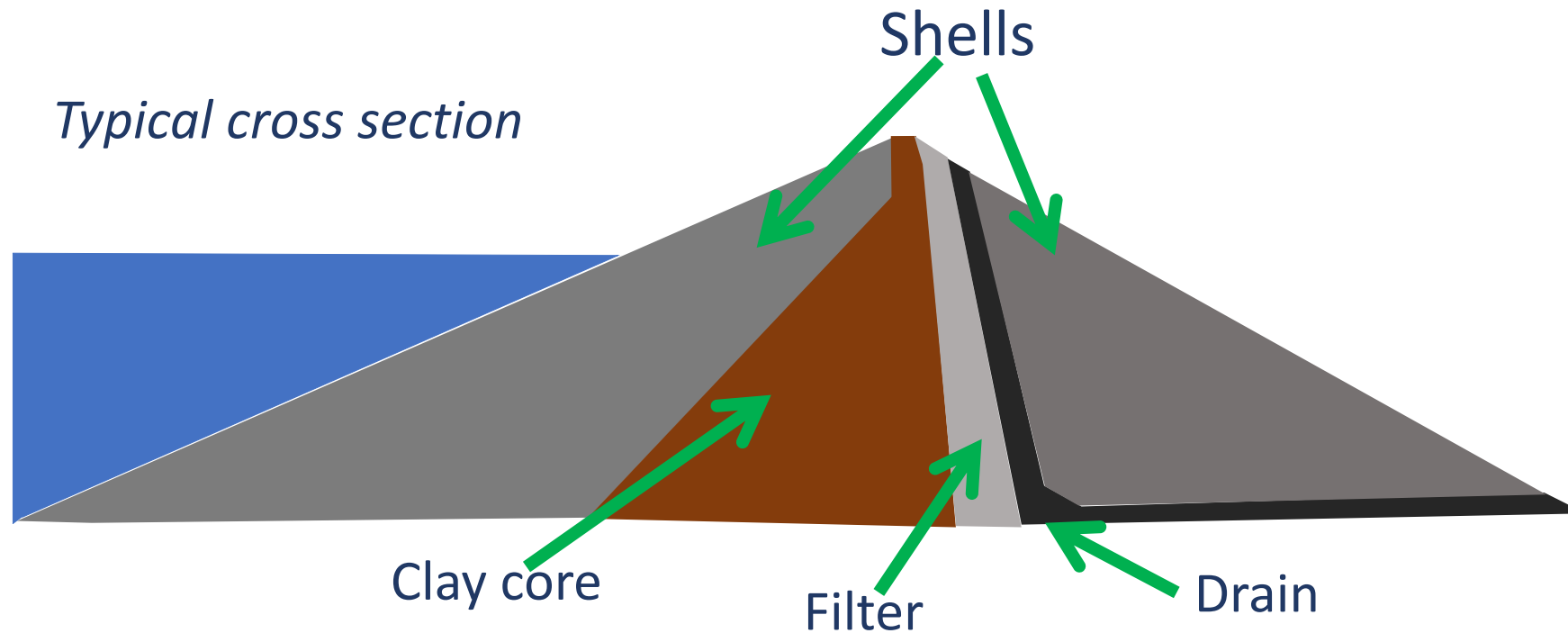


A particle scale perspective on internal erosion and filter design

Catherine O'Sullivan

Joana Fonseca, Kevin Hanley, Kenichi Kawano, Chris Knight, Thomas Shire, Way Way Moinet, Howard Taylor

Application: Embankment Dams



- Dams can be over 100 m high
- Water seeps through dam continuously
- Seeping water can preferentially erode fines

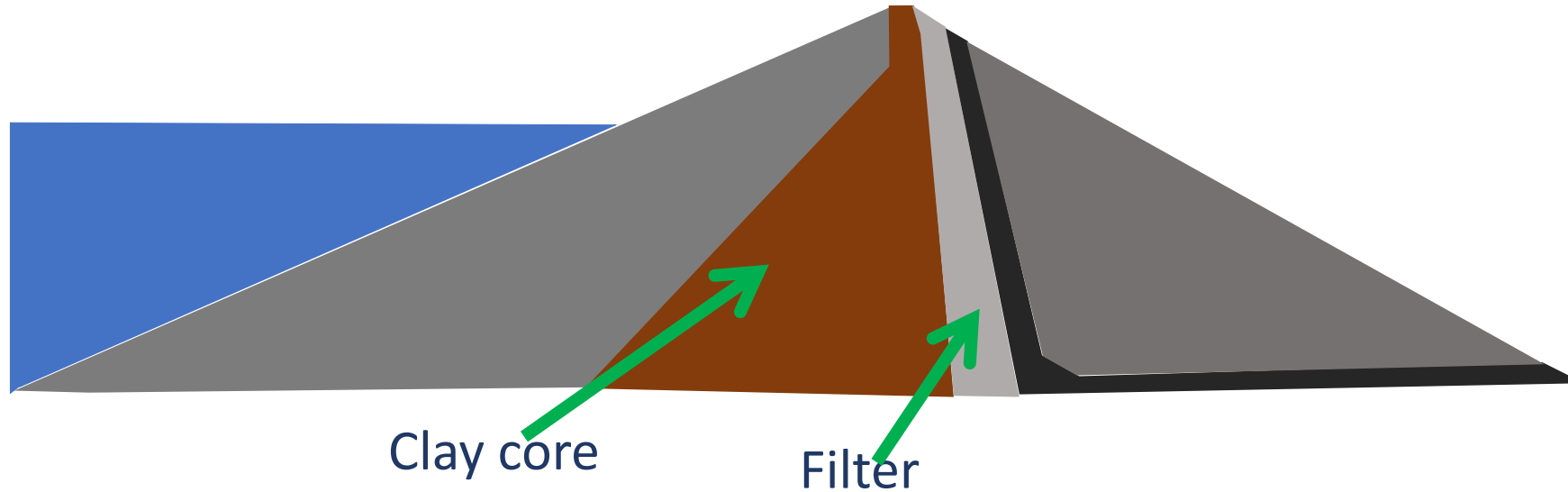
Application: Flood Embankments (Levees)

- Levees – transient water levels, but can be very long.
- Concerned about seepage through embankment and foundations.



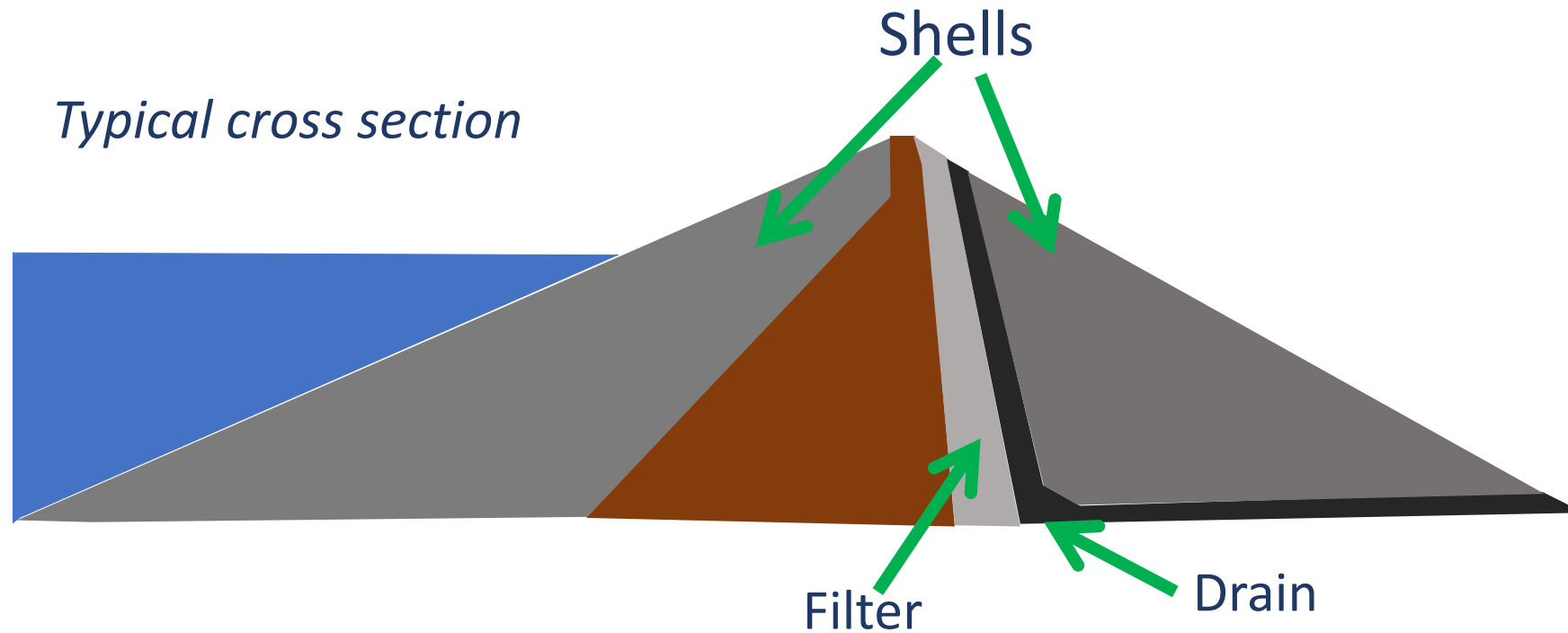
American River Levees California

Question 1



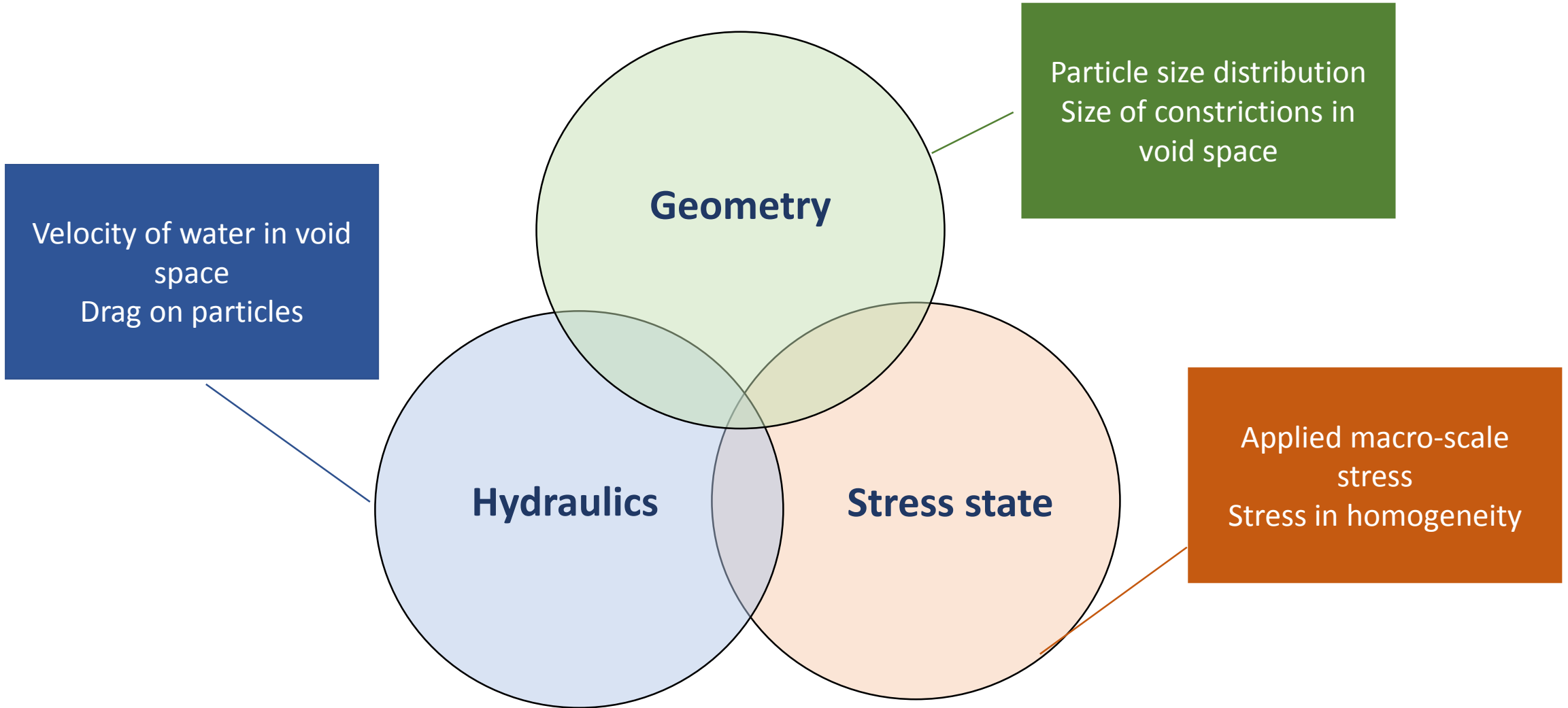
- Is the filter material compatible with the core material?
- Can the filter retain the fine particles in the core?

Question 2



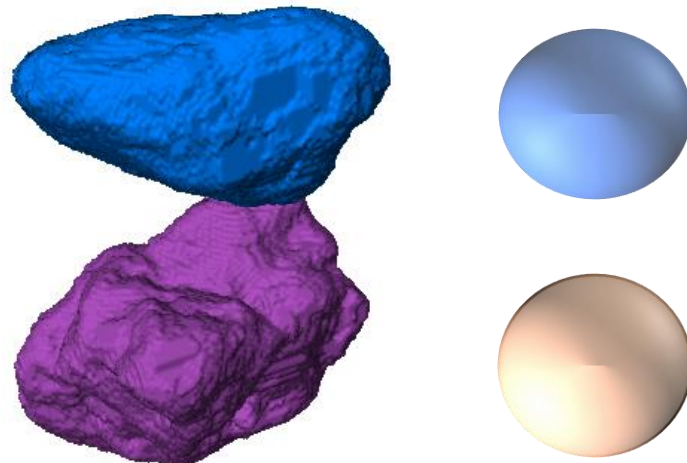
Are the filter, shells and drains internally stable?
Is there a risk of preferential migration of the finer grains?

Factors influencing erosion risk



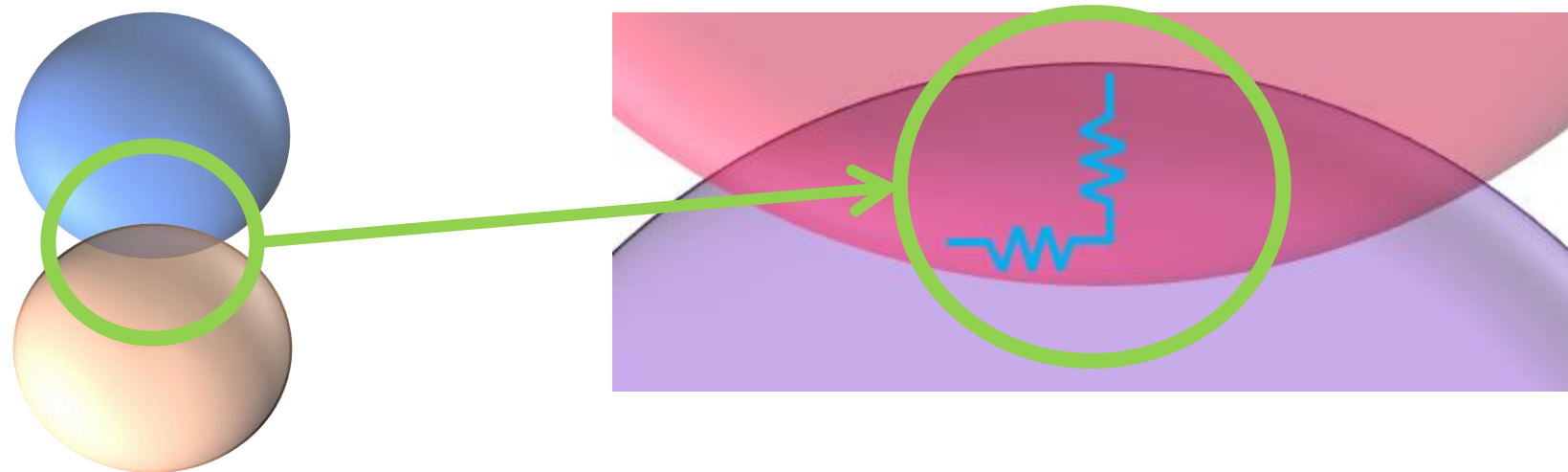
Discrete Element Method (DEM)

- Models grains as geometrically ideal rigid bodies
- Allows contacts to form and break
- Grain deformation modelled as contact overlap



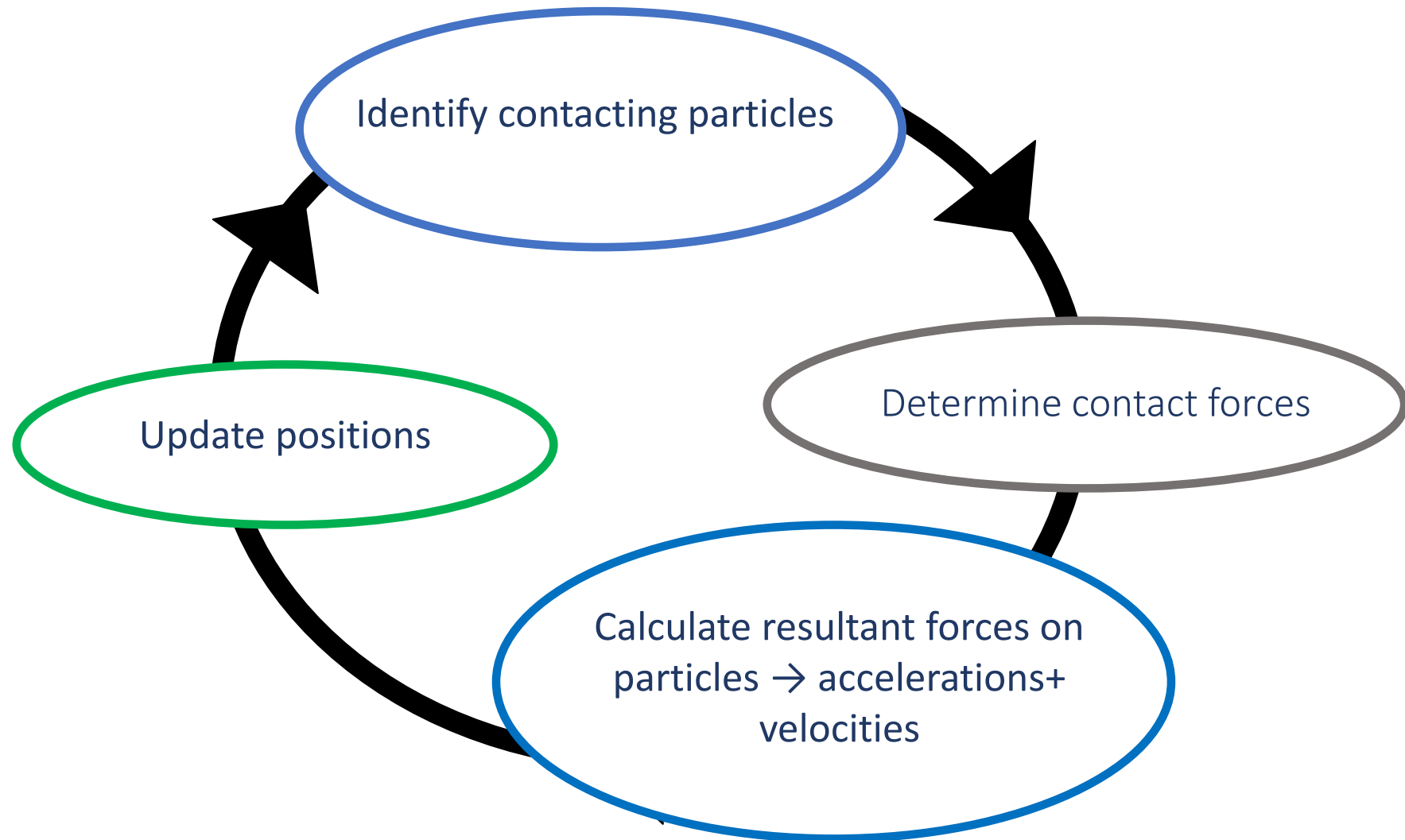
Discrete Element Method (DEM): Contacts

- Contact force calculated using orthogonal normal and shear springs
- Spring deformation calculated from relative motion of contacting particles
- Sliding governed by Coulomb friction : $T_{max} = \mu F_n$
- μ = coefficient of friction
- F_n = contact normal force
- T_{max} = maximum tangential (shear) force



Discrete Element Method (DEM): Calculation process

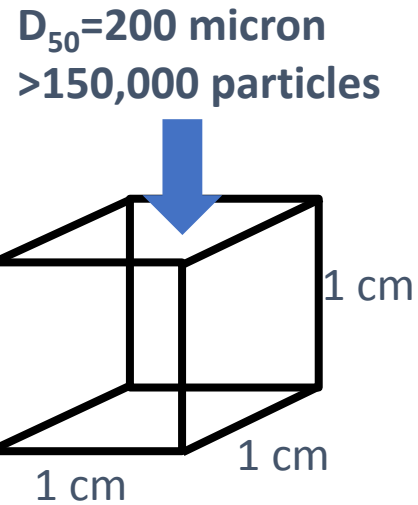
Transient
Calculation – In
Each Time Step:



Discrete Element Method (DEM): Computational cost

High performance computers enable larger samples to be simulated

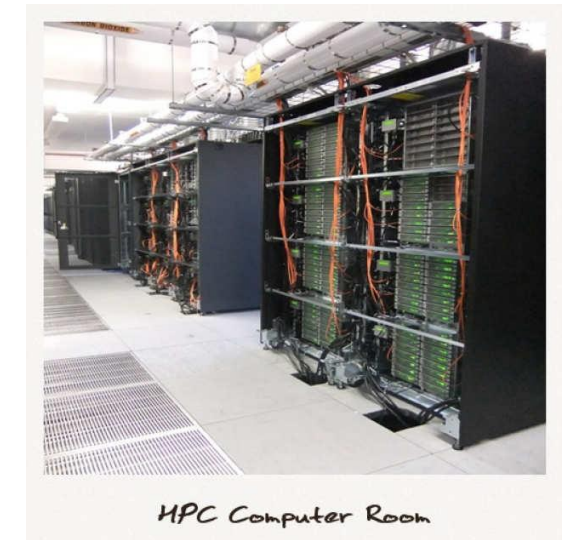
Need to consider large numbers of particles



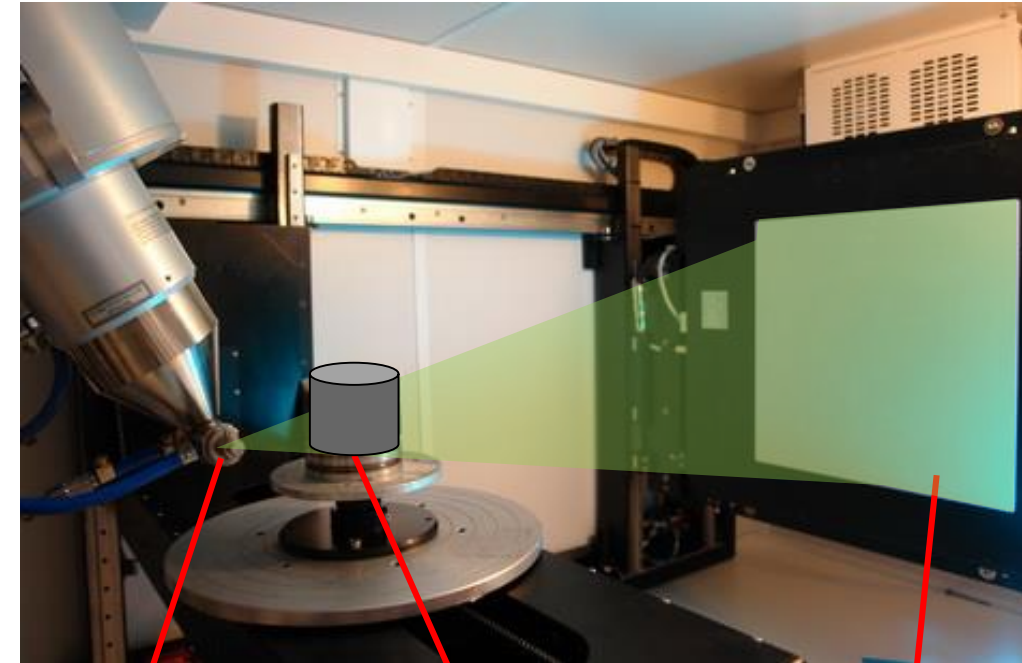
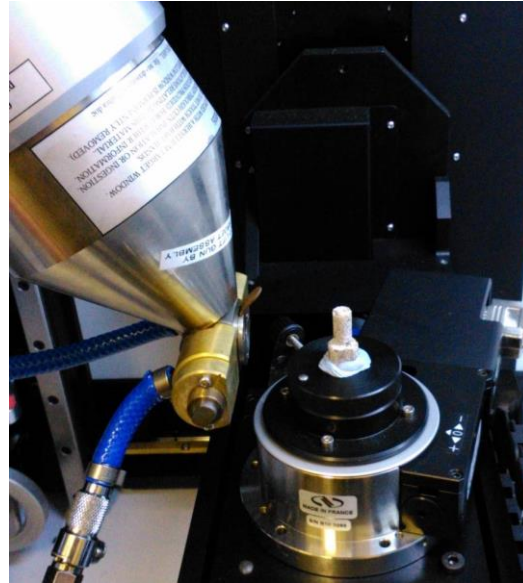
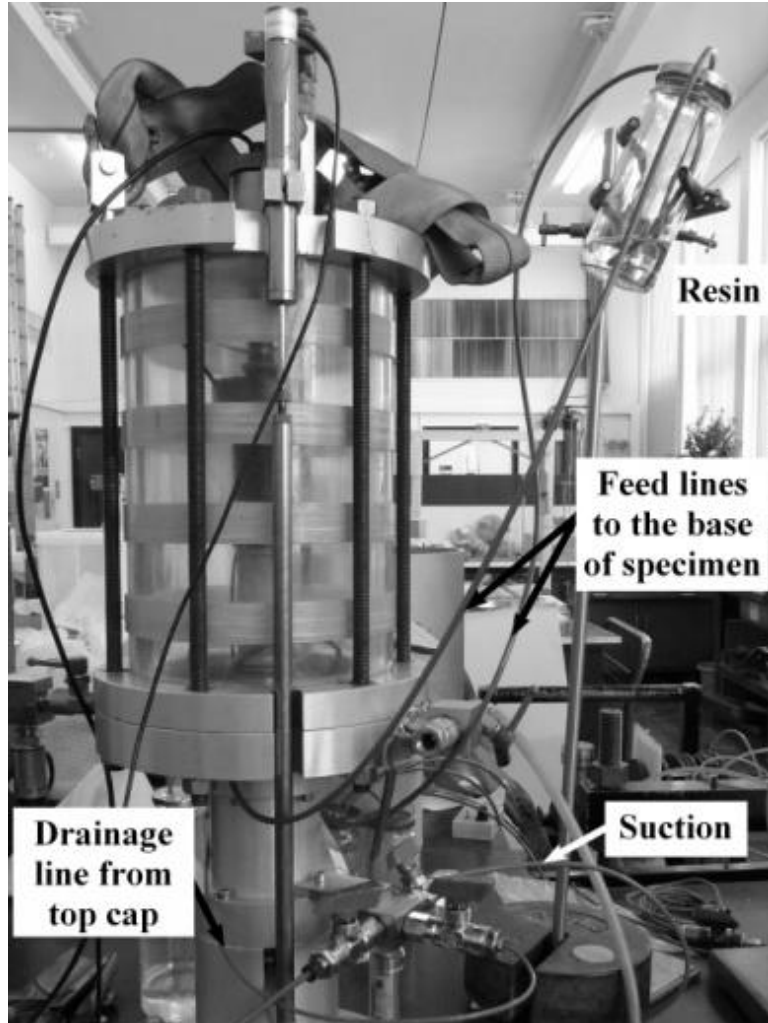
Explicit time integration is conditionally stable



High computational cost



Micro Computed Tomography (microCT)



Micro Computed Tomography (Micro CT)

Research has advanced fundamental understanding of:

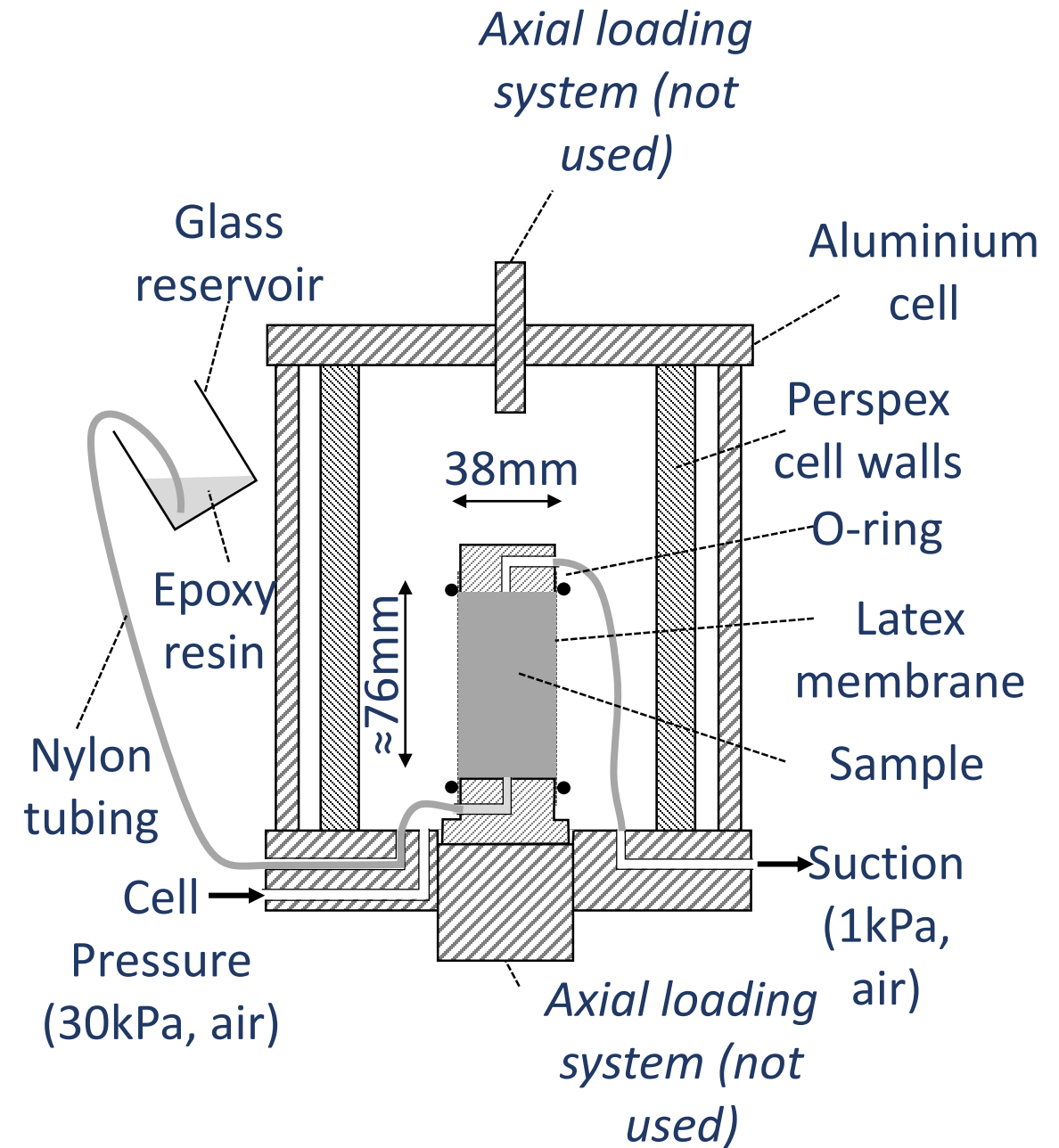
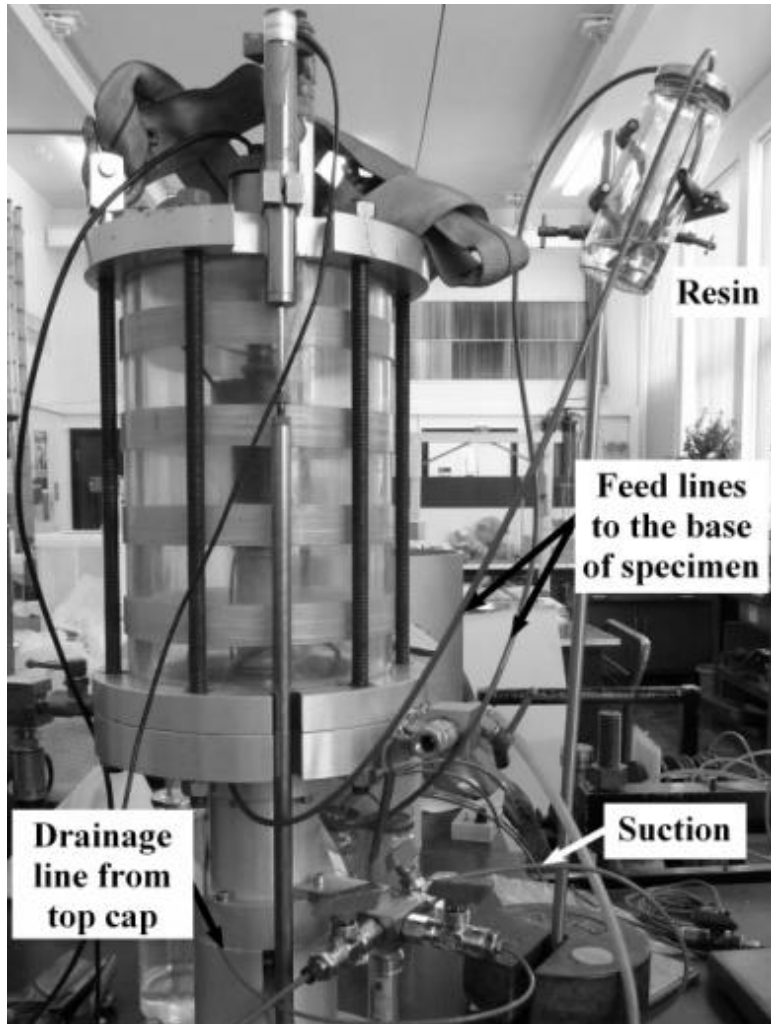
1. Permeability and drag
2. Filtration
3. Internal instability

Permeability and drag

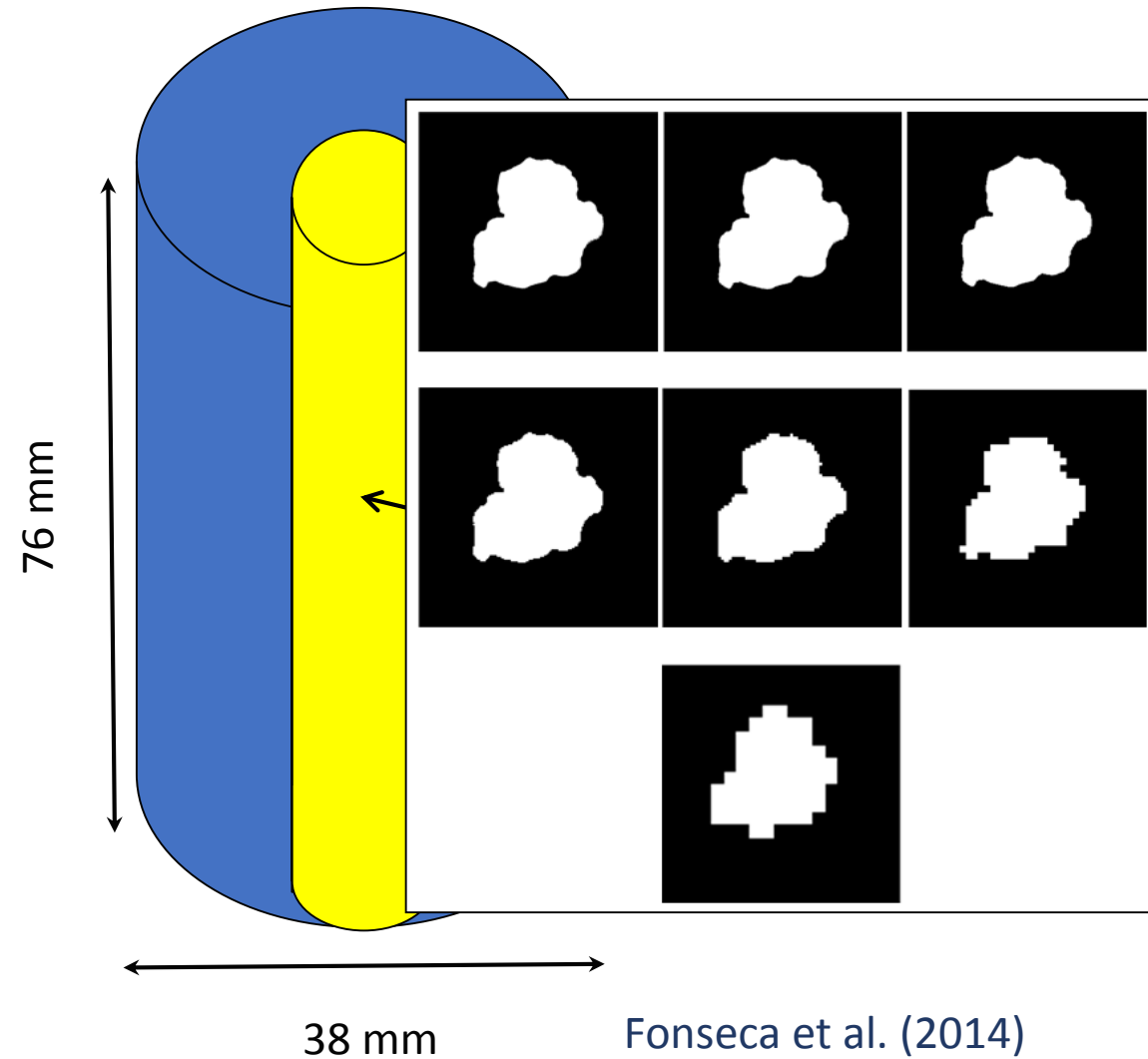
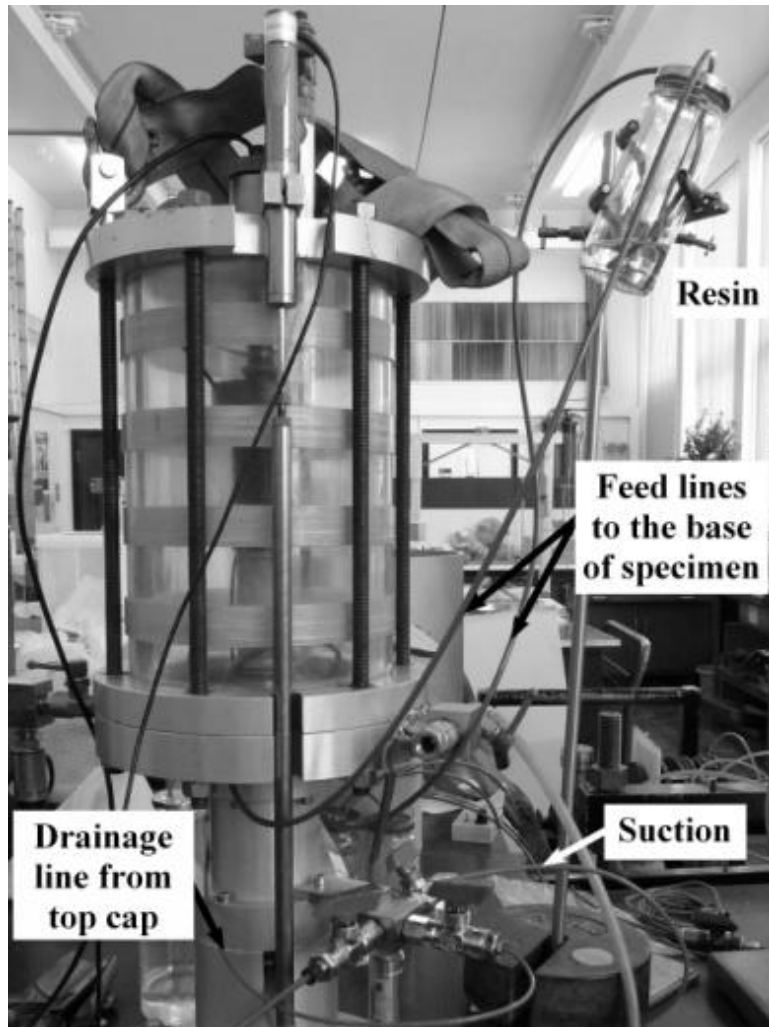
1. How do ideal models compare with actual migration of water through sand?
2. What is the drag on individual grains?

Focus on sub-particle scale

Experimental study



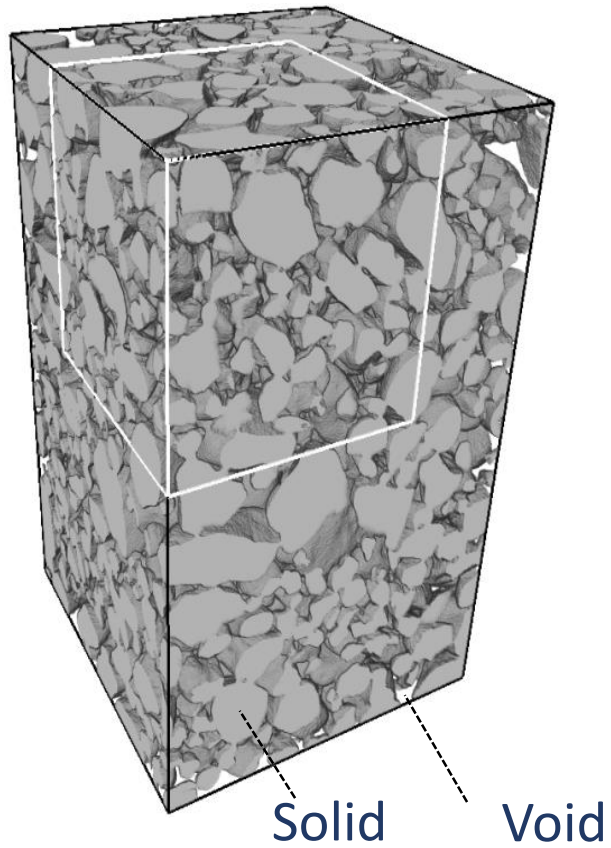
Experimental study



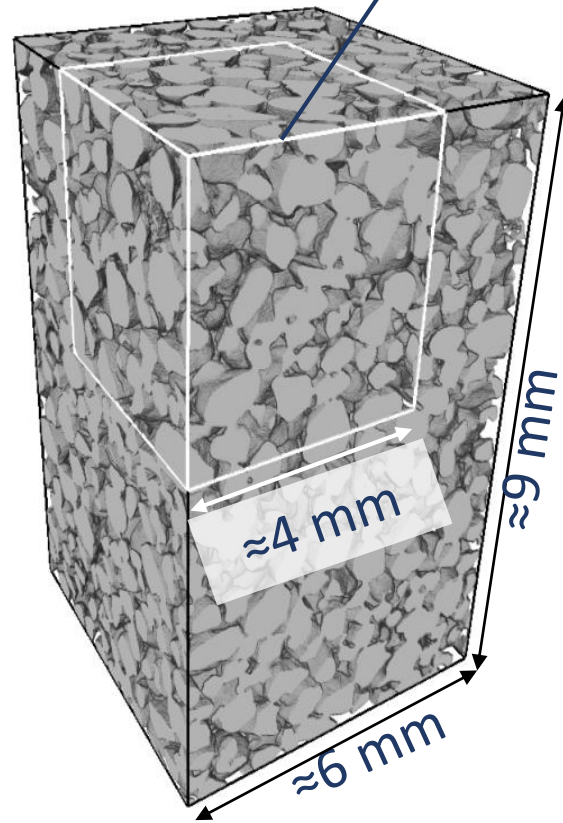
Fonseca et al. (2014)
Géotechnique

Experimental study - Materials

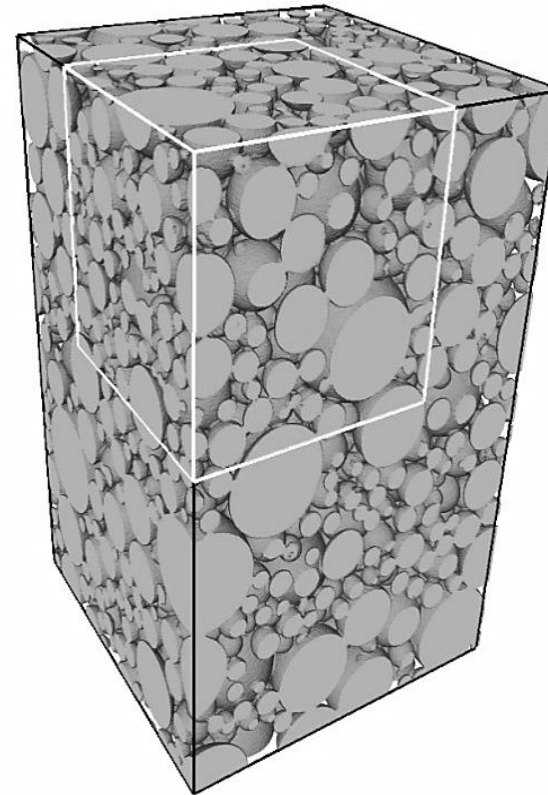
400 vox³ sub-volume used for
CFD analyses



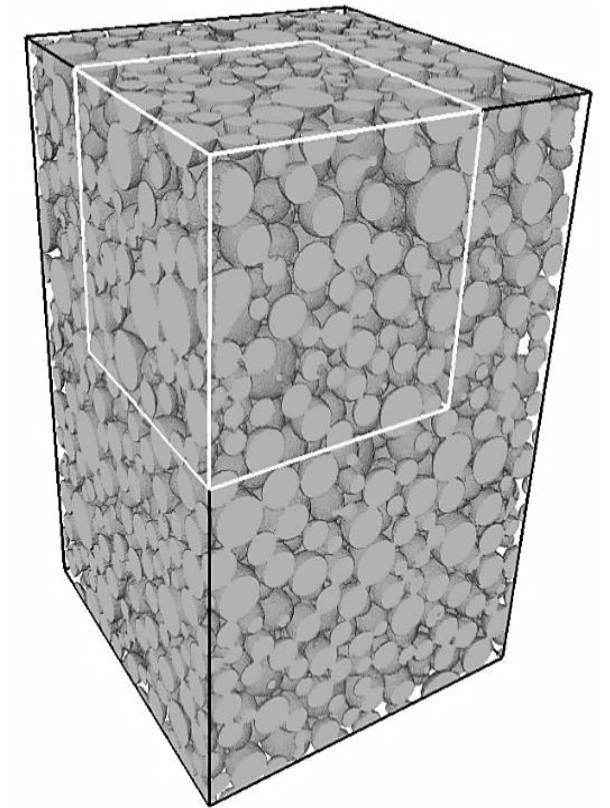
Sand-Cu3



Sand-Cu1.5



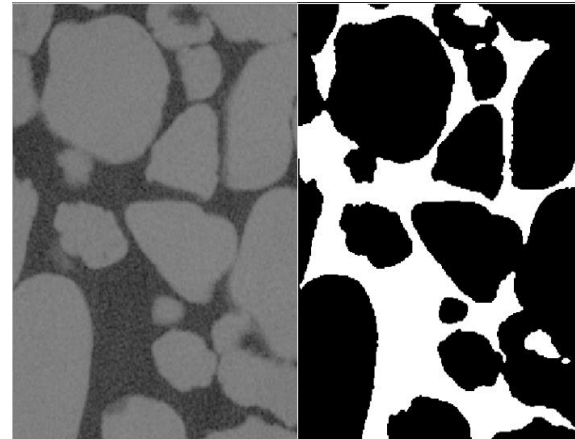
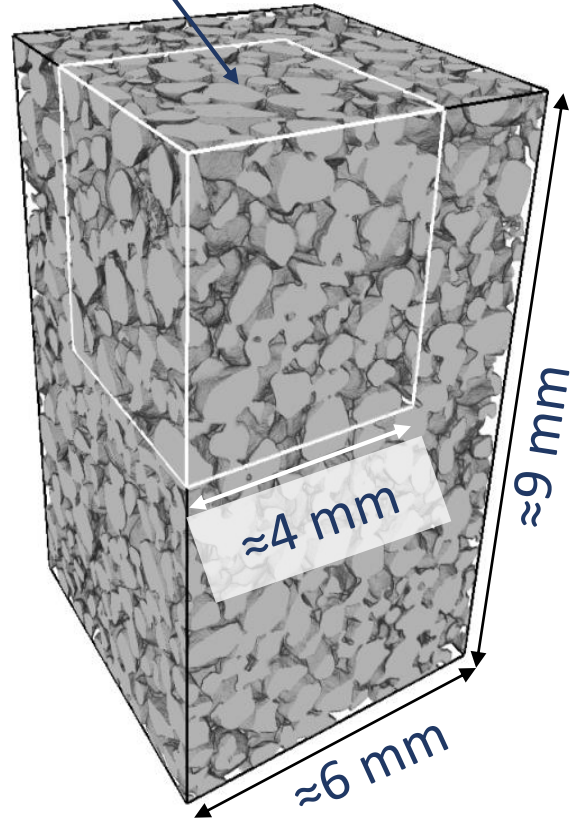
Beads-Cu3



