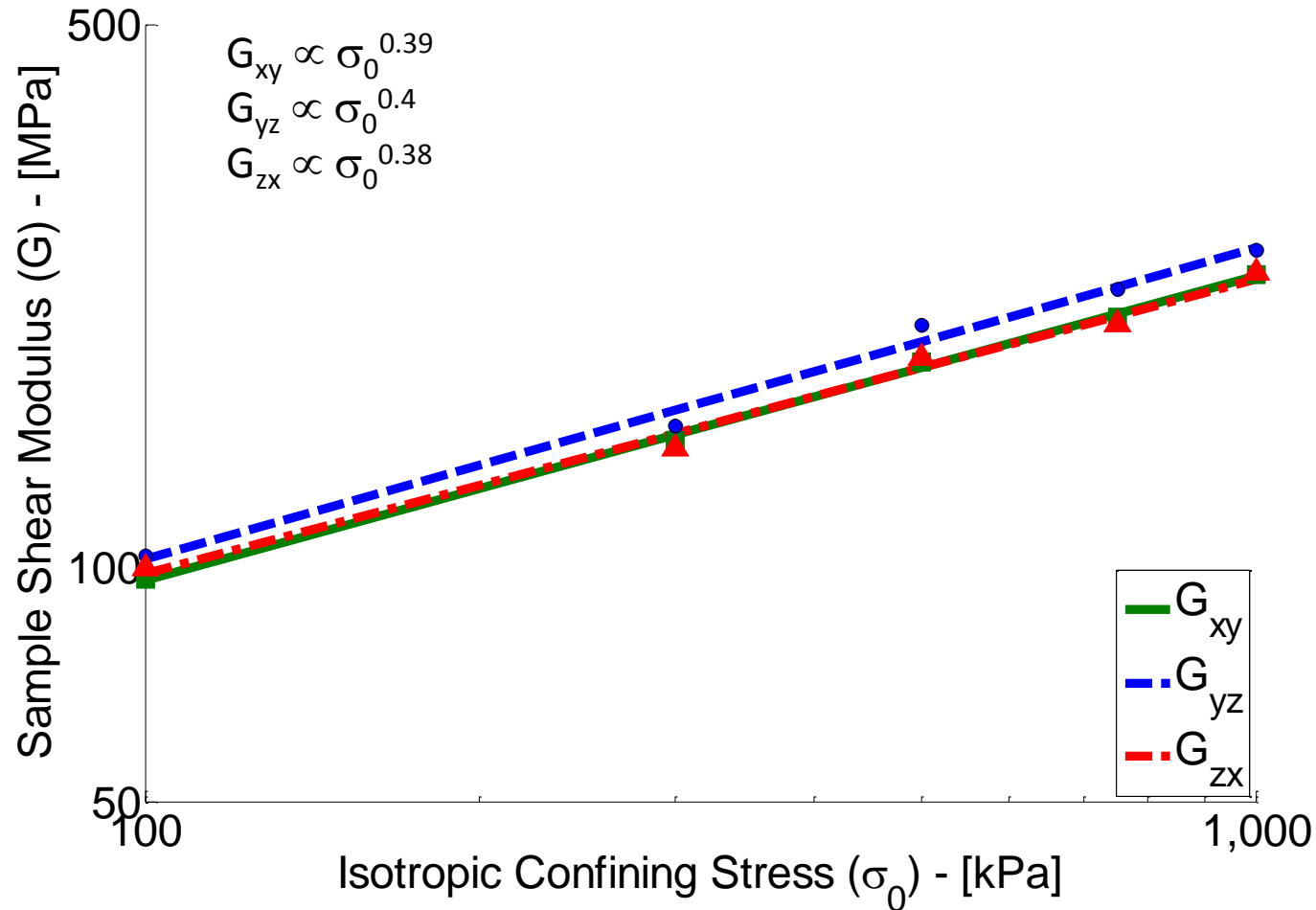


Triaxial test simulations
Simplified Hertz-Mindlin Contact Model
Modified Trubal, periodic cell, spheres
Dr. D. Barreto



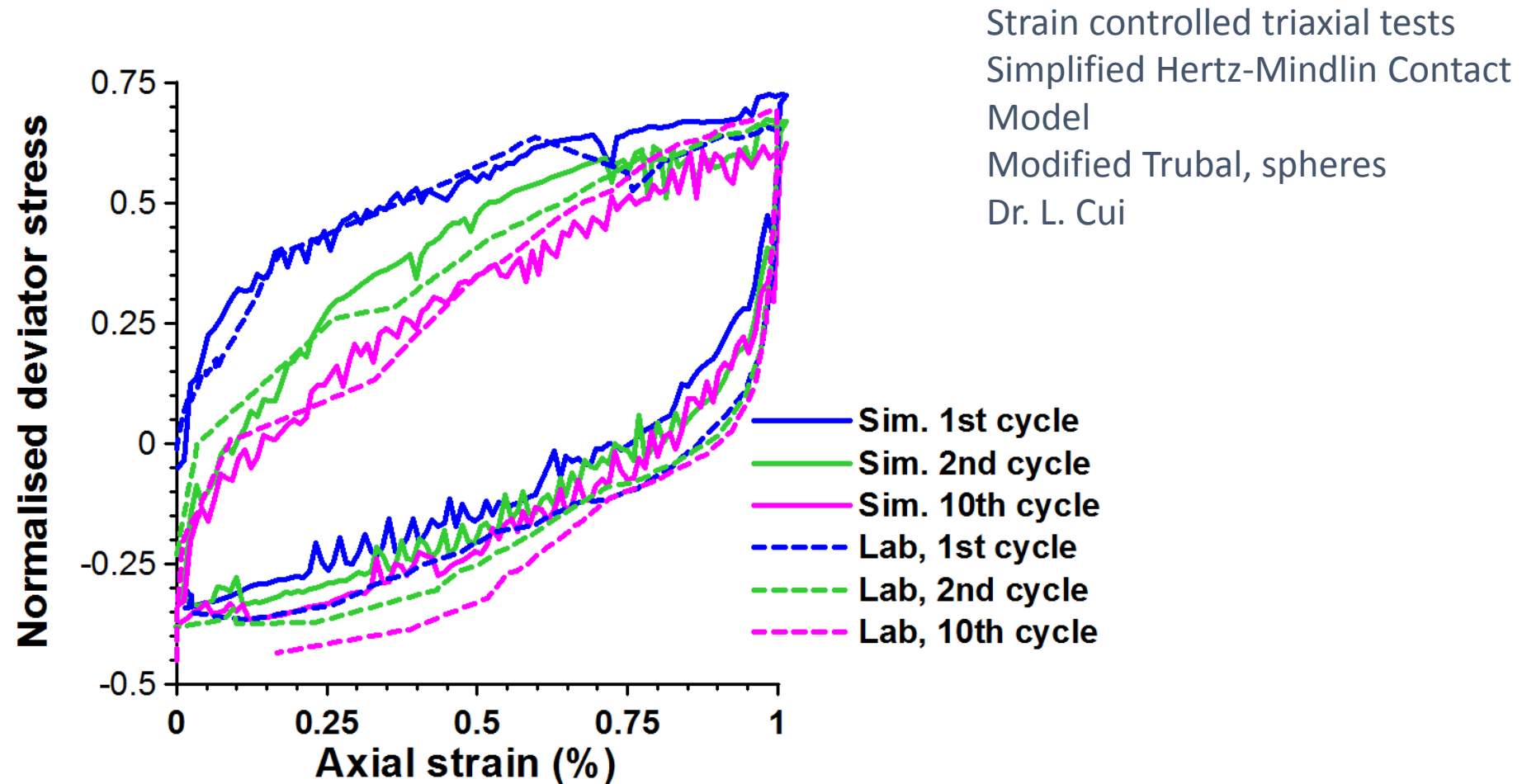
Triaxial test simulation
Parallel Bond Contact Model
PFC 3D spheres
Dr. G. Cheung

DEM can capture the stress-dependency of soil stiffness



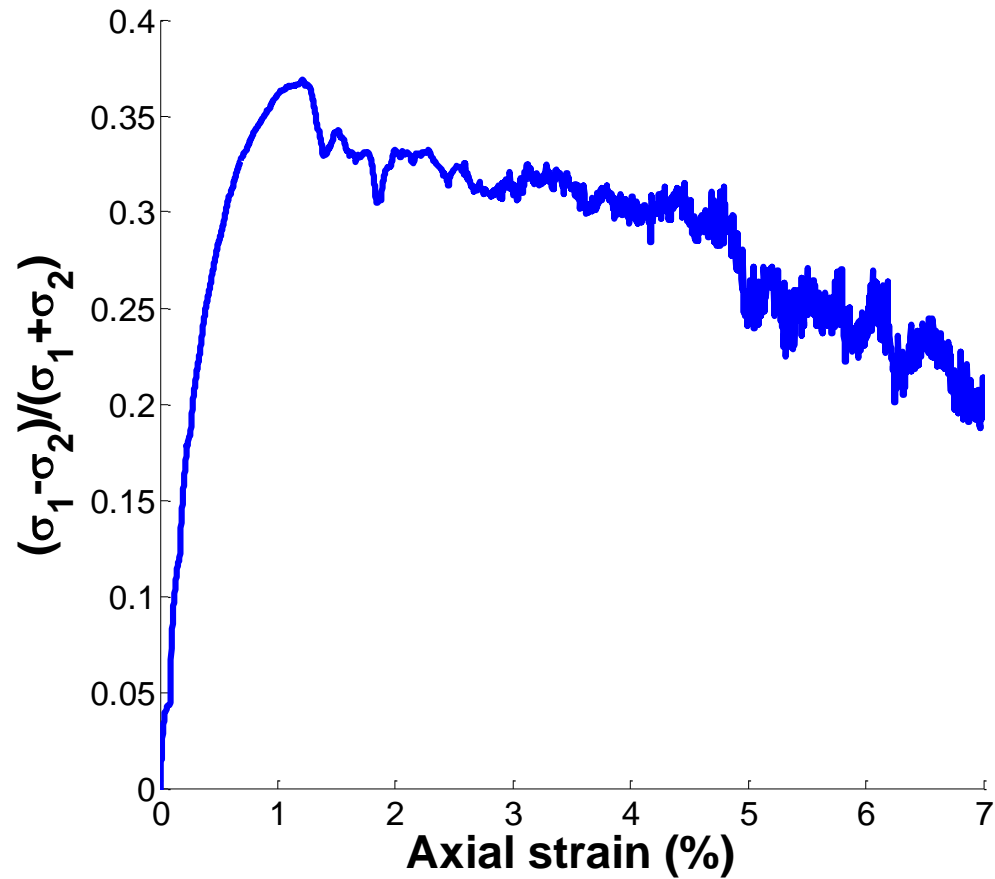
Bender element test
simulation, cubical cell
FCC packing
Simplified Hertz-Mindlin
Contact Model
PFC 3D spheres
Dr. J. O'Donovan

DEM can capture hysteresis in cyclic loading

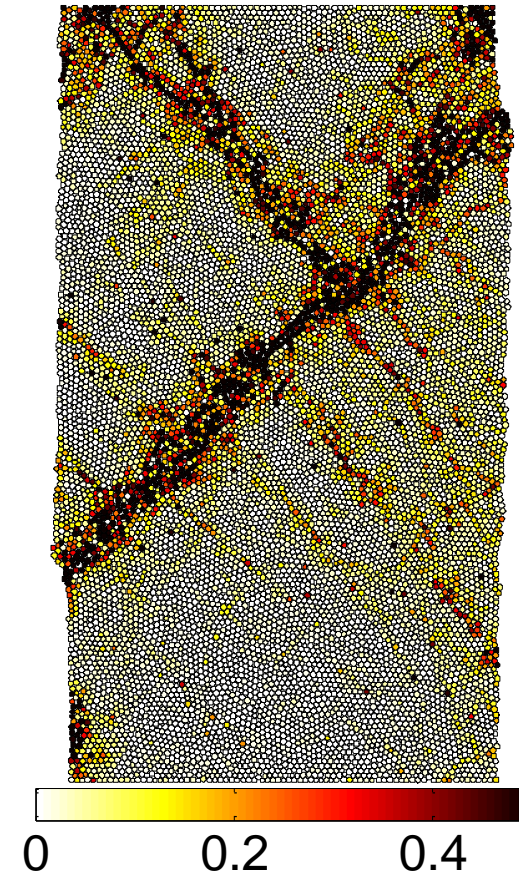


DEM can capture strain softening and localization

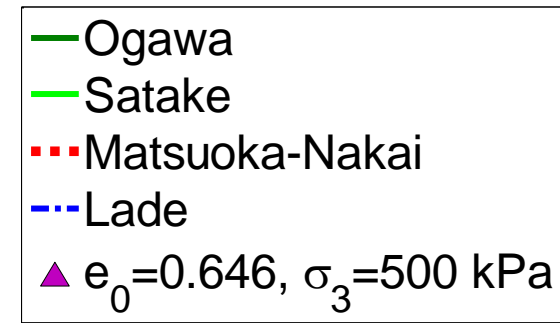
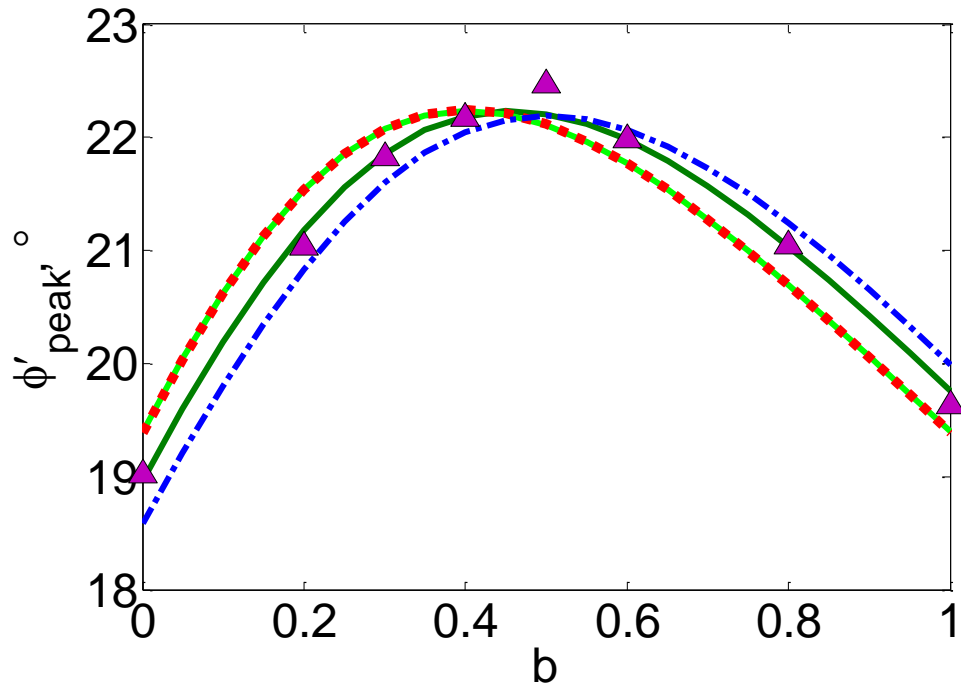
Biaxial compression - 12,512 disks
PFC 2D: Flexible membrane boundary



Disk rotation (rad)
Axial strain=3.7%



DEM can capture 3D stress-space failure criterion



b=0: Triaxial
Compression

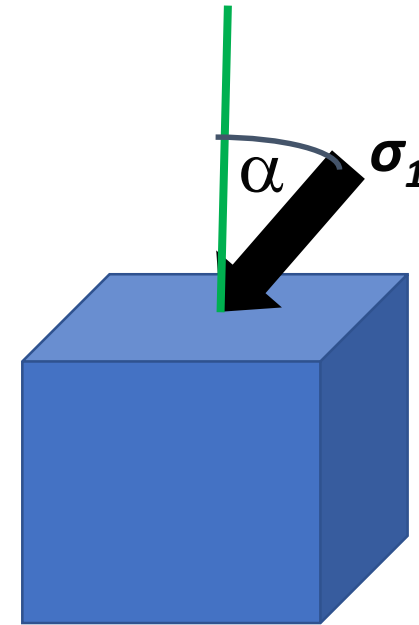
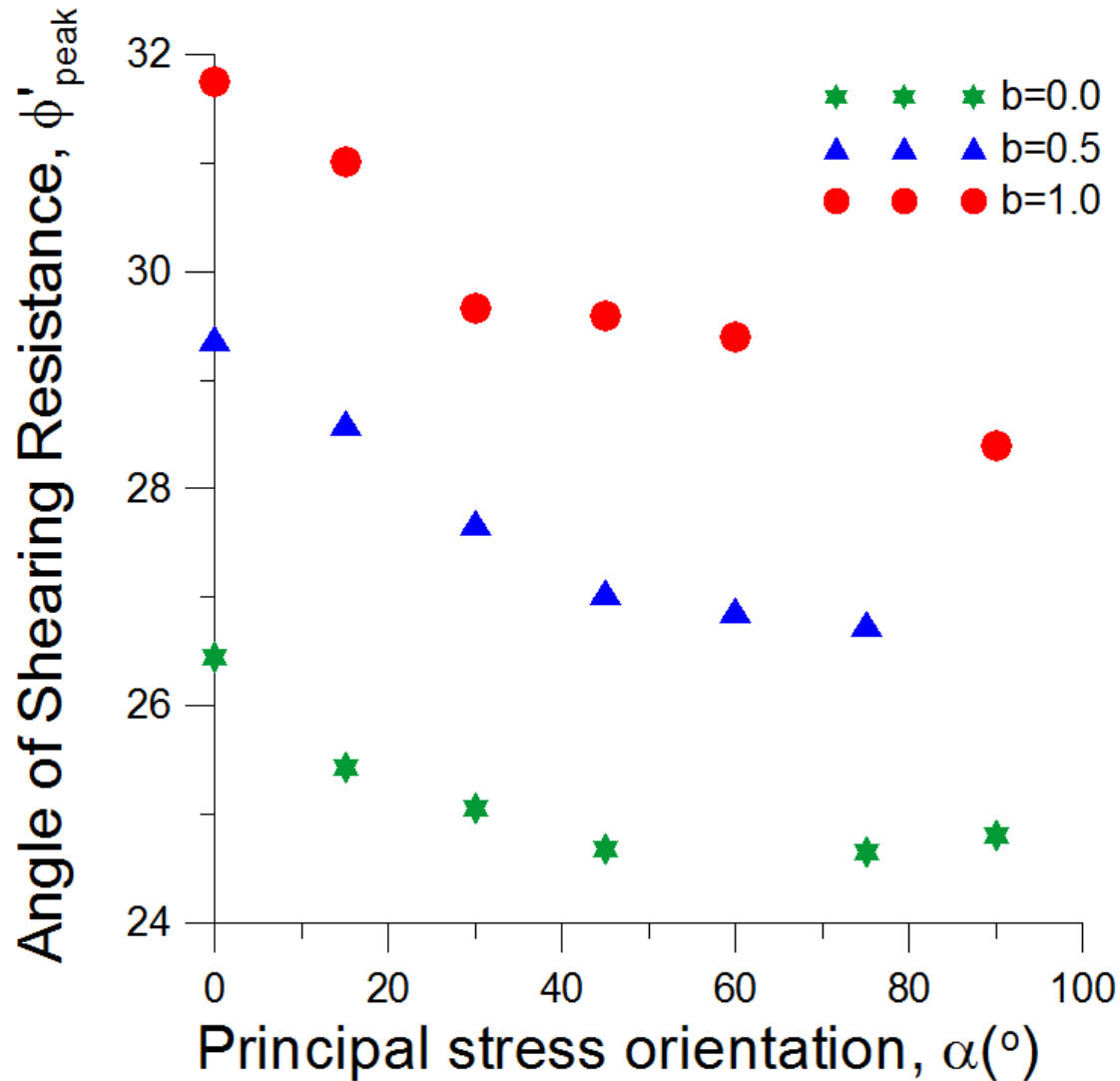
b=1: Triaxial
Extension

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$$

Intermediate
principal stress ratio

Granular LAMMPS
Mr. X. Huang

DEM can capture anisotropy of strength

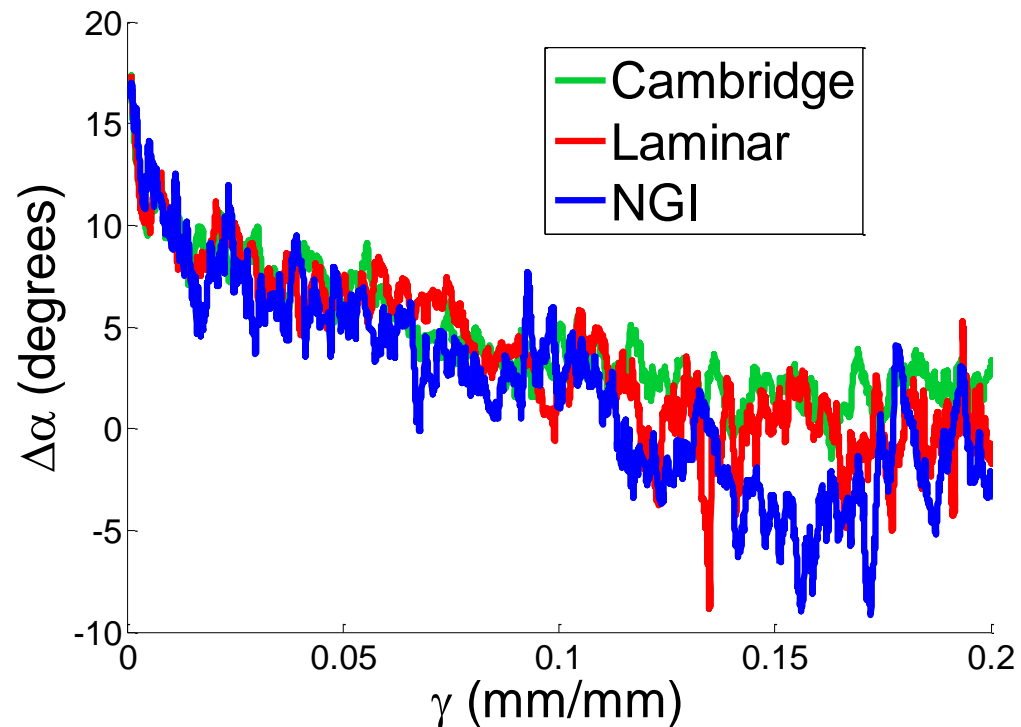


K_0 compressed samples

Modified Trubal

Dr. D. Barreto

DEM can capture the non-coaxiality of stress and strain increments



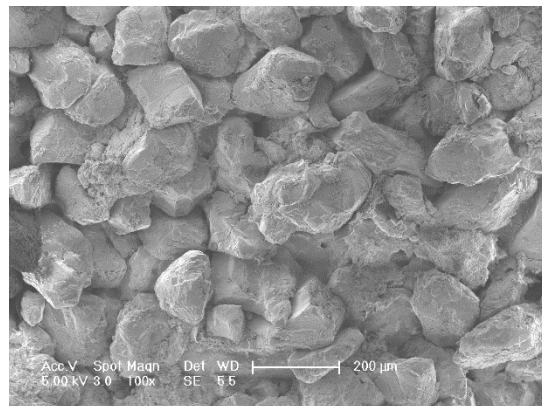
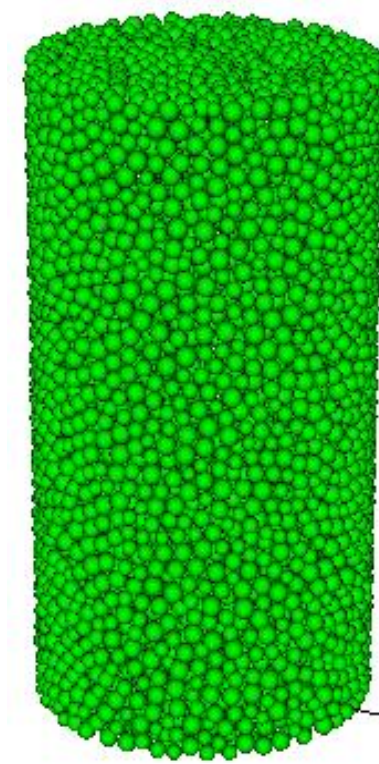
$\Delta\alpha$ = angle between principal stress direction and principal strain increment direction

Simple shear test simulations
PFC 2D
Dr. C. K. Shen

DEM can be calibrated to model sandstone

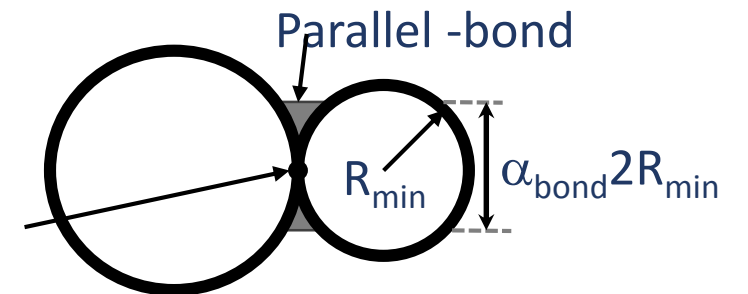


70 MPa Triaxial

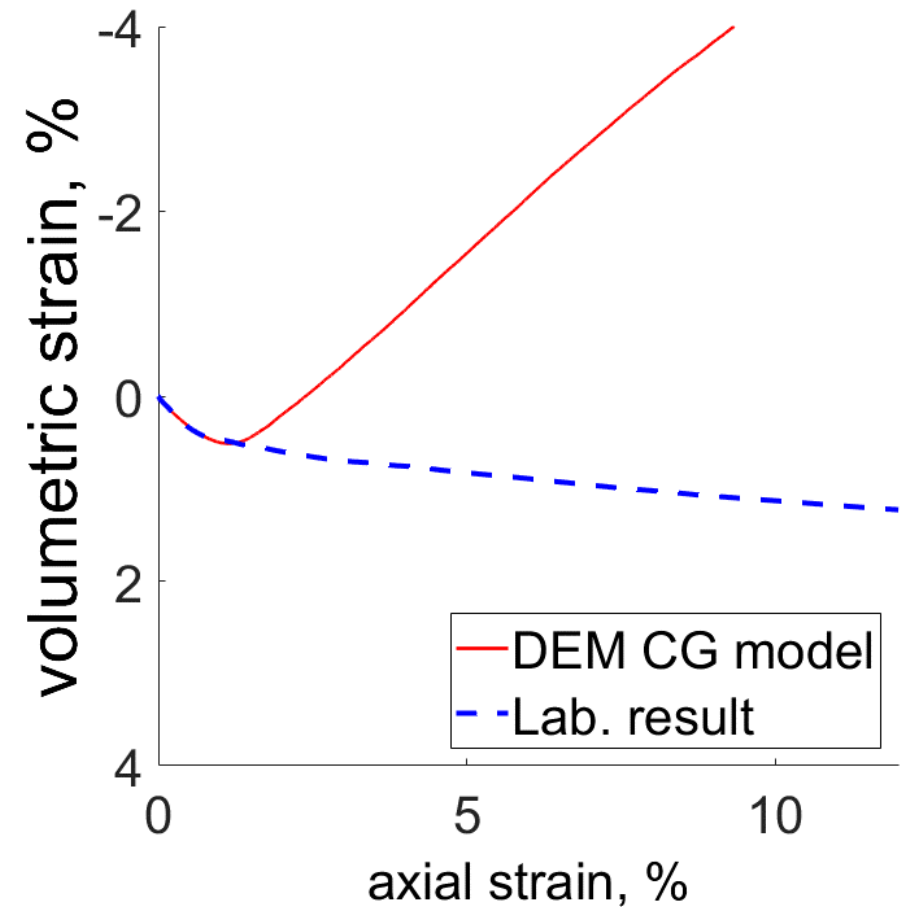
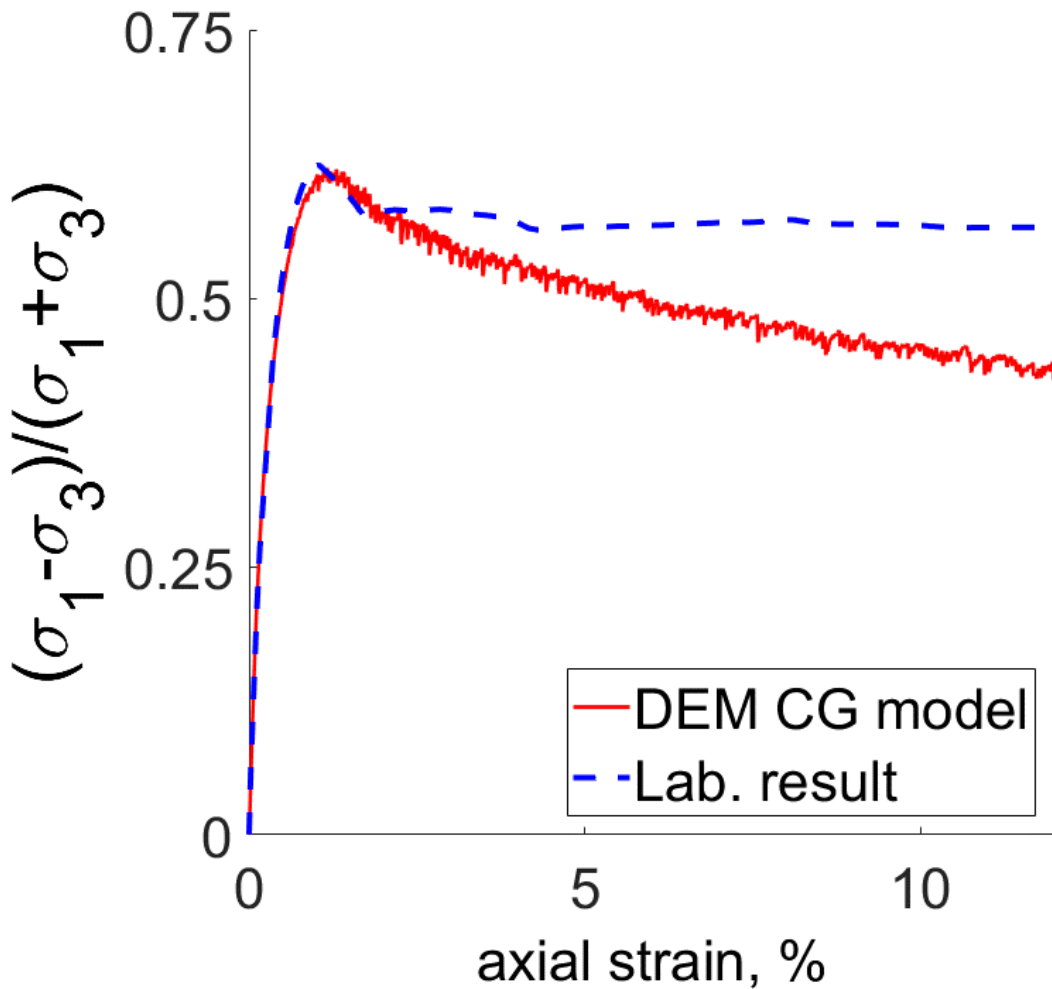


Castlegate sandstone

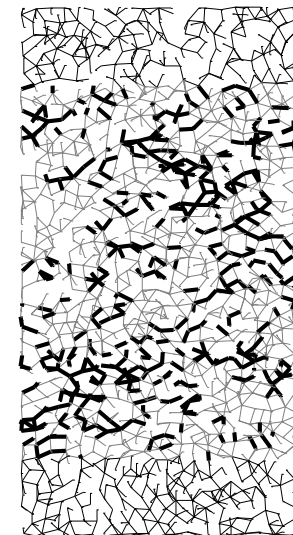
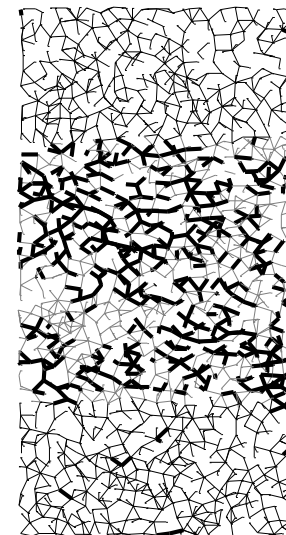
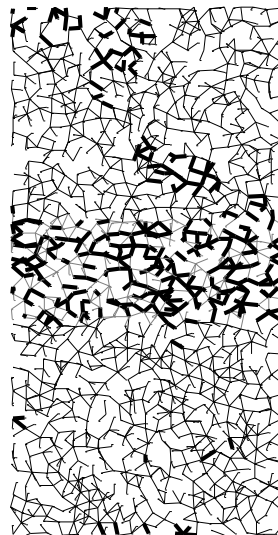
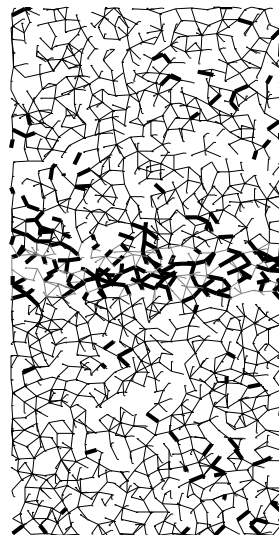
Contact bond at ball-ball contact



