

# Wave propagation in sand: a particle-scale perspective

Catherine O'Sullivan

Masahide Otsubo, John O'Donovan

# Research approach

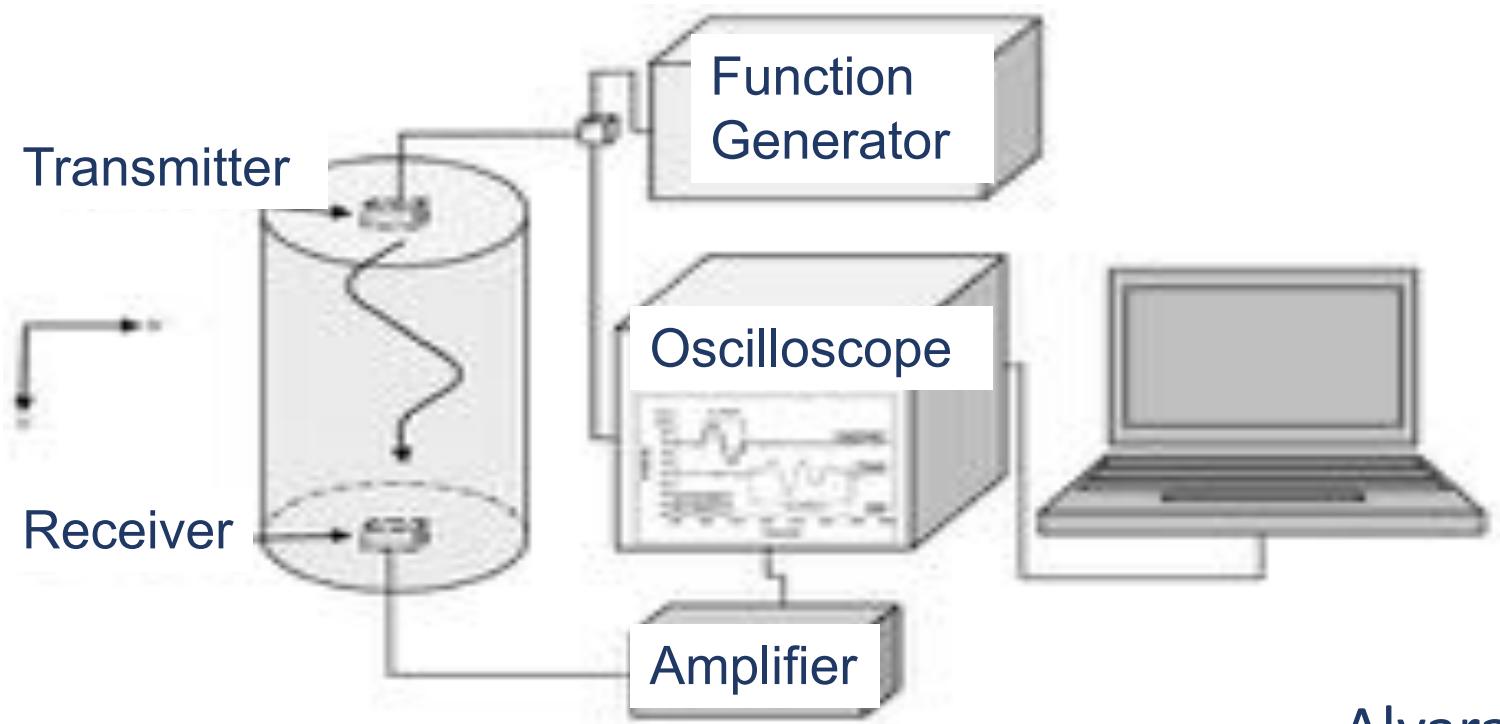
- Philosophy that DEM can provide particle-scale data to explain complex phenomena observed in laboratory experiments.
- DEM enables exploration to develop new approaches to interpret and use the rich set of data available in laboratory geophysical tests.



# Micromechanics of stress wave propagation.

1. Can DEM simulations reproduce experimental observations of stress wave propagation?
2. How reliable are empirical approaches to interpret laboratory geophysics tests?
3. Can DEM simulations inform the design of laboratory geophysics experiments?
4. How does surface roughness effect stiffness?
5. What can we learn about the material fabric from the frequency domain response?
6. What opportunities does the BRISS project present?

# Laboratory Geophysics



$$V_S = d/t_{arr}$$

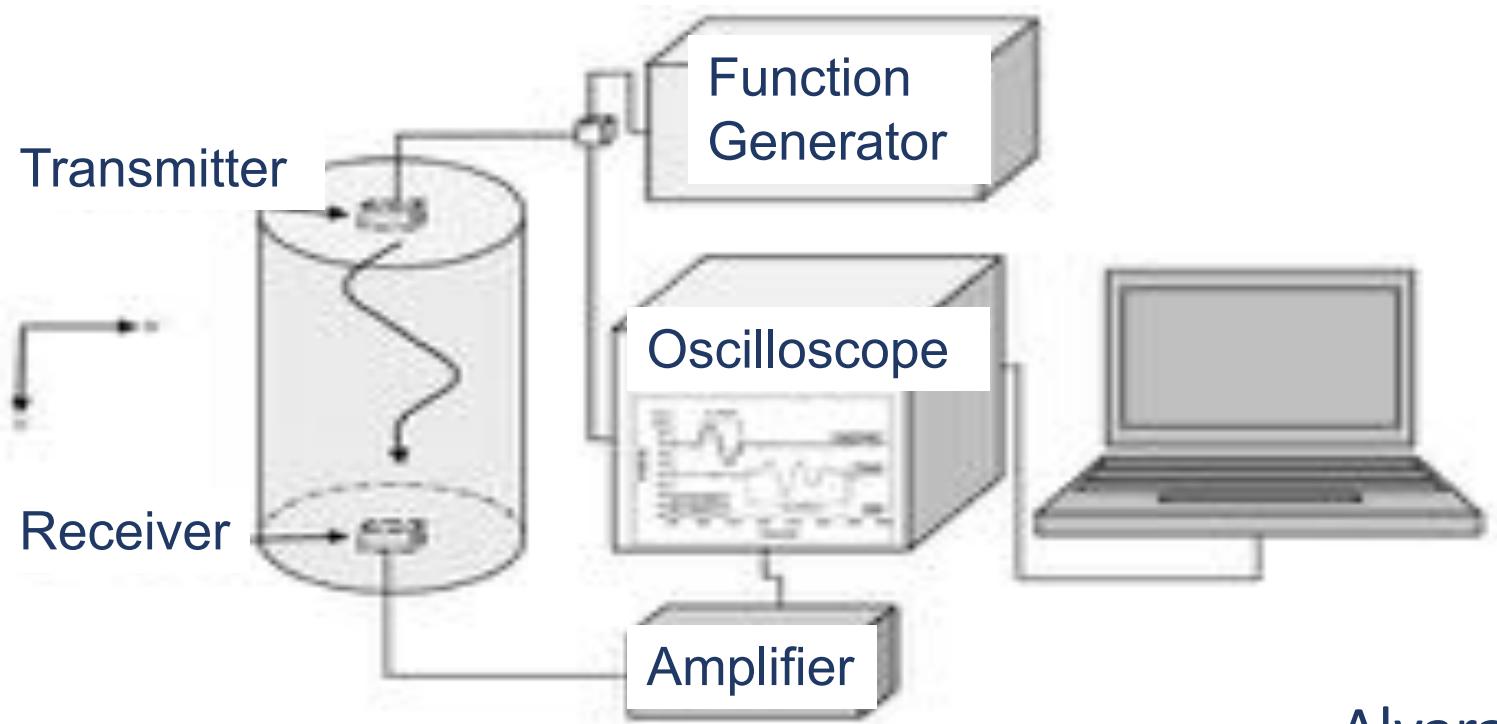
$d$  = travel distance  
 $t_{arr}$  = travel time

Alvarado & Coop (2012)

# Laboratory Geophysics

$$G = \rho V_s^2$$

$G$ = shear stiffness  
 $\rho$ =material density  
 $V_s$ = shear wave velocity



$$V_s = d/t_{arr}$$

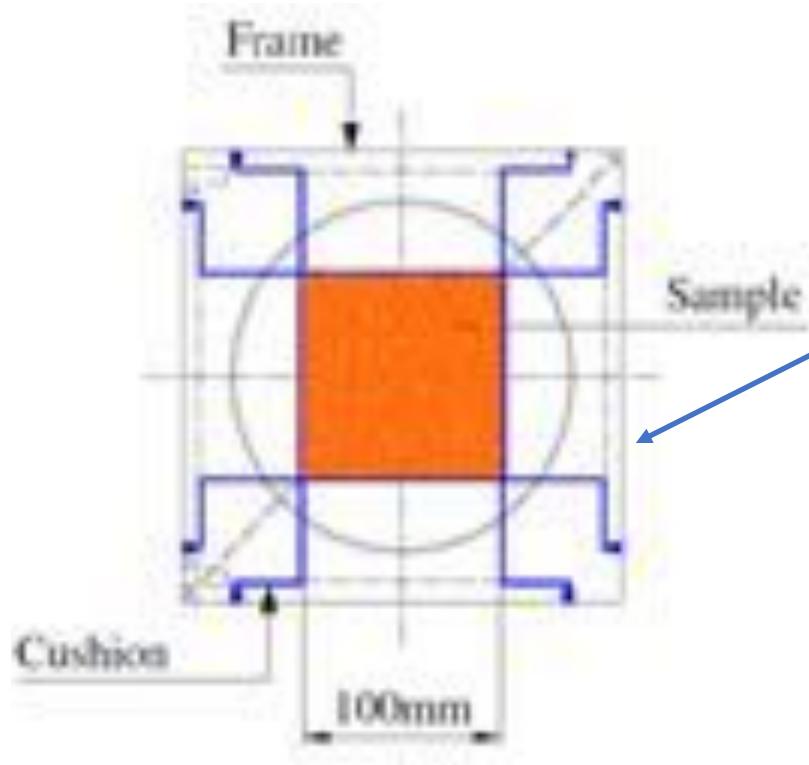
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Alvarado & Coop (2012)

# Micromechanics of stress wave propagation.

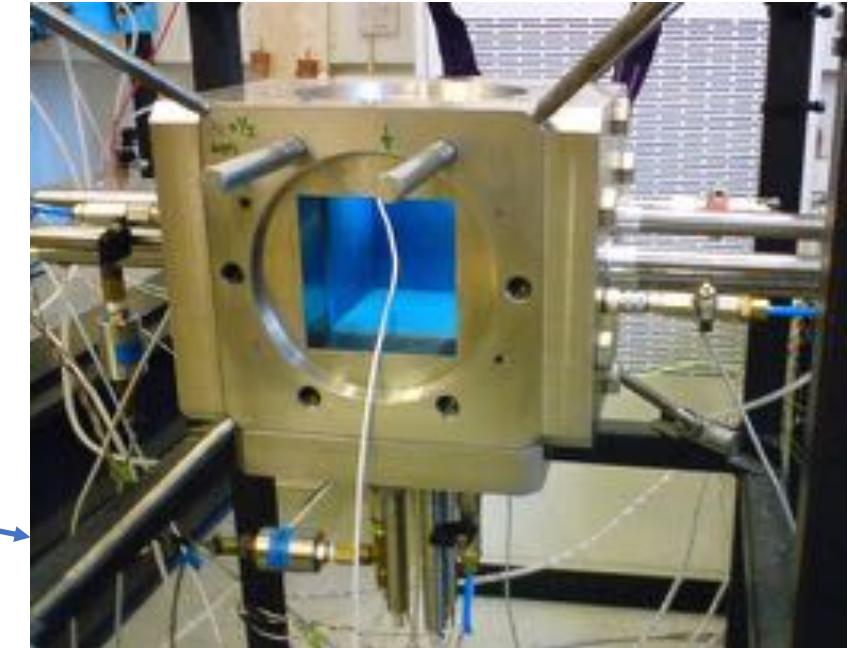
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# Cubical cell apparatus.

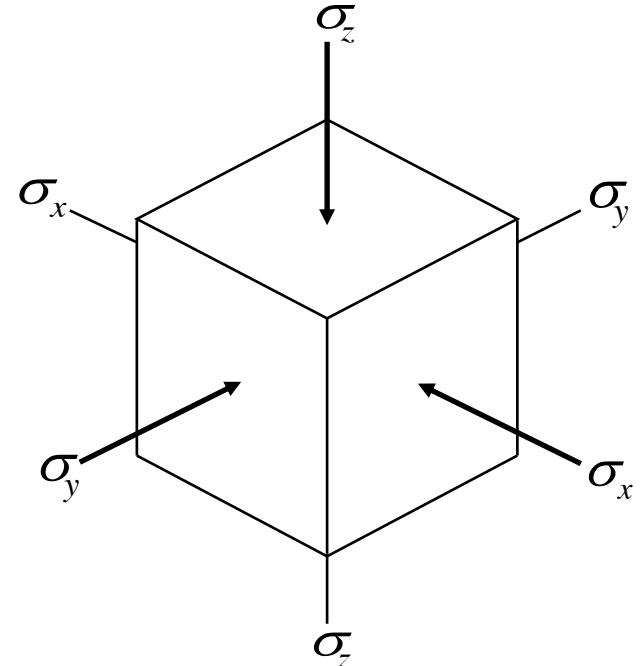


Schematic  
(Sadek, 2006)

Cubical Cell Apparatus  
University of Bristol

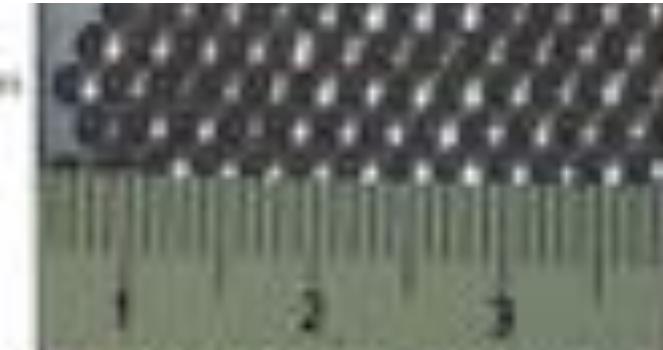


Allows independent control of three principal stresses

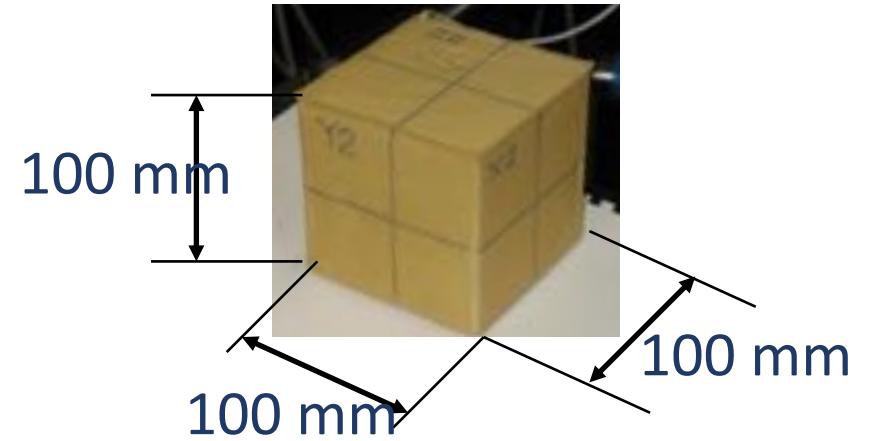


# Cubical cell apparatus – Sample preparation

Borosilicate glass  
ballotini  
2.4 mm – 2.7 mm  
Cavarretta et al. (2012)



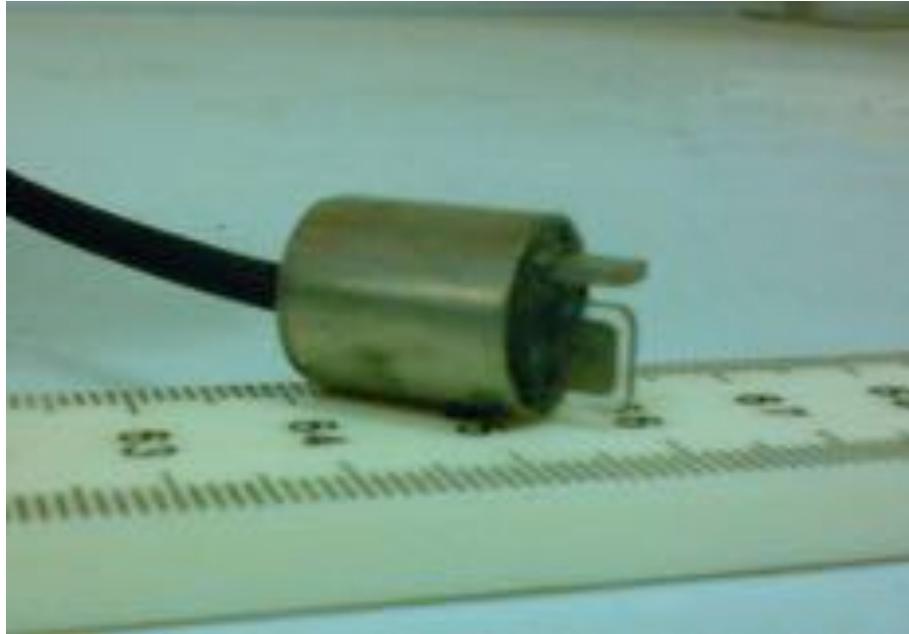
Controlled pluviation into  
membrane lined mould



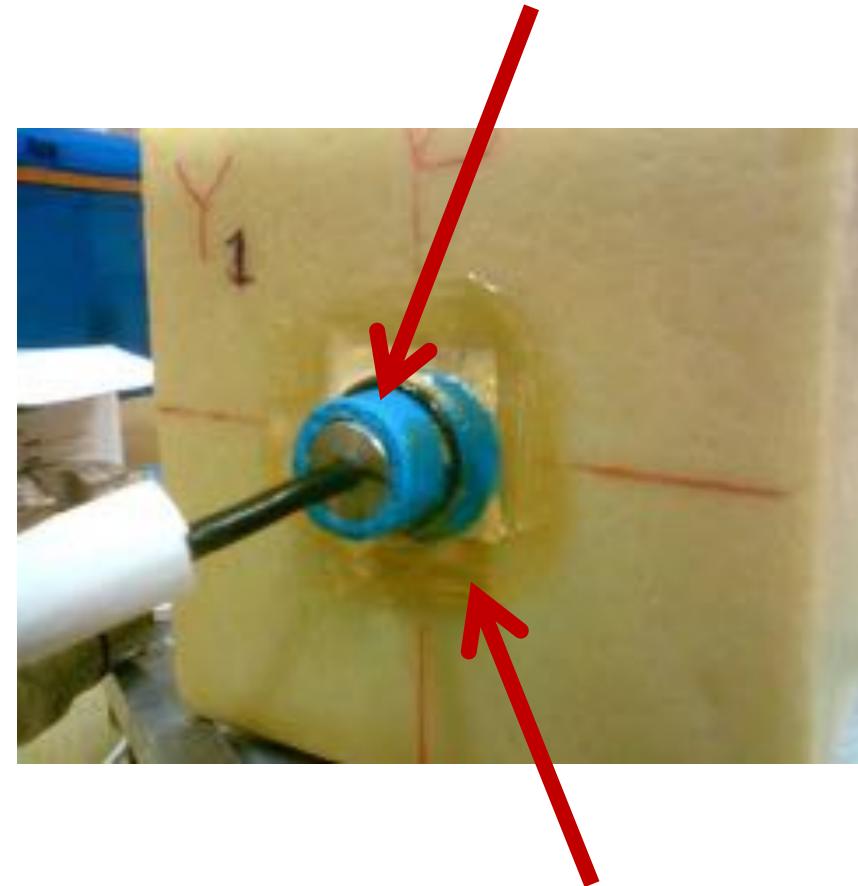
$e_{lab} \approx 0.6$  50 kPa vacuum applied

# Cubical cell apparatus – Bender / Extender Elements

T-shaped bender/extender elements

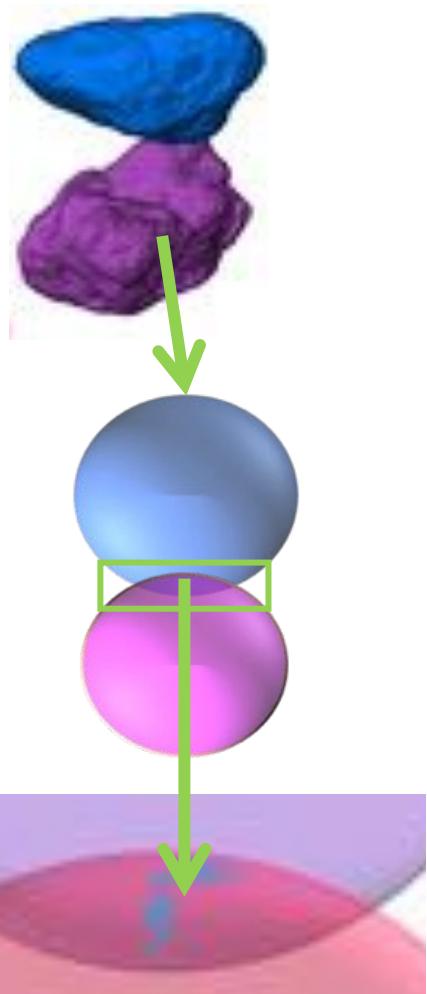


Sealing grommet to prevent loss of vacuum



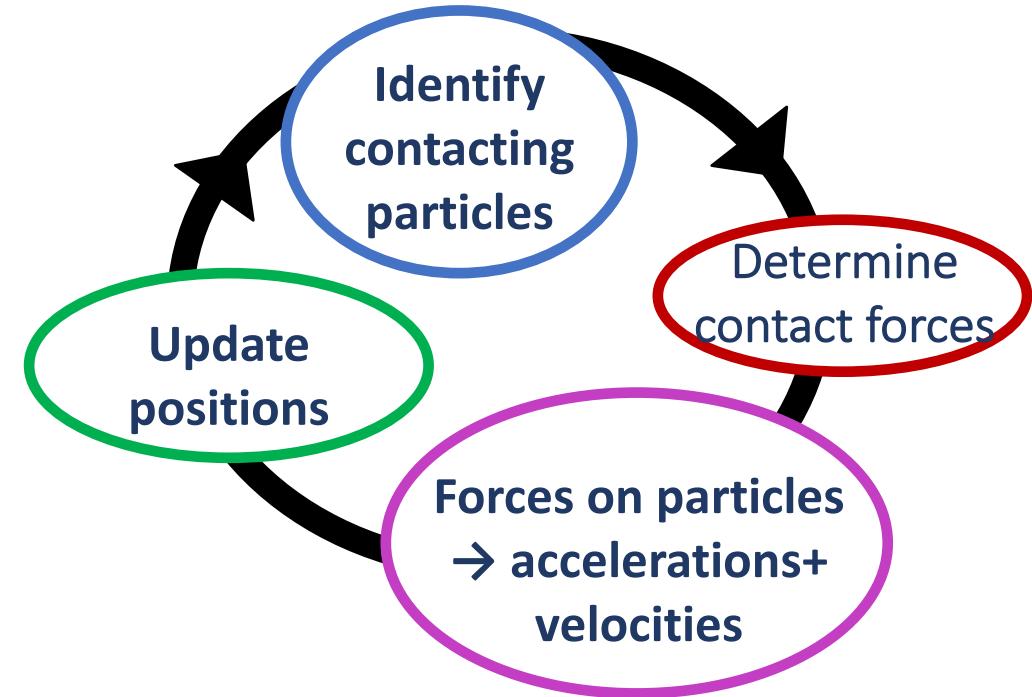
Layers of latex adhesive solution

# Discrete Element Method



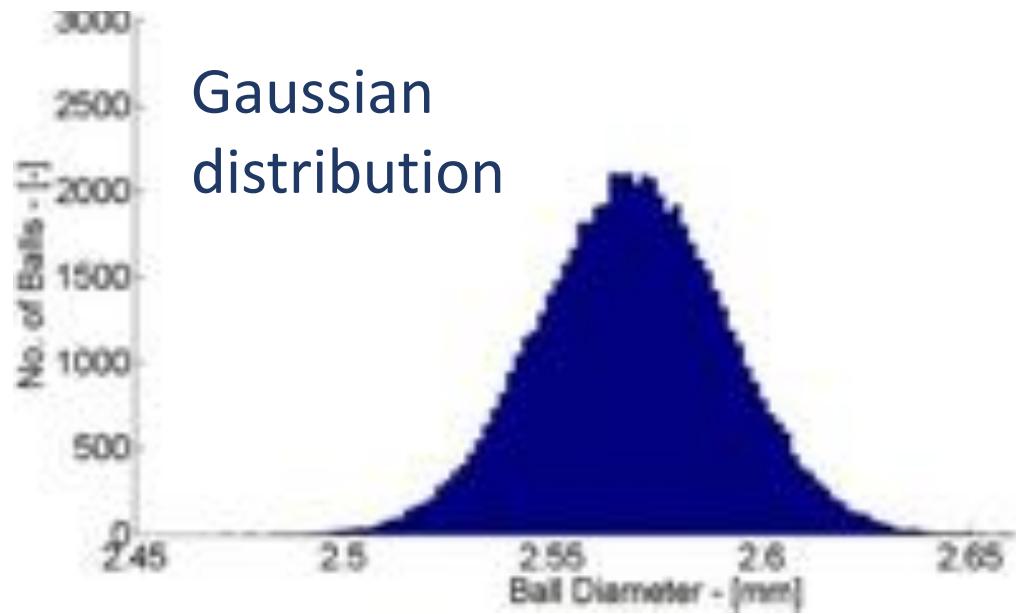
Explicitly models particles.