Beads-Cu1.5

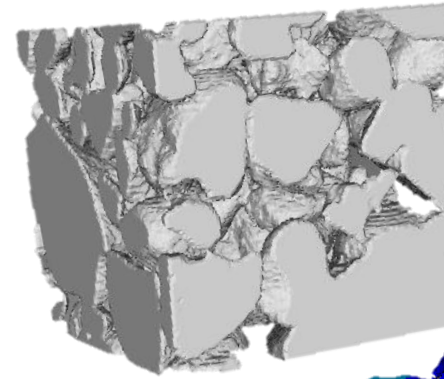
Fluid flow simulations

Sub-volume for
CFD analyses

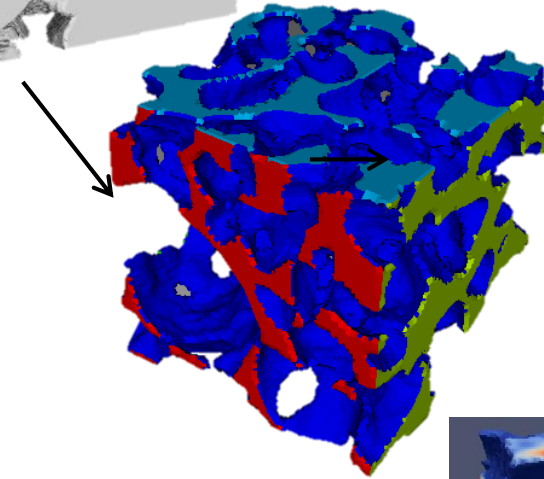


2D Slice from
 μ CT image

Binary image

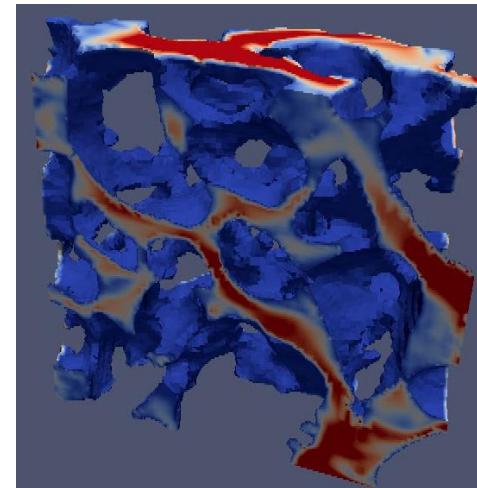


Micro-CT
binary image



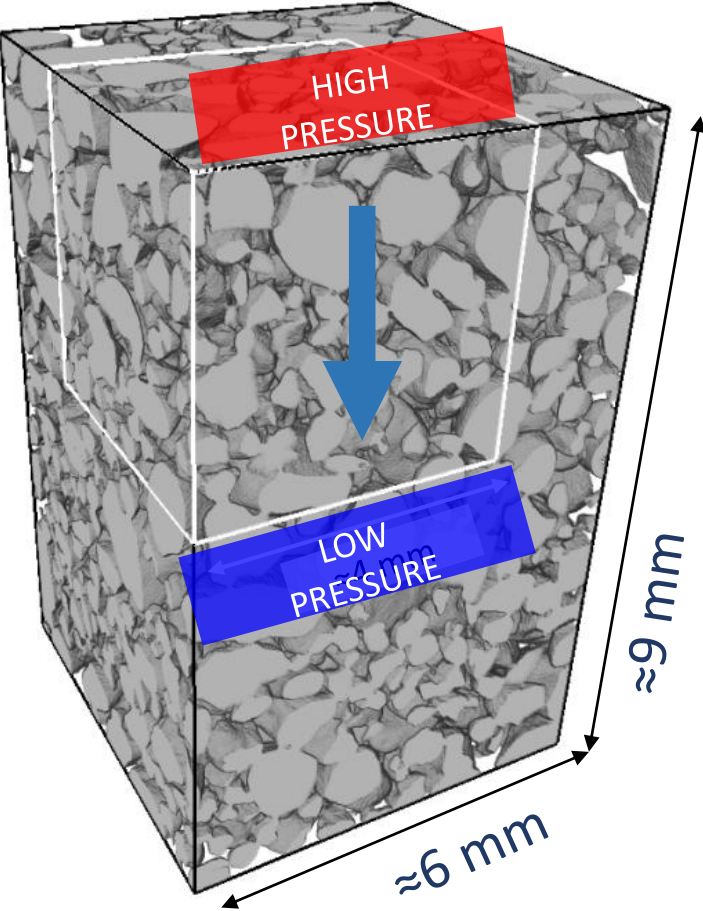
Finite volume
mesh

OpenFOAM
simulation

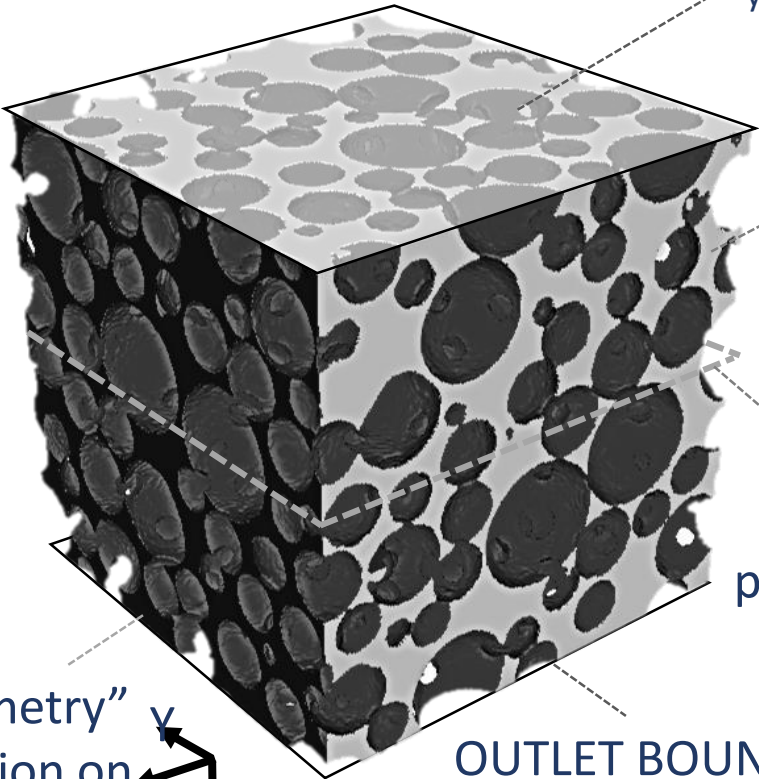


Fluid flow simulations

micro-CT
image (Cu3)



CFD Analysis



INLET BOUNDARY:

$$P_{in} = 0.001 \text{ kPa}$$
$$V_x = 0$$
$$V_y = 0$$

"No slip"
condition
on particle
surfaces

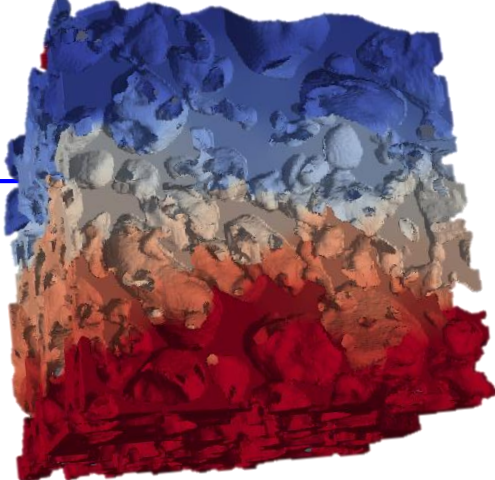
Slice,
perpendicular
to flow
direction

OUTLET BOUNDARY:

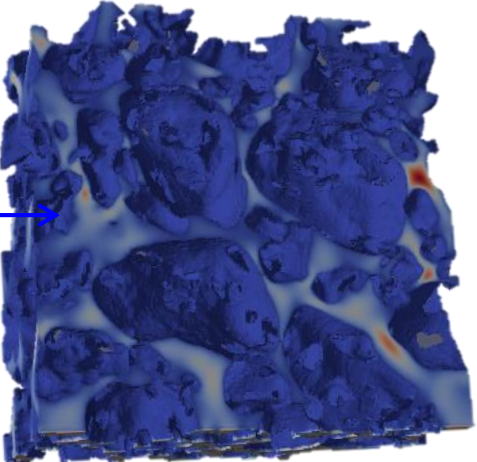
$$P_{out} = 0 \text{ kPa}$$
$$V_x = 0$$
$$V_y = 0$$

CFD output
18-24hrs

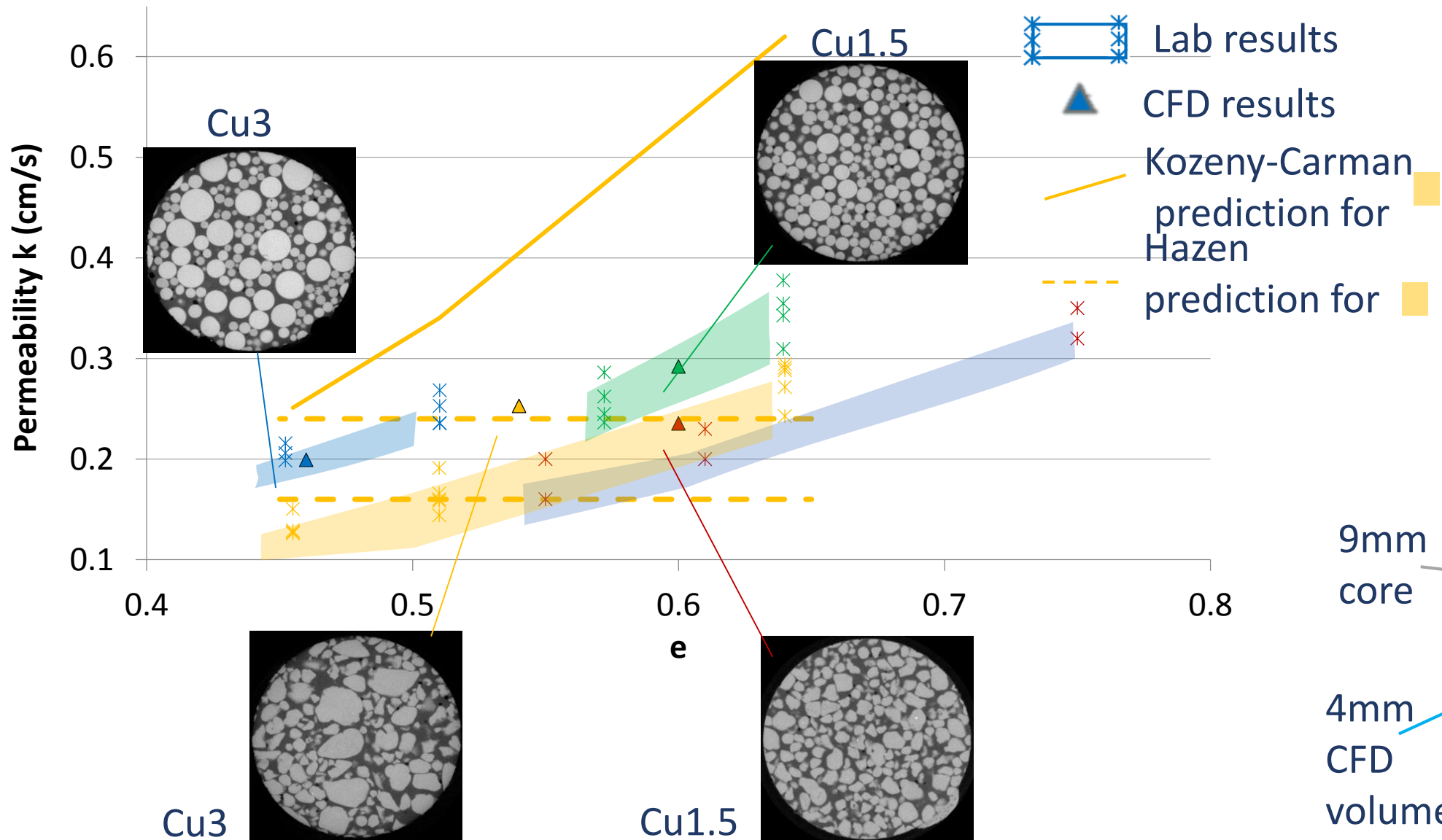
Pressures



Velocities



Comparison of CFD and permeameter data

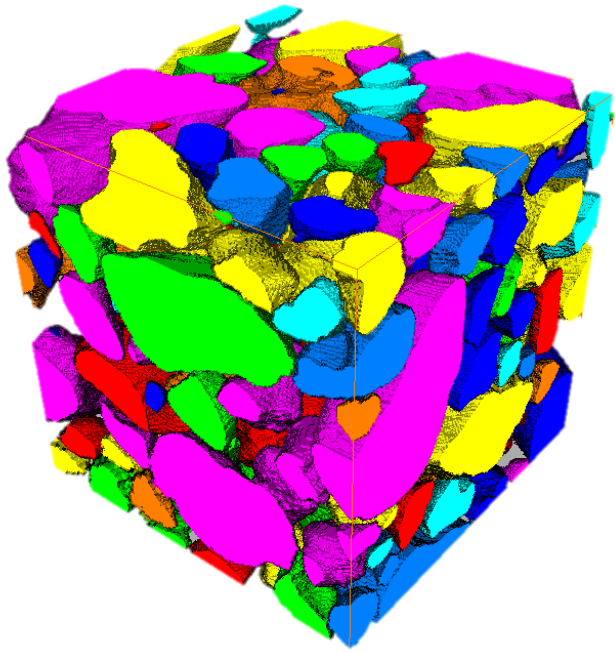


75mm permeameter

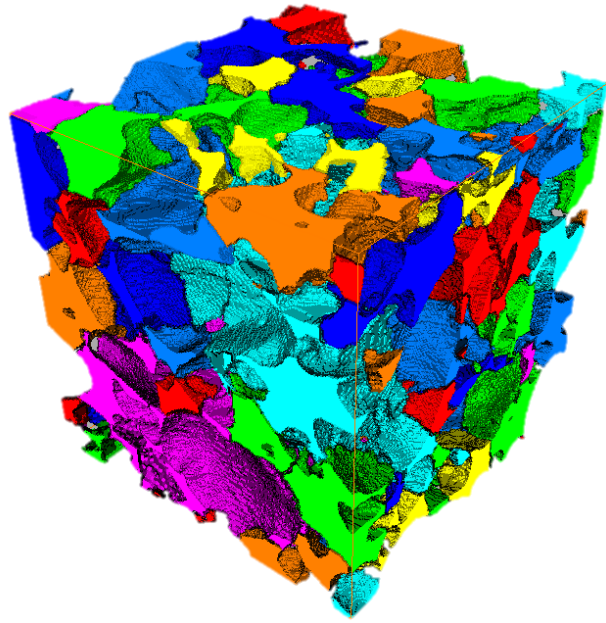


9mm core
4mm CFD volume
38mm triaxial

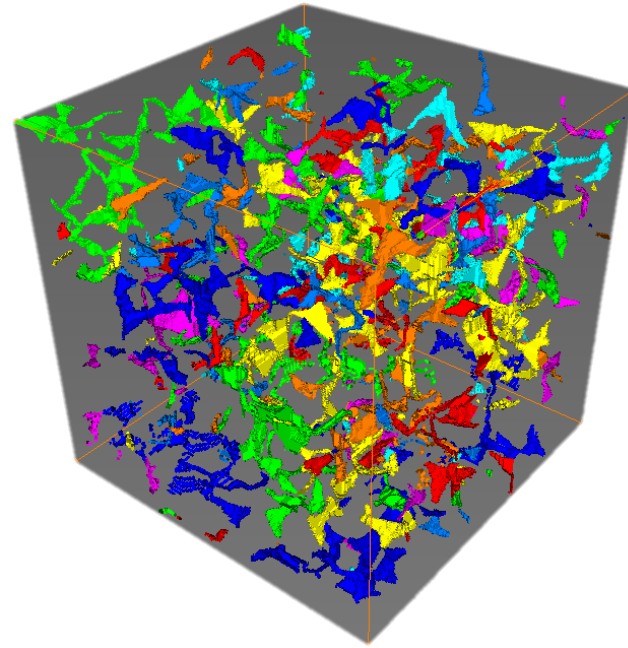
MicroCT: Constrictions in void network – geometrical identification



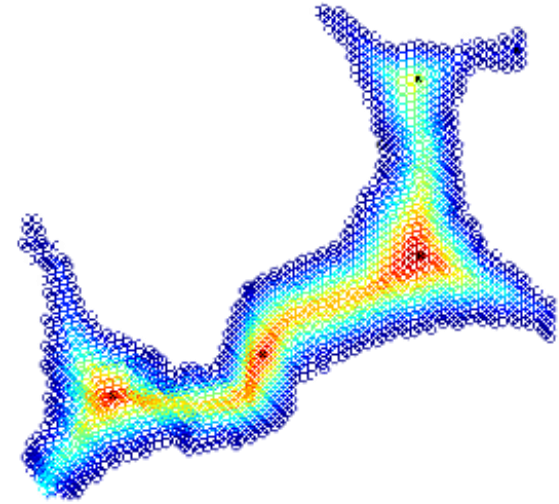
Particles



Voids

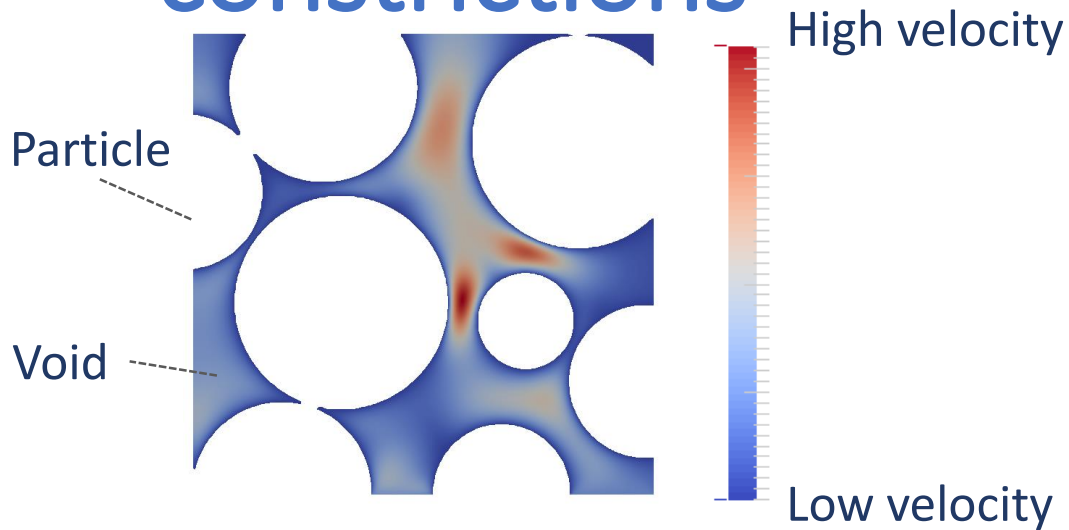


Void Boundaries

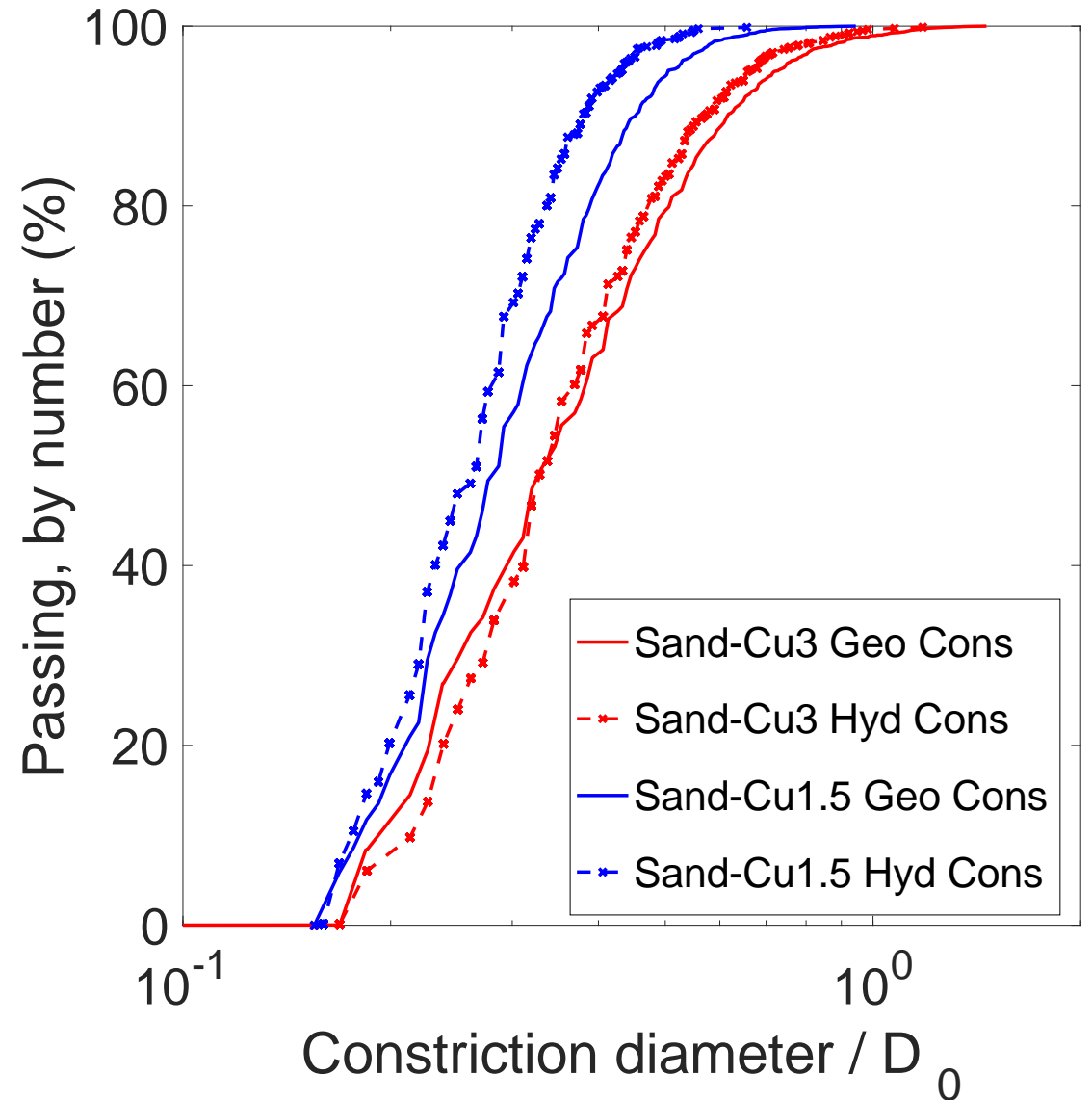
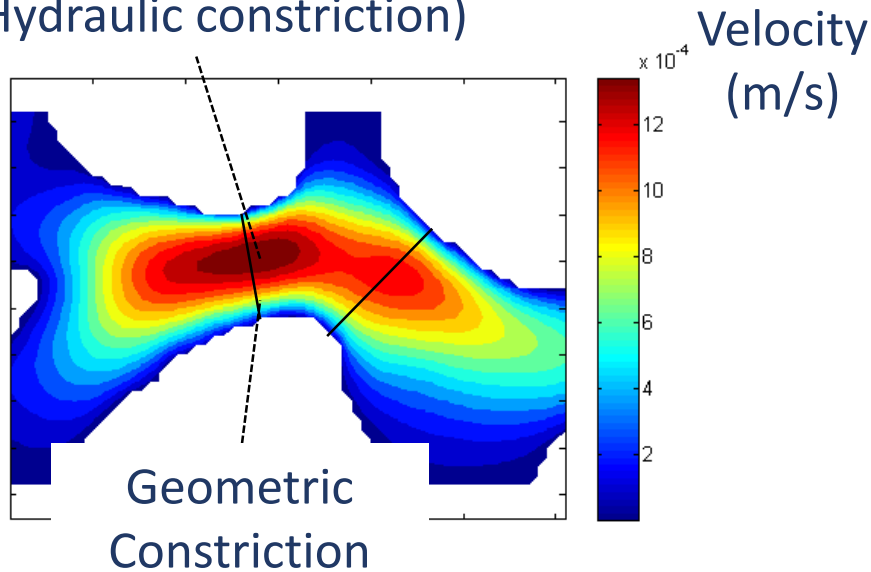


Constrictions local maxima of distances to particles

Comparison of geometric and hydraulic constrictions



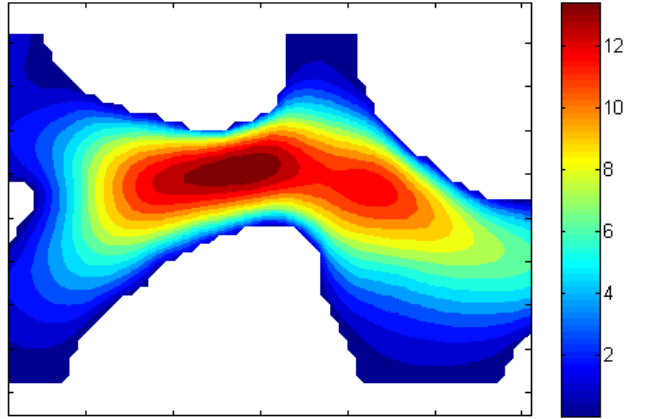
Velocity maximum
(= Hydraulic constriction)



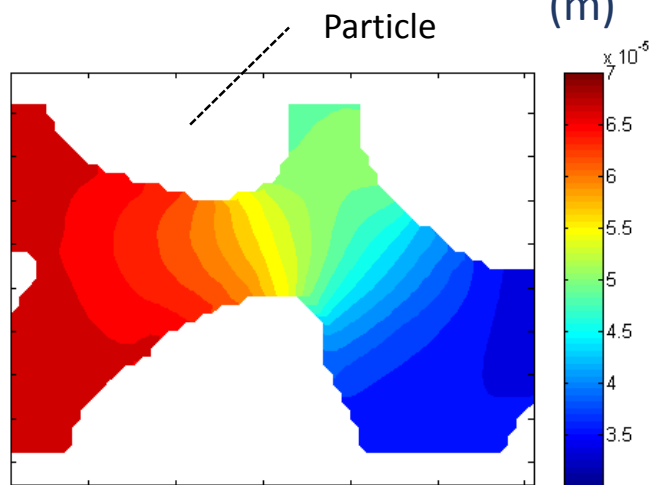
Taylor et al. (2017)

Headloss and streamlines

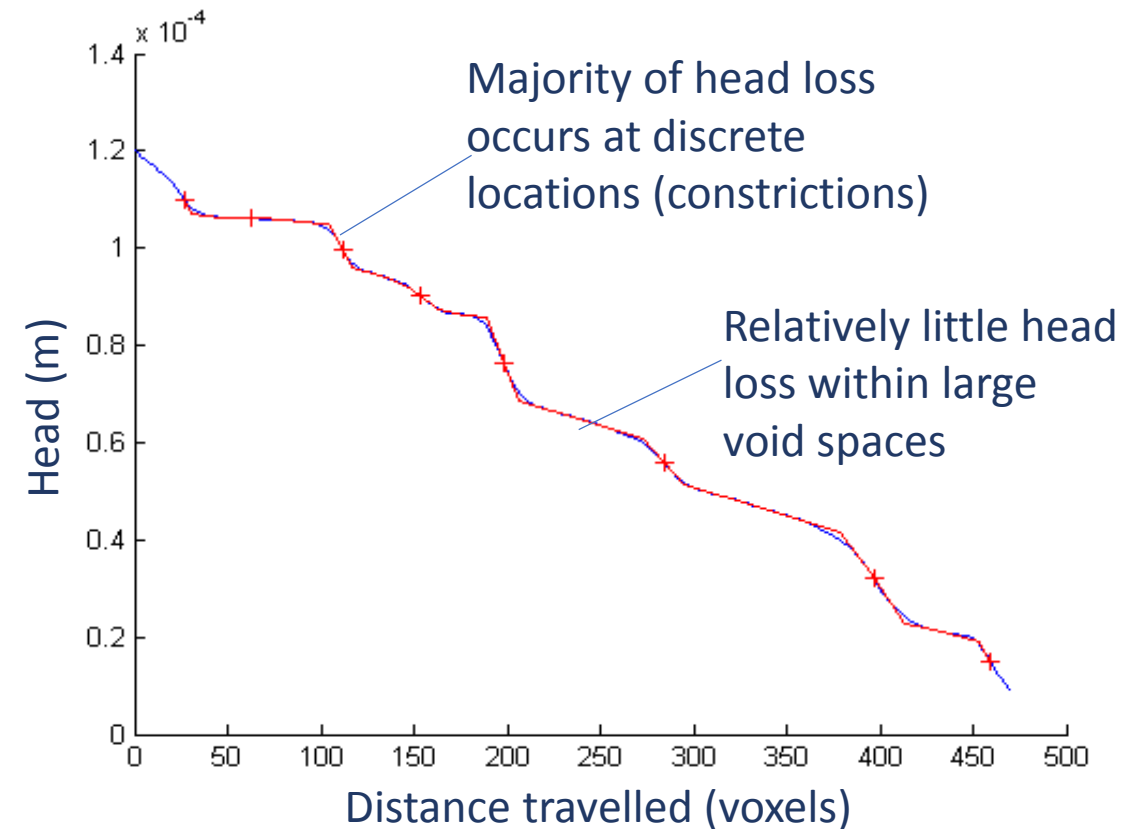
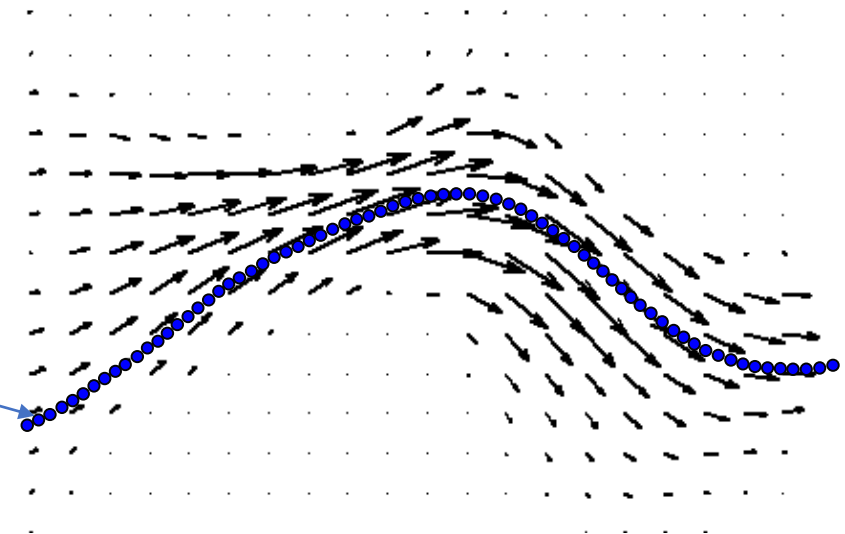
Velocity
(m/s)
 $\times 10^{-4}$



Head
(m)
 $\times 10^{-5}$



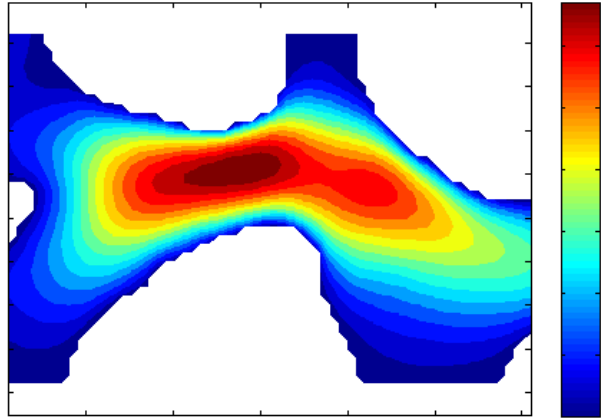
Streamline



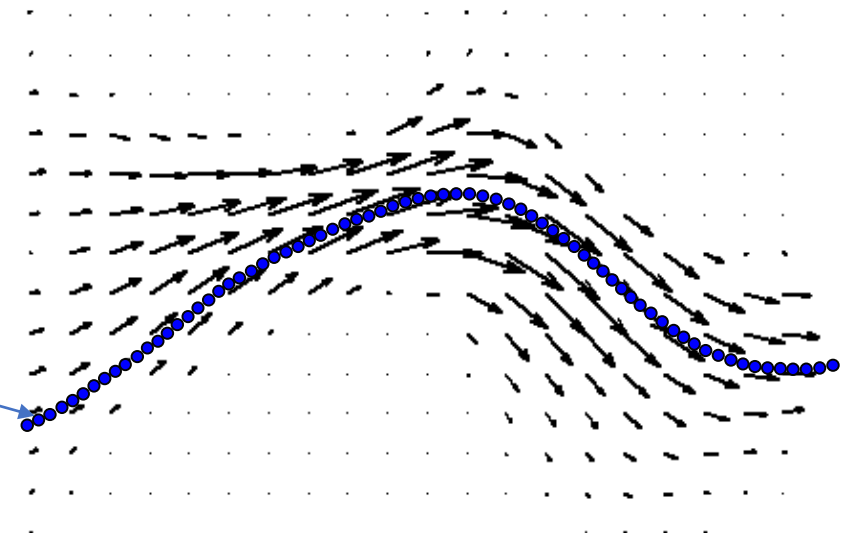
Headloss and streamlines

Velocity
(m/s)

$\times 10^{-4}$

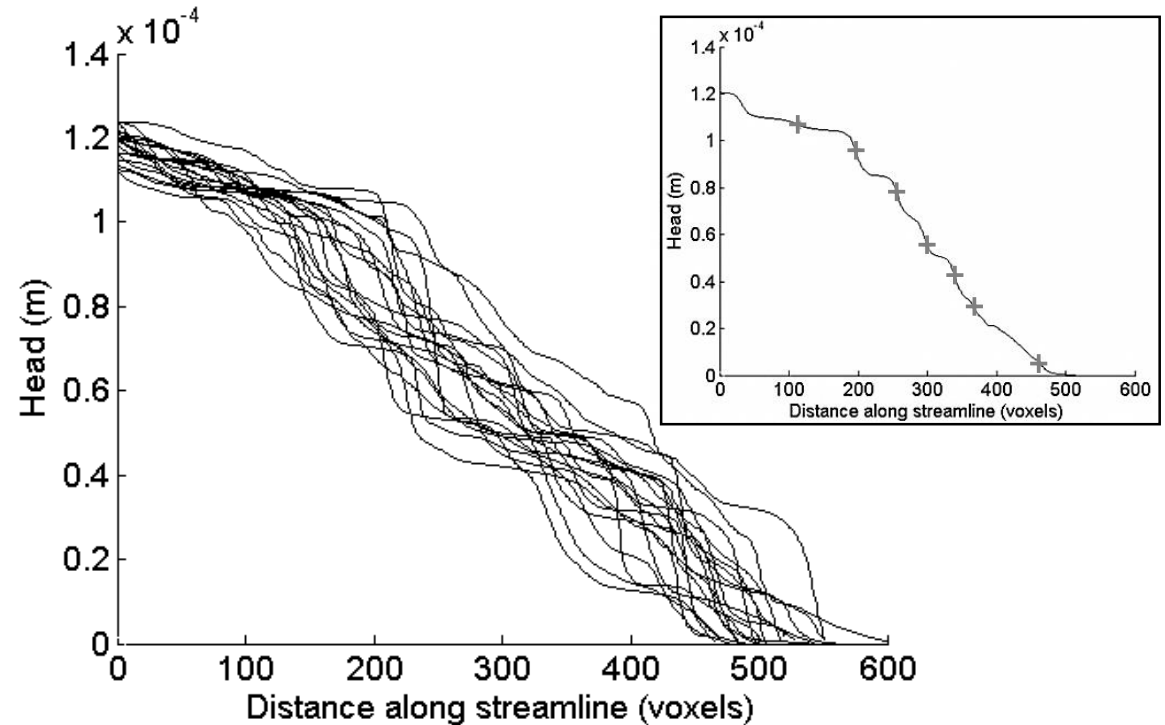
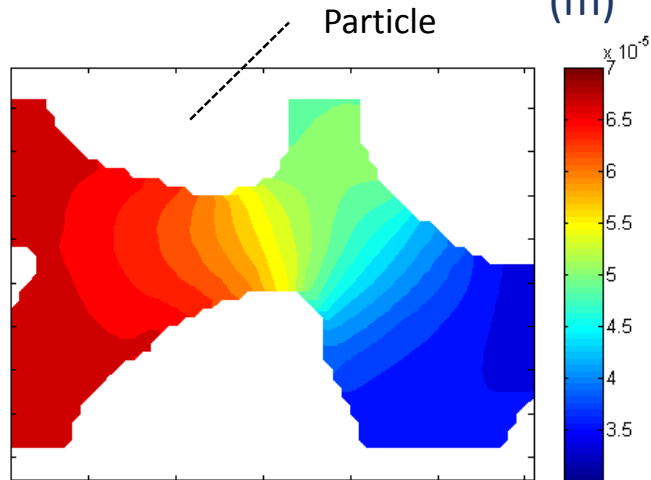


Streamline



Head
(m)

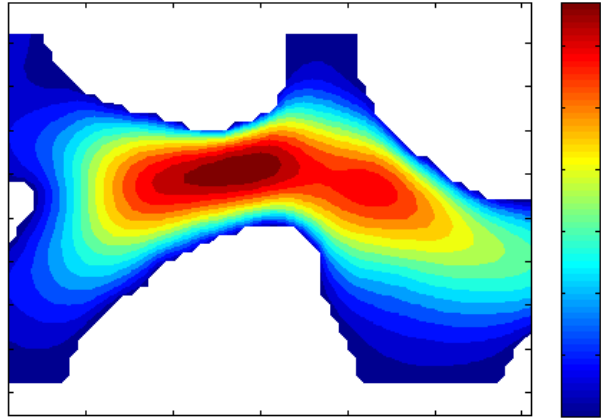
$\times 10^{-5}$



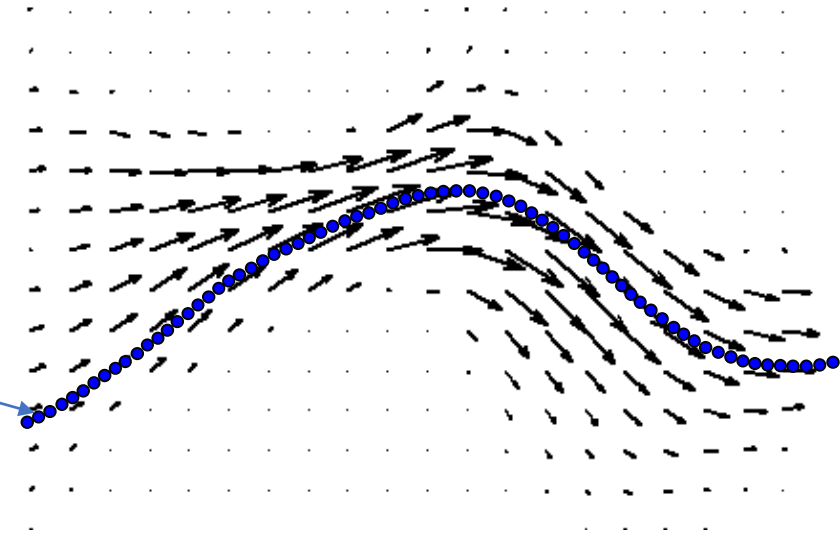
Headloss and streamlines

Velocity
(m/s)

$\times 10^{-4}$

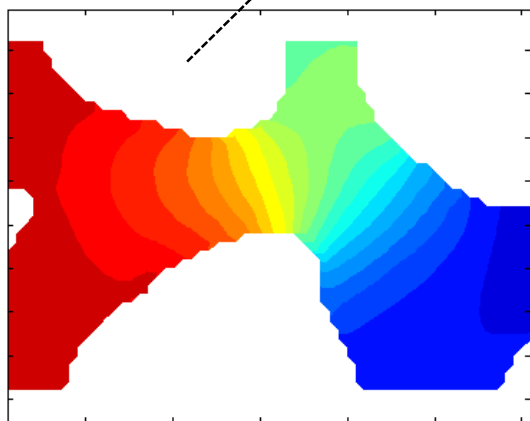


Streamline



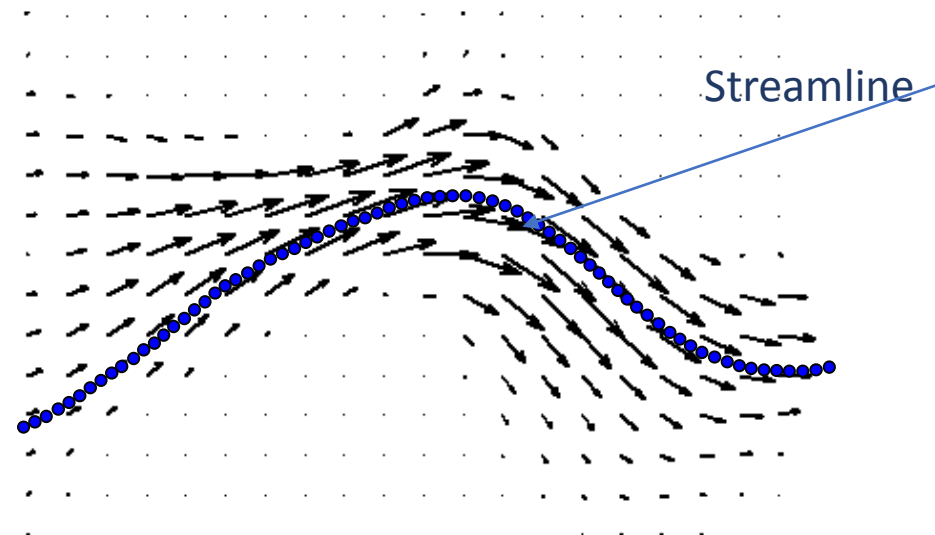
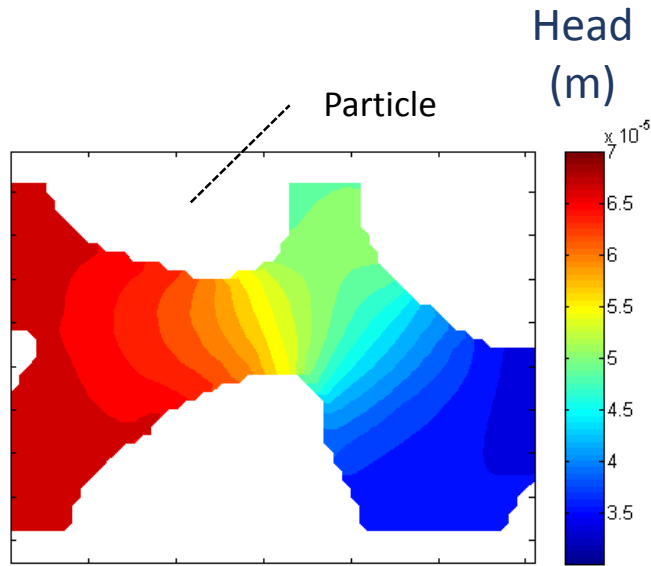
Head
(m)

$\times 10^{-5}$



Material	Proportion of head loss in constrictions	Proportion of length in constrictions
	MEAN (STANDARD DEV.)	
Sand-Cu3[1]	77% (12%)	37% (8%)
Sand-Cu3[2]	77% (11%)	37% (8%)
Sand-Cu1.5	76% (12%)	37% (8%)
Beads-Cu3	77% (12%)	39% (8%)
Beads-Cu1.5	77% (11%)	39% (7%)

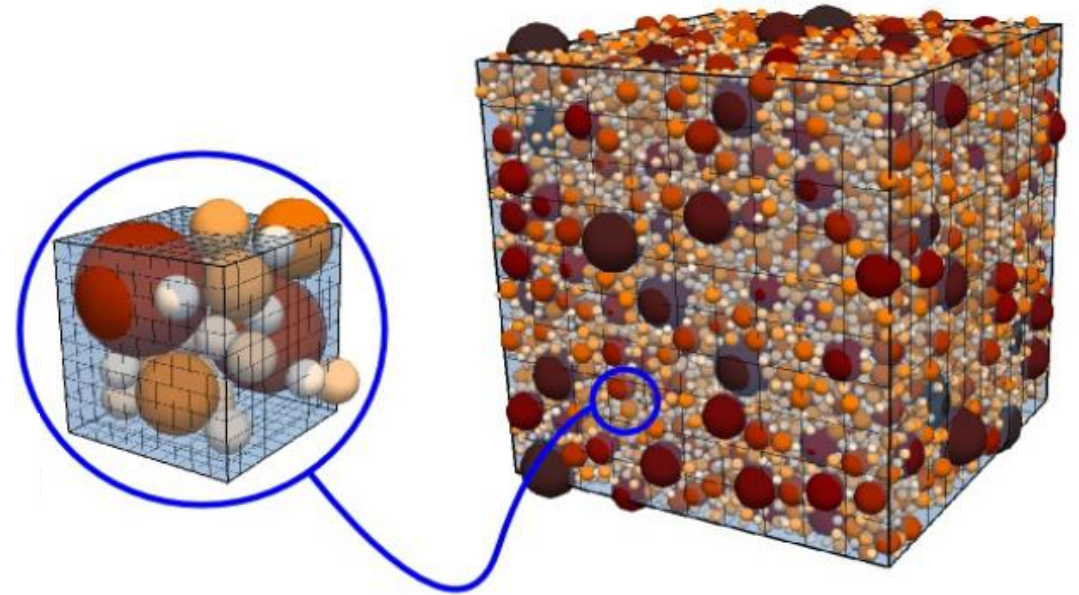
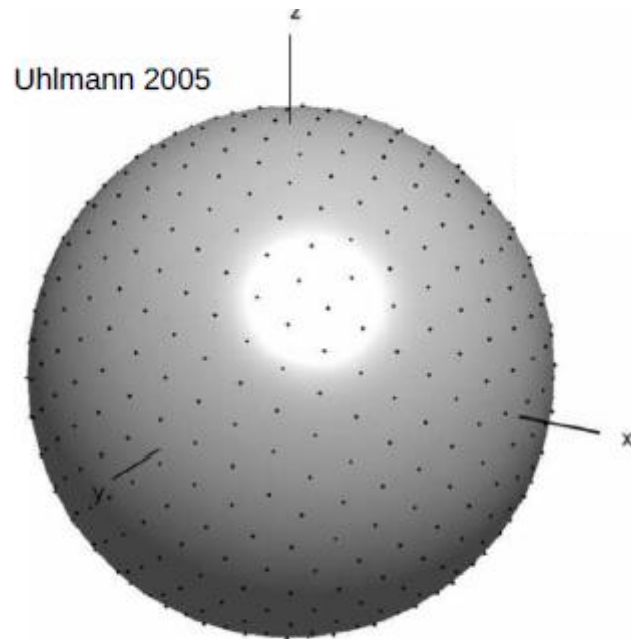
Headloss and streamlines



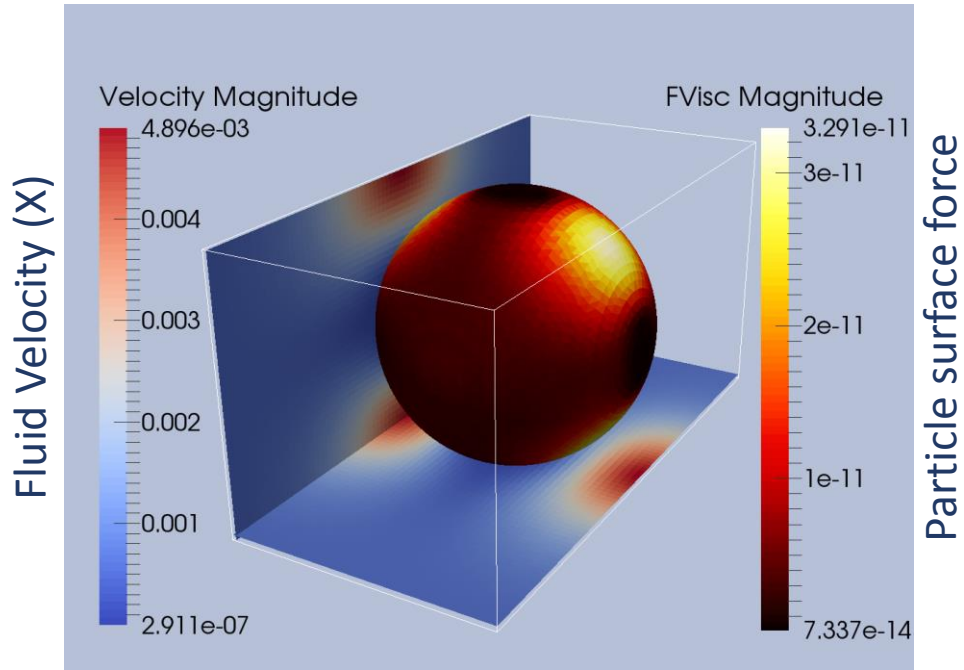
- Sub particle scale modelling of fluid flow has confirmed that constrictions play a key role in determining permeability and local flow velocities
- Permeability and relative permeability influences local velocities within embankment structure
- Velocity and head gradient determine the hydraulic force impacted on particles

Immersed boundary Method (IBM)