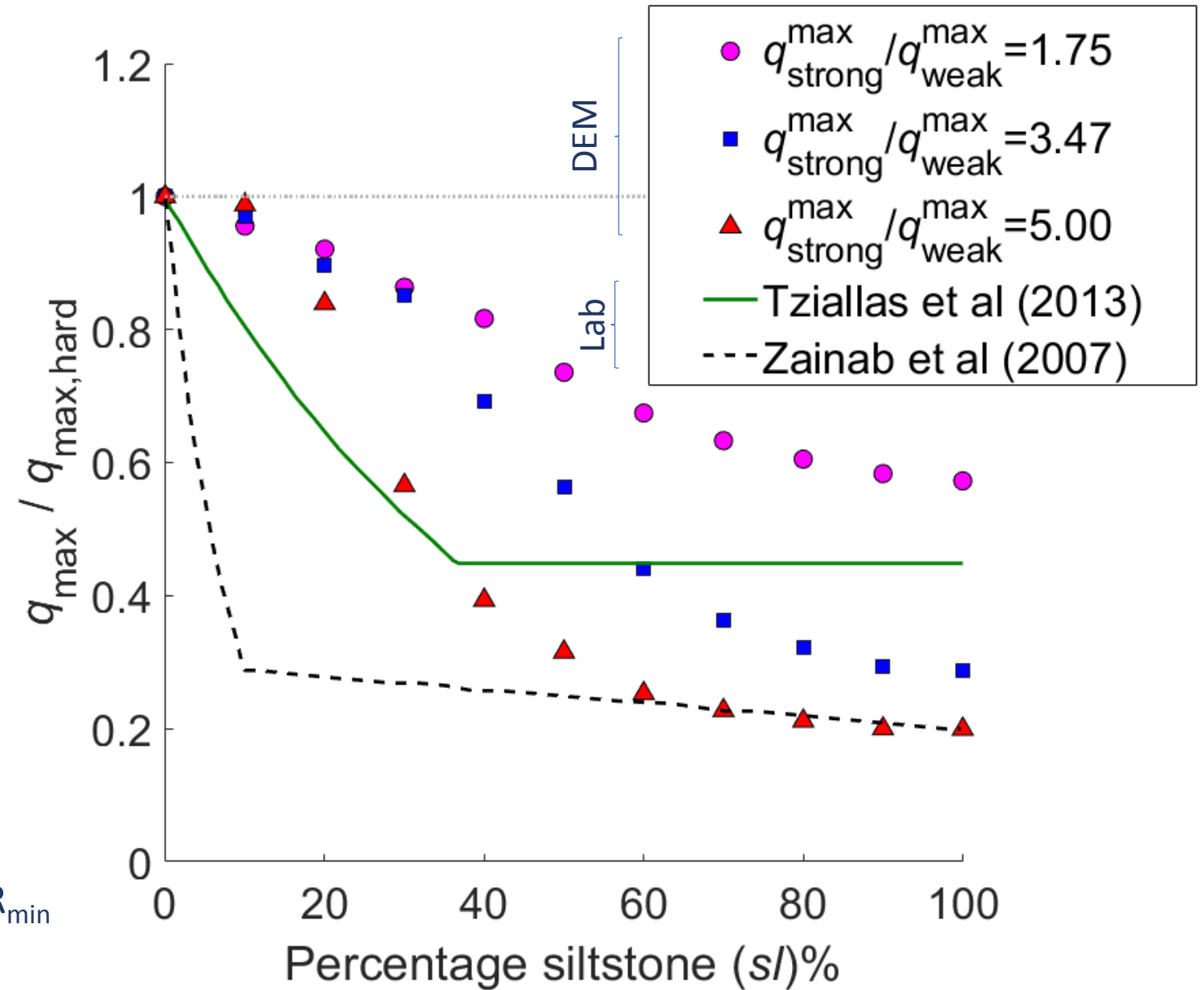
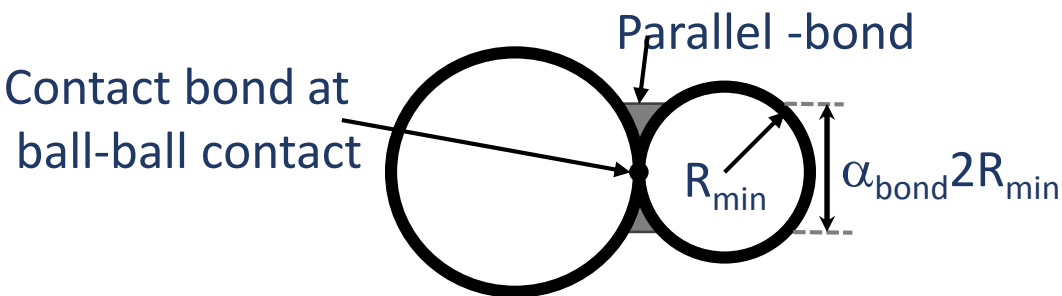
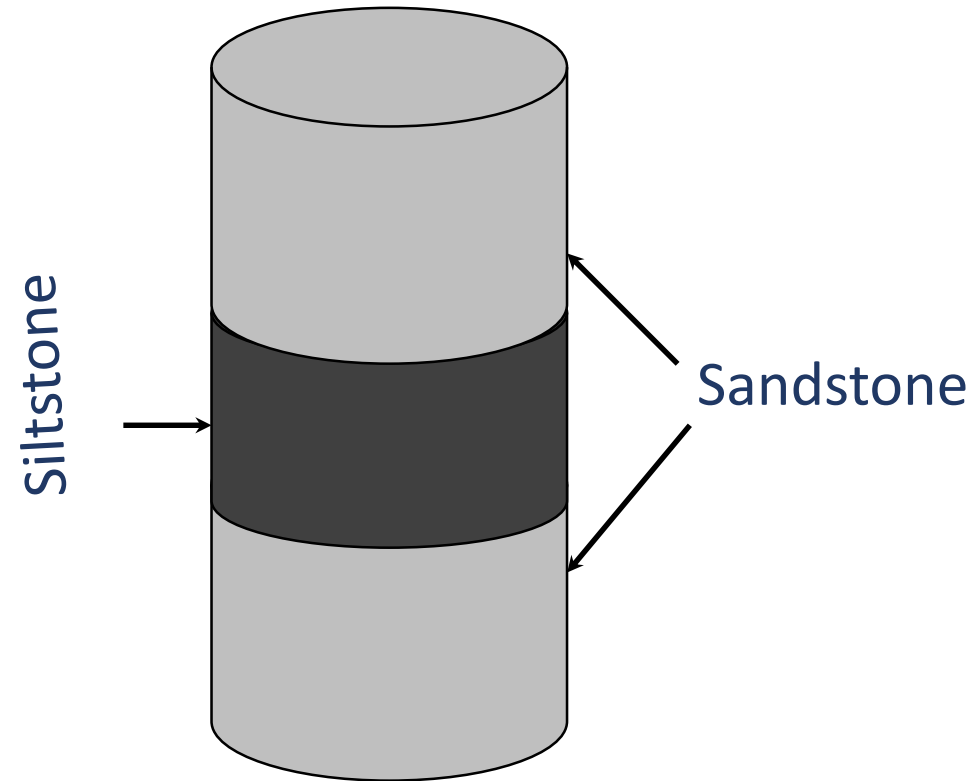
DEM can be calibrated to model sandstone



DEM can be used to simulate interbedded rock layers



DEM can be used to simulate interbedded rock layers



DEM and sand behaviour

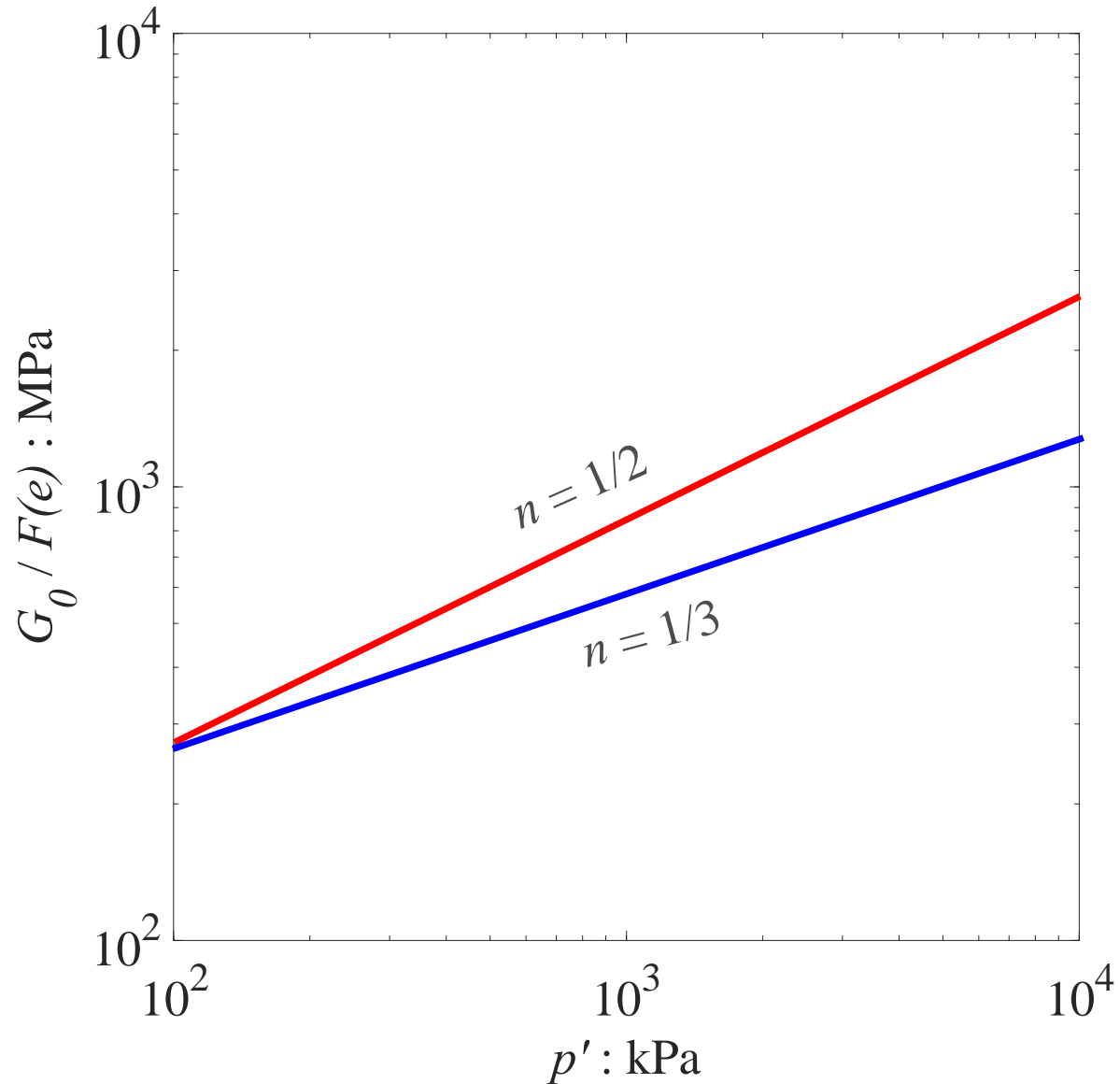
- Even using spherical particles and simple contact models, DEM simulations can capture many of the complex mechanical response characteristics of sand
- The demonstrated ability to capture this behavior means that DEM can be used:
 - To advance understanding of the fundamental mechanisms that underlie the observed response in laboratory element tests
 - In parametric studies to better understand how various particle-scale parameters influence the overall load:deformation response
 - To complement experimental studies e.g. by considering stress and strain states that cannot be attained in experiments

Advancing understanding of load:deformation behaviour.

Three fundamental questions

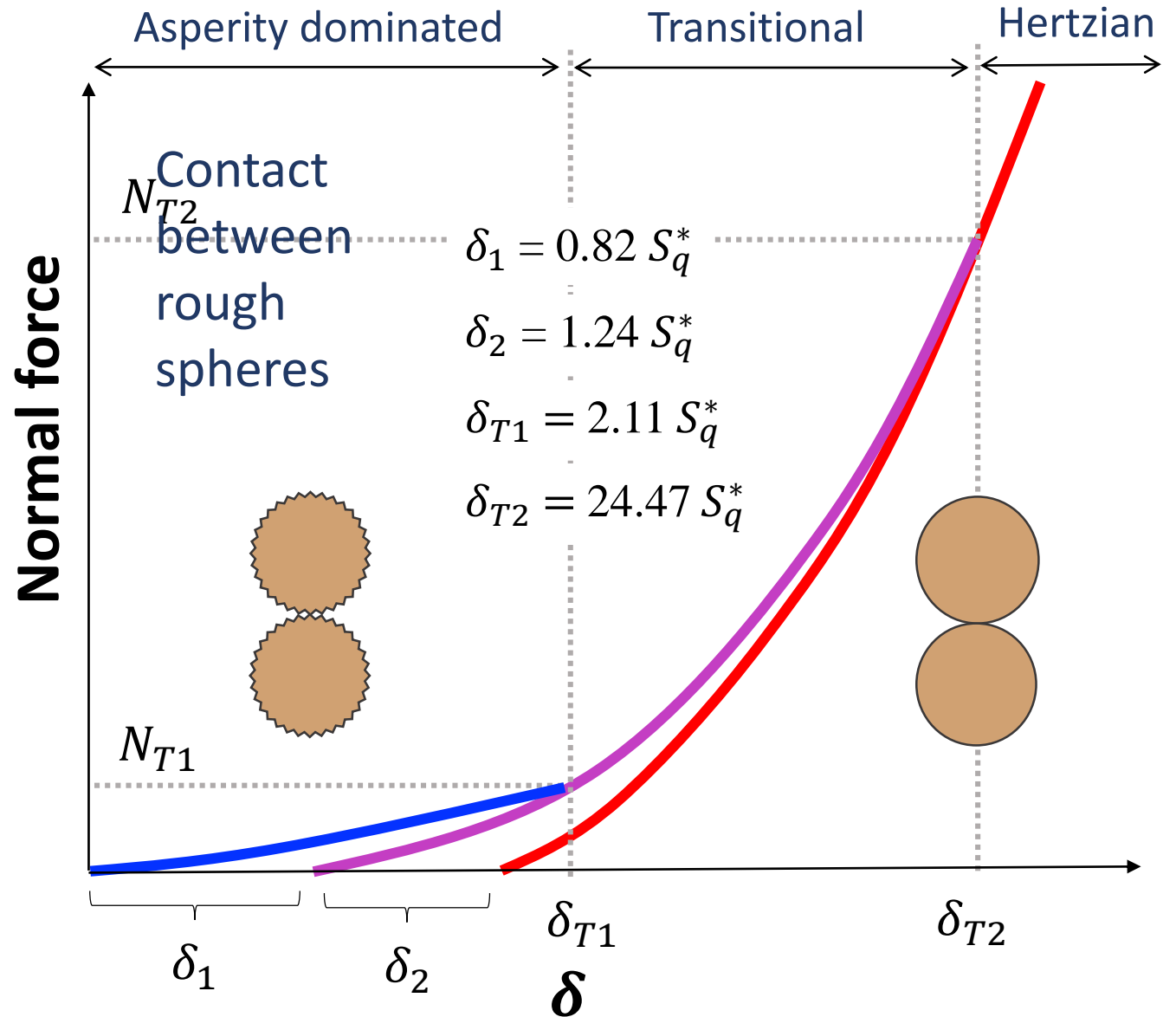
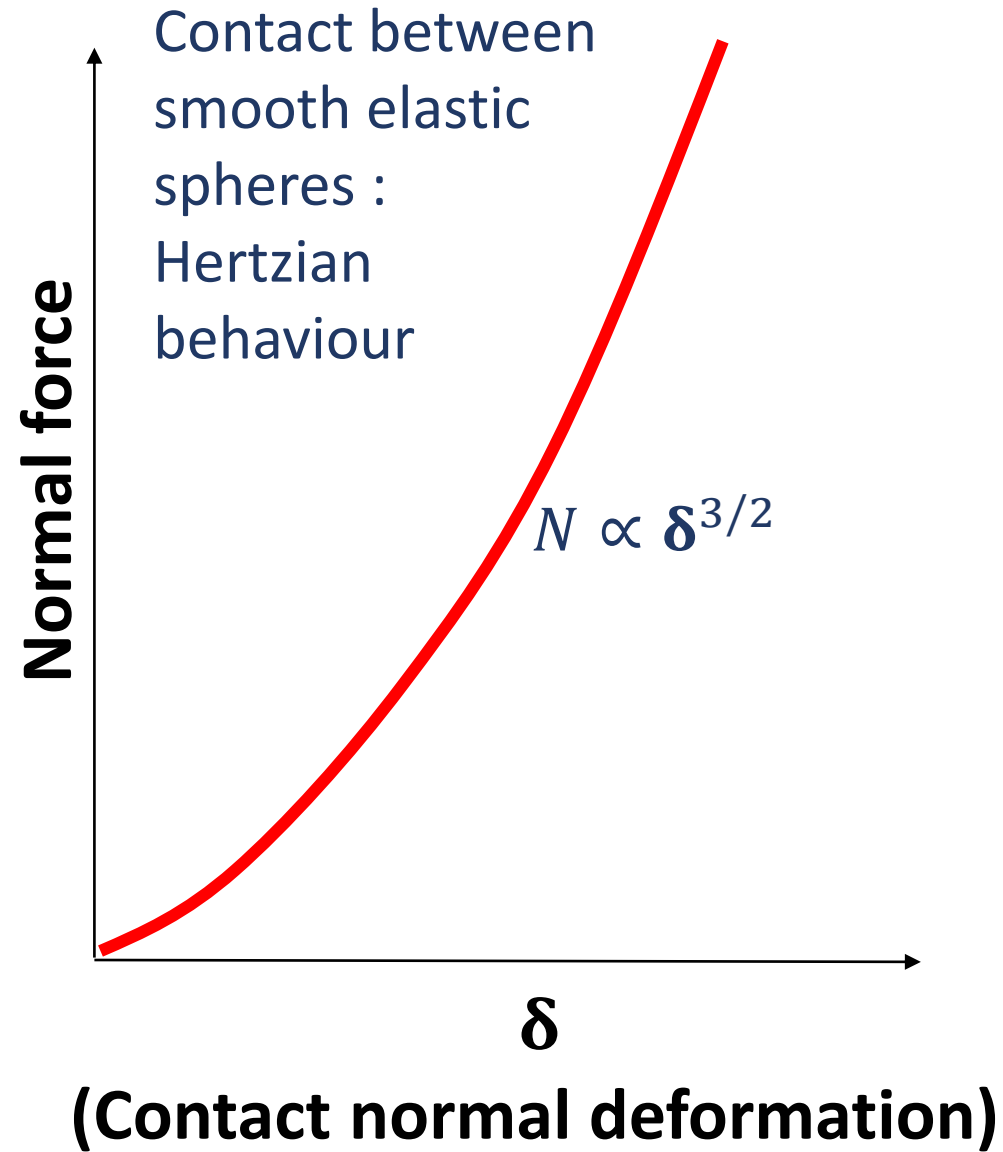
1. What determines the nature of the stress:stiffness relationship?
2. Does the state parameter concept hold under a general stress state?
3. At a given stress level why do we see more crushing in shearing than in compression?

Pressure-Stiffness Relationship



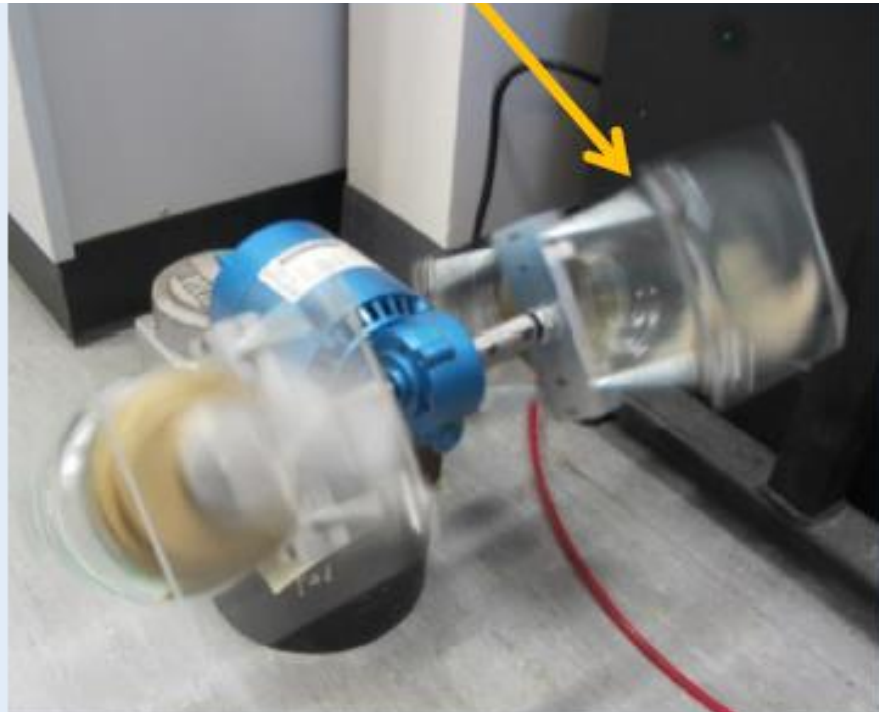
- Elastic theory explains pressure dependency: contact area increases as pressure increases – larger area gives a larger stiffness
- Elastic theory predicts that $G_0 \propto (p')^{1/3}$
- Experimental data for sand gives $G_0 \propto (p')^{1/2}$

Roughness-dependant contact behaviour

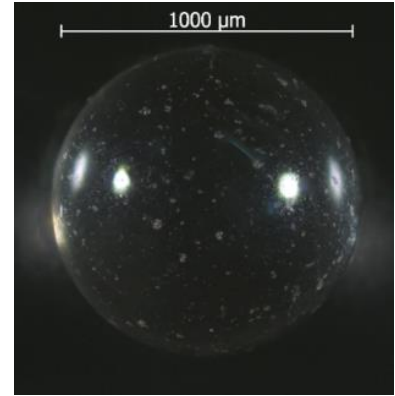


Controlling and Surface Roughness

Ballotini + Toyoura Sand



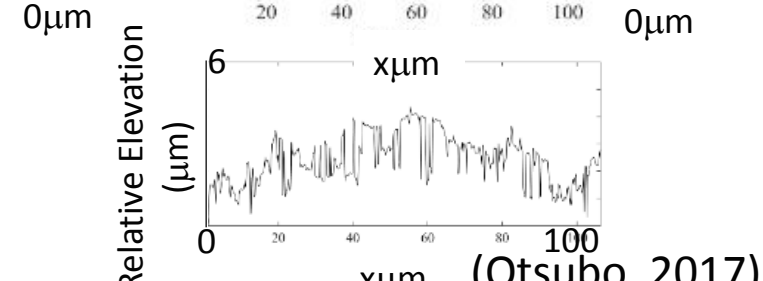
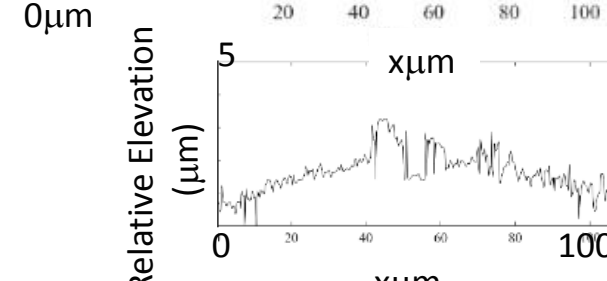
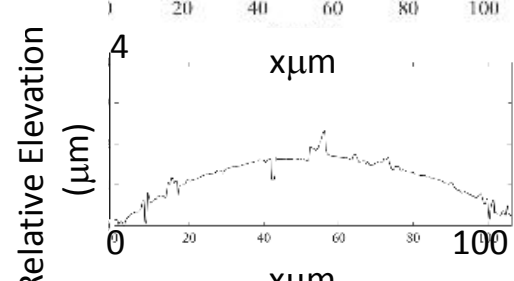
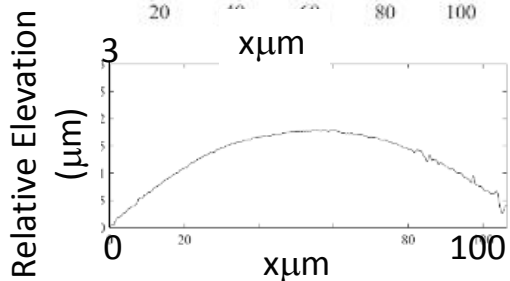
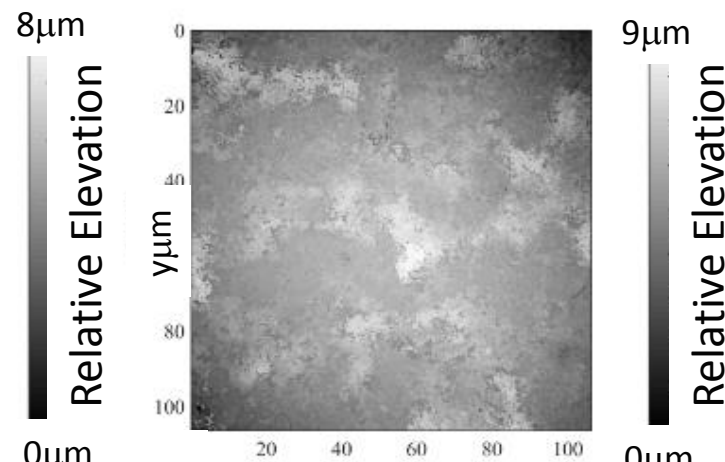
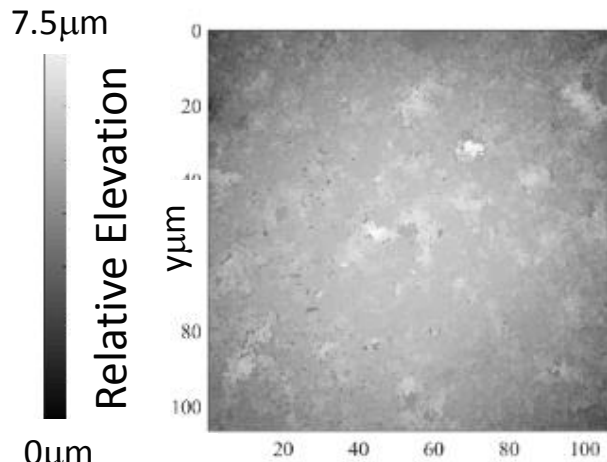
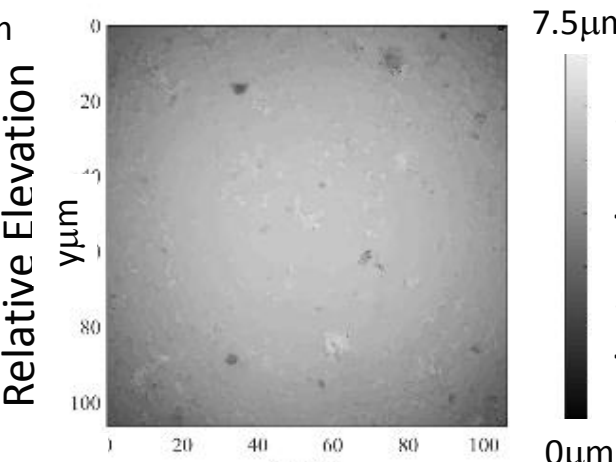
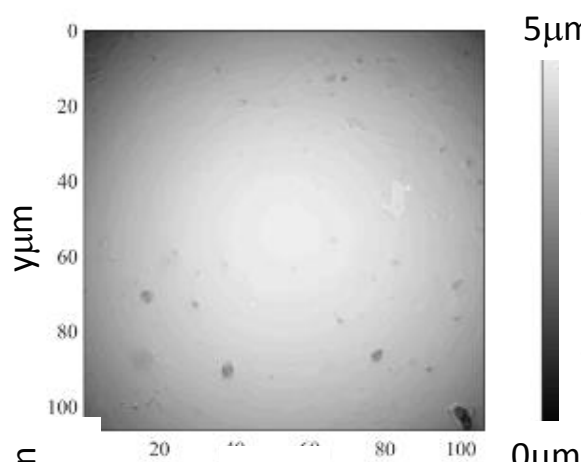
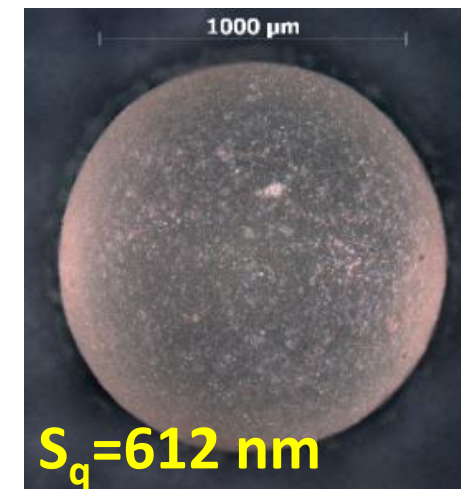
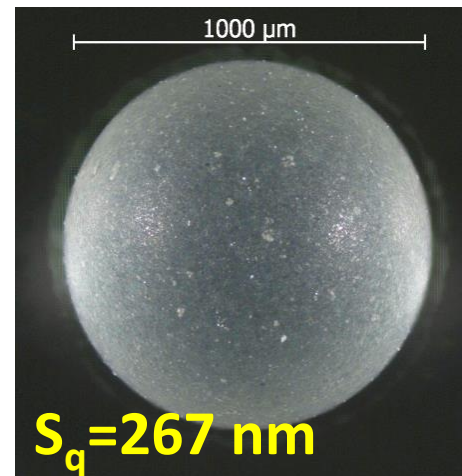
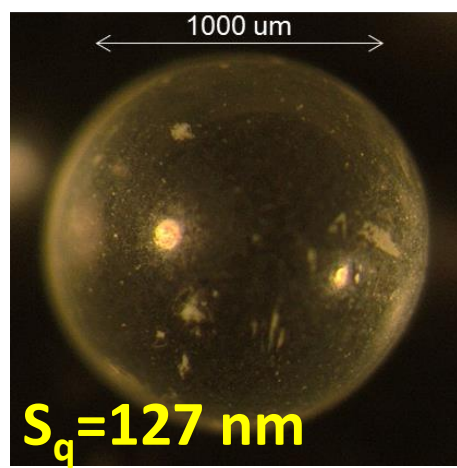
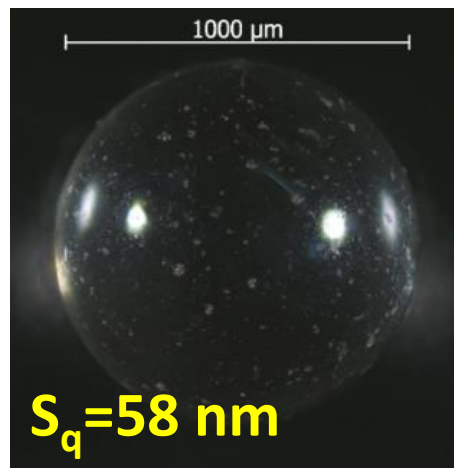
Used spherical glass beads to isolate roughness from shape (form) effects