Idealizes particle shape and contact behaviour



Conceptually simple algorithm – transient analysis – step forward in time

# Discrete Element Method



62,445 particles  
Ave. coord. number: 5.185

$$e_{DEM} \approx 0.67$$

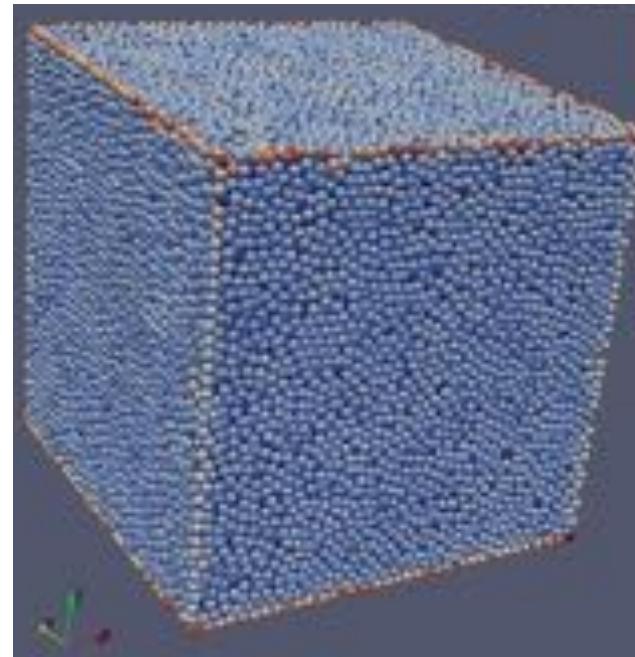
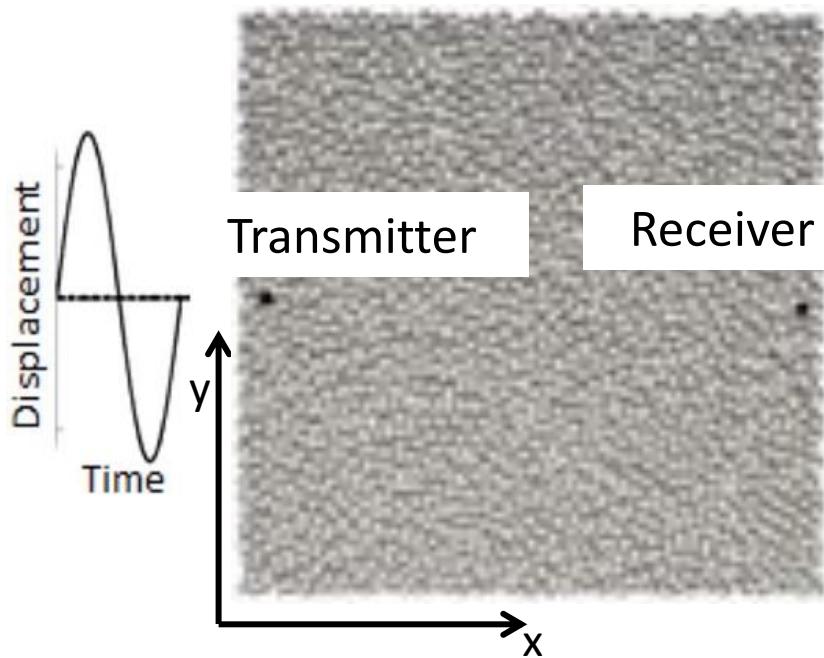


Pluviated using Granular Lammmps

Simulations run in PFC

Elevation

# DEM simulations of wave propagation

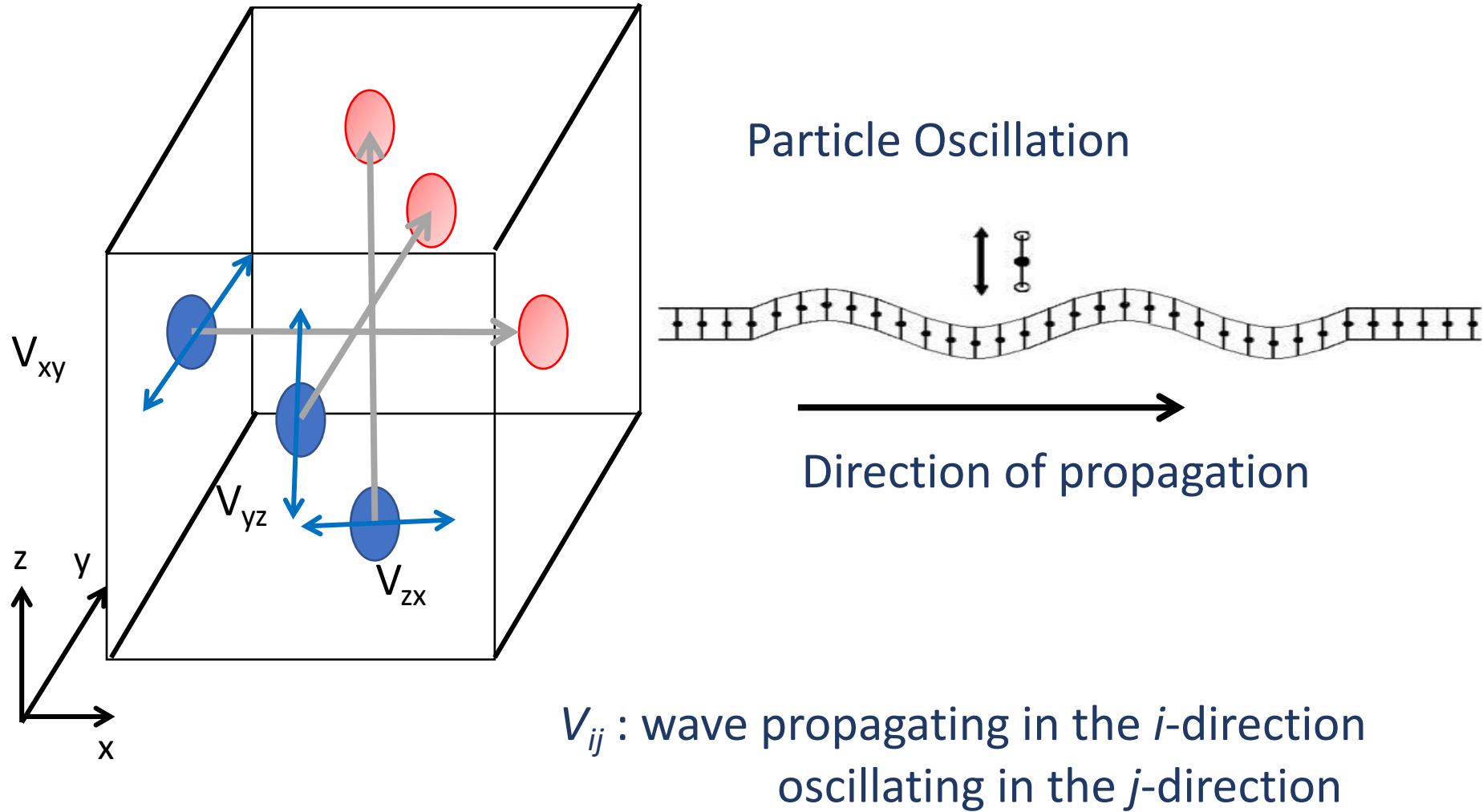


Simulated using PFC 3D

Flexible membrane boundaries

Simplified Hertz-Mindlin contact model – input  $G$  and  $\nu$  for particle  
Bender/extender elements modelled as point sources

# DEM simulations of wave propagation

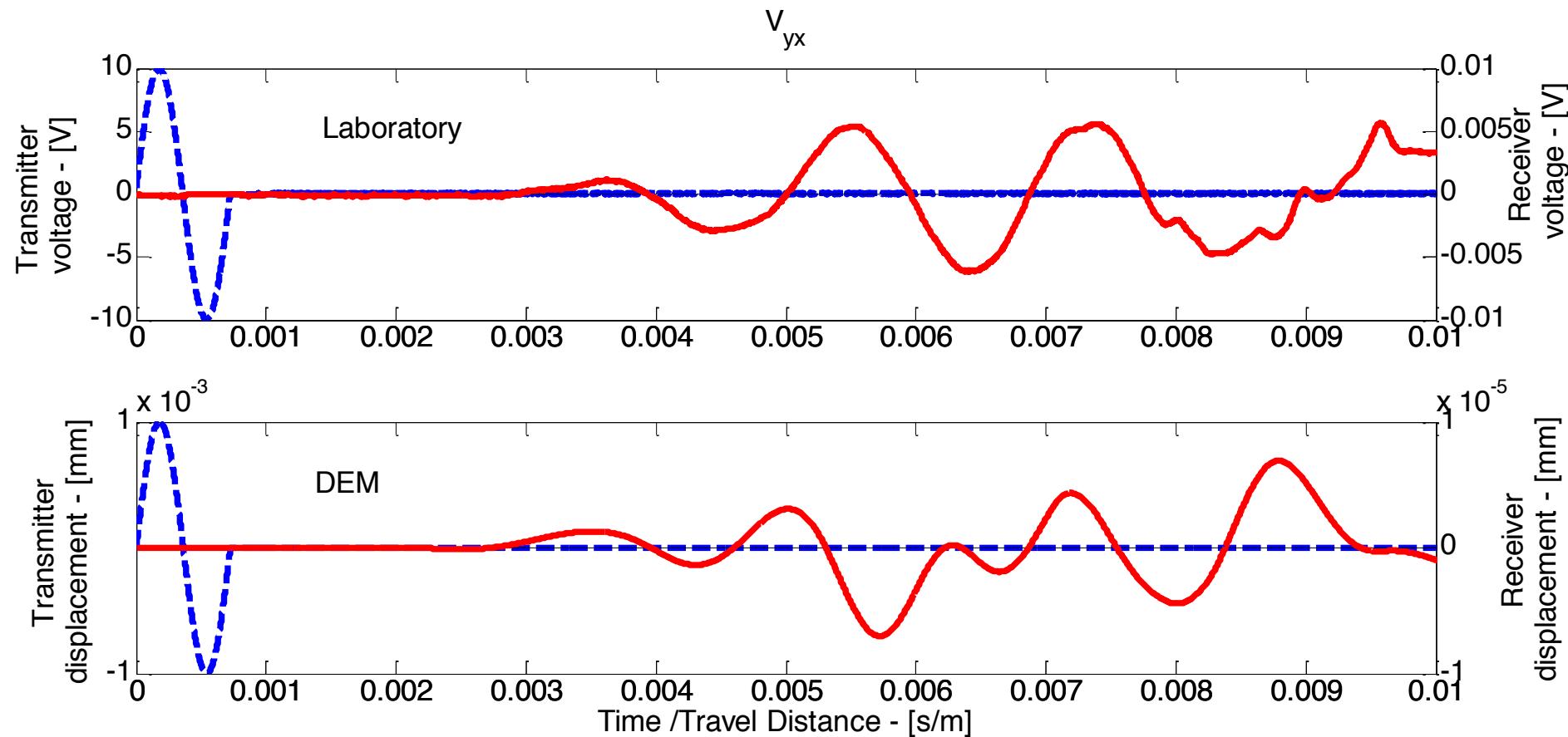


# $V_{yx}$ signals : Time domain

*Isotropic stress of 100 kPa*

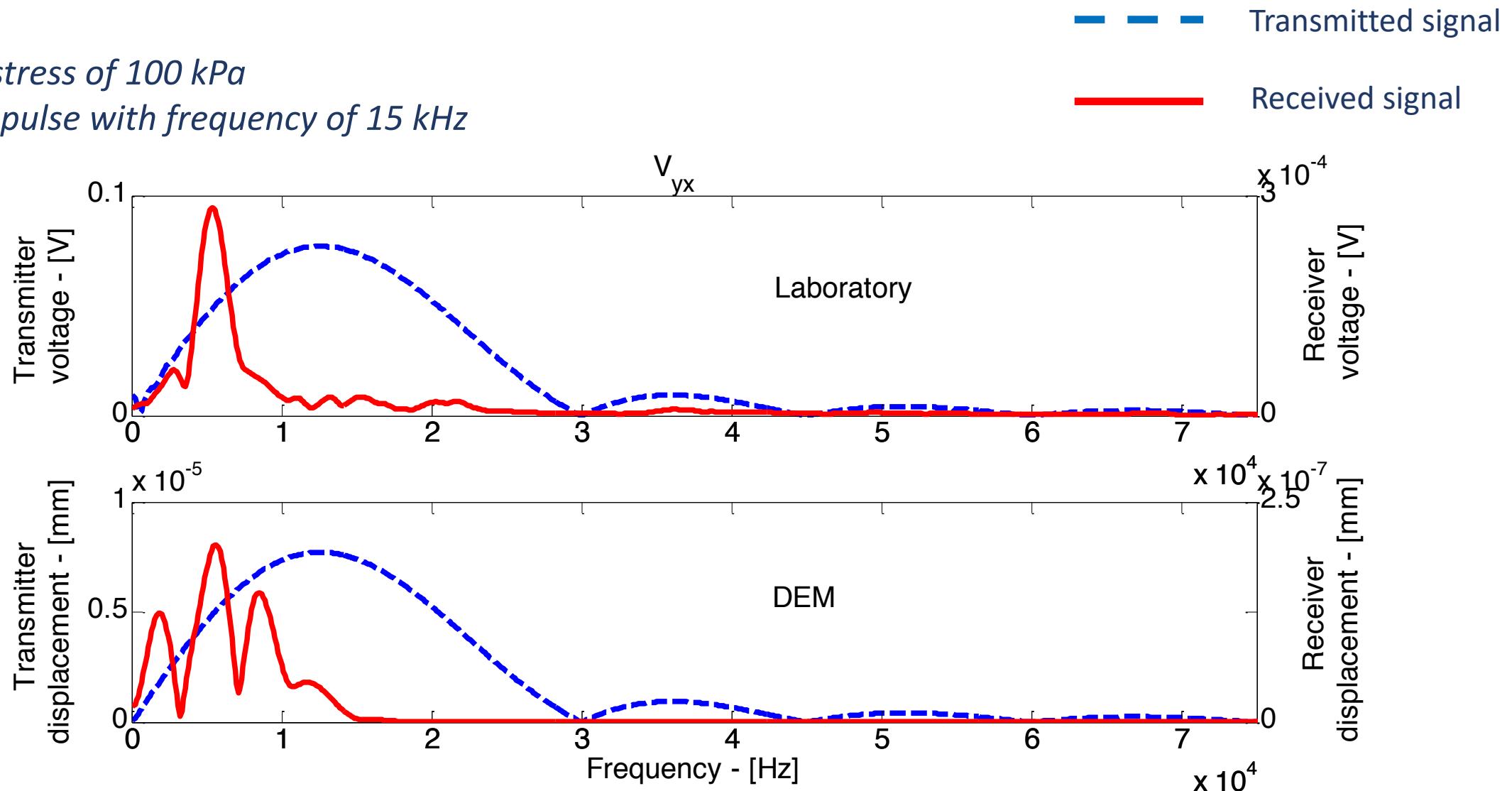
*Sinewave pulse with frequency of 15 kHz*

Transmitted signal  
Received signal

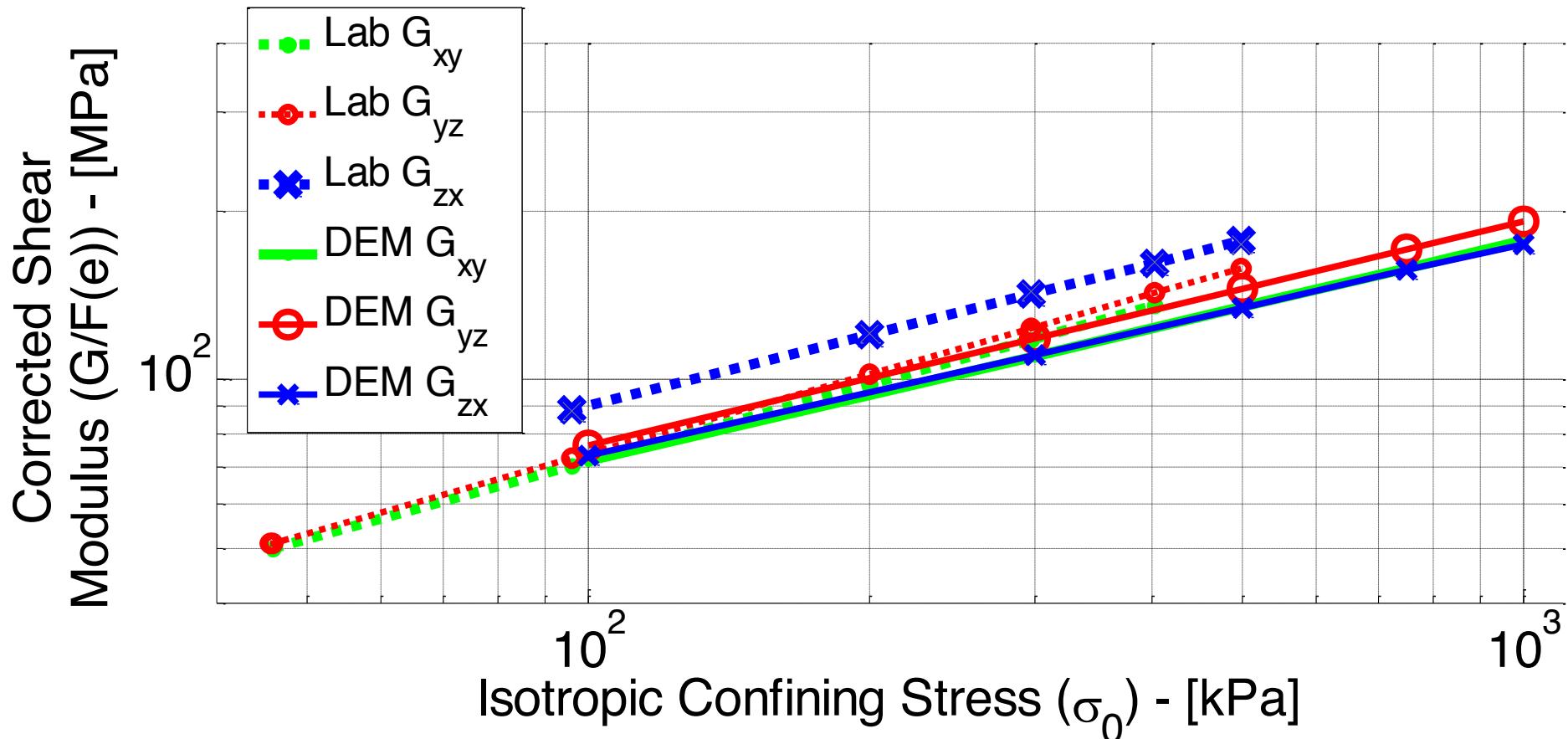


# $V_{yx}$ signals : Frequency domain

Isotropic stress of 100 kPa  
Sinewave pulse with frequency of 15 kHz



# Evolution of shear stiffness with confining pressure



$$G_{xy} = \rho V_{xy}^2$$

$$G \propto \sigma_0^\alpha$$

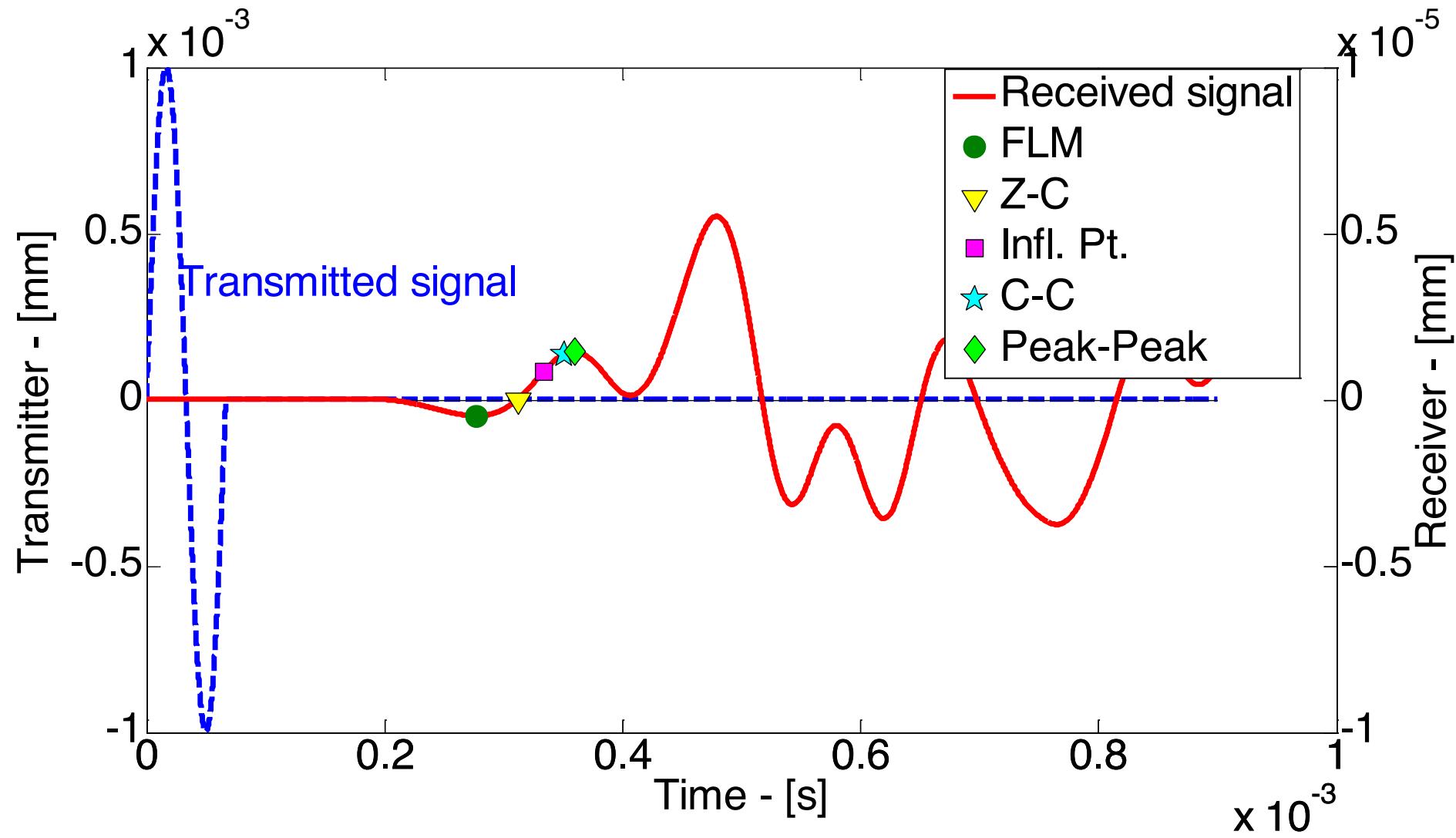
Lab:  $\alpha$  for  $G_{xy}=0.46$   
 $\alpha$  for  $G_{yz}=0.47$   
 $\alpha$  for  $G_{zx}=0.42$

DEM:  $\alpha$  for  $G_{xy}=0.39$   
 $\alpha$  for  $G_{yz}=0.40$   
 $\alpha$  for  $G_{zx}=0.38$