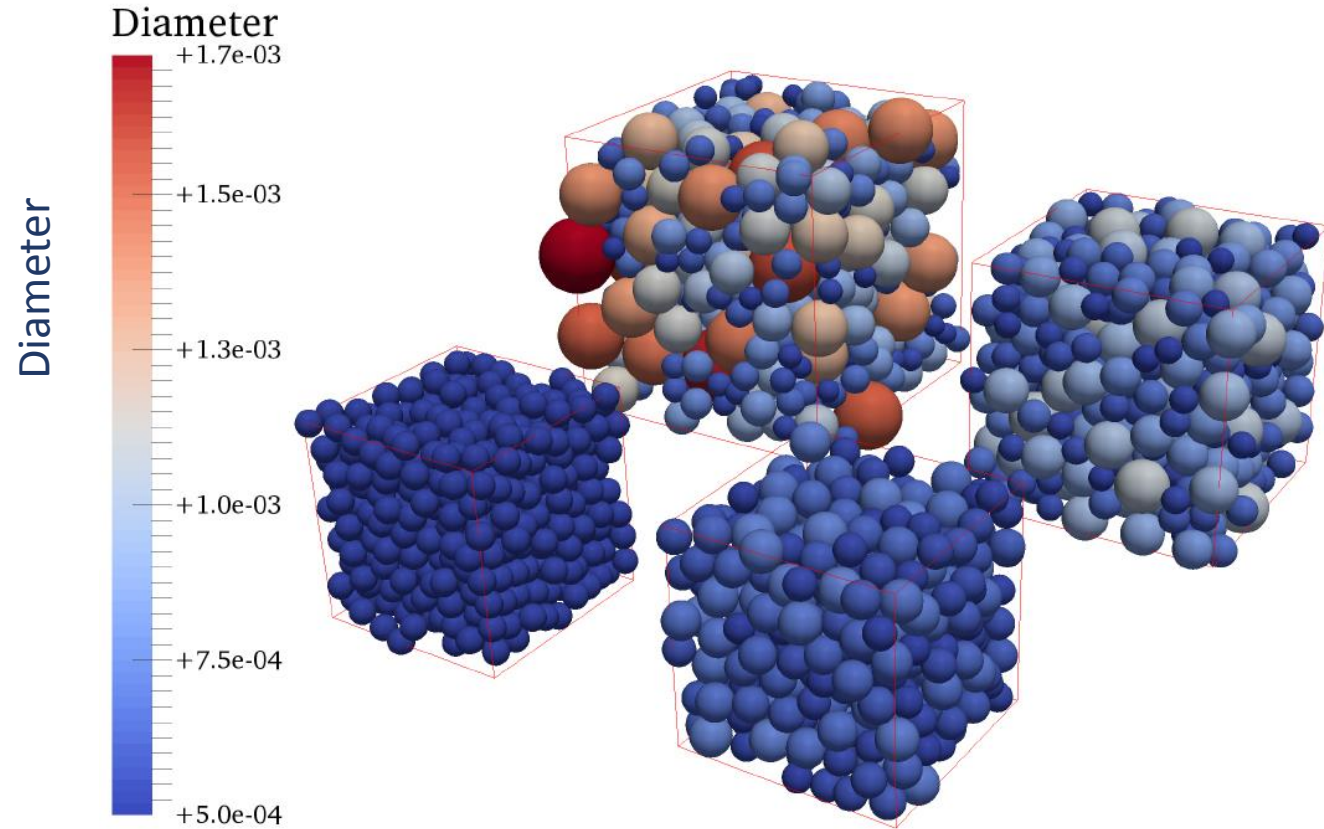
- No-slip no-penetration condition imposed at surface points on immersed objects by applying forces to fluid in neighbouring region.
- Code developed by Prof. Berend van Wachem, Mech. Eng. Imperial / Universität Magdeburg, Germany



Immersed boundary Method (IBM)



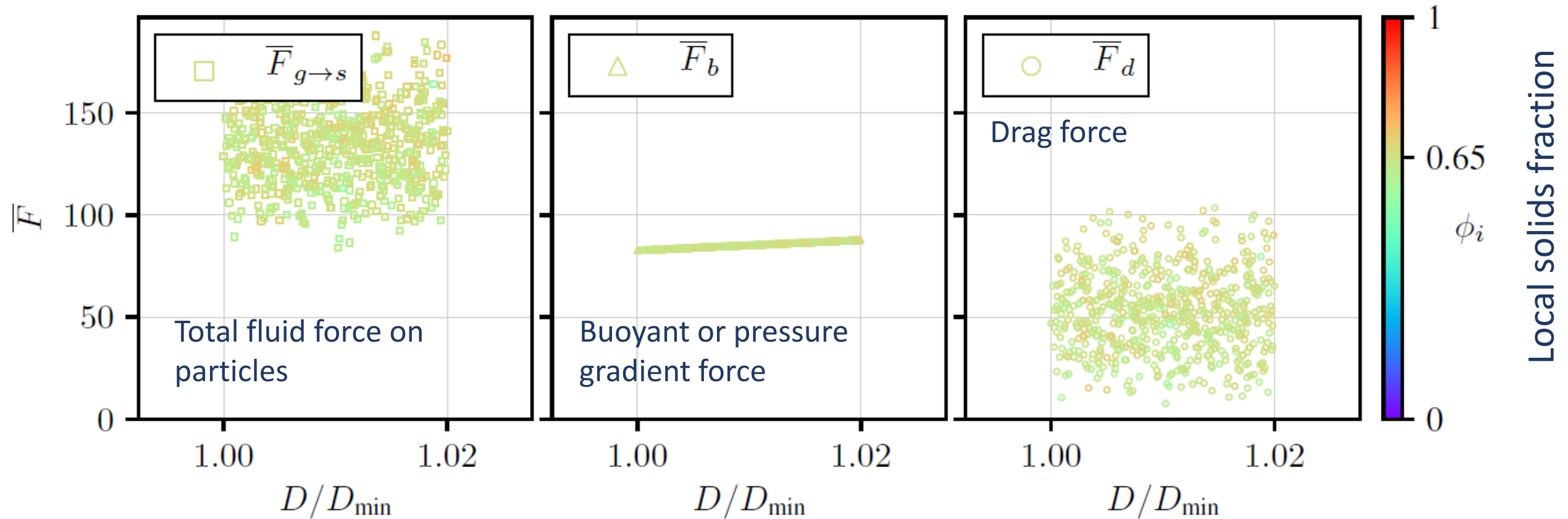
Sub-fluid scale resolution gives detail on pressure on particle surface



Computational cost restricts to small samples and small Cu values (< 2.5)

Fluid forces on particles

Forces are normalized by Stoke's drag force to give \bar{F}

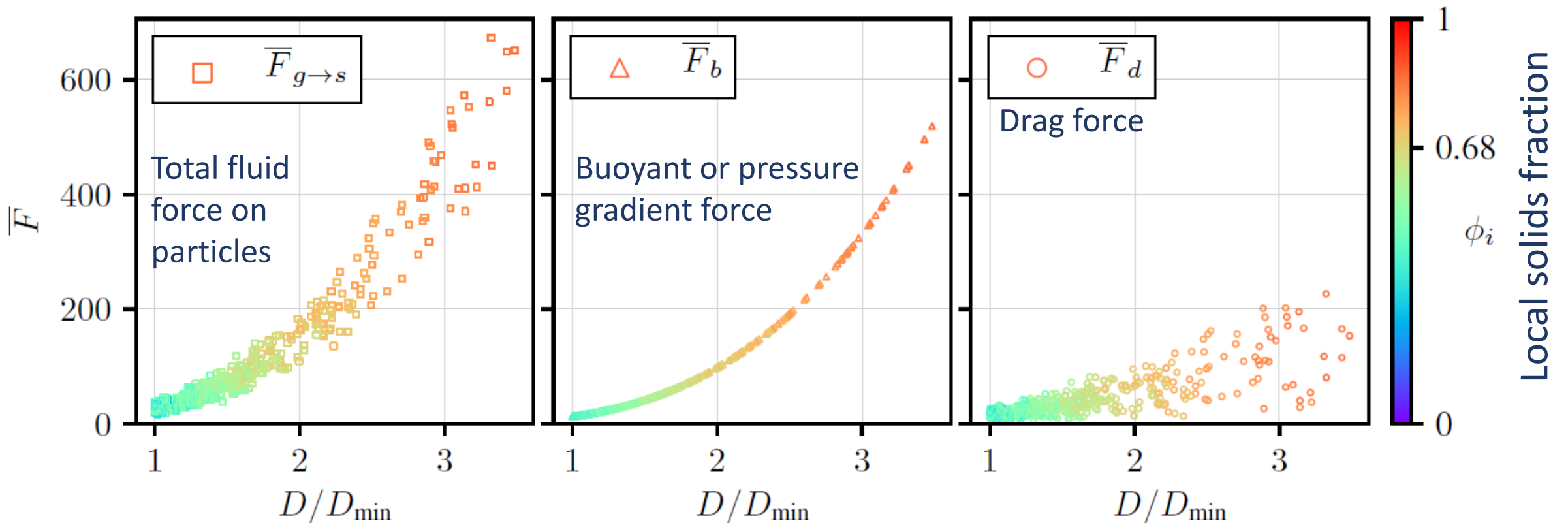


$Cu = 1.01$; void ratio = 0.536

Fluid forces on particles

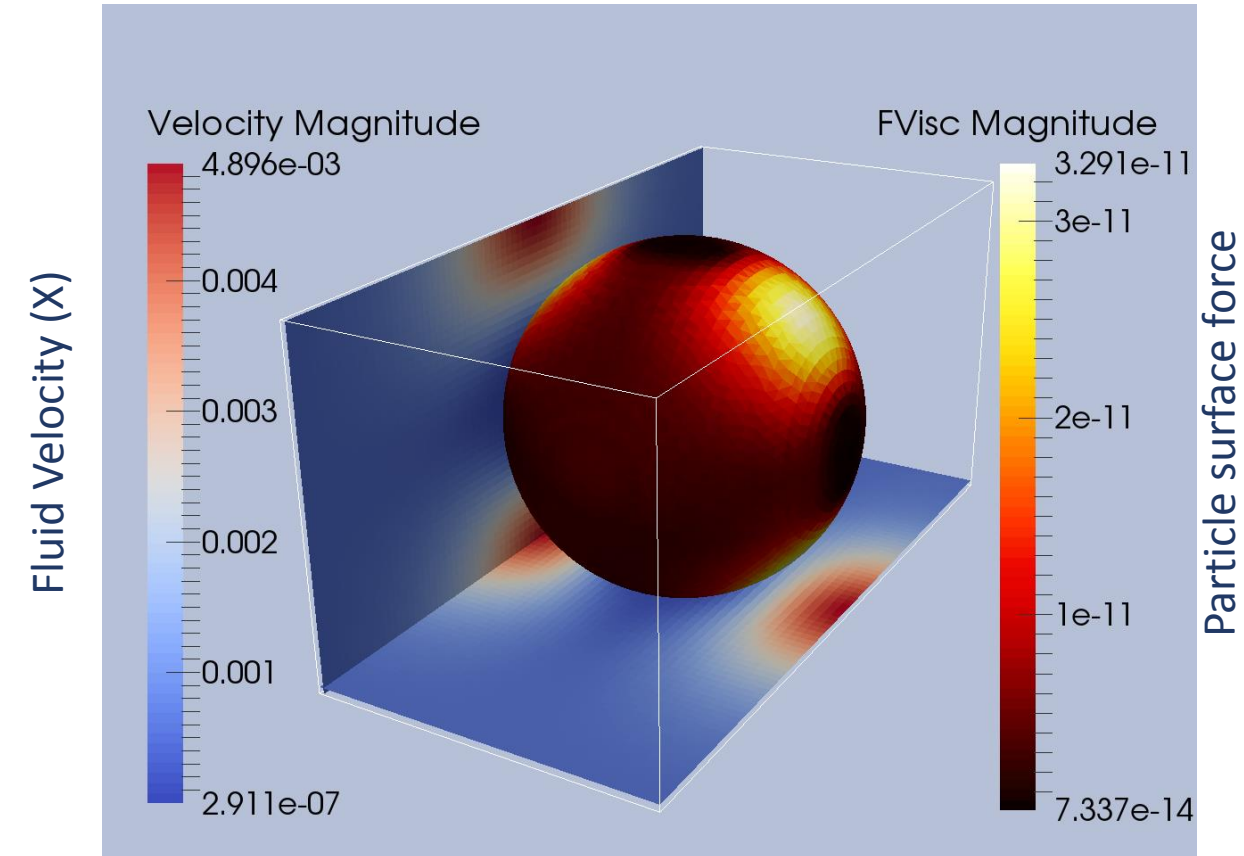
Stoke's drag: $F_{\text{stoke}} = 6\pi\eta Rv$
 η = viscosity
 R = particle radius
 V = relative velocity

Forces are normalized by Stoke's drag force to give \bar{F}



$Cu = 2.0$; void ratio = 0.468

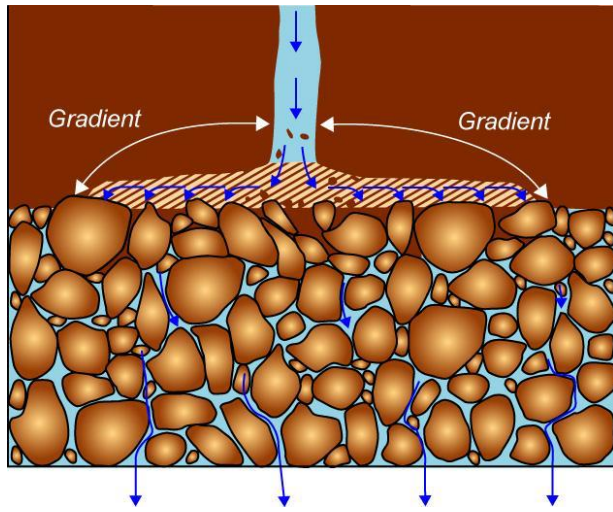
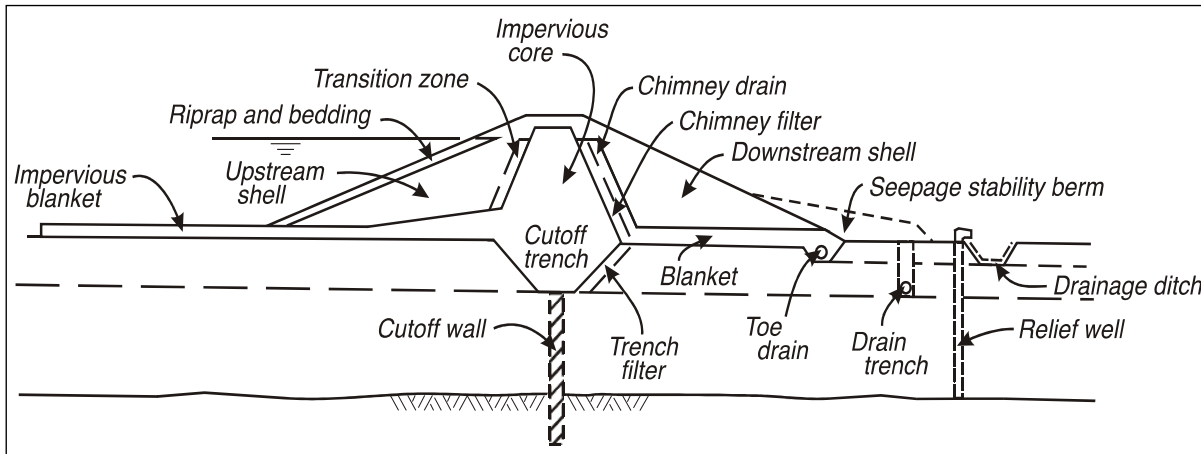
Fluid forces on particles



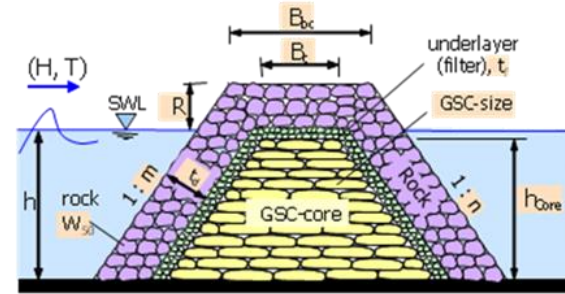
- IBM simulations show force in polydisperse systems strongly influenced by individual particle diameters
- Particle diameter is linked to local packing density
- Buoyancy force / pressure gradient force is becomes significantly larger than the drag force (which is linked to fluid flow velocity) as the Cu increases

Filtration: base – filter compatibility

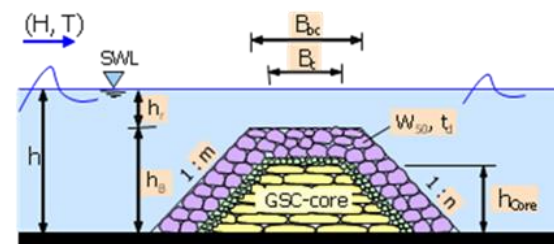
Embankment dams



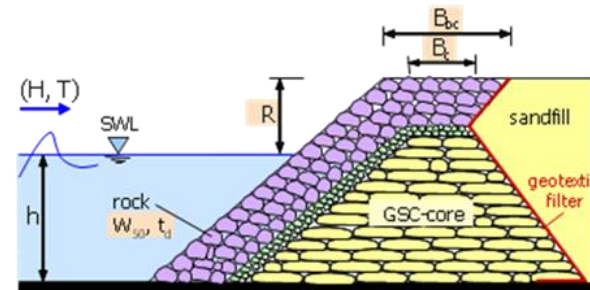
Rubble Mound Breakwaters



(a1) Rubble mound breakwater with GSCore



(b1) Submerged breakwater with GSCore

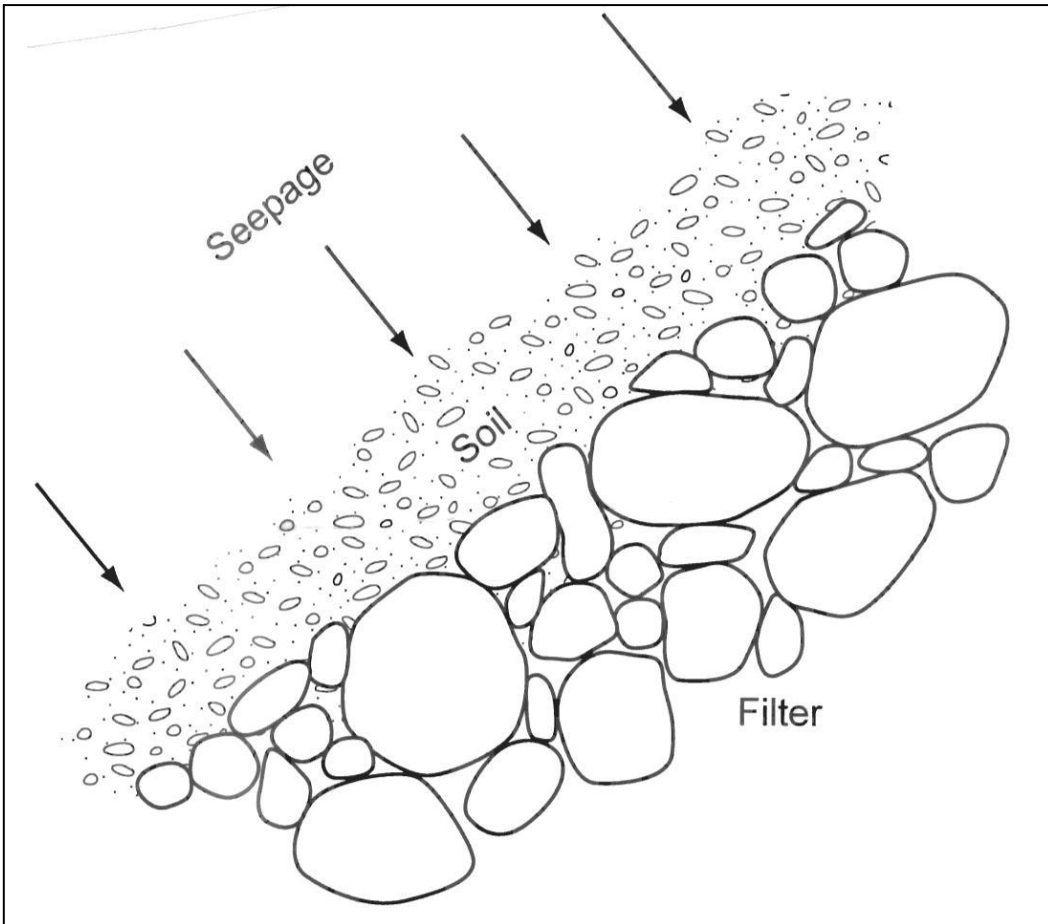


(c1) Seawall with GSCore and backfill
<https://www.tu-braunschweig.de/Medien-DB/hyku-mi/geocore1.png>



HR Wallingford Images:
https://www.innovationresearchfocus.org.uk/Issues/88/IRF88_HR1.html

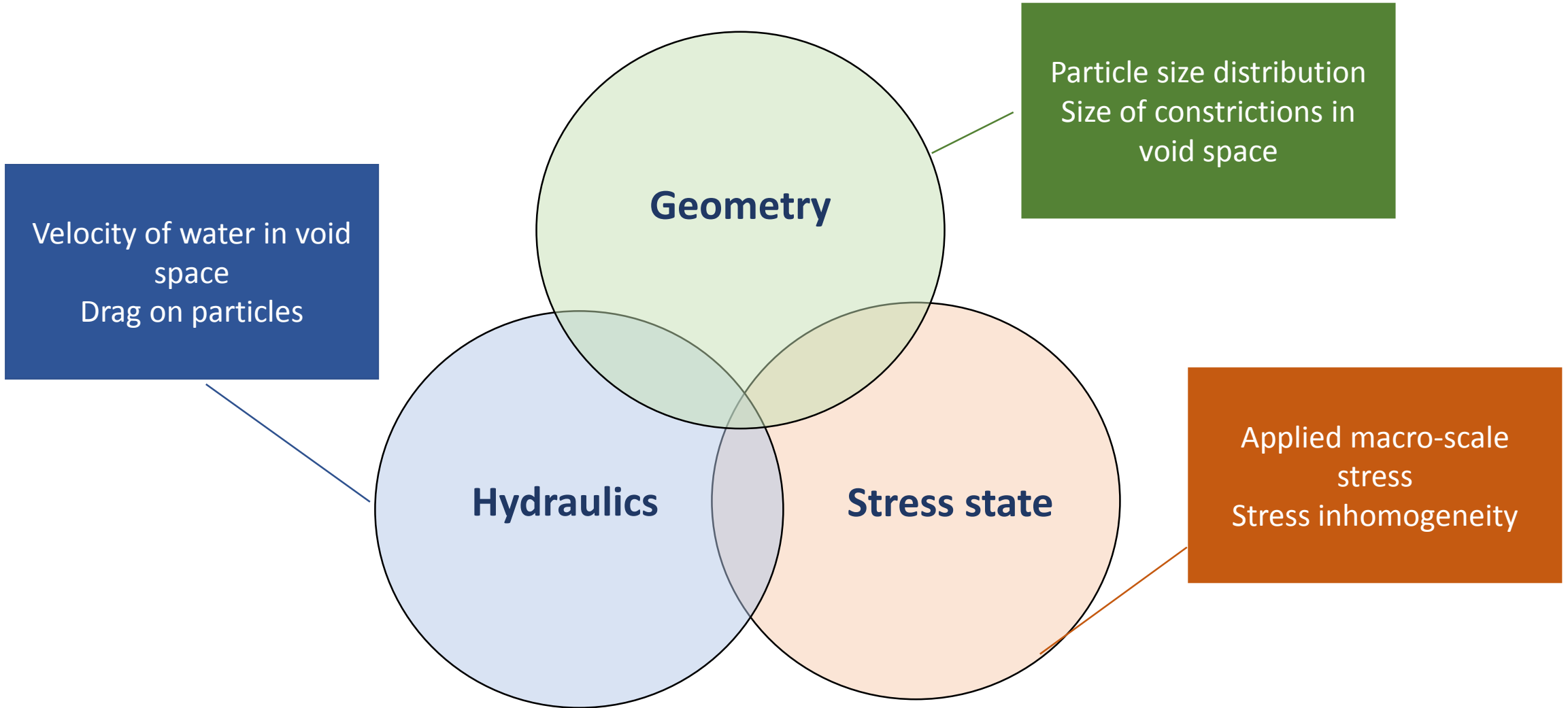
Filtration: base – filter compatibility



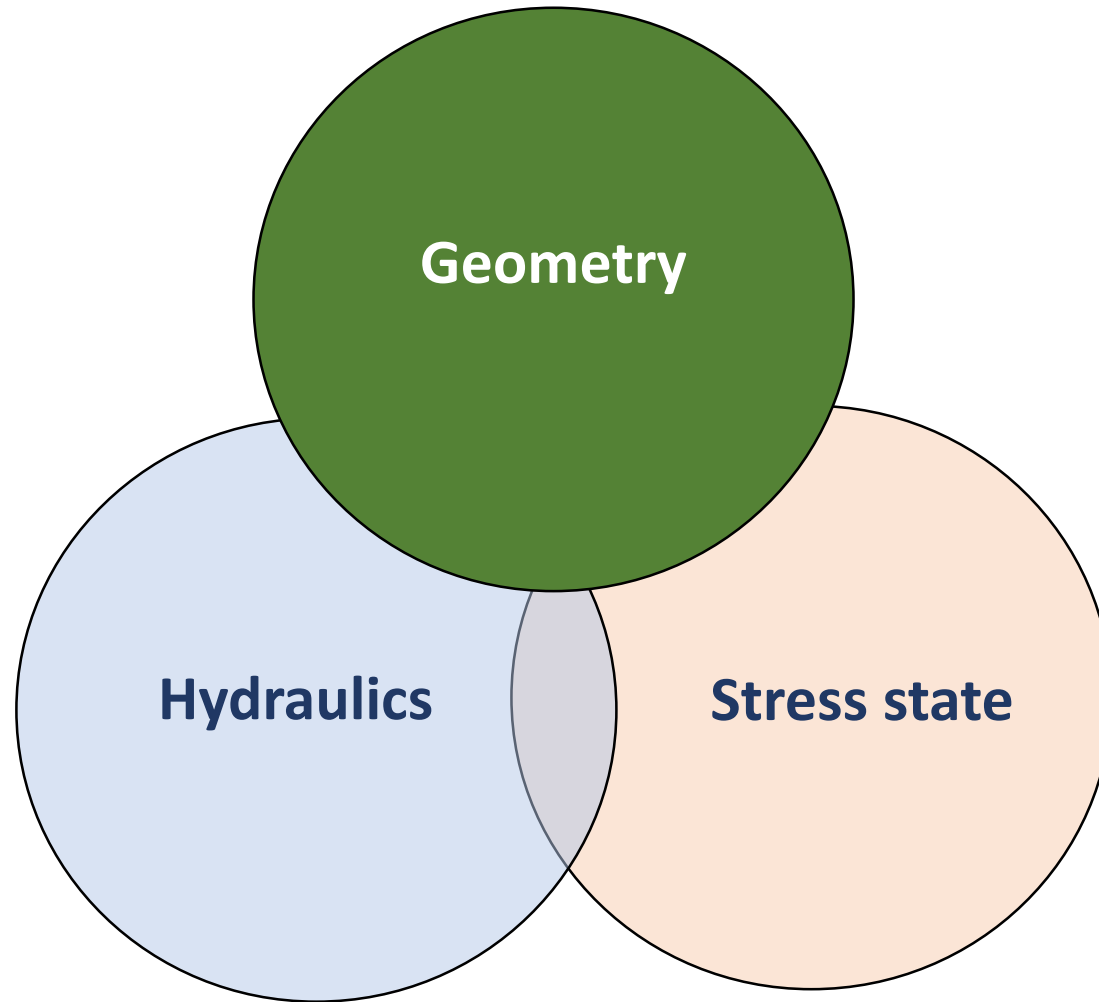
(FEMA, 2011)

- Filter should retain finer base material
- D_{15} often used as a means to estimate filter constriction sizes
- Originates from Terzaghi's filter rule (Sherard & Dunnigan, 1989; ICOLD, 2015)
- Supported by macro-scale filtration experiments (Kenney et al., 1985)
- D_{10} is linked to permeability (e.g. Hazen correlation)

Factors influencing erosion risk

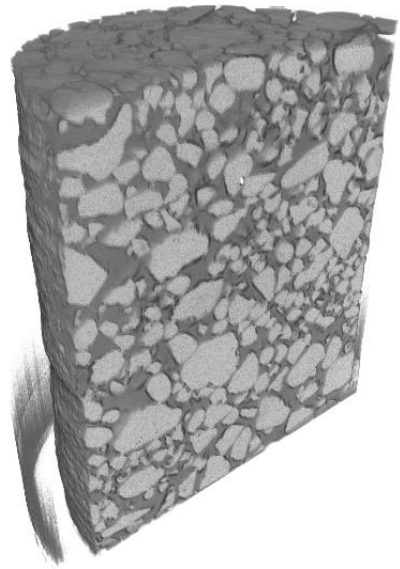


Factors influencing erosion risk



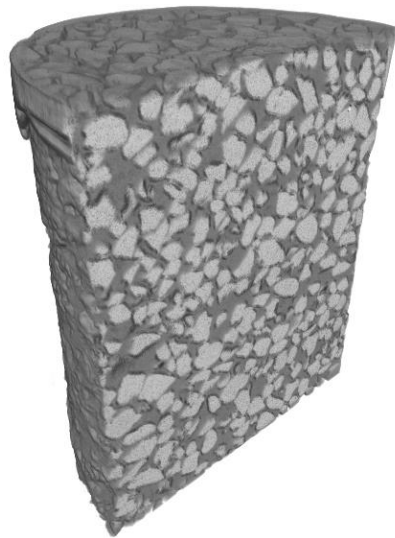
Filtration – Samples Considered

Laboratory Experiments

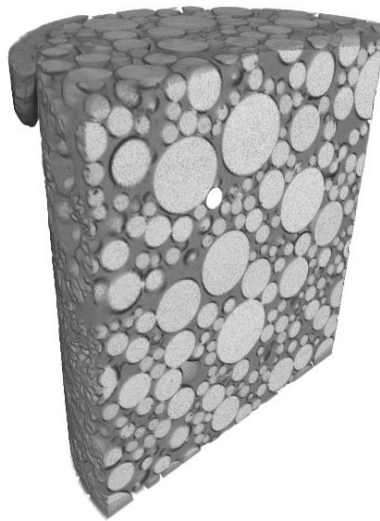


Leighton Buzzard
Sand
 $C_u=3$

(Taylor, 2017)

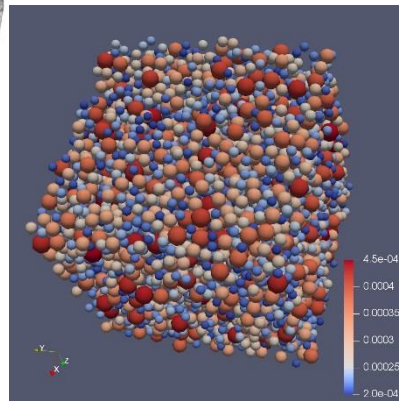


Leighton Buzzard
Sand
 $C_u=1.5$



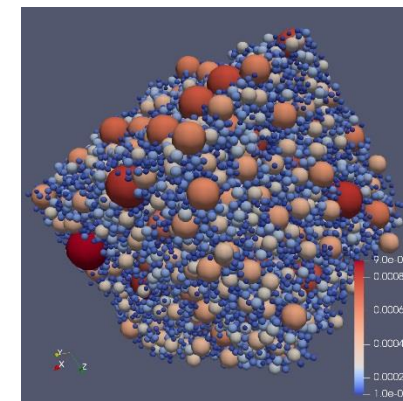
Glass Beads
 $C_u=3$

DEM Simulations

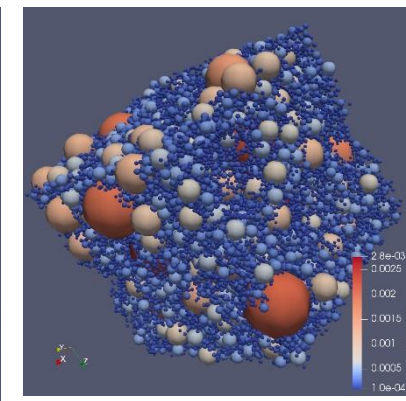


Spheres
 $C_u=1.2$

(Shire, 2018)

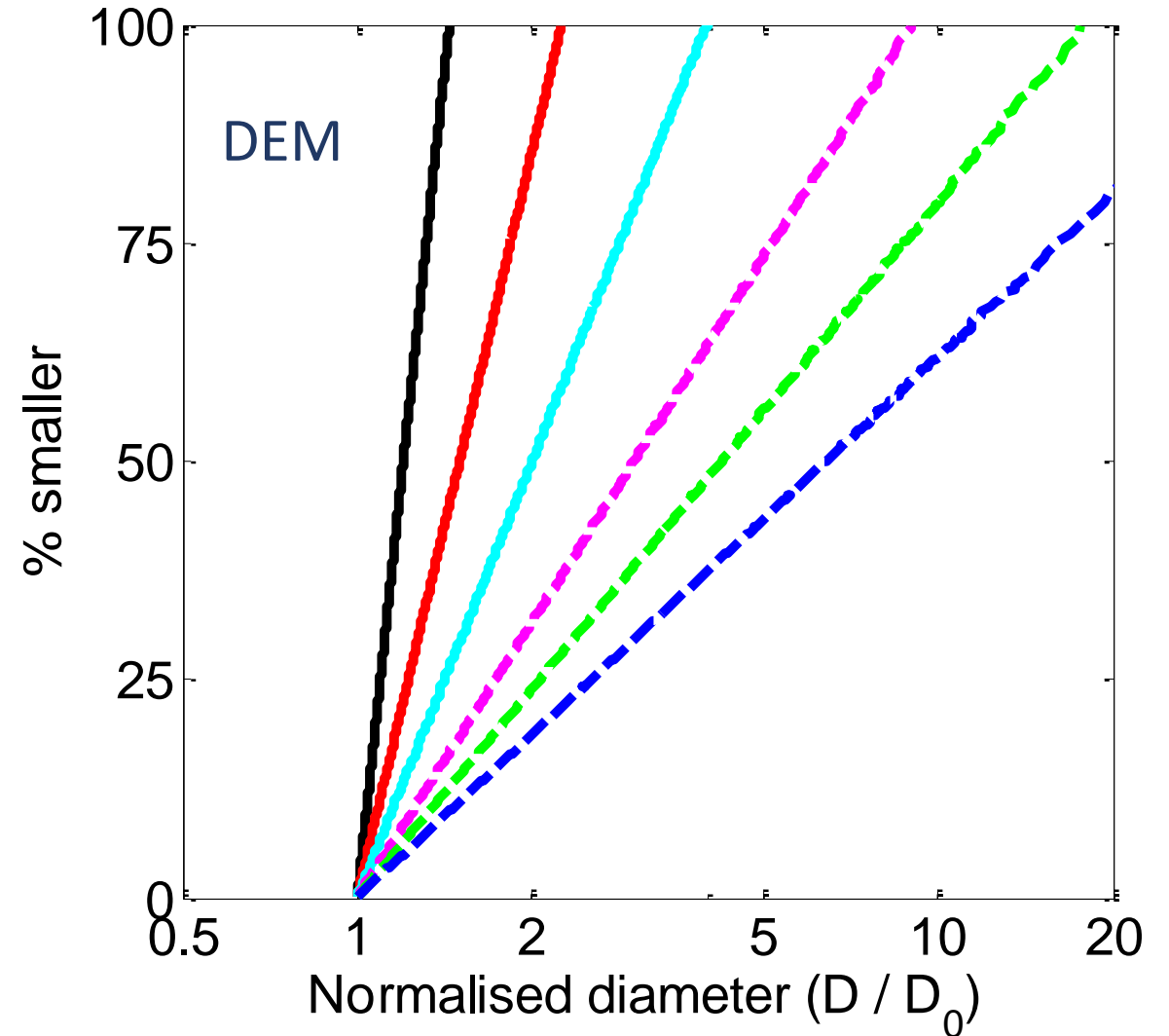
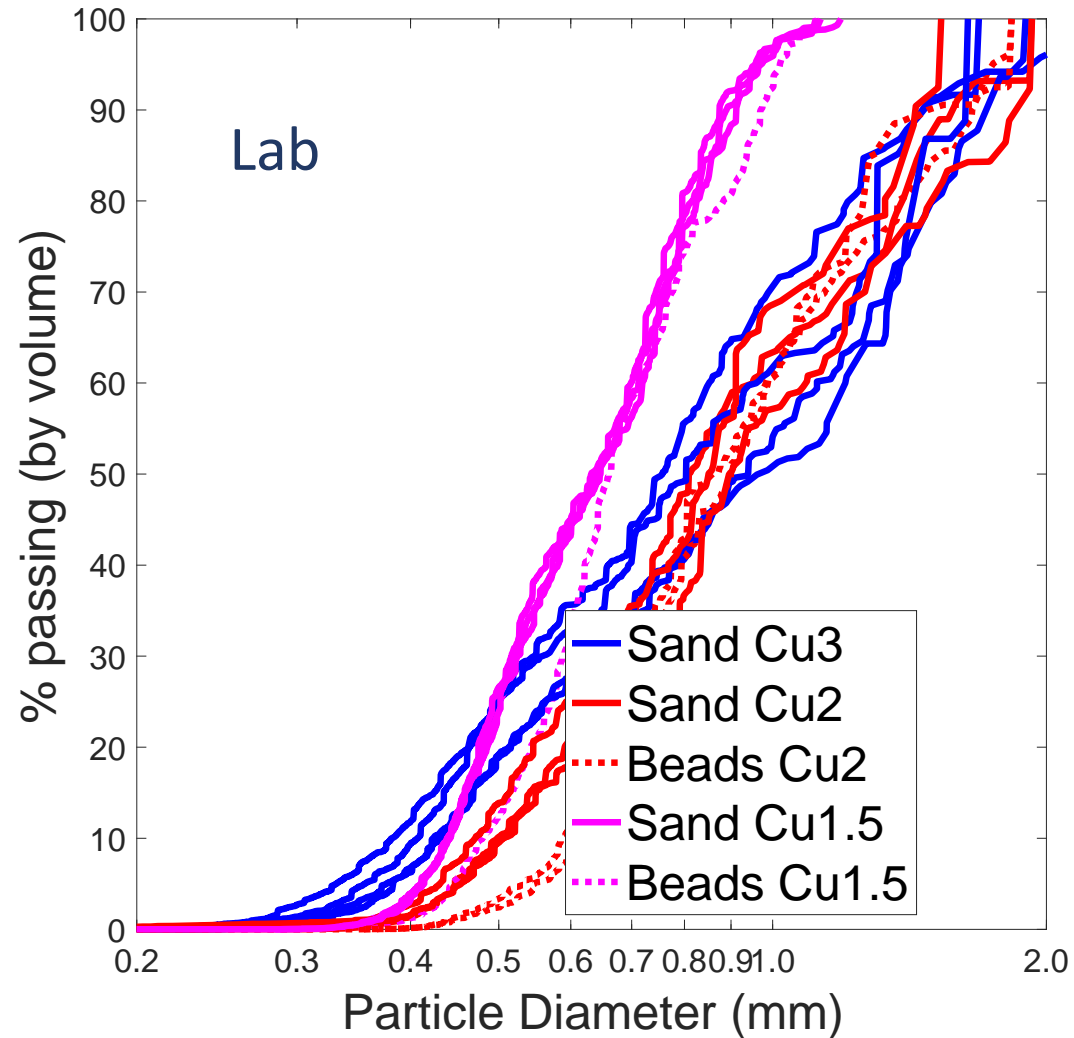


Spheres
 $C_u=3.0$

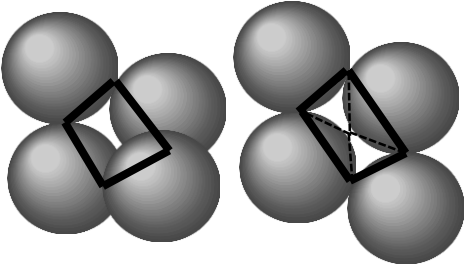
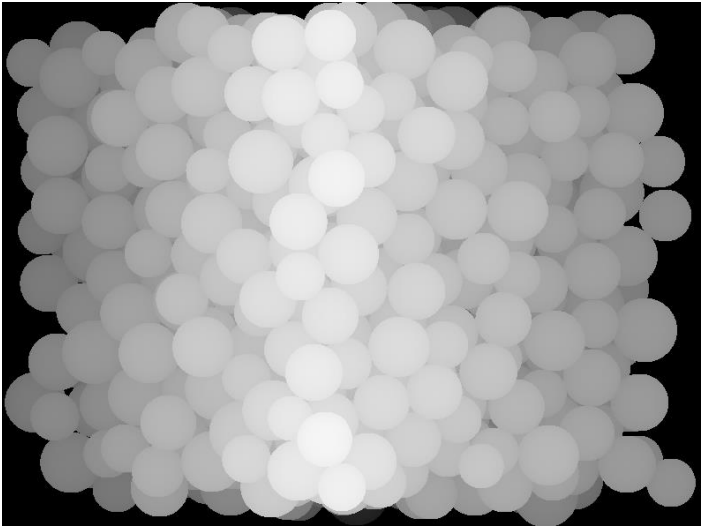


Spheres
 $C_u=6.0$

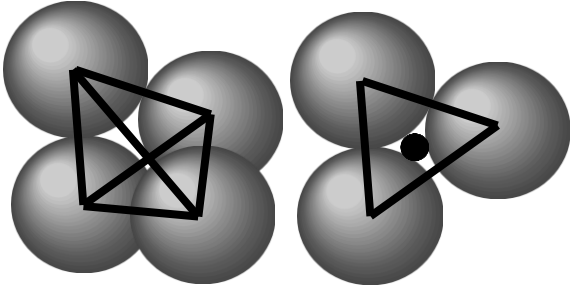
Filtration – Samples Considered



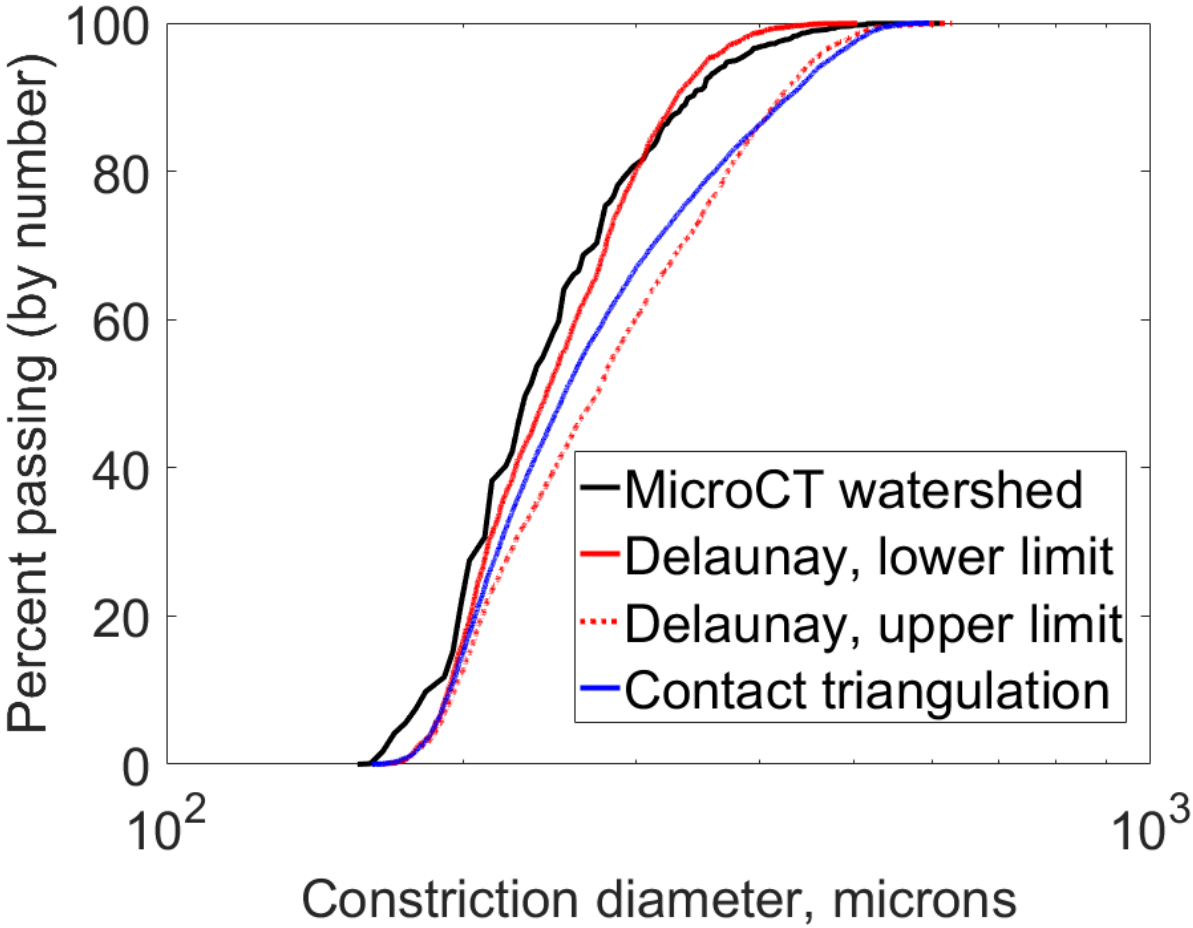
MicroCT: DEM Boundaries



Contact triangulation



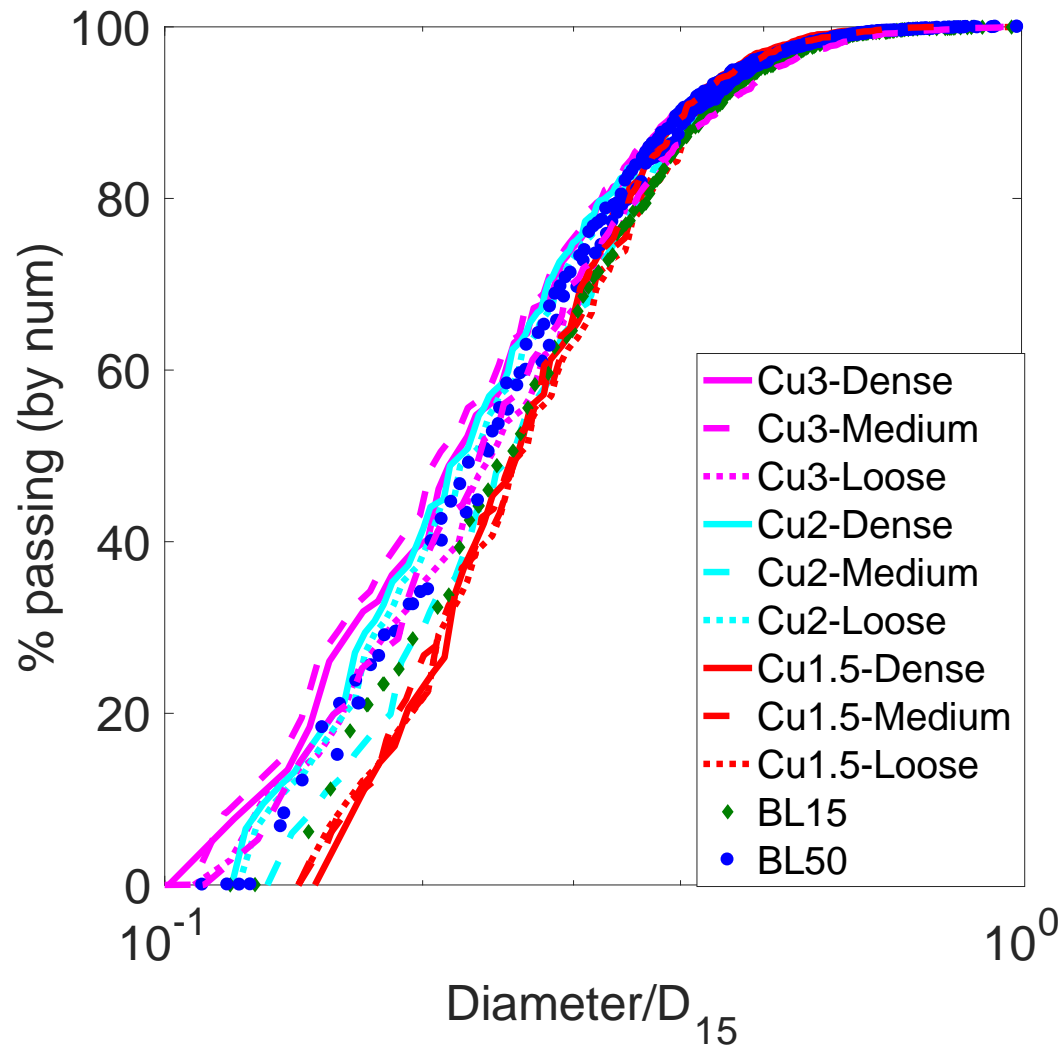
Delaunay triangulation of particles



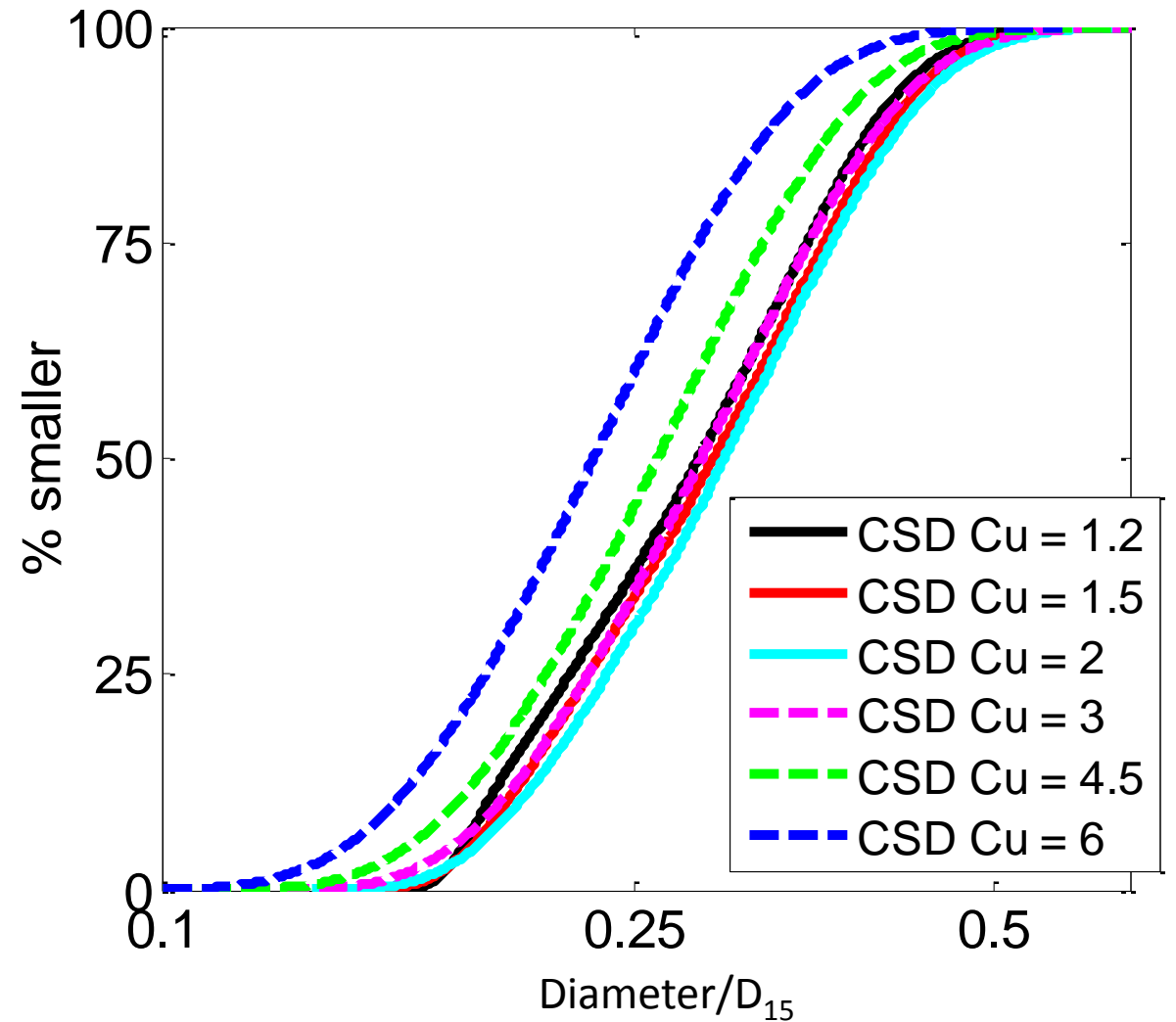
Shire et al. (2016)