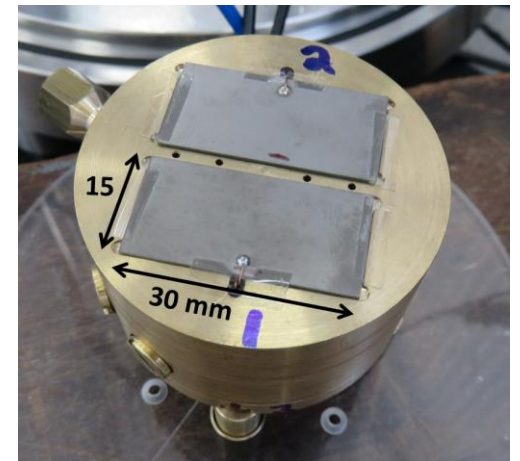
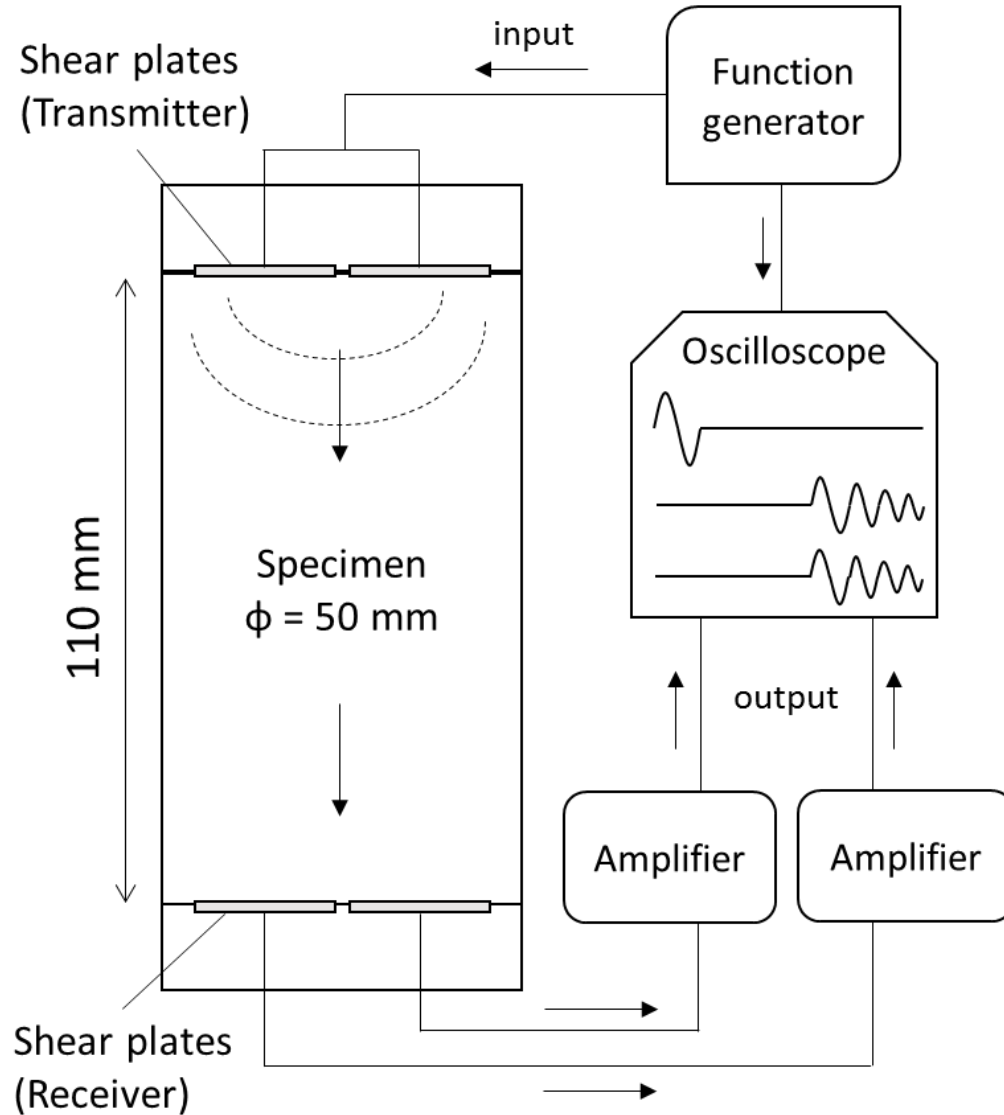
Measured roughness using interferometry



Roughened Ballotini

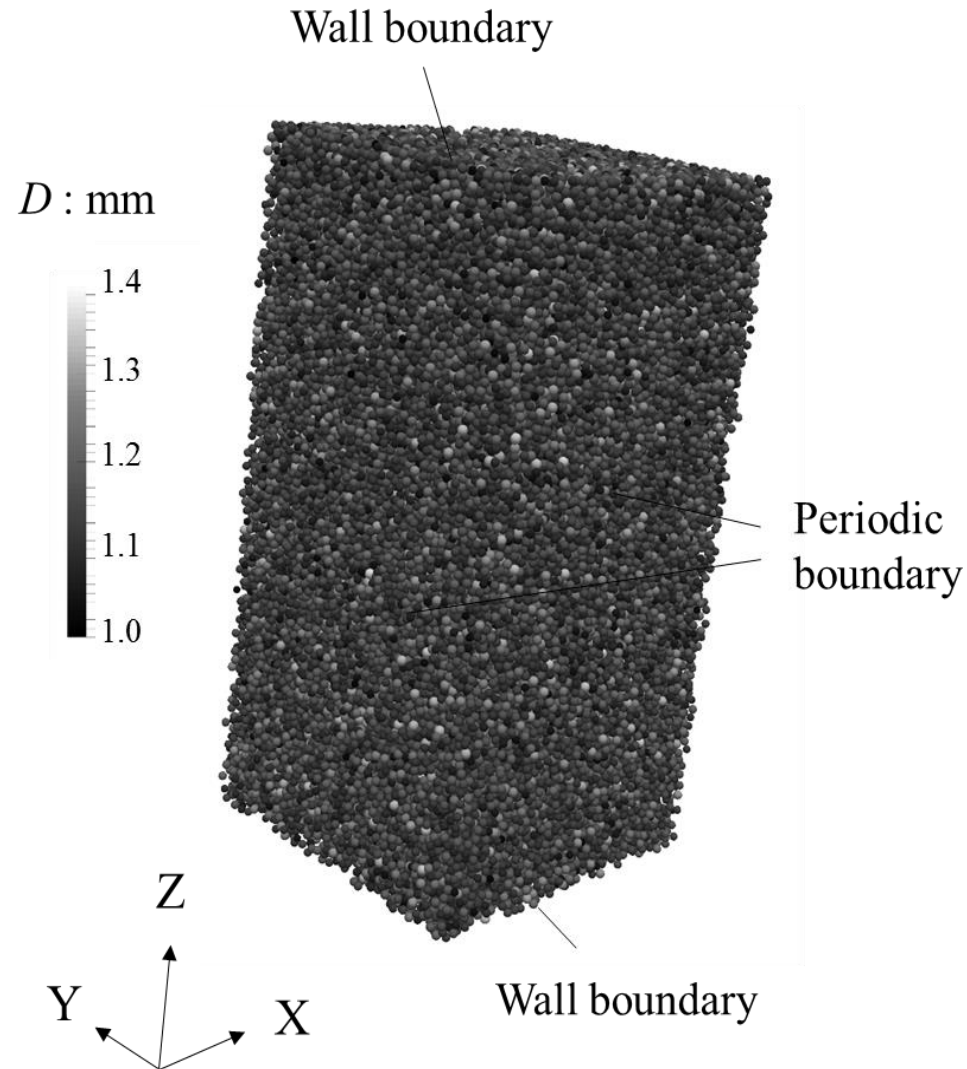


Quantifying Stiffness in Laboratory



(Otsubo, 2017)

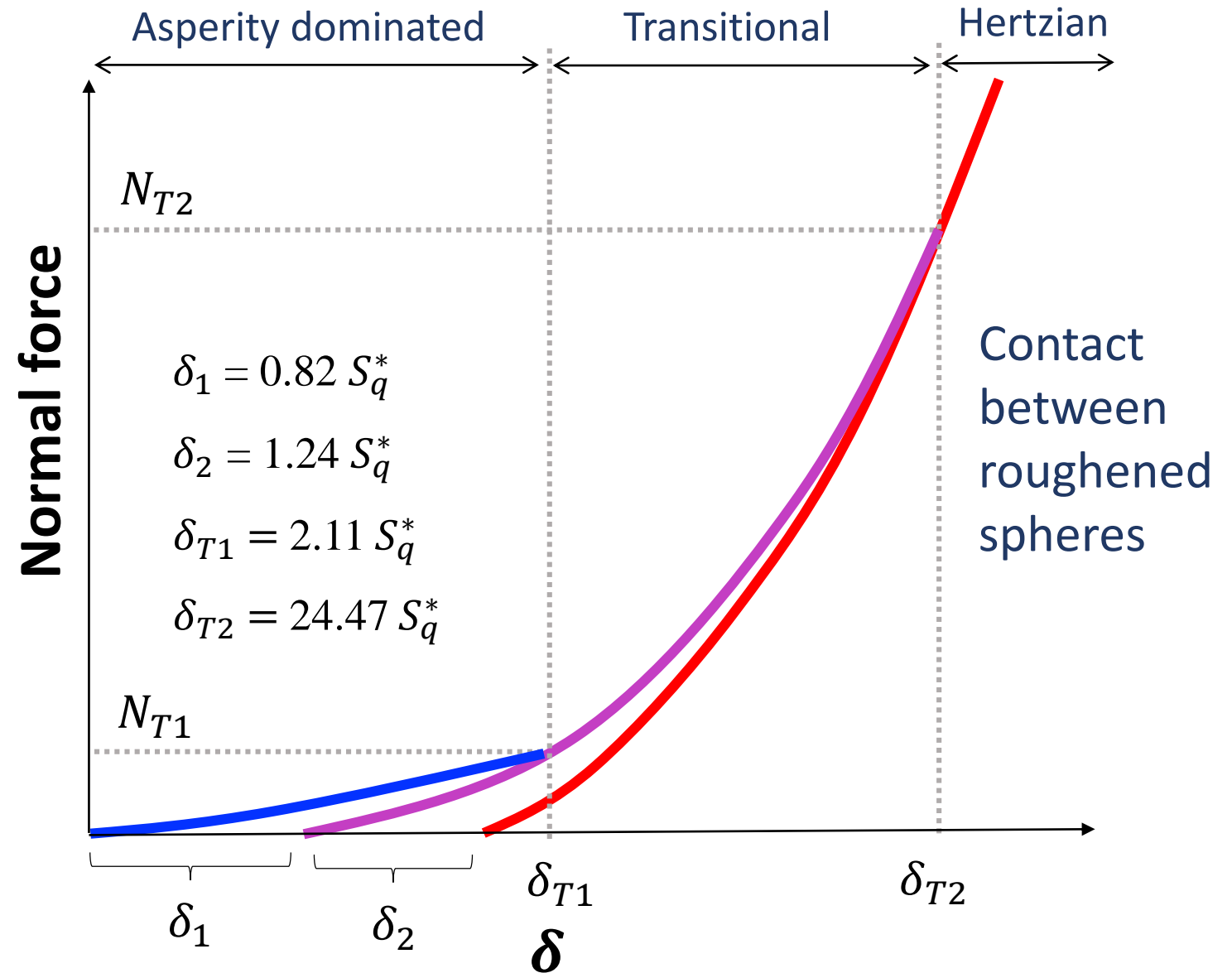
Simulating Tests using DEM



- DEM code: LAMMPS
- 155,165 particles
- Diameter : $D = 2.54$ mm (mono-size)
- Particle shear modulus : $G_p = 25$ GPa
- Particle Poisson's ratio : $\nu_p = 0.2$
- Inter-particle friction :
- $\mu = 0.0$ (dense) 0.15 (loose)
- $p' = 100$ kPa to 10 MPa
- Planar shear waves

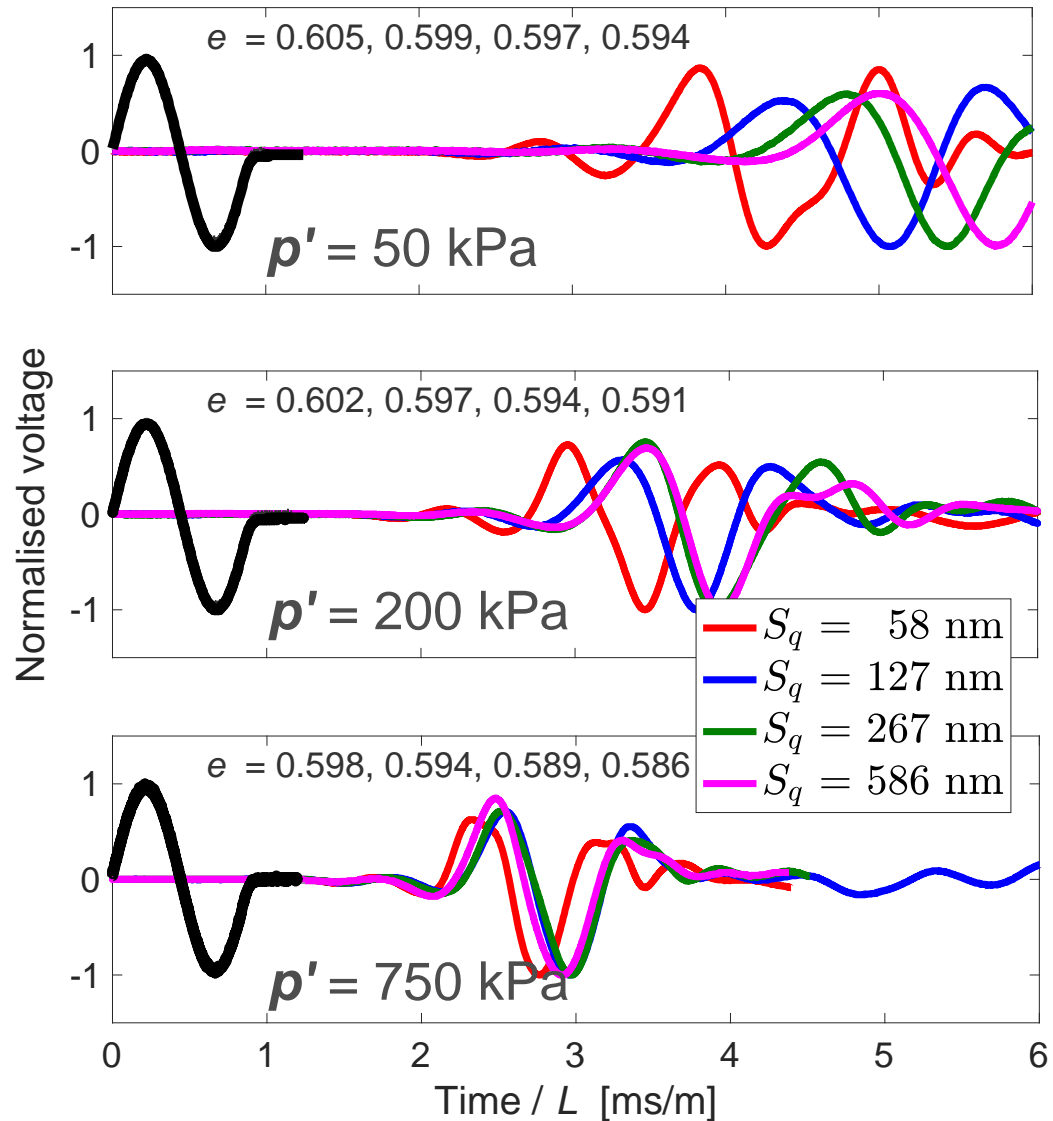
Roughness-dependant contact behaviour

DEM Contact Model

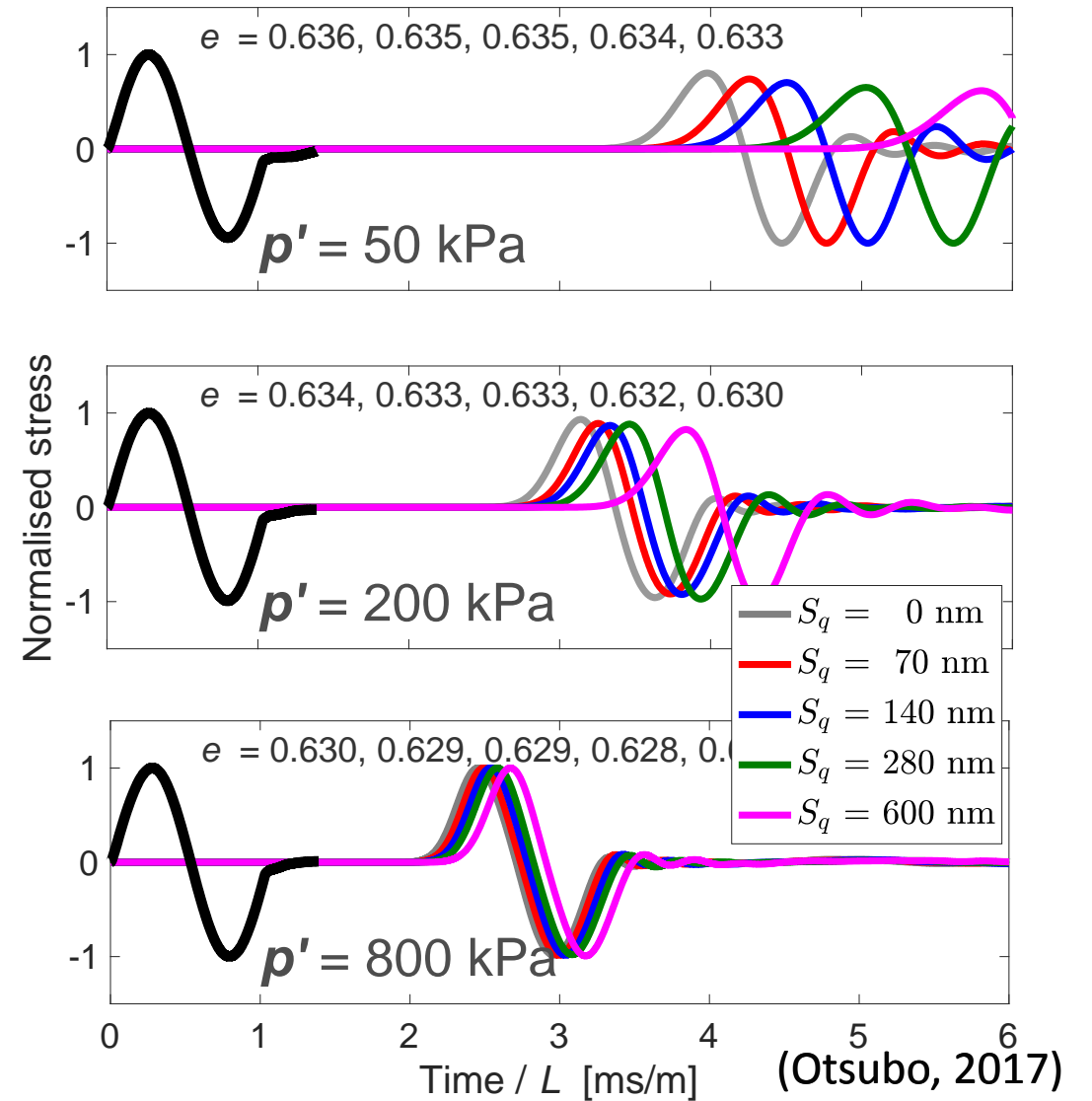


Surface Roughness and Stiffness

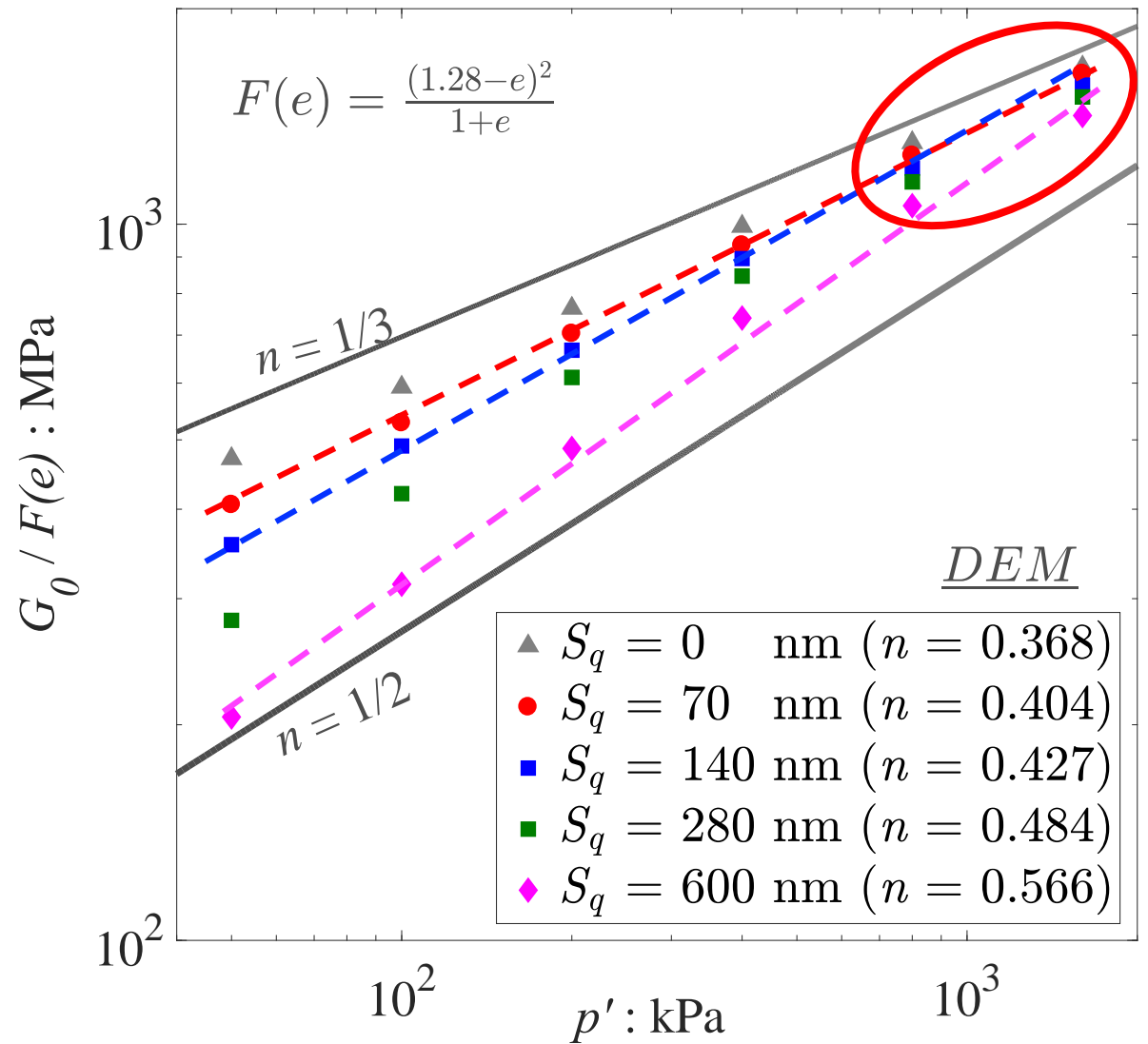
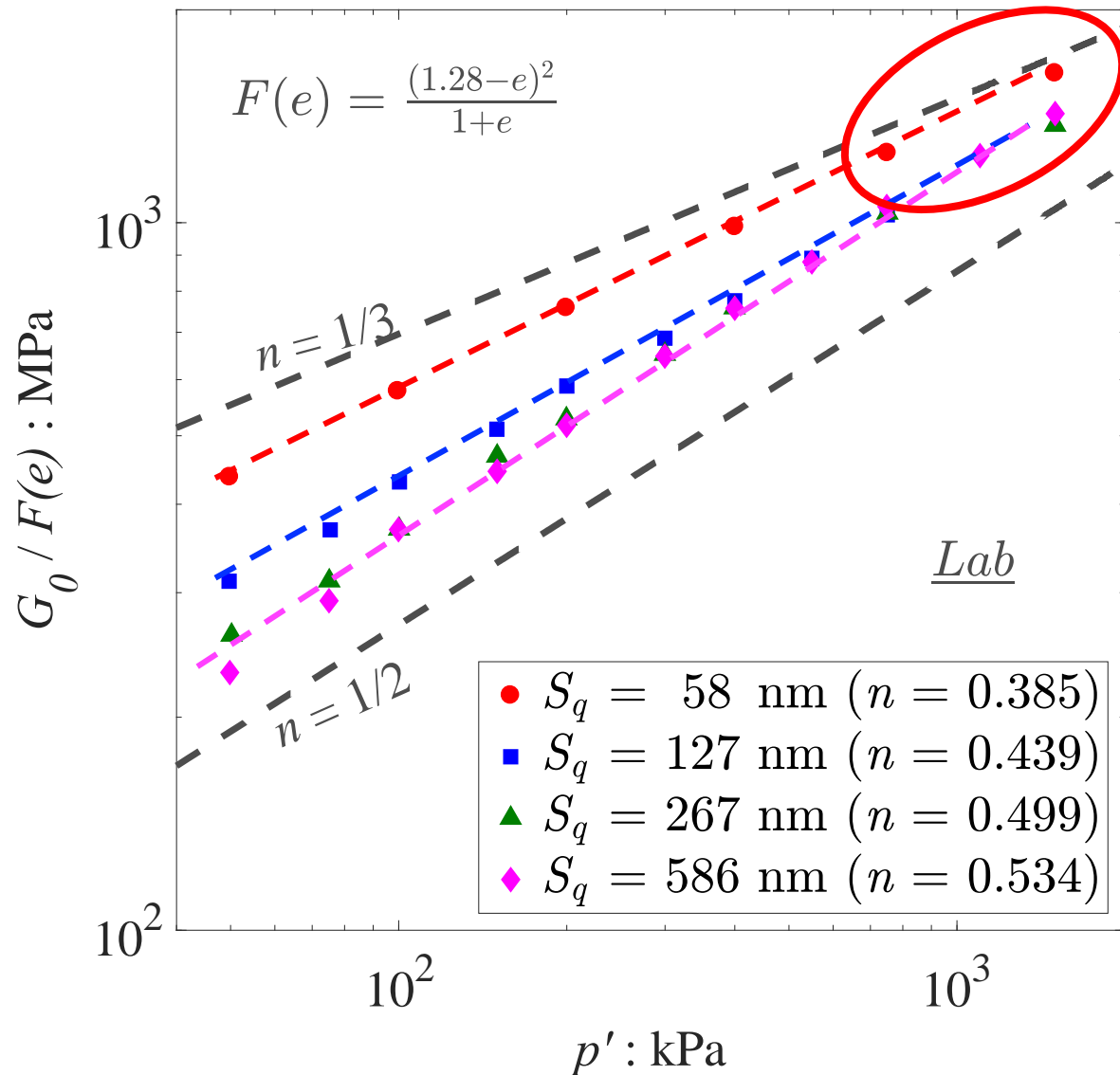
Experiments



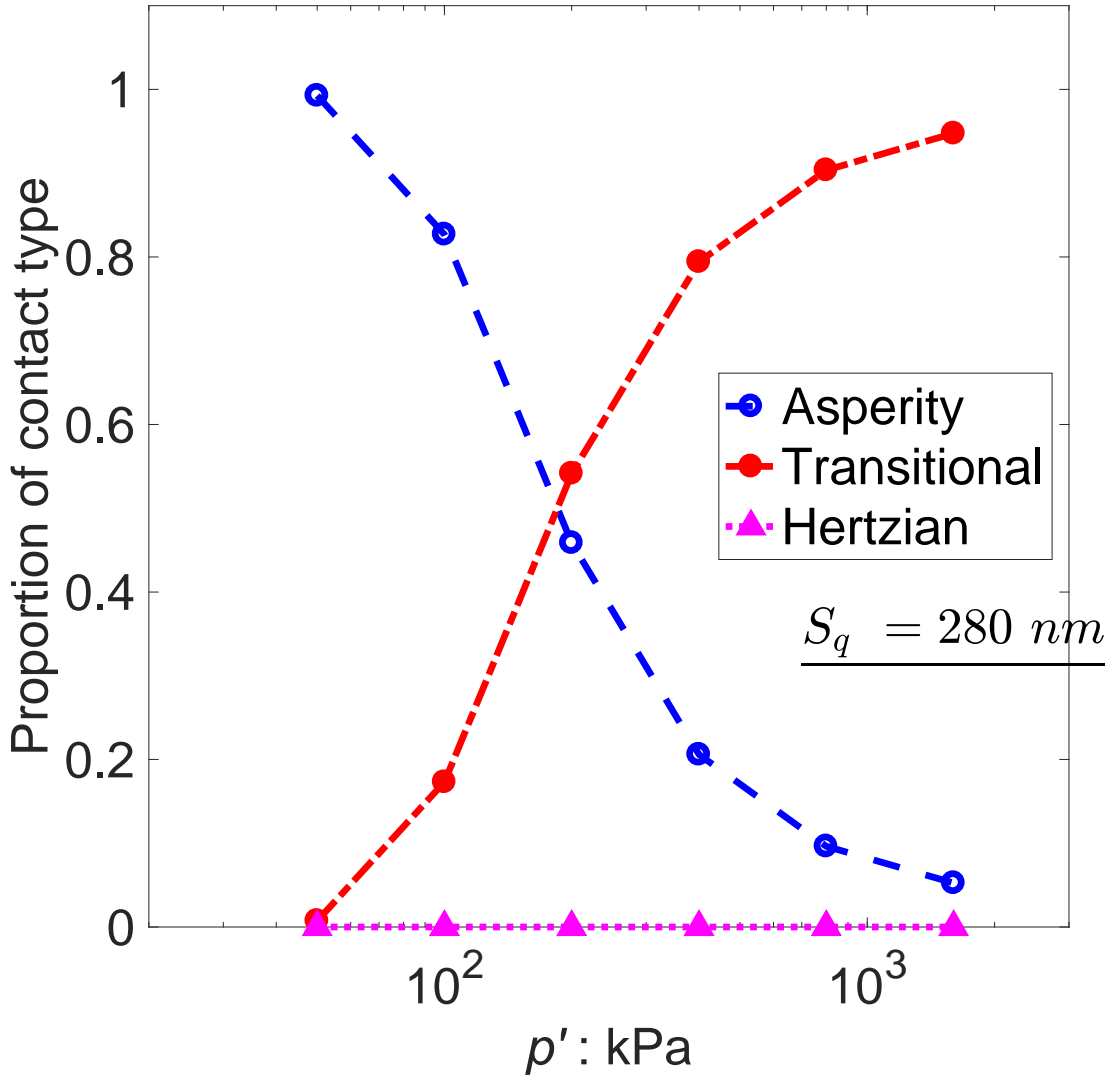
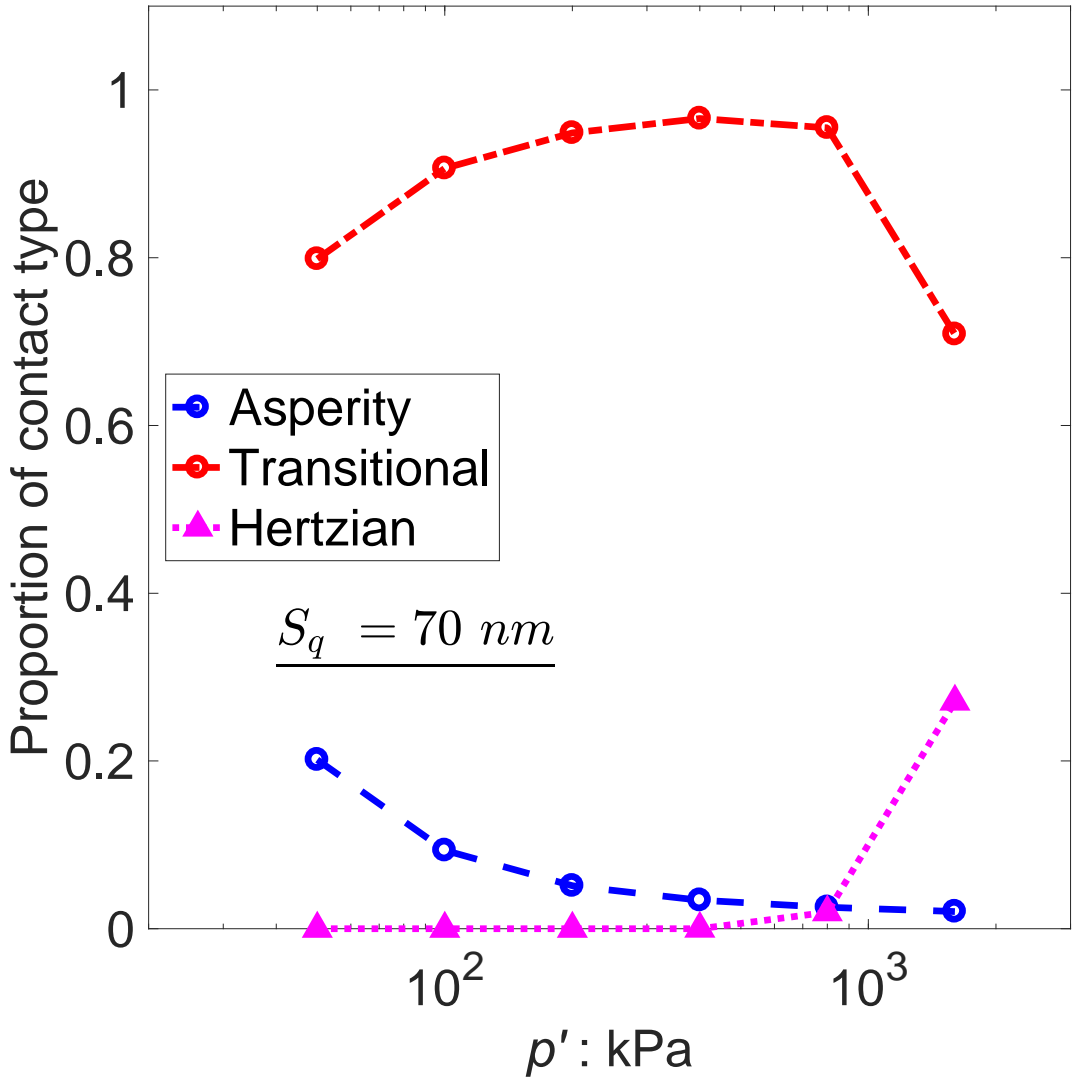
DEM simulations



Surface Roughness and Stiffness

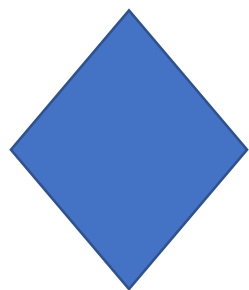


DEM data on Contact Evolution



What determines the nature of the stress:stiffness relationship?