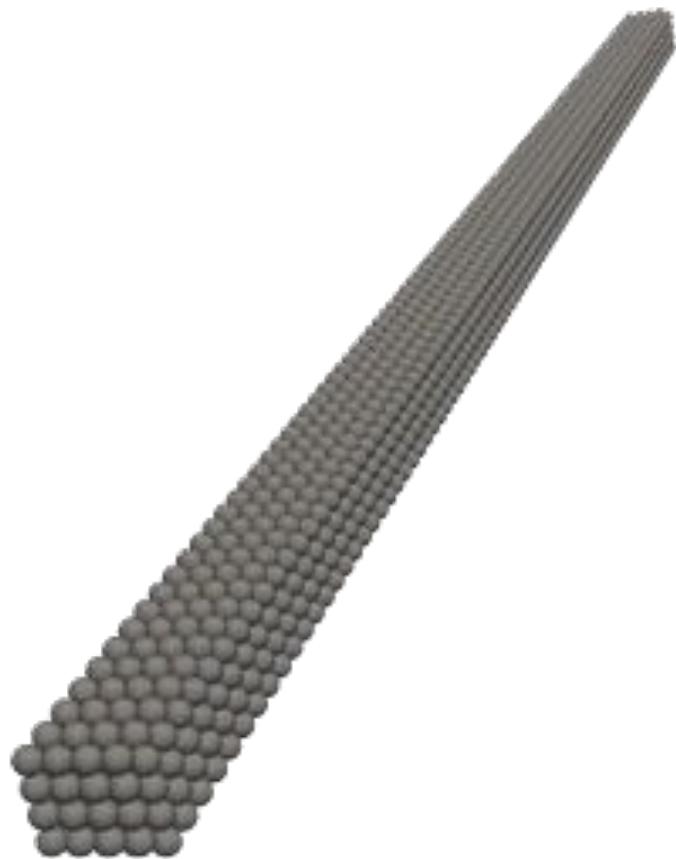
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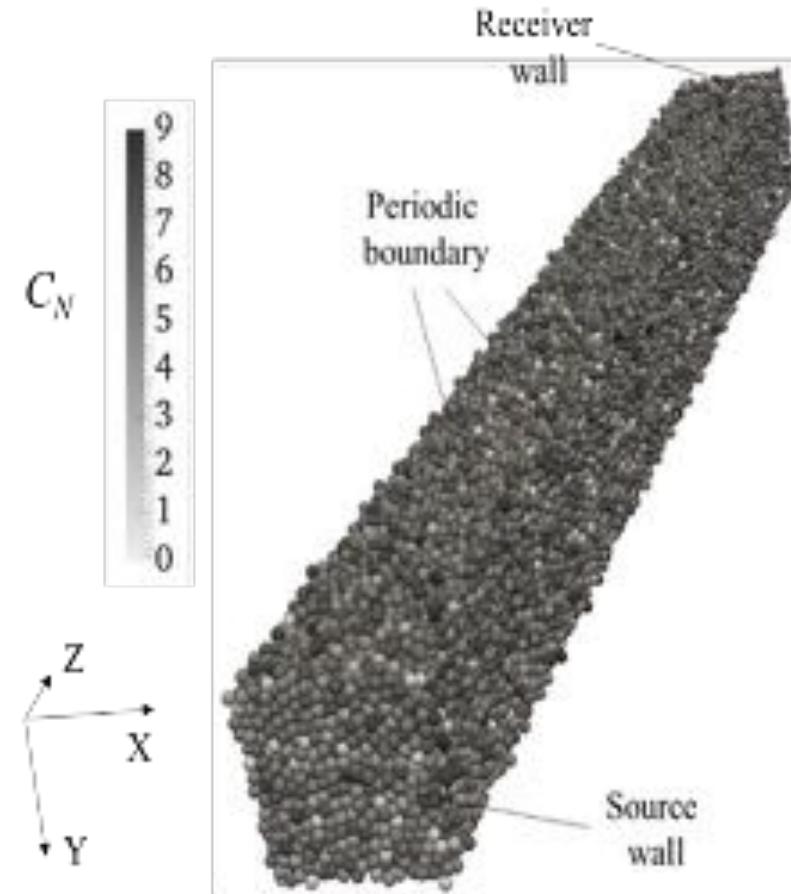
# Bender element test interpretation



# Bender element interpretation – DEM samples

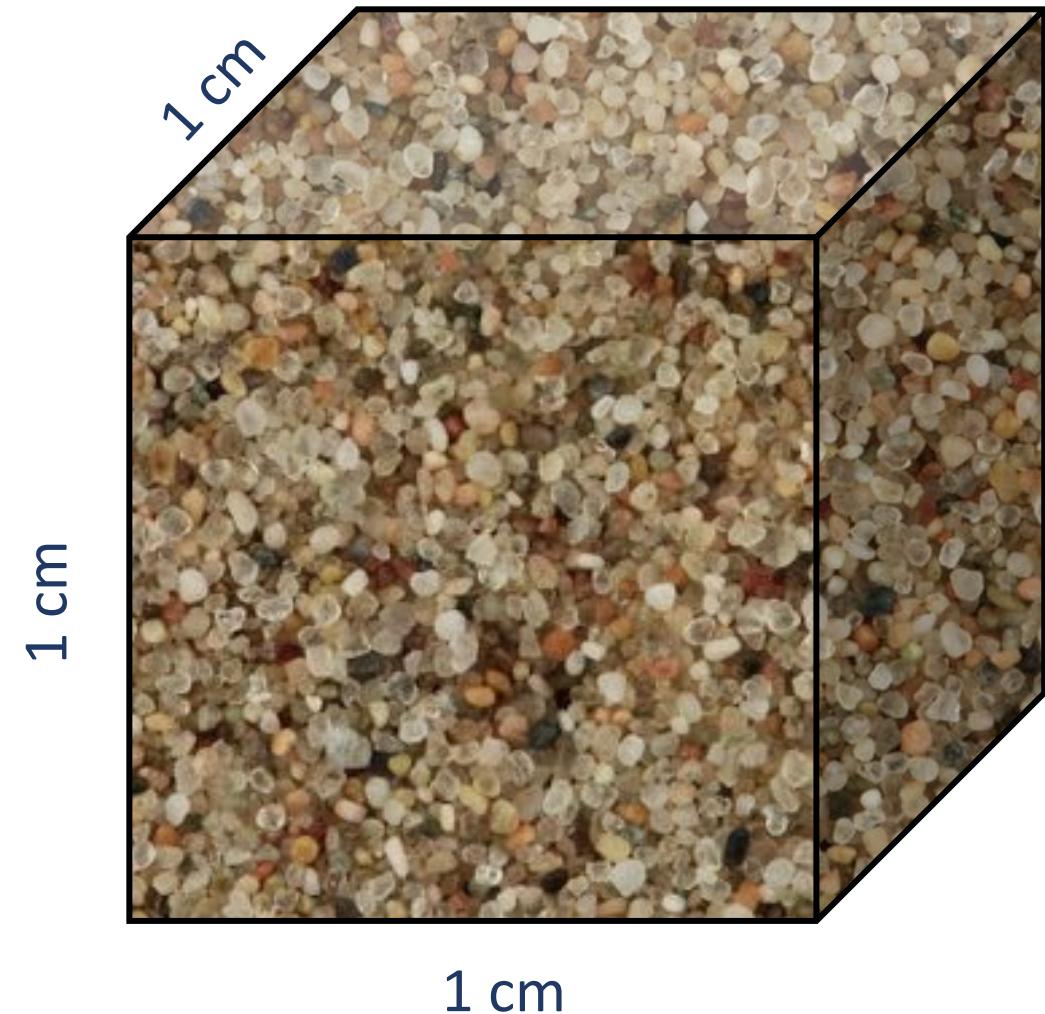
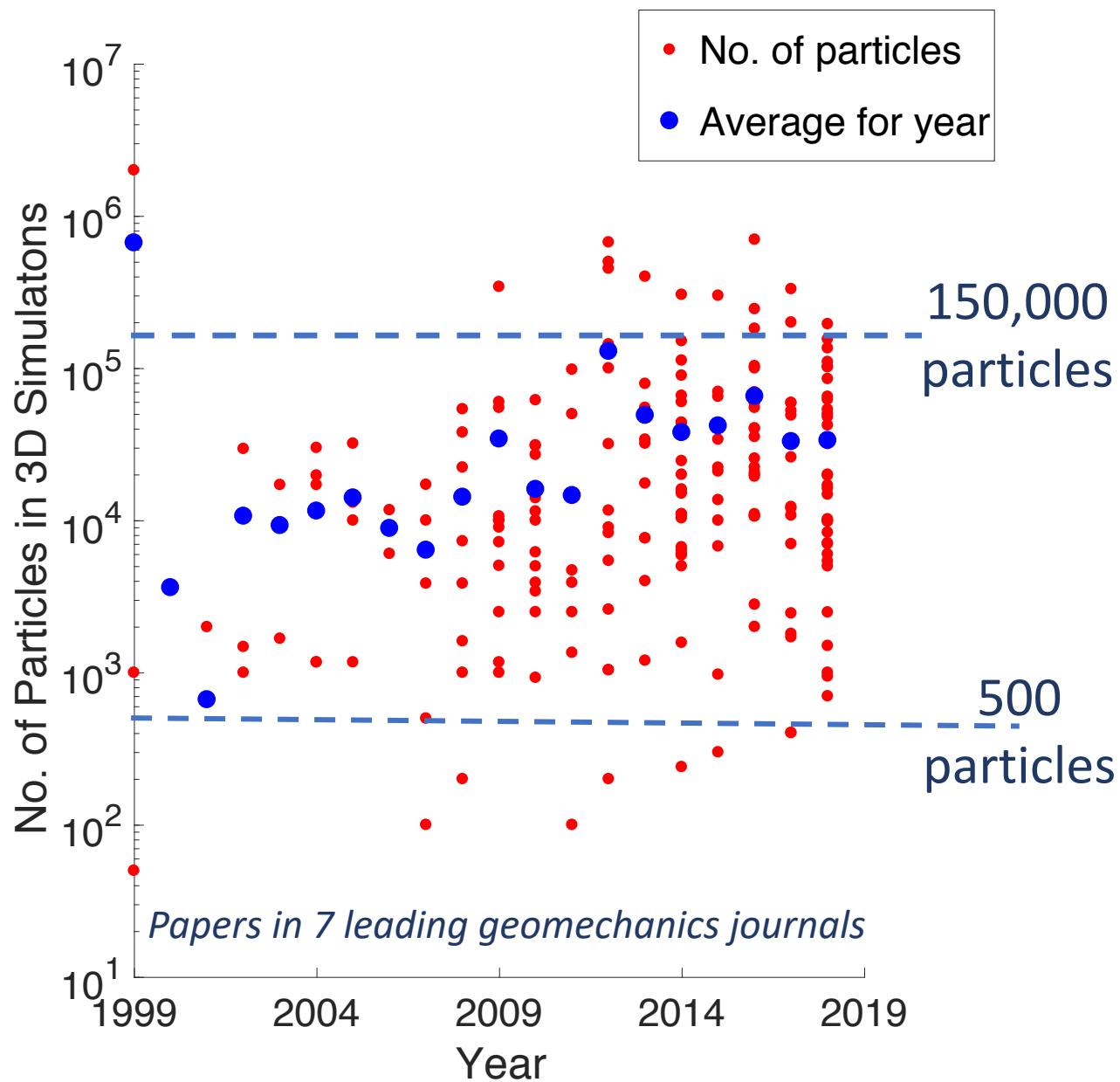


FCC Sample: 3,200 particles (4x4x200 layers)

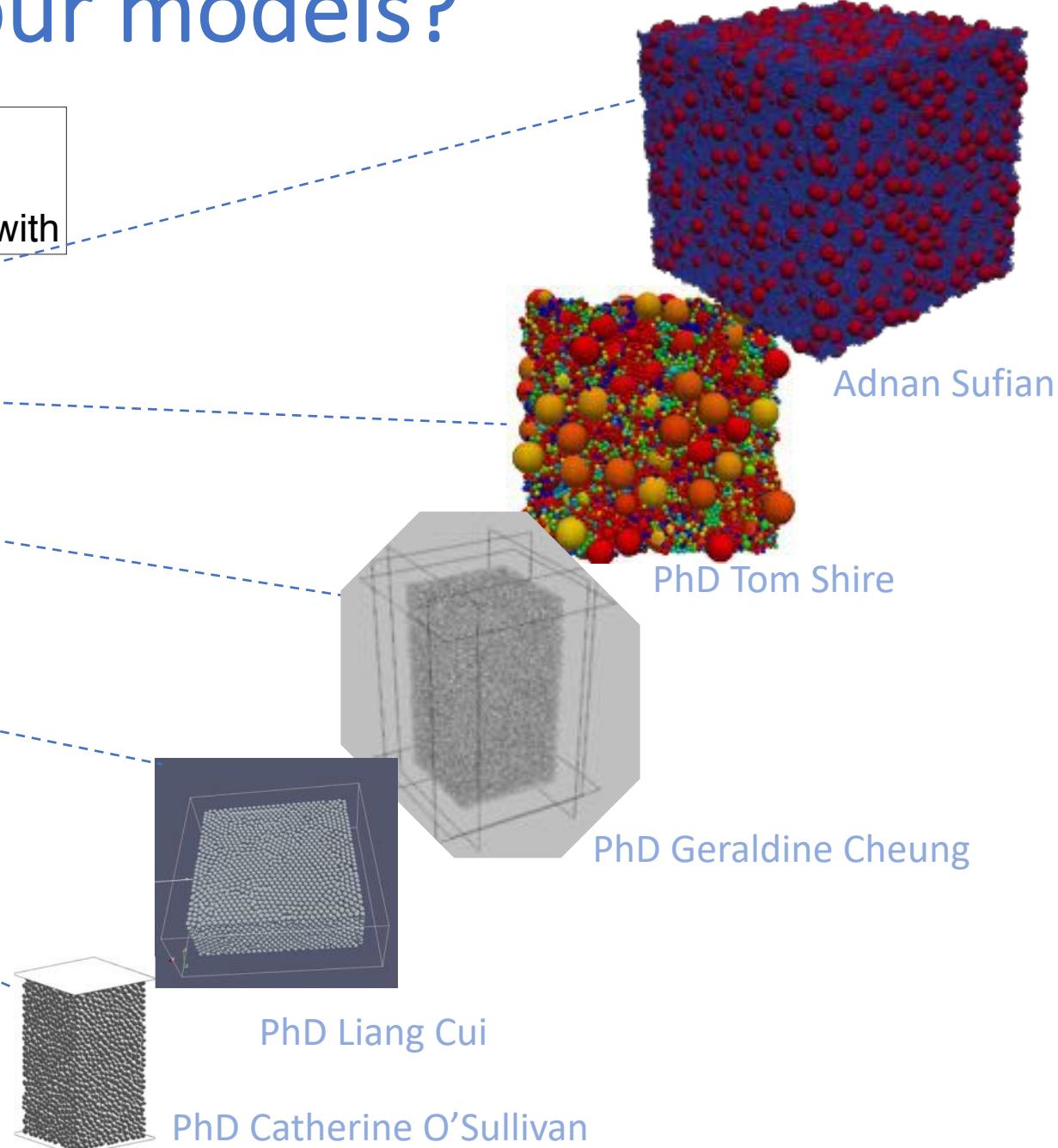
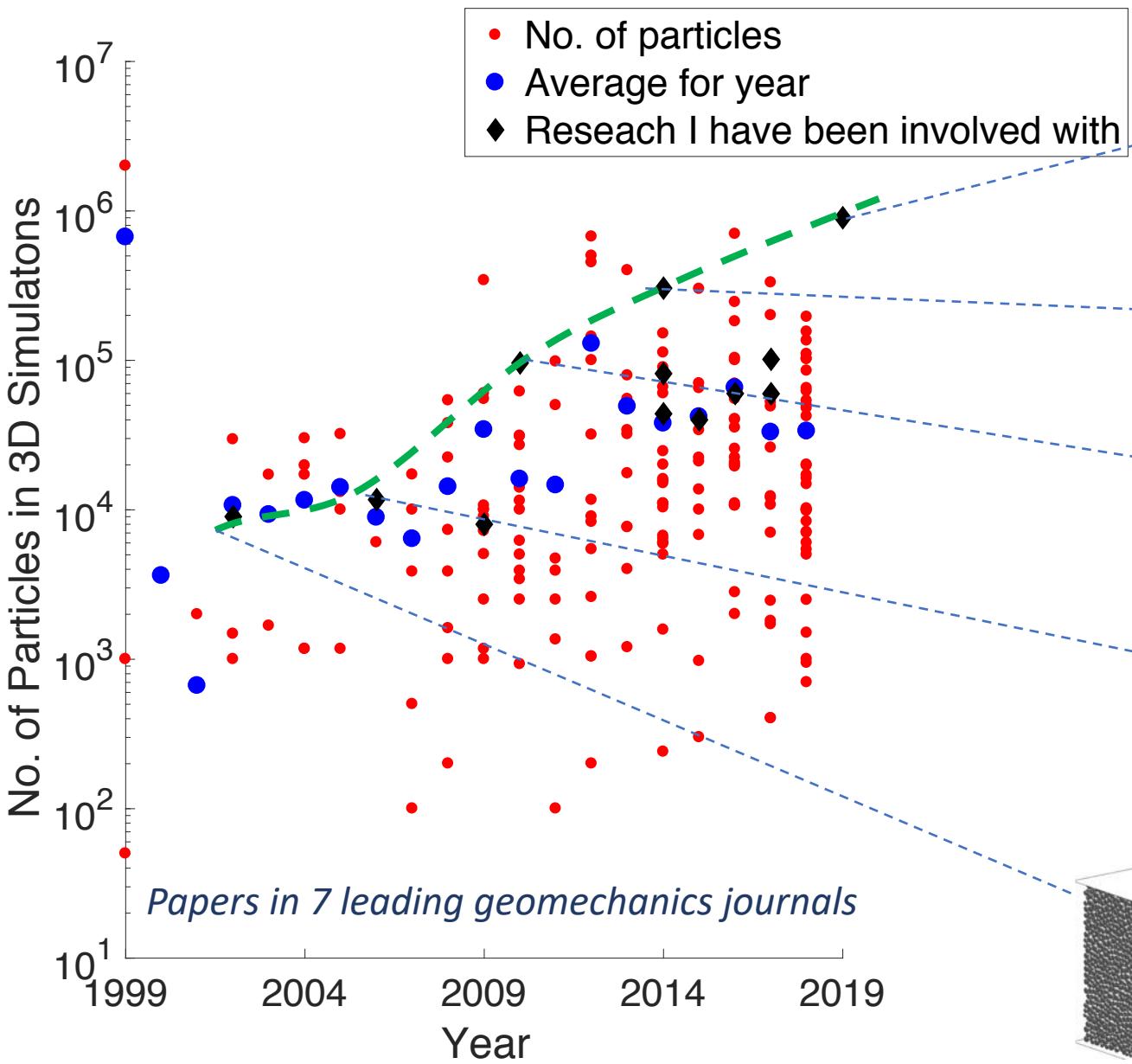


Random monodisperse sample:  
35,201 particles  
sample lengths (L) 141D to 144D

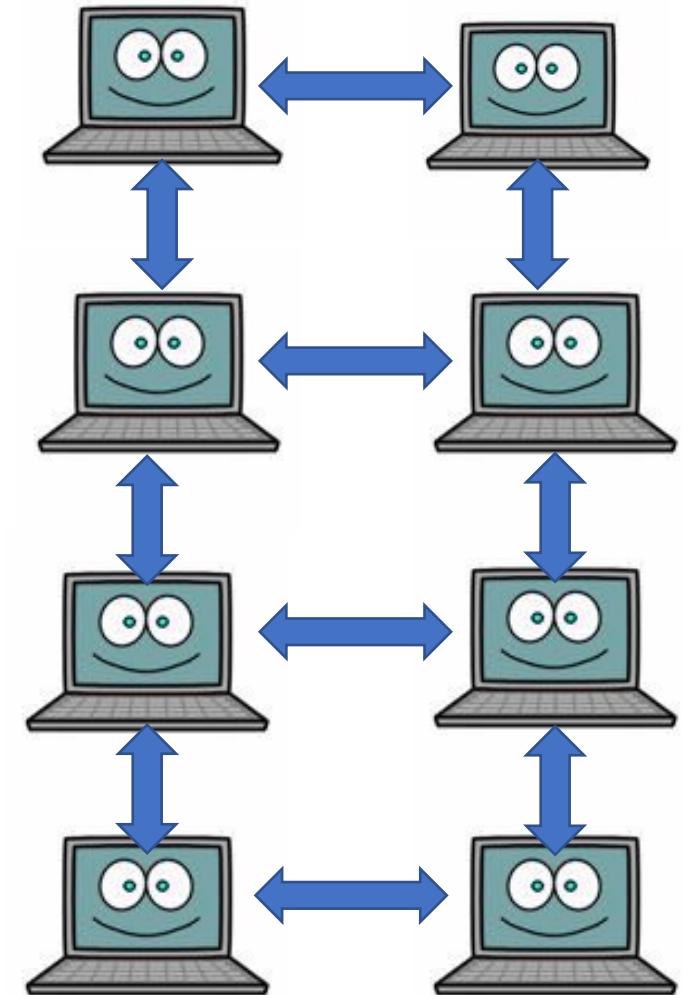
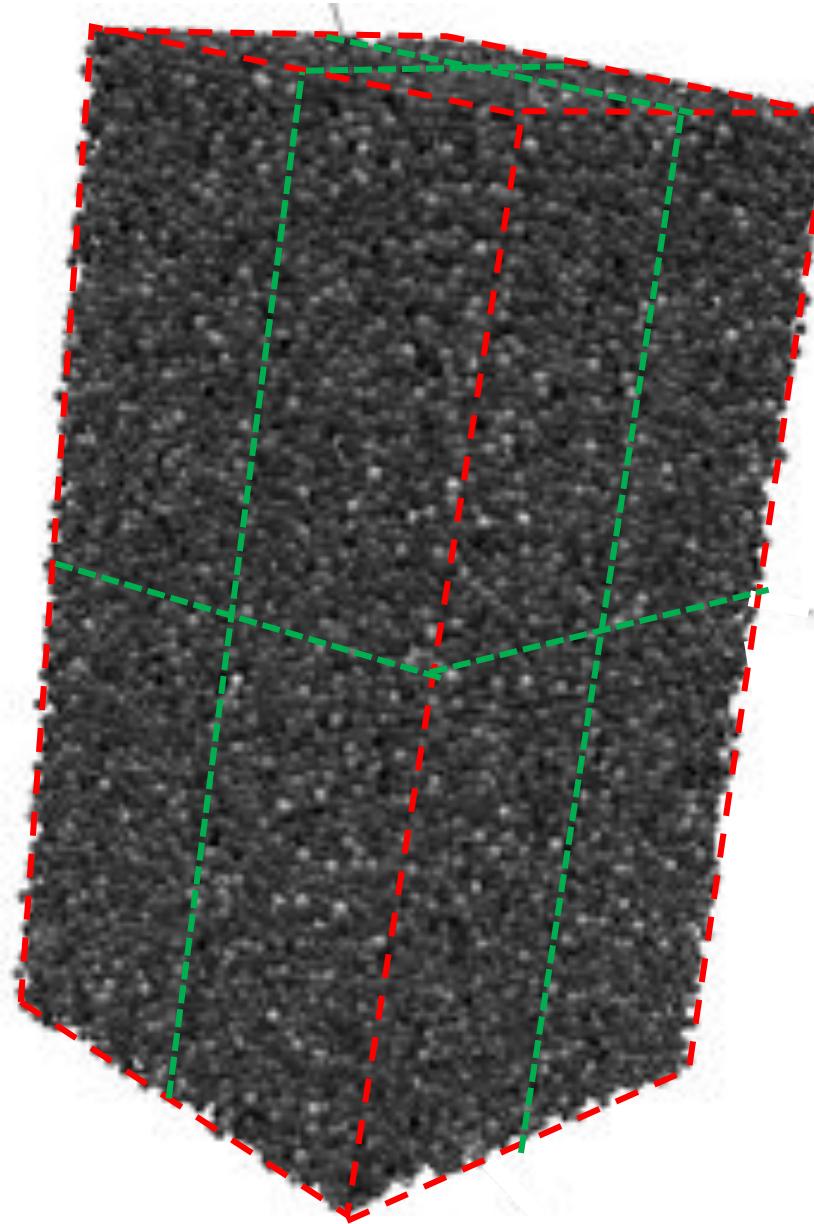
# How many particles in our models?