Filtration - Constriction Sizes

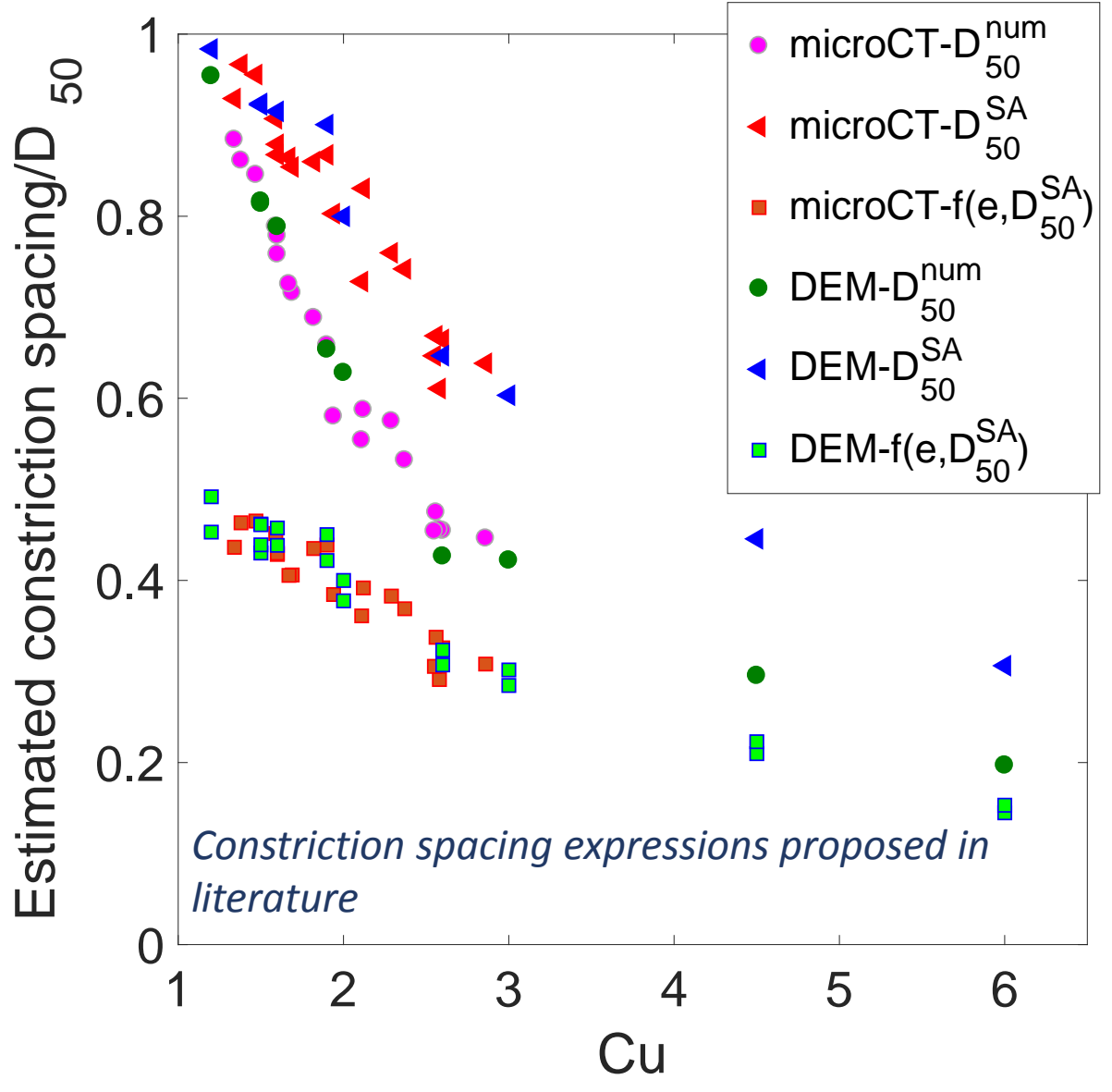
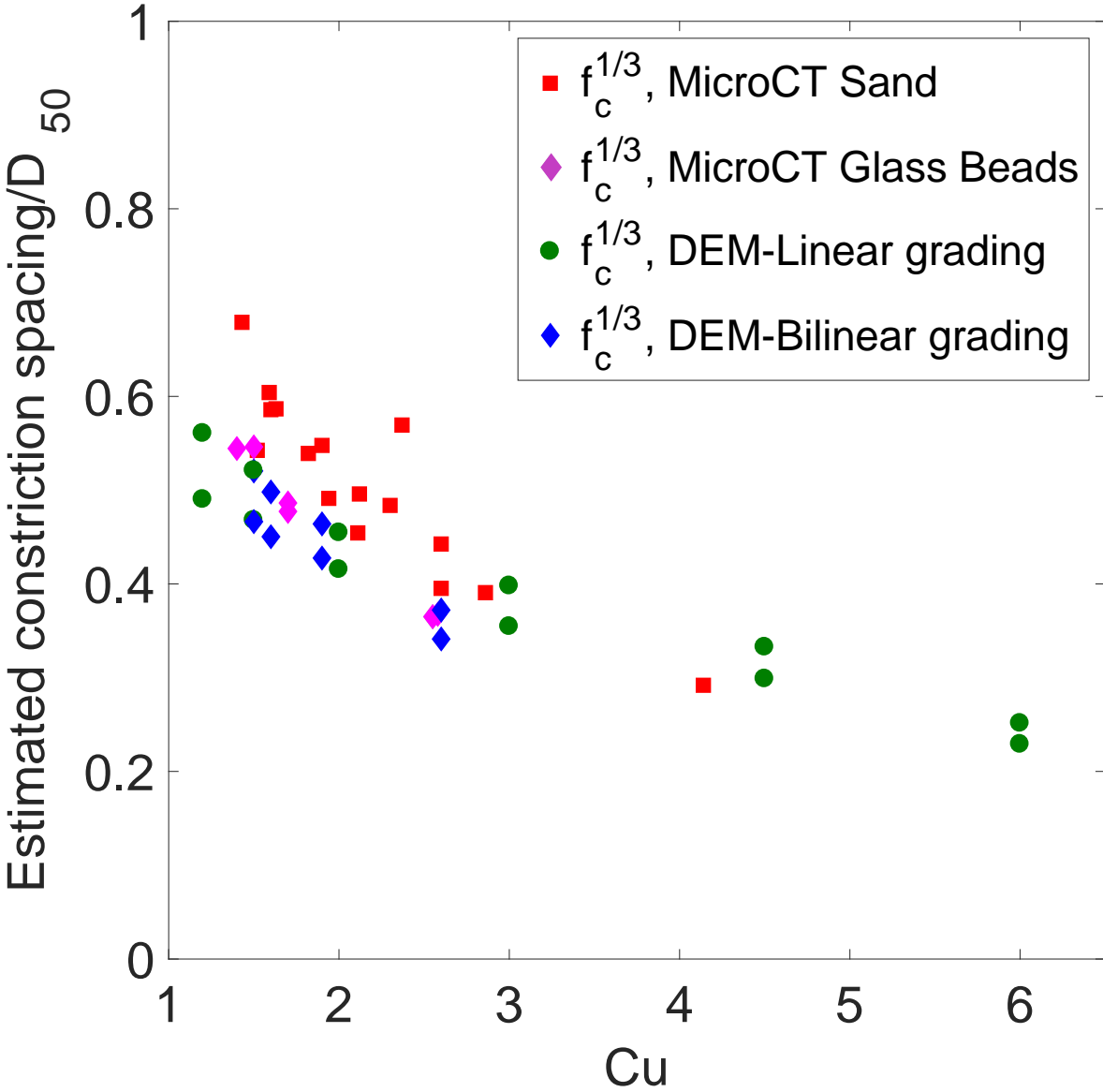
MicroCT Experimental data



DEM Simulation data

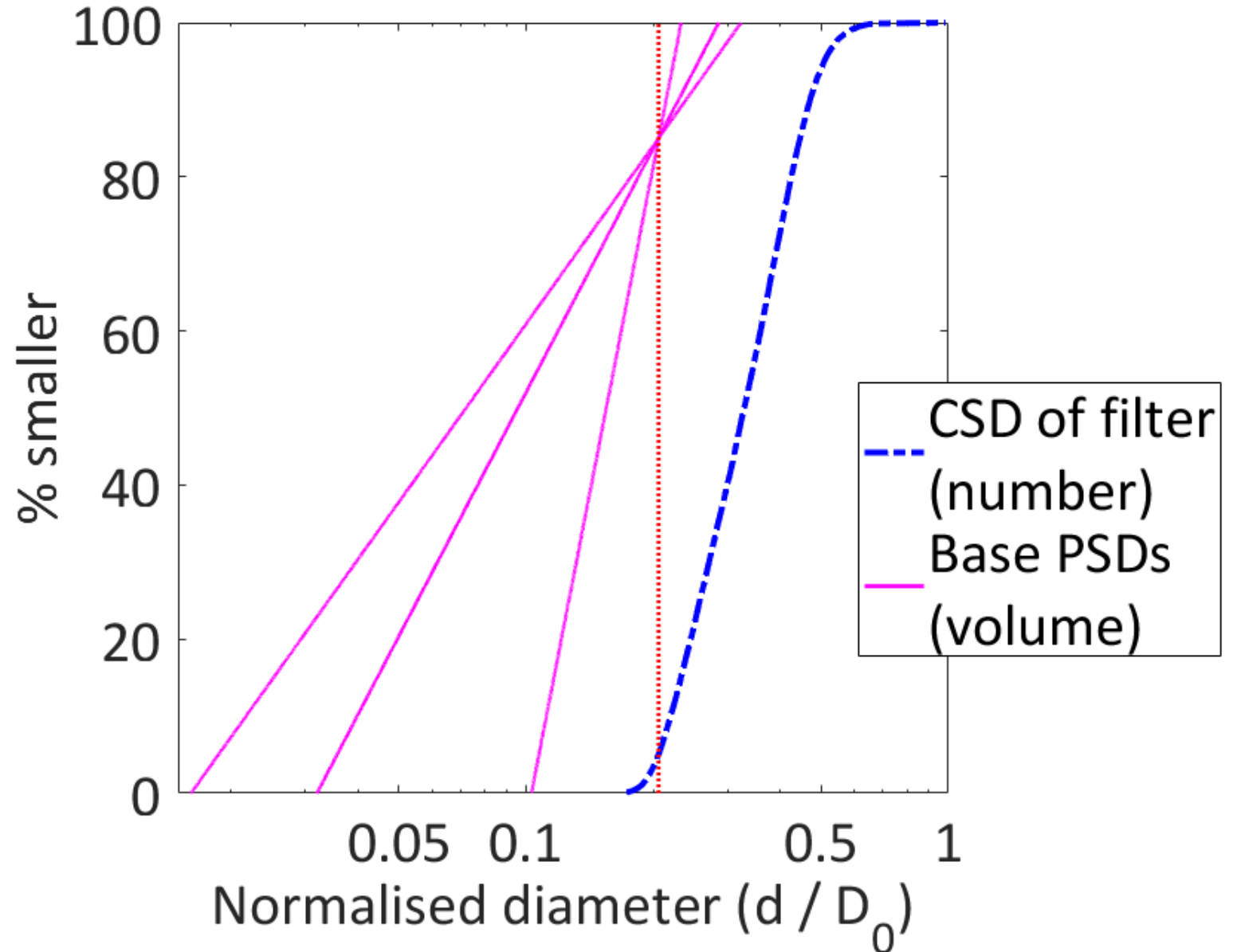


Filtration – Constriction Density / Spacing

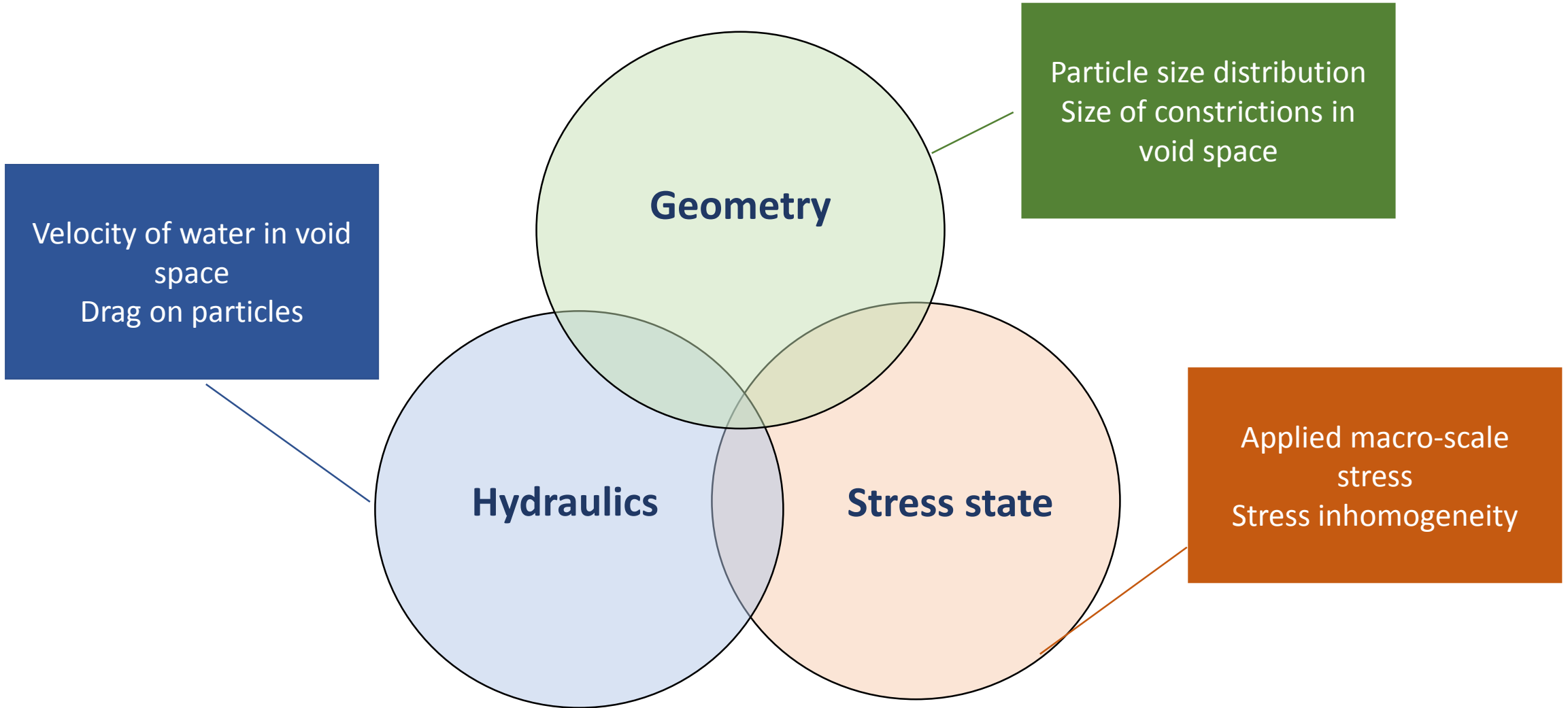


Filtration – Network model

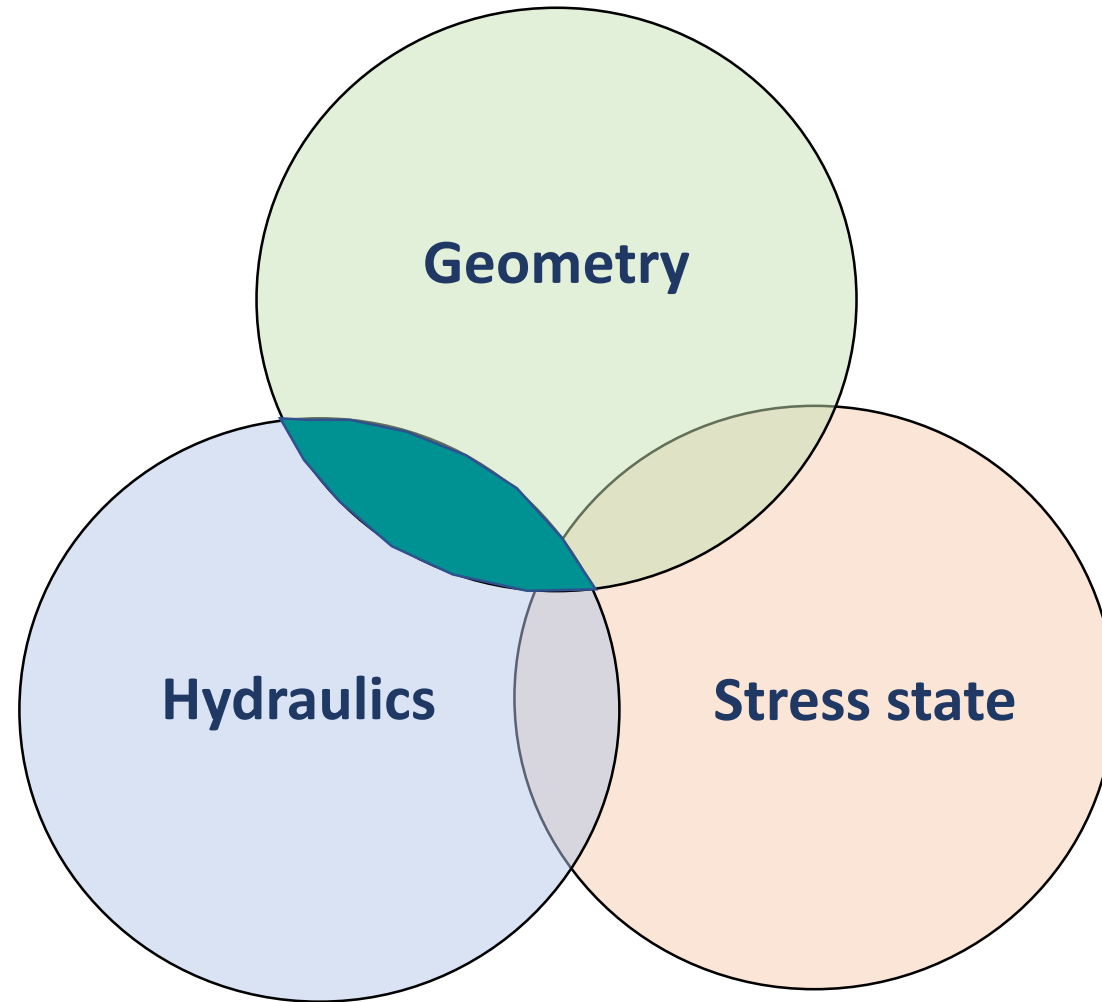
Can't judge a filter's effectiveness simply by visual comparison of the CSD of the filter and the PSD of the base material to be retained



Factors influencing erosion risk

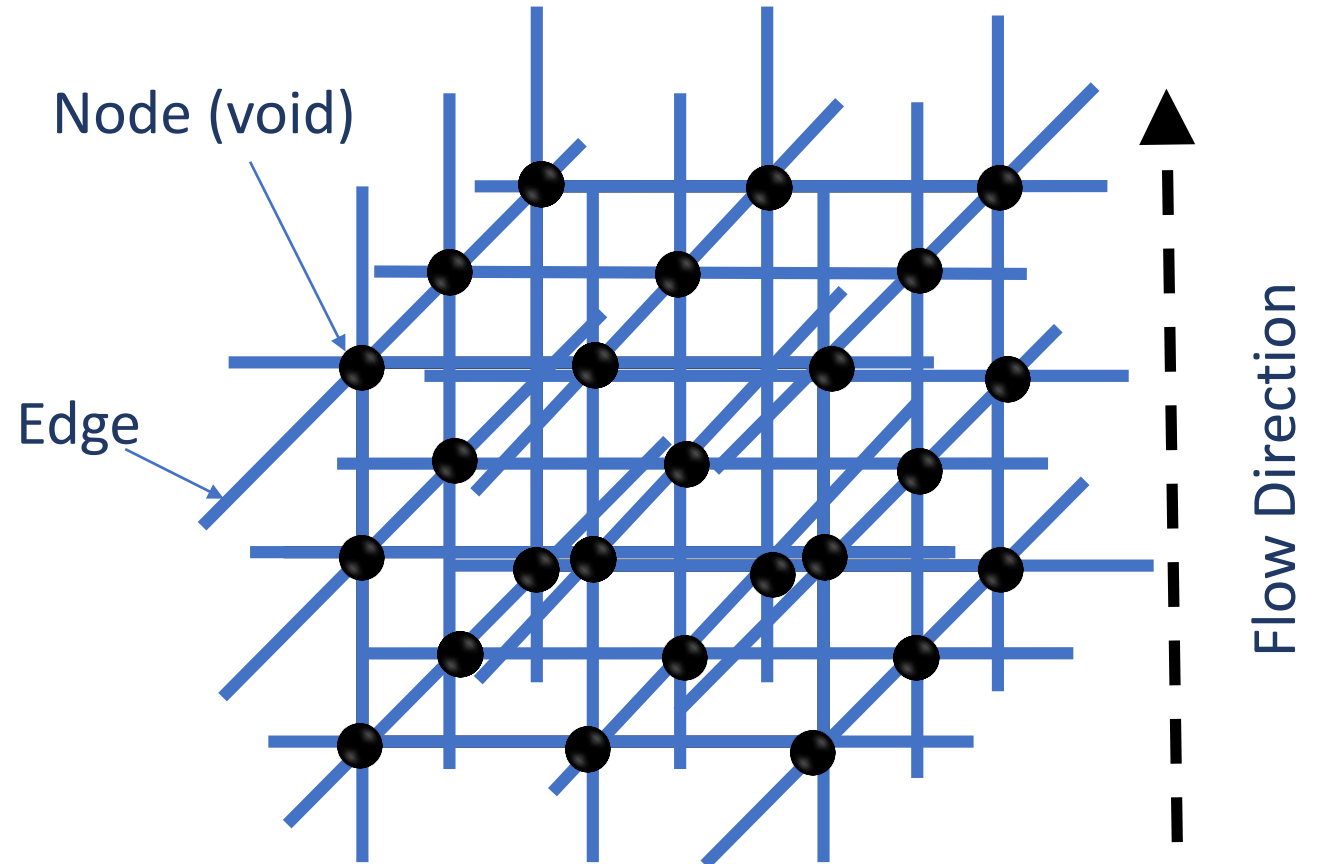


Factors influencing erosion risk



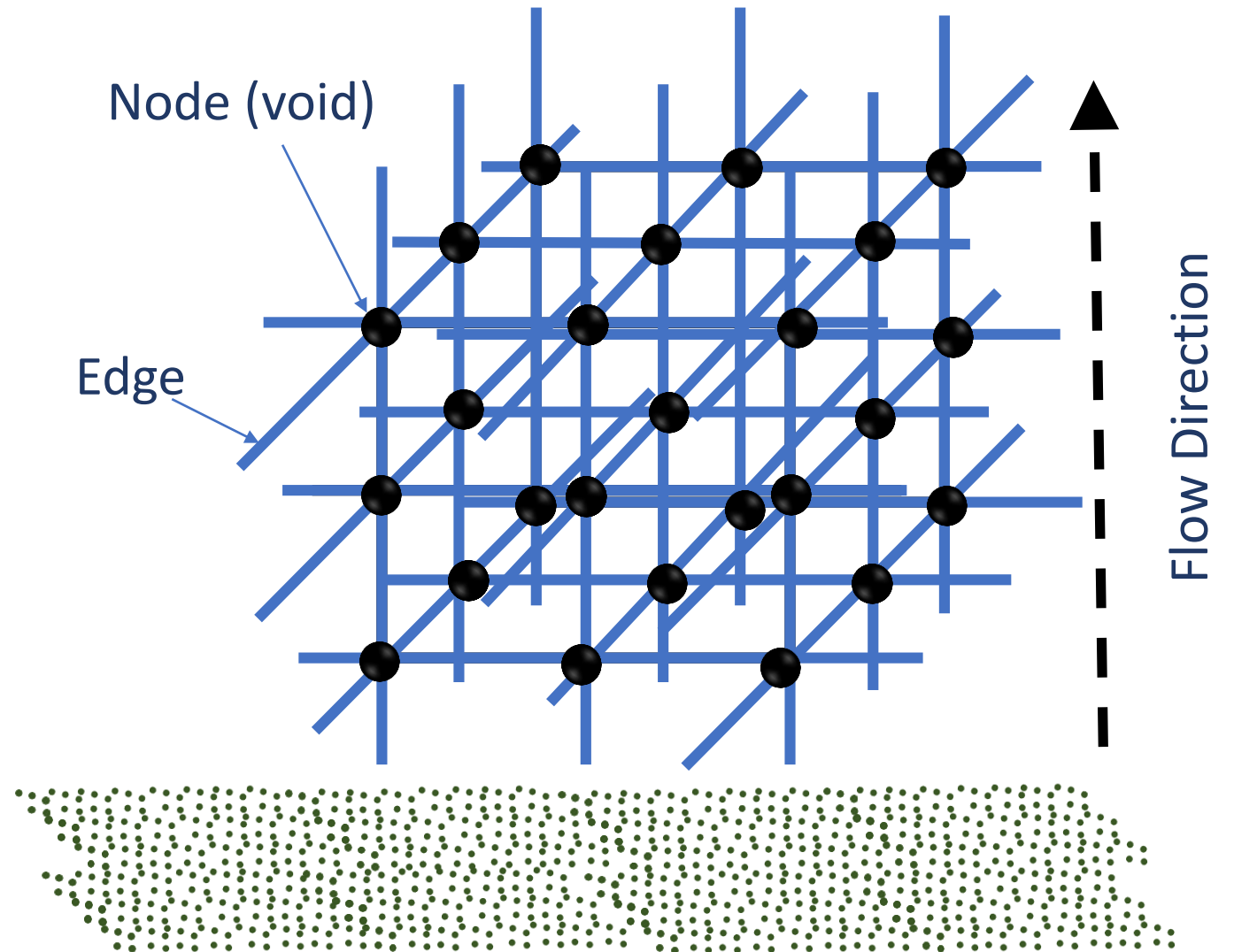
Filtration – Network model

- Network model – lattice topology
- Nodes = individual voids
- Edges = inter void connections
- Edge diameters = constriction diameters



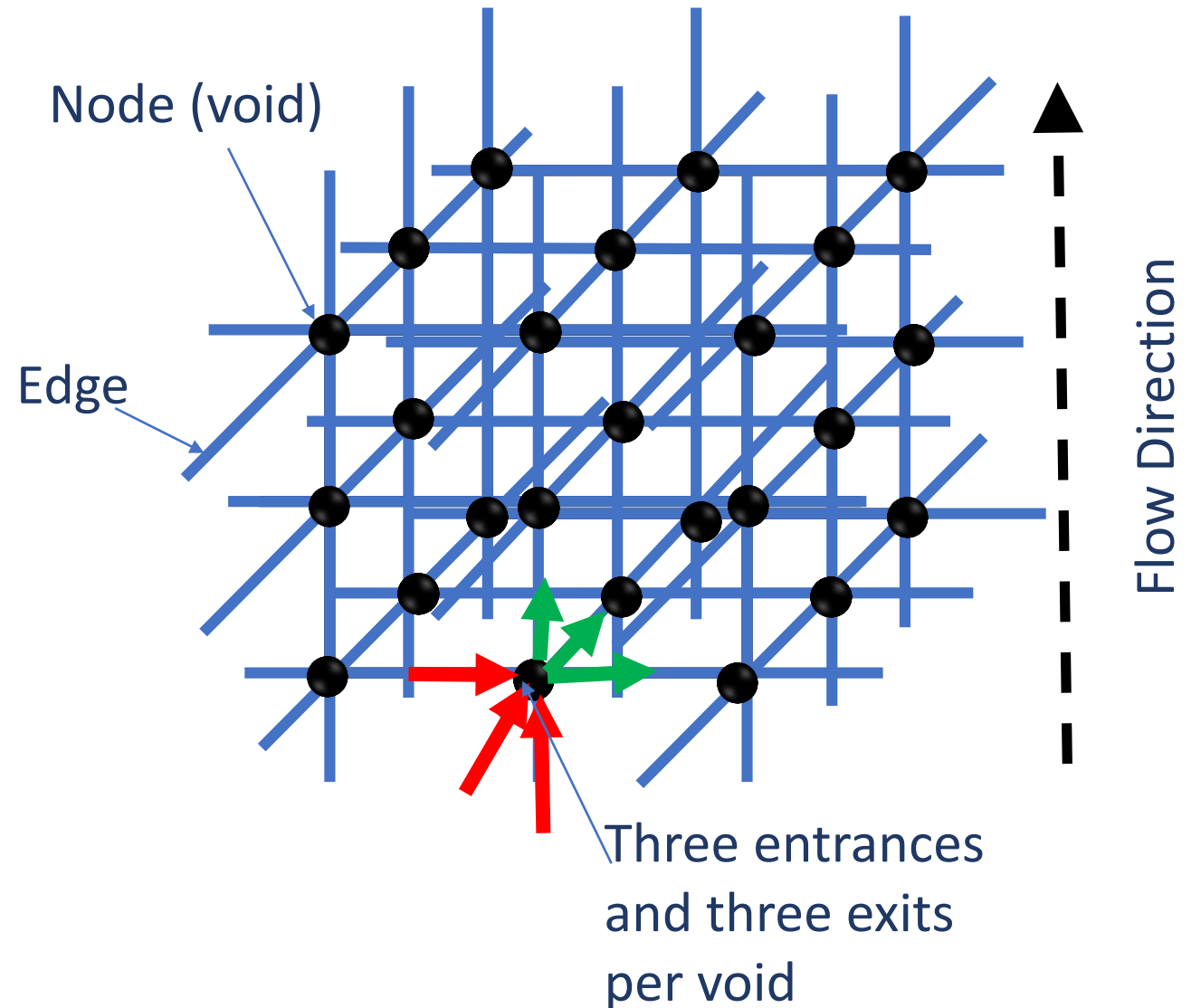
Filtration – Network model

- Simulates migration of finer base particles through network
- Fluid flow not explicitly considered
- Simple algorithm means up to 400 million base particles could be considered on a desktop pc

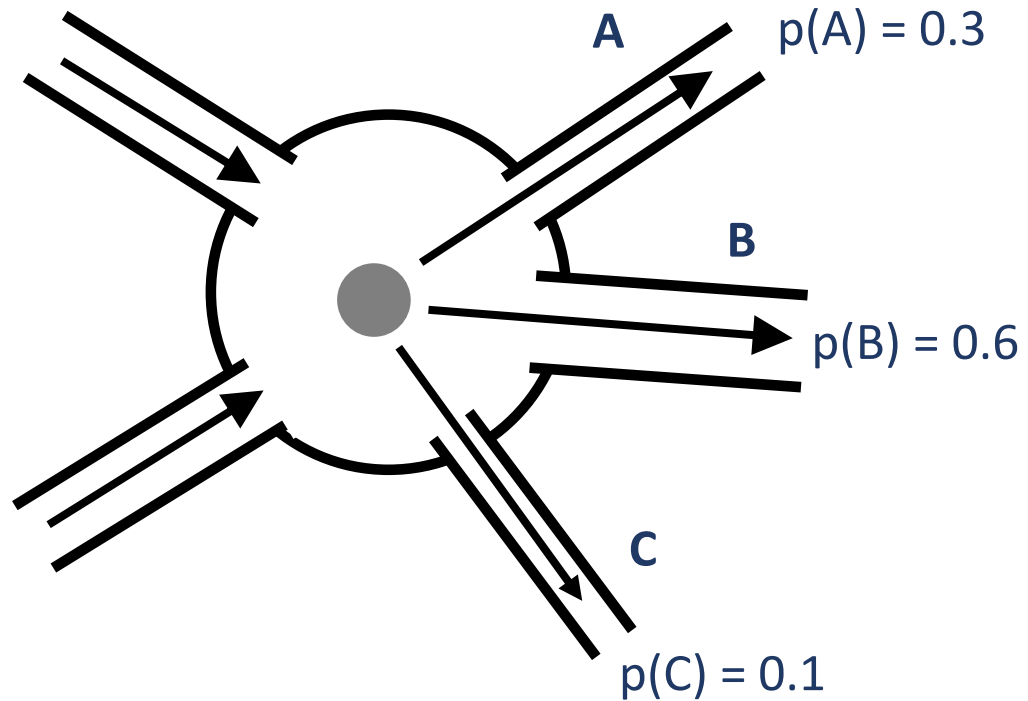


Filtration – Network model

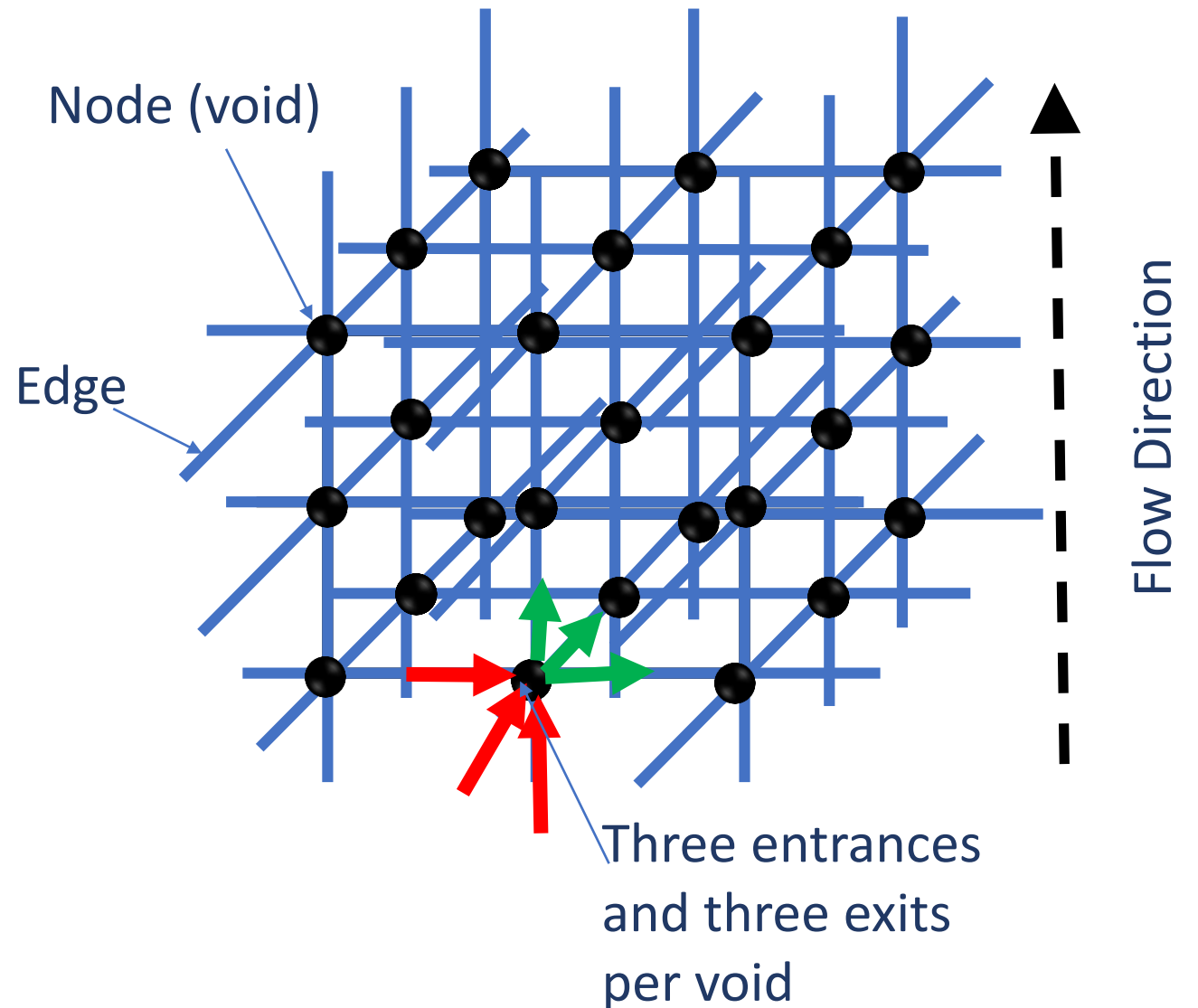
- Network model – lattice topology
- Nodes = individual voids
- Edges = inter void connections
- Edge diameters = constriction diameters



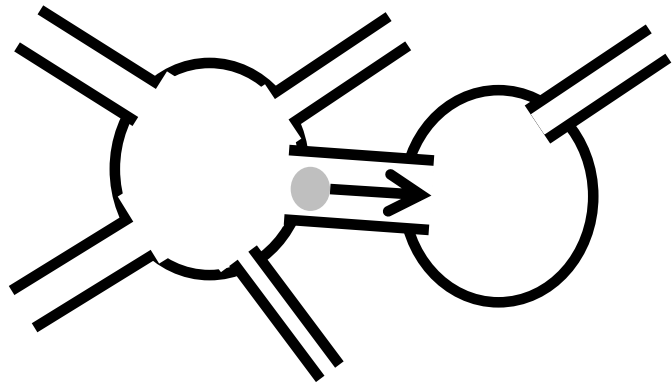
Area based random walk



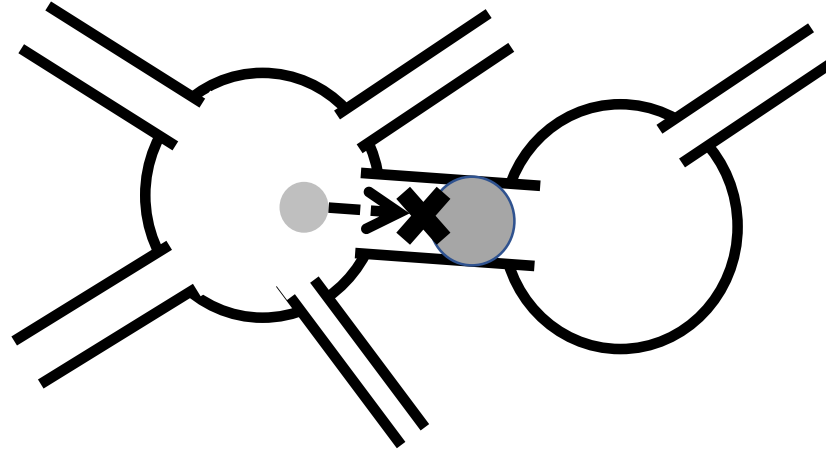
Likelihood of selecting a target edge to move through depends on constriction area



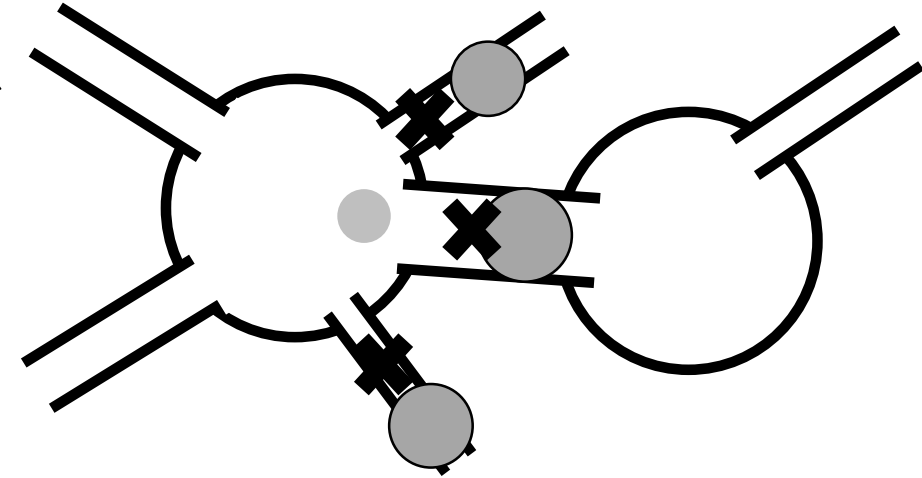
“Random walk” of base particles through network



Base particle moves through constriction

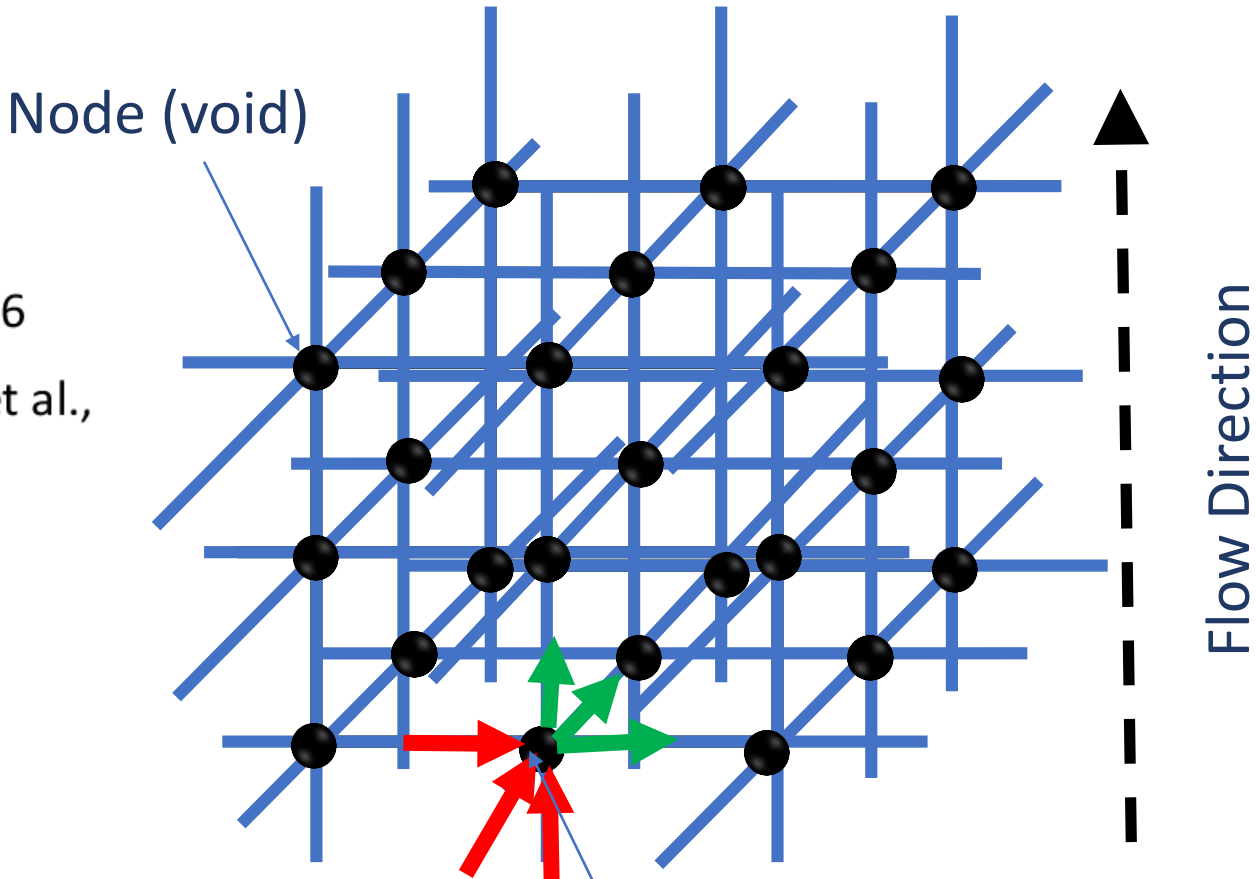
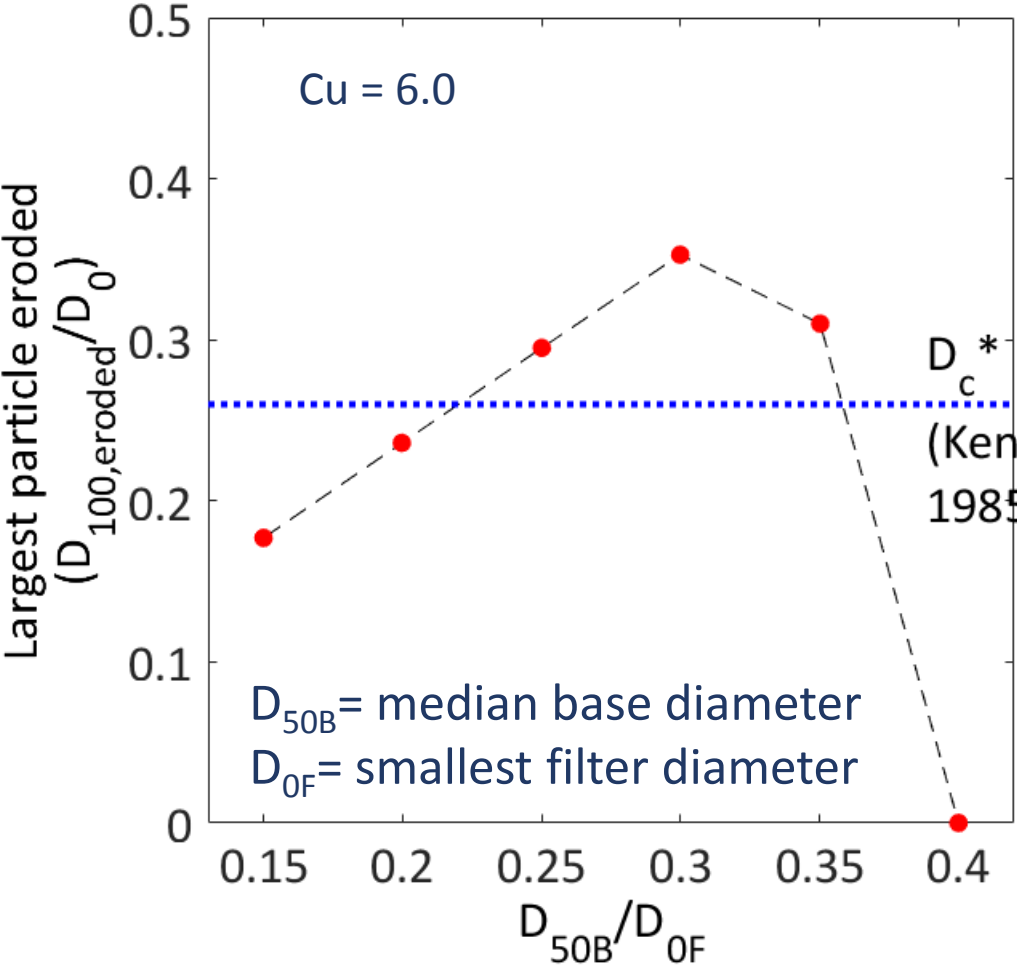


Base particle retained + constriction blocked



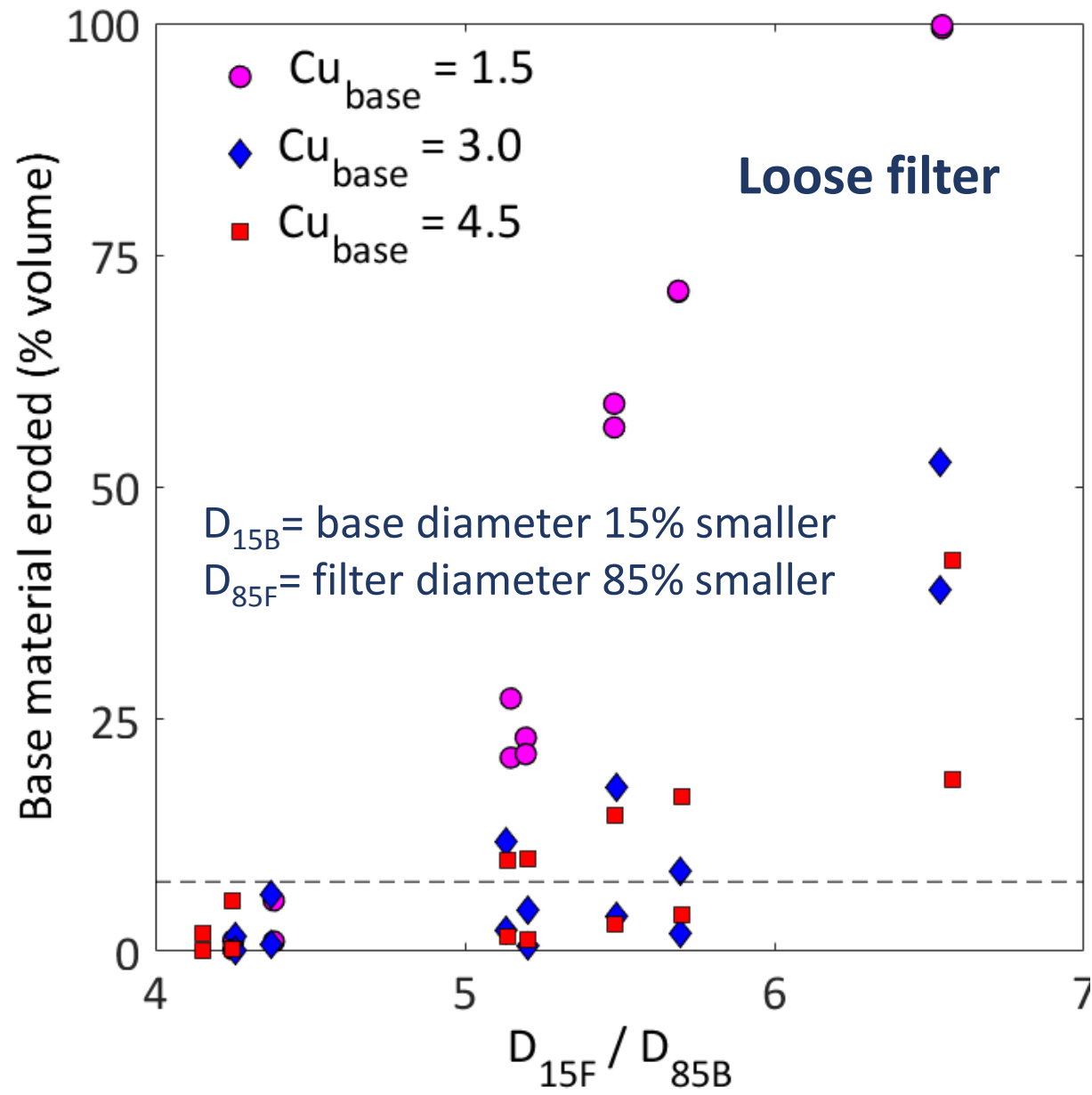
Base particle retained in void

Filtration – Network model

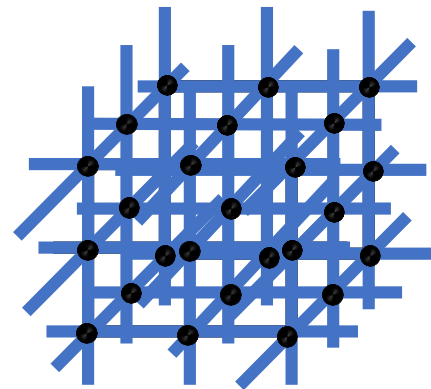


Filter $Cu = 1.2, 3, 6$, largest base particle eroded agrees with experimental data