- Complementary DEM + experimental research study showed that the exponent in the stress:stiffness relationship is influenced by surface roughness.
- Conclusions are robust as they are confirmed in lab and with simulations.
- DEM provided insight beyond that attainable in the laboratory by enabling classification of the contacts in each virtual sample.



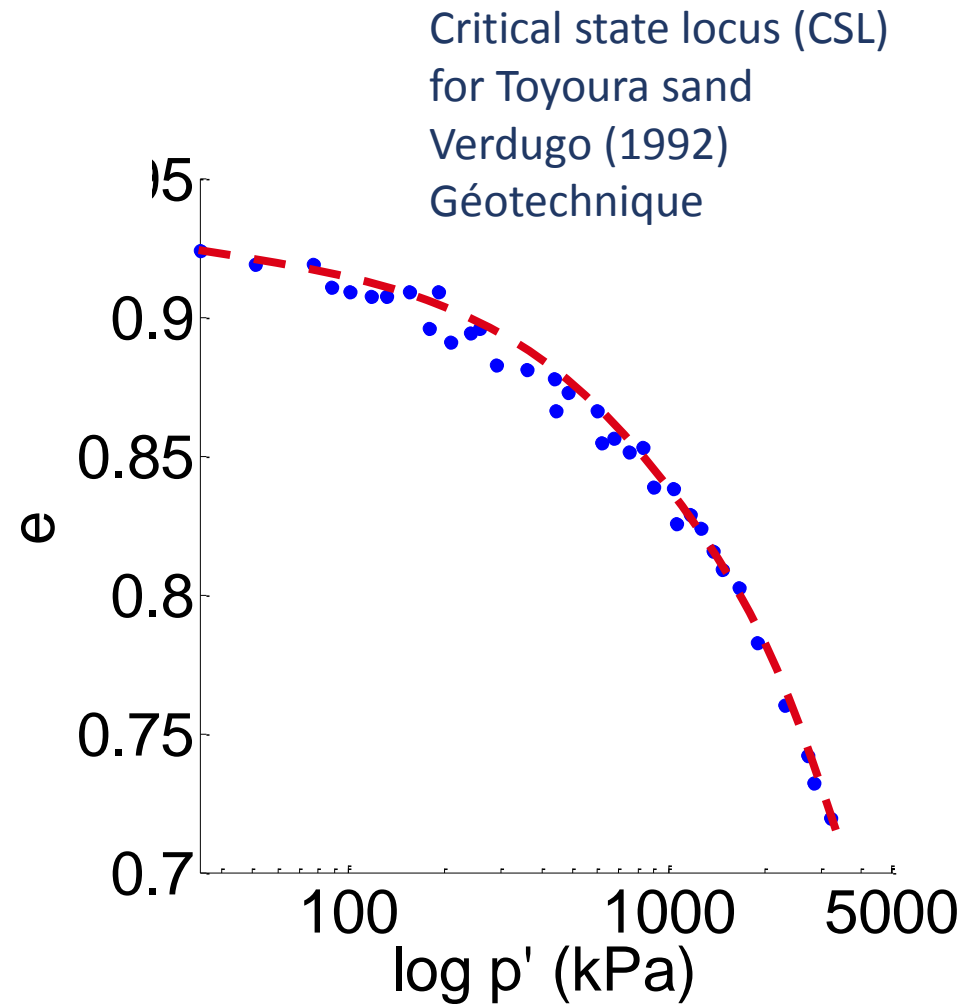
Does the state parameter concept hold under a general stress state?

- Been and Jefferies state parameter concept is integrated in many constitutive models for sand behaviour.
- It is not easy to experimentally explore whether the quantitative relationships proposed by Been and Jefferies hold in a general stress state.
- This lack of understanding compromises our ability to use the developed constitutive models outside of axi-symmetric (triaxial) stress states.

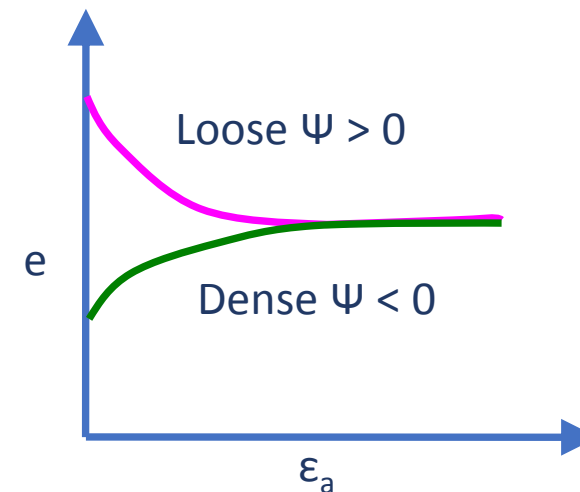
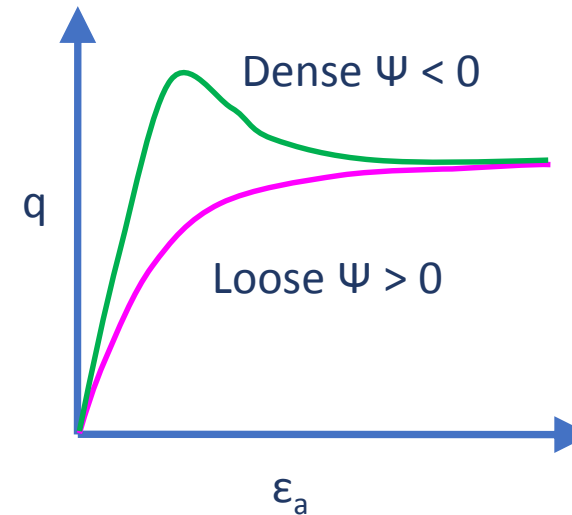
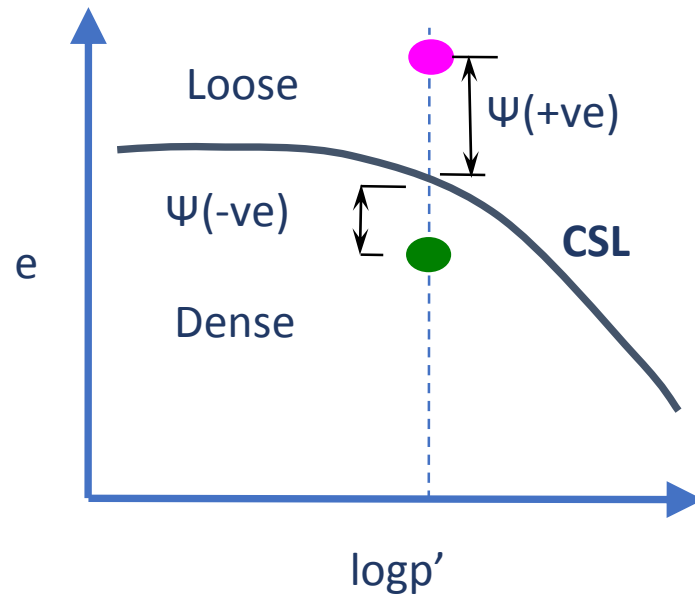
State Parameter for Sand: ψ

Critical state is a state where the soil deforms at constant stress and constant void ratio

Been and Jefferies (1985)
Géotechnique



State Parameter for Sand: ψ



e void ratio

q deviatoric stress

p' mean effective stress

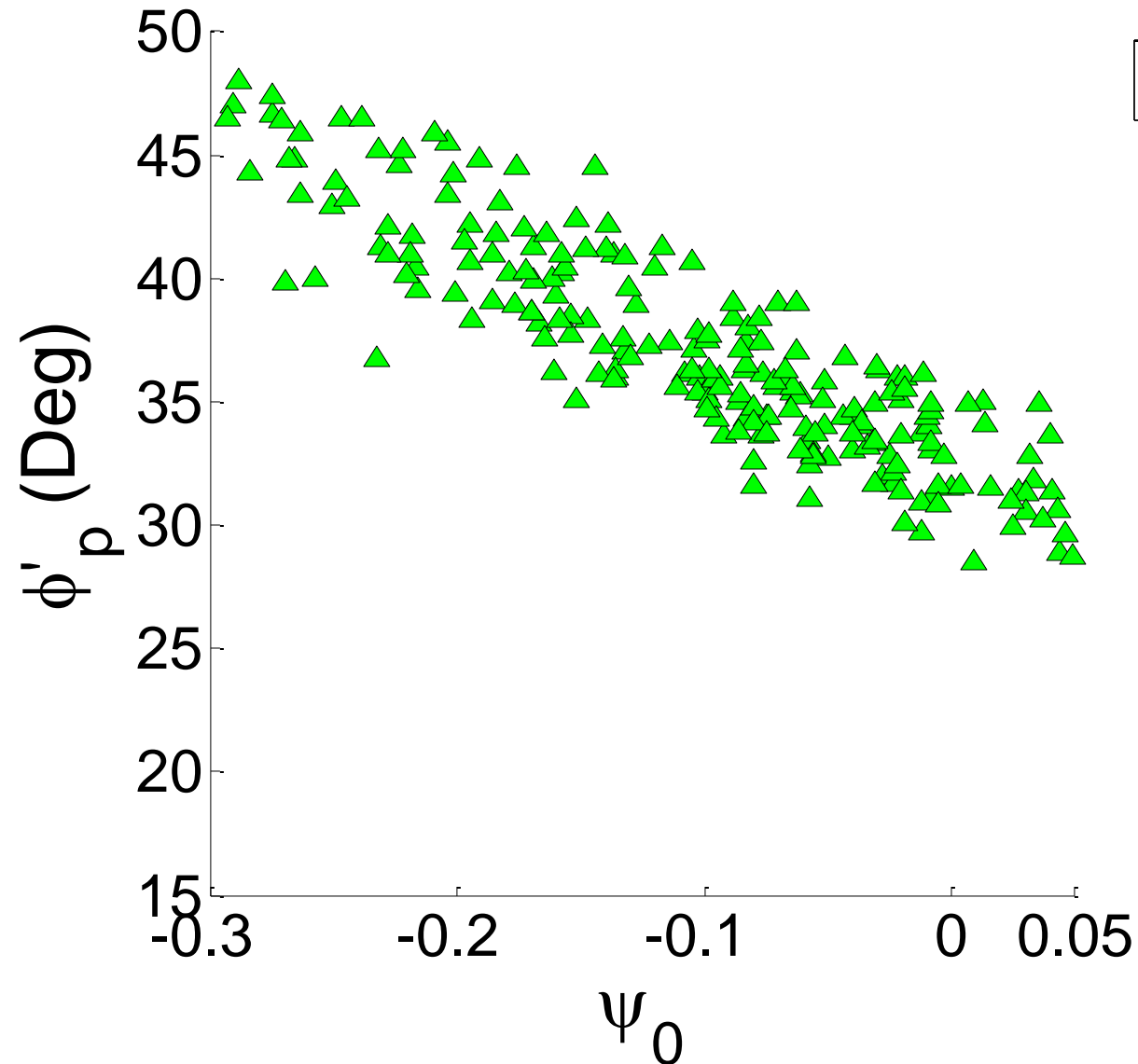
$$q = \sigma'_1 - \sigma'_3$$

$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$$

(triaxial case)

Been and Jefferies (1985)
Géotechnique

Variation in Φ'_p with Ψ_0



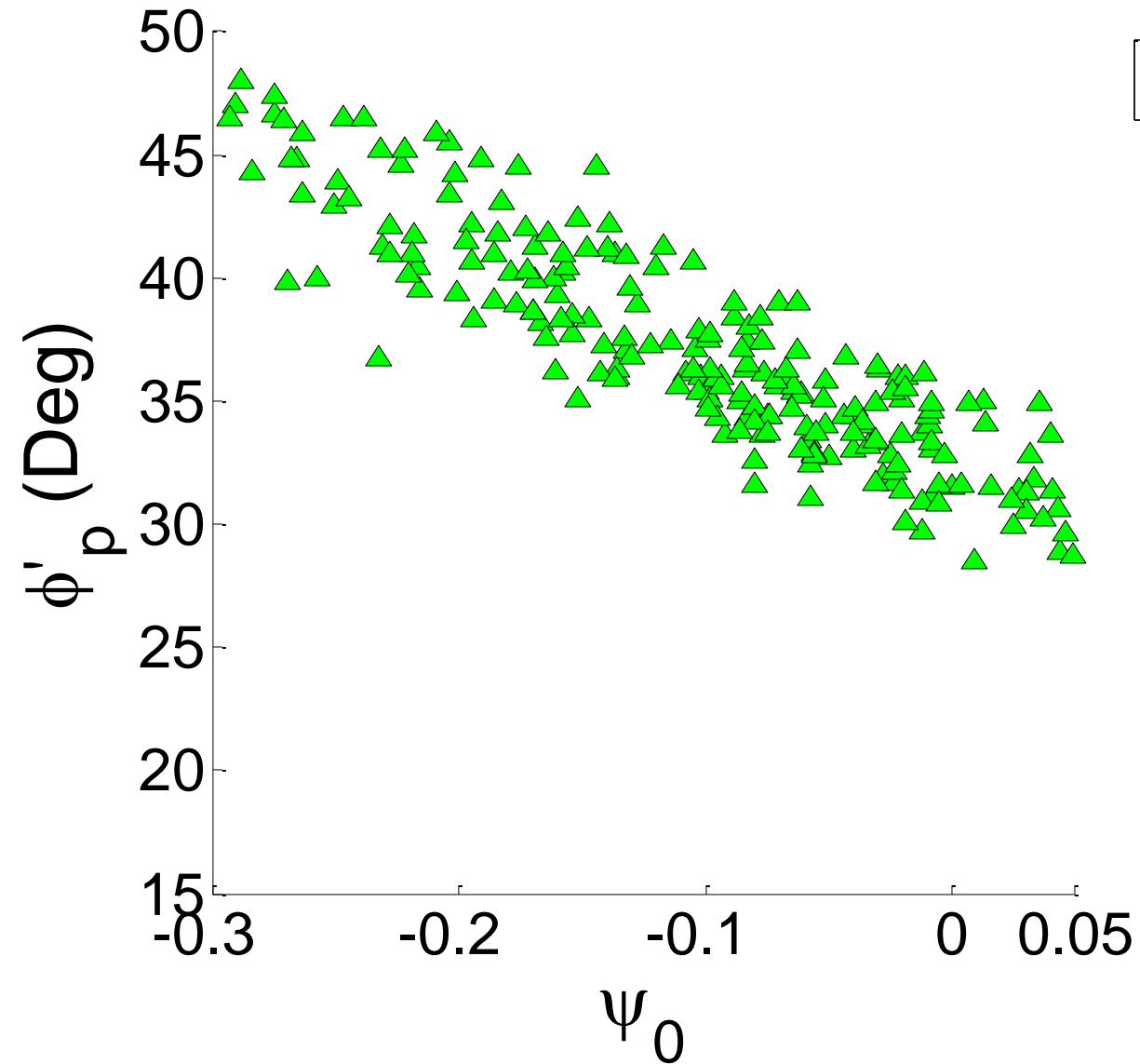
▲ Jefferies and Been

Experimental data
from Jefferies and
Been (2006)

ϕ'_p = peak angle of shearing
resistance
= peak "friction" angle

Ψ_0 = initial state parameter

Variation in Φ'_p with Ψ_0

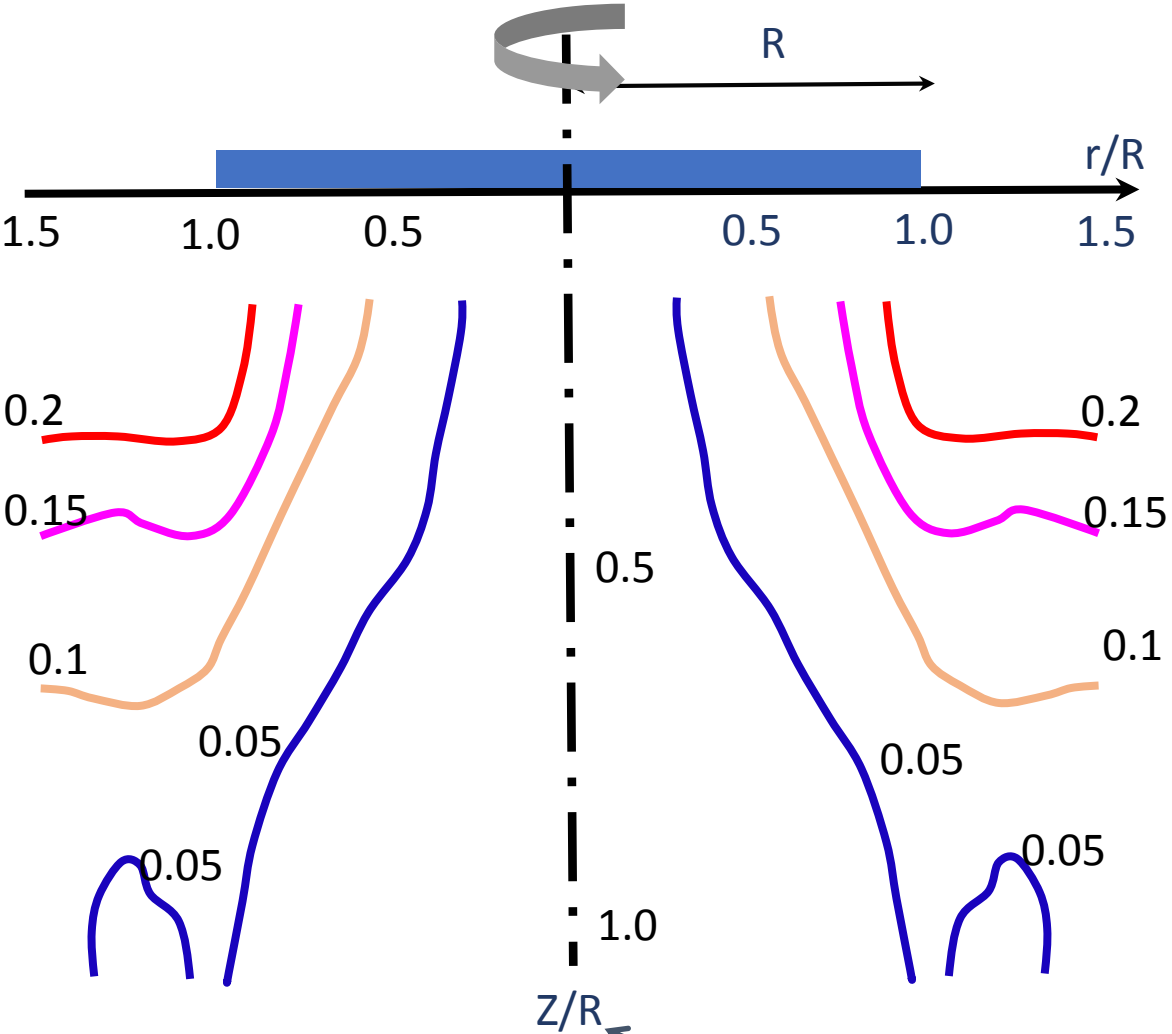


▲ Jefferies and Been



Triaxial cell at IC

In Situ Stresses



Stress increments beneath a circular footing on isotropic, linear elastic foundation

Contours of
$$\frac{\Delta\sigma'_2 - \Delta\sigma'_3}{\Delta\sigma'_1 - \Delta\sigma'_3}$$

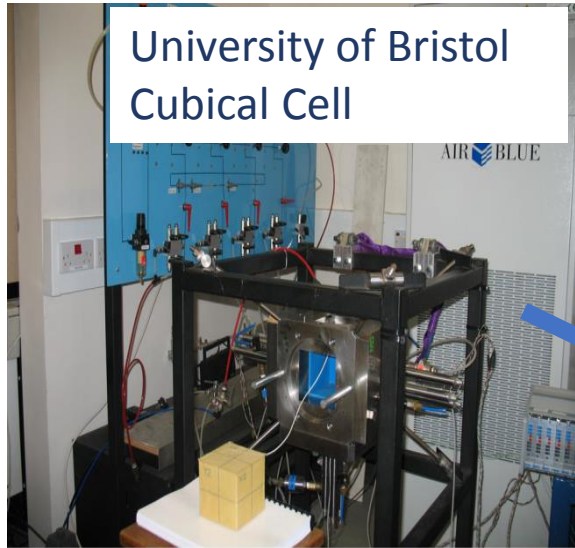
σ'_1 =major principal stress
 σ'_2 =intermediate principal stress
 σ'_3 =minor principal stress

$\sigma'_1 > \sigma'_2 > \sigma'_3$

Hight (1983)

- Along CL:
- $\Delta\sigma'_2 = \Delta\sigma'_3$
 - Triaxial conditions

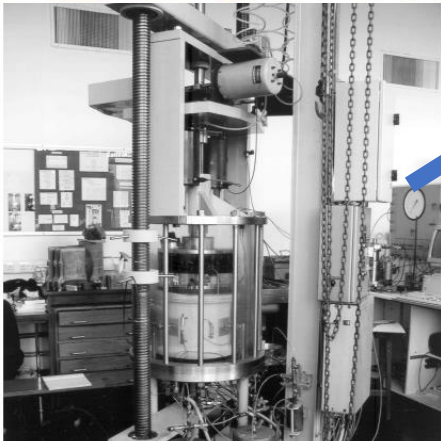
Variation in Soil Strength with b



Can control b

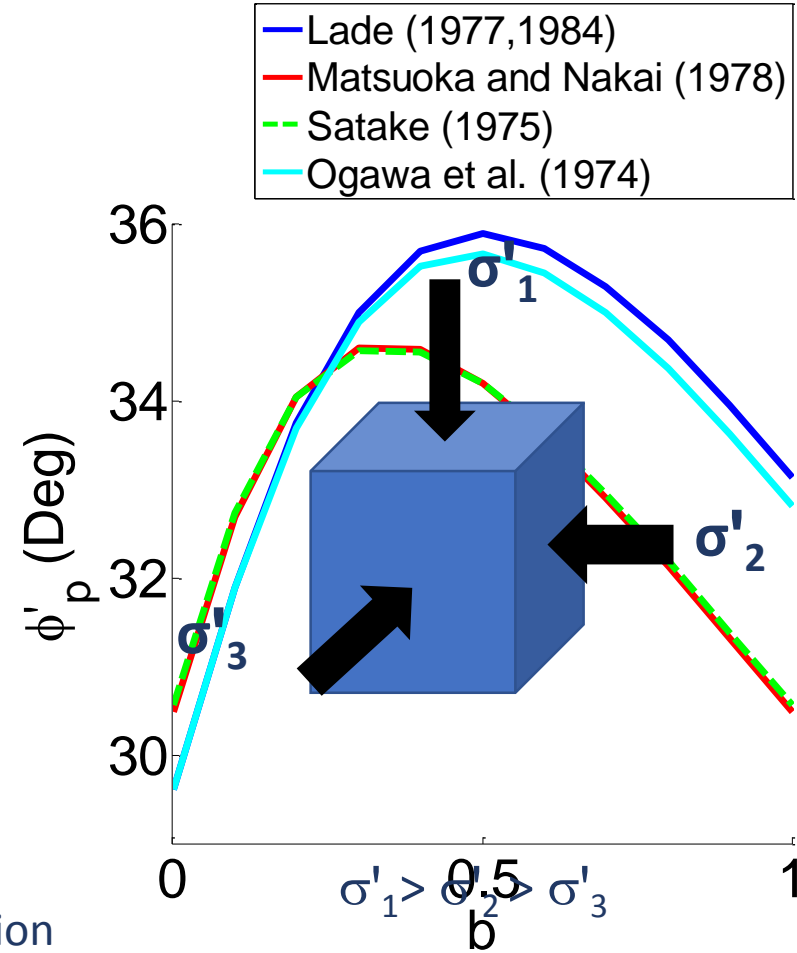
$b =$ intermediate stress ratio

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3}$$



Large Hollow Cylinder Apparatus at IC

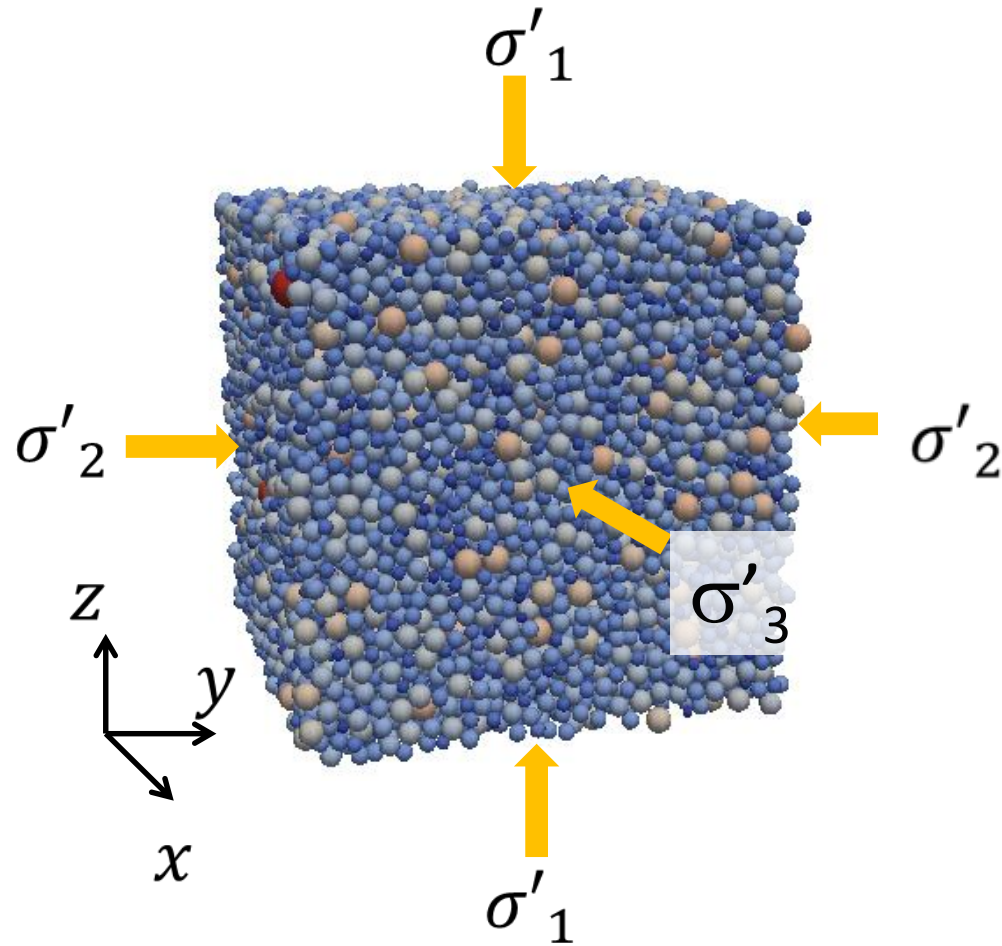
- $b=0$: Triaxial compression
- $b=1$: Triaxial extension
- $b \approx 0.5$: Plane strain



$\phi'_p =$ peak angle of shearing resistance
= peak "friction" angle

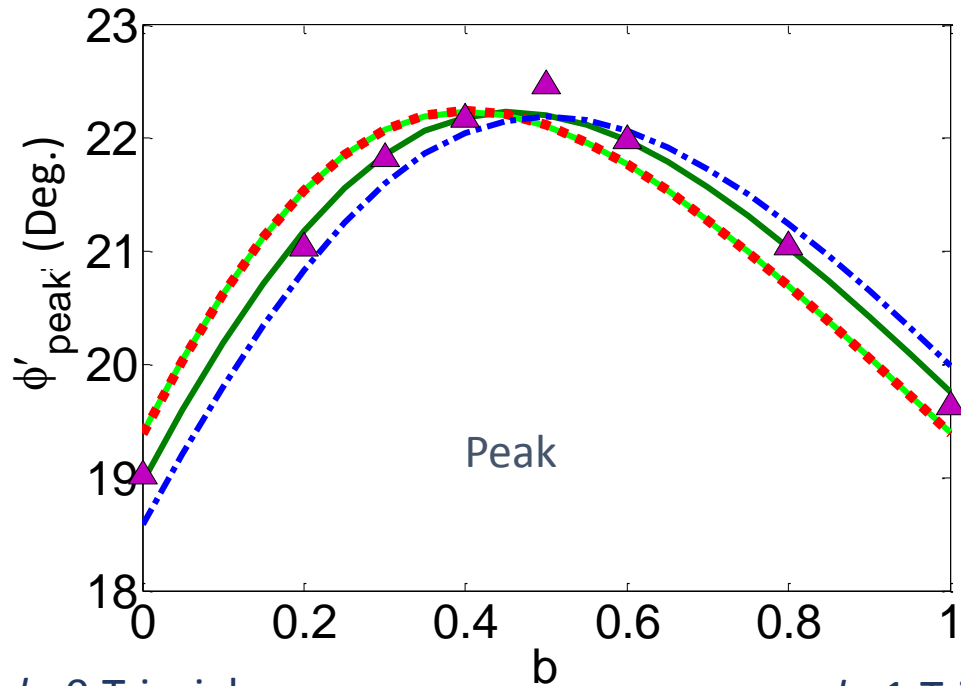
O'Sullivan et al. (2015); Géotechnique

DEM Analysis of ψ for a 3D Stress State

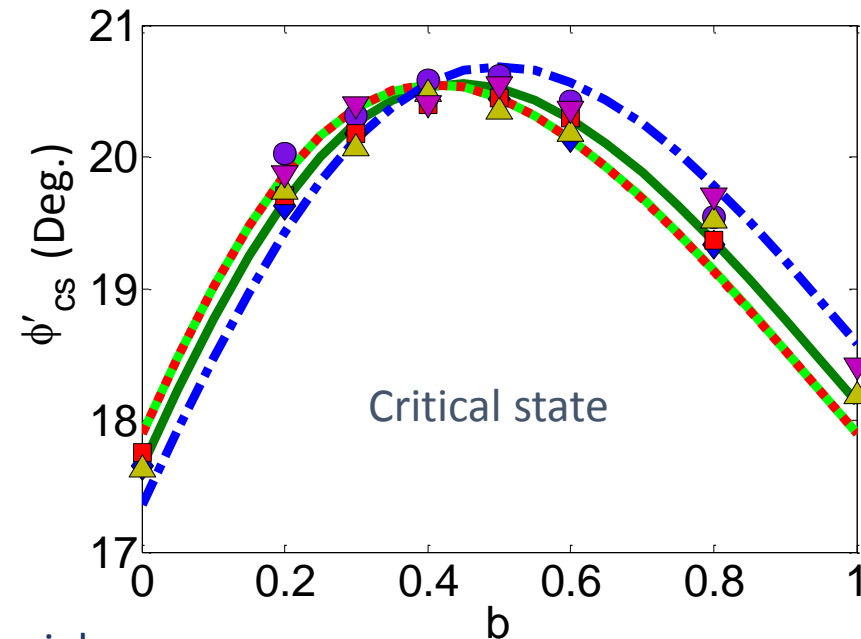
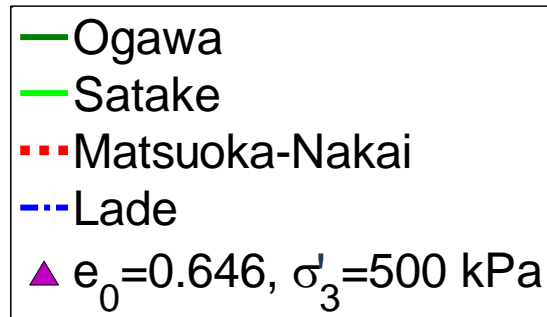