# How many particles in our models?



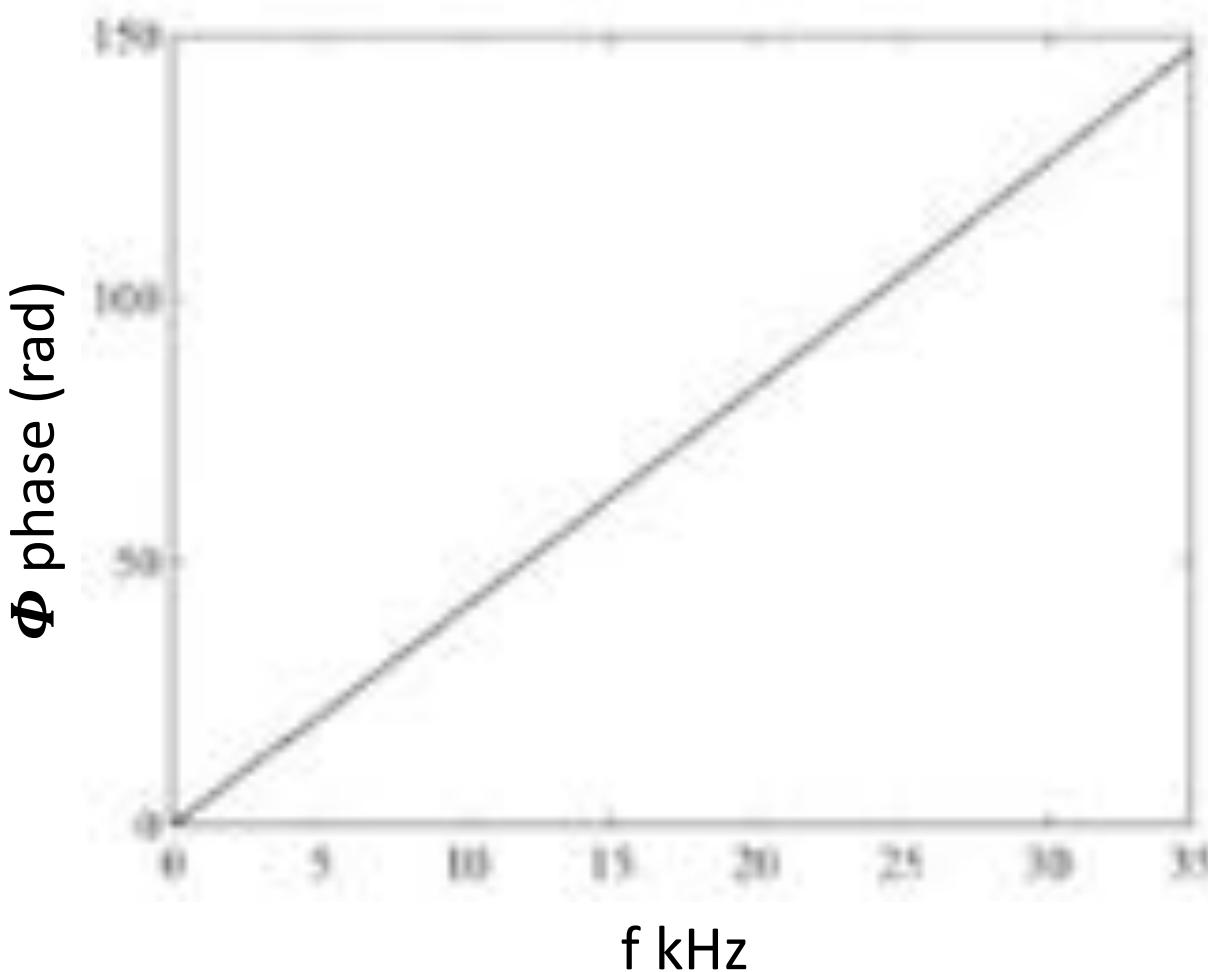
# High performance computers



# Stacked phase

Secant slope → phase velocity

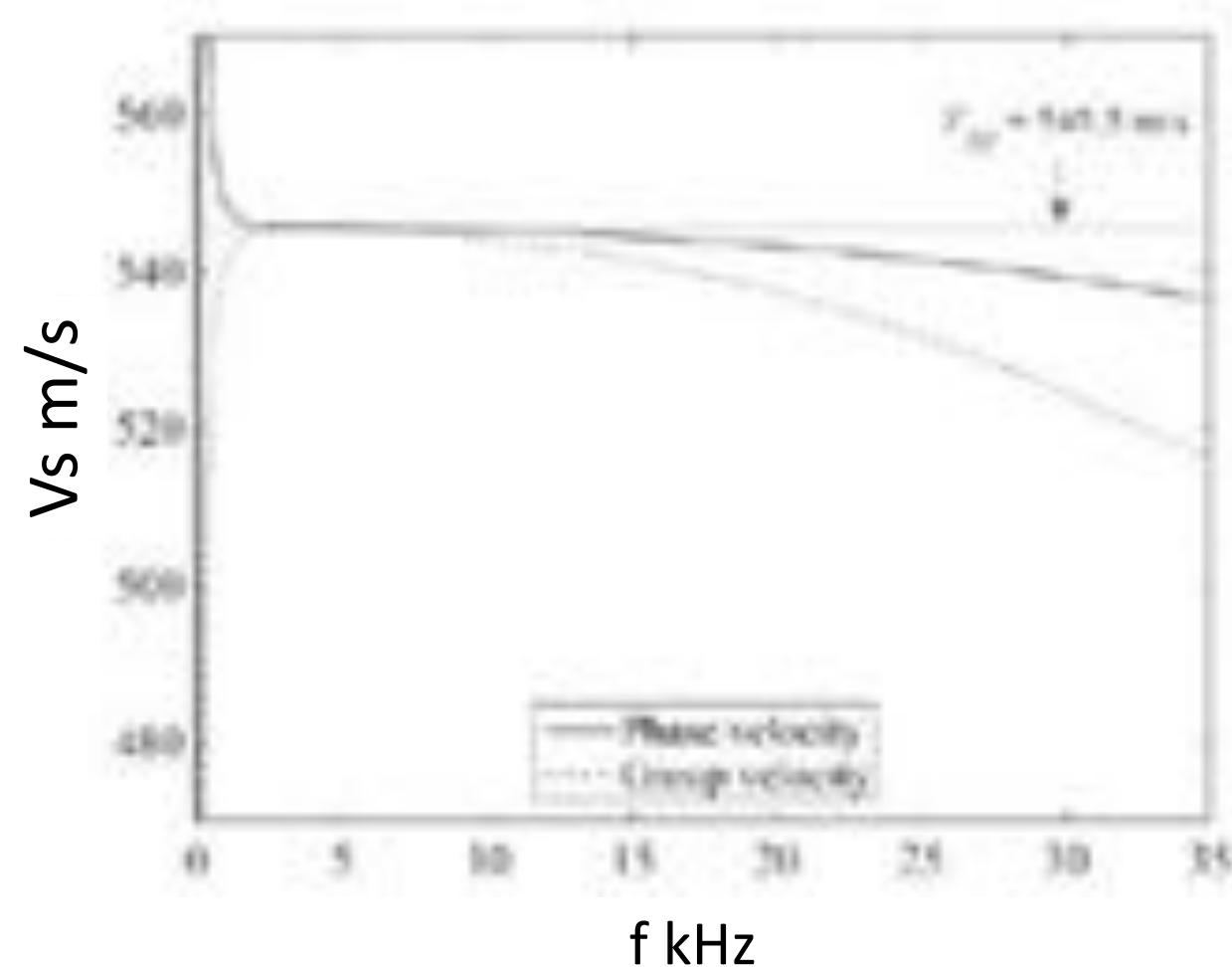
Tangent slope → group velocity



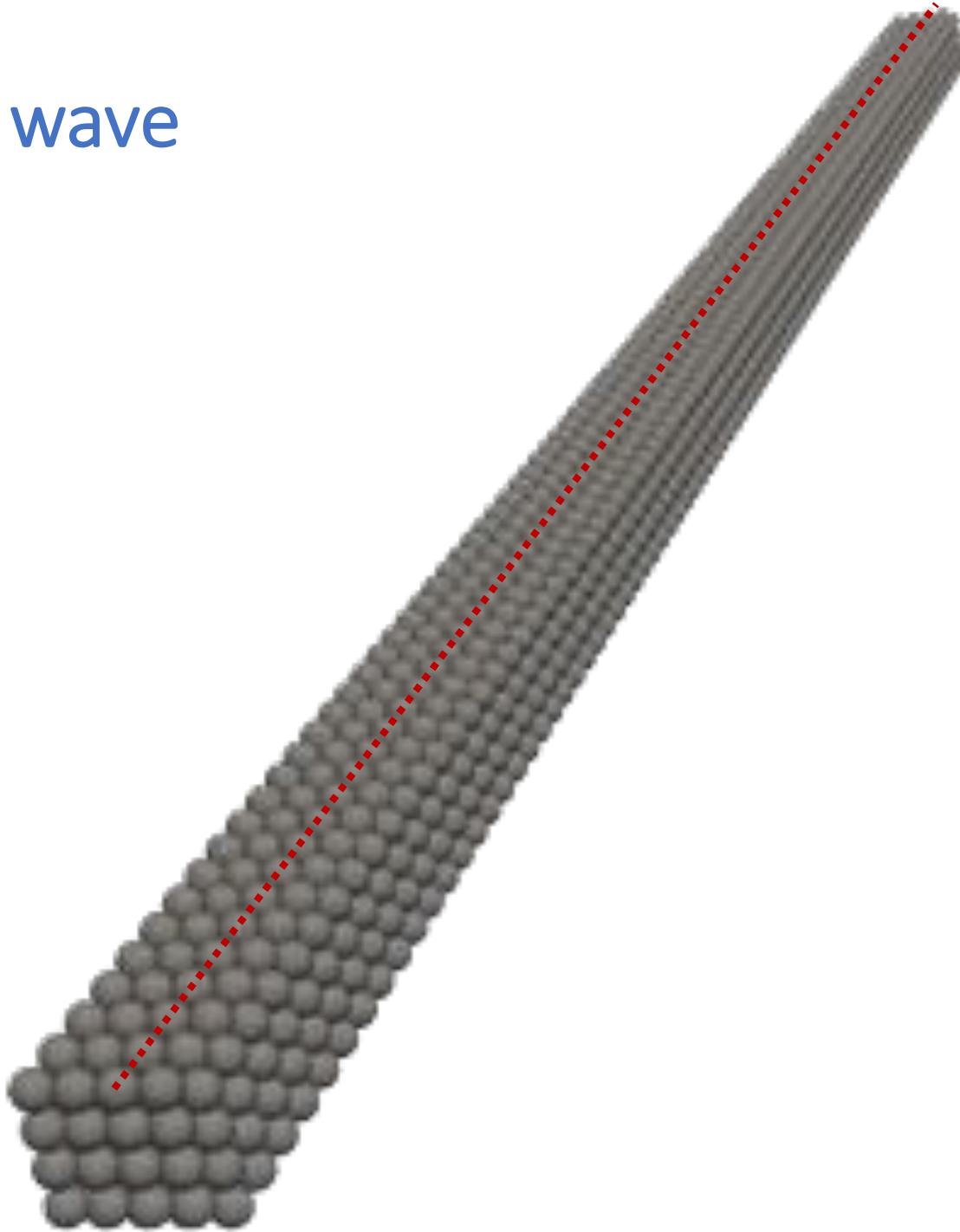
Velocities approach long-wave limit as frequency decreases



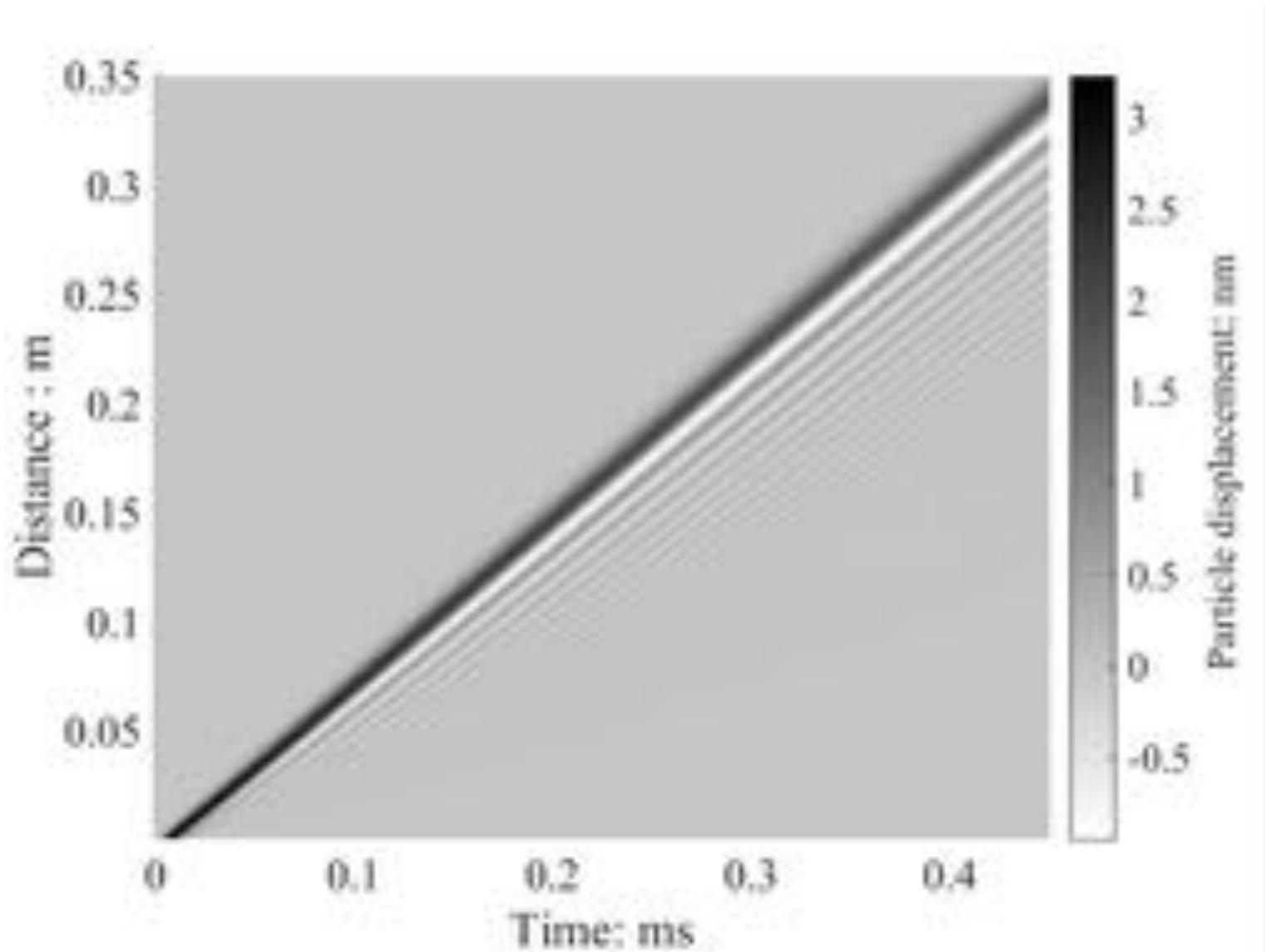
$V_s, sp$



# Analysis of propagating wave



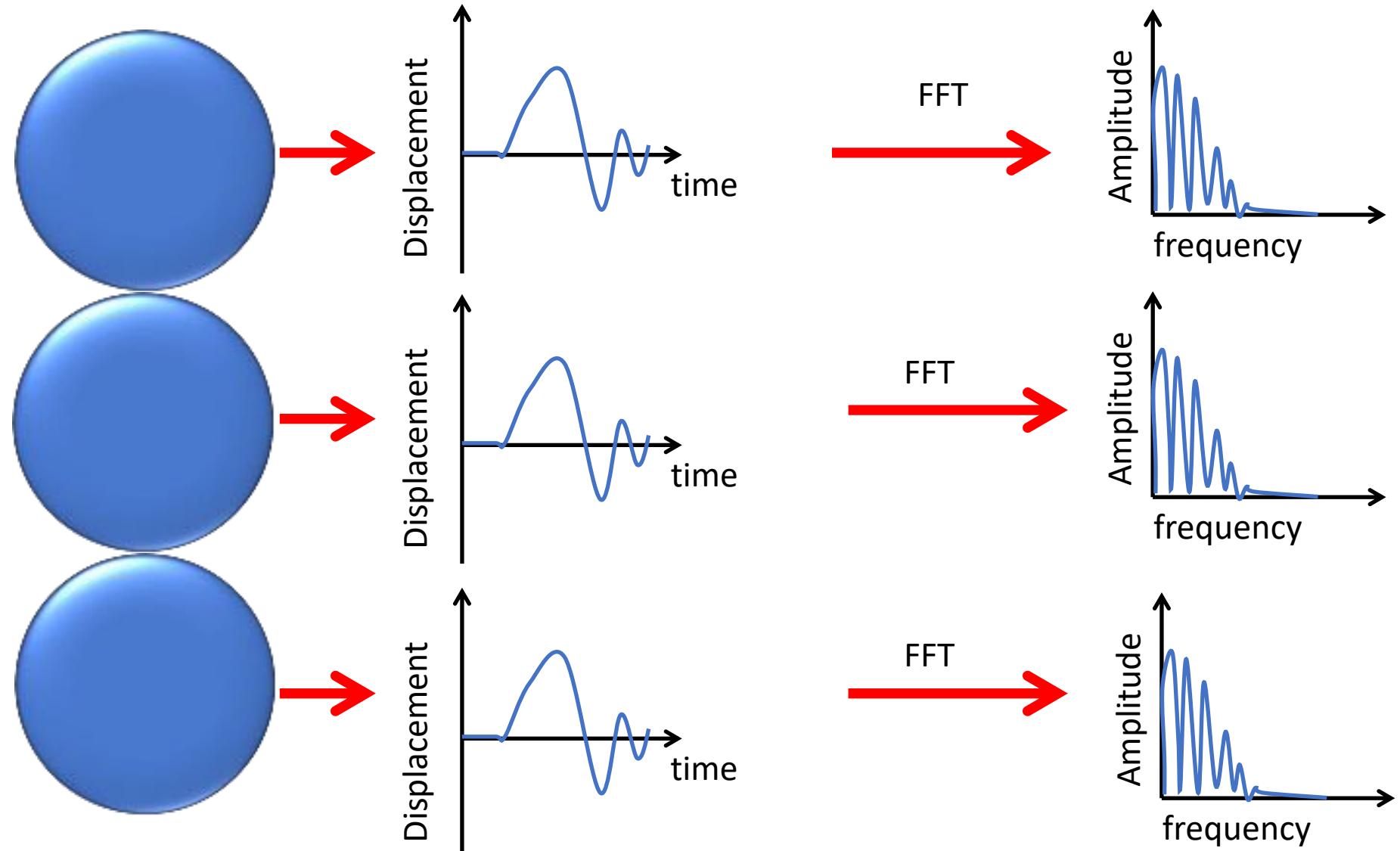
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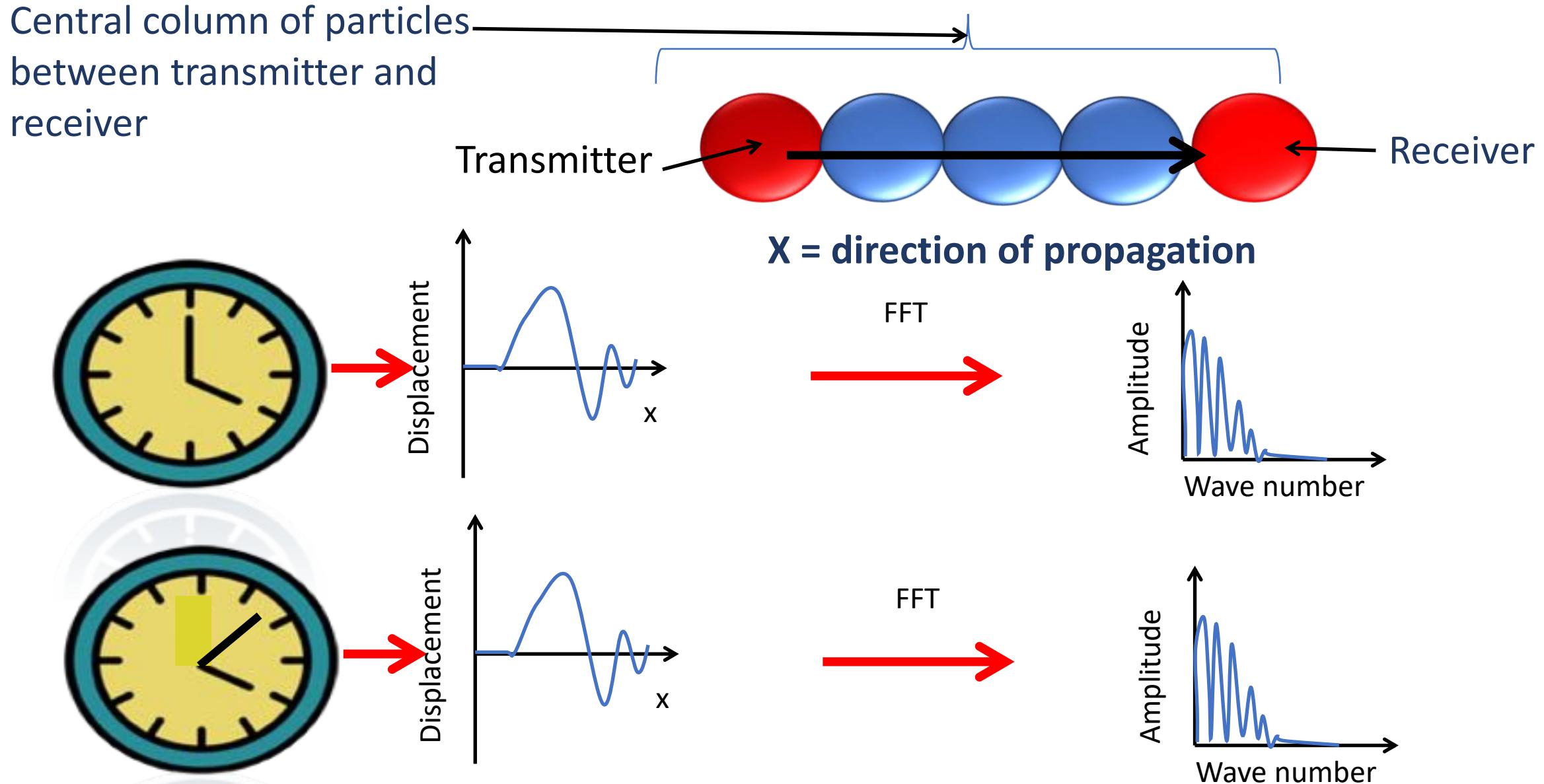
# Analysis of propagating wave: Frequency Domain

Central  
column of  
particles  
between  
transmitter  
and receiver

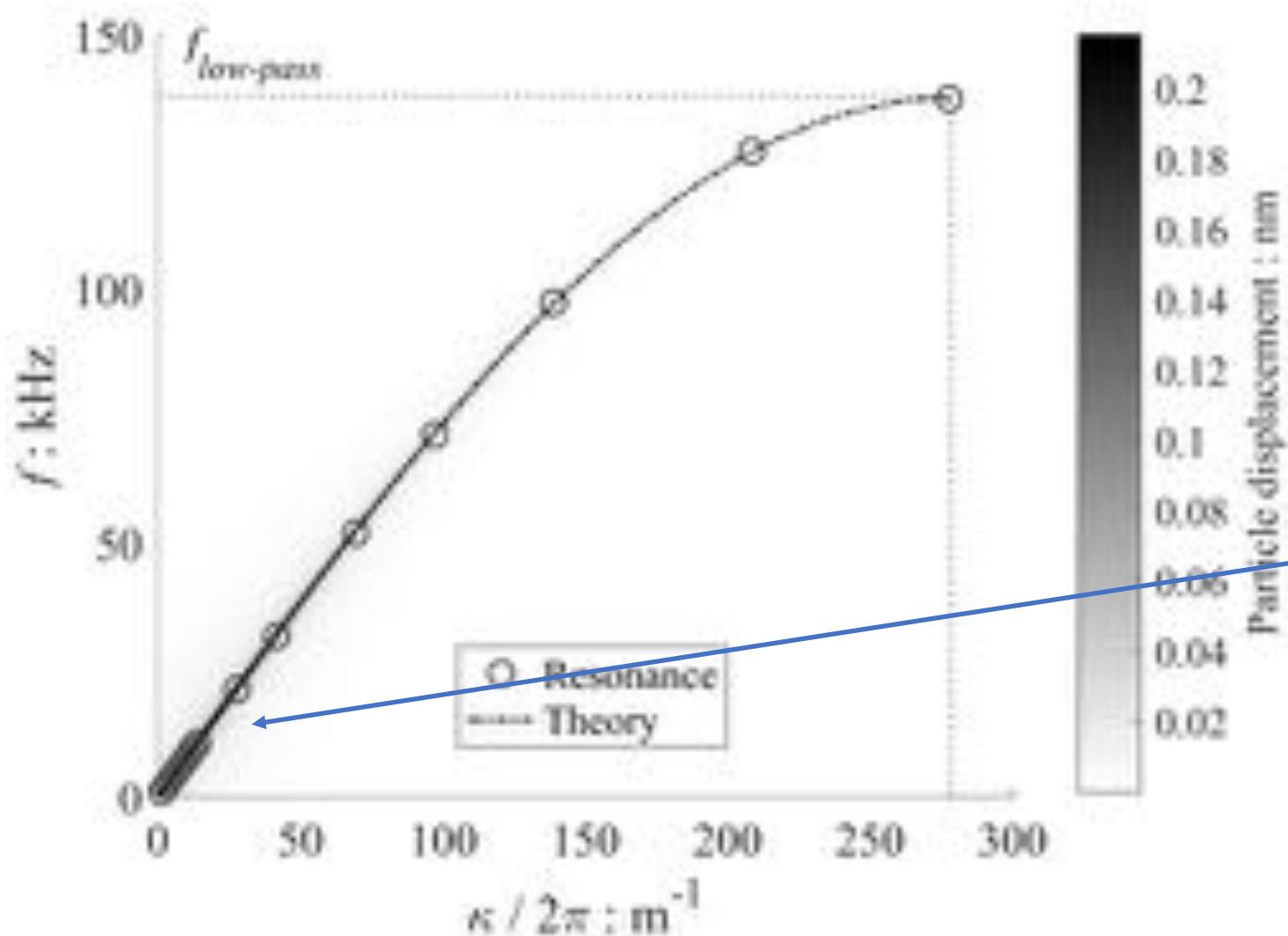
Mouraille, O.  
University of Twente



# Analysis of propagating wave: Frequency Domain



# Analysis of propagating wave



Slope gives long wave velocity

# Measured wave velocities

Contact model	$S_q$ μm	$\sigma^*$ MPa	$e$	$\overline{C_N}$	$V_{S,dL,w}$ m/s	$V_{S,P-P}$ m/s		$V_{S,SP}$ m/s	$V_{S,dispersion}$ m/s
HM	0	0.1	0.353	12	545	542	(-0.4%)	545	(-0.2%)
		0.2	0.352	12	612	610	(-0.4%)	612	(0.0%)
		0.3	0.352	12	656	652	(-0.5%)	655	(-0.1%)
		0.5	0.352	12	715	710	(-0.7%)	713	(-0.2%)
		1	0.351	12	804	796	(-1.0%)	801	(-0.3%)
		10	0.341	12	1175	1167	(-0.7%)	1171	(-0.3%)