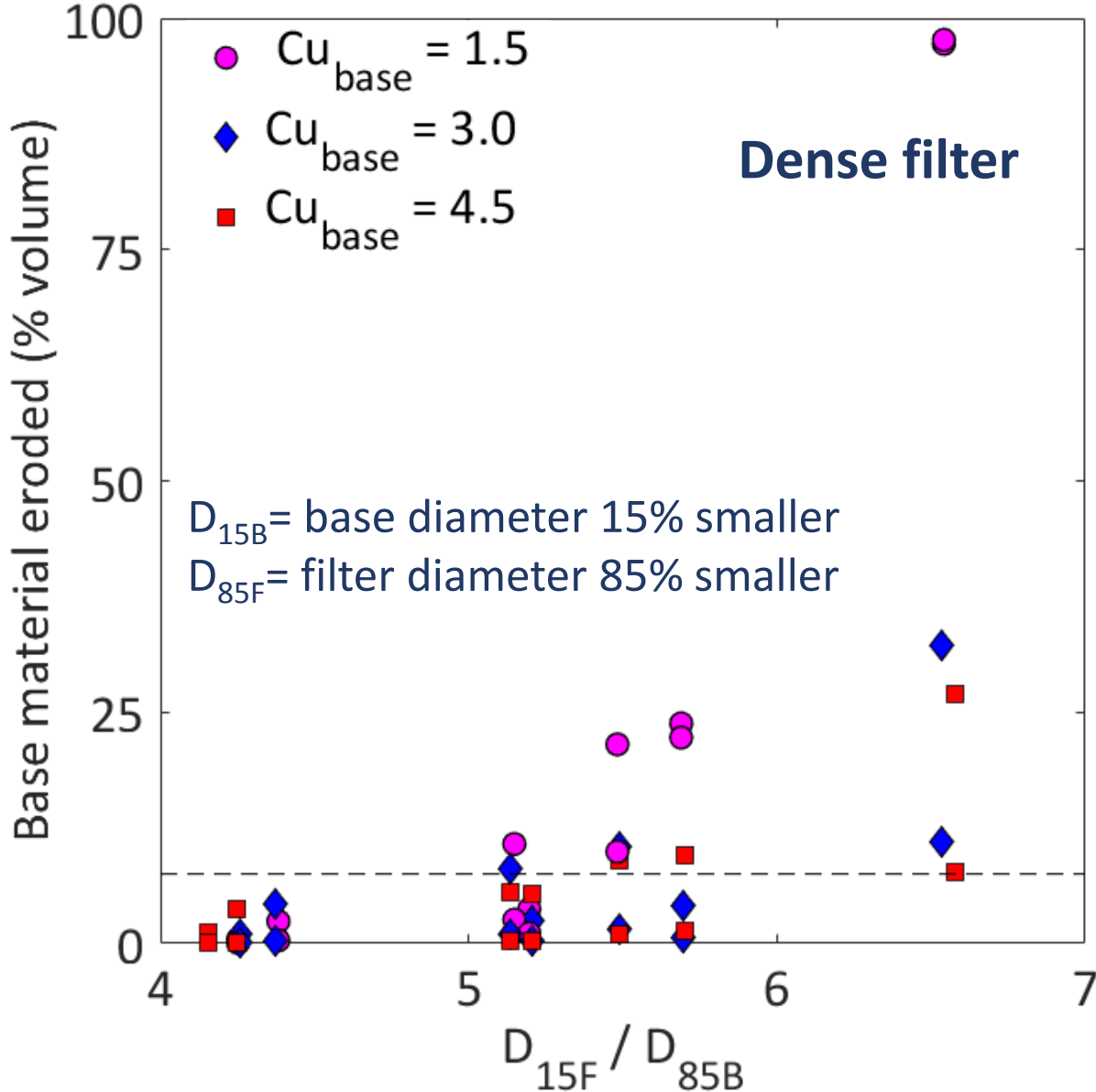
Filtration – Network model



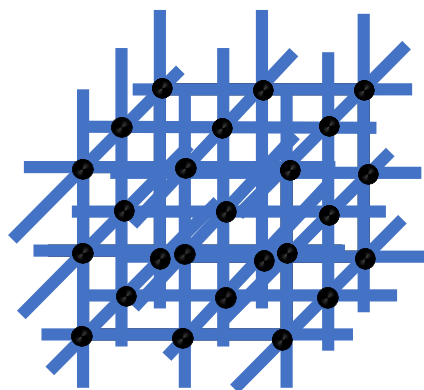
- Cu Filter = 1.5 and 3.0
- Network model that considers only constriction sizes and not full void space topology confirms experimental observation that filter characteristic diameter (D_{15F}) controls filtration



Filtration – Network model

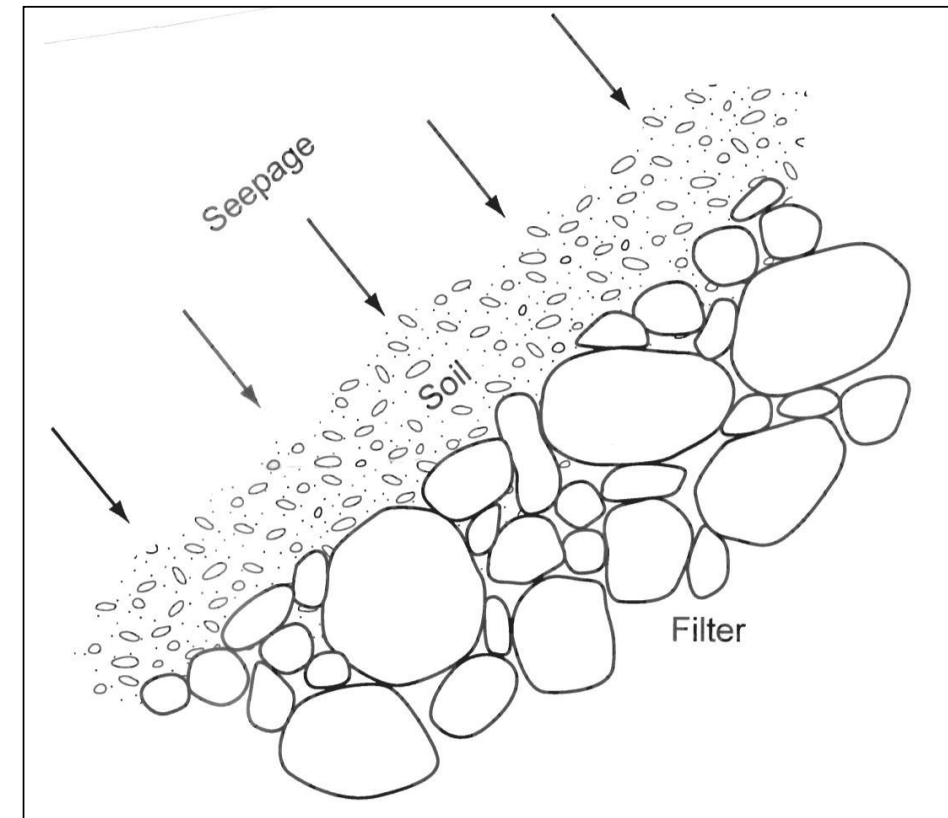


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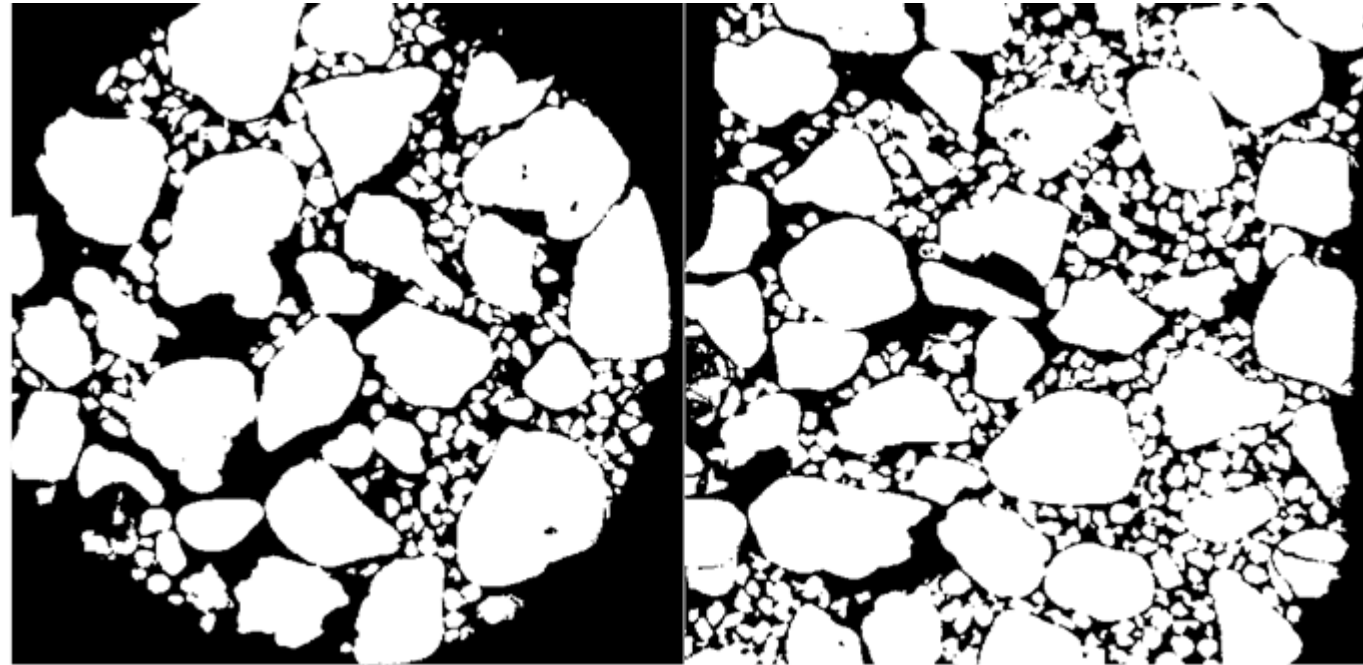
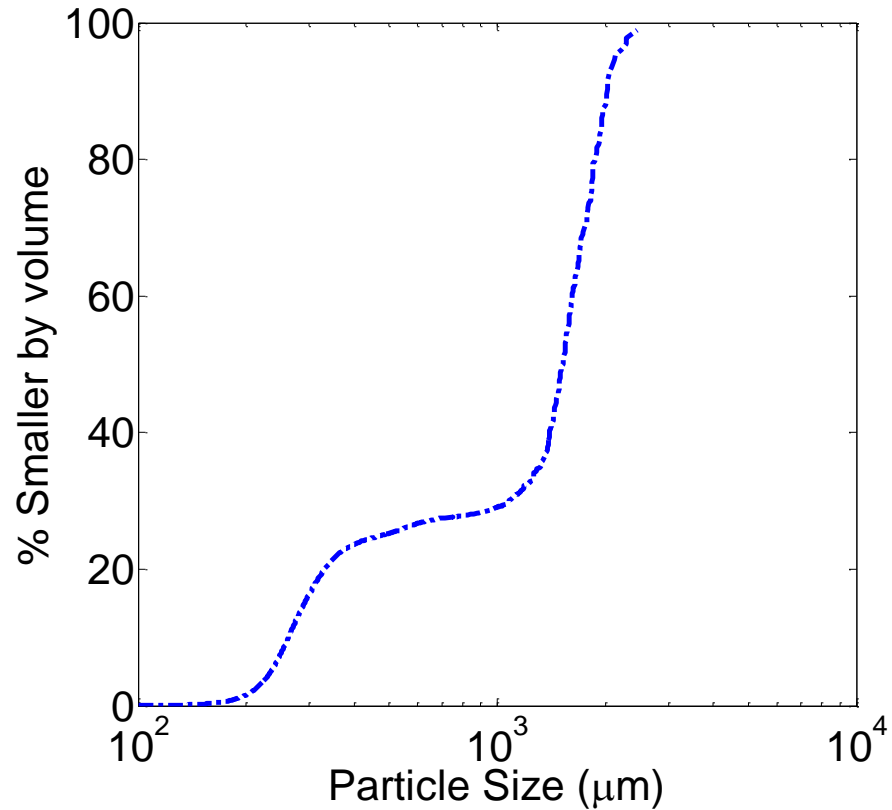
Filtration

- Particle scale analyses support empirically derived guidelines for engineering design
- Direct measurement of constrictions confirms there is a characteristic particle diameter that is indicative of constriction sizes
- Network modelling confirms D_{15F}/D_{85B} rule in filter design is reasonable
- Particle scale modelling and measurement gives insight into constriction density



(FEMA, 2011)

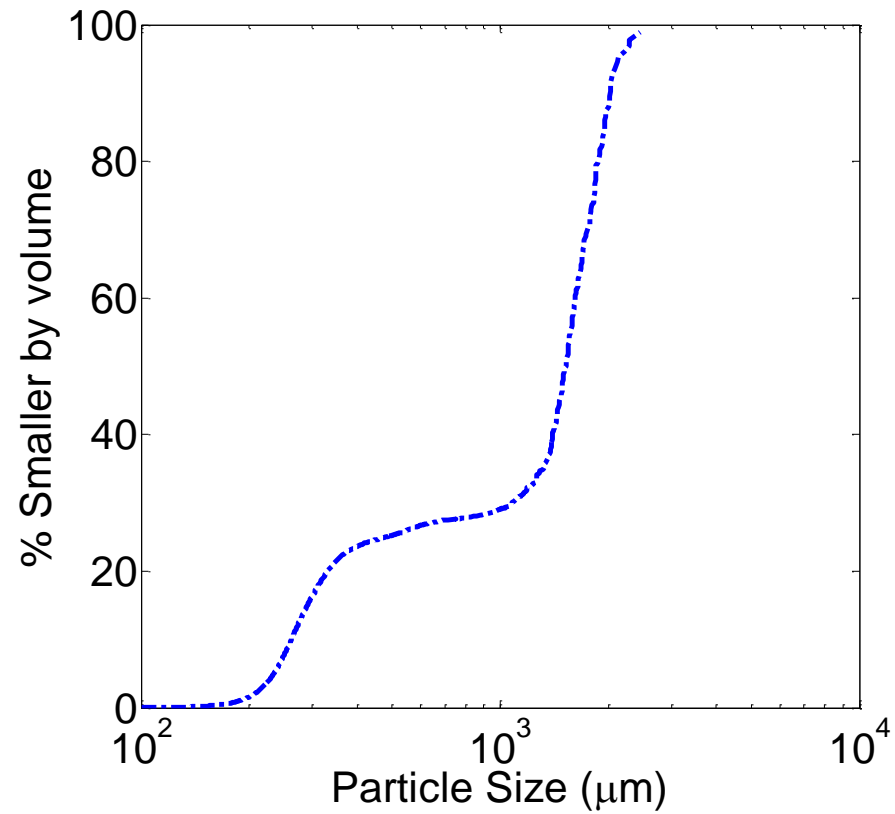
Internal Instability



Fonseca et al. (2014) Géotechnique

- In gap graded materials erosion can happen at low hydraulic gradients

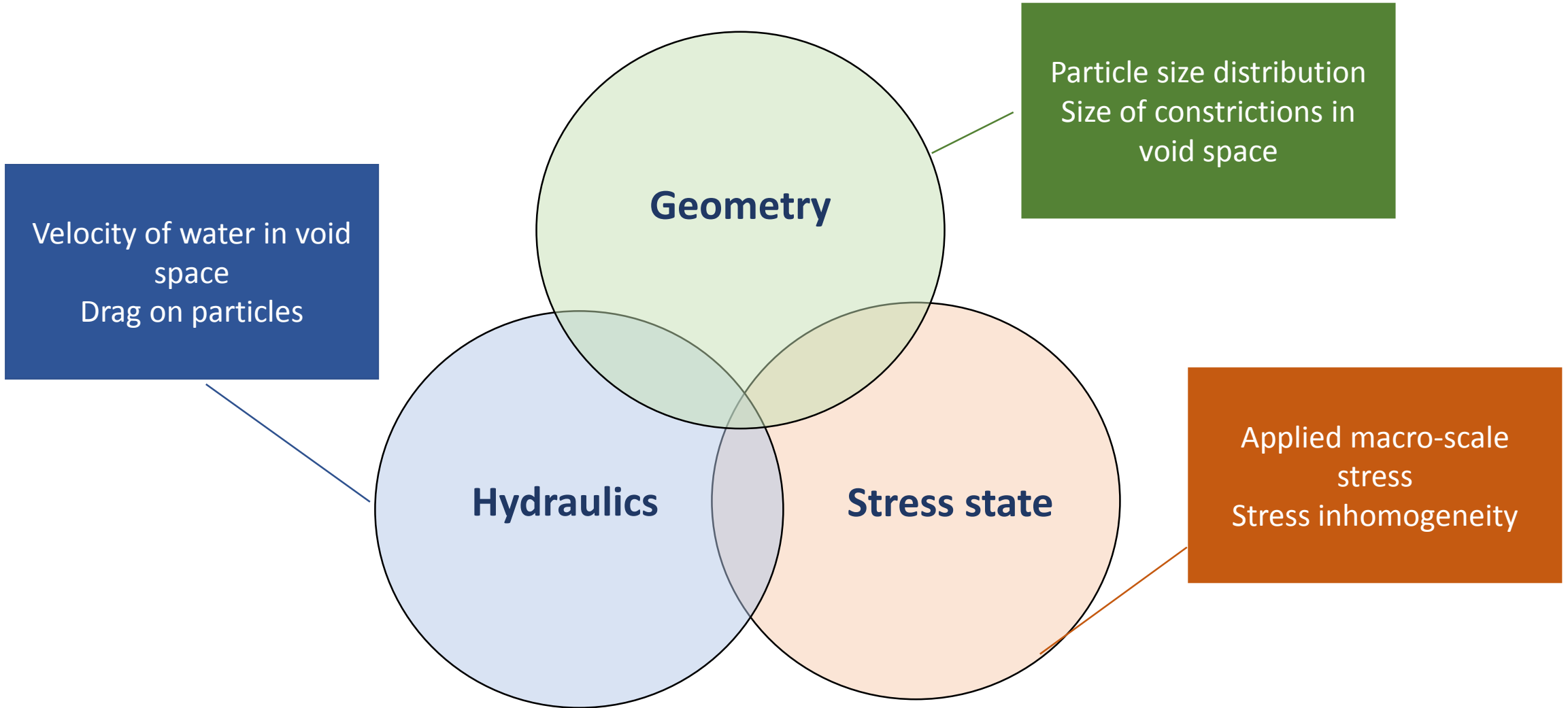
Internal Instability



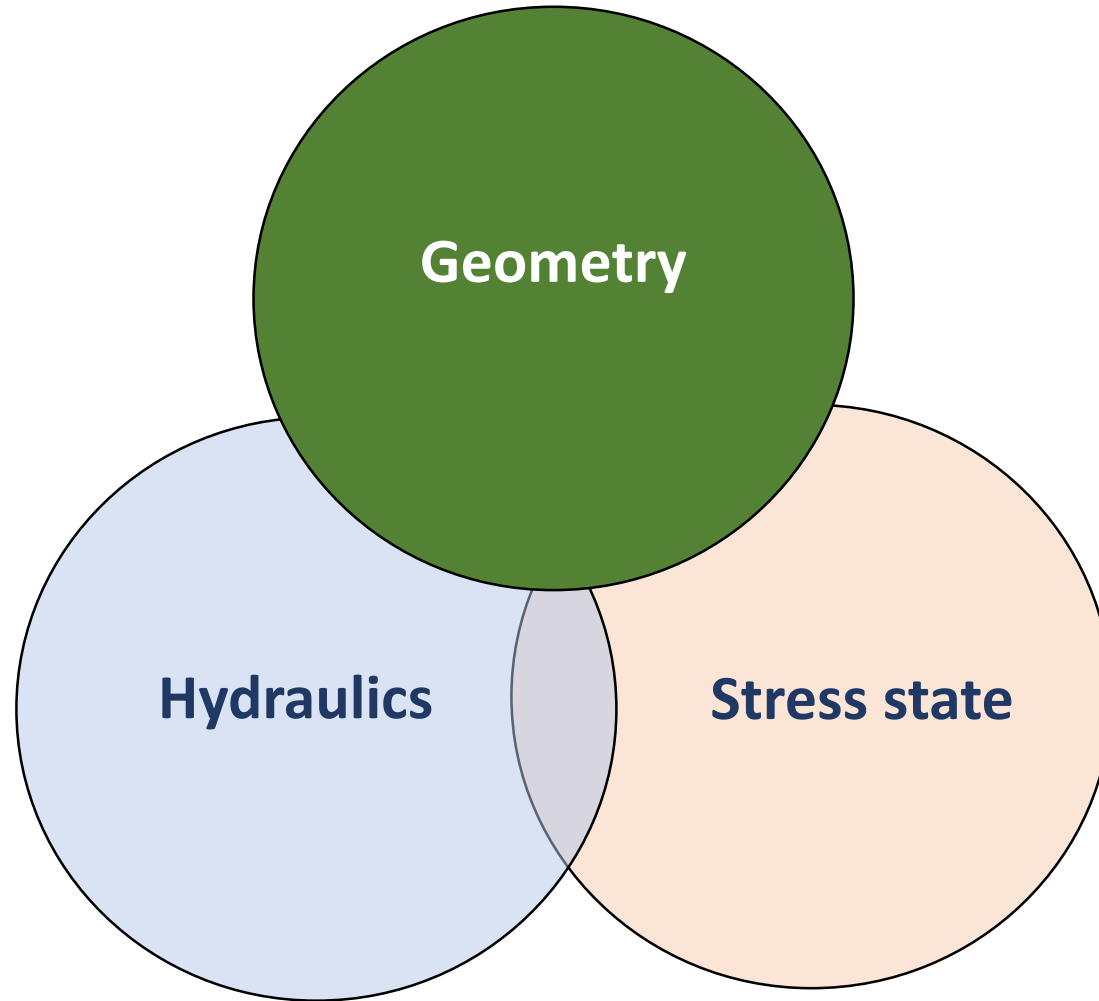
Robert Negri MSc

- In gap graded materials erosion can happen at low hydraulic gradients

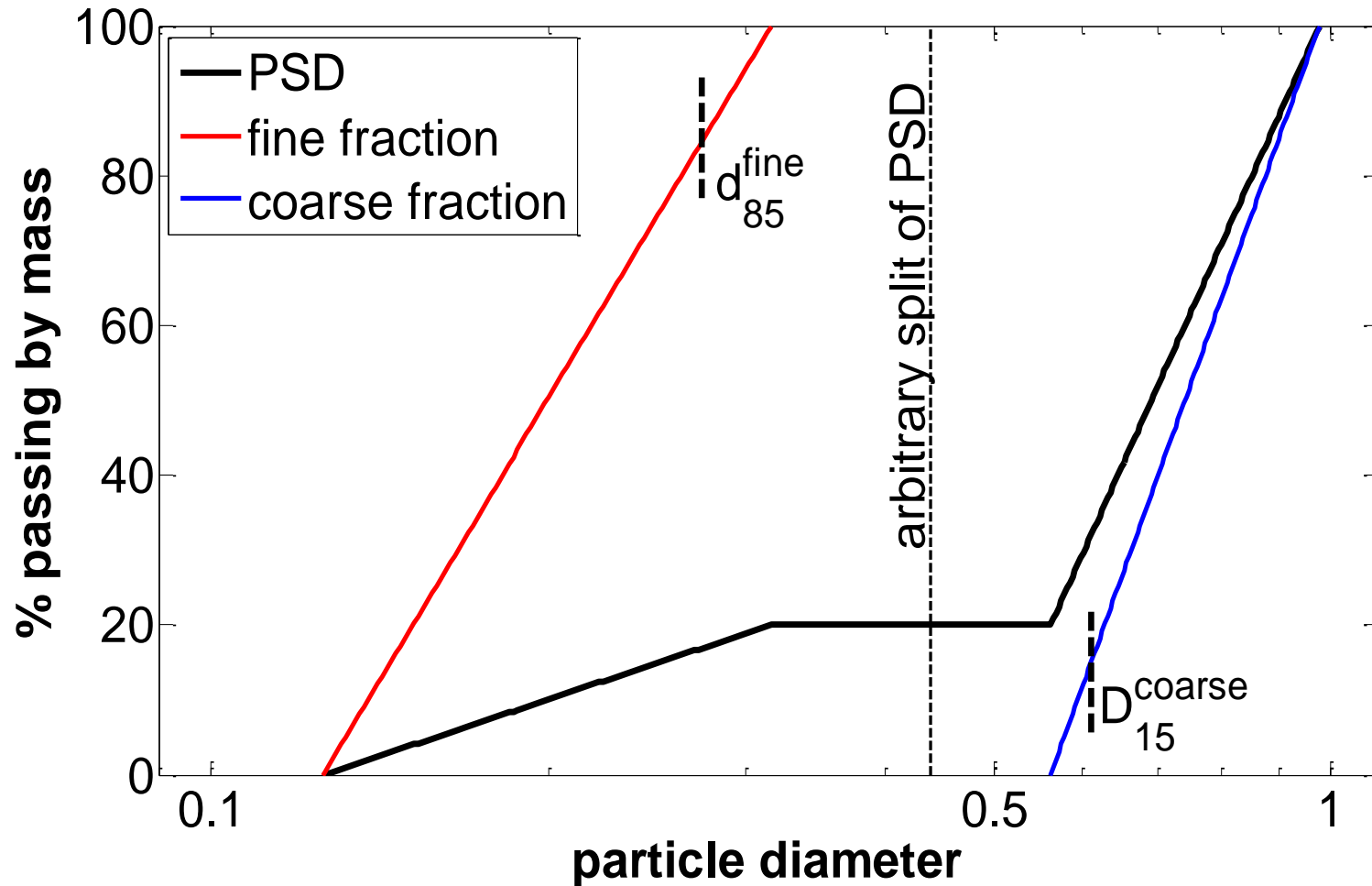
Factors influencing erosion risk



Factors influencing erosion risk



Empirical Filter Criteria: Kézdi (1979)



Split PSD into coarse and fine “PSDs”

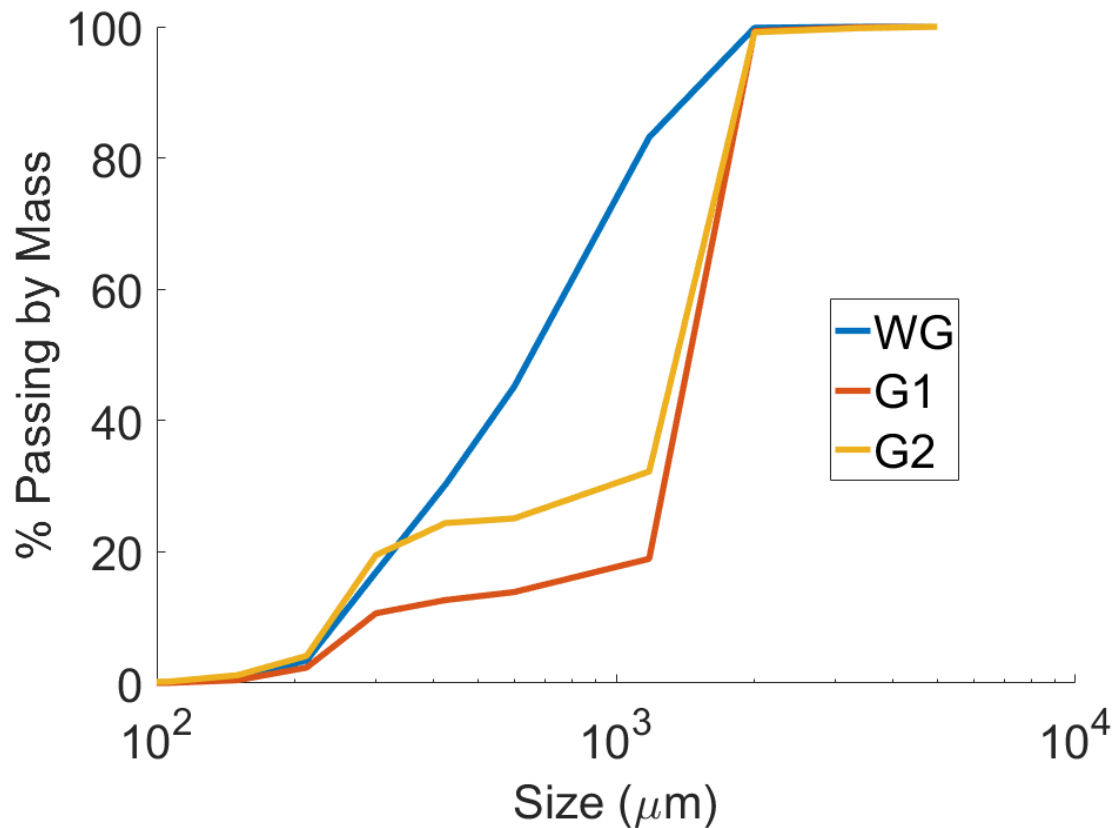
Stable if: $d_{85}^{fine} > (D_{15}^{coarse} / 4)$

i.e. if

$D_{15}^{coarse} / d_{85}^{fine} < 4$

Relates to Terzaghi filter rule

Internal Instability: μ CT study materials



Leighton Buzzard Sand

WG – Well graded

G1:

86%: $2360\mu\text{m} > D > 1180\mu\text{m}$

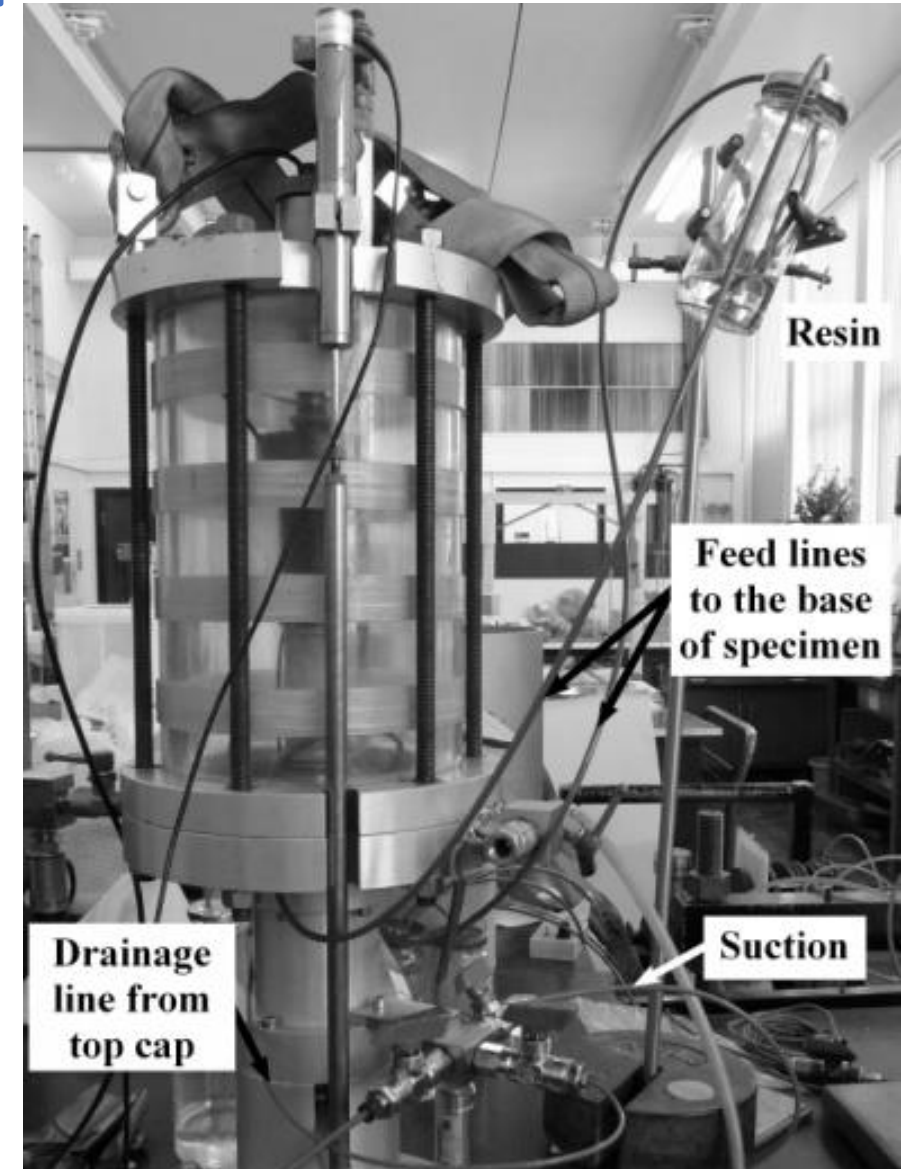
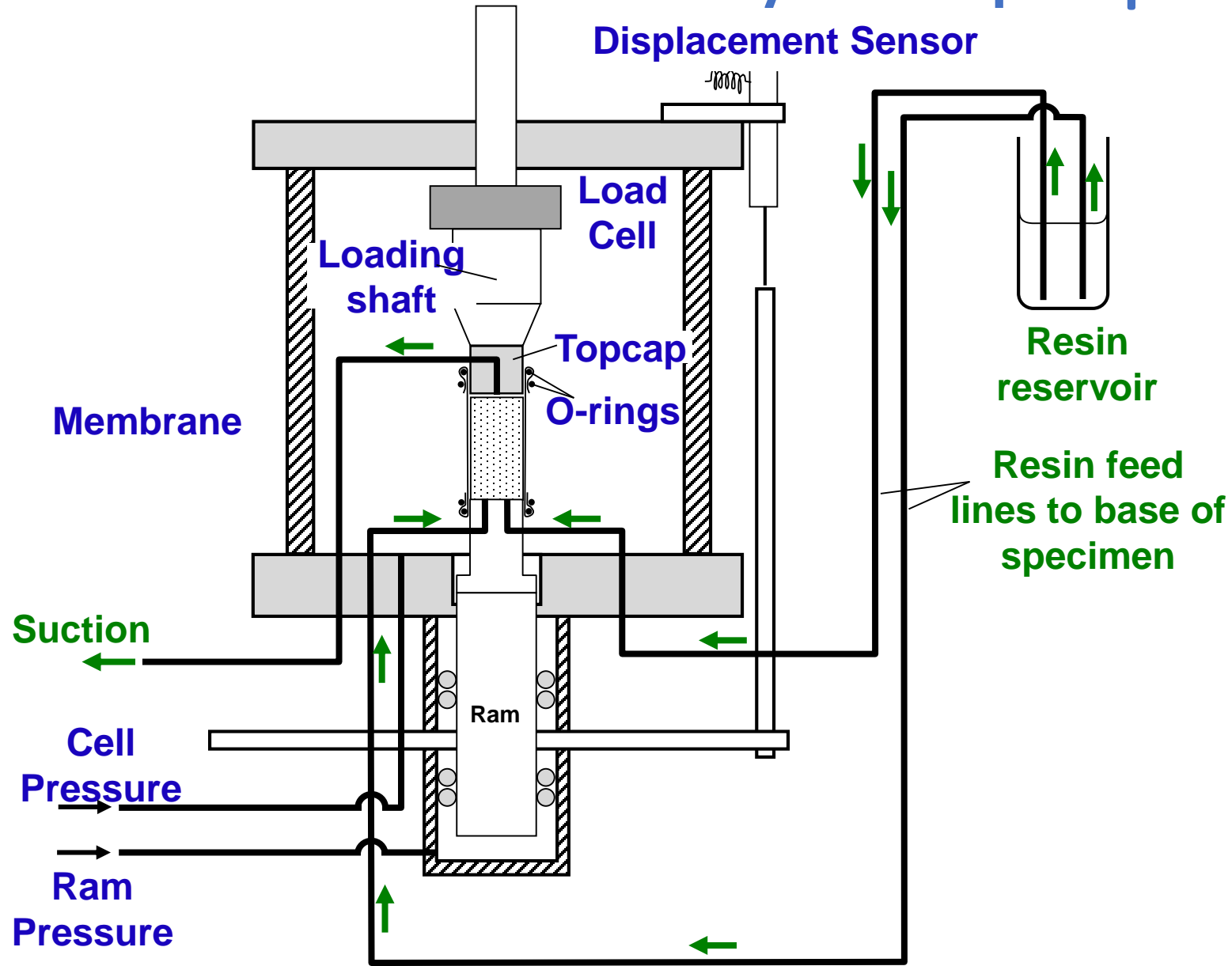
12%: $300\mu\text{m} > D > 150\mu\text{m}$

G2:

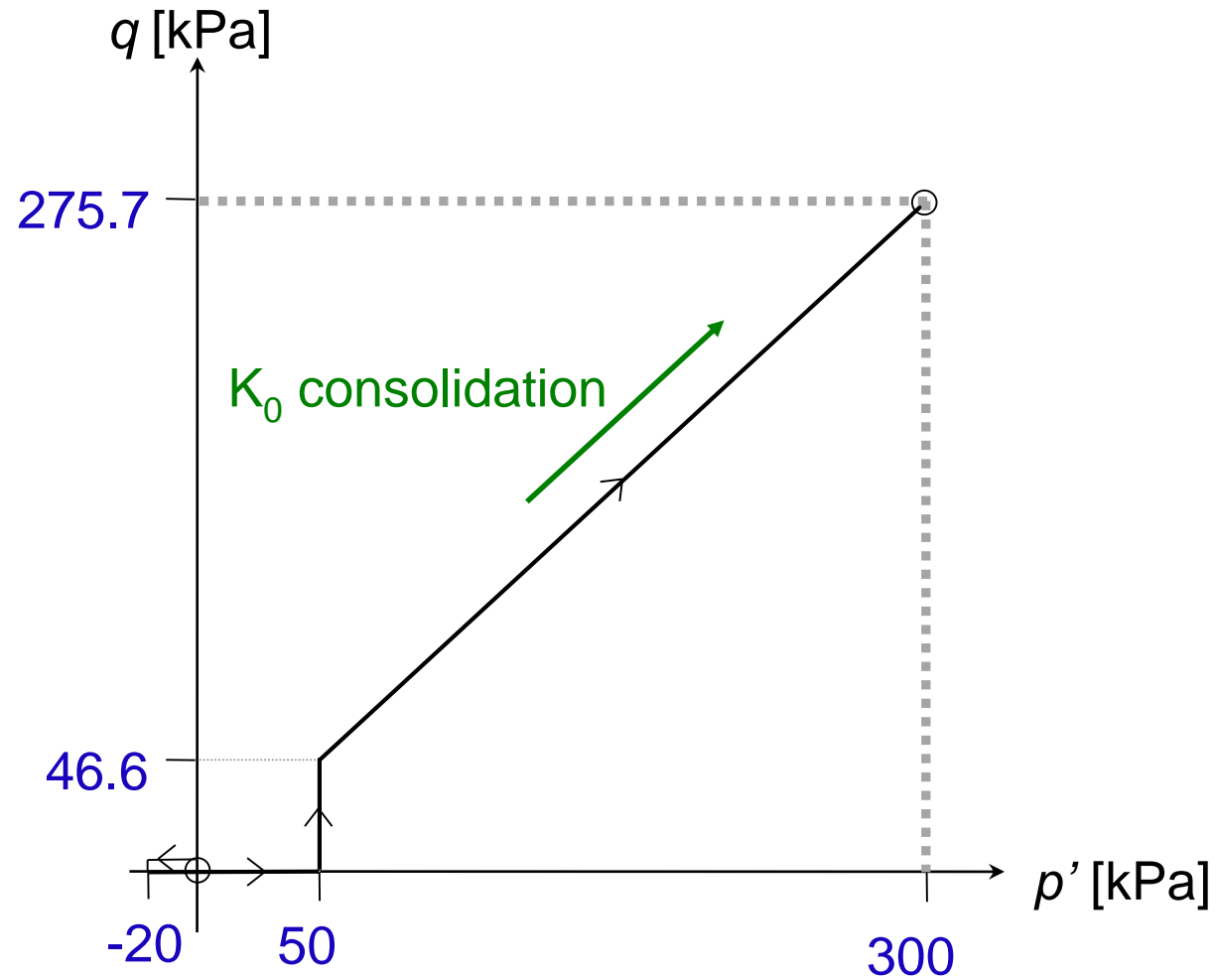
73%: $2360\mu\text{m} > D > 1180\mu\text{m}$

24%: $300\mu\text{m} > D > 150\mu\text{m}$

Internal Instability: sample preparation



Internal Instability: stress path

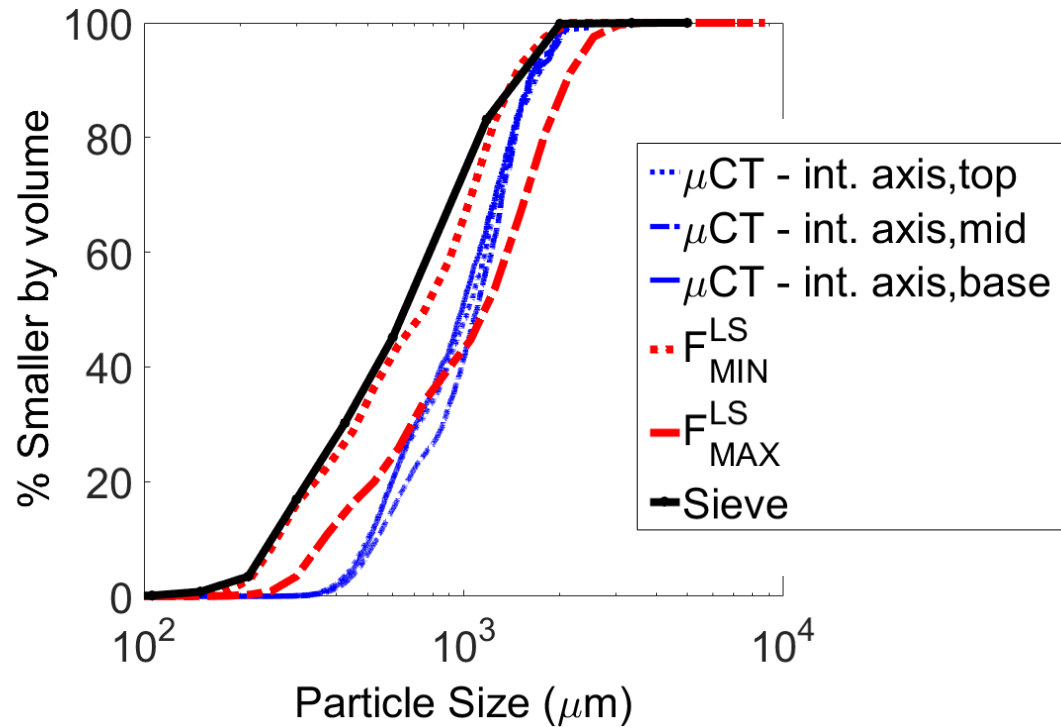


K_0 consolidation

$K_0 = 0.43$

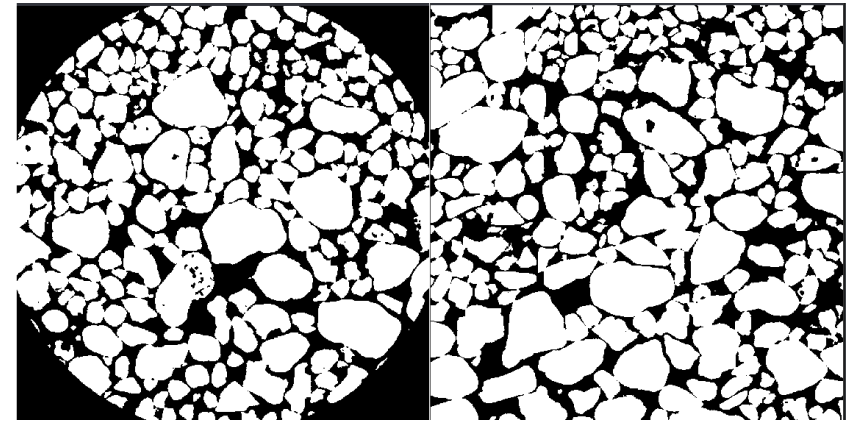
$\phi' = 35^\circ$

Well graded sample



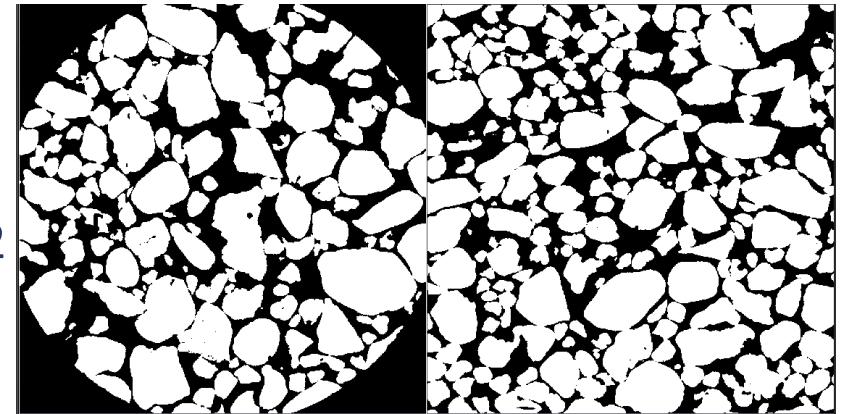
WG Top

$$D_{15}^{coarse}/d_{85}^{fine}=1.56$$



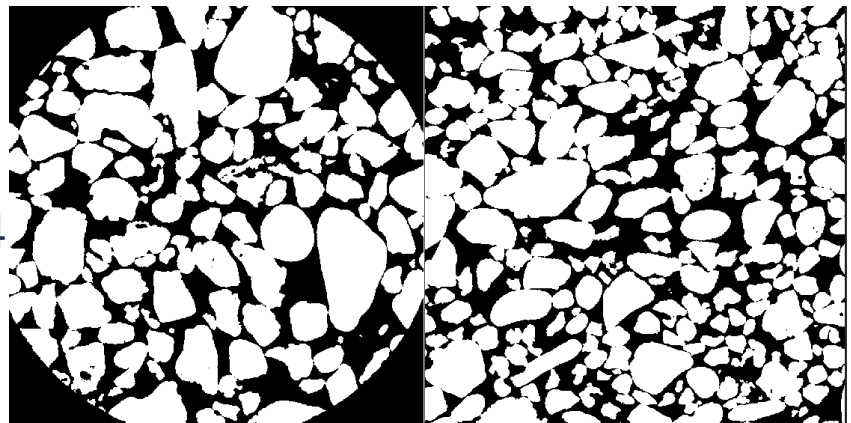
WG Middle

$$D_{15}^{coarse}/d_{85}^{fine}=1.62$$

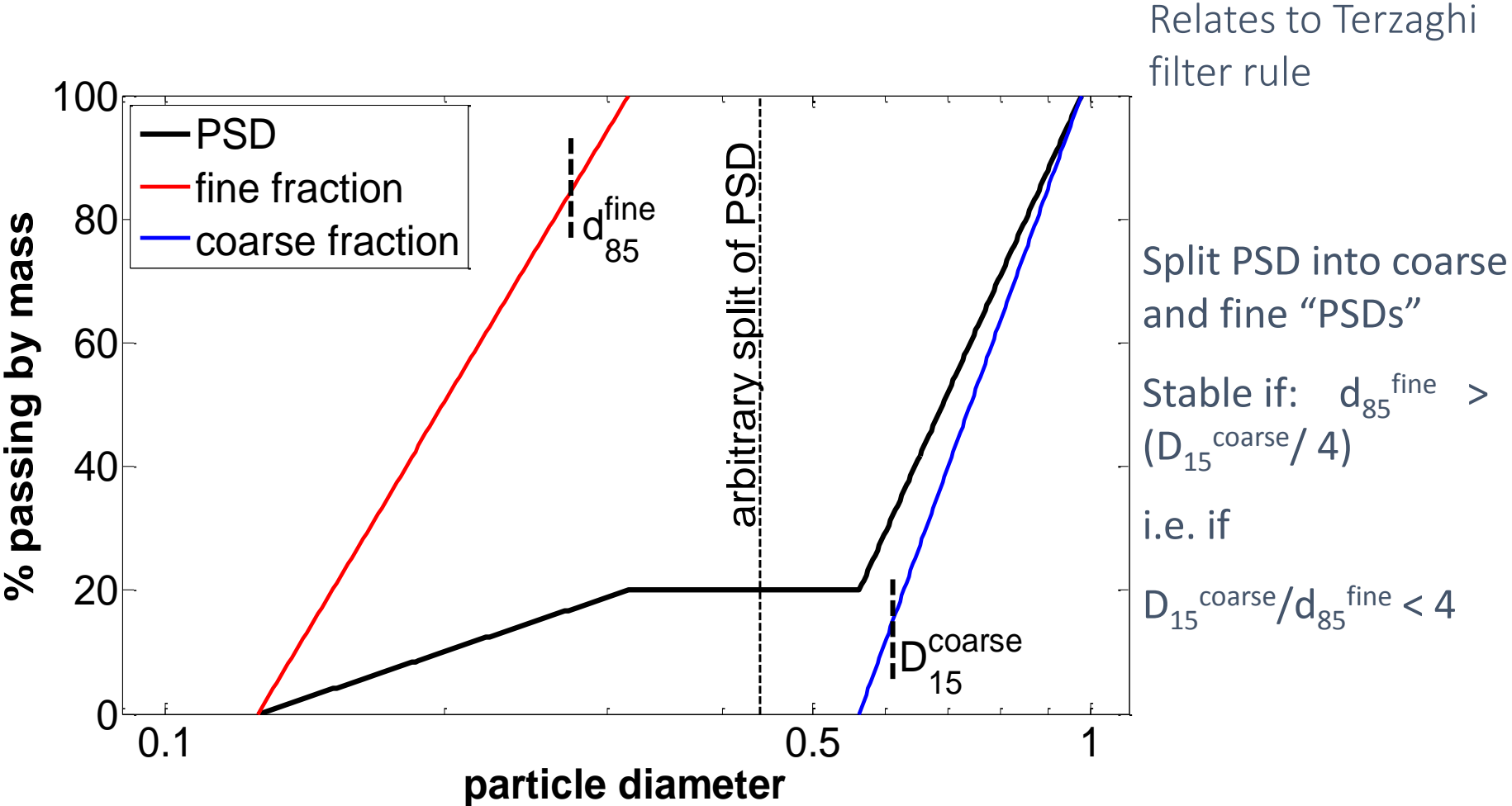


WG Bottom

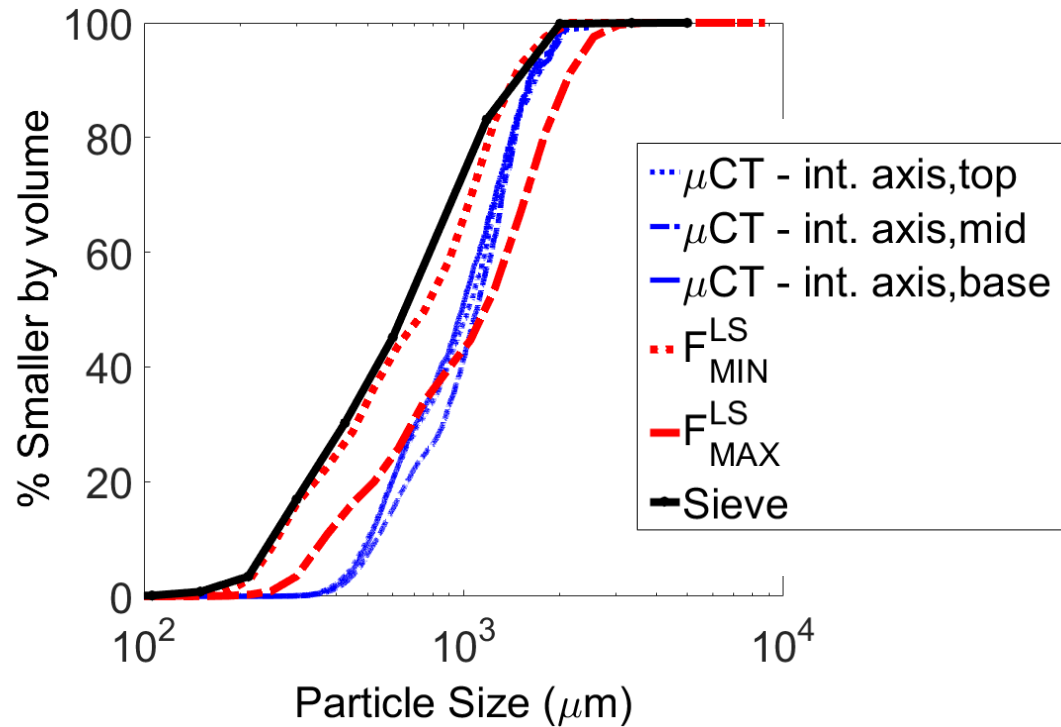
$$D_{15}^{coarse}/d_{85}^{fine}=1.54$$



Internal Instability: Filter criterion Kézdi (1979)