- 20,164 spherical particles
- Periodic boundaries
- Grading similar to Toyoura sand
- Friction coefficient $\mu=0.25$
- Controlled b
- Deformed to large strain
- Varied initial state parameter (ψ_0)
- Most of simulations used LAMMPS

Variation in Φ'_p and Φ'_{cs} with b



$b=0$: Triaxial
Compression



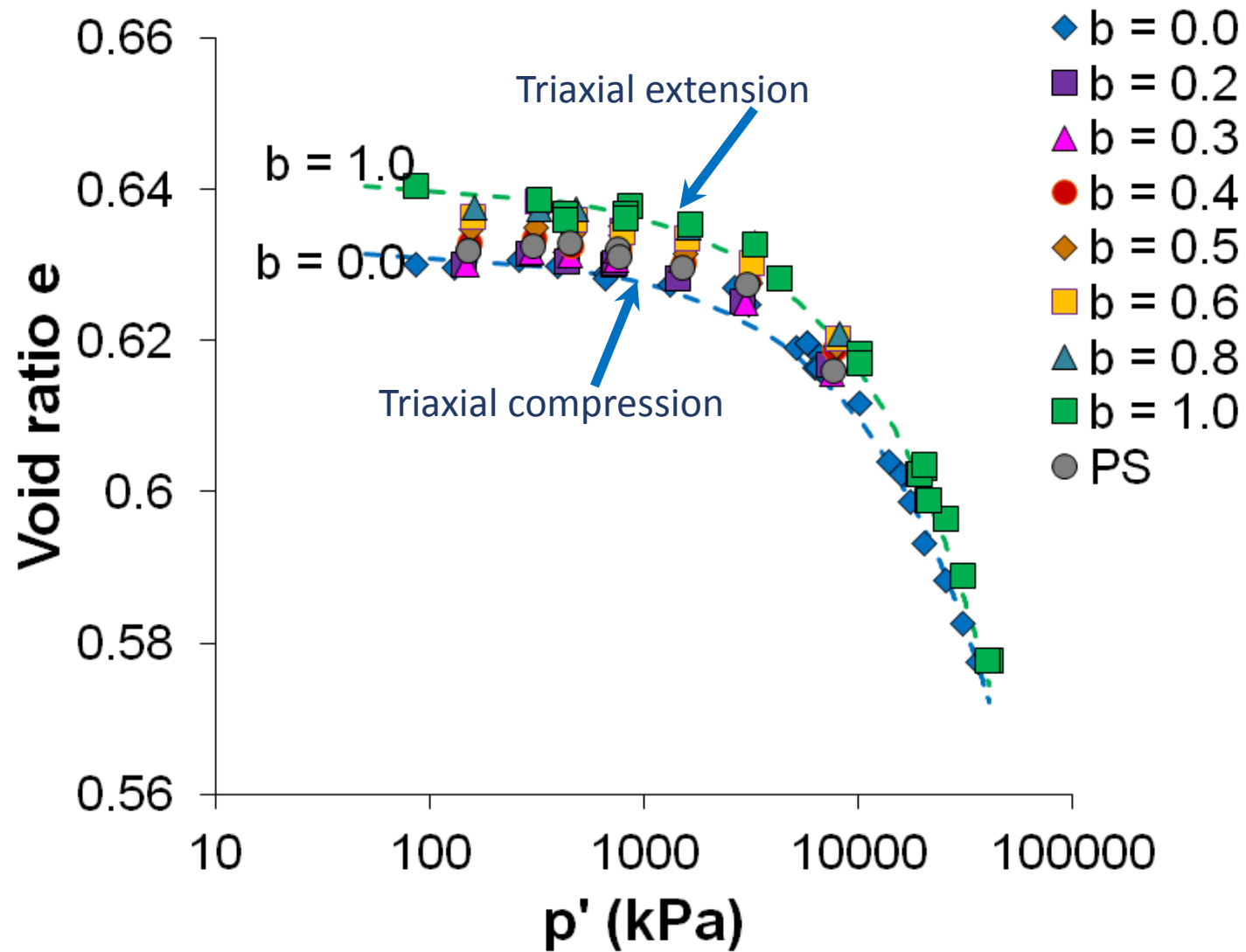
$b=1$: Triaxial
Extension

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3}$$

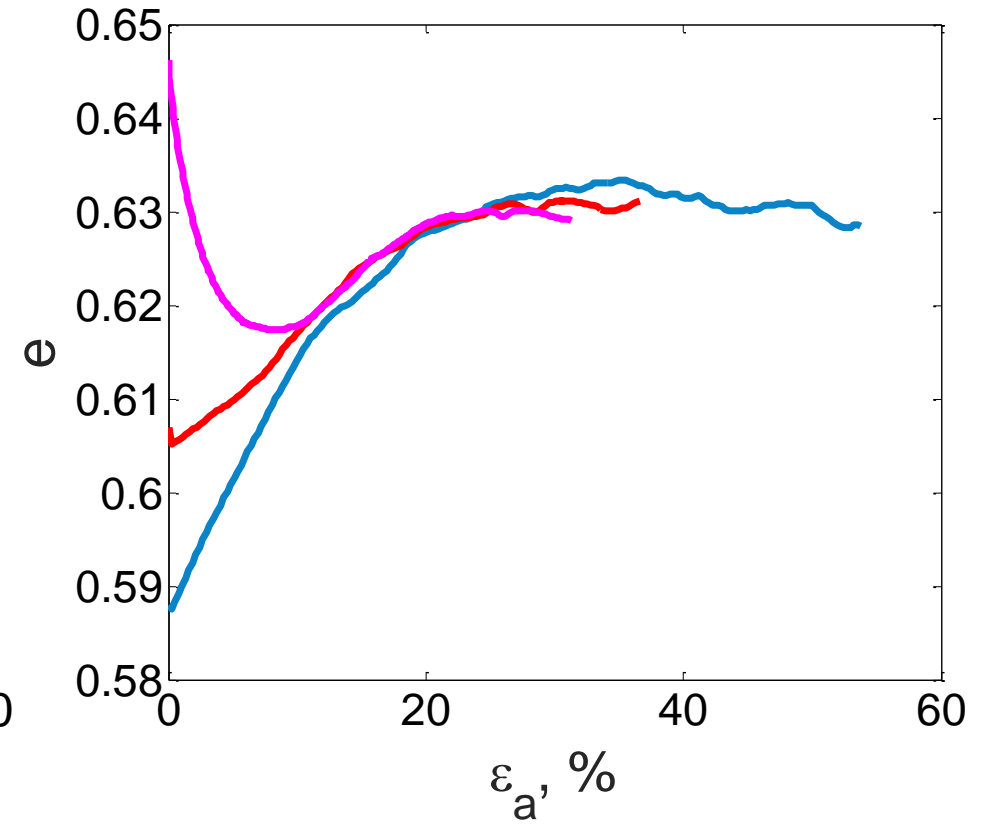
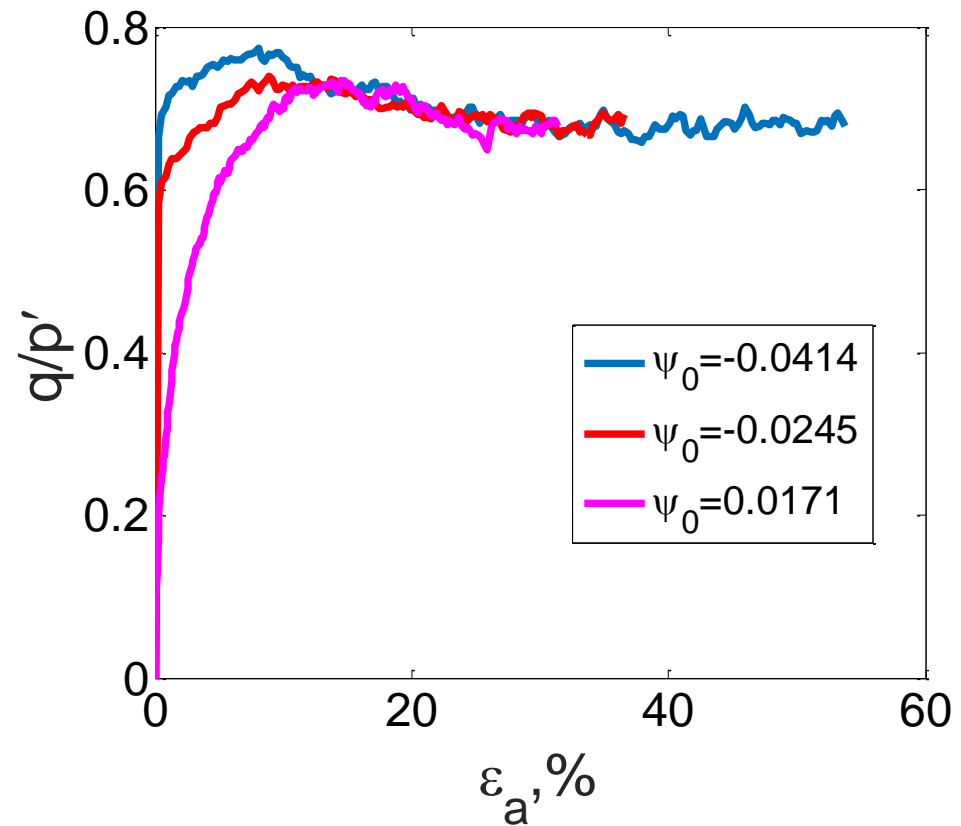
Huang et al. (2014)
Granular Matter

- \blacklozenge $e_0=0.533, \sigma'_3=100$ kPa
- \bullet $e_0=0.612, \sigma'_3=5000$ kPa
- \blacktriangledown $e_0=0.625, \sigma'_3=2000$ kPa
- \blacksquare $e_0=0.625, \sigma'_3=1000$ kPa
- \blacktriangle $e_0=0.646, \sigma'_3=500$ kPa

Sensitivity of CSL locus to b



State-Dependent Response



$\psi > 0$ loose
 $\psi < 0$ dense

$$q = \sigma'_1 - \sigma'_3$$

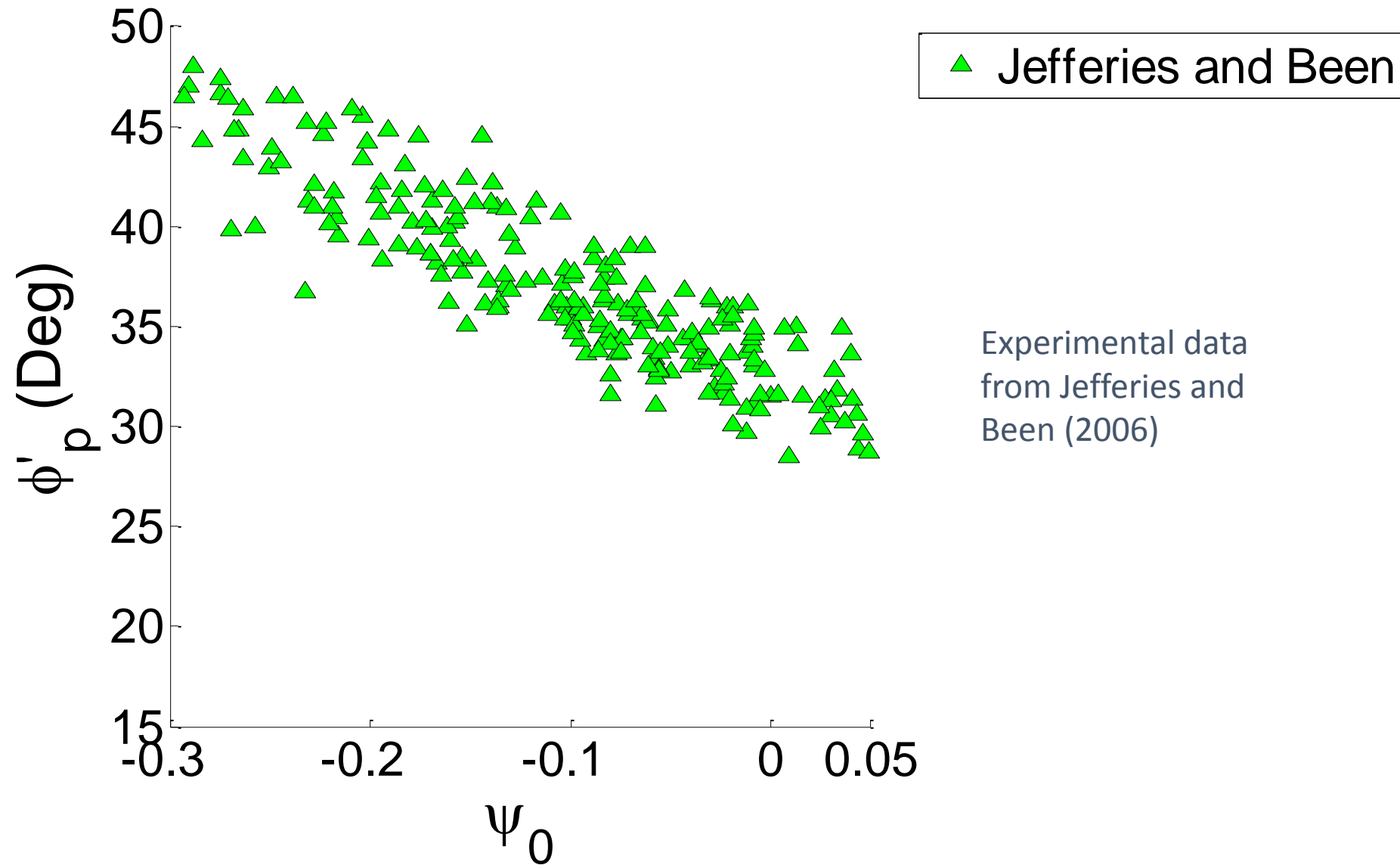
$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$$

(triaxial)

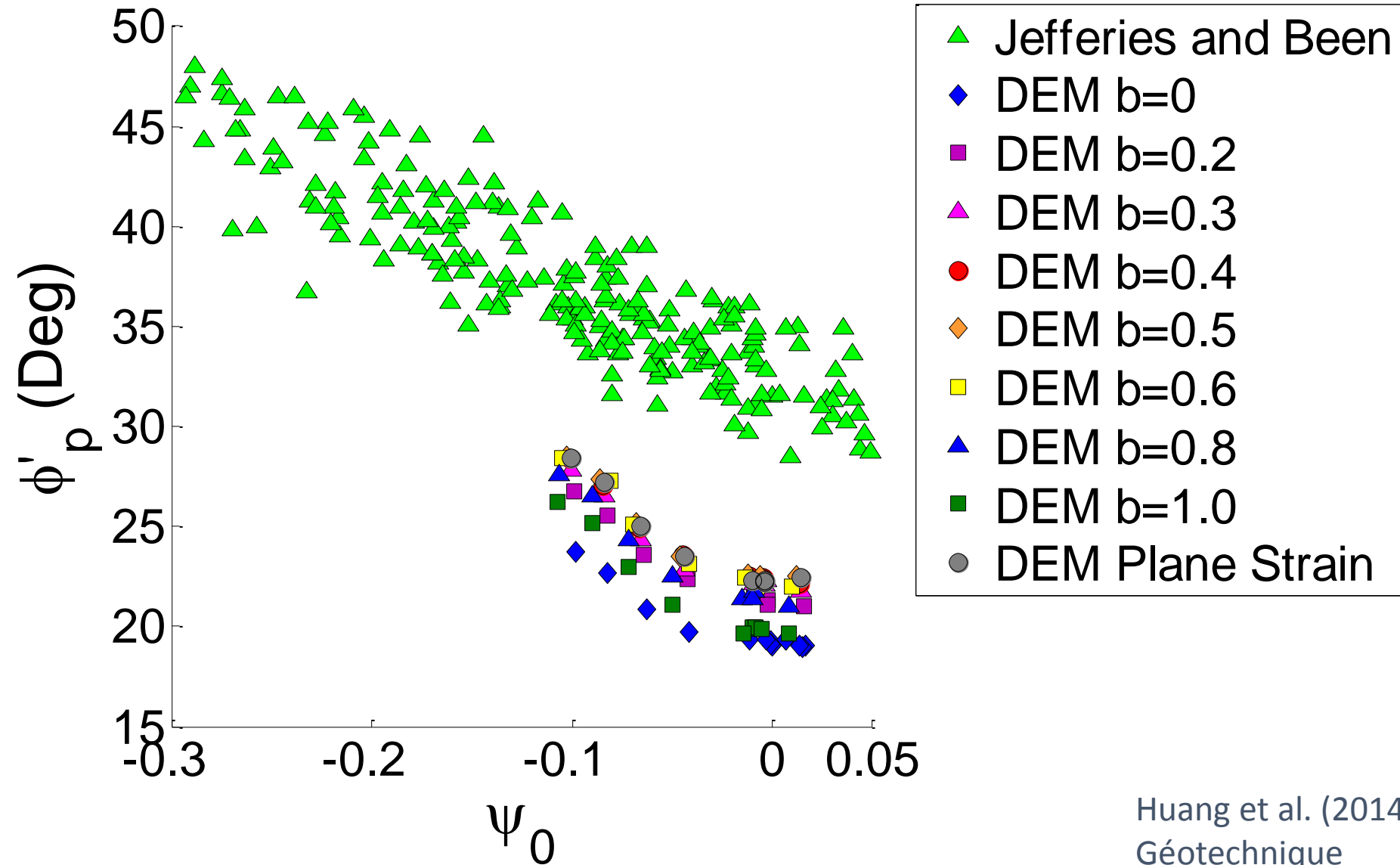
$\sigma'_3 = 500 \text{ kPa}$

Huang et al. (2014)
 Géotechnique

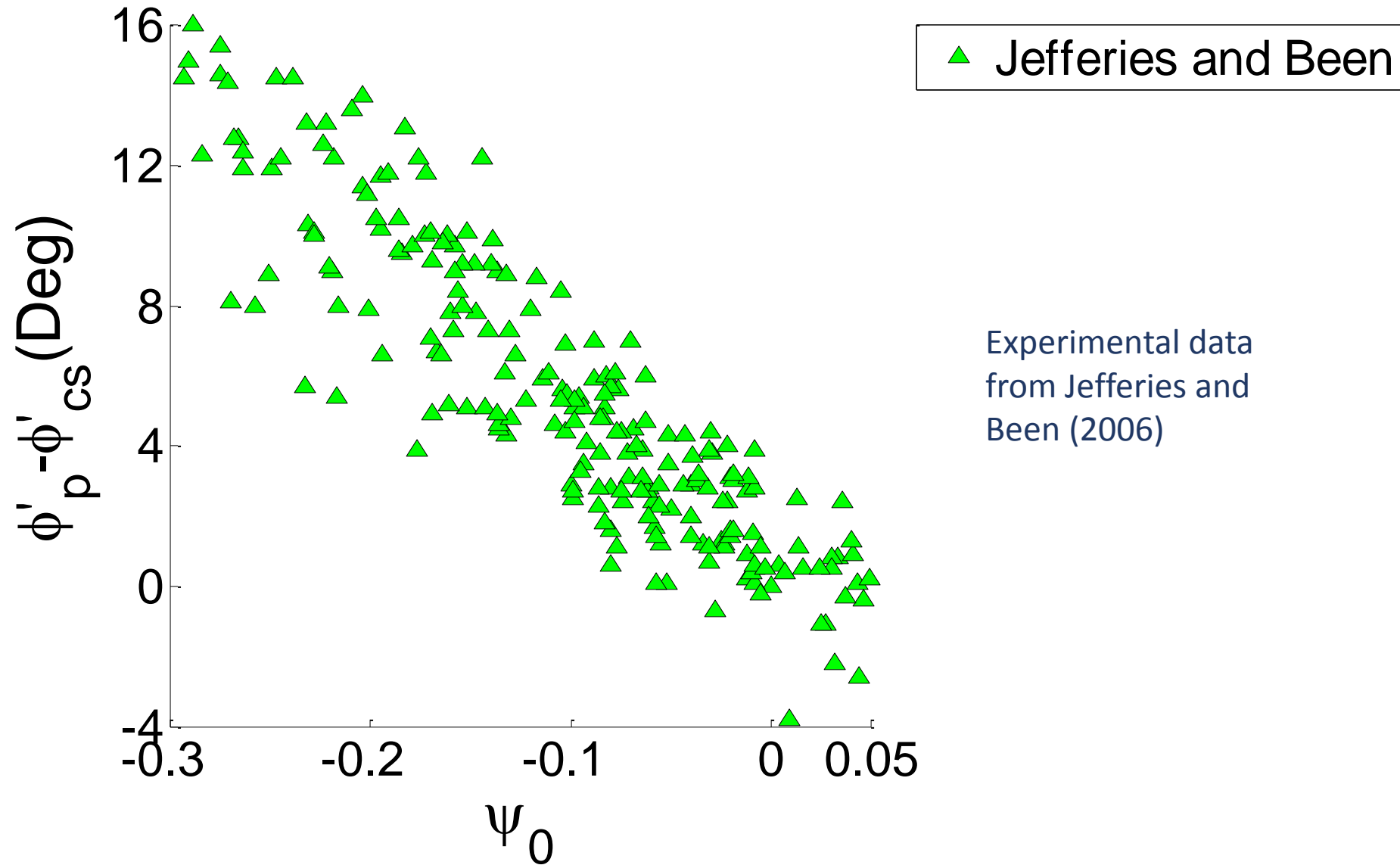
Variation in Φ'_p with Ψ_0



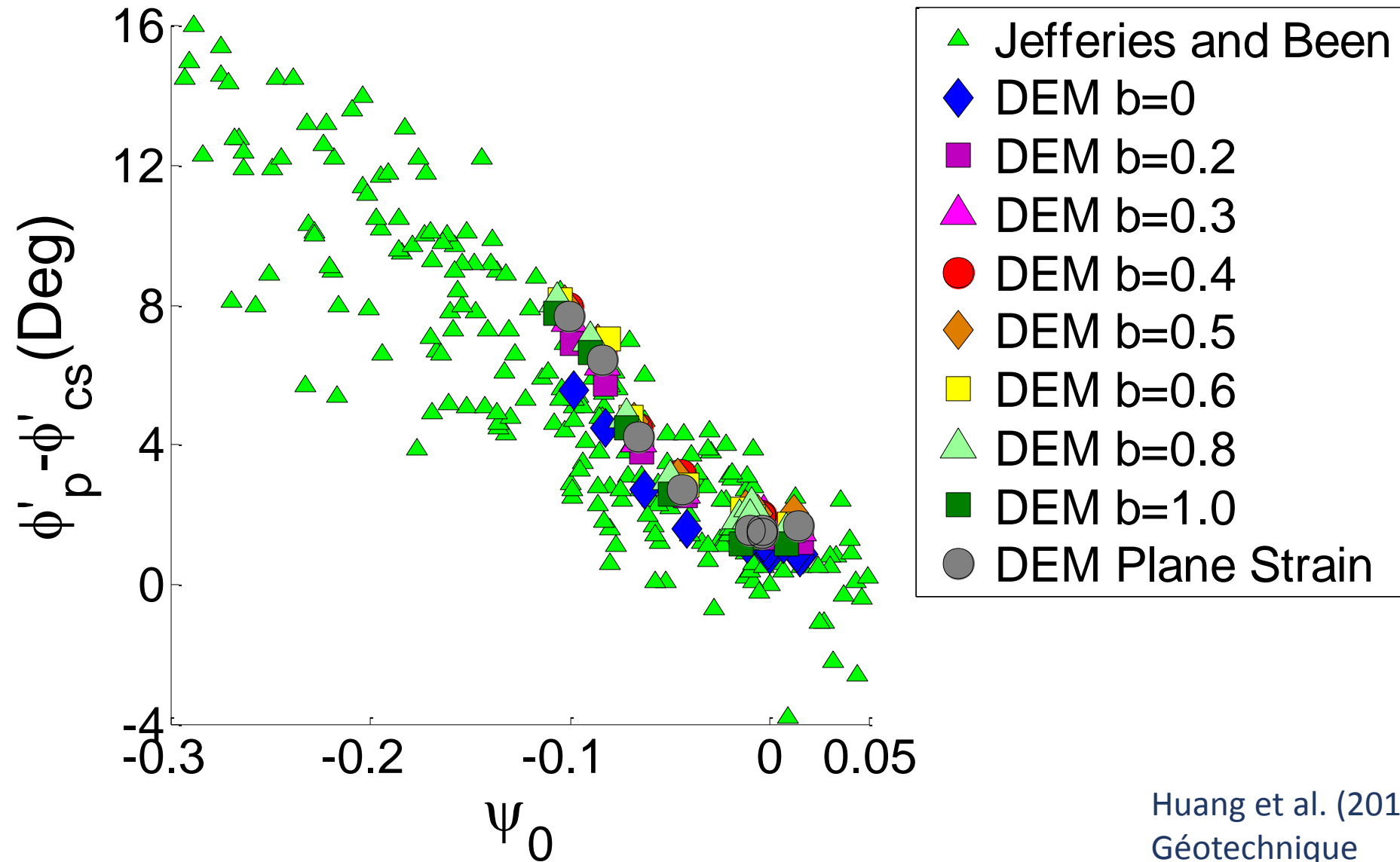
Variation in Φ'_p with Ψ_0



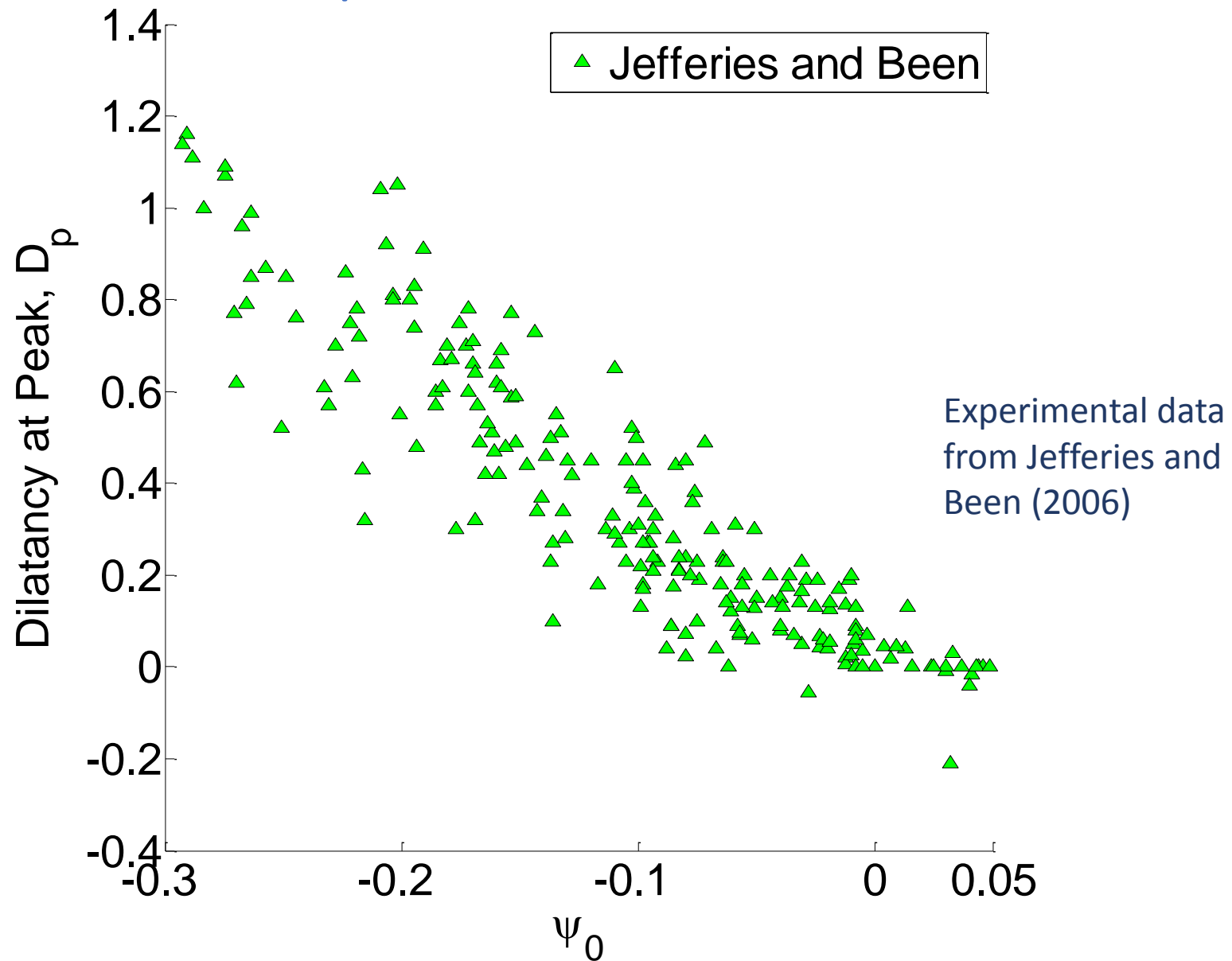
Variation in $\Phi'_p - \Phi'_{cs}$ with Ψ_0



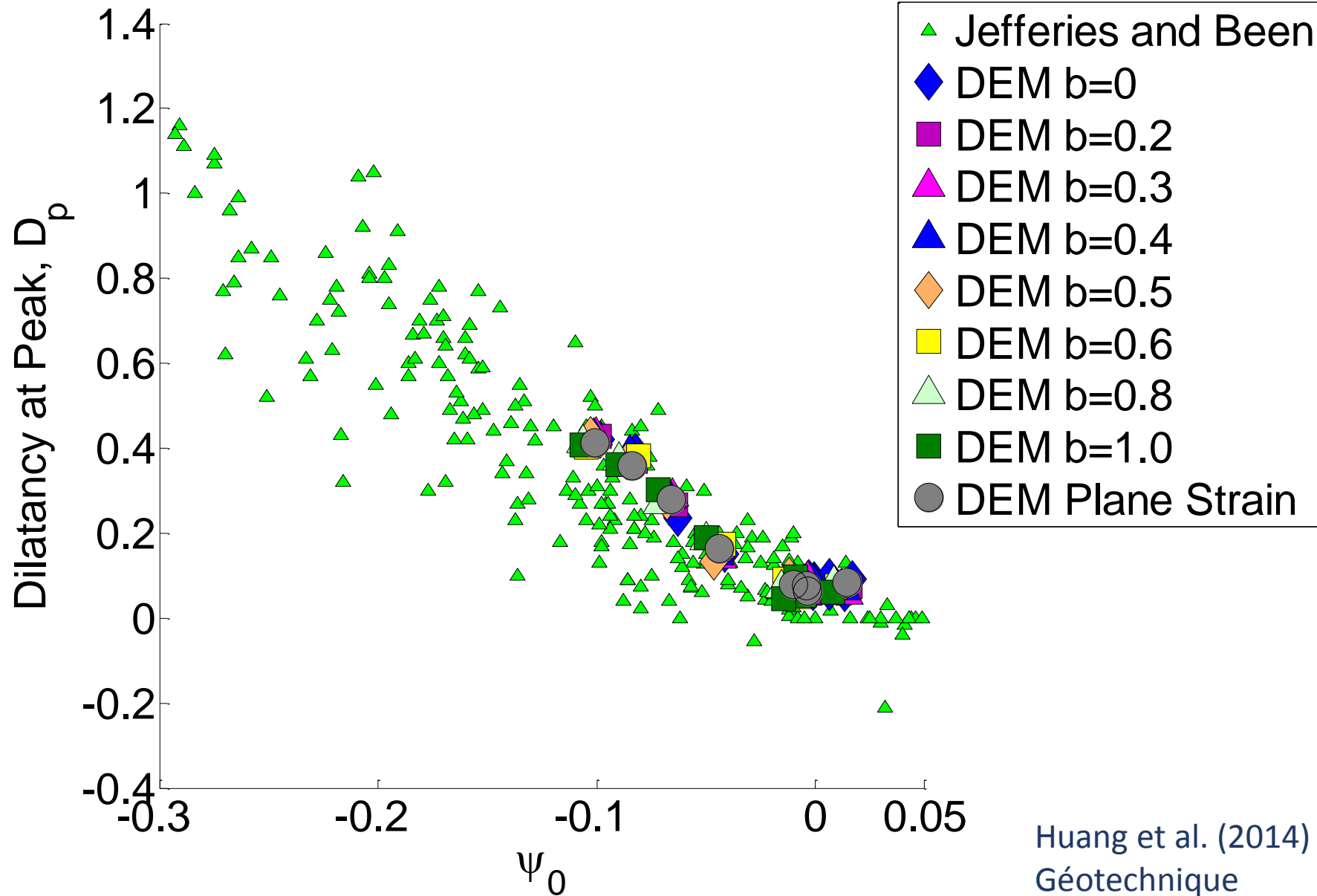
Variation in $\Phi'_p - \Phi'_{cs}$ with ψ_0