# Measured wave velocities

Packing	Contact model	$S_q$ μm	$\sigma'$ MPa	$e$	$\bar{C}_N$	$V_{S,dL/dt}$ m/s	$V_{S,P,P}$ m/s	$V_{S,SP}$ m/s	$V_{S,dispersion}$ m/s
RDP	HM	0	0.1	0.544	6.02	365	368 (0.9%)	373 (2.1%)	370 (1.3%)
			0.2	0.543	6.07	411	415 (0.8%)	420 (2.1%)	417 (1.4%)
			0.3	0.543	6.1	442	445 (0.7%)	447 (1.1%)	447 (1.1%)
			0.5	0.542	6.15	483	486 (0.6%)	495 (2.4%)	488 (1.1%)
			1	0.54	6.24	546	549 (0.5%)	560 (2.5%)	552 (1.0%)
			10	0.519	6.69	831	831 (0.0%)	849 (2.2%)	837 (0.8%)
RLP	HM	0	0.1	0.646	5.03	301	306 (1.7%)	303 (0.8%)	306 (1.6%)
			0.2	0.645	5.12	343	349 (1.7%)	340 (-0.8%)	348 (1.4%)
			0.3	0.644	5.17	371	377 (1.5%)	371 (0.0%)	376 (1.4%)
			0.5	0.643	5.25	410	416 (1.5%)	409 (-0.2%)	416 (1.4%)
			1	0.64	5.36	470	476 (1.3%)	466 (-0.7%)	476 (1.4%)
			10	0.613	5.91	749	752 (0.5%)	753 (0.5%)	756 (1.0%)

# Measured wave velocities

Packing	Contact model	$S_g$ μm	$\sigma'$ MPa	$e$	$\bar{C}_N$	$V_{S,dL/dr}$ m/s	$V_{S,P,P}$ m/s	$V_{S,SP}$ m/s	$V_{S,dispersion}$ m/s
RDP	0.5	0.1	0.544	6.13	324	327	(0.99)	328	(1.5%)
		0.2	0.543	6.16	382	384	(0.59)	389	(1.9%)
		0.3	0.543	6.19	414	417	(0.09)	420	(1.3%)
		0.5	0.542	6.23	457	460	(0.59)	465	(1.7%)
		1	0.539	6.3	523	525	(0.39)	535	(2.2%)
RM	1	10	0.518	6.7	827	825	(-0.29)	840	(1.0%)
		0.1	0.541	6.16	282	284	(0.79)	285	(0.9%)
		0.2	0.54	6.2	349	351	(0.59)	355	(1.9%)
		0.3	0.539	6.22	393	395	(0.49)	398	(1.3%)
		0.5	0.538	6.25	445	446	(0.39)	453	(1.7%)
	10	0.1	0.536	6.31	514	514	(0.09)	516	(0.6%)
		0.2	0.514	6.77	813	811	(-0.29)	810	(0.79)
		0.3	0.514	6.77	813	811	(-0.29)	810	(0.79)
		0.5	0.514	6.77	813	811	(-0.29)	810	(0.79)
		1	0.514	6.77	813	811	(-0.29)	810	(0.79)

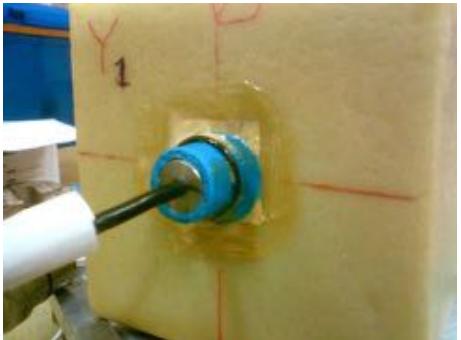
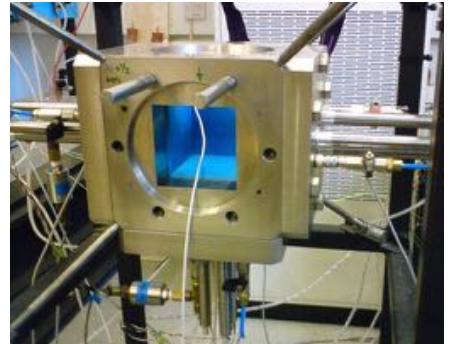
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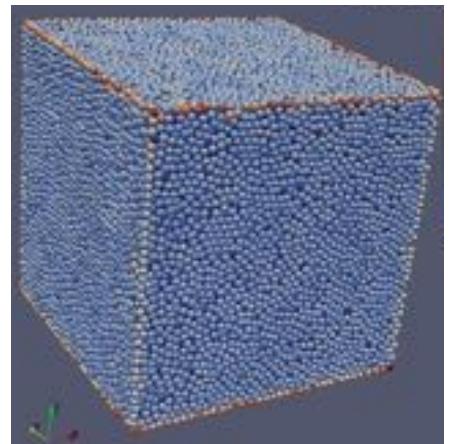
# Bender element testing



Bender elements – piezo electric elements



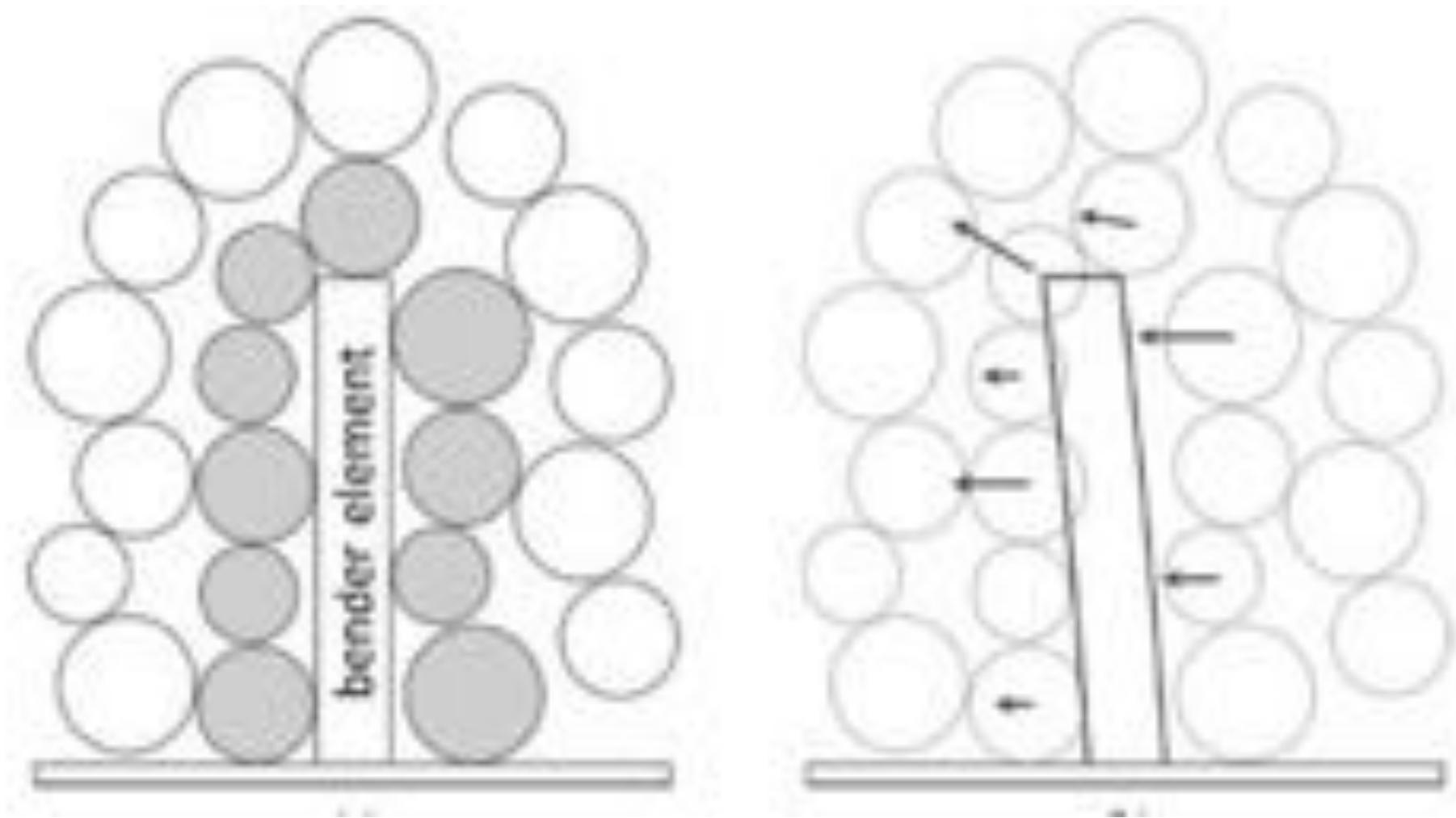
Cubical cell University of Bristol



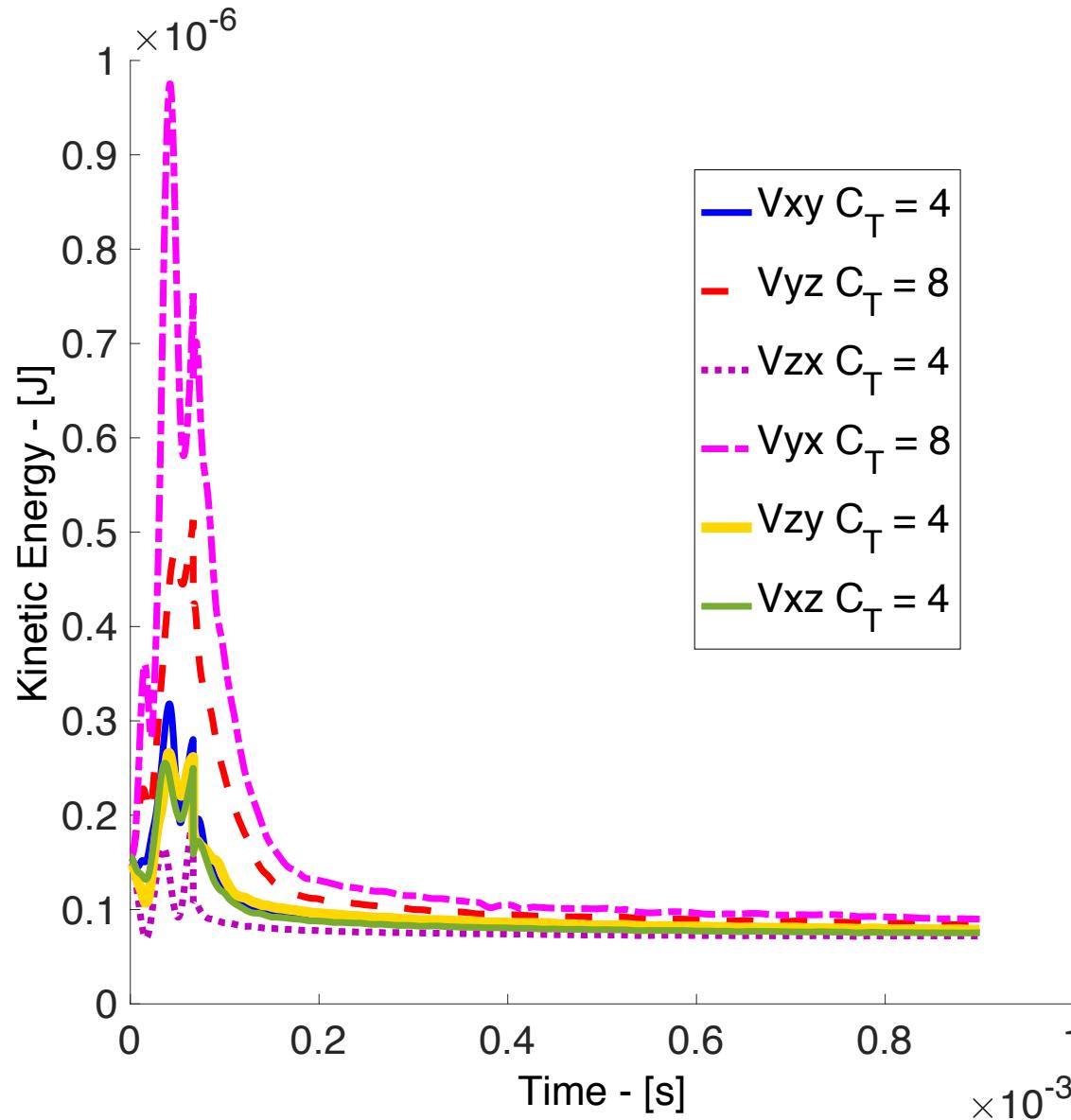
Bender element modelled as point source (particle)

DEM simulation of cubical cell

# Bender element testing

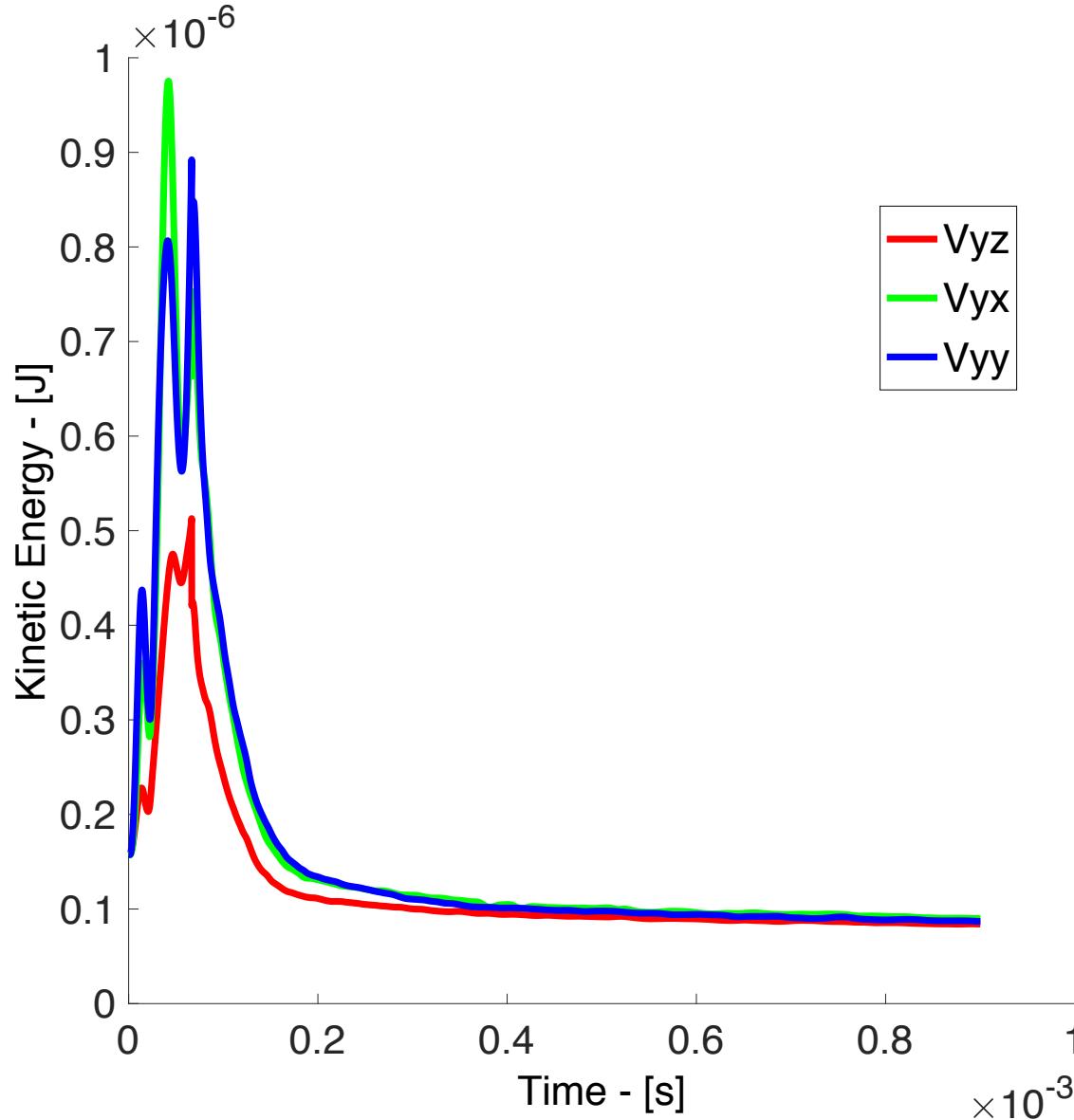


# Influence of transmitter connectivity



Transmitter grain

# Influence of transmitter connectivity



Local fabric tensor:

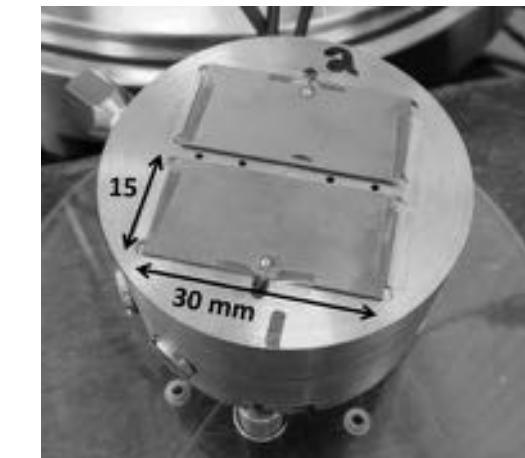
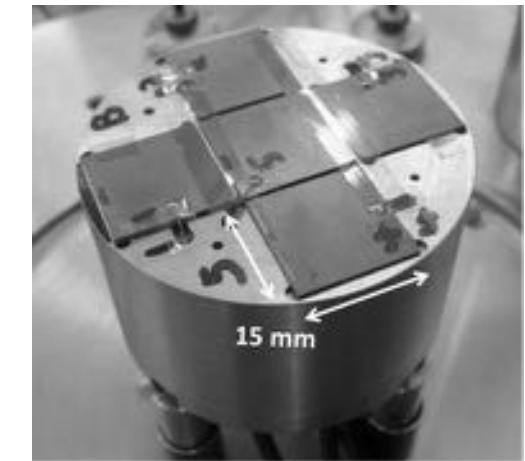
$$\Phi_{yz}=0.207$$
$$\Phi_{yx}=0.407$$
$$\Phi_{yy}=0.386$$


Transmitter grain

# Alternative approach to laboratory geophysics



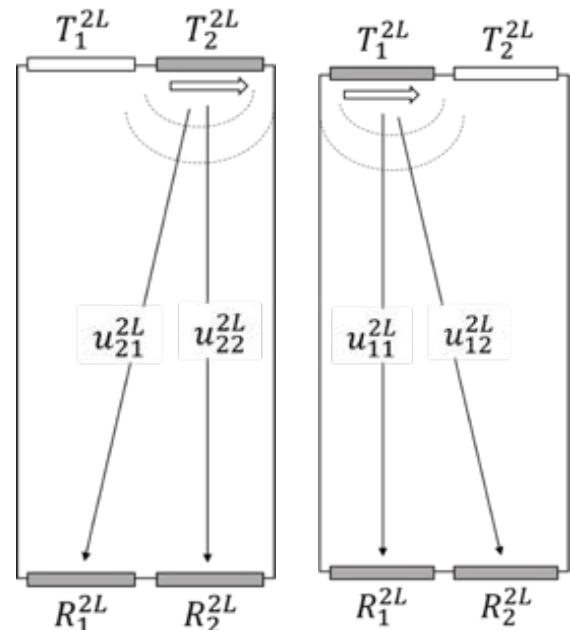
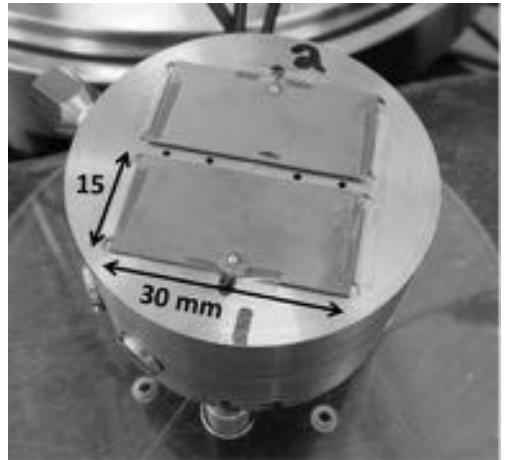
Bender elements



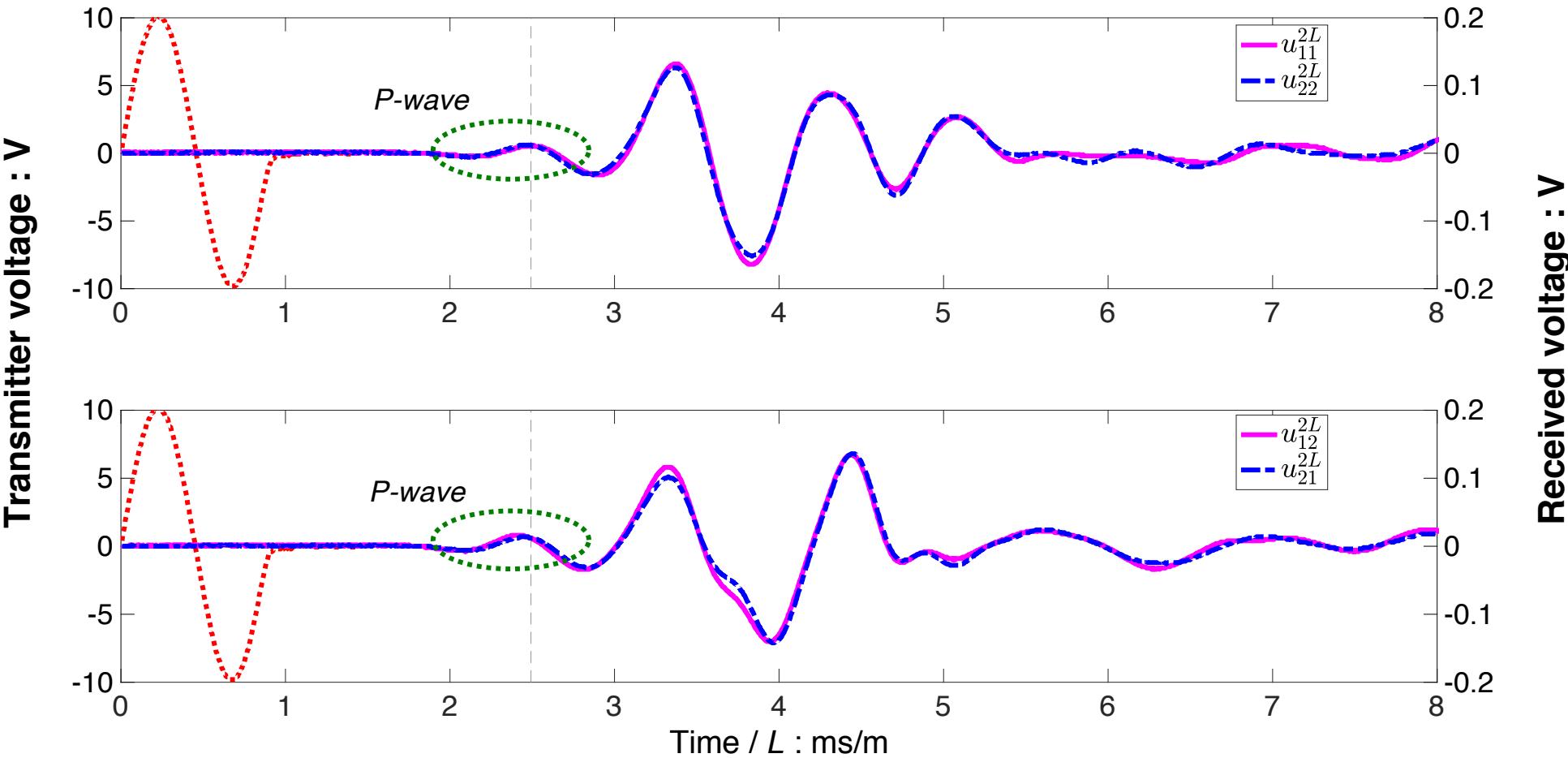
Shear plates

Otsubo (2016)