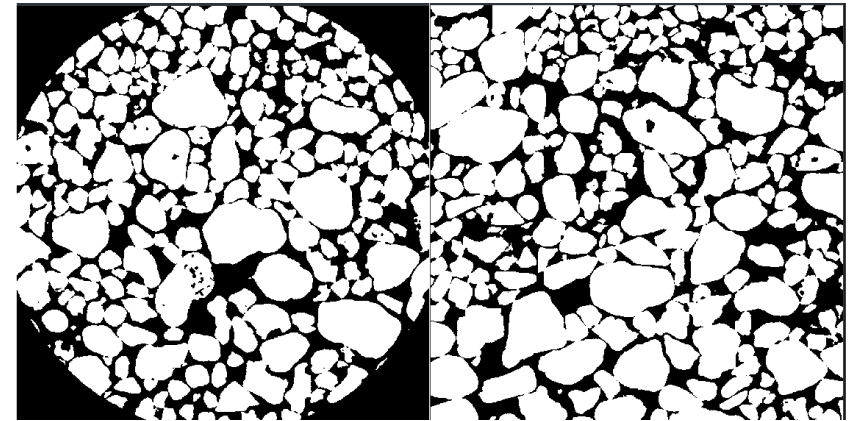


Well graded sample



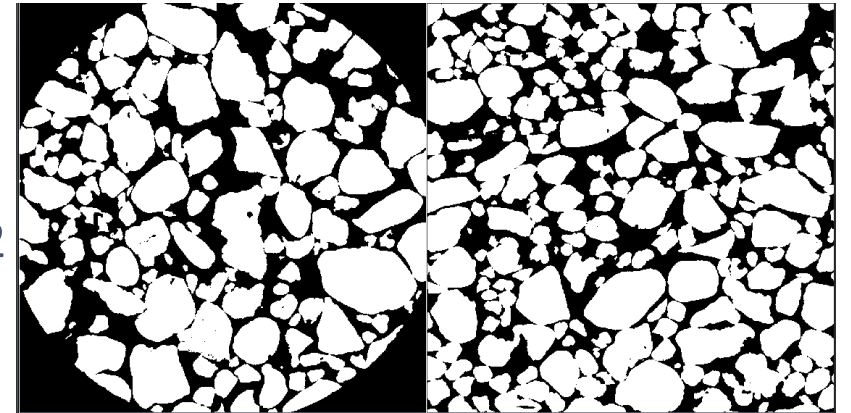
WG Top

$$D_{15}^{coarse}/d_{85}^{fine}=1.56$$



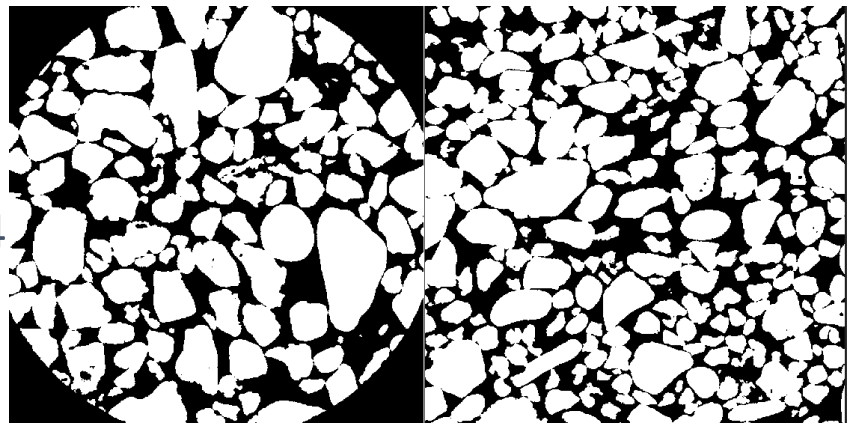
WG Middle

$$D_{15}^{coarse}/d_{85}^{fine}=1.62$$



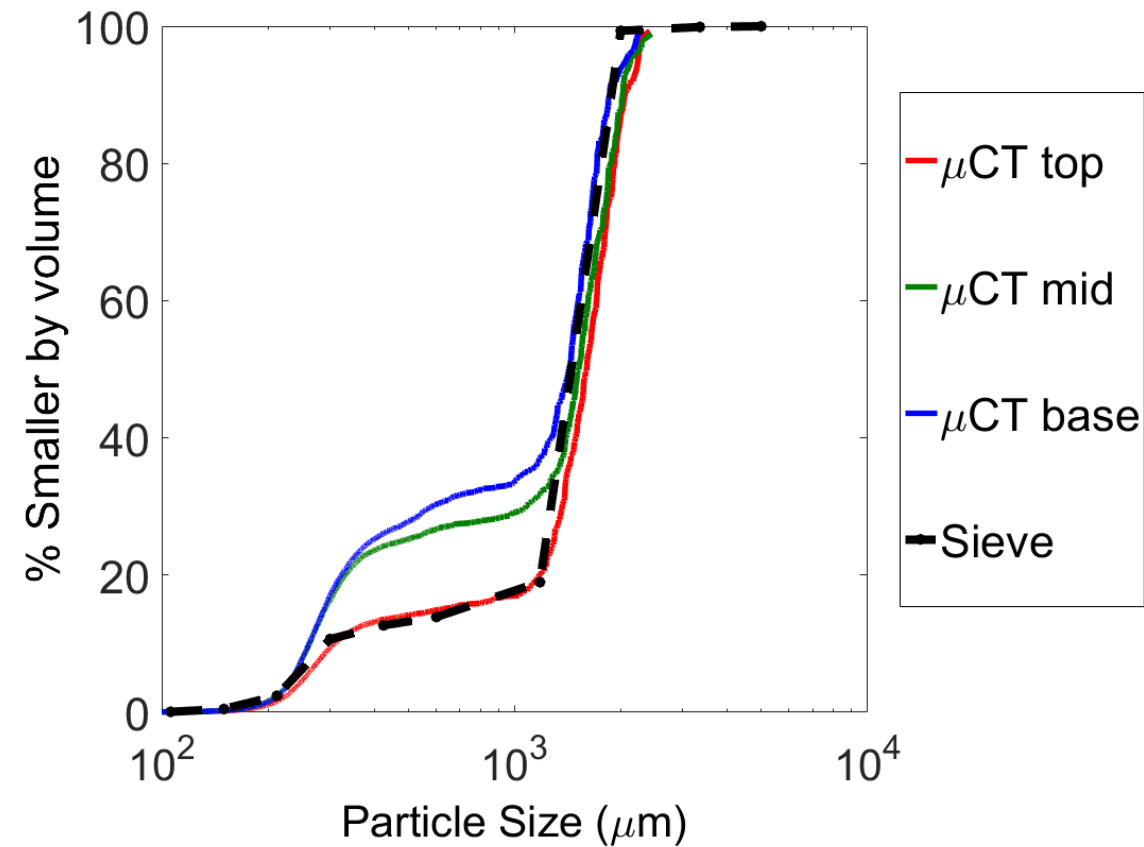
WG Bottom

$$D_{15}^{coarse}/d_{85}^{fine}=1.54$$



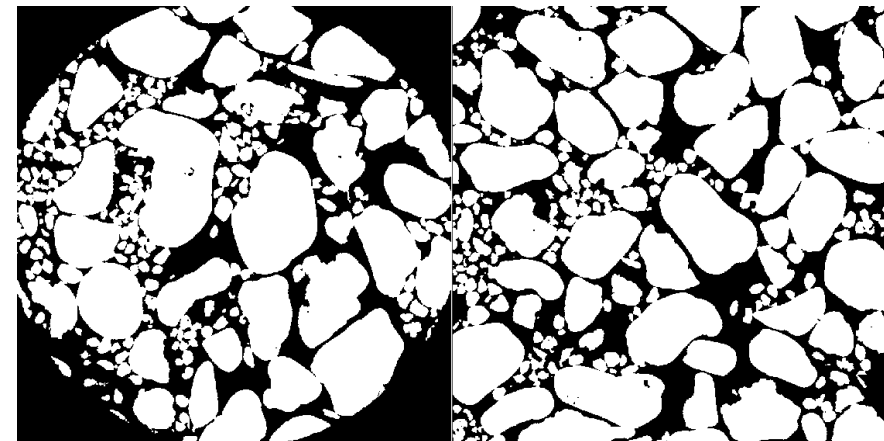
Sample G1

12%: 300mm>D>150mm



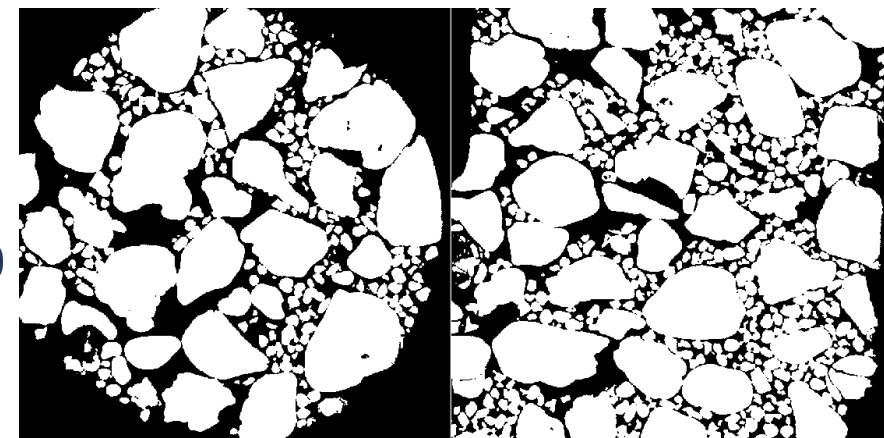
G1 Top

$$D_{15}^{\text{coarse}}/d_{85}^{\text{fine}}=4.66$$



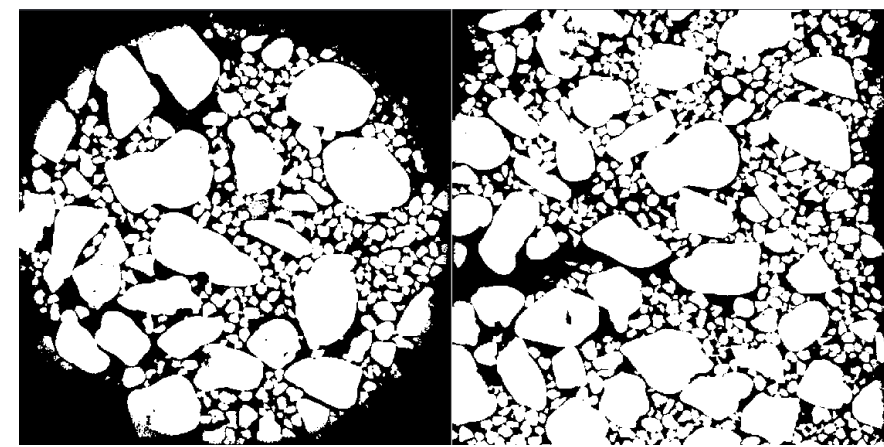
G1 Middle

$$D_{15}^{\text{coarse}}/d_{85}^{\text{fine}}=3.90$$

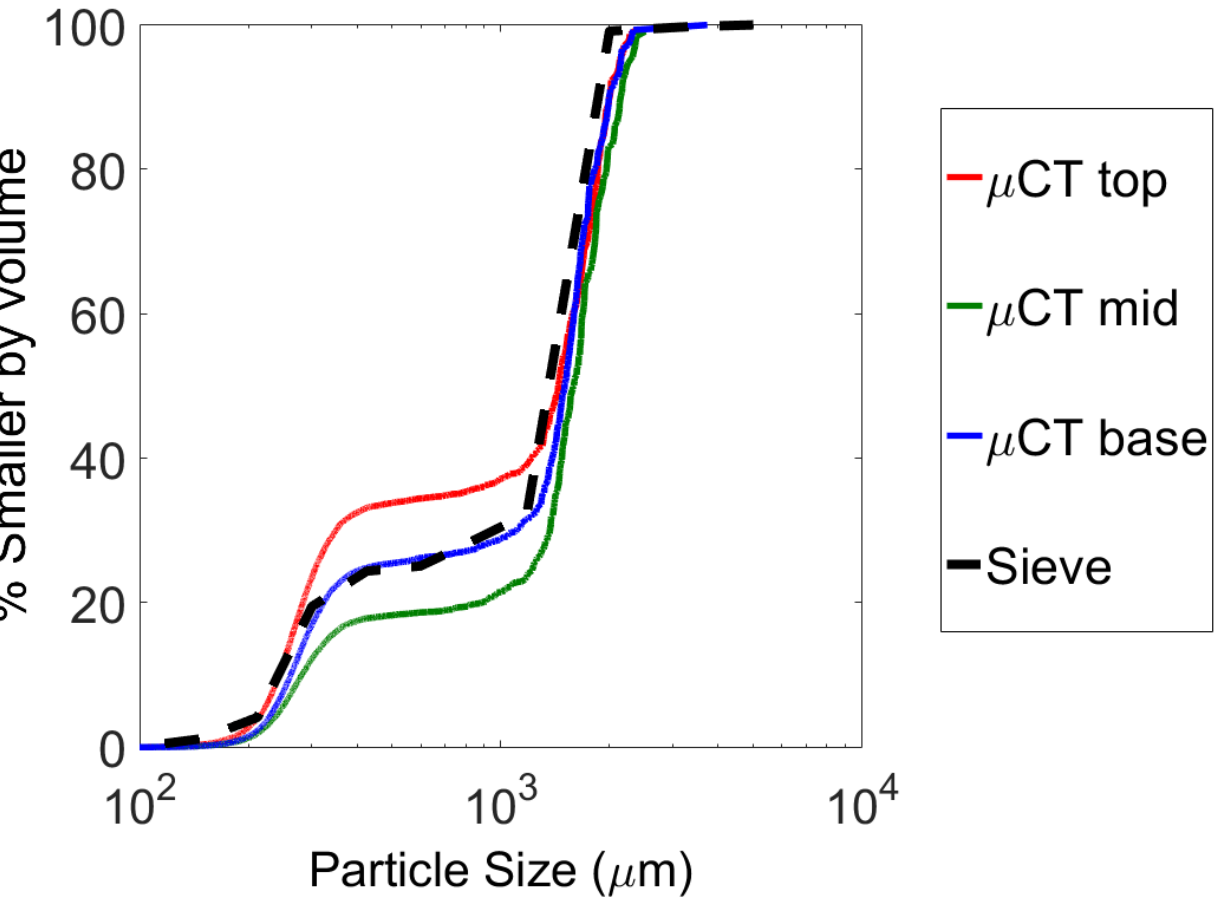


G1 Bottom

$$D_{15}^{\text{coarse}}/d_{85}^{\text{fine}}=3.30$$



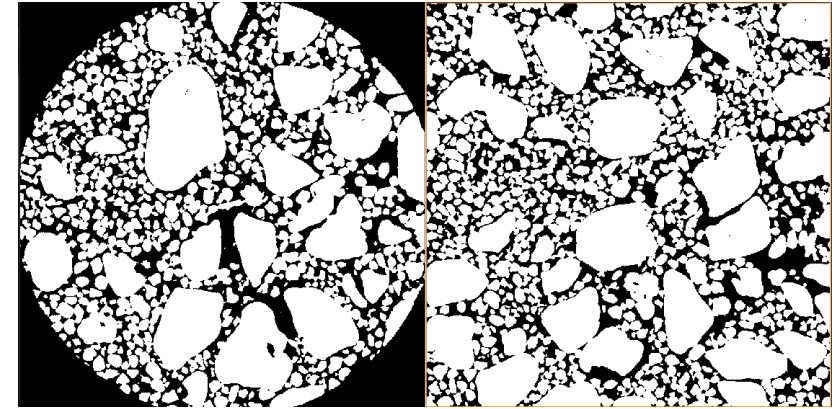
Sample G2



(24%: $300\mu\text{m} > D > 150\mu\text{m}$)

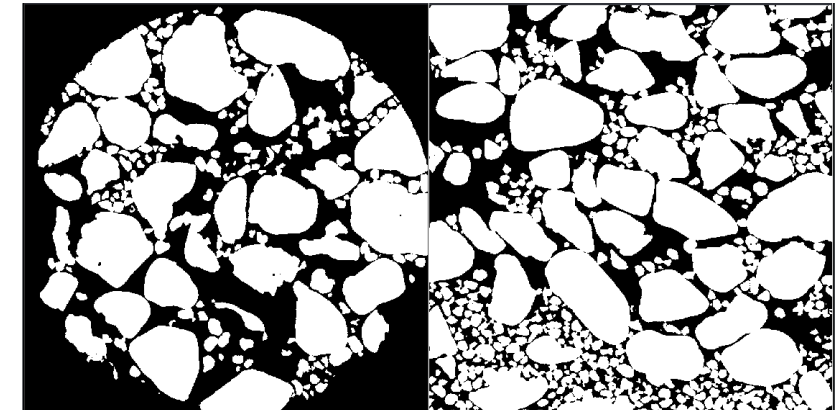
G2 Top

$$D_{15}^{\text{coarse}}/d_{85}^{\text{fine}}=4.01$$



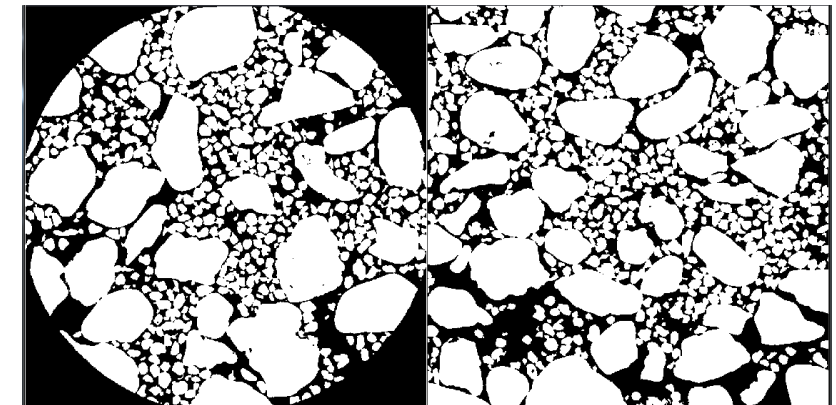
G2 Middle

$$D_{15}^{\text{coarse}}/d_{85}^{\text{fine}}=4.29$$



G2 Bottom

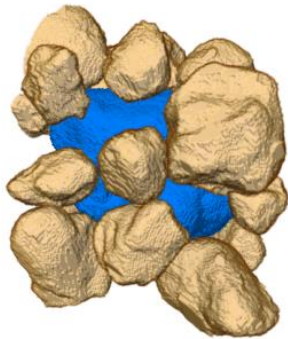
$$D_{15}^{\text{coarse}}/d_{85}^{\text{fine}}=4.07$$



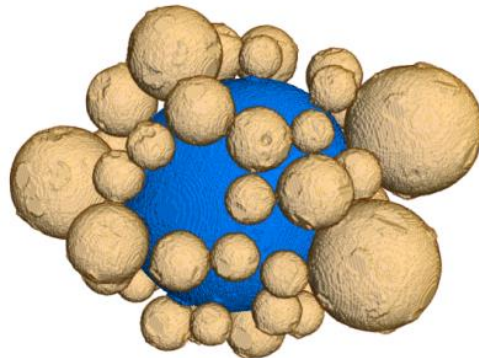
Coordination number

N_c = Coordination number

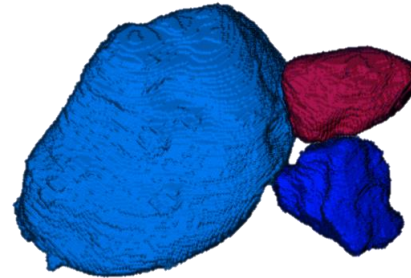
No of contacts per particle



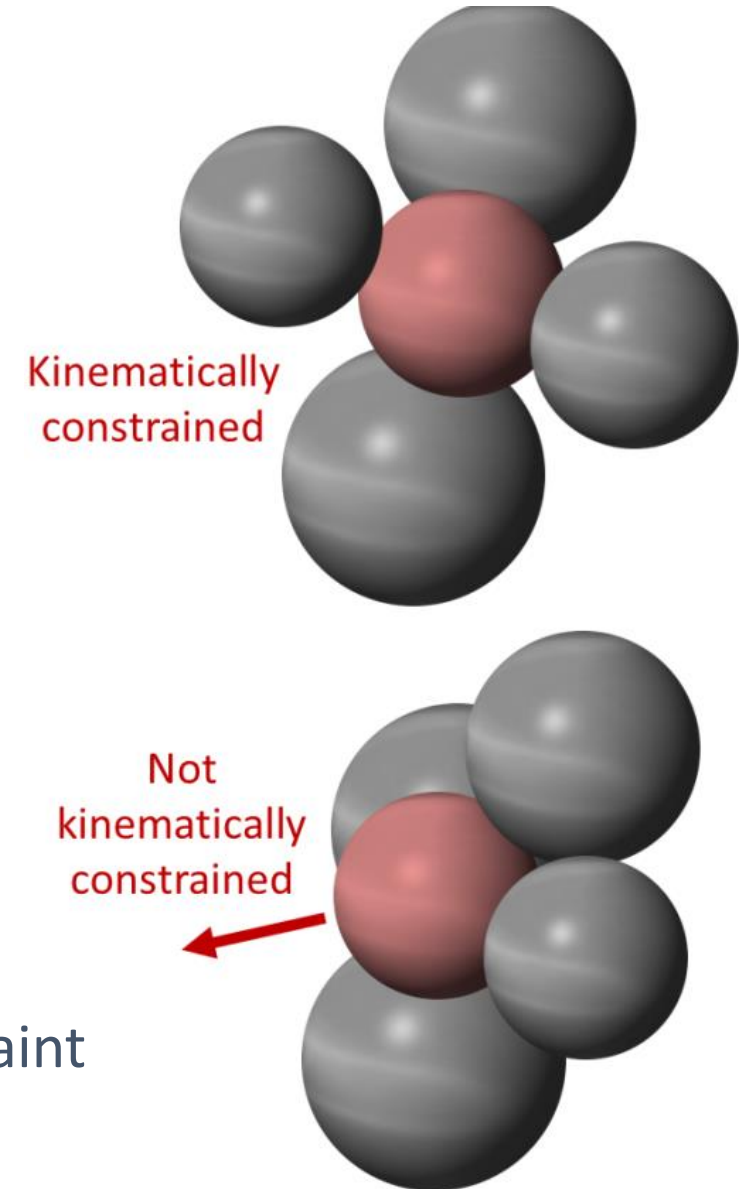
Leighton Buzzard
Sand
Blue particle
20 contacts



Glass beads
Blue particle
50 contacts

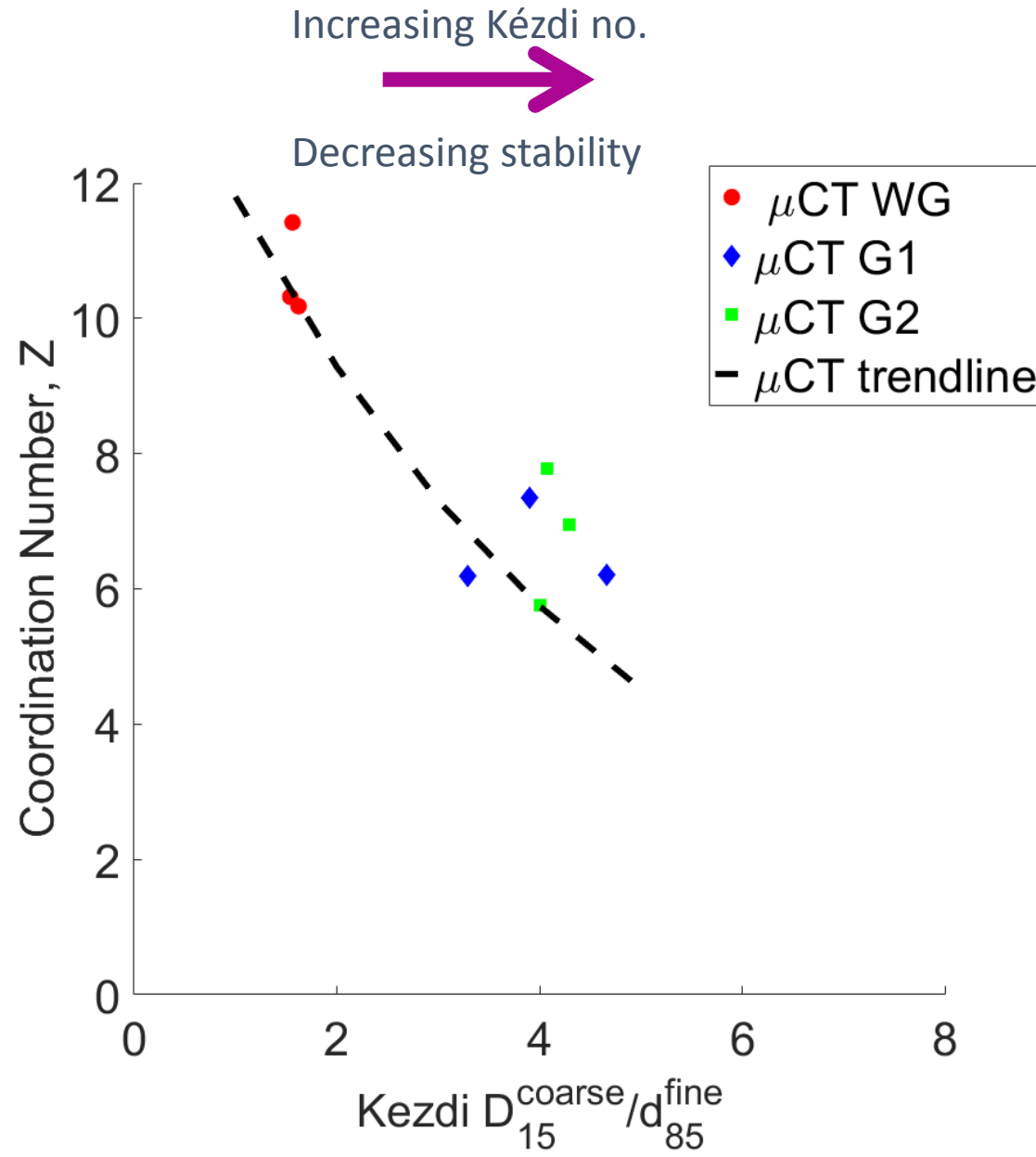


Leighton Buzzard
Sand
Blue particle
2 contacts



No of contacts gives indication of kinematic constraint

Variation in Coordination No. with Kézdi Ratio



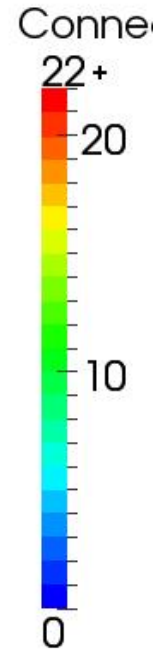
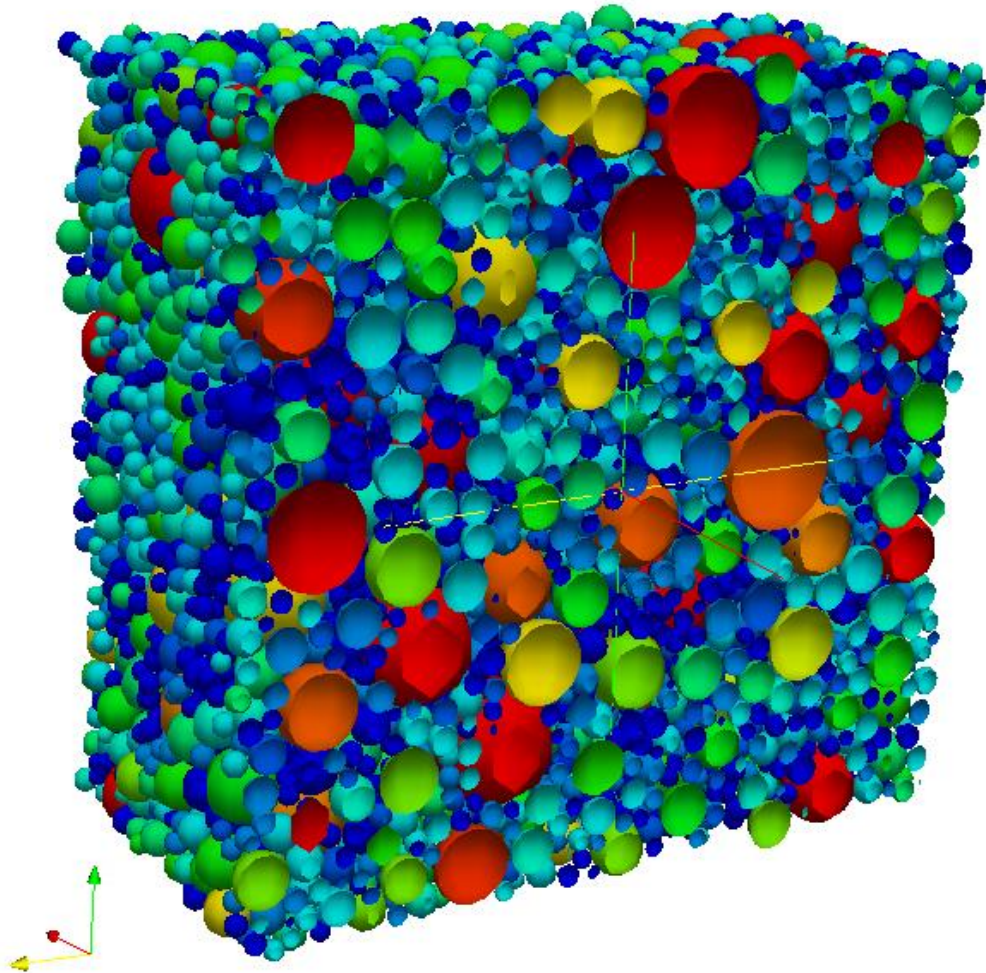
Fonseca et al. (2014)

Géotechnique

Shire and O'Sullivan (2013)

Acta Geotechnica

Discrete element method simulations



Spherical particles

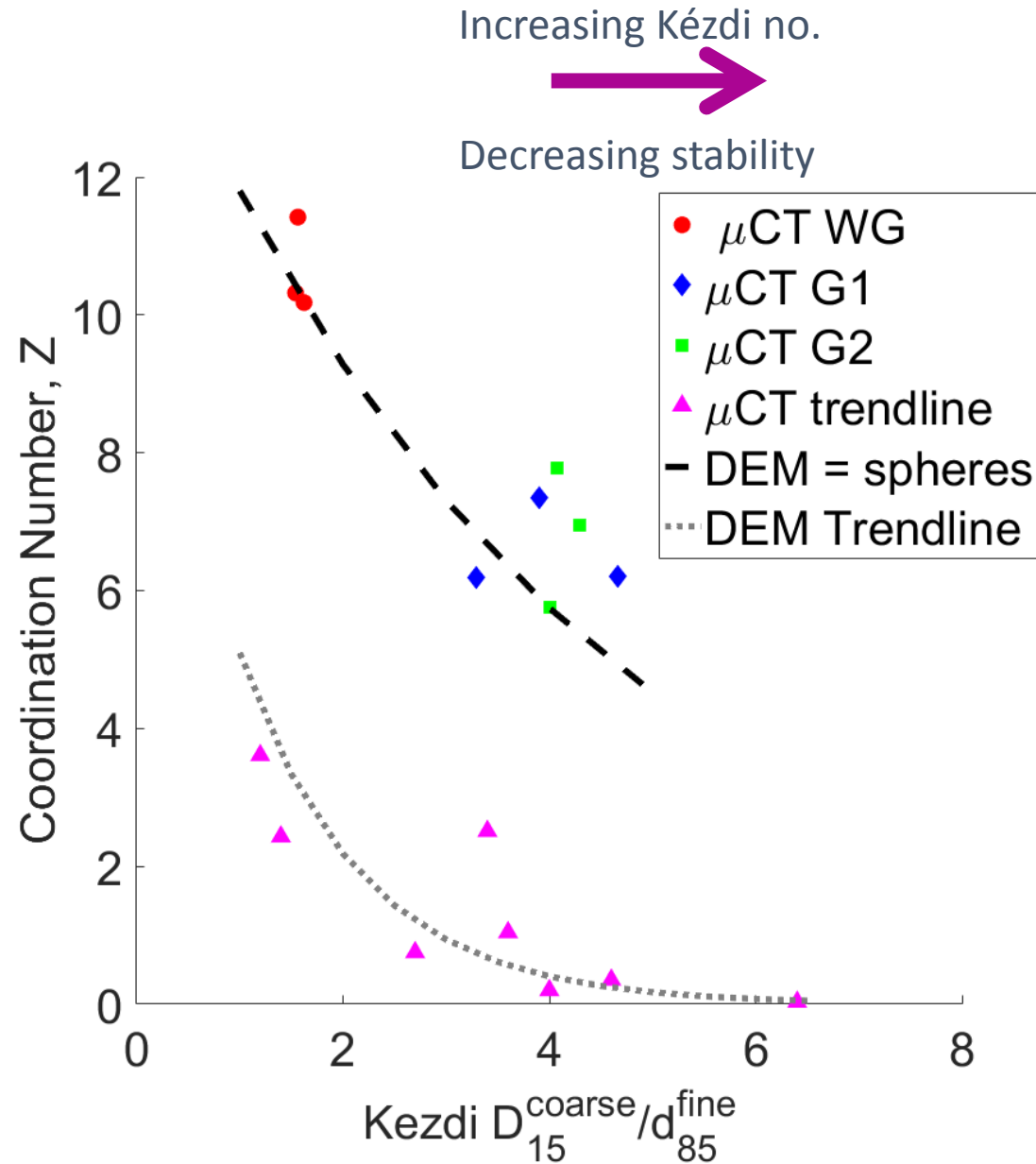
Simple contact models

Isotropic samples

Gravity neglected

Shire and O'Sullivan (2013)
Acta Geotechnica

Variation in Coordination No. with Kézdi Ratio



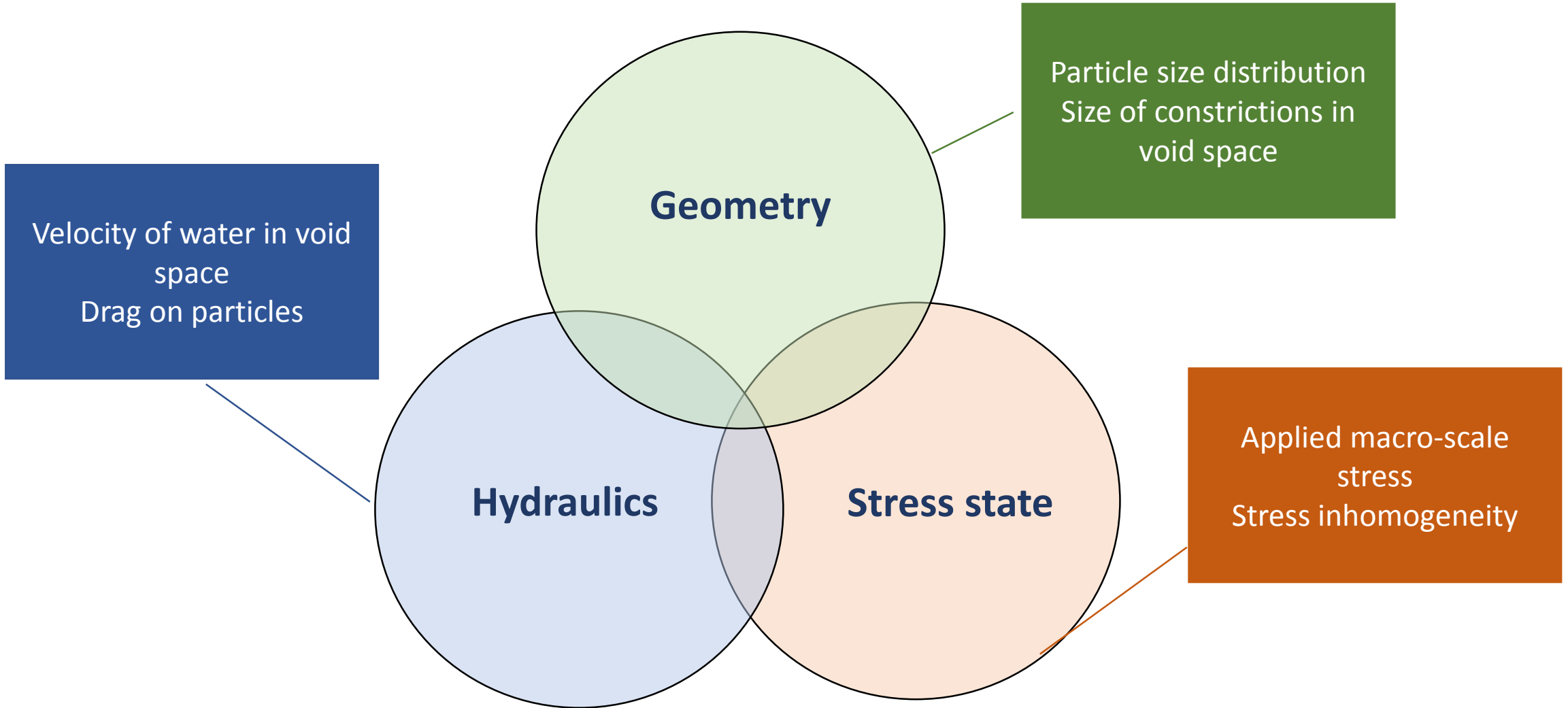
Fonseca et al. (2014)

Géotechnique

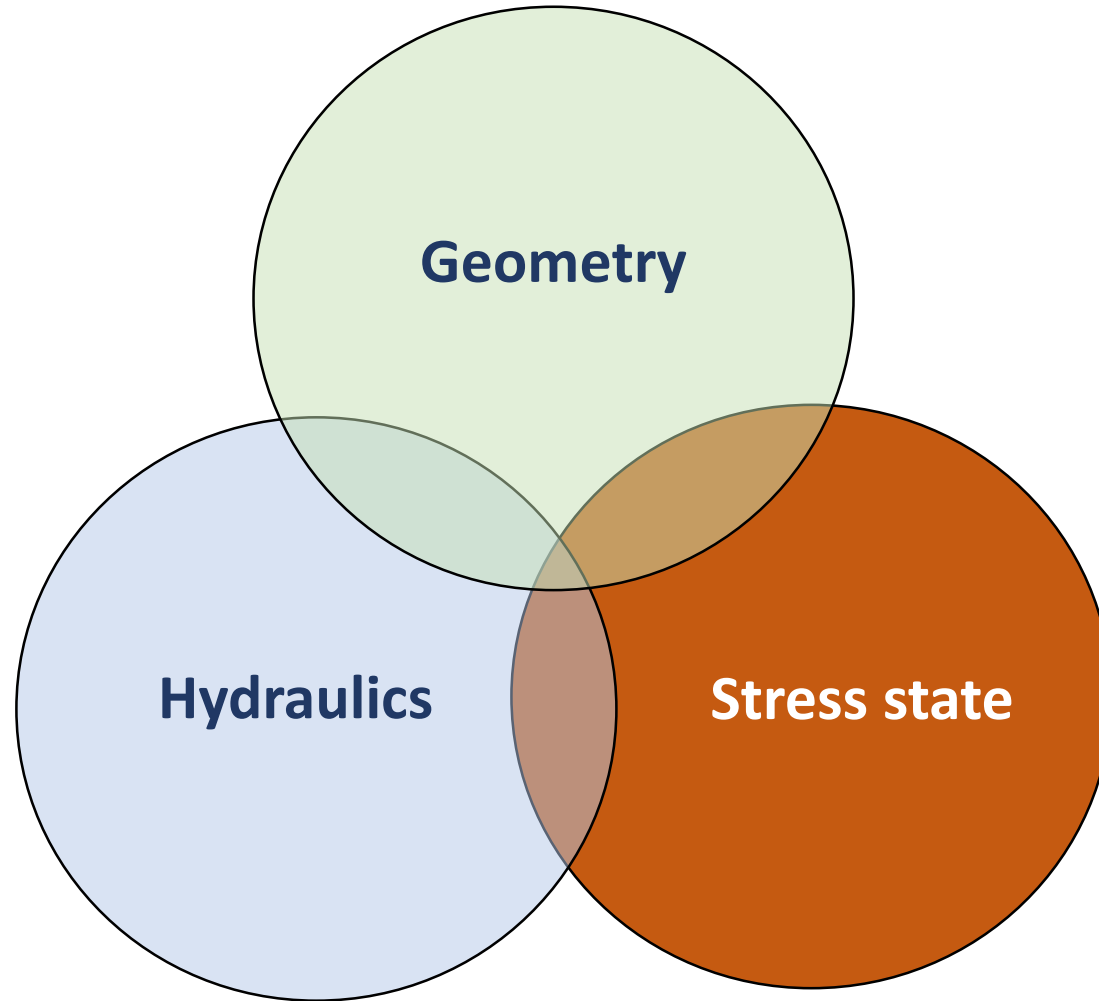
Shire and O'Sullivan (2013)

Acta Geotechnica

Factors influencing erosion risk



Factors influencing erosion risk



Stress Partition - α

- Hypothesis to explain erosion at low hydraulic gradients
- Based on observations of permeameter tests
- Coarse matrix transfers most of stress
- Finer grains carry reduced effective stress:

$$\sigma'_{\text{fines}} = \alpha \times \sigma'$$



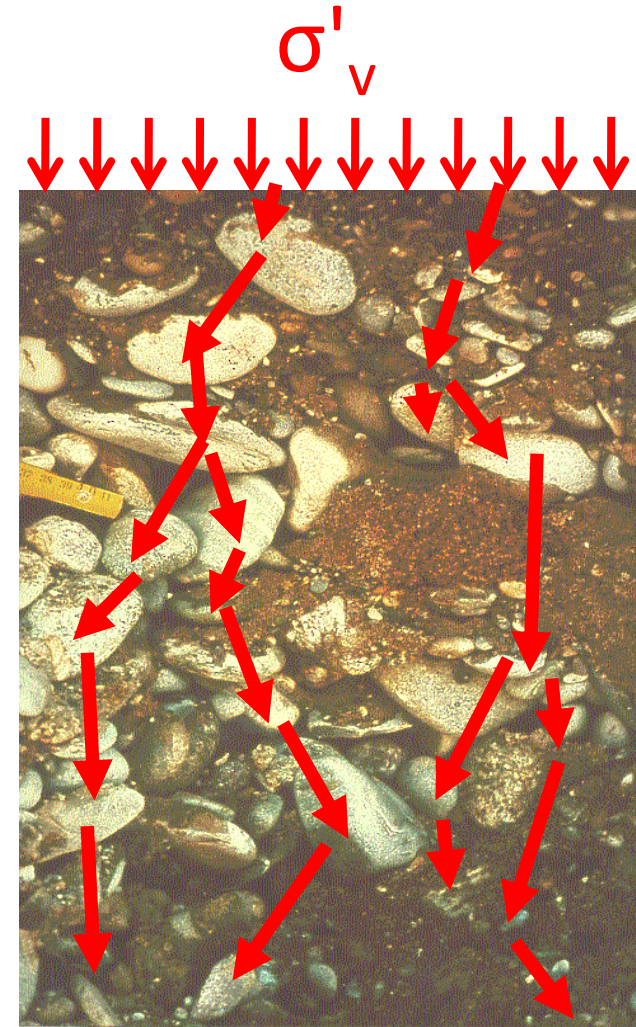
Skempton and Brogan (1994)
Géotechnique

Stress Partition - α

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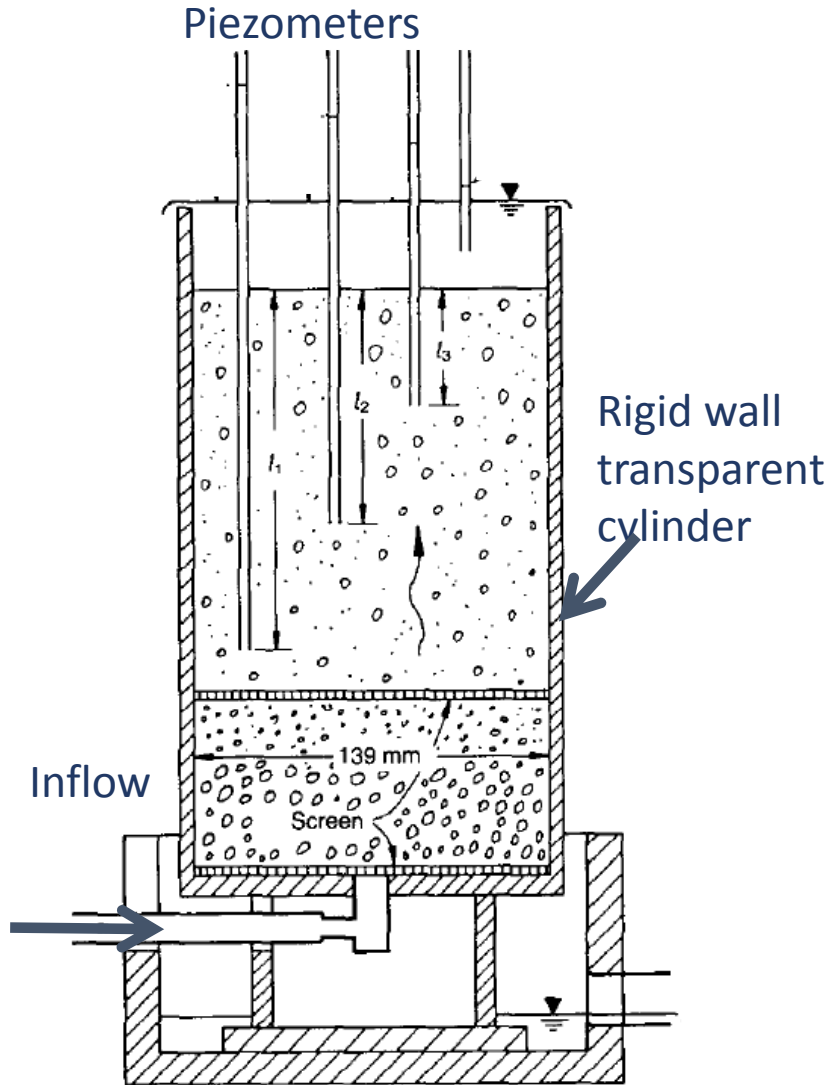
$$\sigma'_{\text{fines}} = \alpha \times \sigma'$$