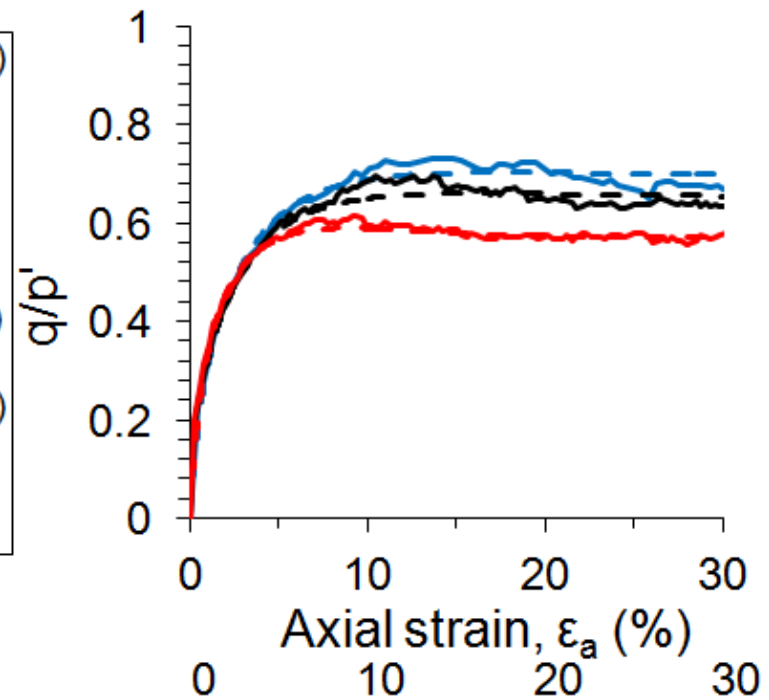
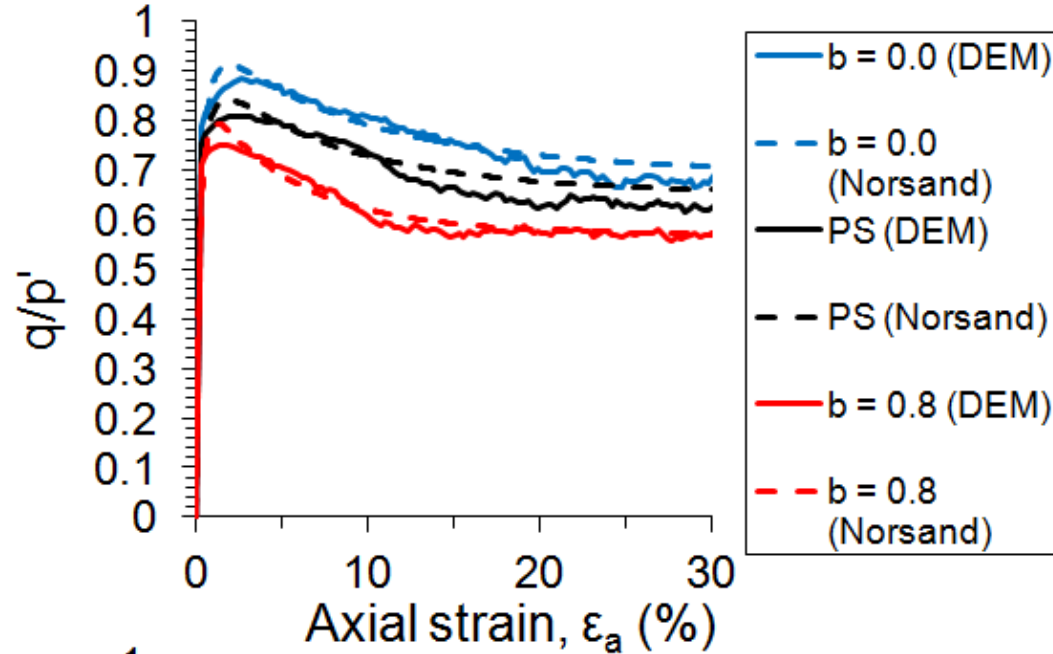
Variation in D_p with ψ_0



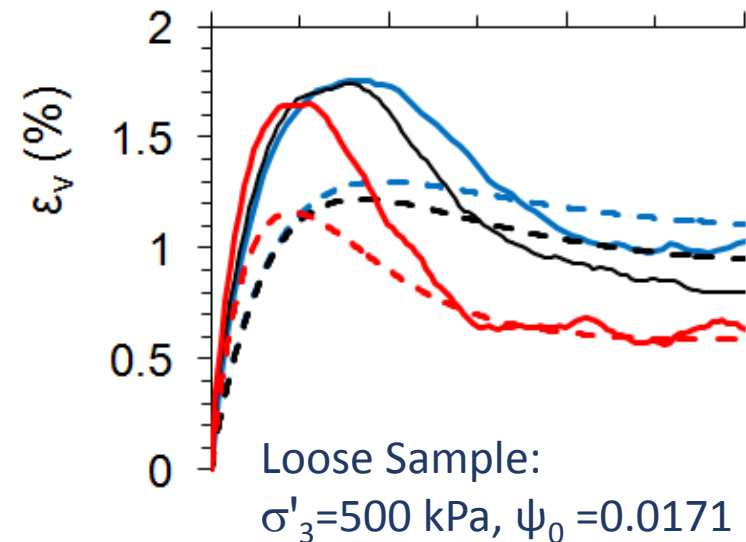
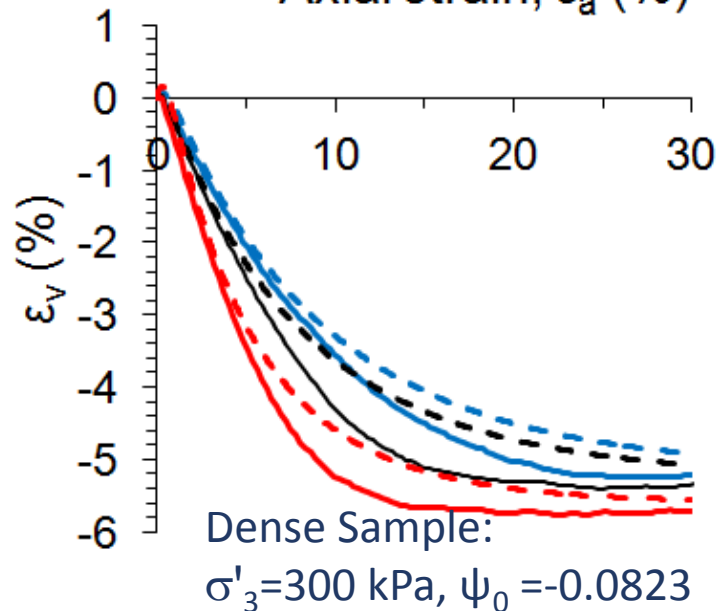
Variation in D_p with ψ_0



Consideration of NORSAND model

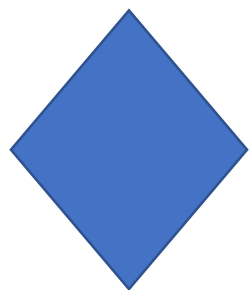


Huang et al. (2014)
Géotechnique



Does the state parameter concept hold under a general stress state?

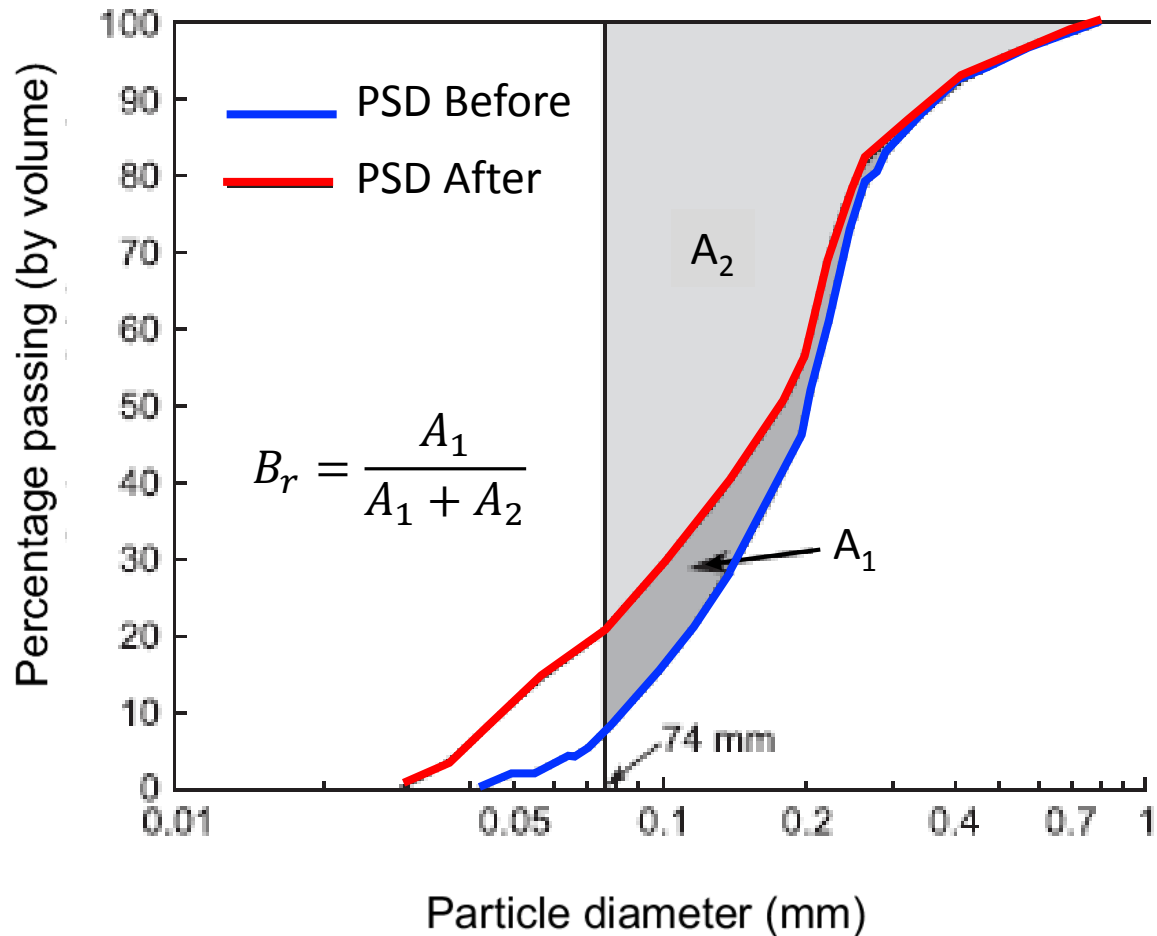
- Been and Jefferies state parameter concept is integrated in many constitutive models for sand behaviour.
- DEM simulations show that the quantitative relationships proposed by Been and Jefferies hold in a true-triaxial stress state.
- Constitutive models can be calibrated to capture the DEM data and give confidence in their predictive capacity outside of axi-symmetric (triaxial) stress states.



At a given stress level why do we see more crushing in shearing than in compression?

- Can DEM capture the trends reported from experimental studies?
- Can the particle-scale data available in the DEM simulations explain the experimental observations?

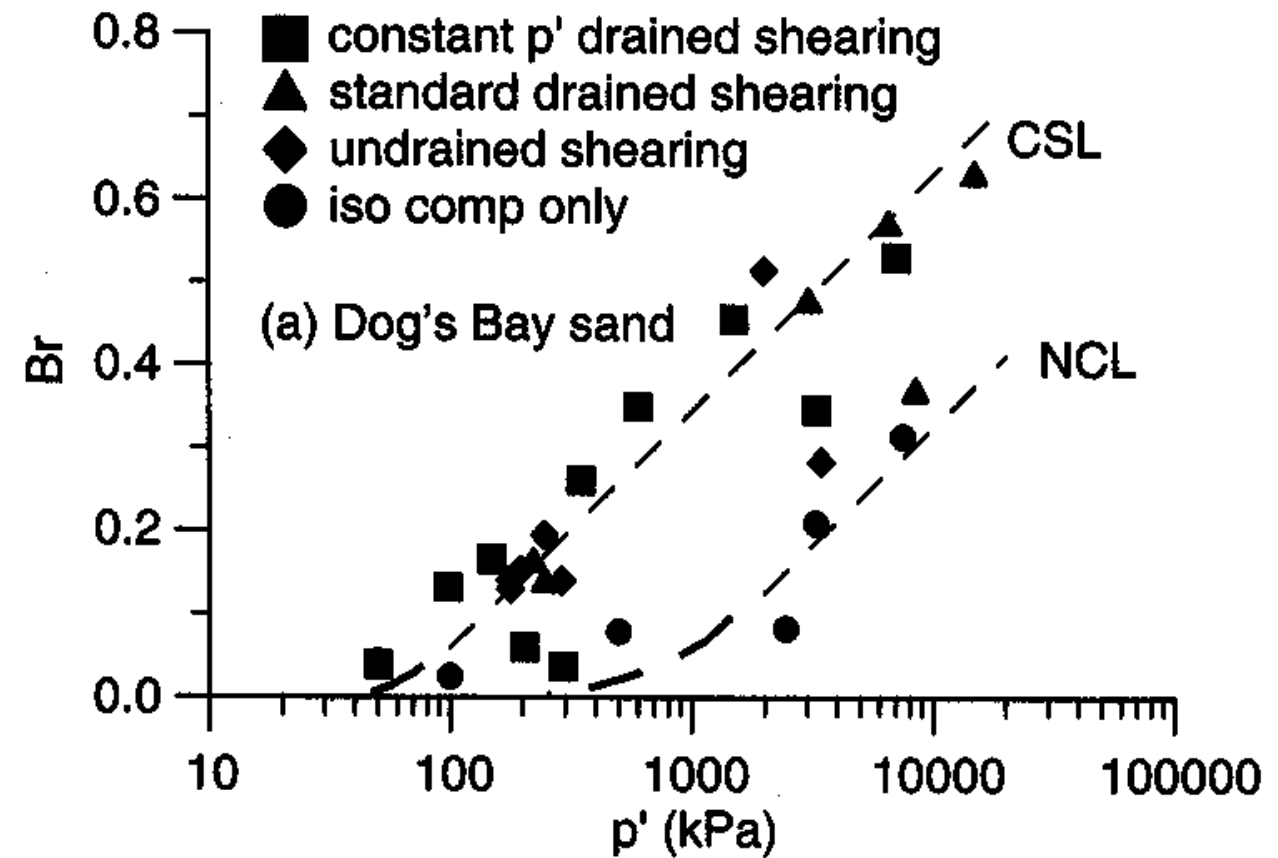
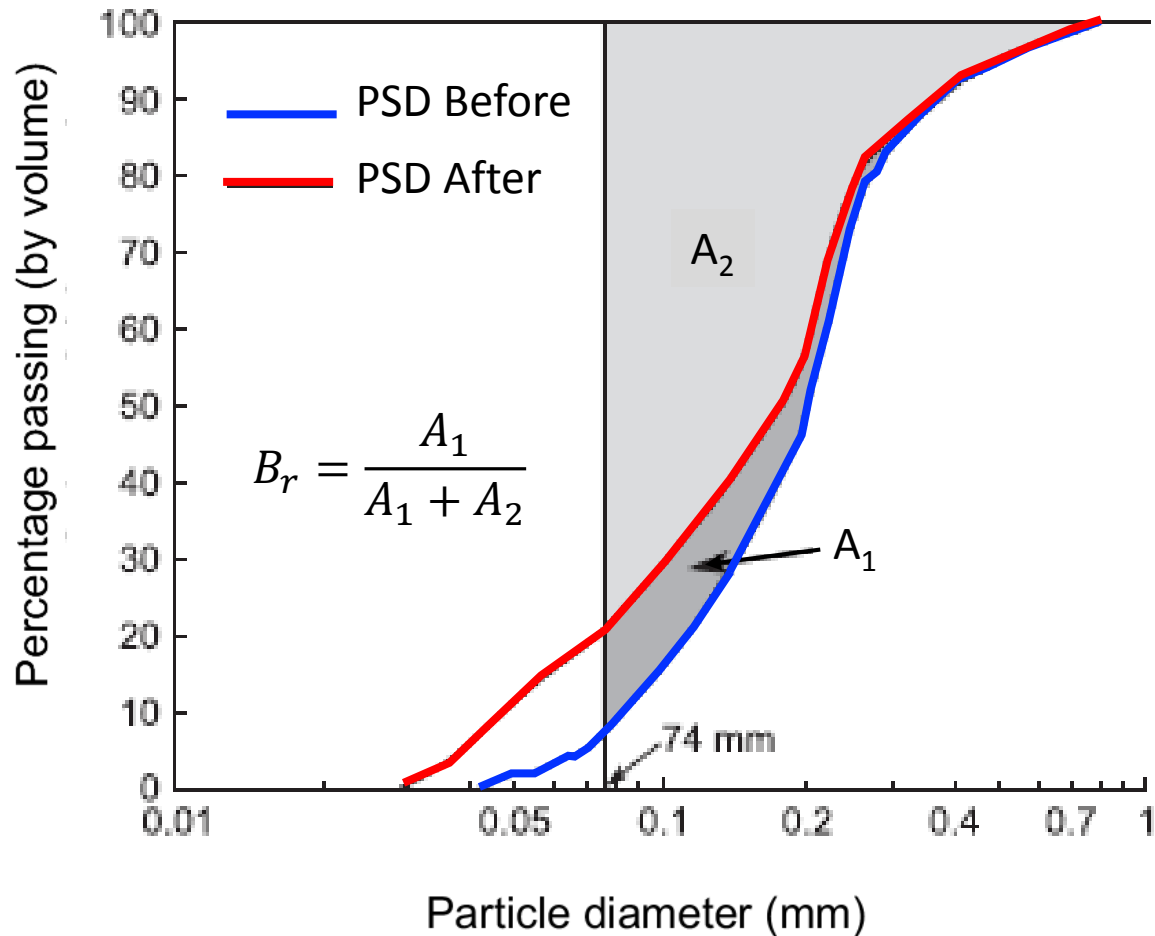
Influence of crushing on load:deformation behaviour



Use of Hardin's relative breakage to quantify crushing

Influence of crushing on load:deformation behaviour

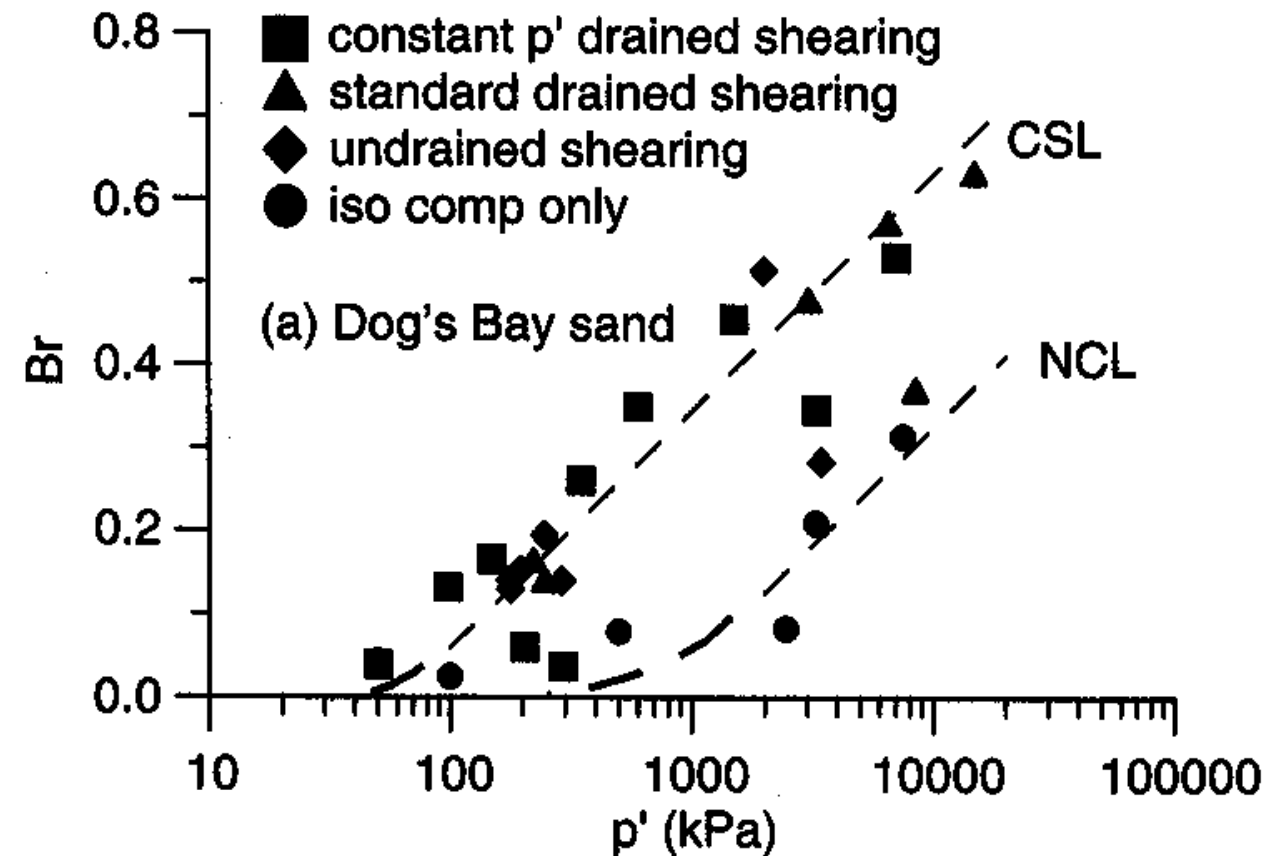
Coop and Lee (1993)



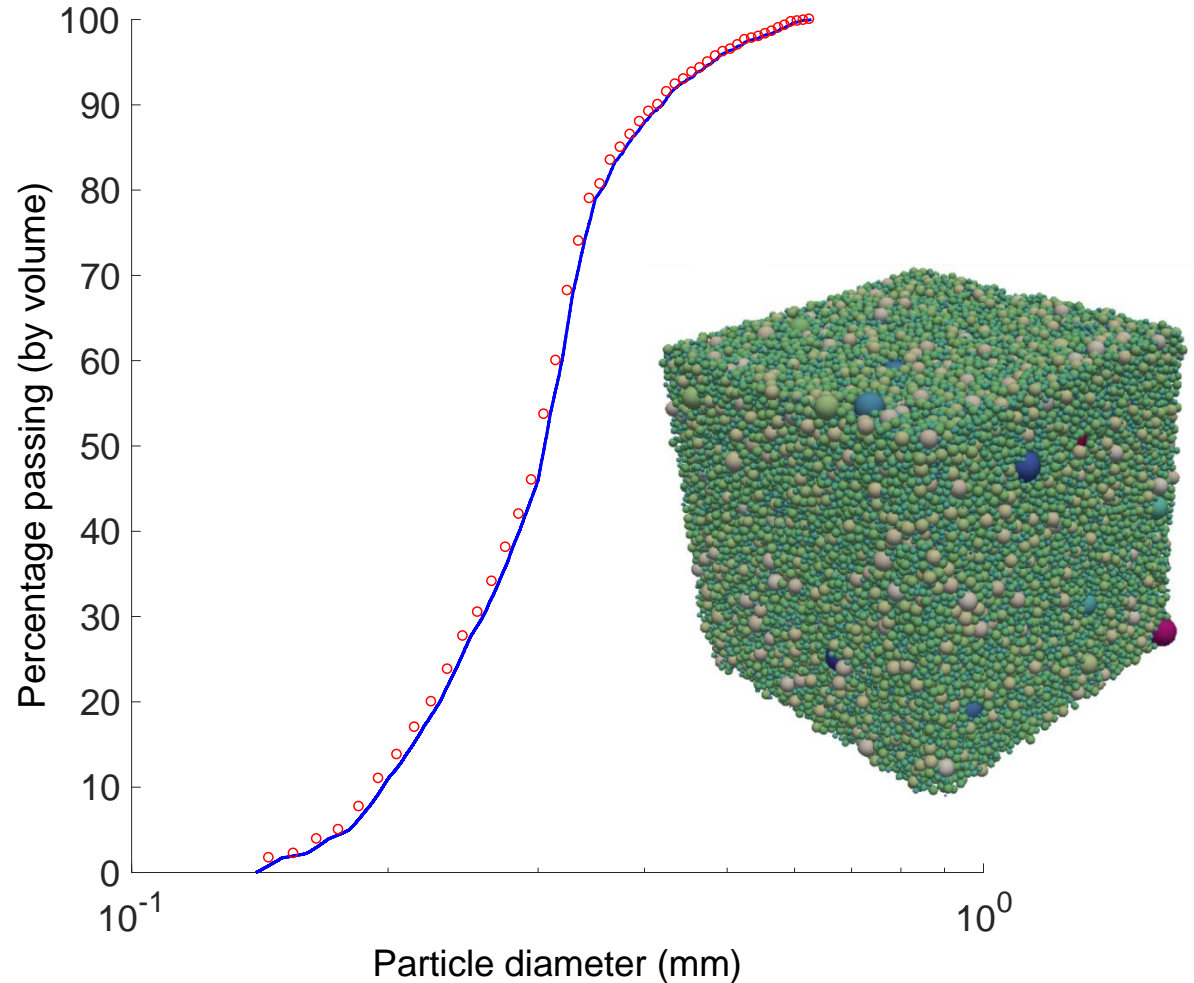
Influence of crushing on load:deformation behaviour

Coop and Lee (1993)

1. Can a DEM simulation capture difference in extent of crushing on CSL when compared with NCL at the same stress level?
2. Can DEM simulation data explain the difference in extent of crushing?

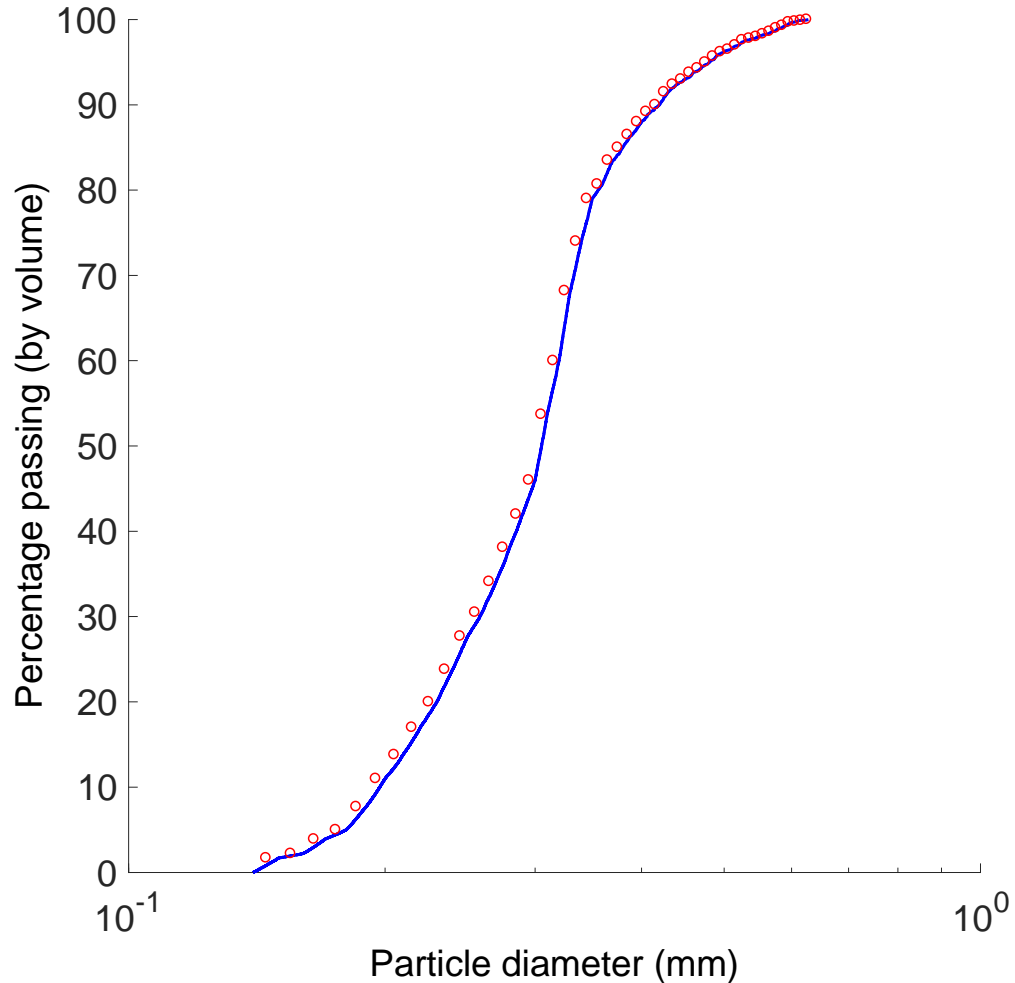


Influence of crushing on load:deformation behaviour

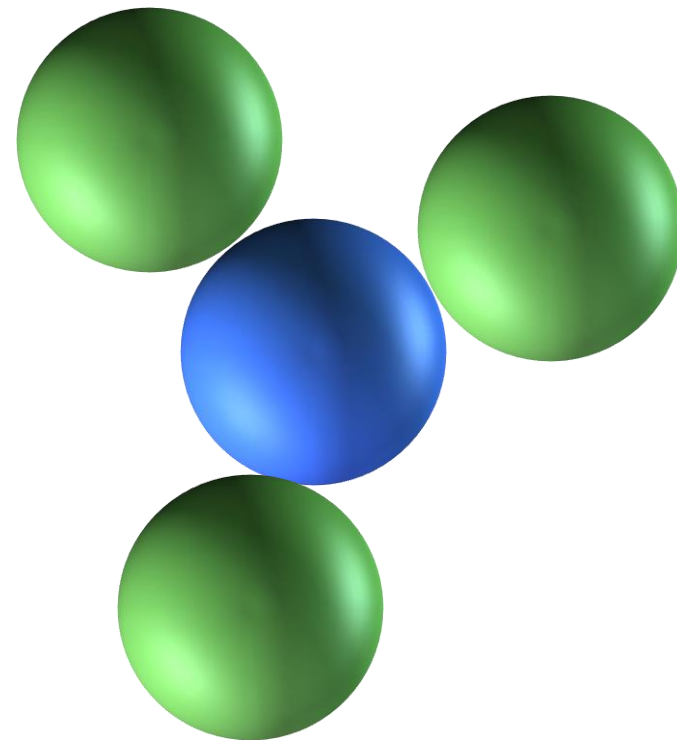


- DEM simulations used particle size distribution similar to Dunkirk sand
- Isotropic compression in a periodic cell
- Strain controlled triaxial compression with a constant σ'_3