# Assessing shear plate technology

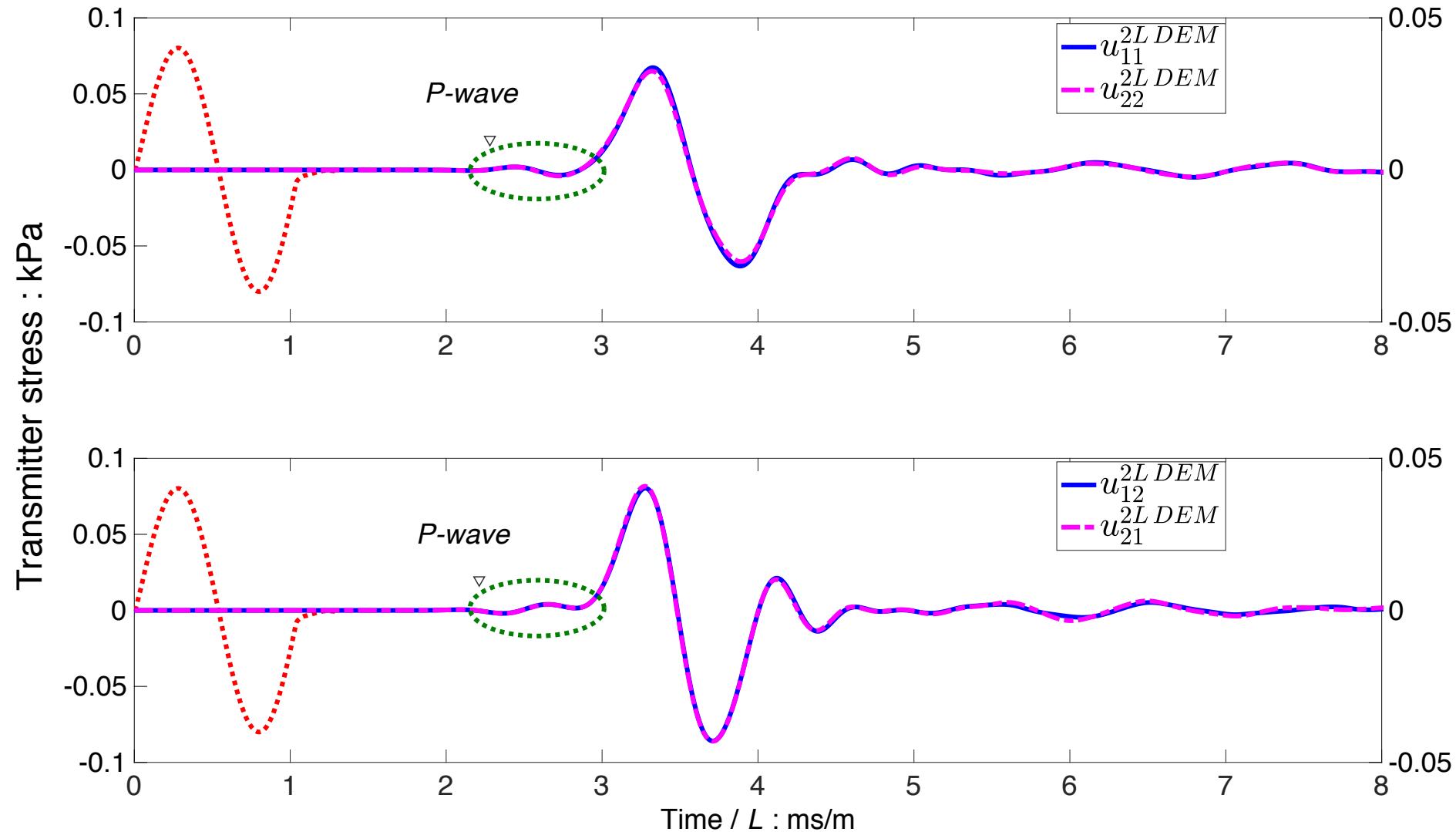
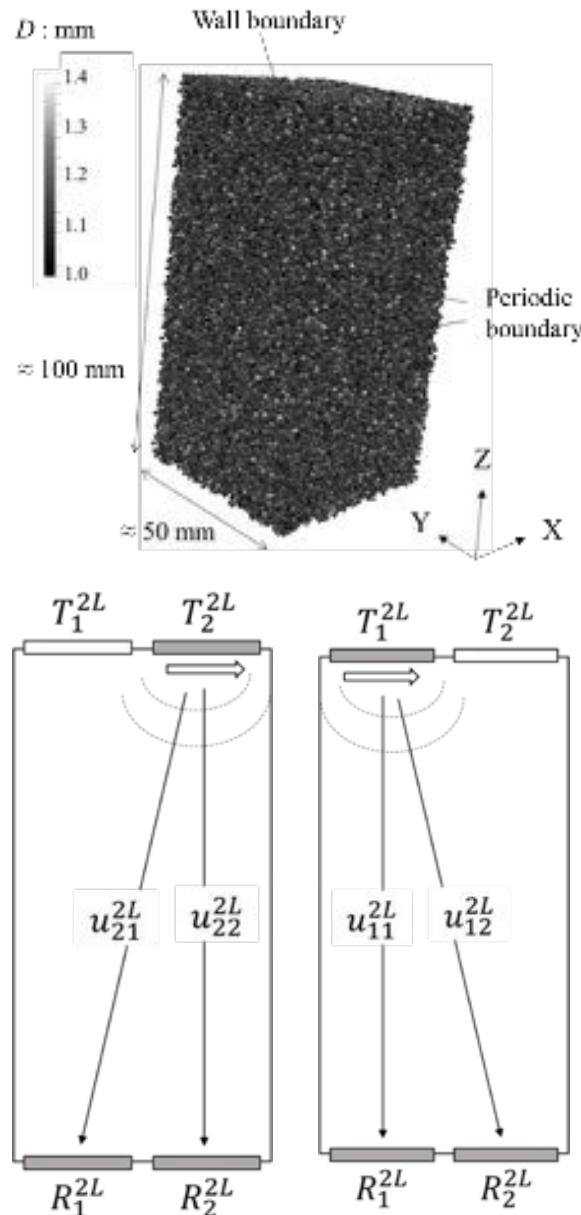


Experimental data

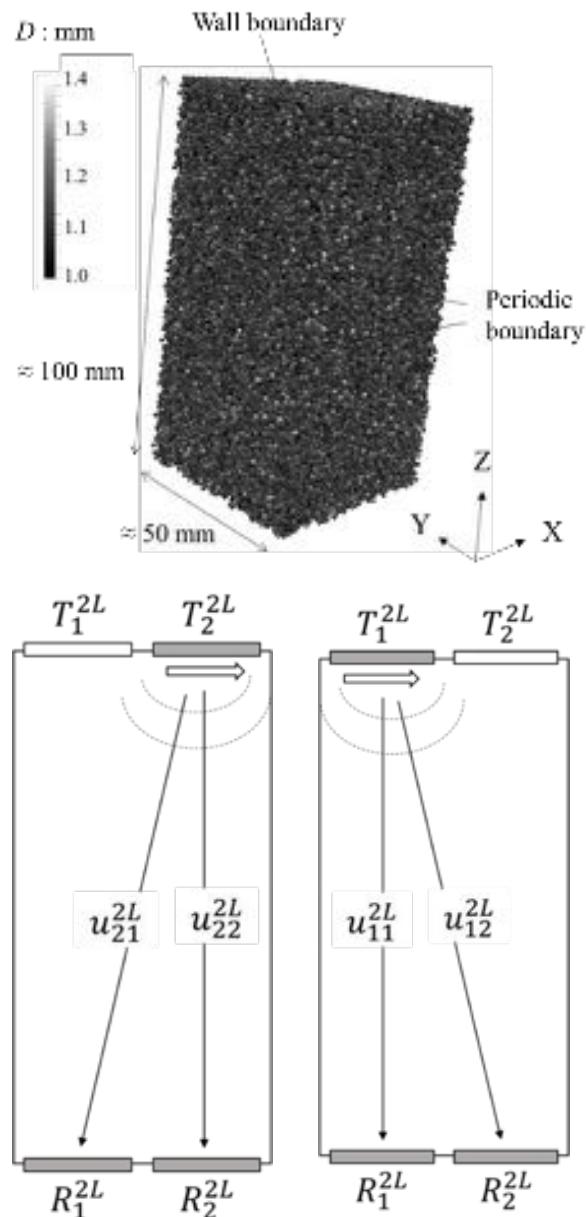


# Assessing shear plate technology

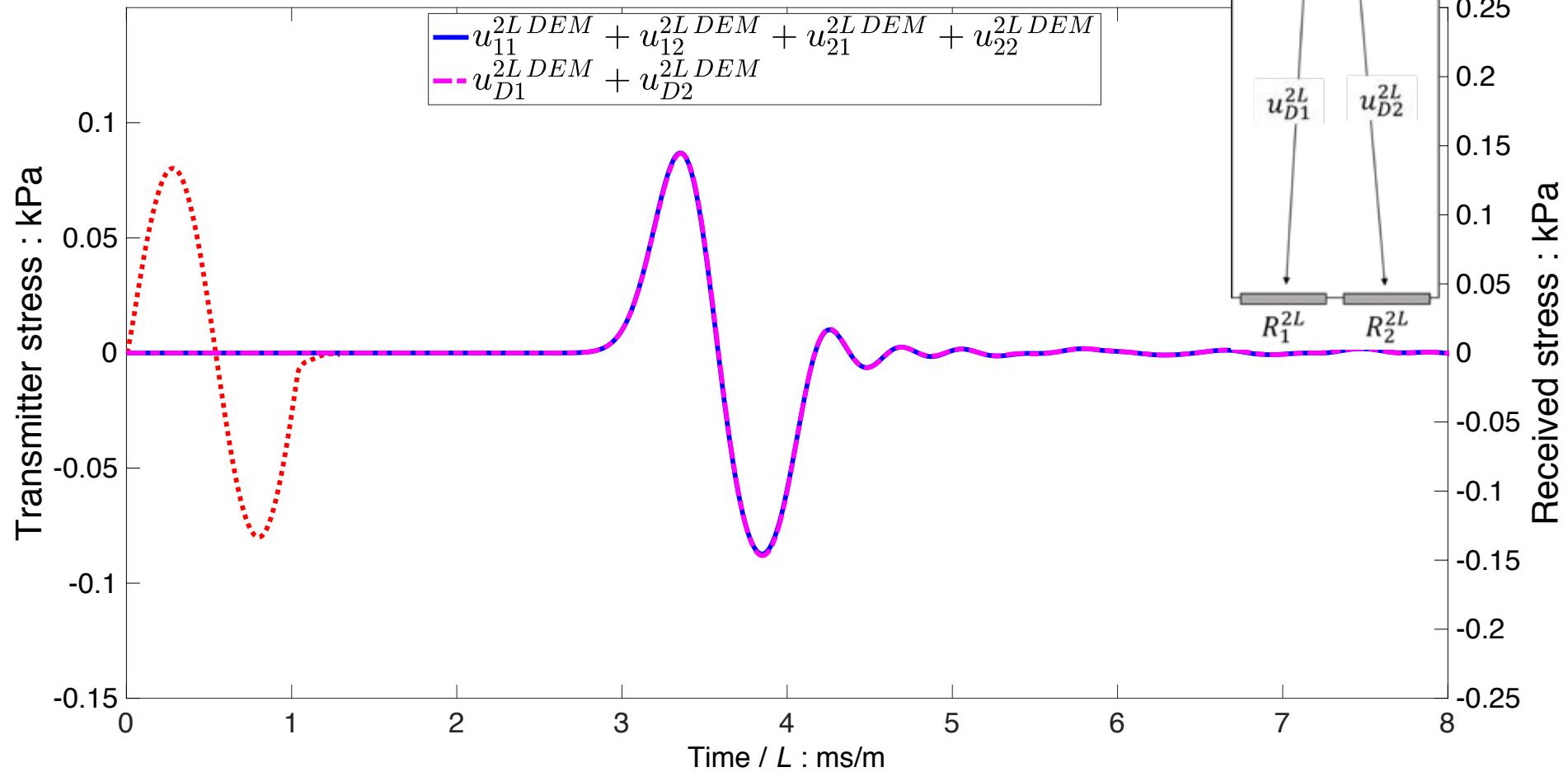
DEM data



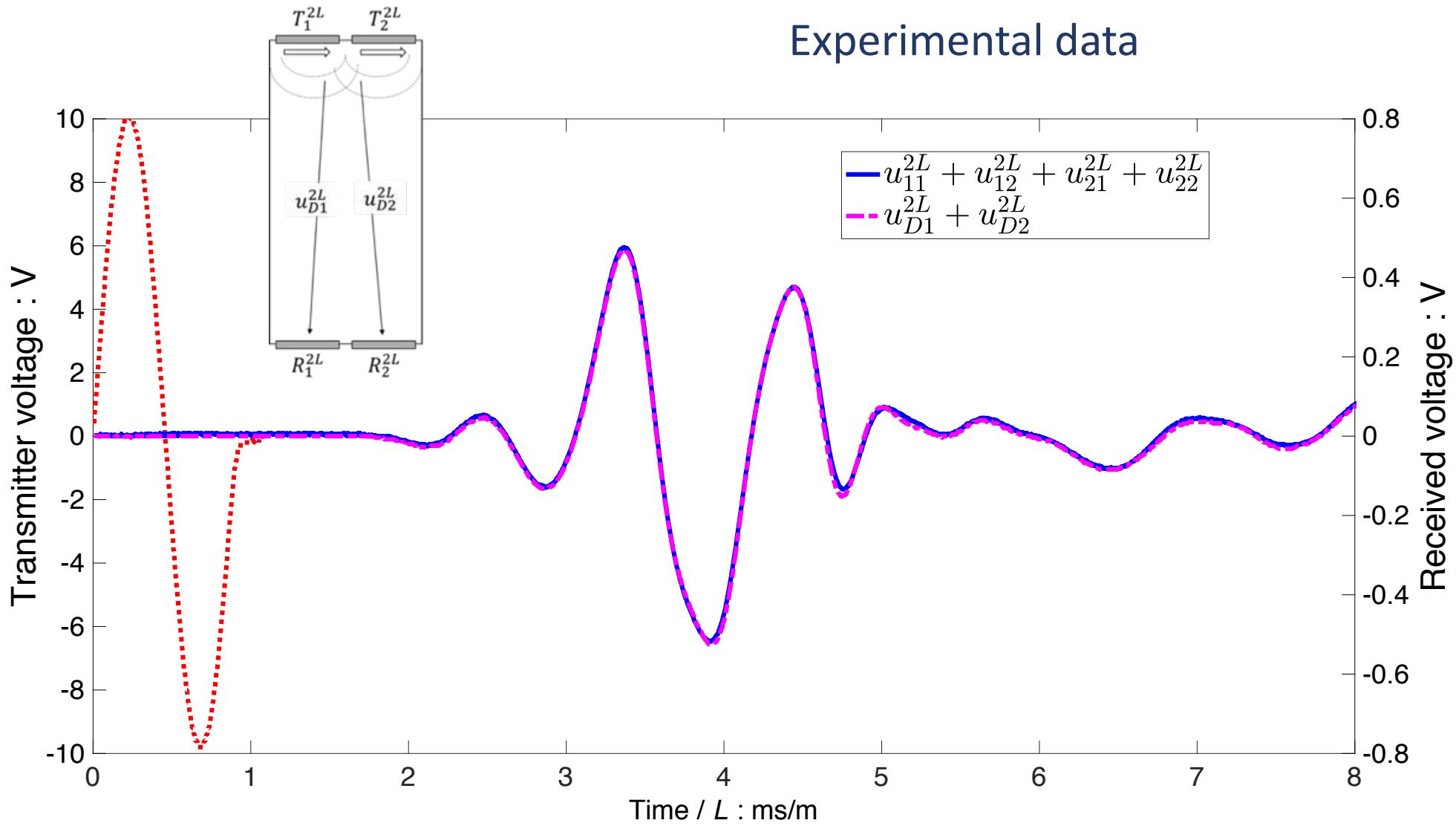
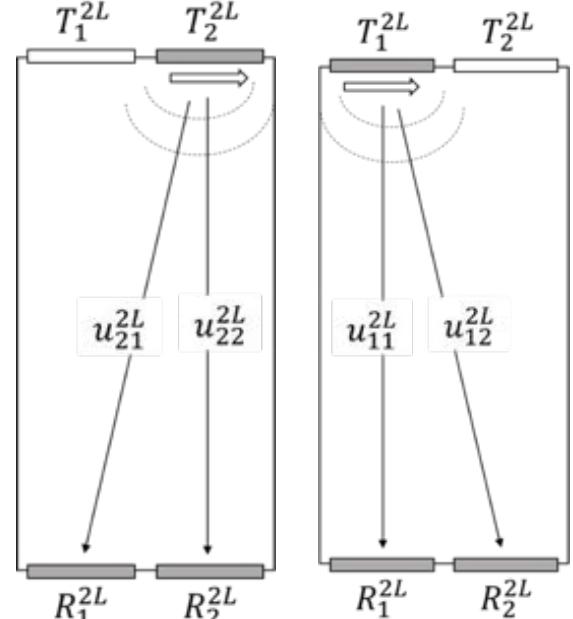
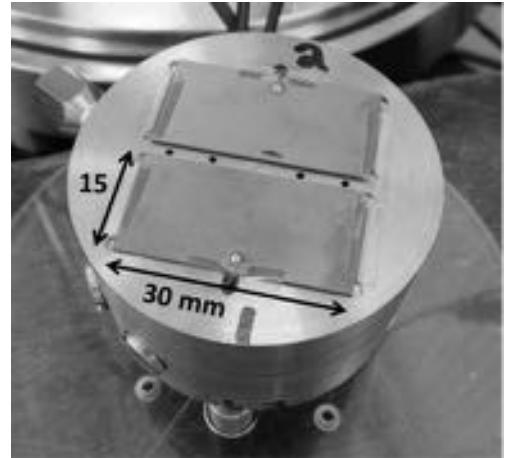
# Assessing shear plate technology



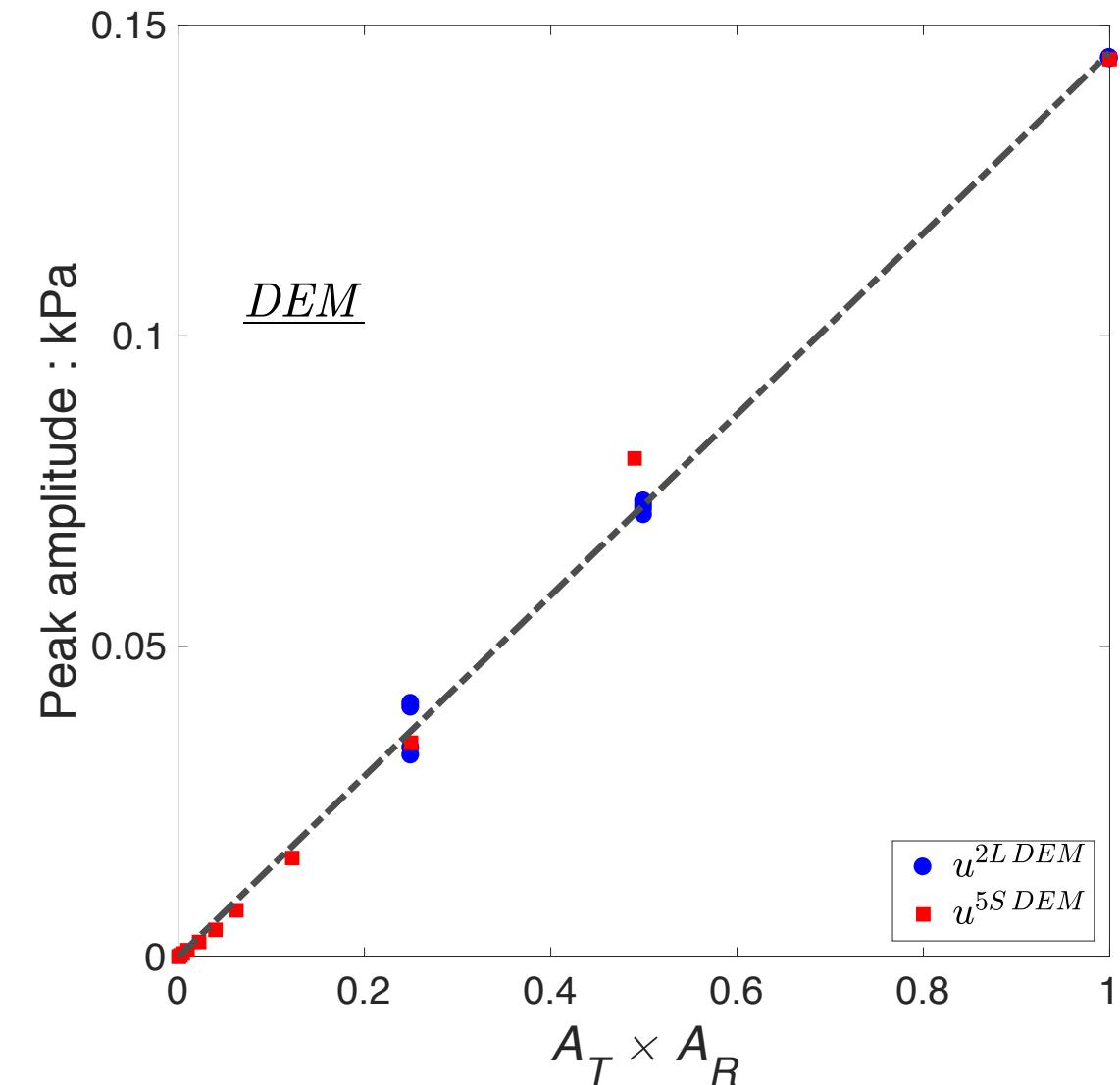
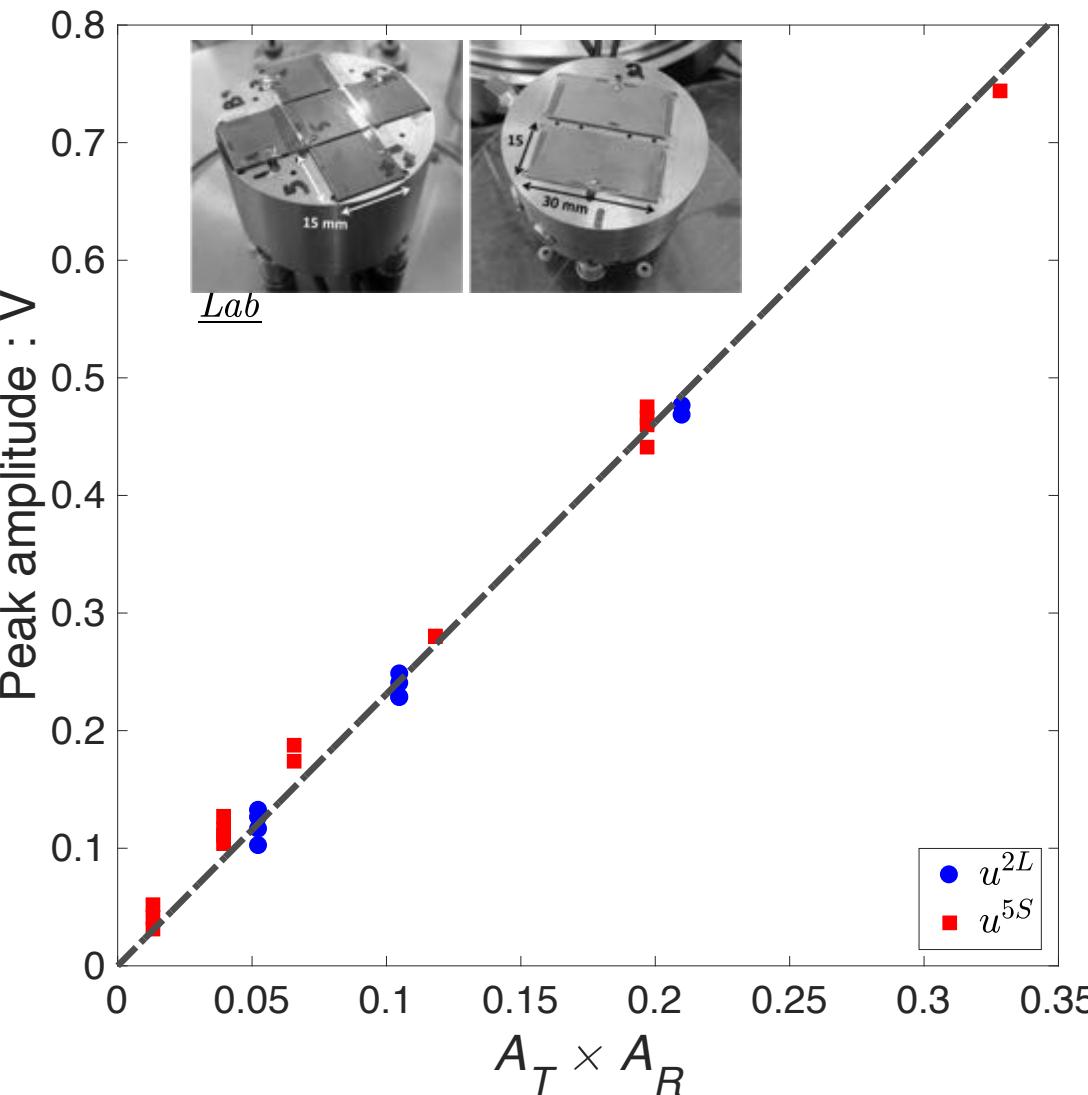
DEM data