Skempton and Brogan (1994)
Géotechnique

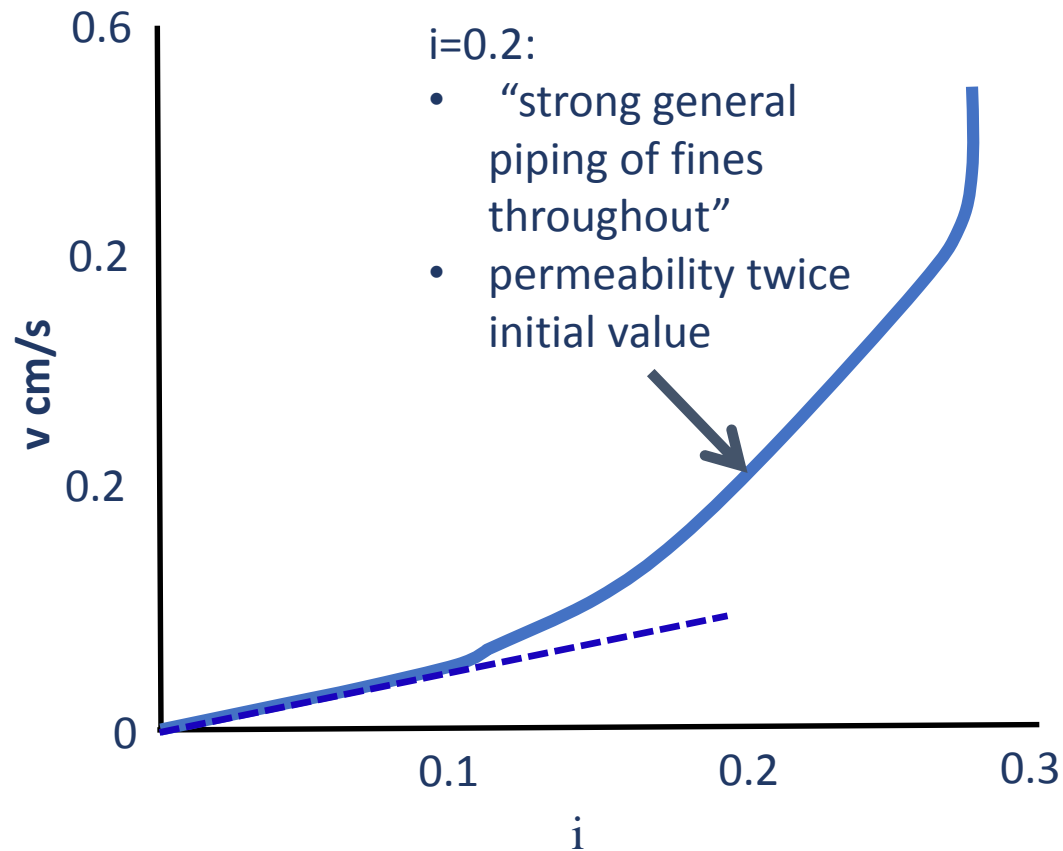


Skempton and Brogan Permeameter Experiments

Skempton and Brogan (1994)
Géotechnique



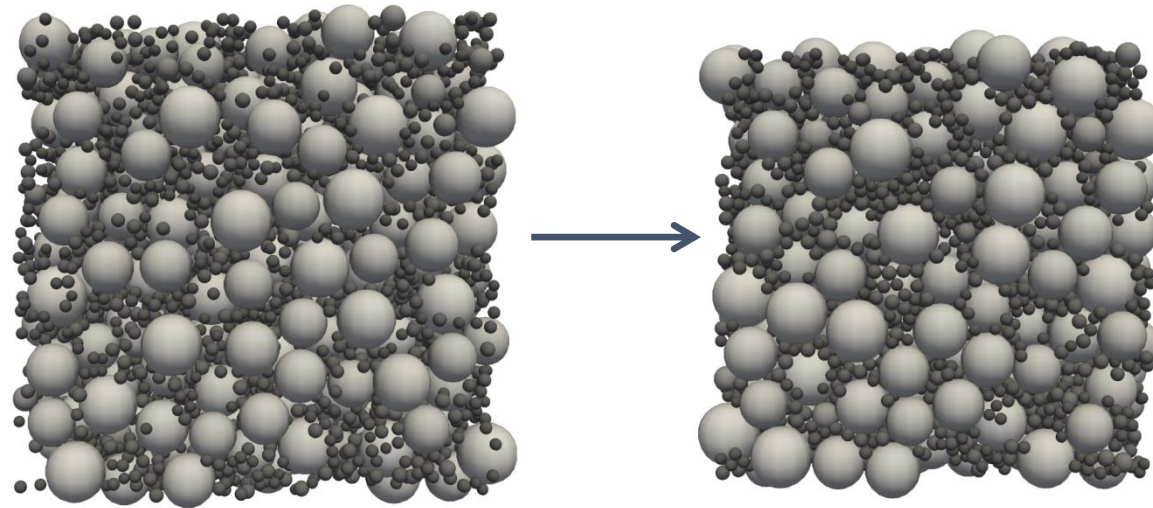
Sample A



α is indirectly calculated

$$\alpha = i_{\text{crit}} / i_{\text{crit(heave)}}$$

DEM Simulations to Investigate Instability



- DEM code granular LAMMPS with periodic boundaries
- Isotropic compression at to $p' = 50\text{kPa}$
- Sample density controlled using inter particle friction (μ):

$\mu = 0.0$ (Dense)

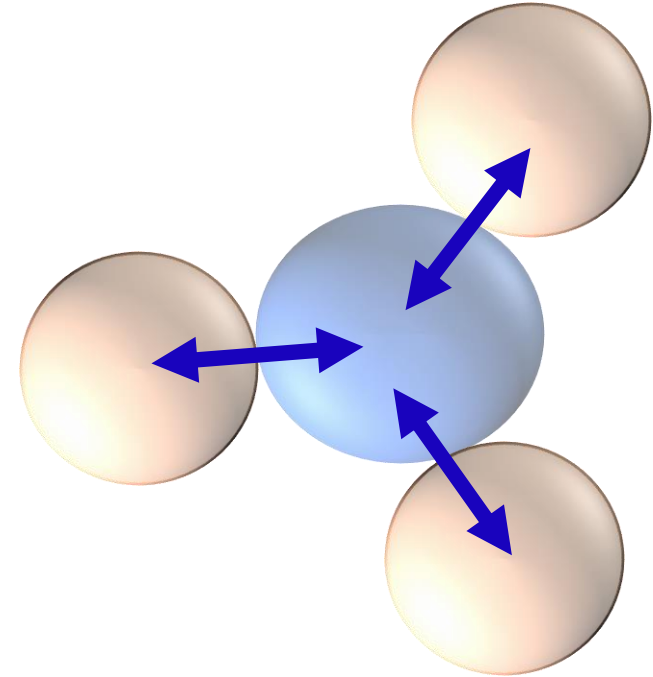
$\mu = 0.1$ (Medium dense)

$\mu = 0.3$ (Loose)

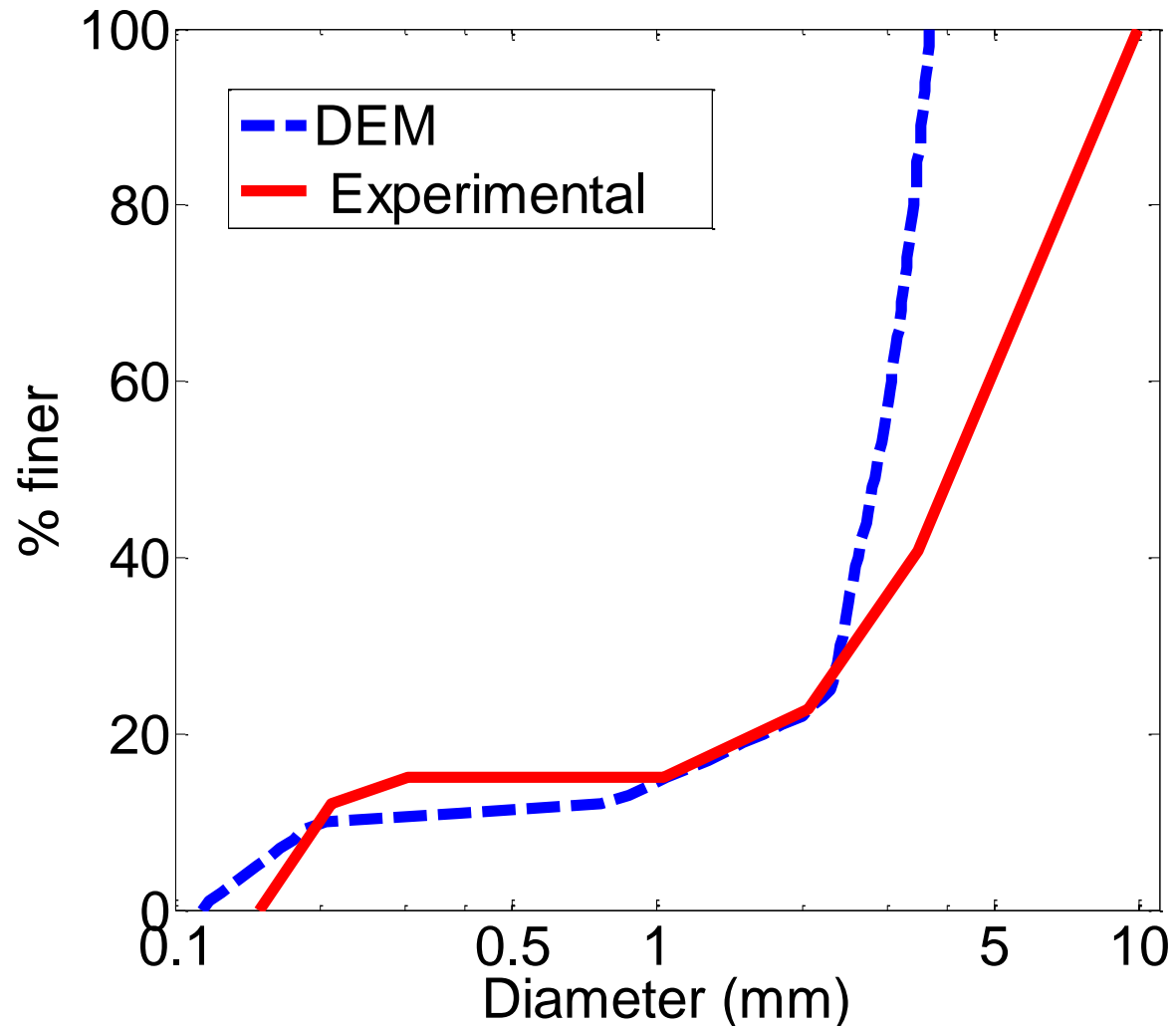
α – DEM Calculations

$$\alpha = \frac{p'_{fine}}{p'}$$

- p' =overall mean effective stress
- p'_{fine} =mean effective stress in finer fraction
- p' and p'_{fine} can be directly obtained from a summation of contact forces in DEM



Skempton and Brogan Sample A: comparison of α values



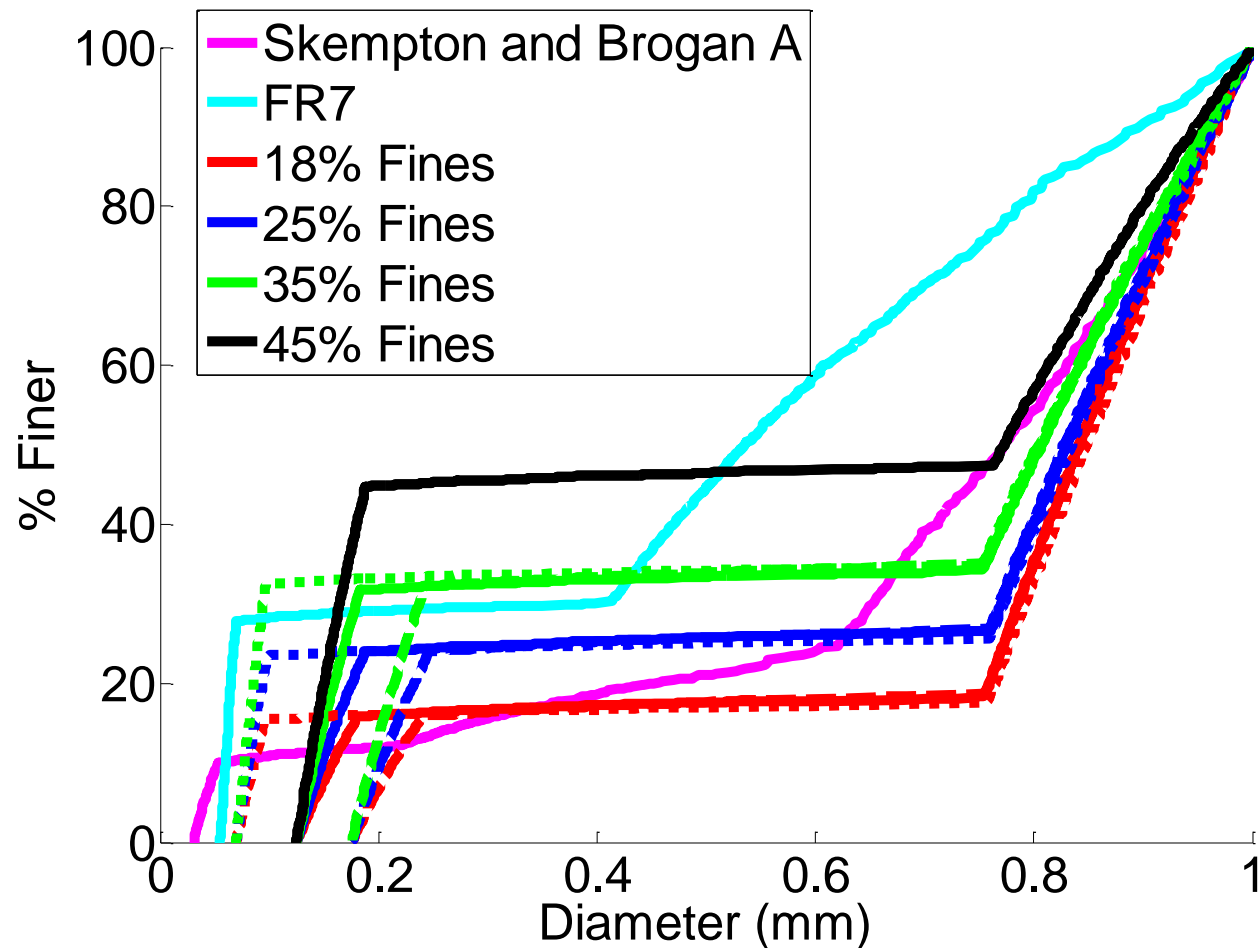
Density	α_{DEM}
Loose	0.15
Medium	0.06
Dense	0.04

$$\alpha_{\text{experiment}} = 0.18$$

Experimental sample placed moist with no densification

Shire et al. (2014) ASCE
JGGE

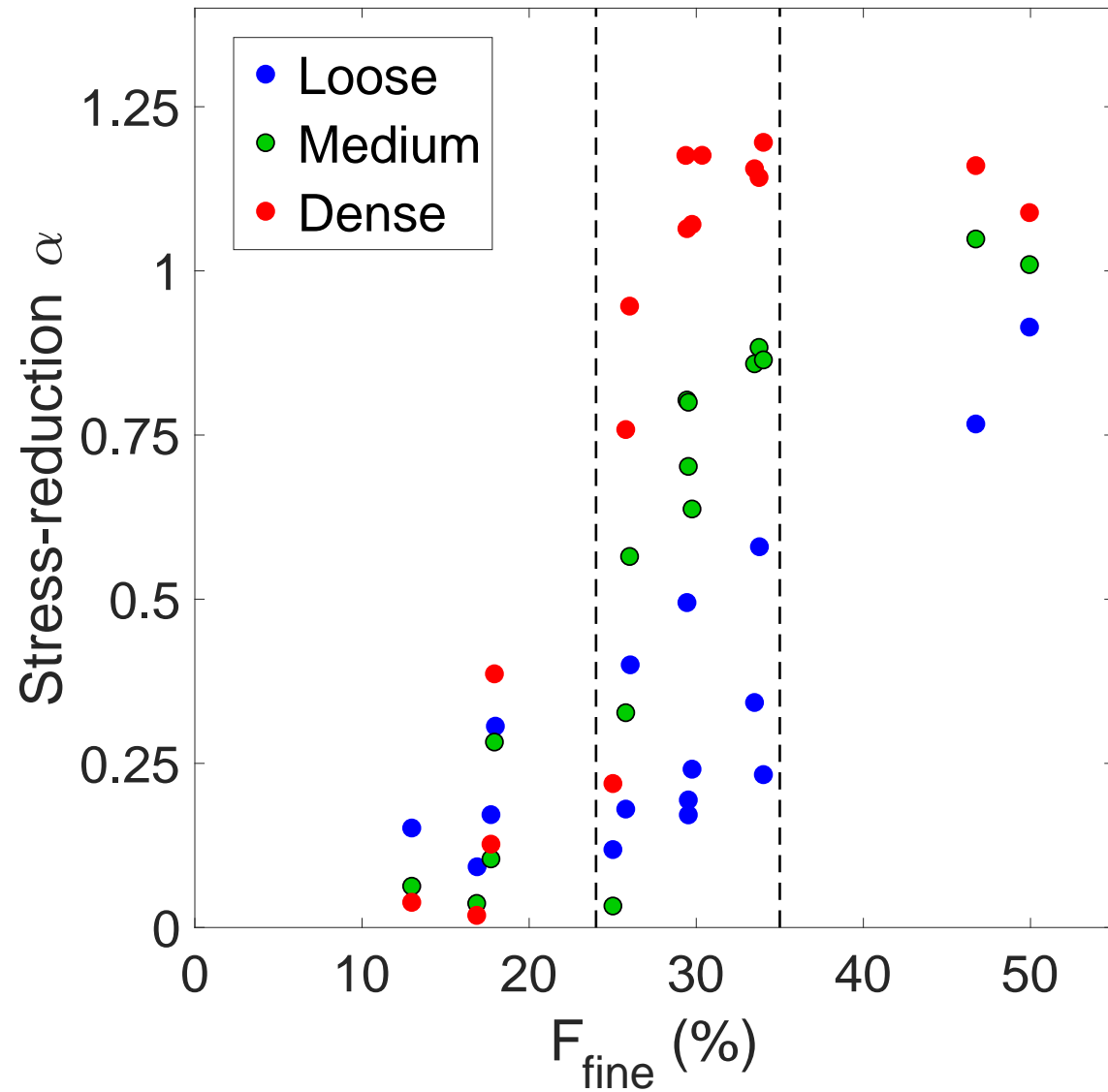
Link between α and particle size distribution



Looked at a range of gap graded materials

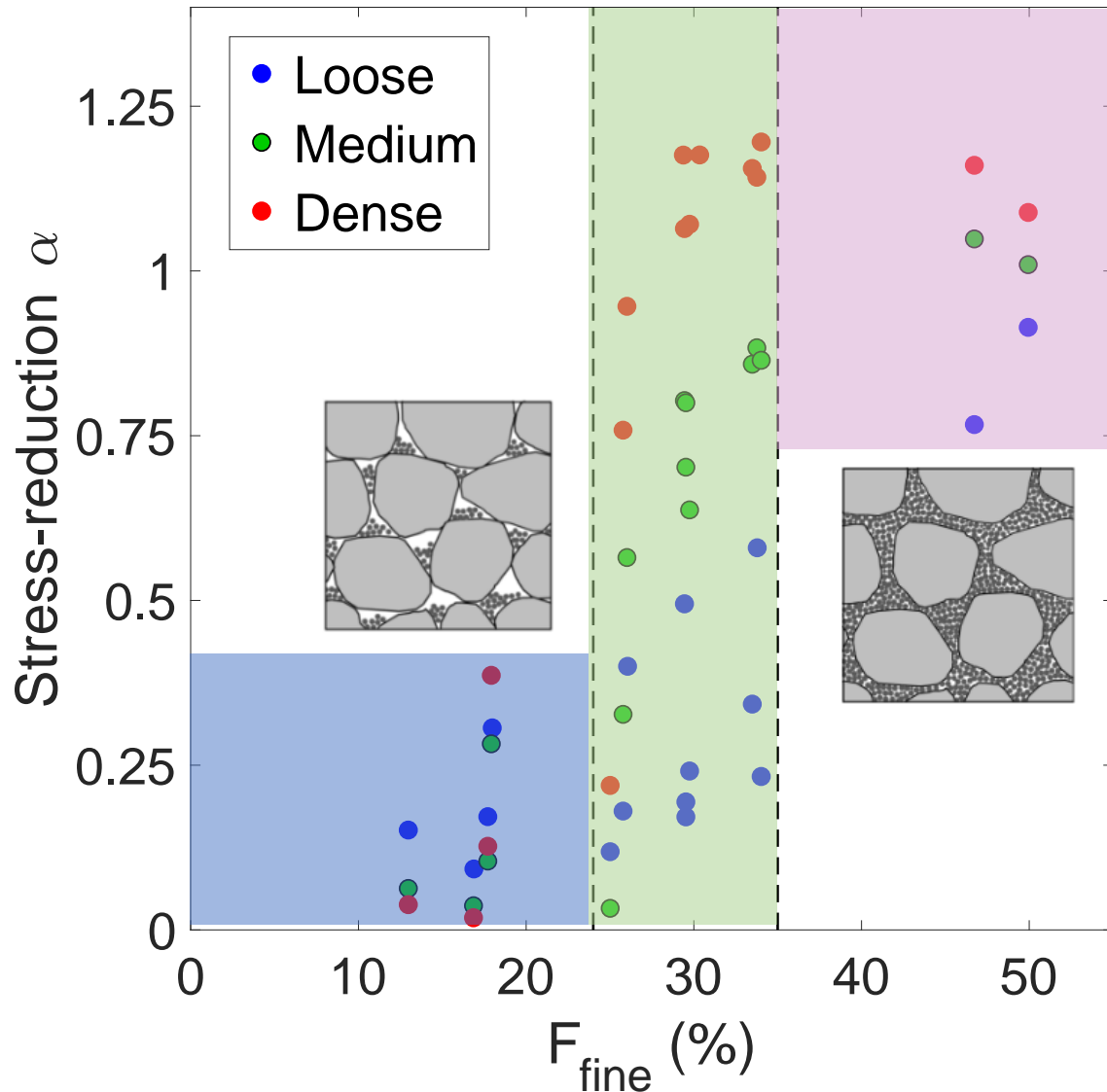
Density varied for all samples

Variation in α with Fines Content (F_{fine})



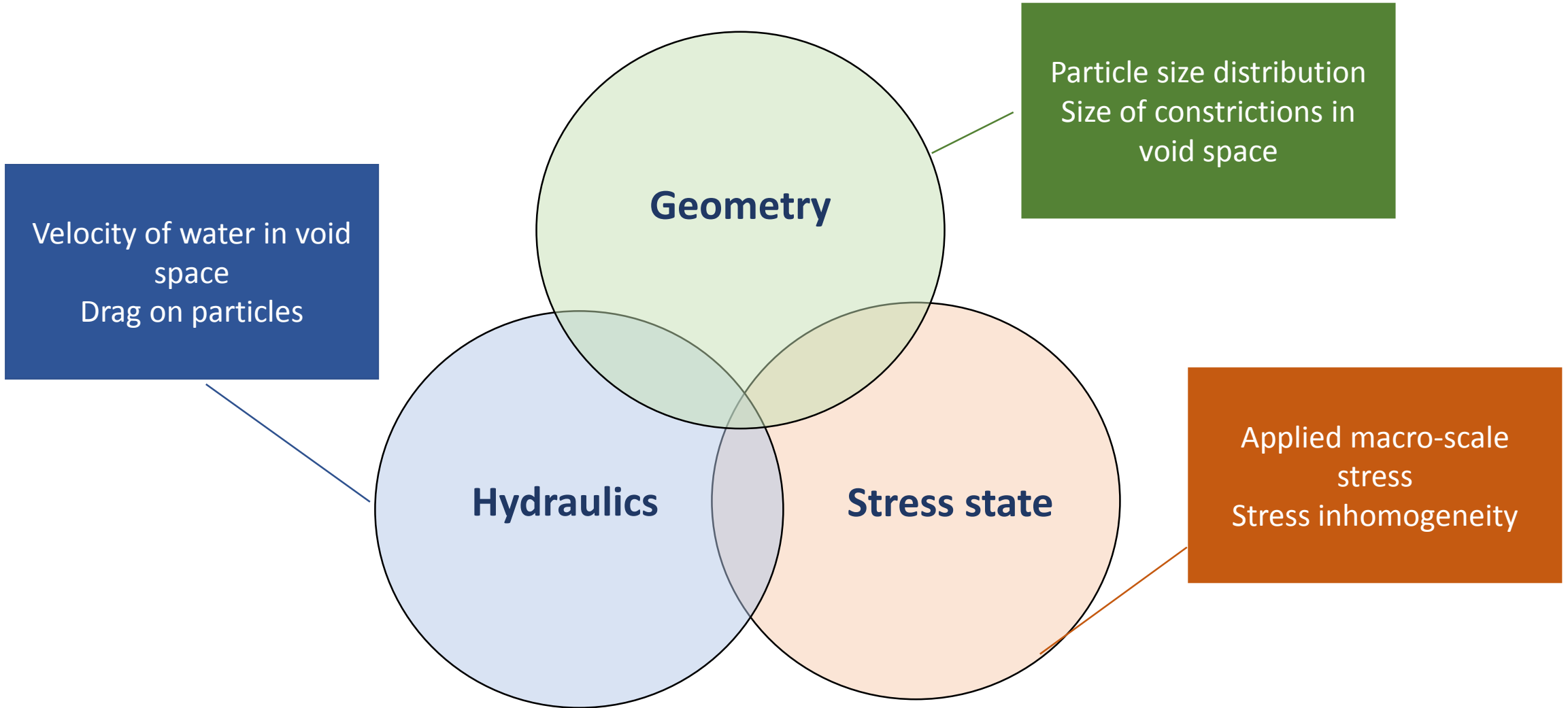
$$\sigma'_{\text{fines}} = \alpha \times \sigma'$$

Variation in α with Fines Content (F_{fine})

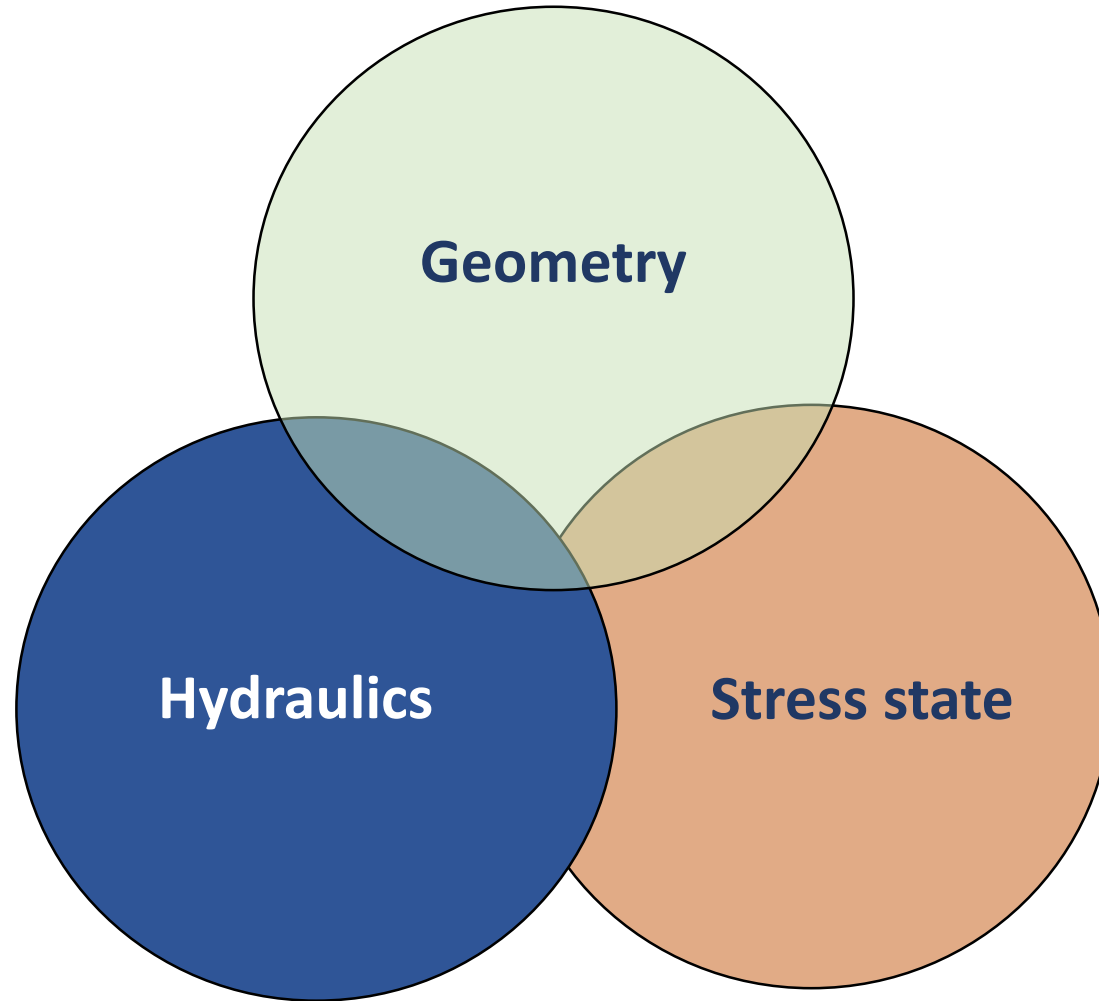


- Critical fines content where fines just fill voids: $F_{\text{fine}}=24-29\%$
- Finer fraction separates coarse fraction particles: $F_{\text{fine}}=35\%$
- Confirms hypotheses of Skempton and Brogan (1994)

Factors influencing erosion risk

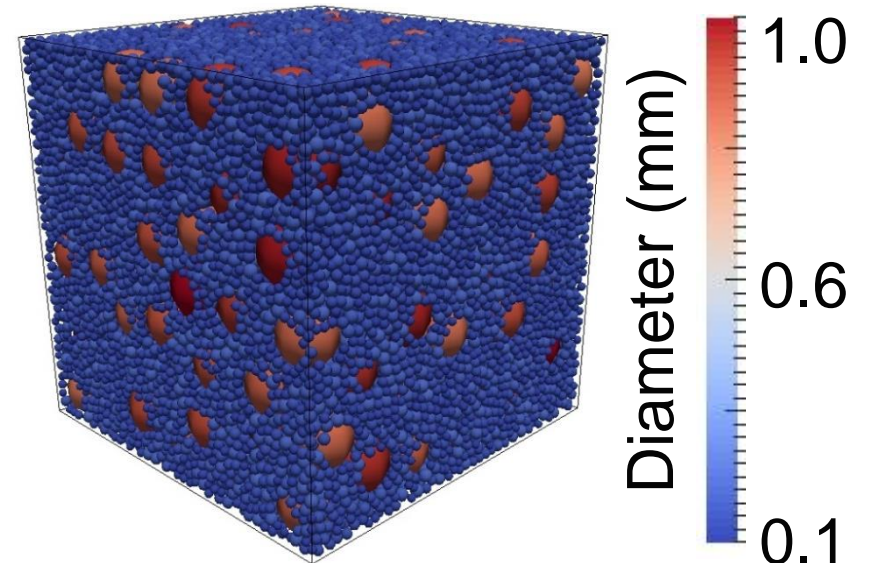
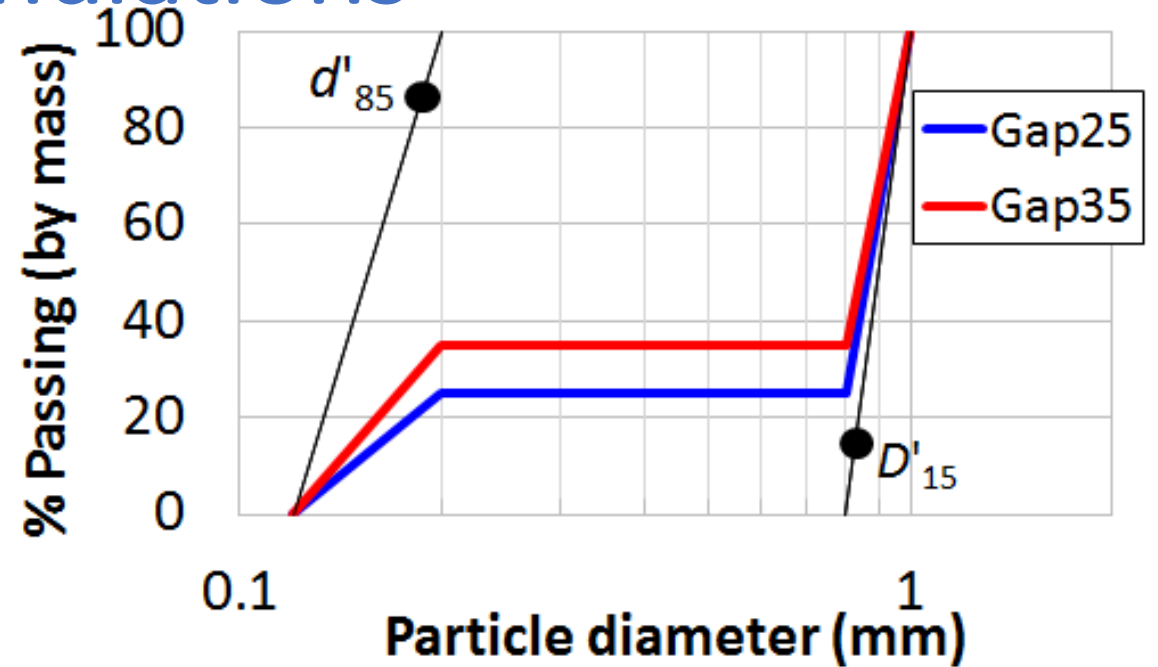


Factors influencing erosion risk



Permeameter test simulations

- PFC 3D Coupled with CCFD
- Circa 30,000 particles
- Di Felice drag expression
- Particle assembly: 6.1 mm cube
- Fluid cell size: 1.2 mm

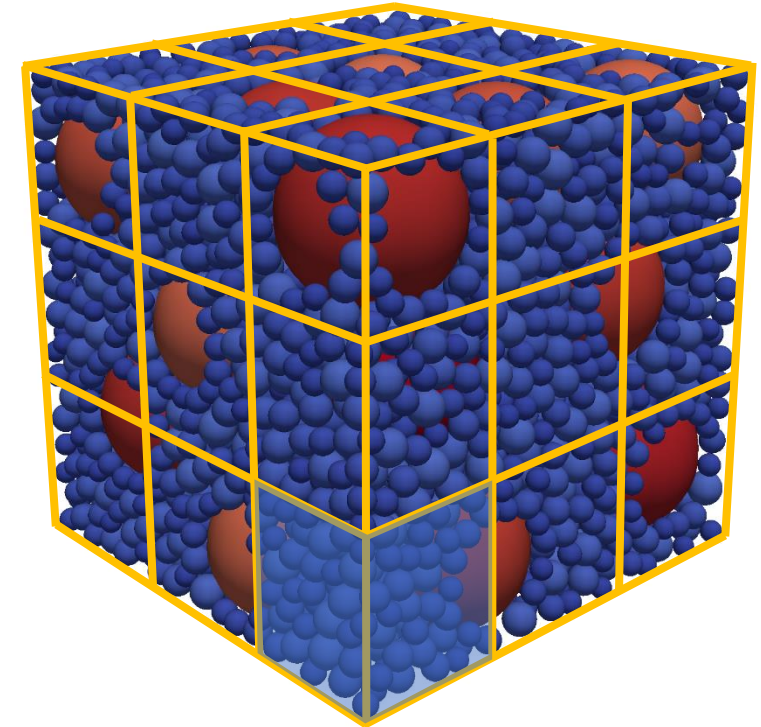


Permeameter test simulations

Combination of DEM (PFC3D) and CFD (CCFD)

- DEM for soil particles
- CFD for water seepage

Coarse grid method proposed by Tsuji



Data exchange

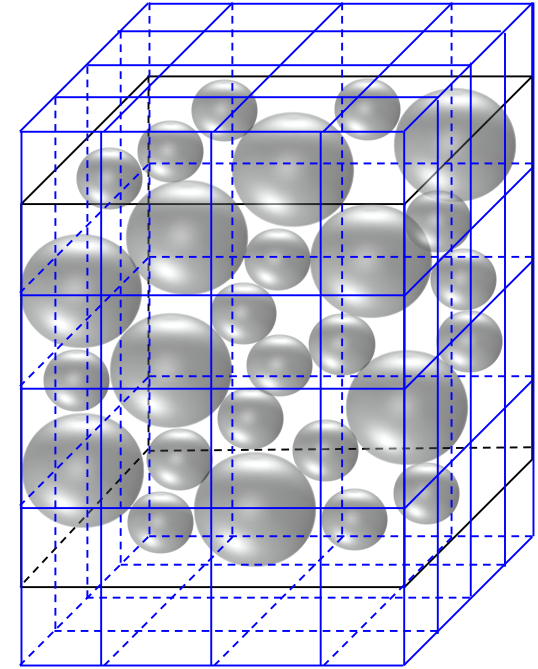
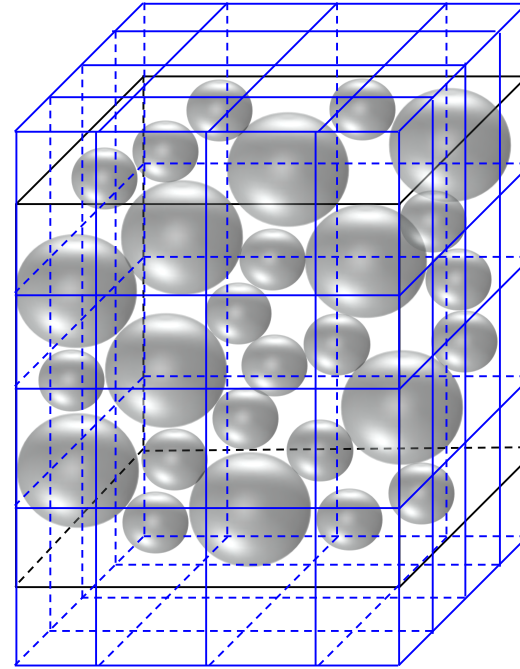
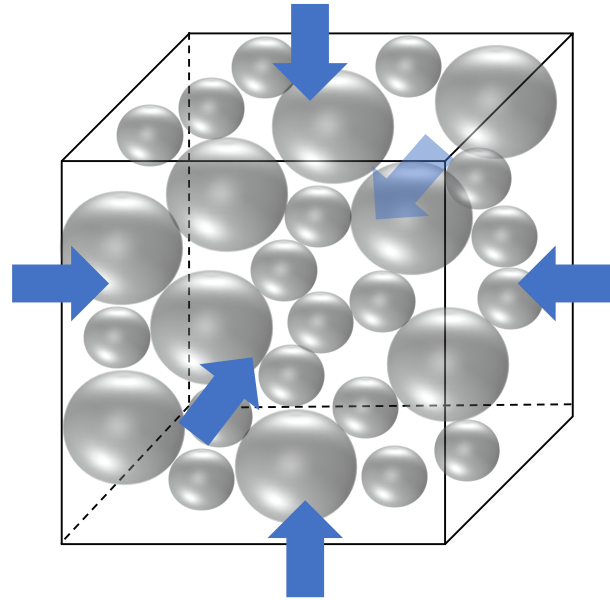
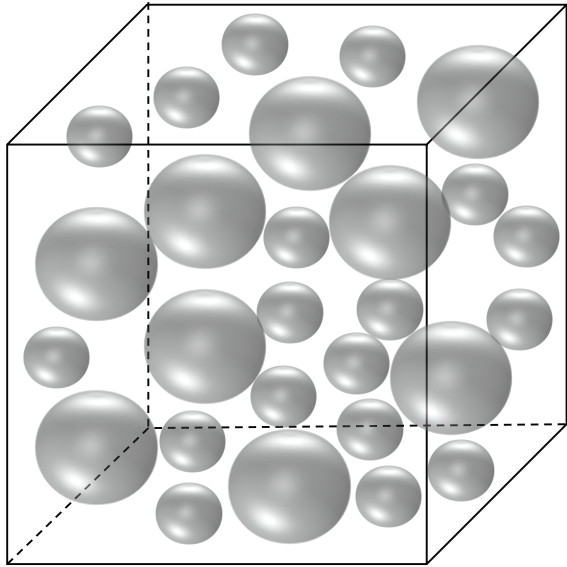


-Porosity
-Drag force

-Fluid velocity
-Fluid pressure gradient

(Tsuji et al., 1993, Xu and Yu, 1997)

Permeameter test simulations



Create non-contacting
cloud of spheres

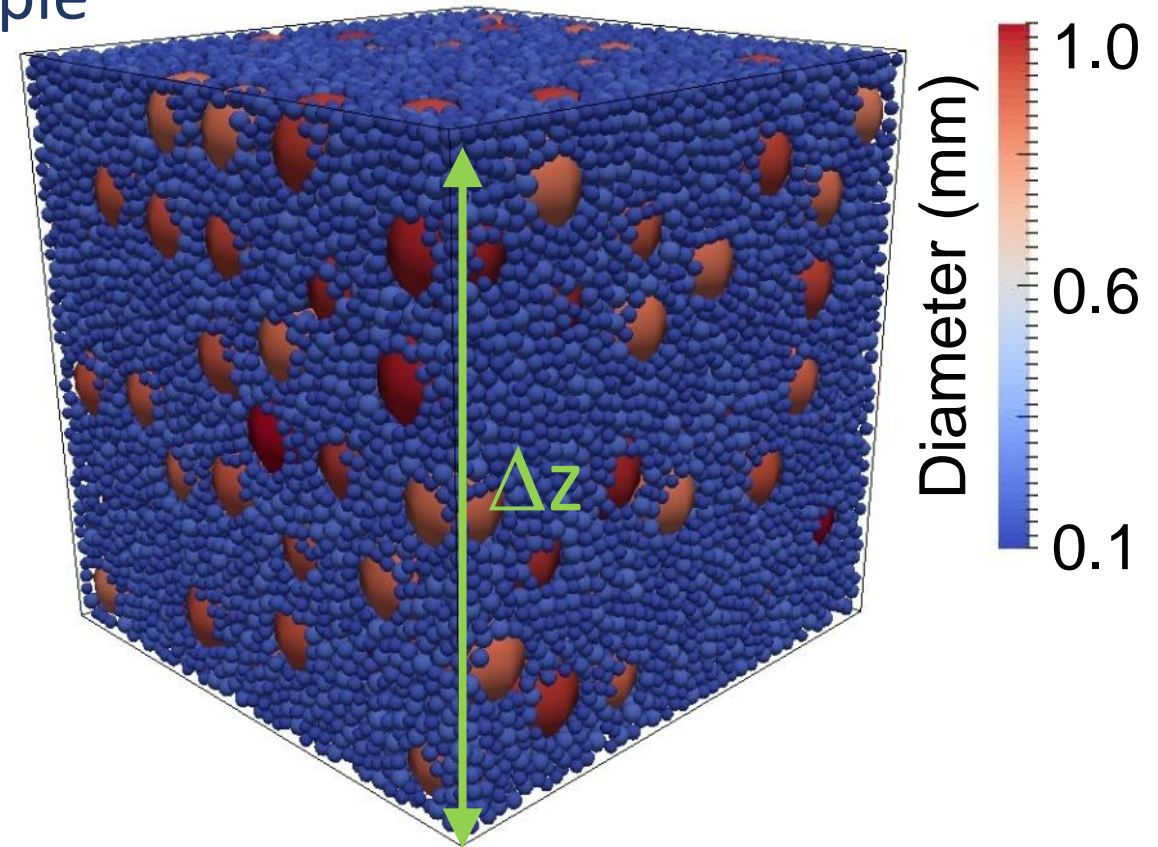
Compress to 50kPa,
Apply gravity

Create fluid mesh,
Fix boundaries,
Fix particle positions,
Apply pressure gradient

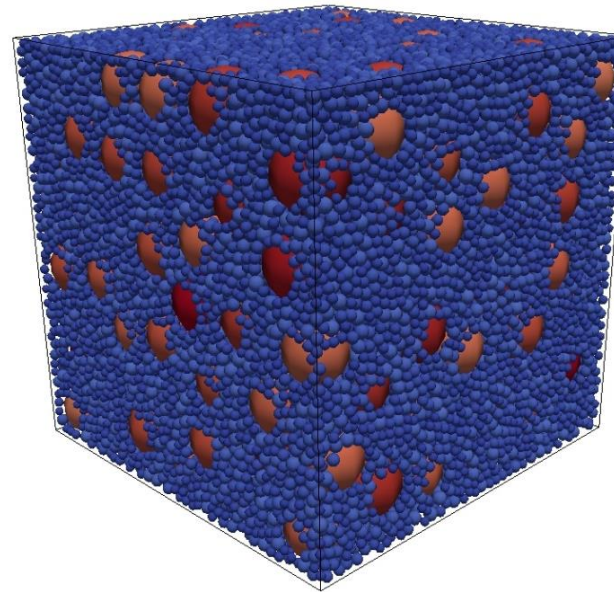
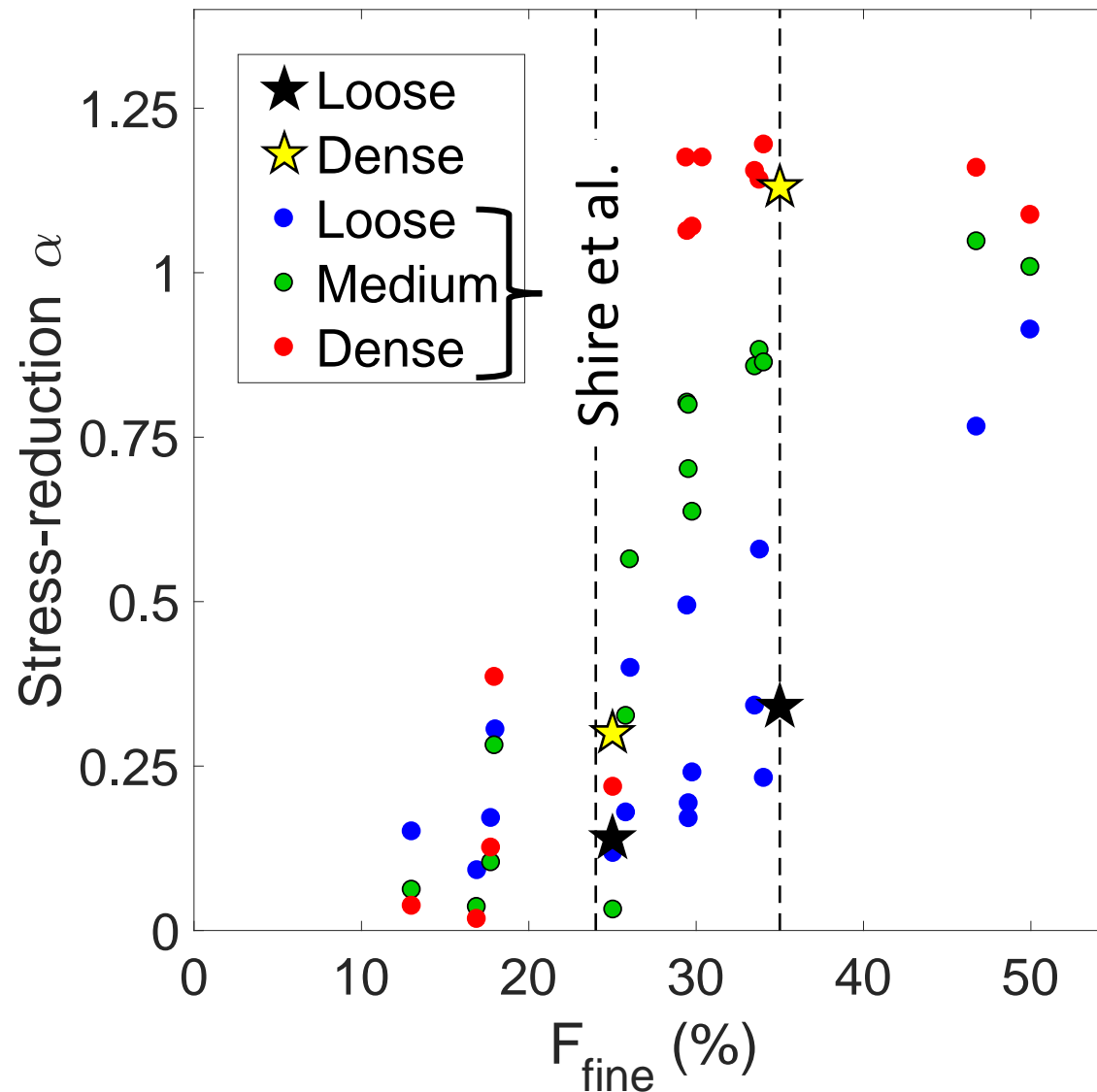
Steady state fluid,
Release particles,
Monitor response

Permeameter test simulations

- Applied pressure differential across sample (Δp)
- Increased hydraulic gradient (i) in steps
- As samples small
- $$i = \frac{\Delta h}{\Delta z} \approx \frac{\Delta p}{\gamma_w \Delta z}$$
- Δh =head drop across sample
- γ_w = unit weight of water
- Simulation gives permeability $k \approx 5 \times 10^{-3}$ m/s



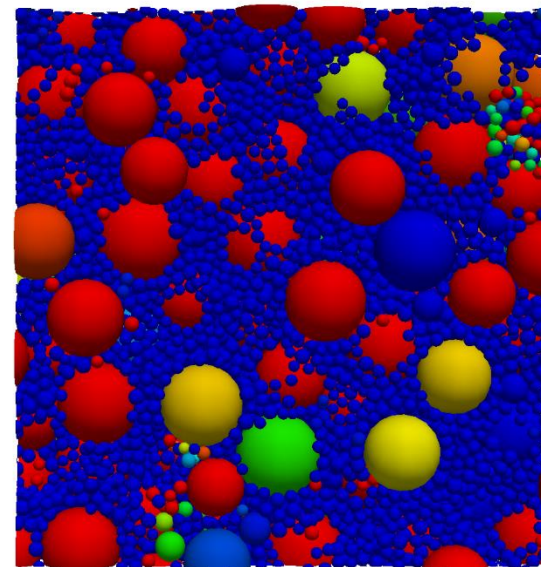
Virtual permeameter test samples



Current study:

PFC+CCFD

Rigid walls

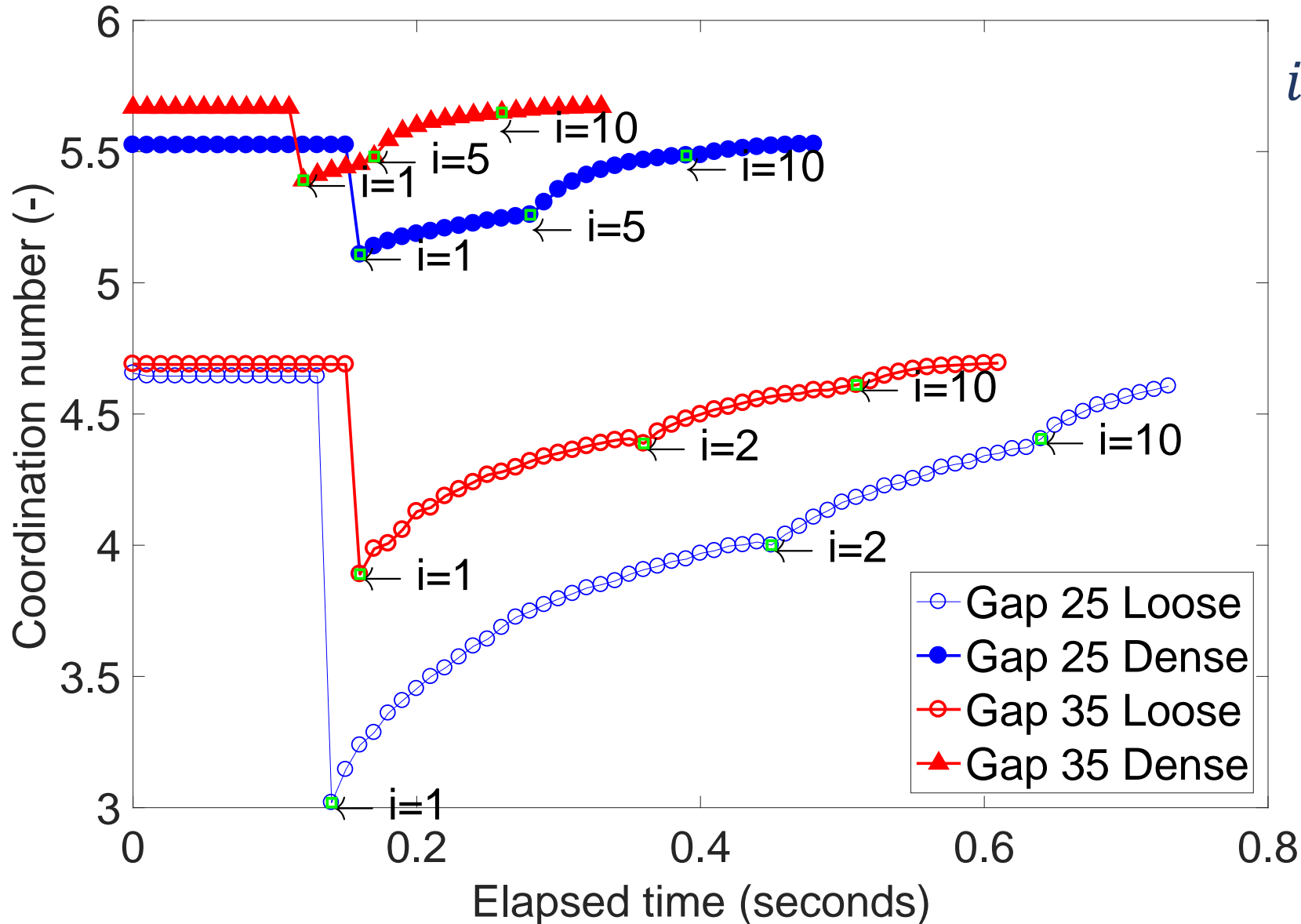


Shire et al.