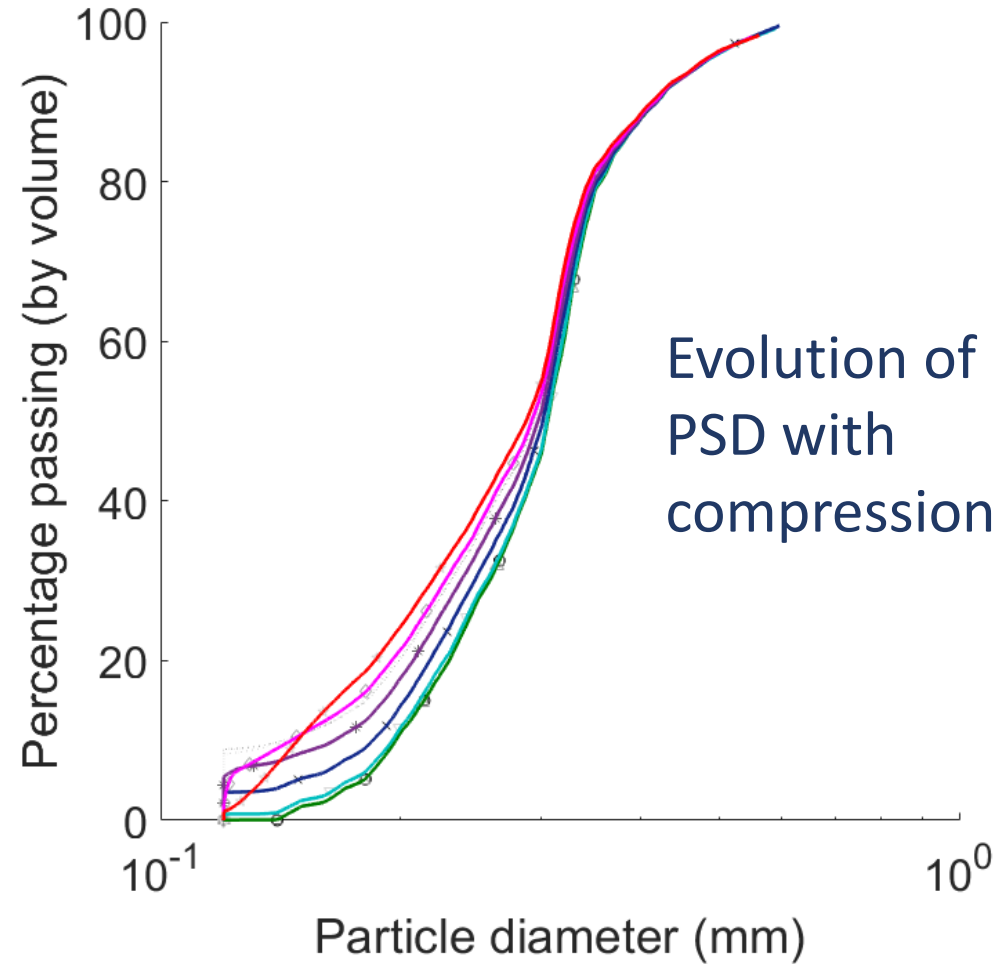
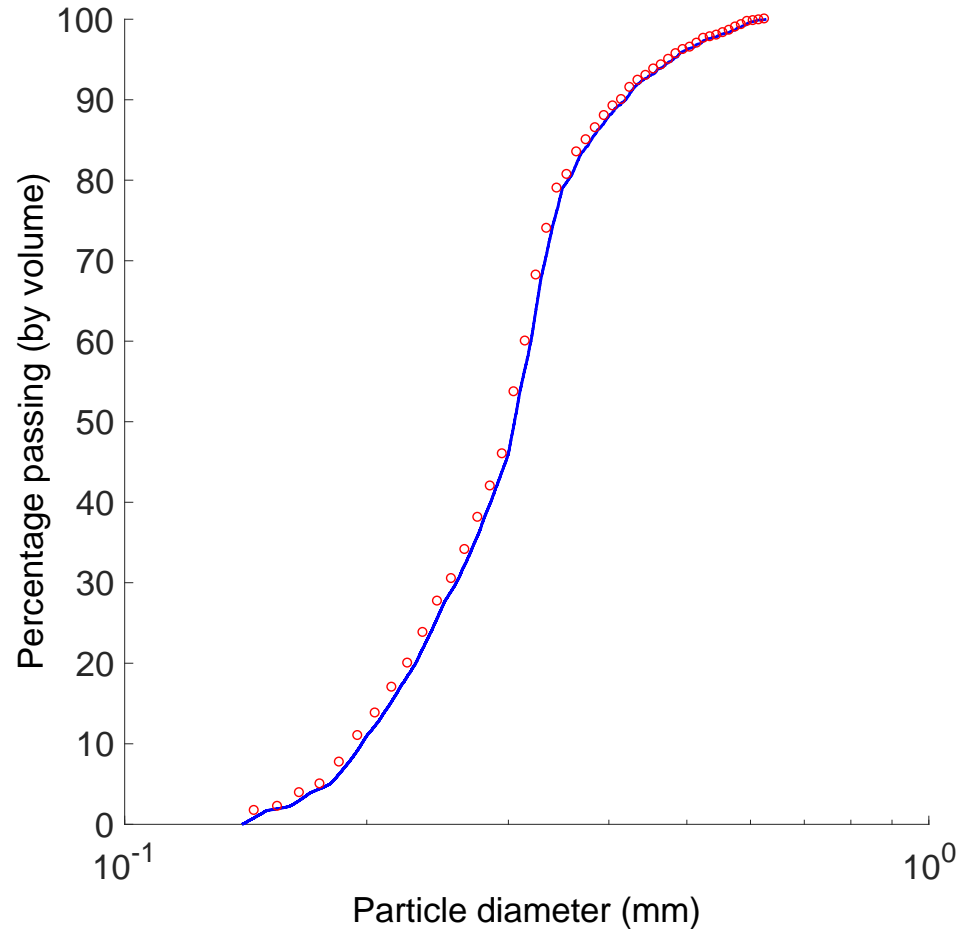
Influence of crushing on load:deformation behaviour



- Maximum force crushing criterion
- Particle shrunk to loose all contacts if criterion reached

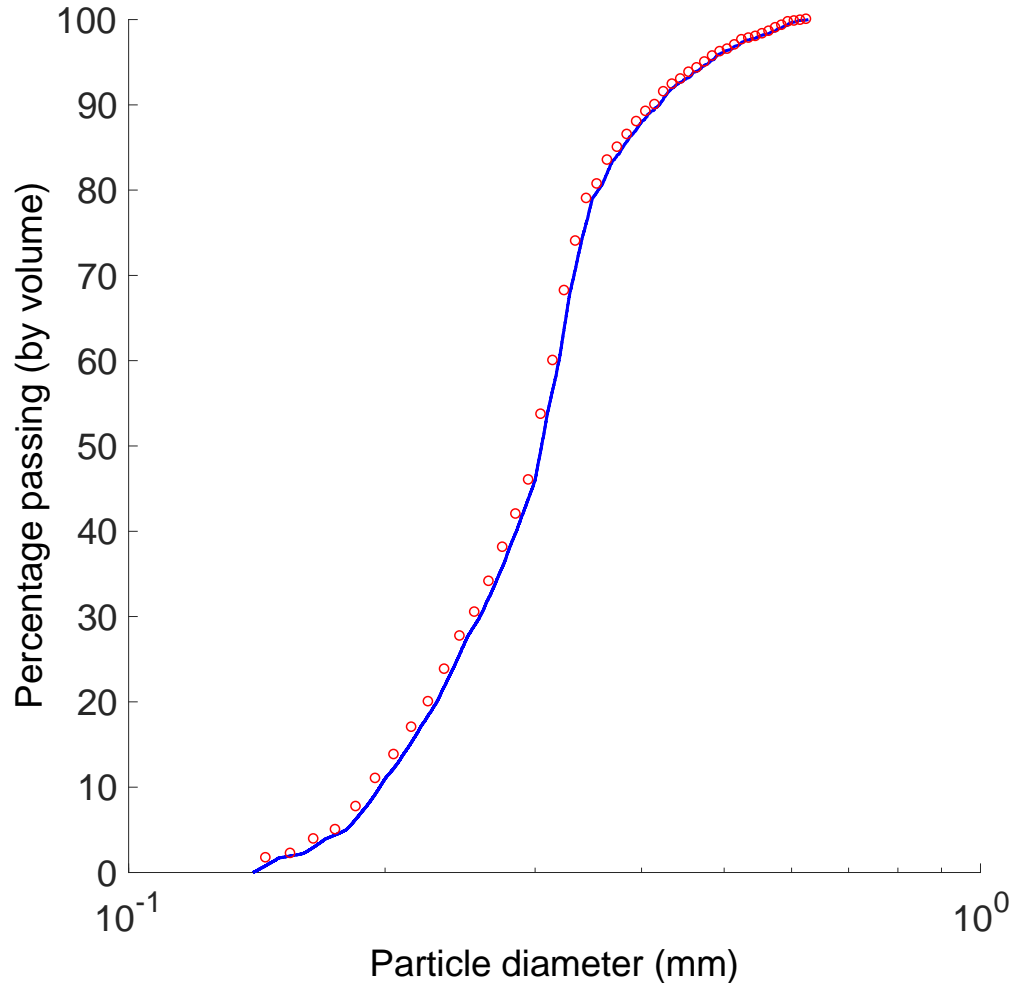


Influence of crushing on load:deformation behaviour

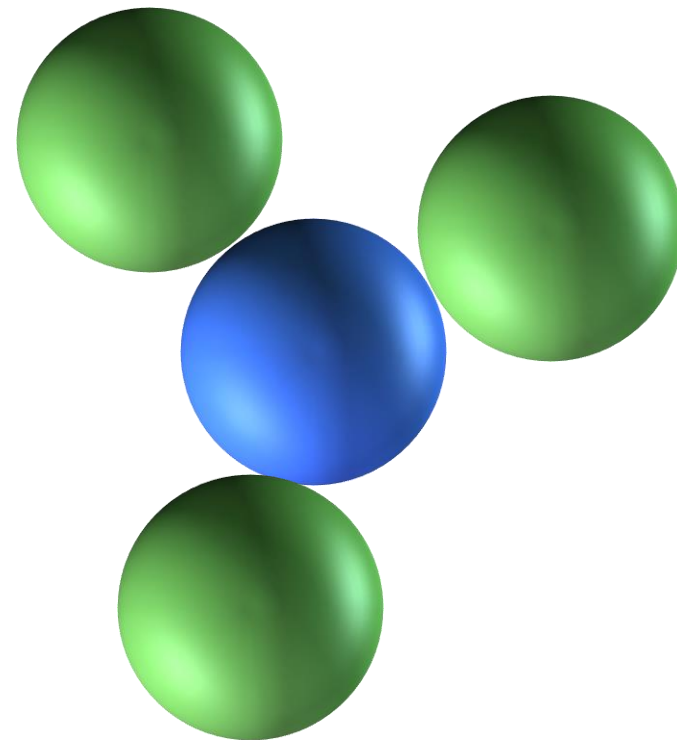


- ▽ 8 MPa
- × 16 MPa
- * 24 MPa
- ◇ 32 MPa
- ☆ 40 MPa
- ⋯ 32 MPa (Lo)
- ⋯ 40 MPa (Lo)
- data1
- data2
- 4 MPa
- 8 MPa
- 16 MPa
- 24 MPa
- 32 MPa
- 40 MPa

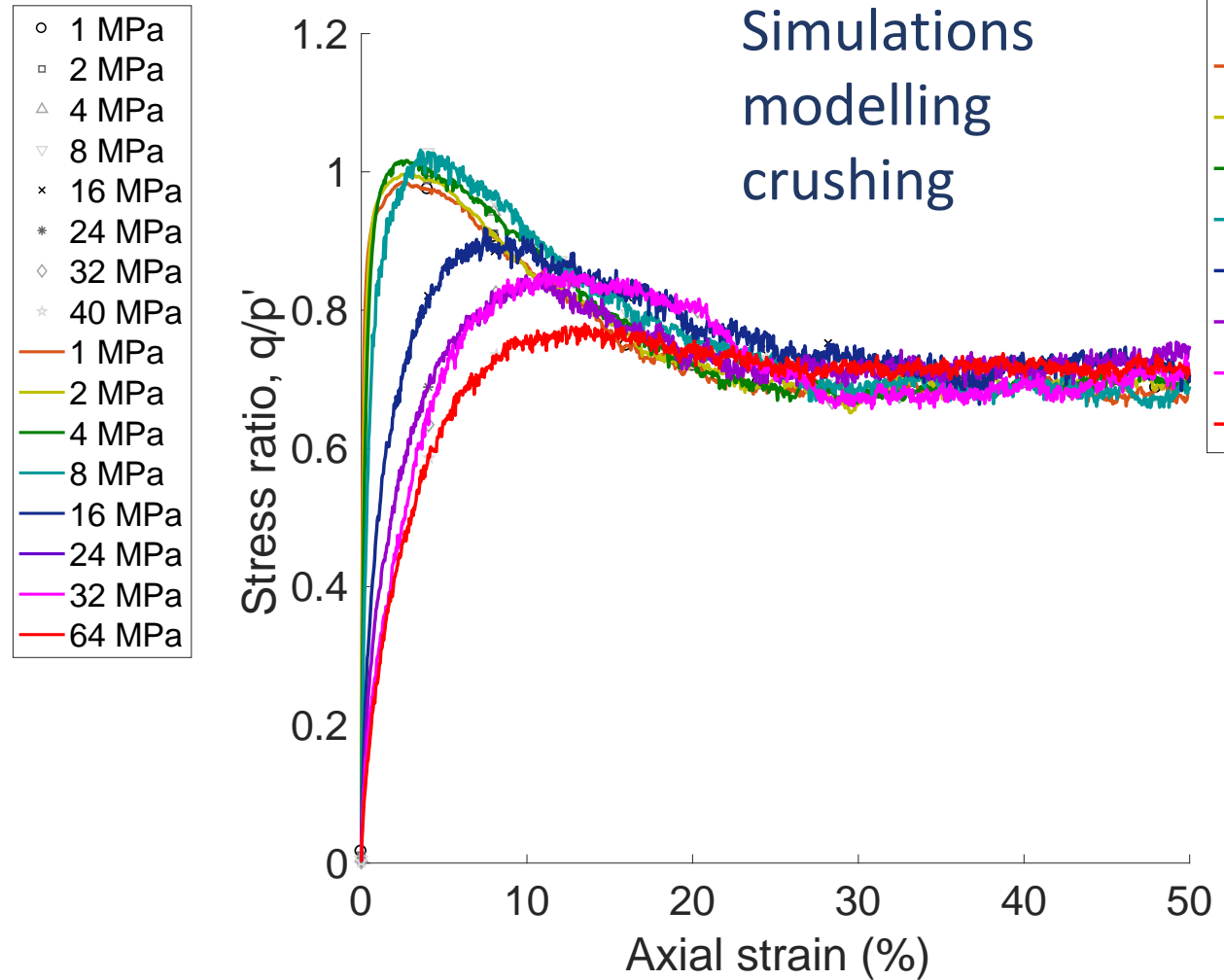
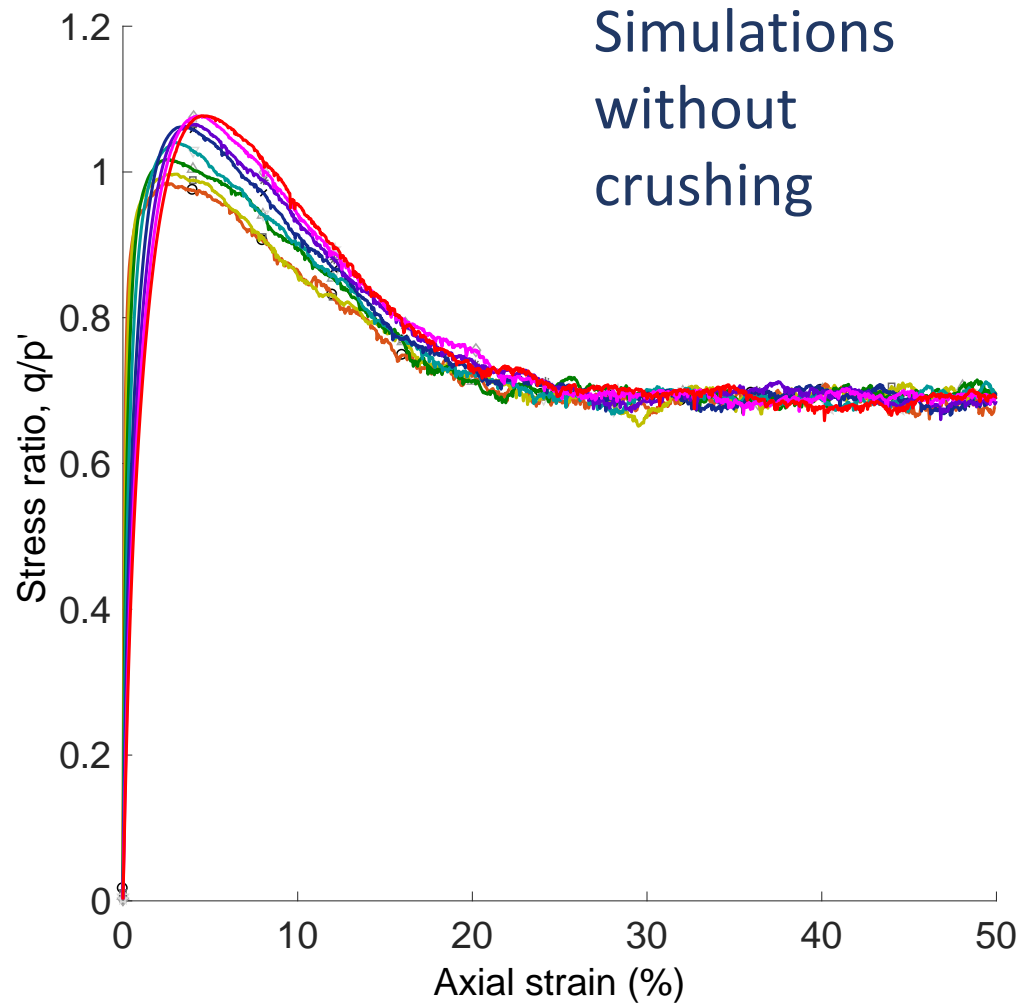
Influence of crushing on load:deformation behaviour



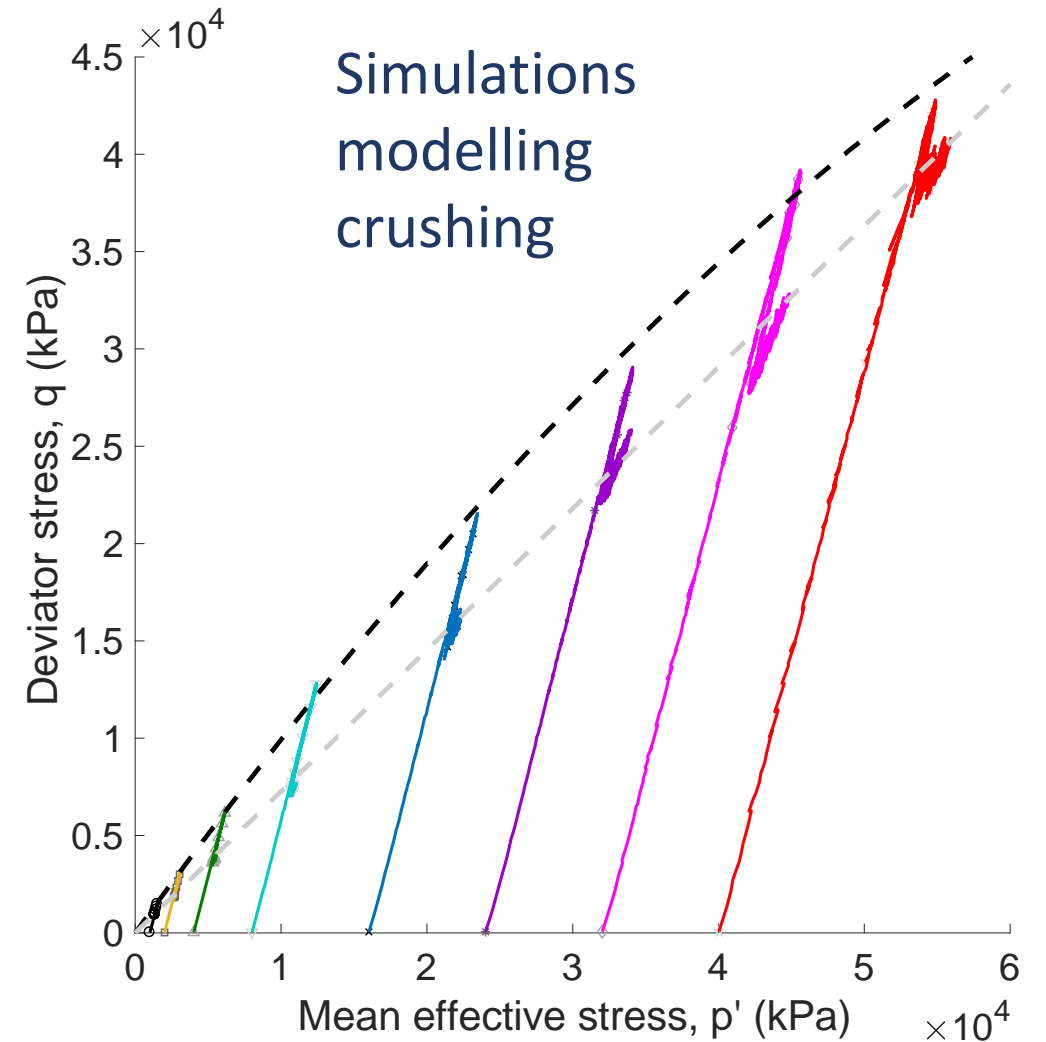
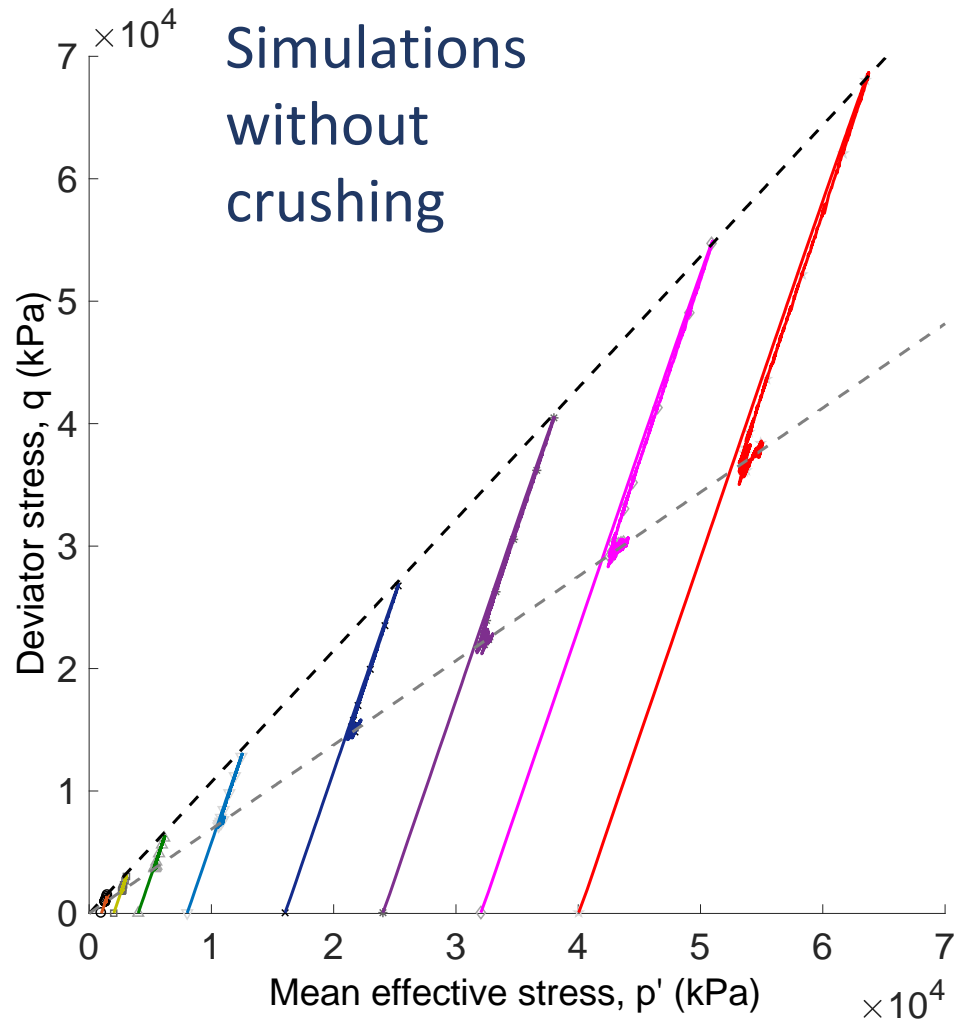
- Maximum force crushing criterion
- Particle shrunk to loose all contacts if criterion reached



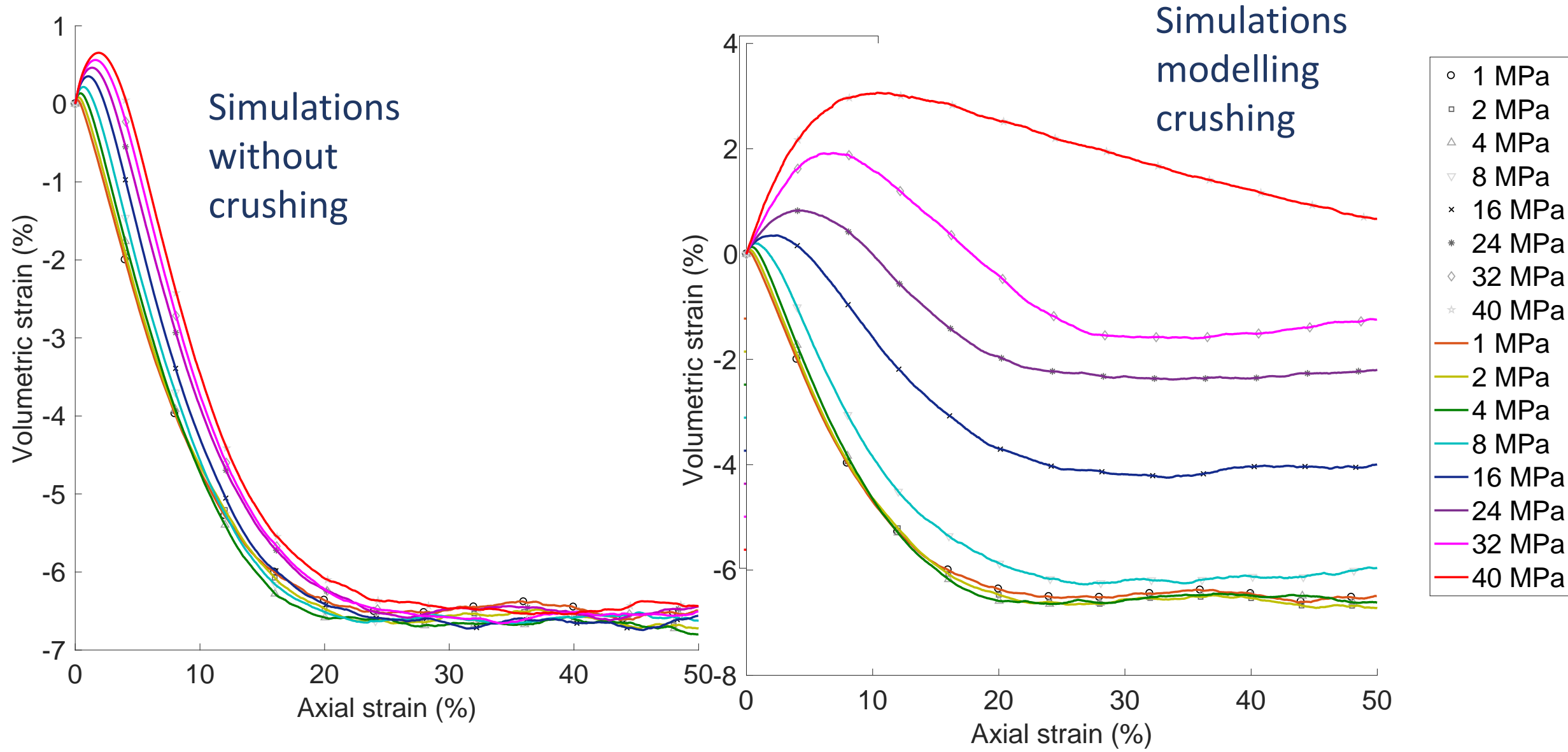
Influence of crushing on load:deformation behaviour



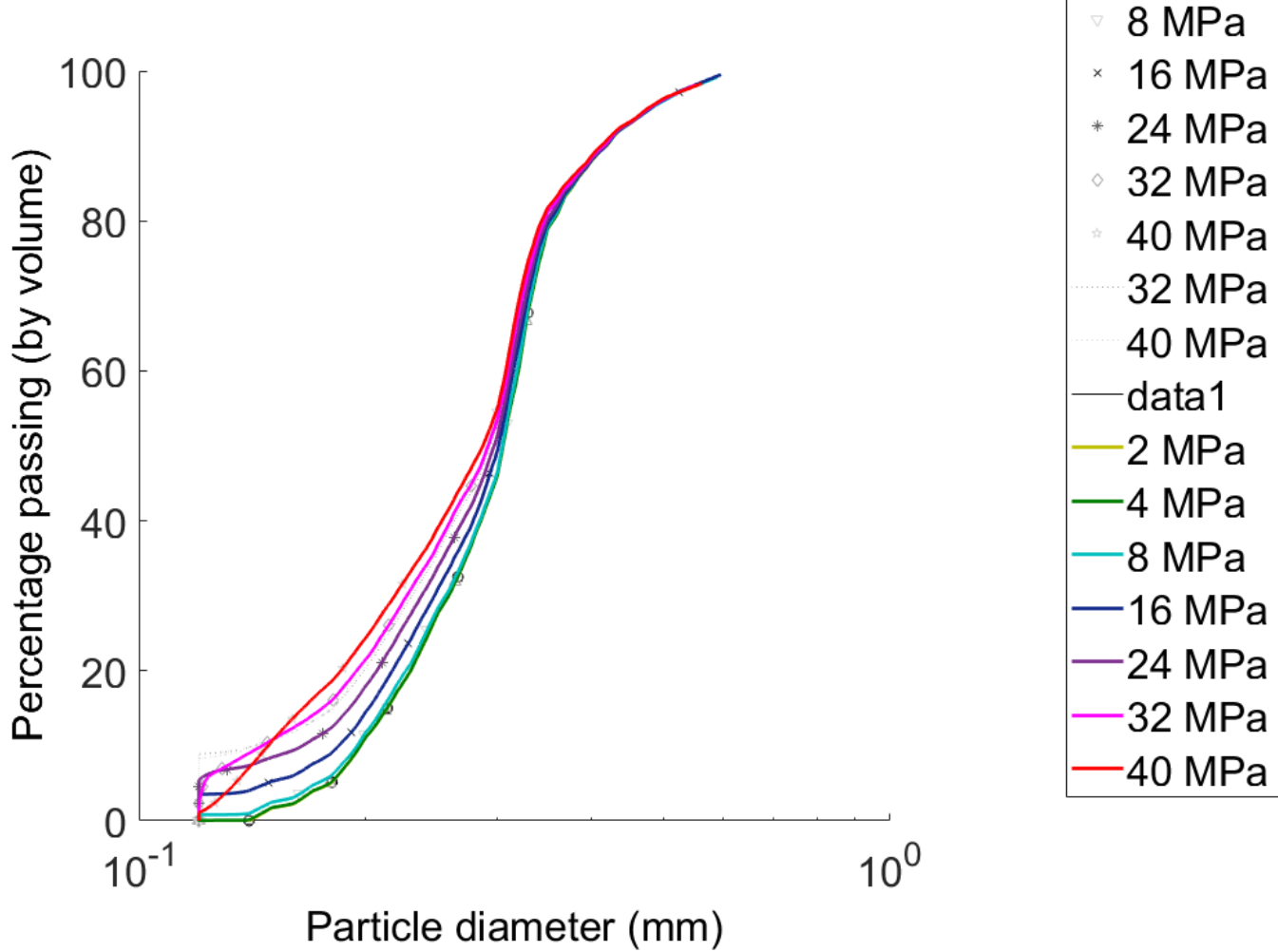
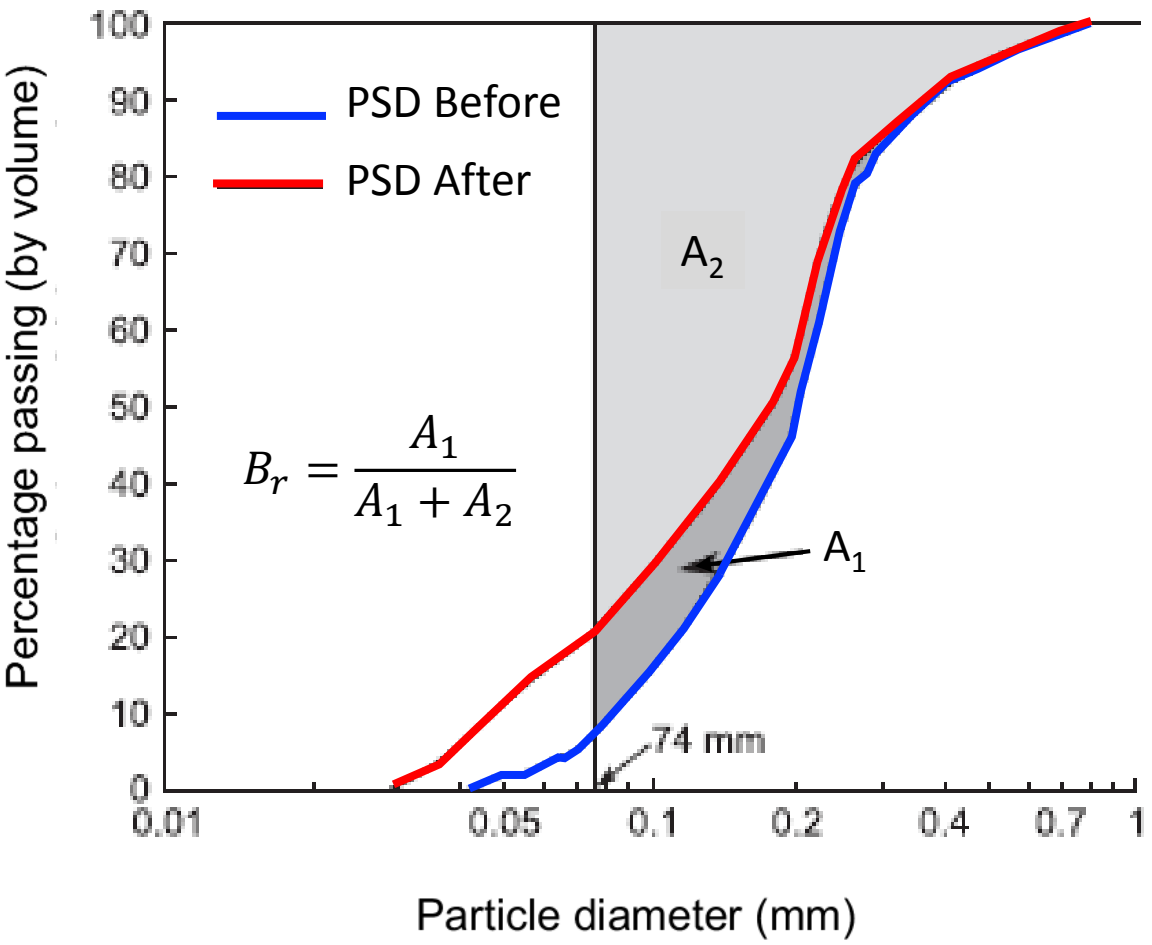
Influence of crushing on load:deformation behaviour