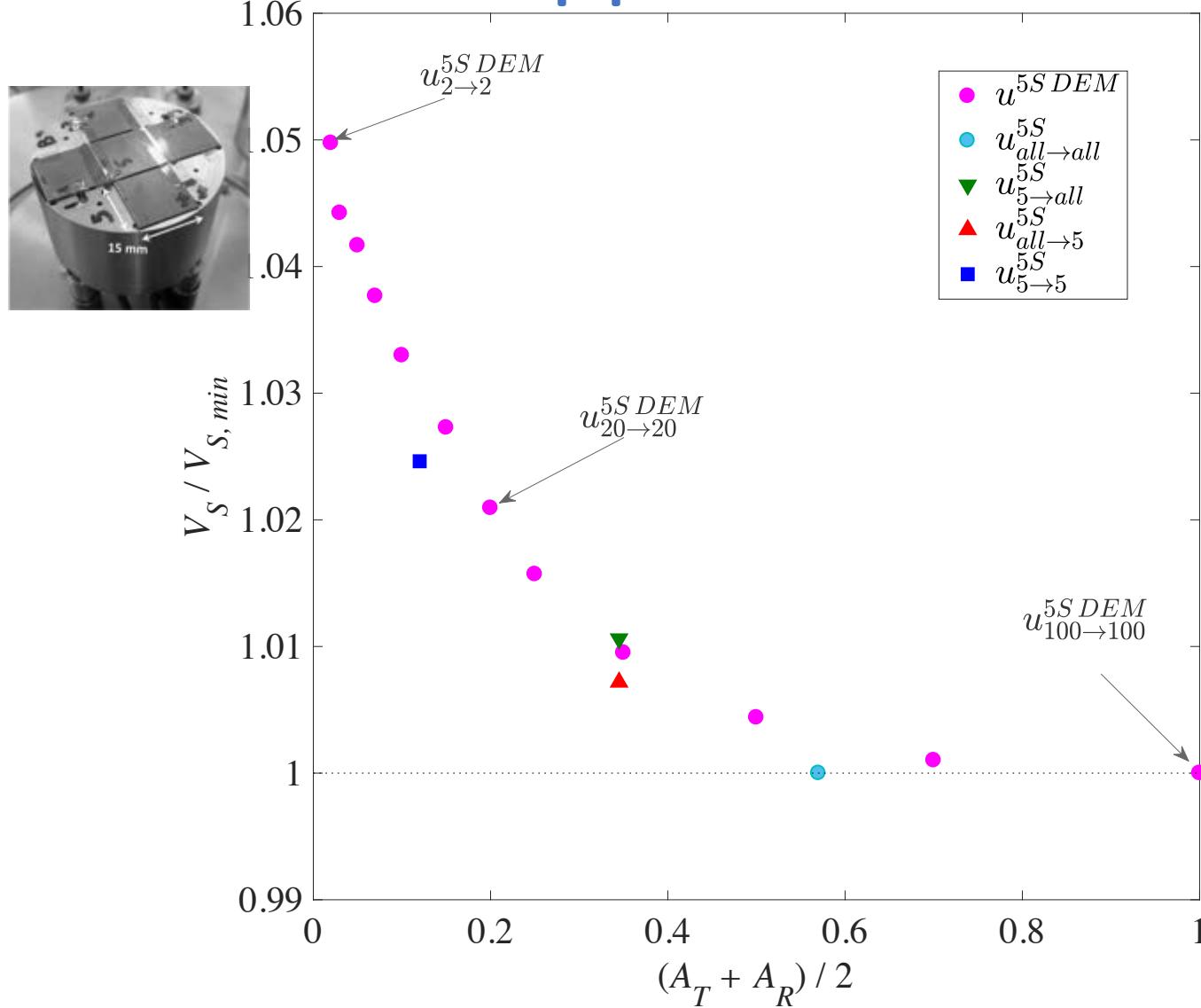
# Assessing shear plate technology



# Alternative approach to laboratory geophysics



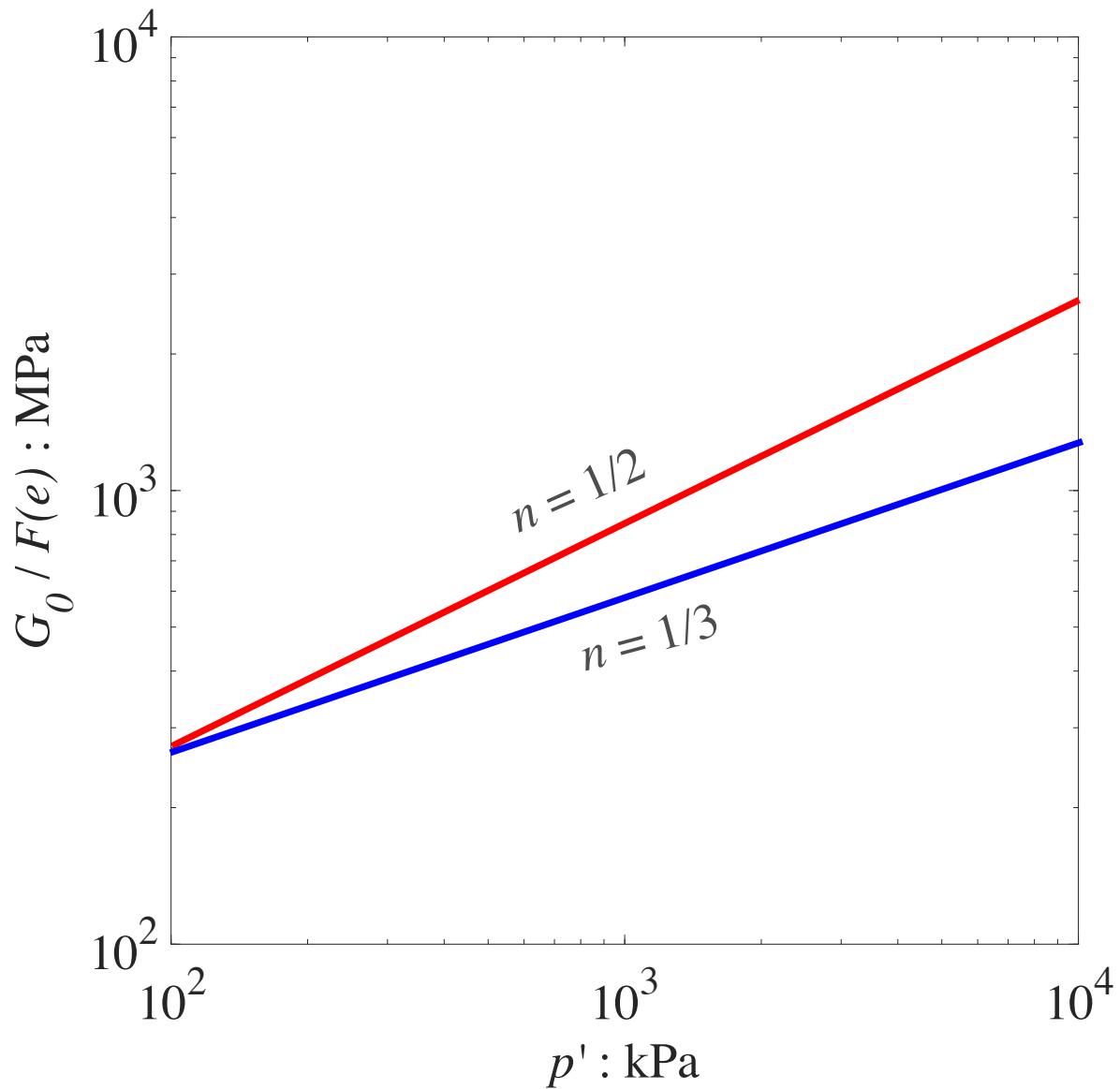
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5. What can we learn about the material fabric from the frequency domain response?
6. What opportunities does the BRISS project present?

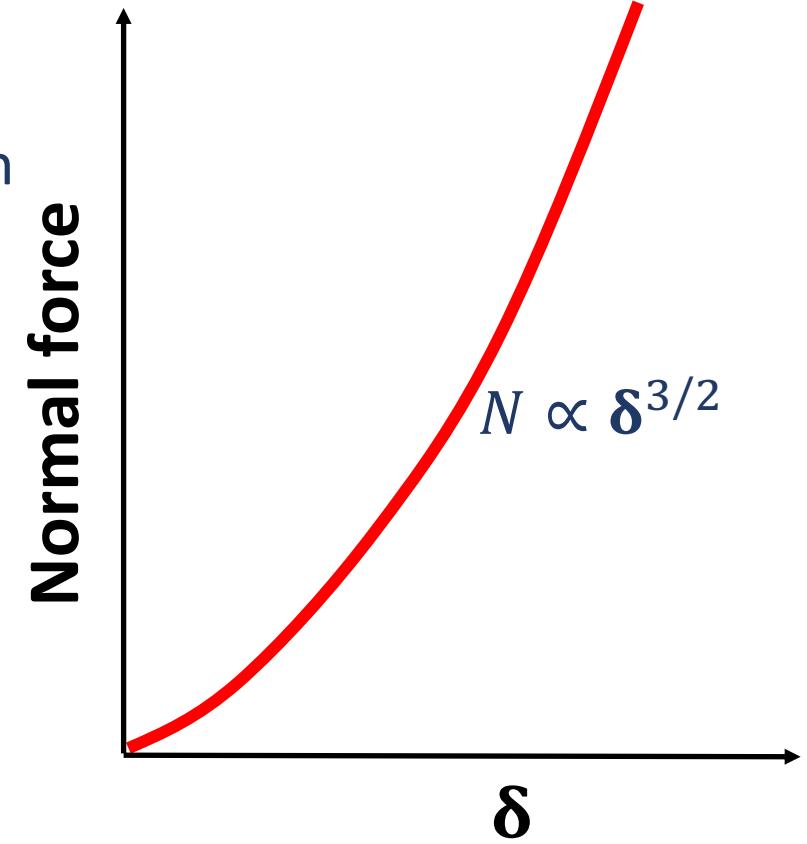
# Pressure-Stiffness Relationship



- Elastic theory explains pressure dependency: contact area increases as pressure increases – larger area gives a larger stiffness
- Elastic theory predicts that  $G_0 \propto (p')^{1/3}$
- Experimental data for sand gives  $G_0 \propto (p')^{1/2}$

# Hertz Mindlin contact model for normal spring

- Implemented in many (most?) DEM codes
- Originates from consideration of perfectly smooth spheres
- Normal force given by  $N = \frac{4}{3} E_p^{*1/2} a \delta$ 
  - $a = \sqrt{\delta R^*}$  = radius of contact area
  - $\delta$  = particle overlap
  - $R^*$  = equivalent particle radius
  - $E_p^*$  = equivalent particle Young's modulus



# Simplified Hertz Mindlin contact model for tangent spring

- Implemented in many (most?) DEM codes
- Does not consider partial slip or loading history
- Originates from consideration of perfectly smooth spheres
- Tangential stiffness given by  $k_T^{HM} = 8aG_p^*$ 
  - $a = \sqrt{\delta R^*}$  = radius of contact area
  - $R^*$  = equivalent particle radius
  - $\delta$  = particle overlap
  - $G_p^*$  = equivalent particle shear modulus

# Real contact interaction

## Surface roughness

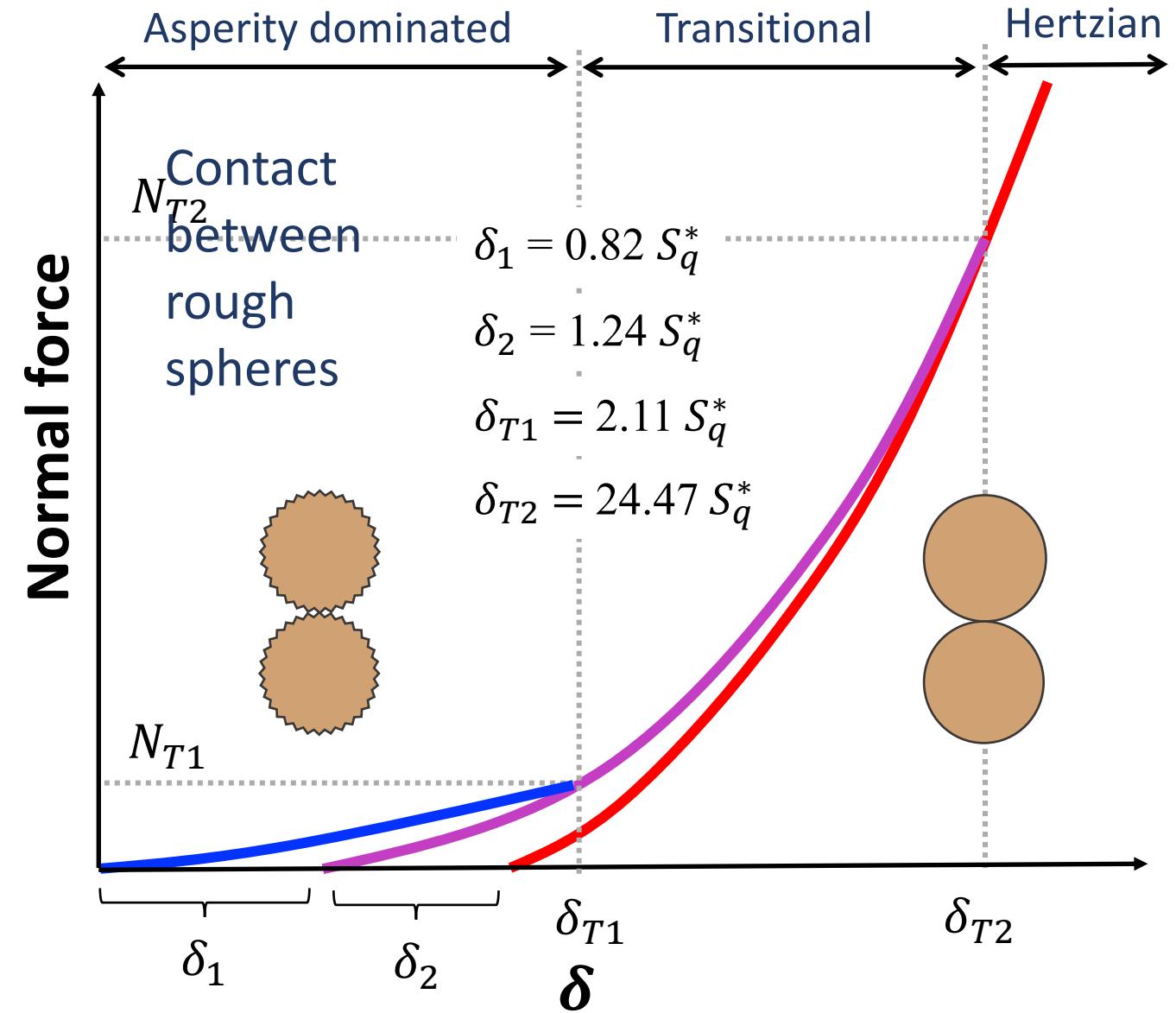
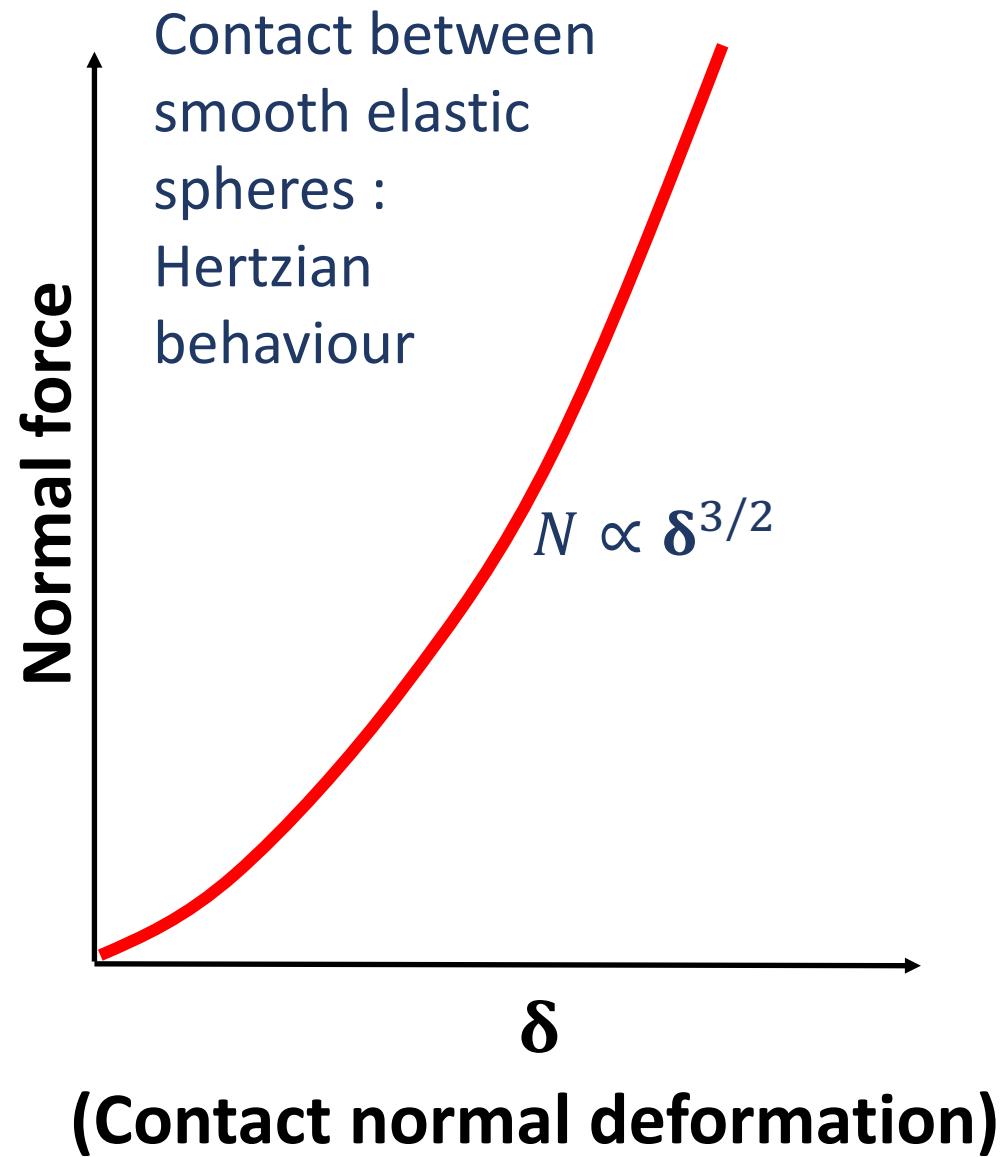
- Reduces normal stiffness
- Introduces hysteresis

## Partial slip

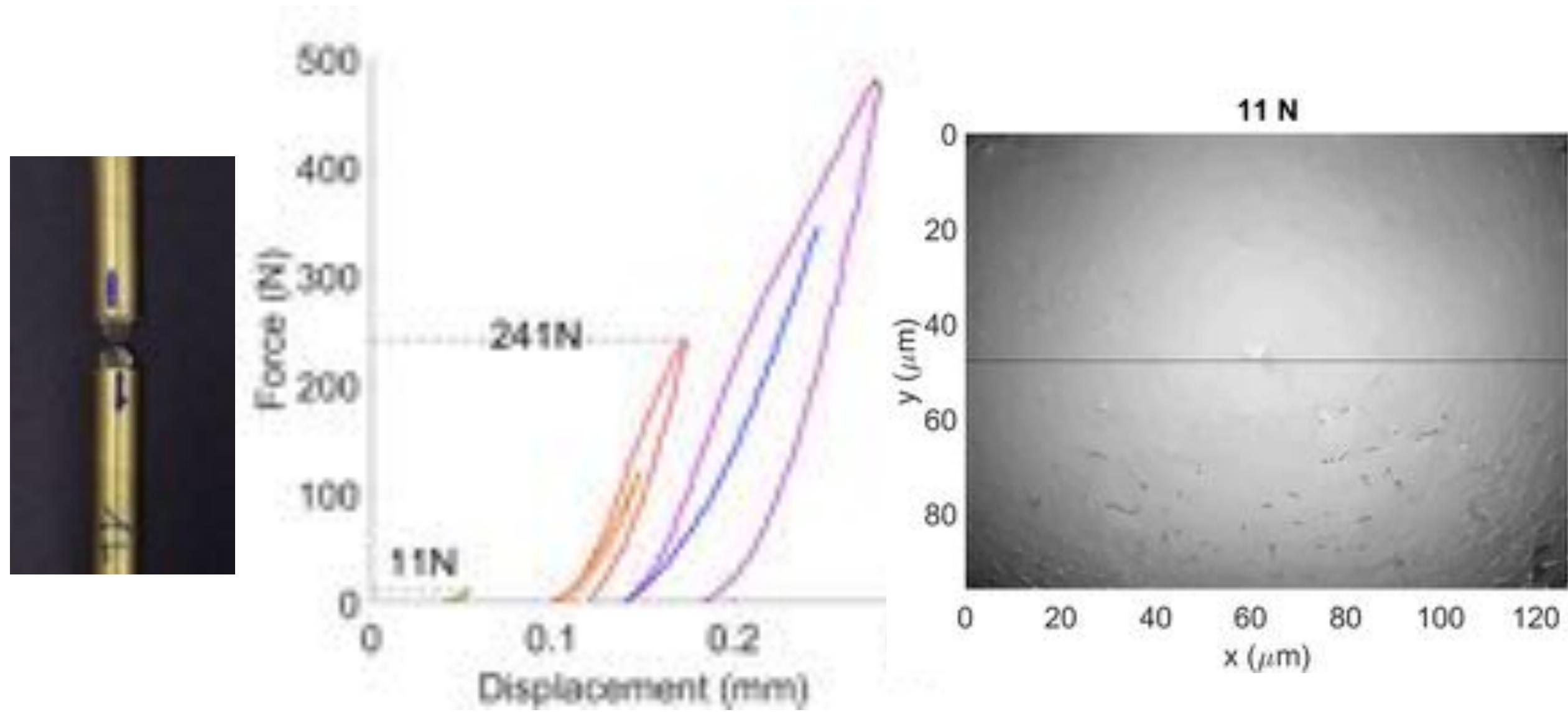
- Influences tangential stiffness
- Introduces hysteresis
- Introduces load-history effect