LAMMPS

Periodic boundaries

Virtual permeameter test – sample response



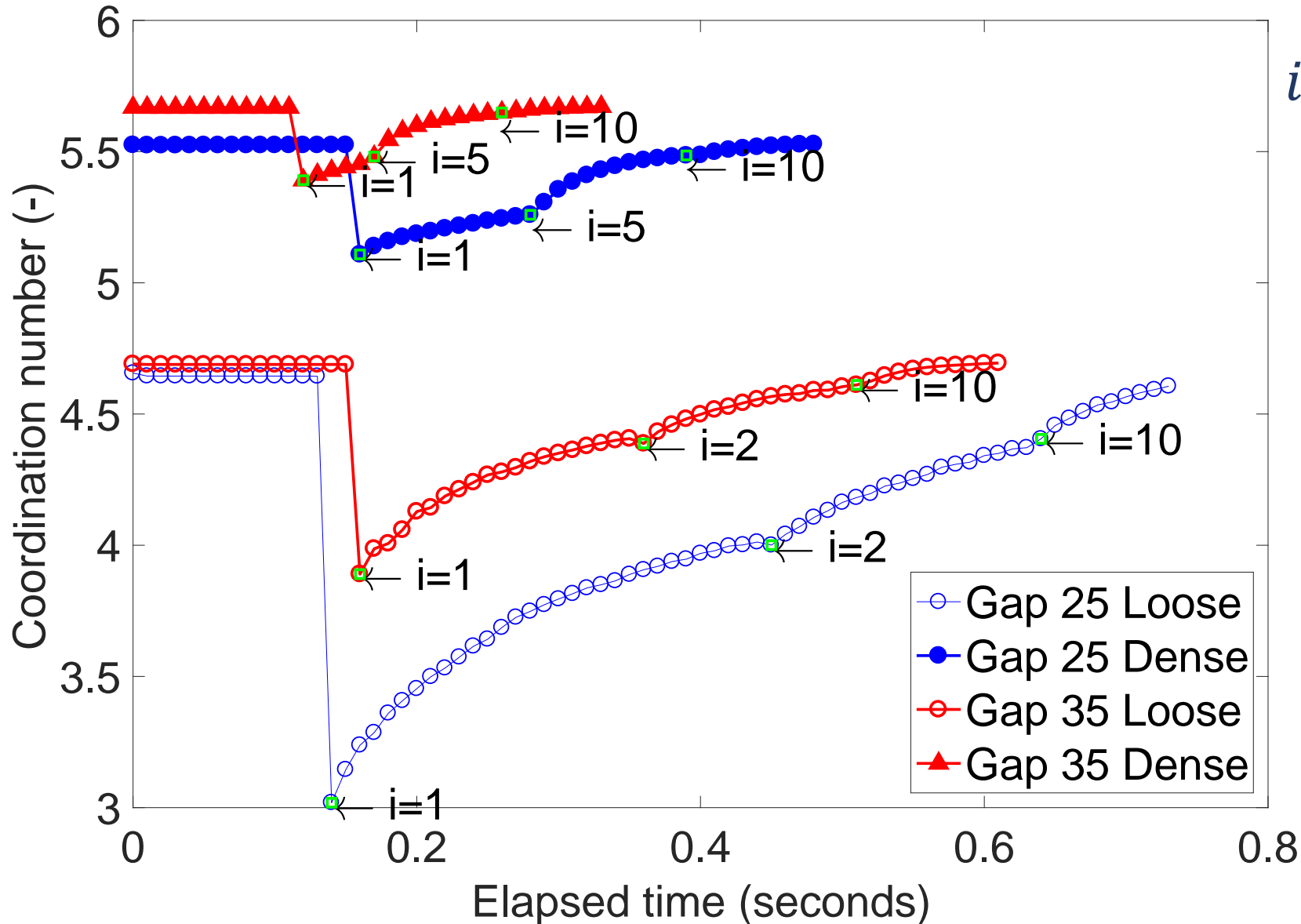
i = hydraulic gradient

$$i = \frac{\Delta h}{\Delta z} \approx \frac{\Delta p}{\gamma_w \Delta z}$$

N_c = Coordination number

Average no of contacts
per particle

Virtual permeameter test – sample response

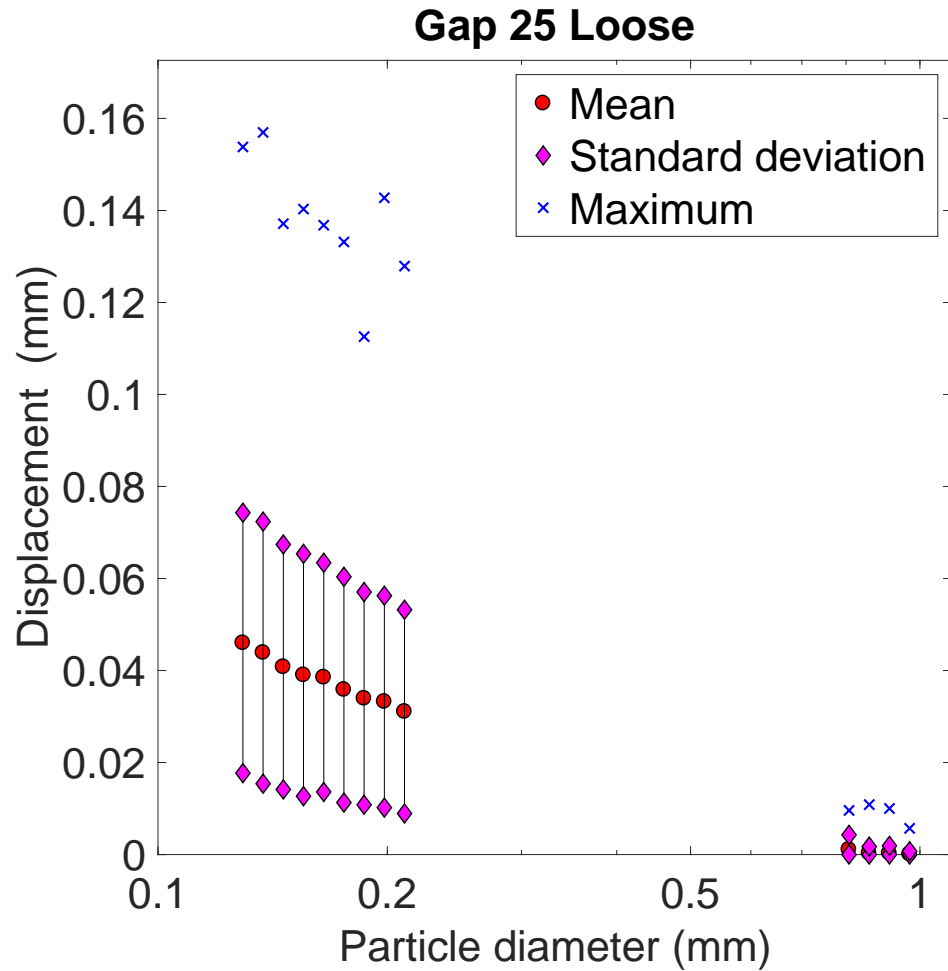


i = hydraulic gradient

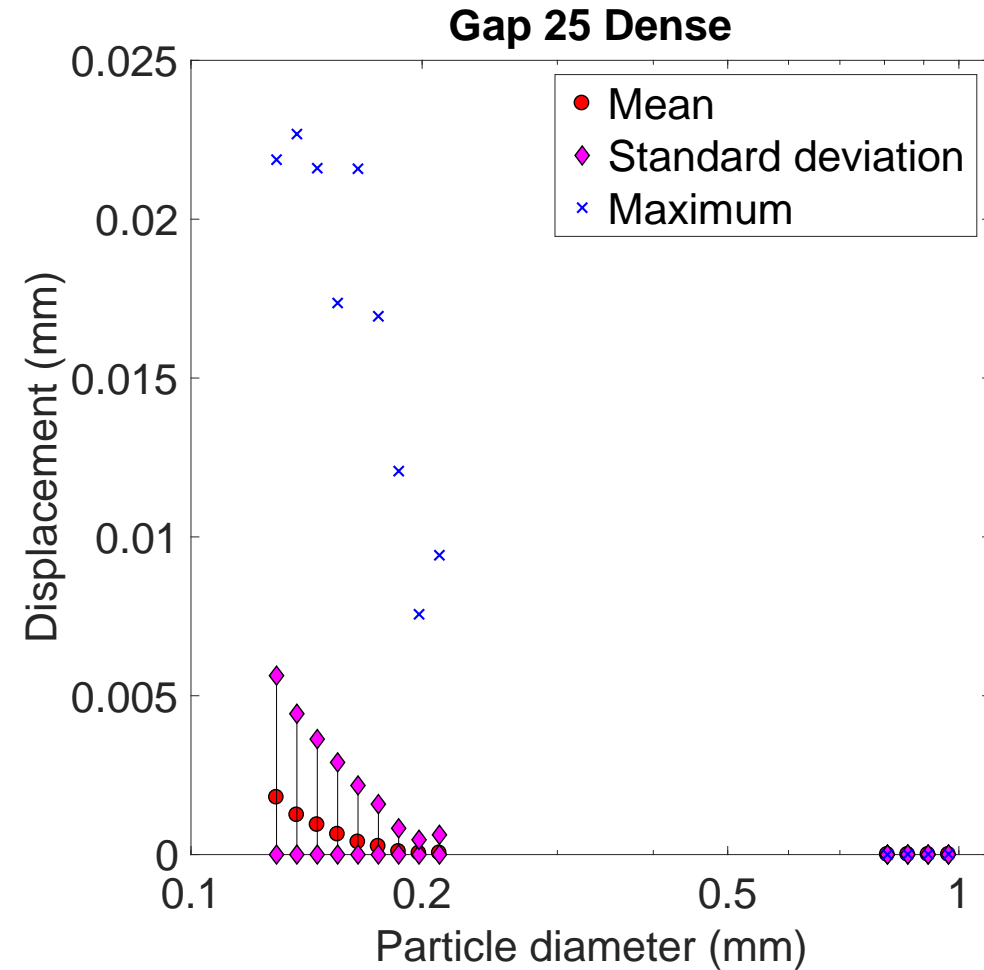
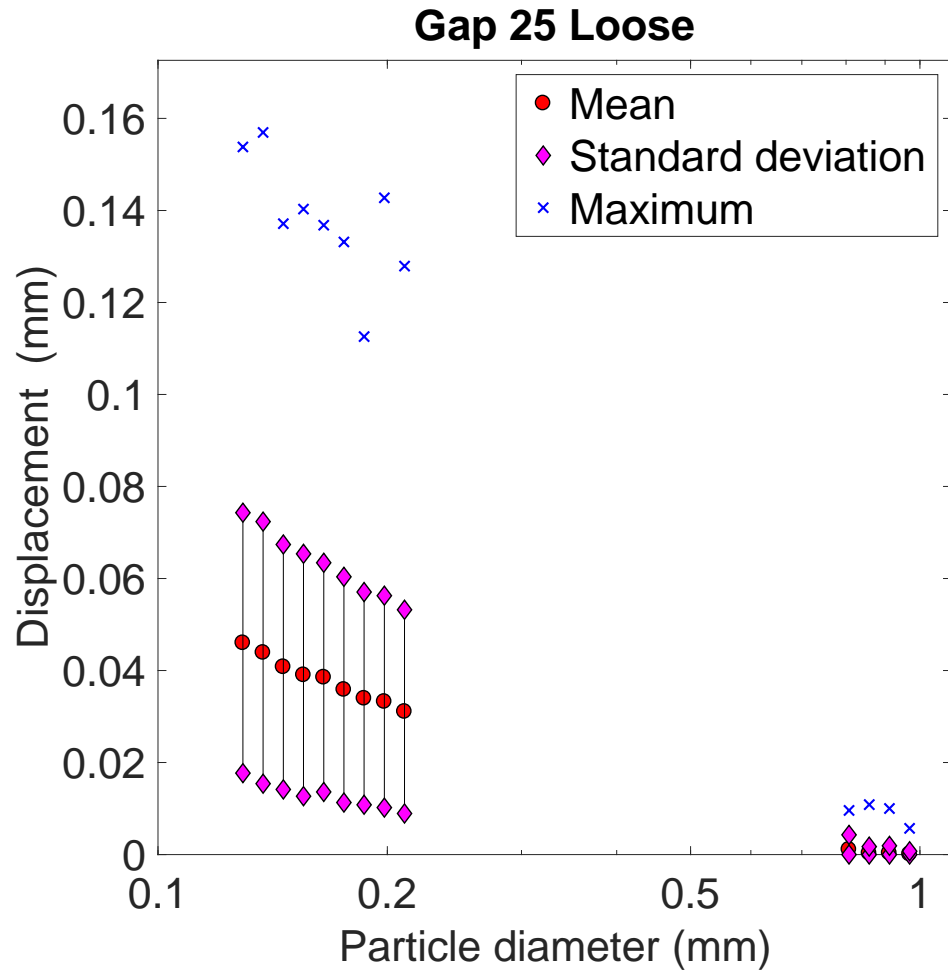
$$i = \frac{\Delta h}{\Delta z} \approx \frac{\Delta p}{\gamma_w \Delta z}$$

Focus on $i = 1$ for detailed analyses \rightarrow *initiation of erosion*

Particle displacements – for $i = 1$



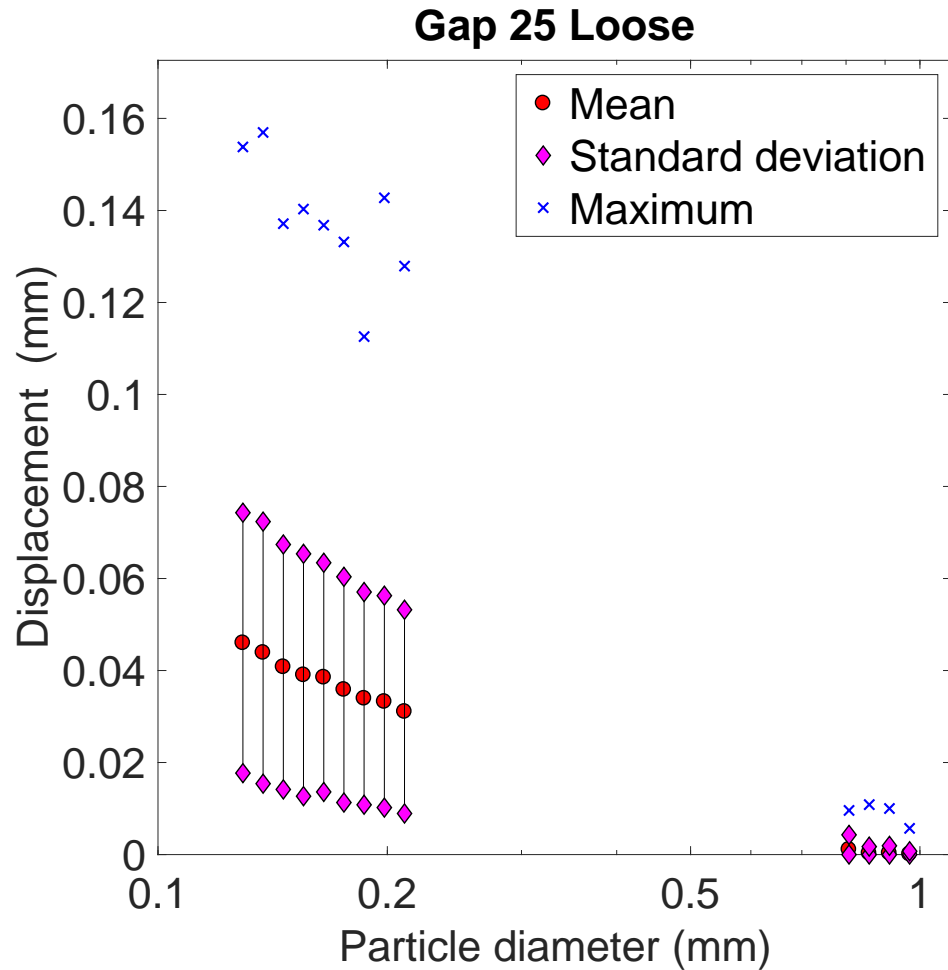
Particle displacements – for $i = 1$



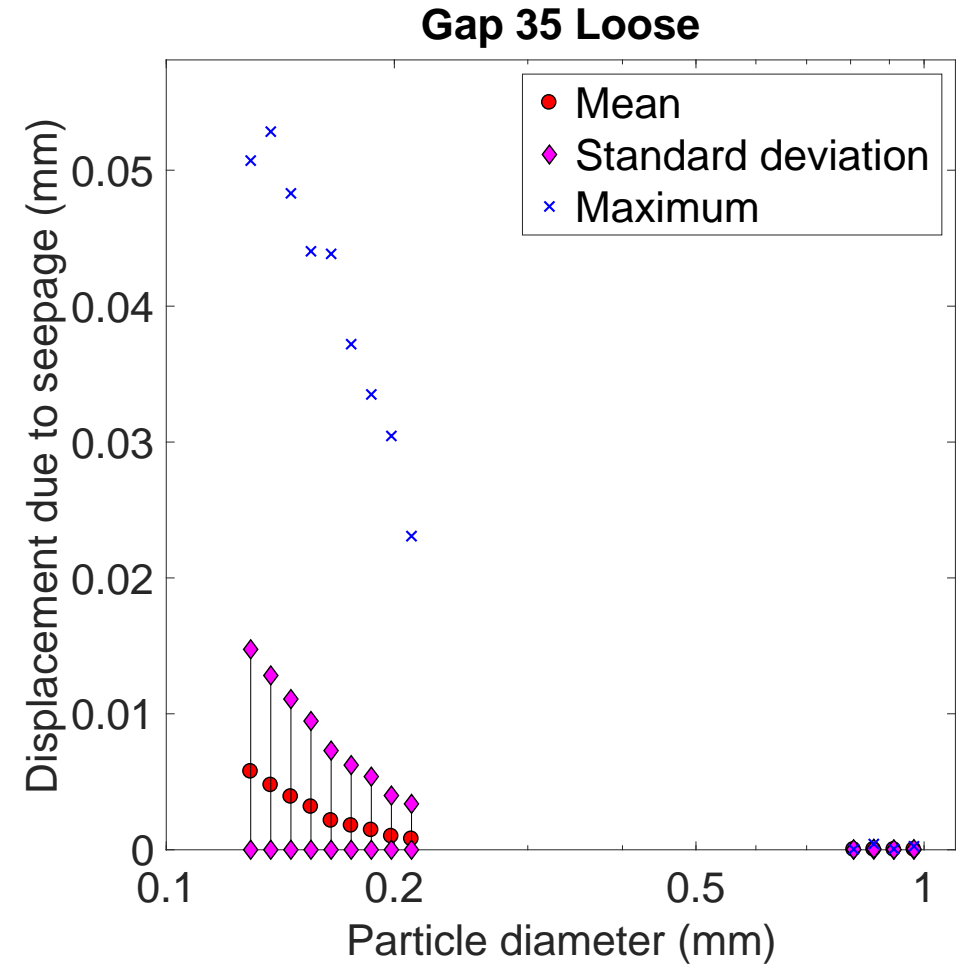
Increase in density



Particle displacements – for $i = 1$



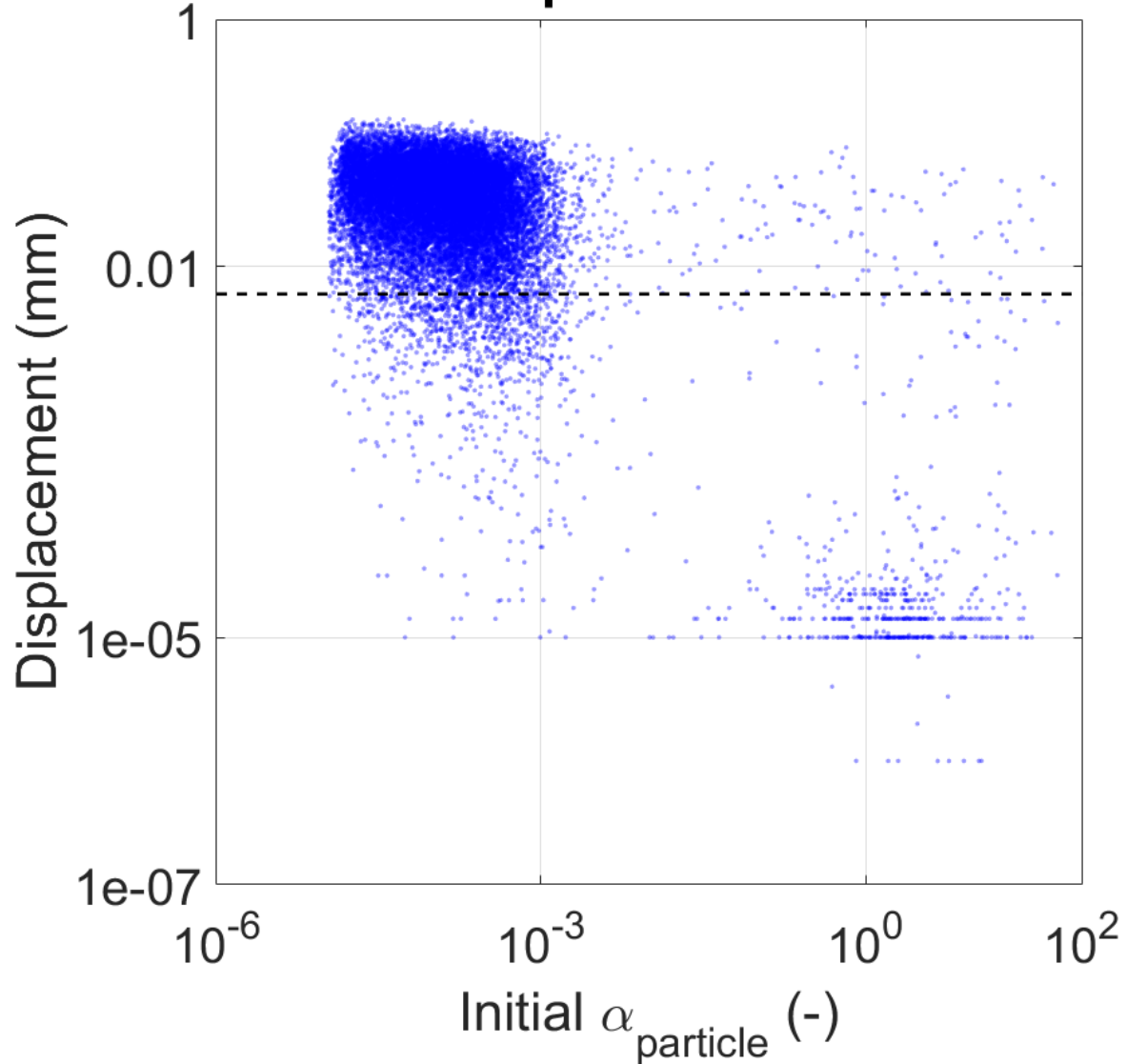
Just underfilled



Just overfilled

Particle displacements – for $i = 1$

Gap 25 Loose



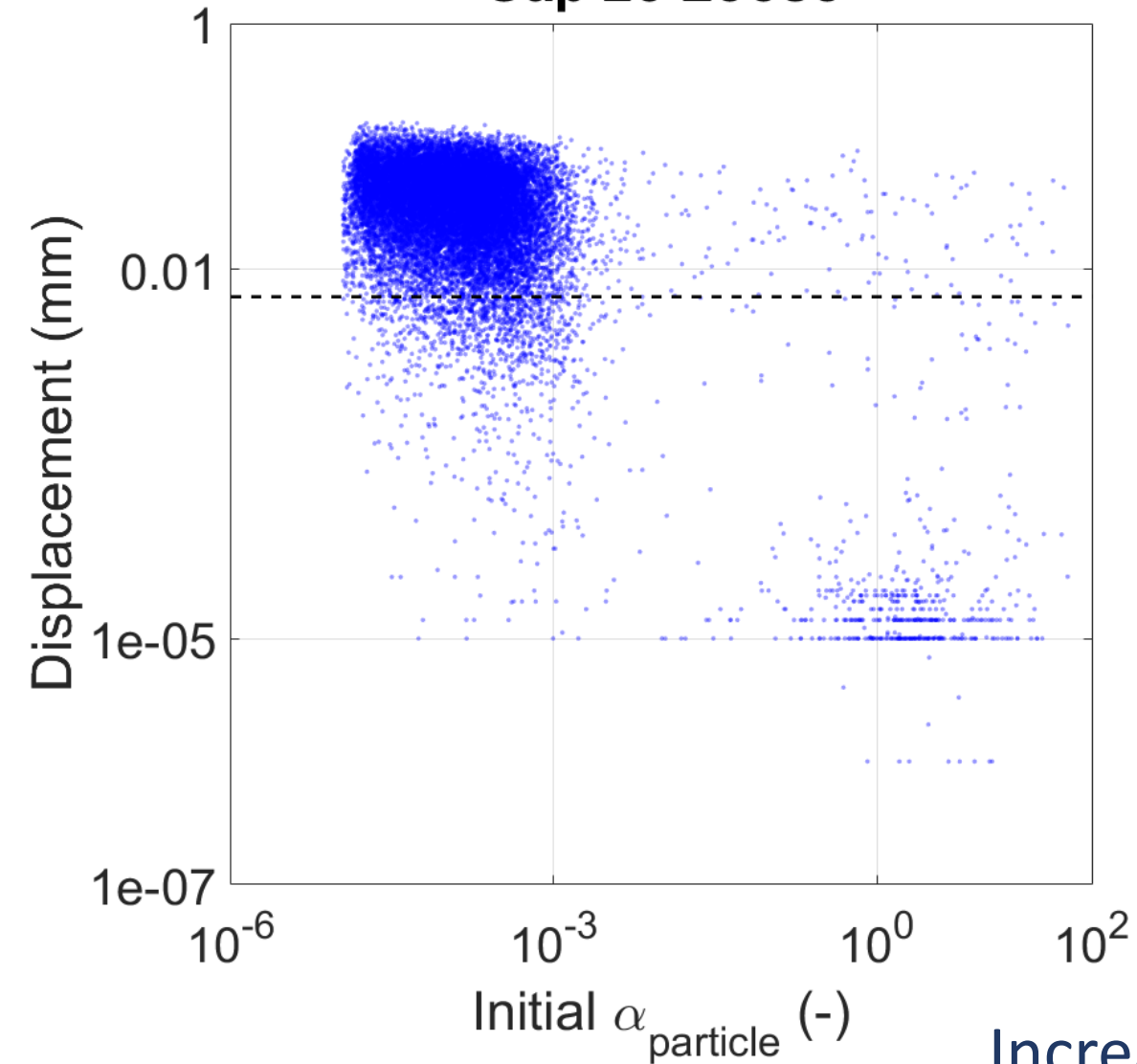
$$\alpha_{particle} = \frac{\sigma_{particle}}{\sigma_{overall}}$$

$\sigma_{particle}$ = average stress in a particle

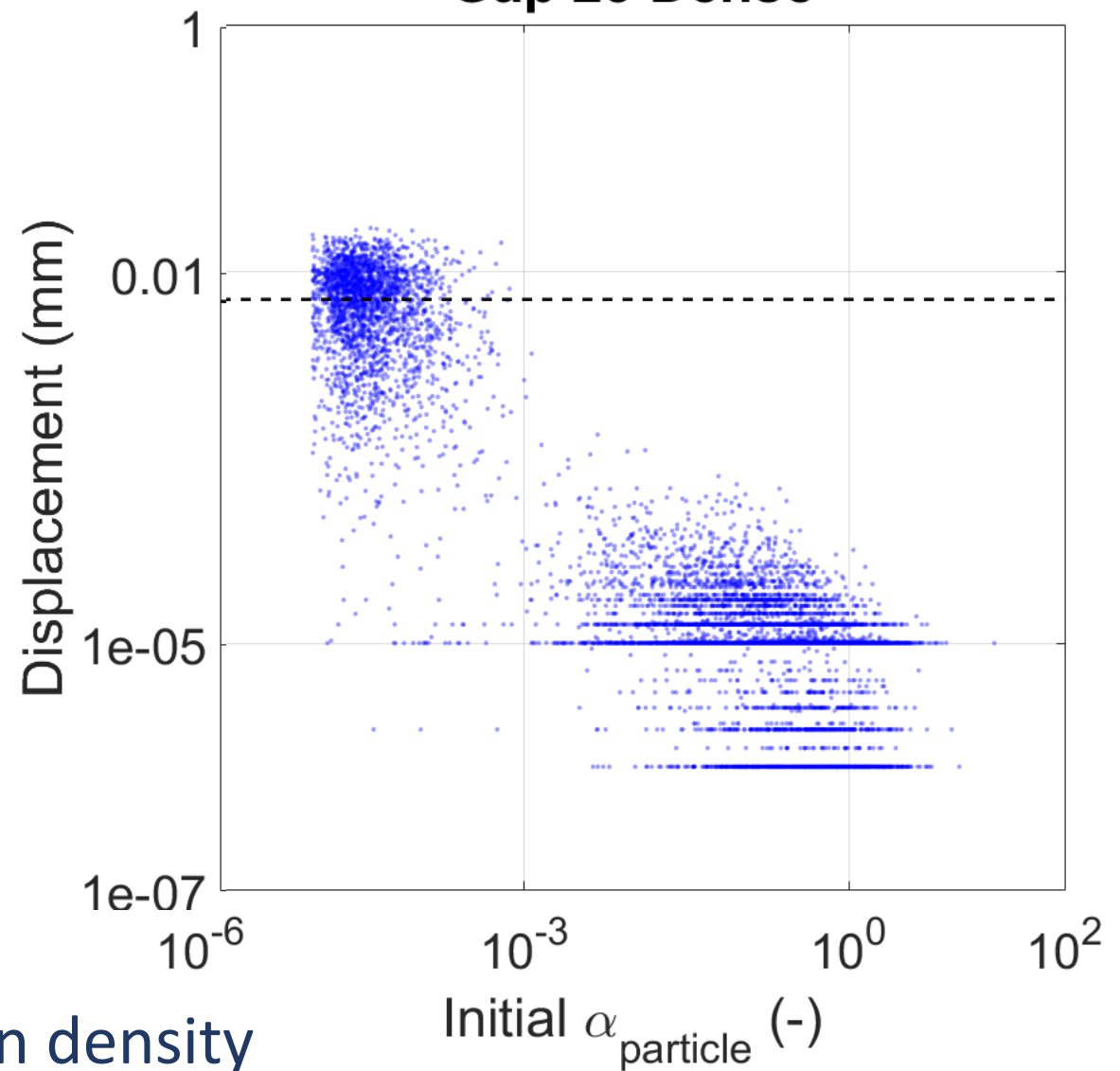
$\sigma_{overall}$ = overall sample stress

Particle displacements – for $i = 1$

Gap 25 Loose



Gap 25 Dense

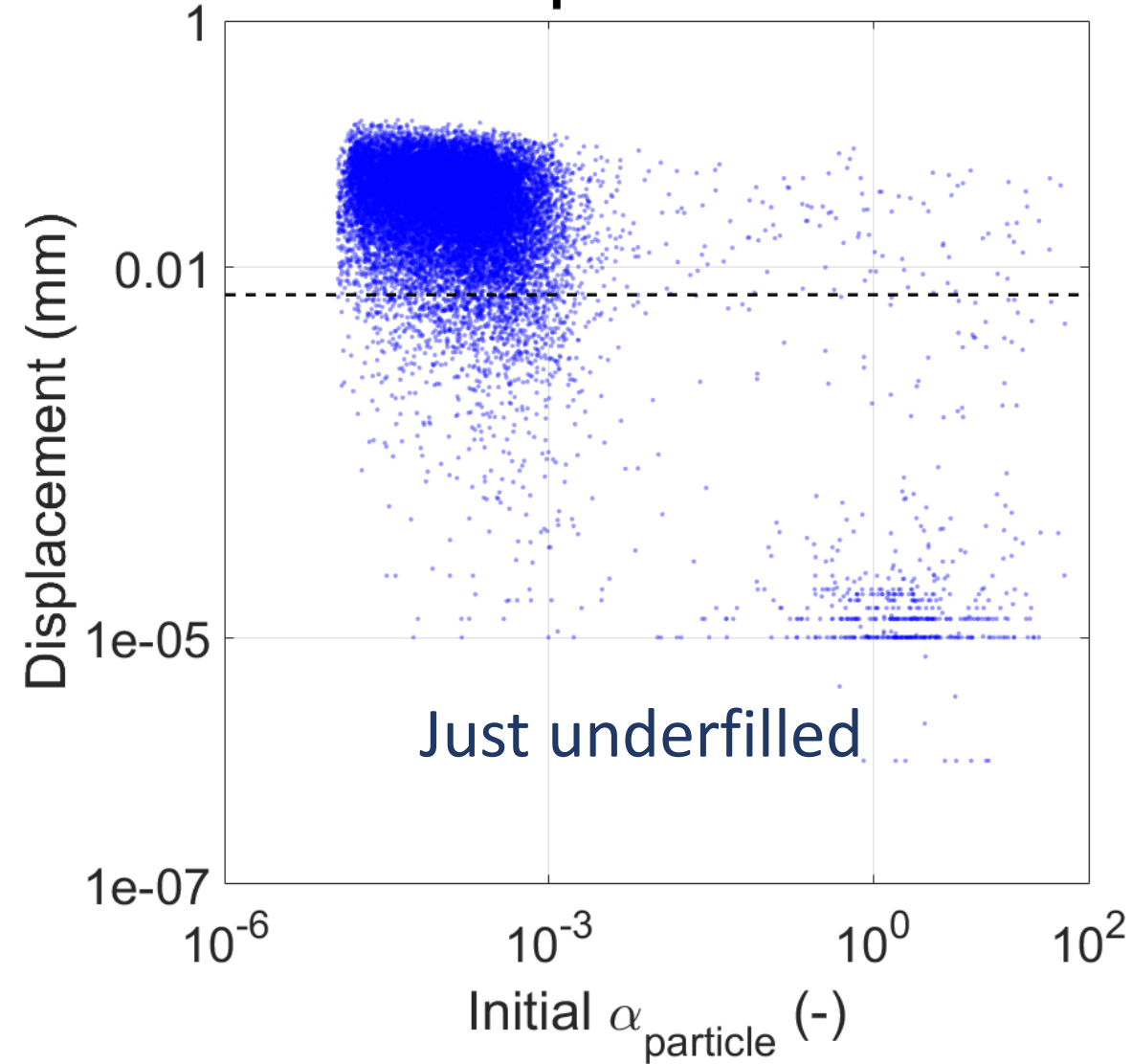


Increase in density

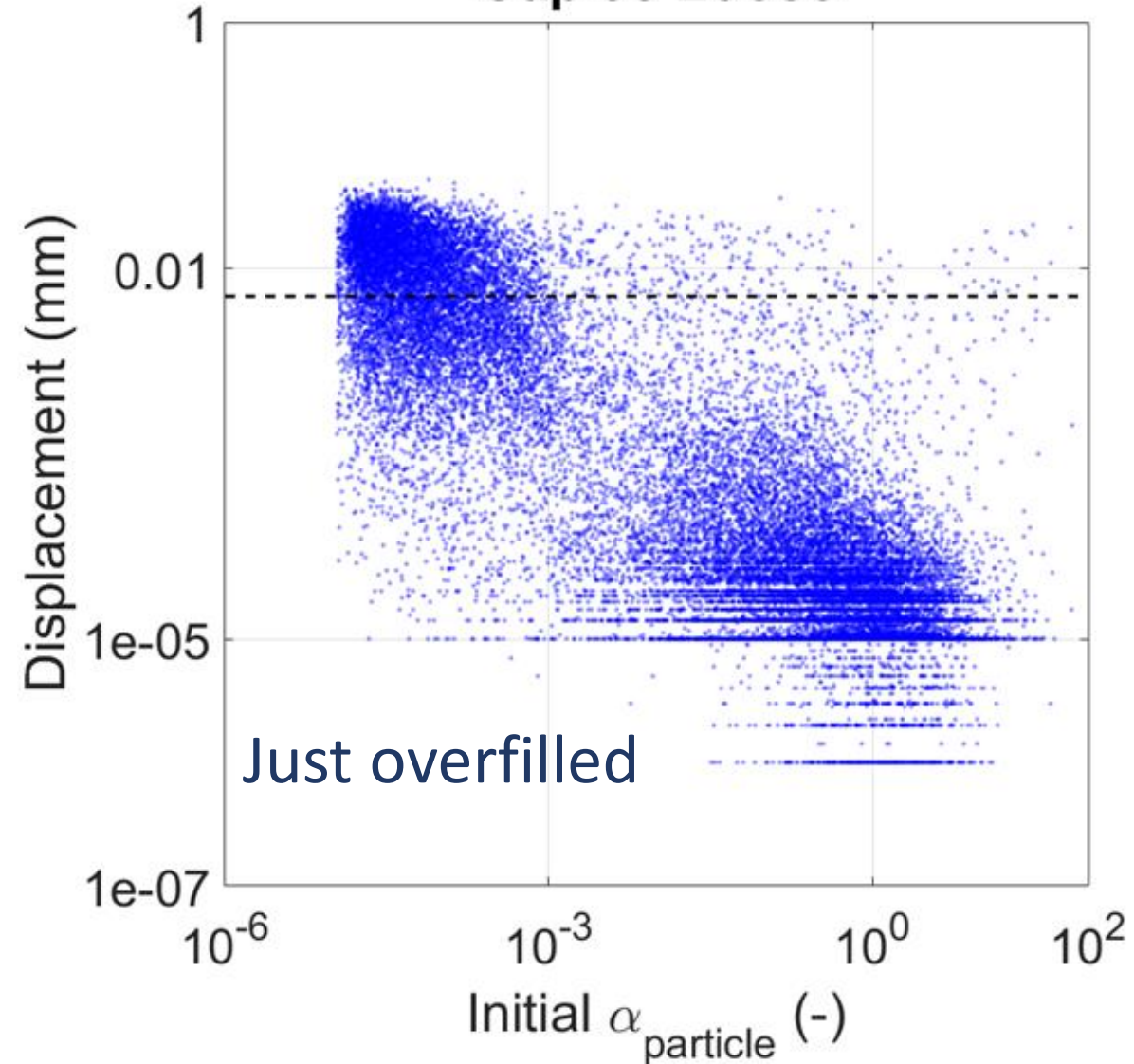


Particle displacements – for $i = 1$

Gap 25 Loose

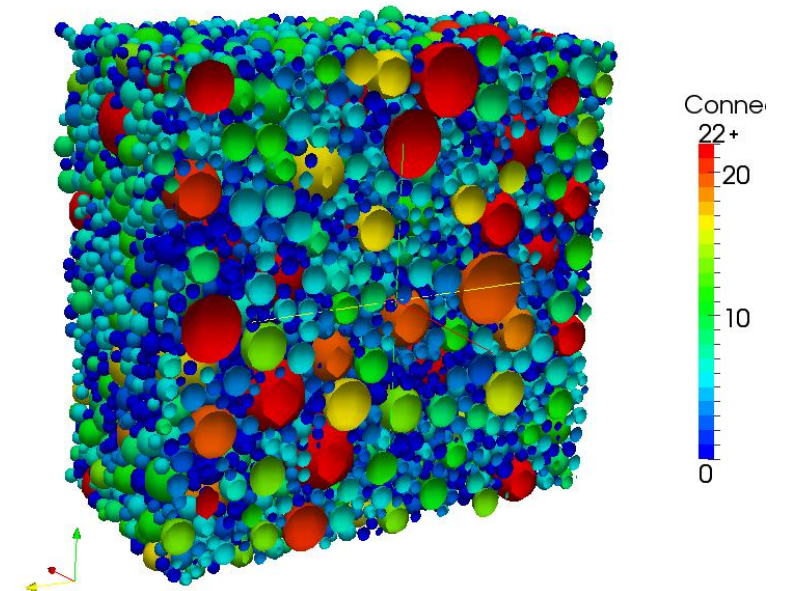
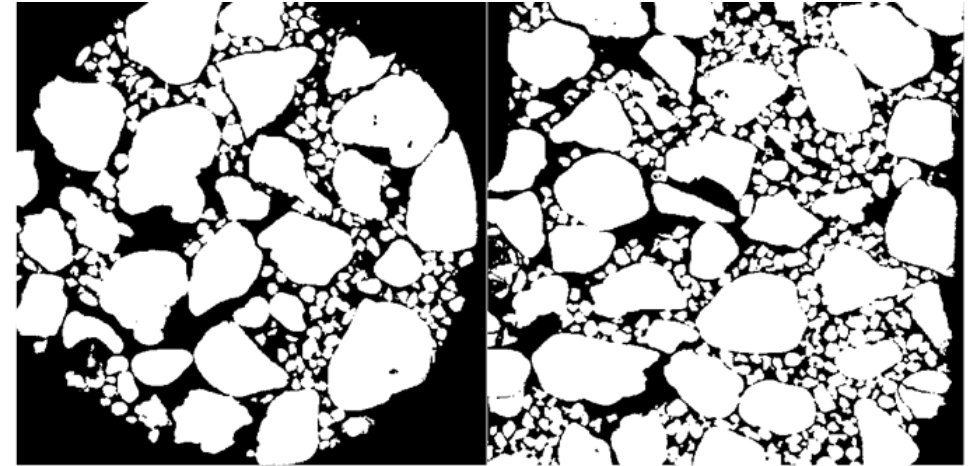


Gap 35 Loose



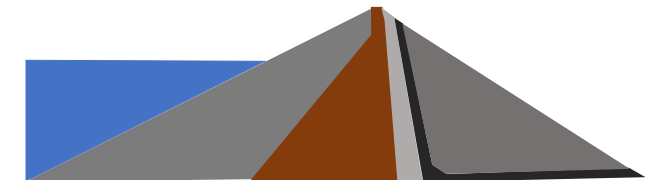
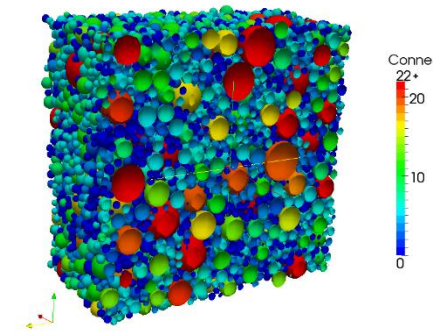
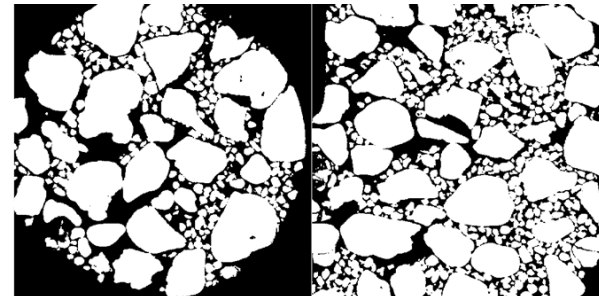
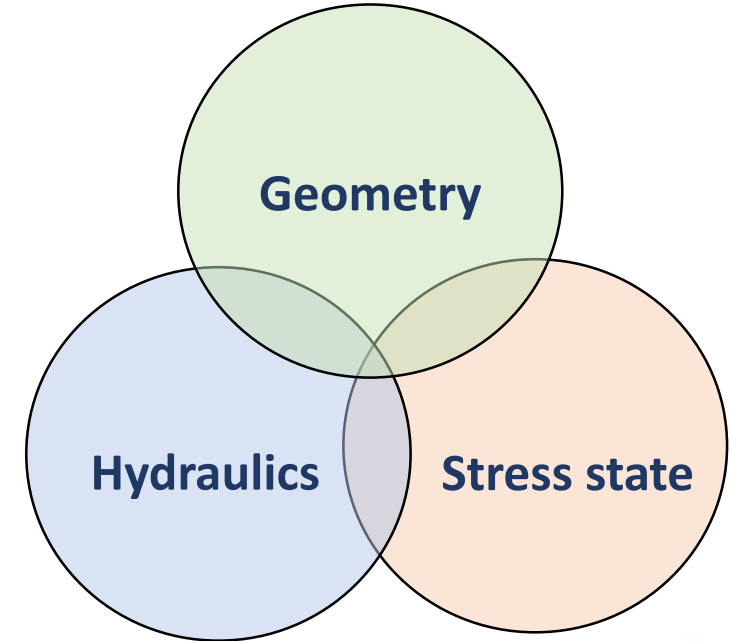
Internal instability

- microCT + DEM confirmed a link between the Kézdi criterion to assess internal instability risk and the contact density within the samples
- DEM simulations confirmed a link between the proportion of stress carried by the finer grains and the fines content.
- For fines contents between 25% and 35% susceptibility to internal instability depends on packing density
- Coupled DEM + CFD simulations confirmed a link between the stress carried by the finer grains and the likelihood of grain migration under seepage flow



Conclusions

- Considerations of permeability, filter compatibility and internal instability are important in dam and embankment design and maintenance
- Geometry / particle scale topology of materials; stress state and fluid:particle interaction determine behaviour
- Particle-scale characterization and simulation can improve understanding leading to more robust design guidance



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