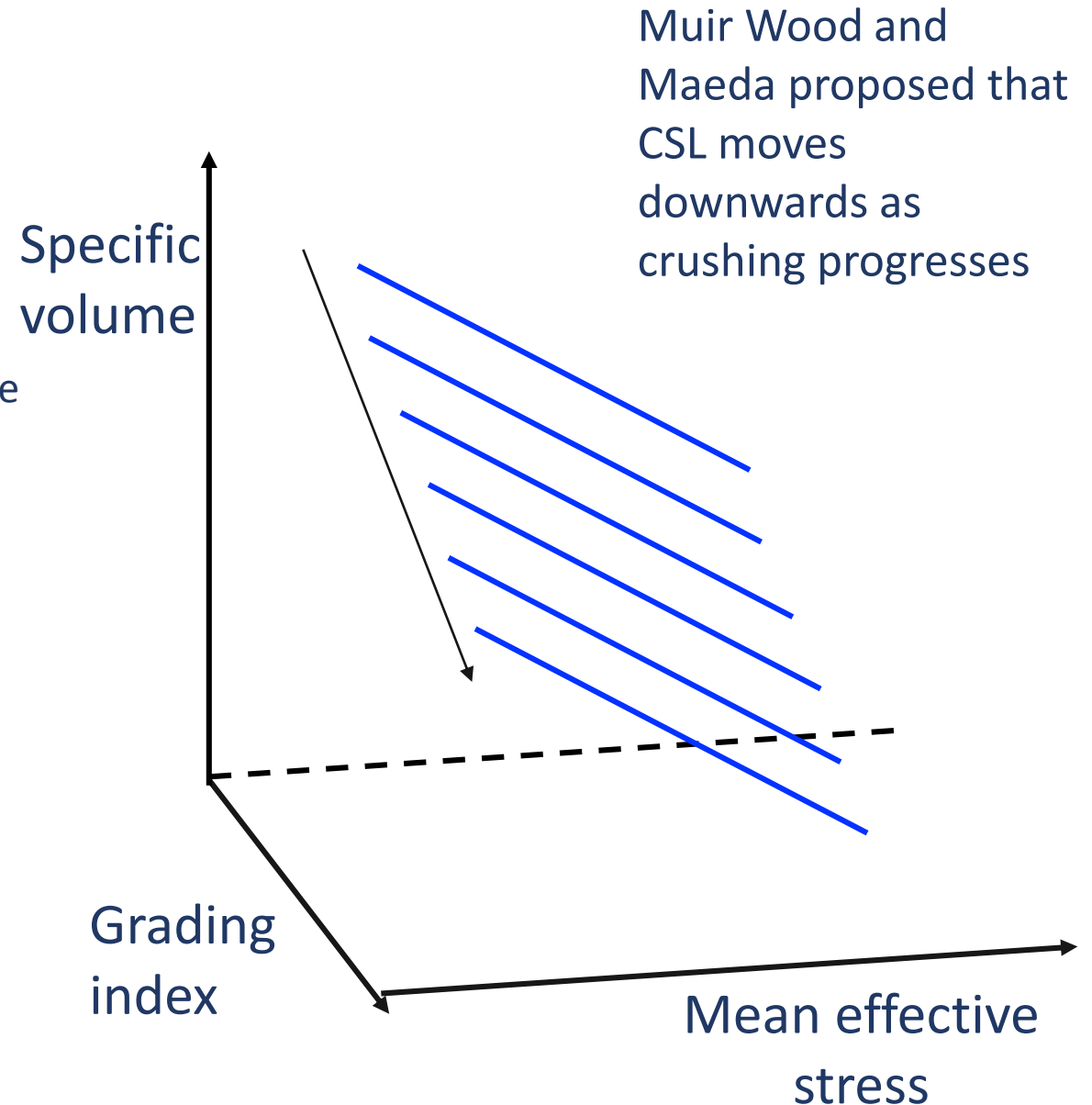
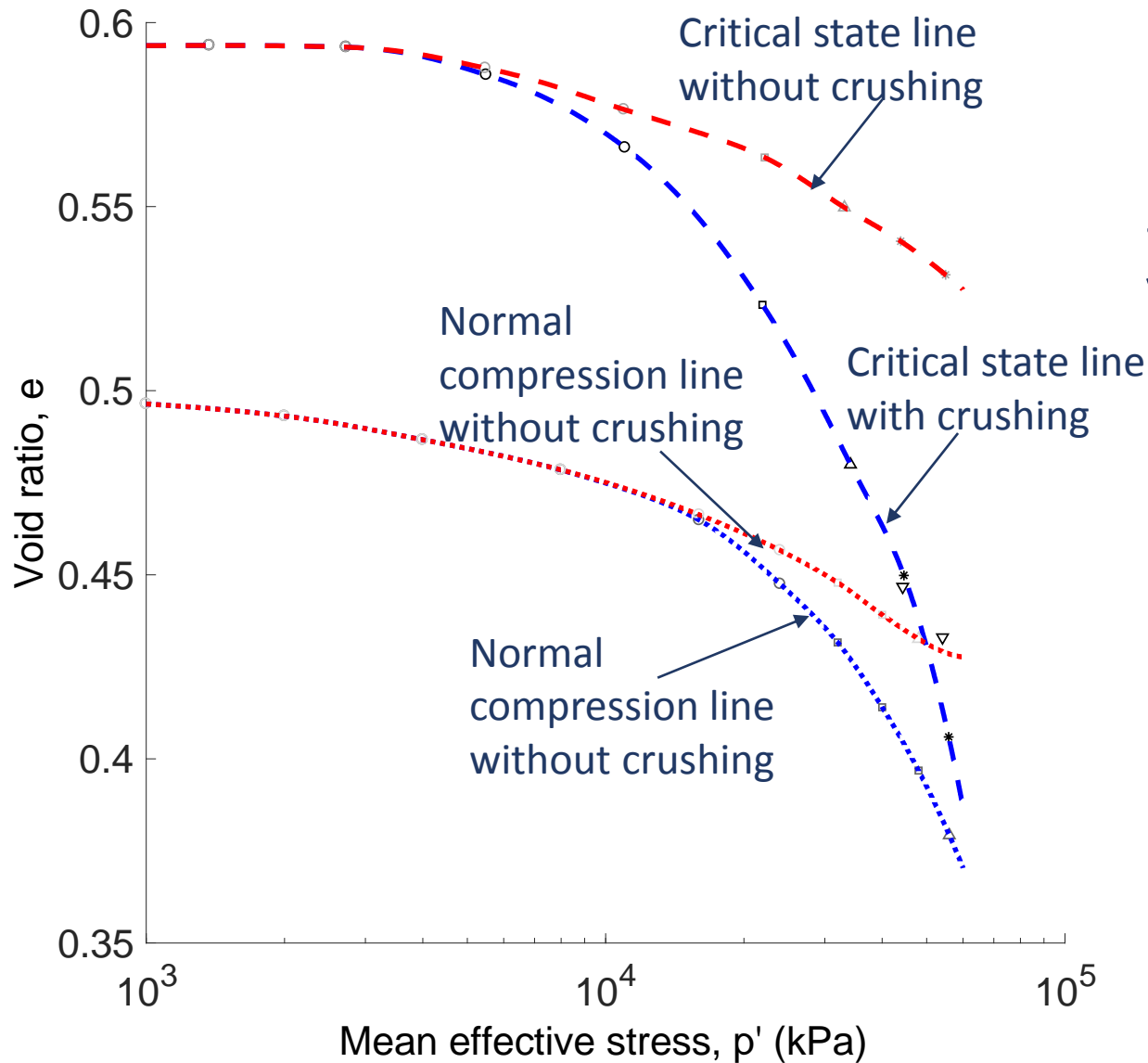
Influence of crushing on load:deformation behaviour



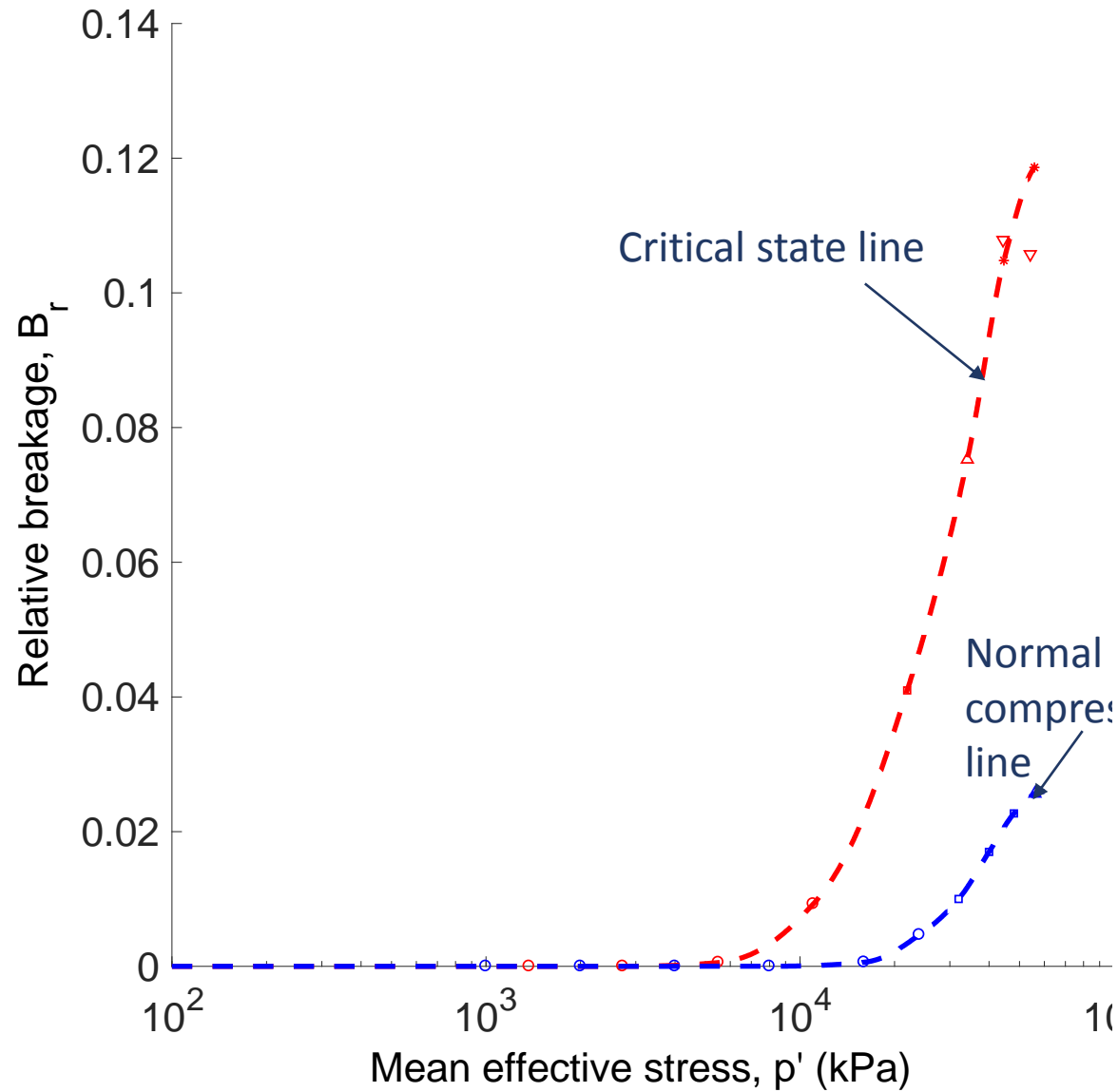
Influence of crushing on load:deformation behaviour



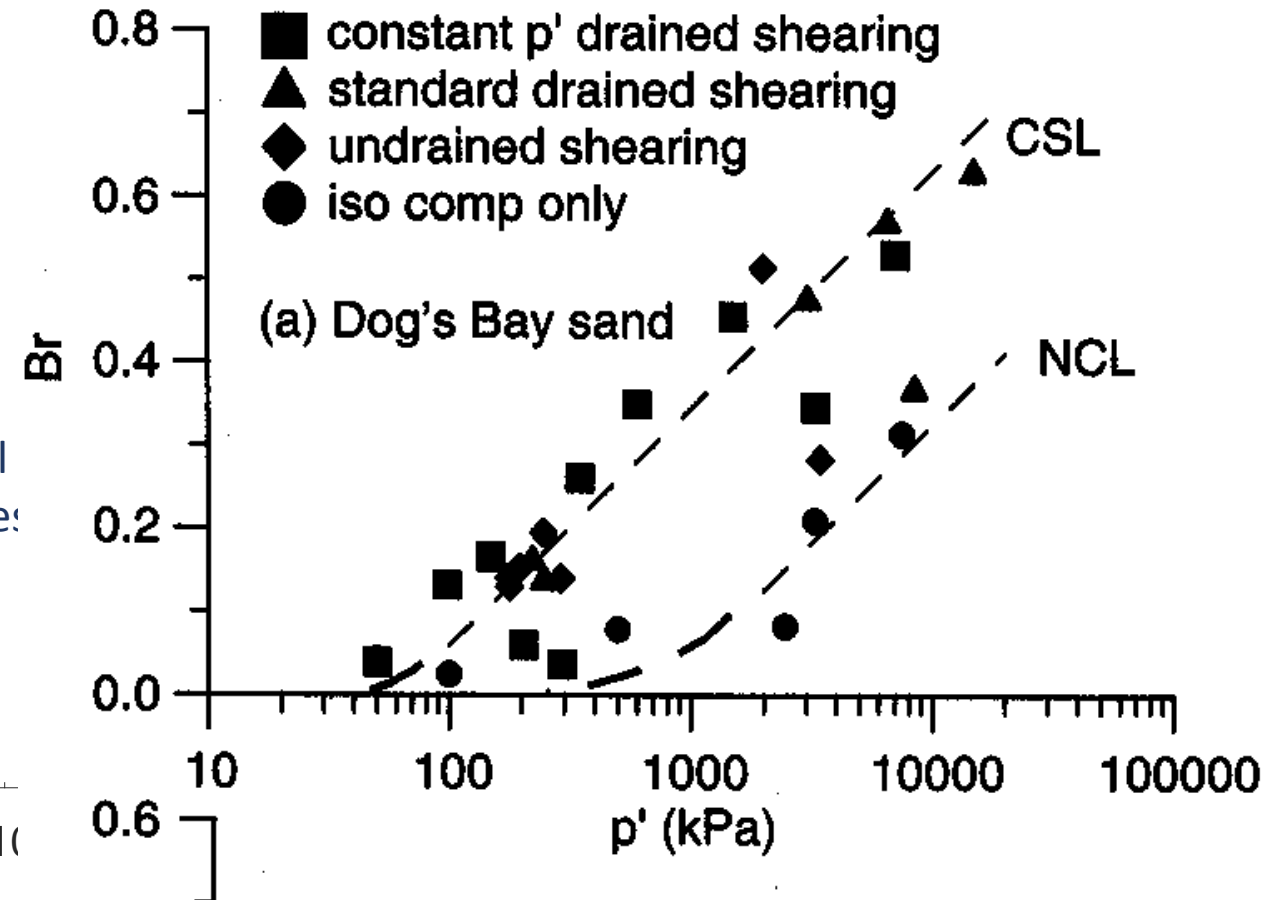
Influence of crushing on load:deformation behaviour



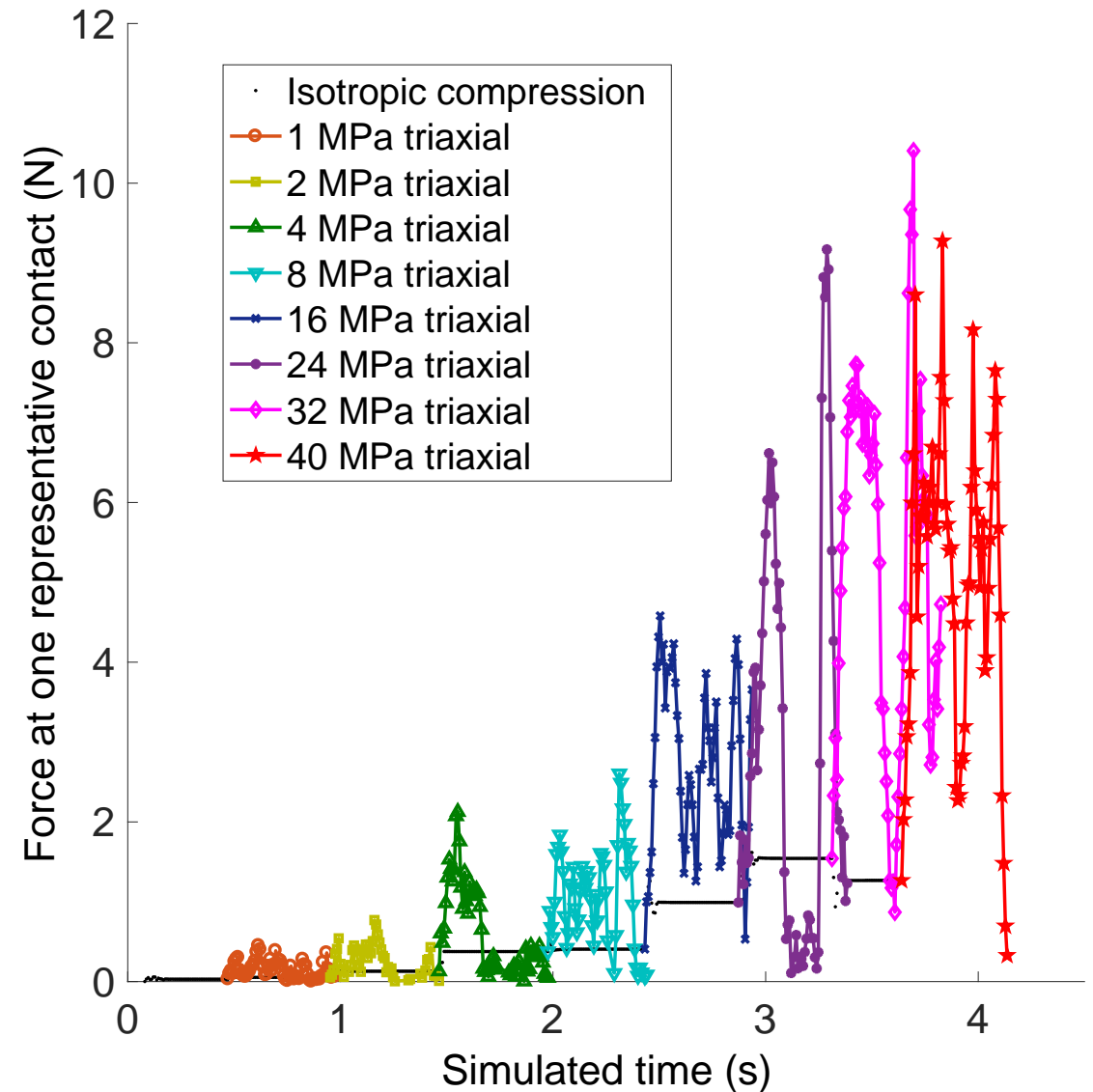
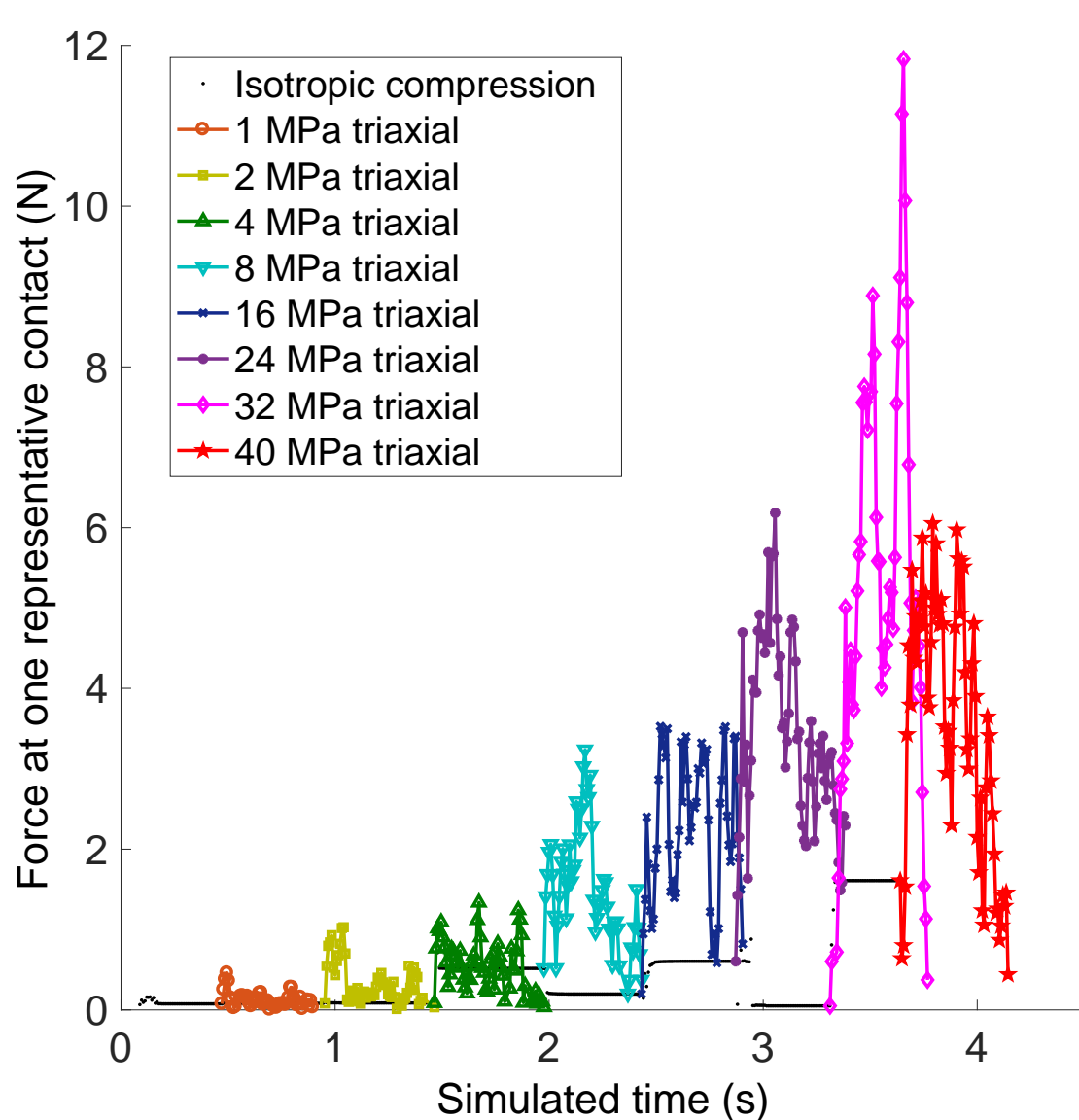
Influence of crushing on load:deformation behaviour



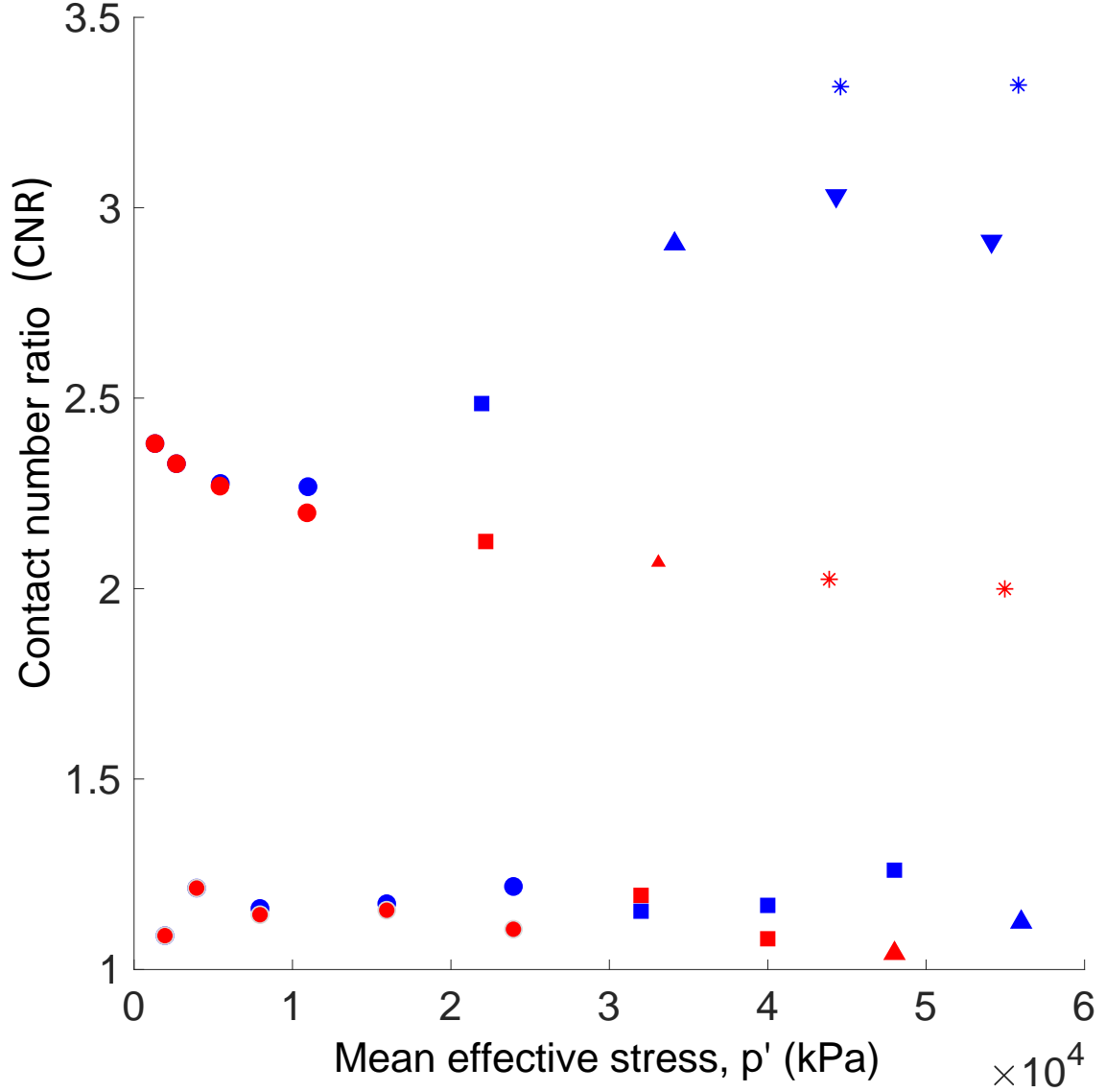
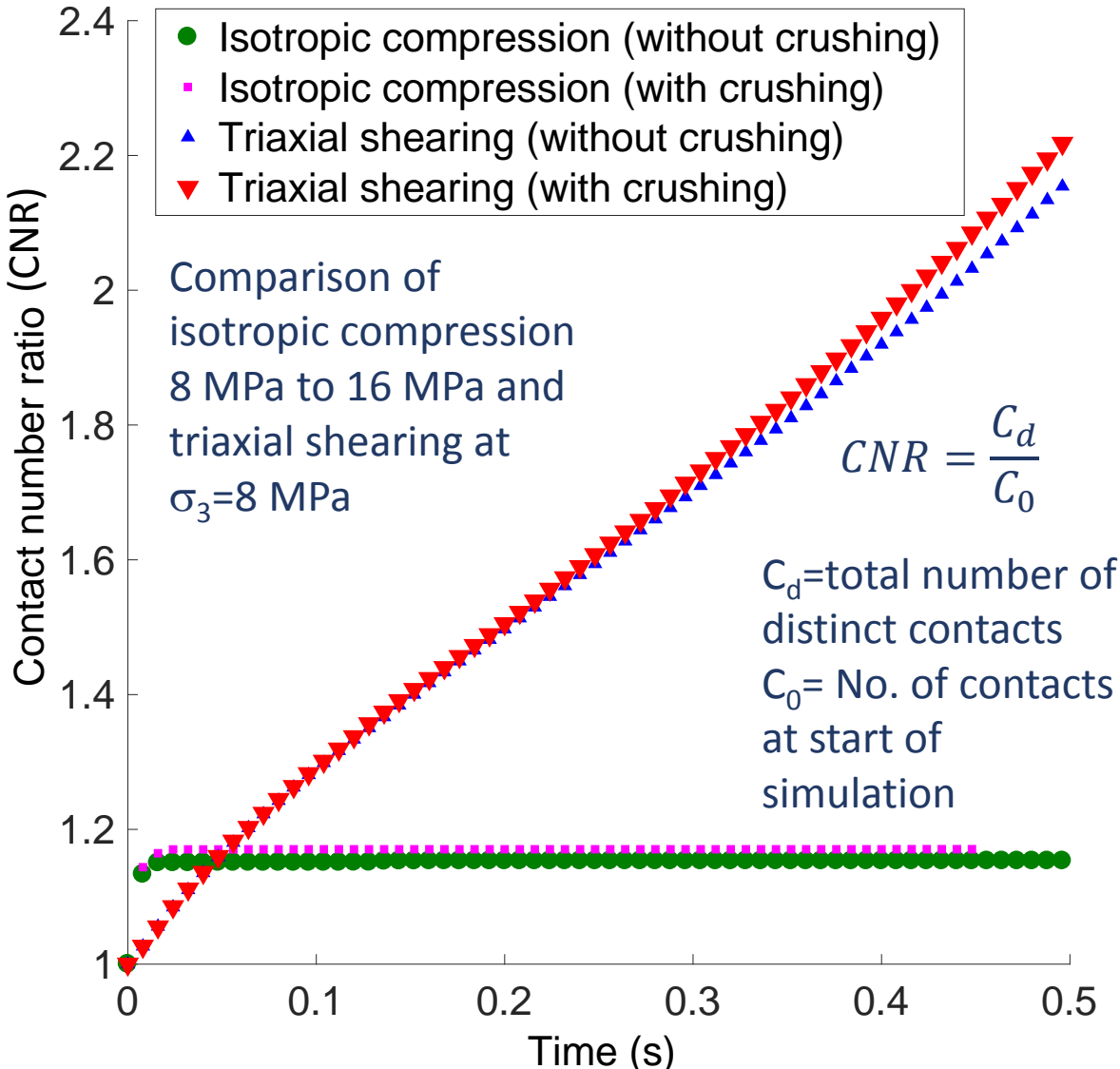
Coop and Lee (1993)



Influence of crushing on load:deformation behaviour

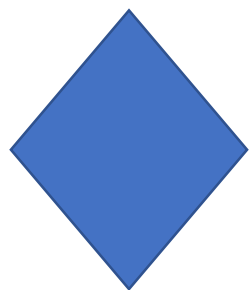


Influence of crushing on load:deformation behaviour



At a given stress level why do we see more crushing in shearing than in compression?

- DEM can capture the trends reported from experimental studies.
- The particle-scale data available in the DEM simulations show that there is significantly more variation in the forces experienced at contacts during shearing than during compression.
- The particle-scale data also show that contact evolution (formation of new contacts) is greater in shearing than in compression.



Conclusions

- DEM can capture many of the complex response phenomena associated with sand behavior.
- DEM can generate the particle scale data to explain the overall mechanical response observed in laboratory element tests.
- DEM can be used to complement experimental research by considering frameworks and constitutive models under stress states that cannot be attained experimentally.

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