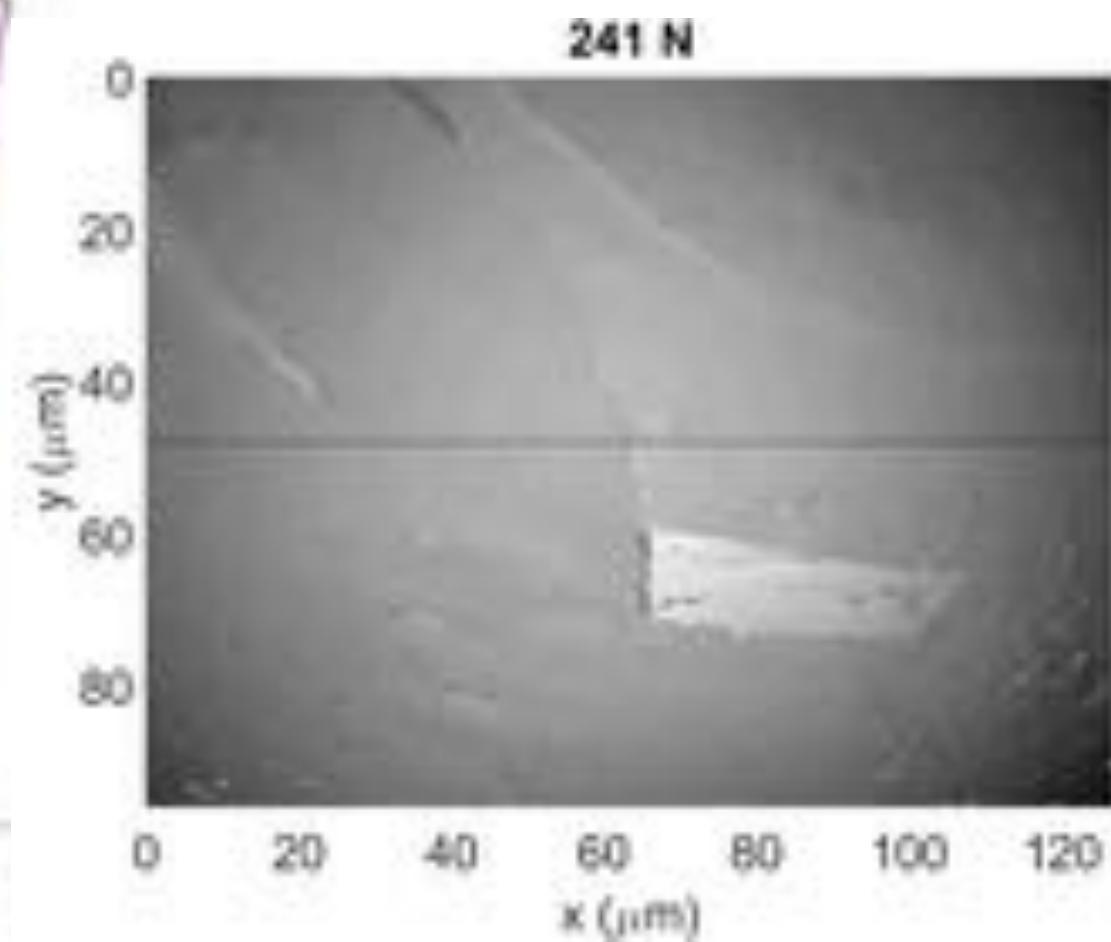
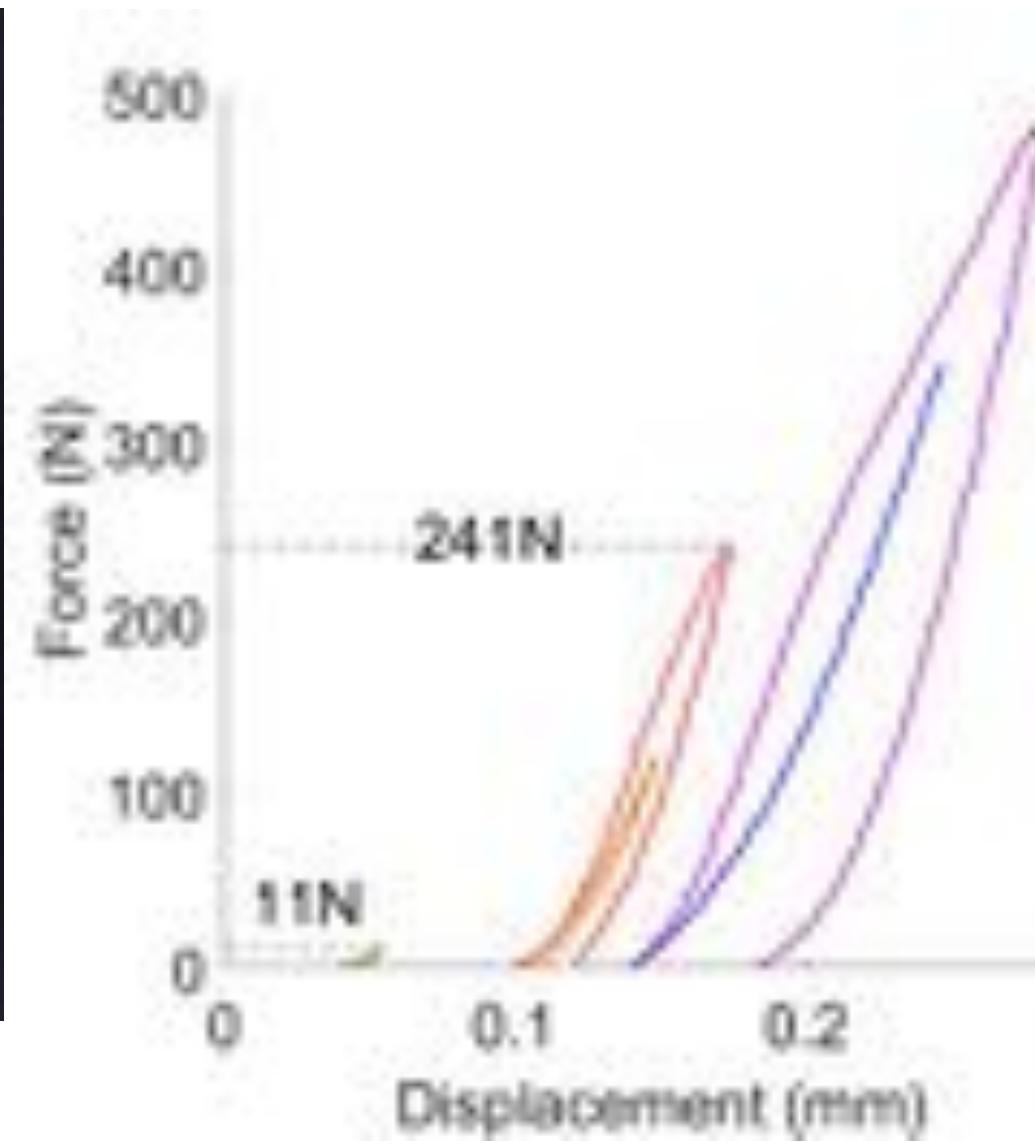
# Roughness-dependant contact behaviour



# Rough surface effects



# Rough surface effects



# Quantifying surface roughness

Optical interferometry



Surface roughness measure

RMS  
roughness

$$S_q = \sqrt{\frac{1}{n} \sum_{i=1}^n Z_i^2}$$

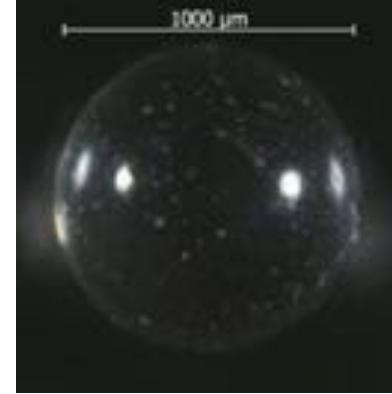


# Controlling and Surface Roughness

Ballotini + Toyoura Sand



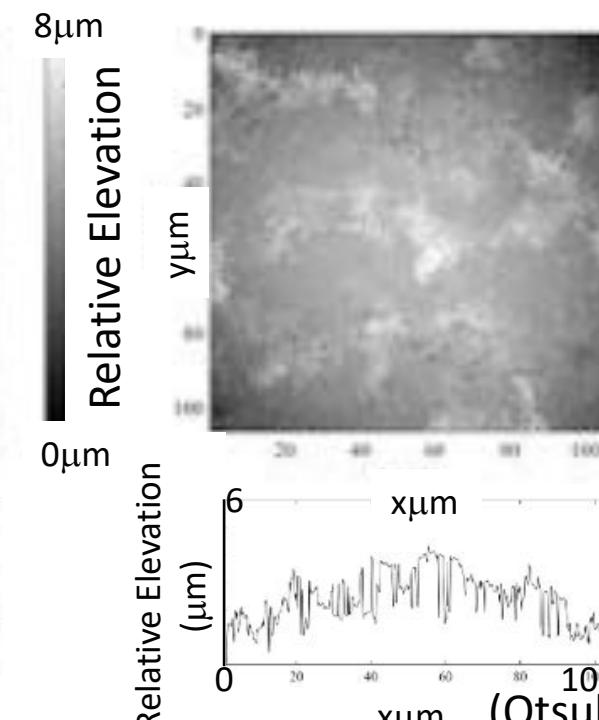
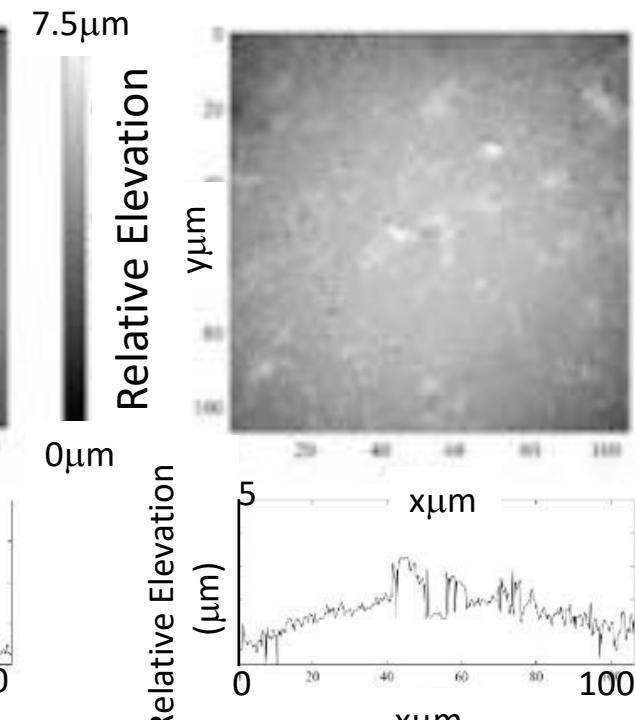
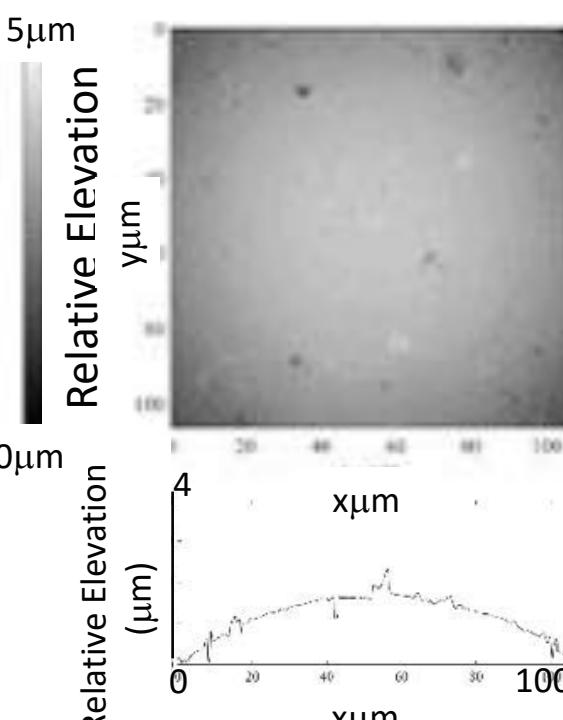
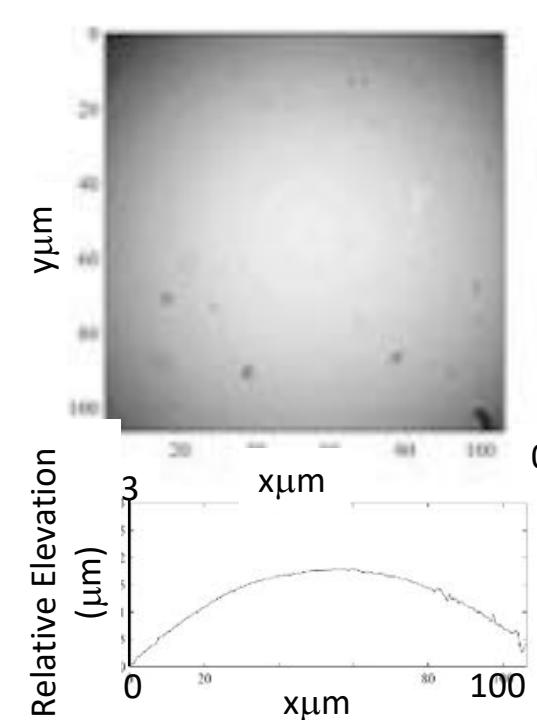
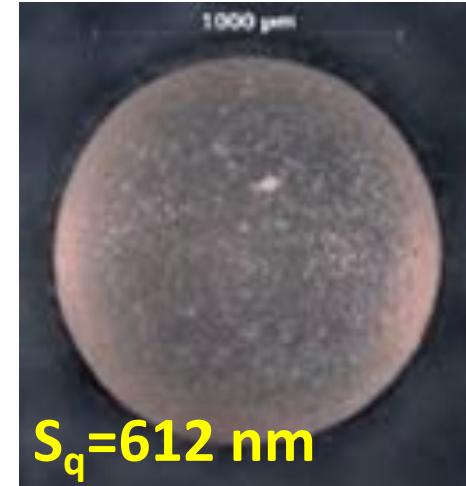
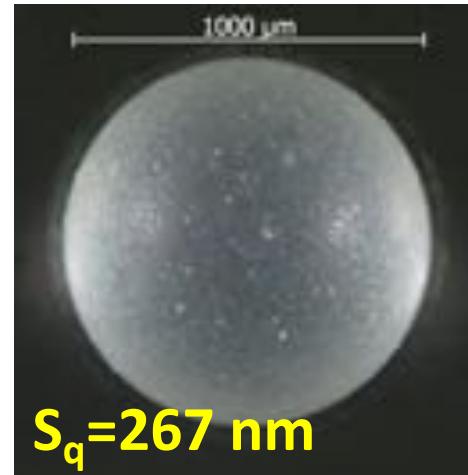
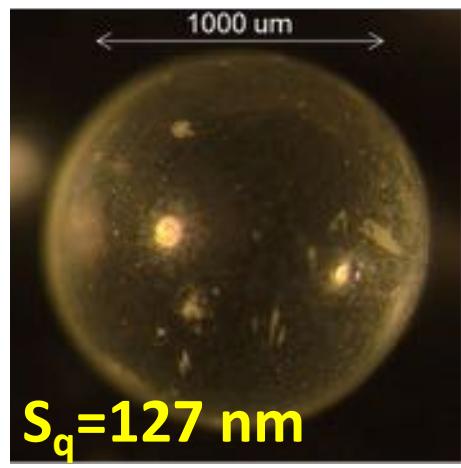
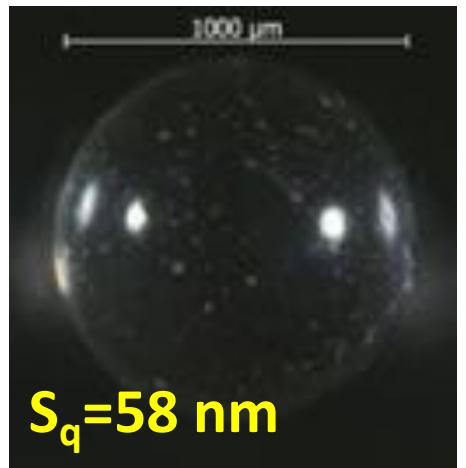
Used spherical glass beads to isolate roughness from shape (form) effects



Measured roughness using interferometry



# Roughened Ballotini



9μm  
Relative Elevation

0μm

Relative Elevation

0μm

Relative Elevation

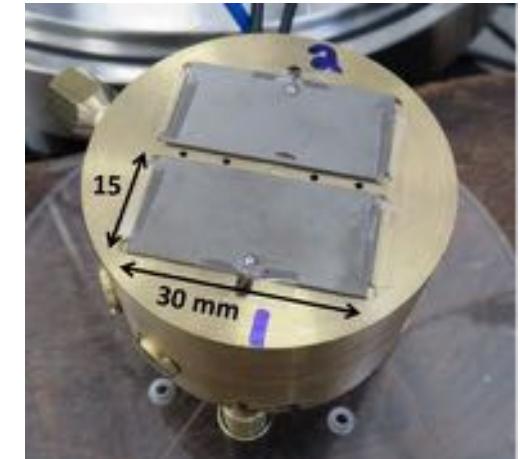
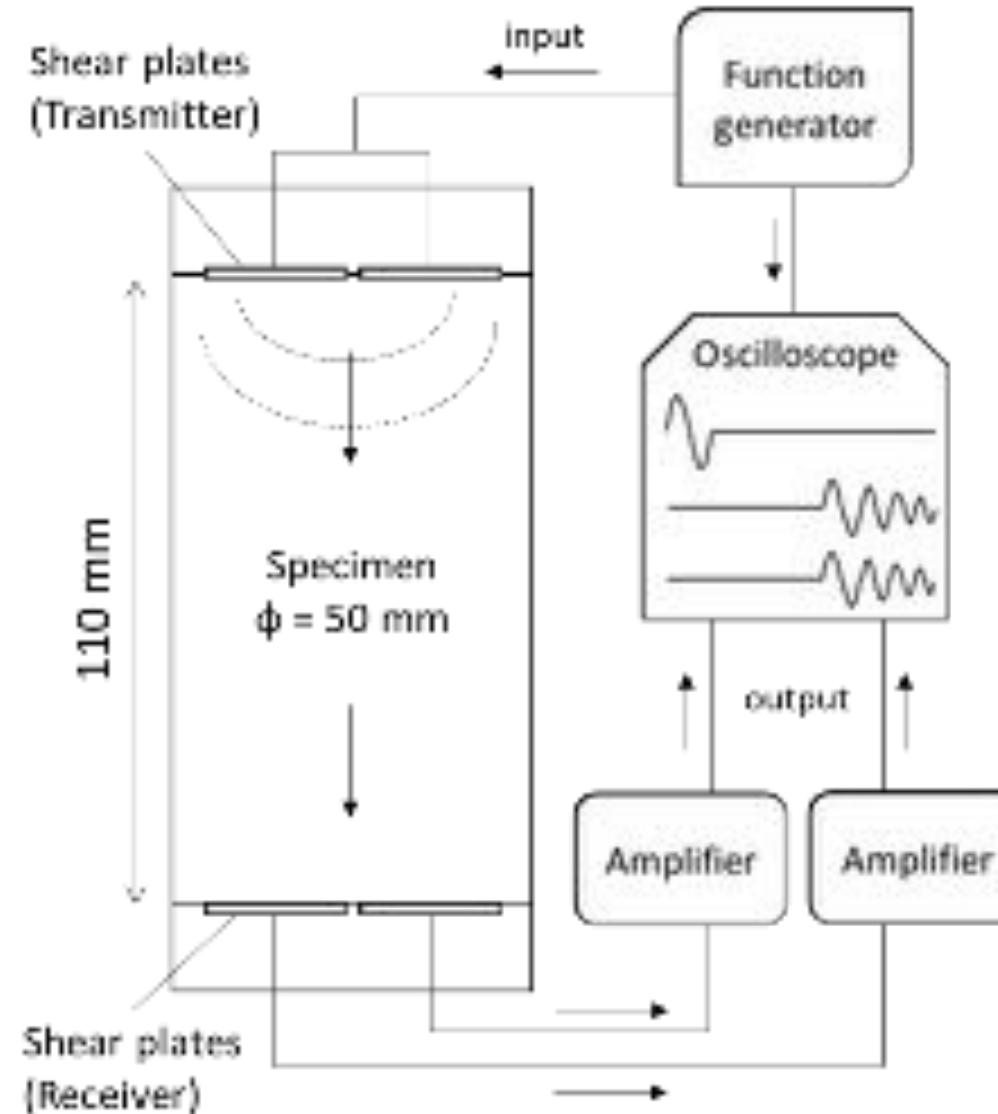
0μm

Relative Elevation

0μm

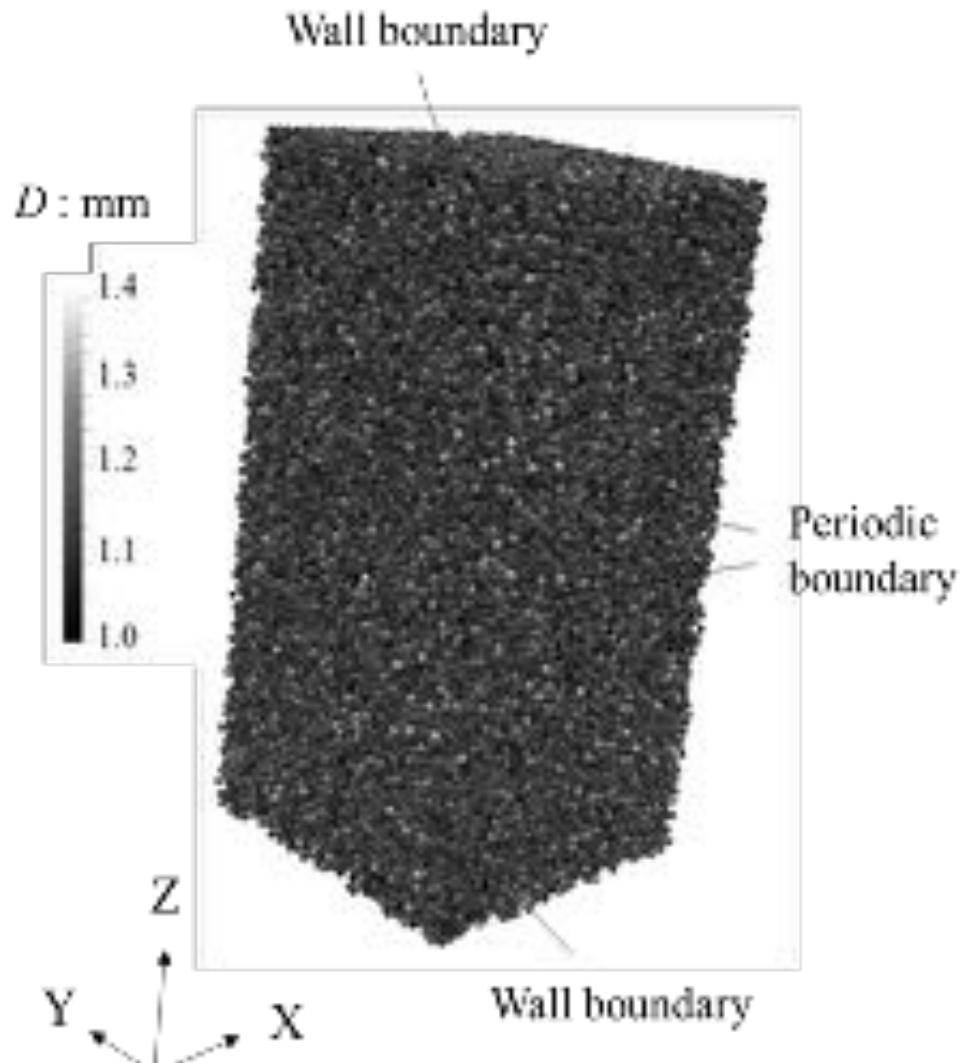
(Otsubo, 2017)

# Quantifying Stiffness in Laboratory



(Otsubo, 2017)

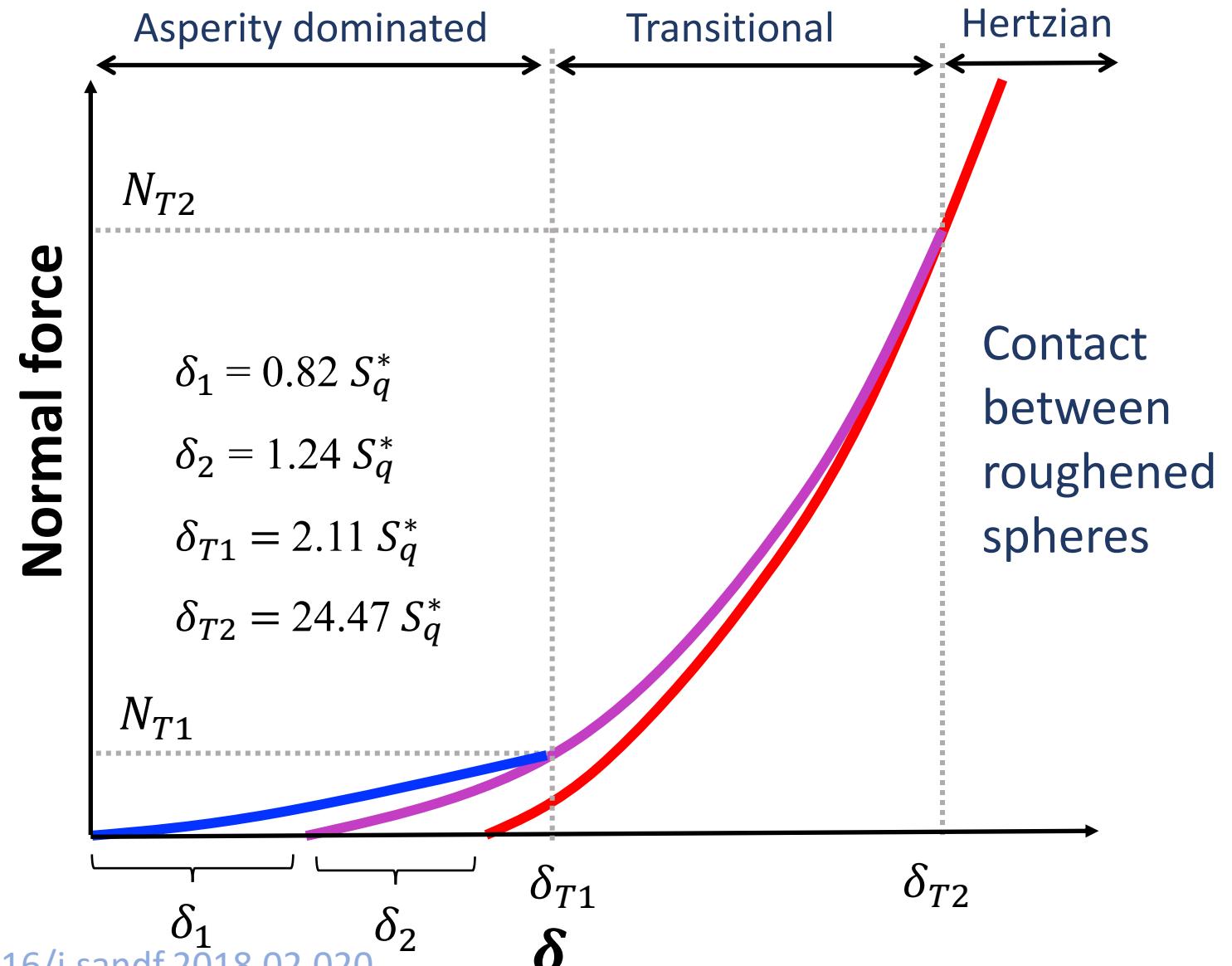
# Simulating Tests using DEM



- DEM code: LAMMPS
- 155,165 particles
- Diameter :  $D = 2.54$  mm (mono-size)
- Particle shear modulus :  $G_p = 25$  GPa
- Particle Poisson's ratio :  $\nu_p = 0.2$
- Inter-particle friction :
- $\mu = 0.0$  (dense) 0.15 (loose)
- $p' = 100$ kPa to 10MPa
- Planar shear waves

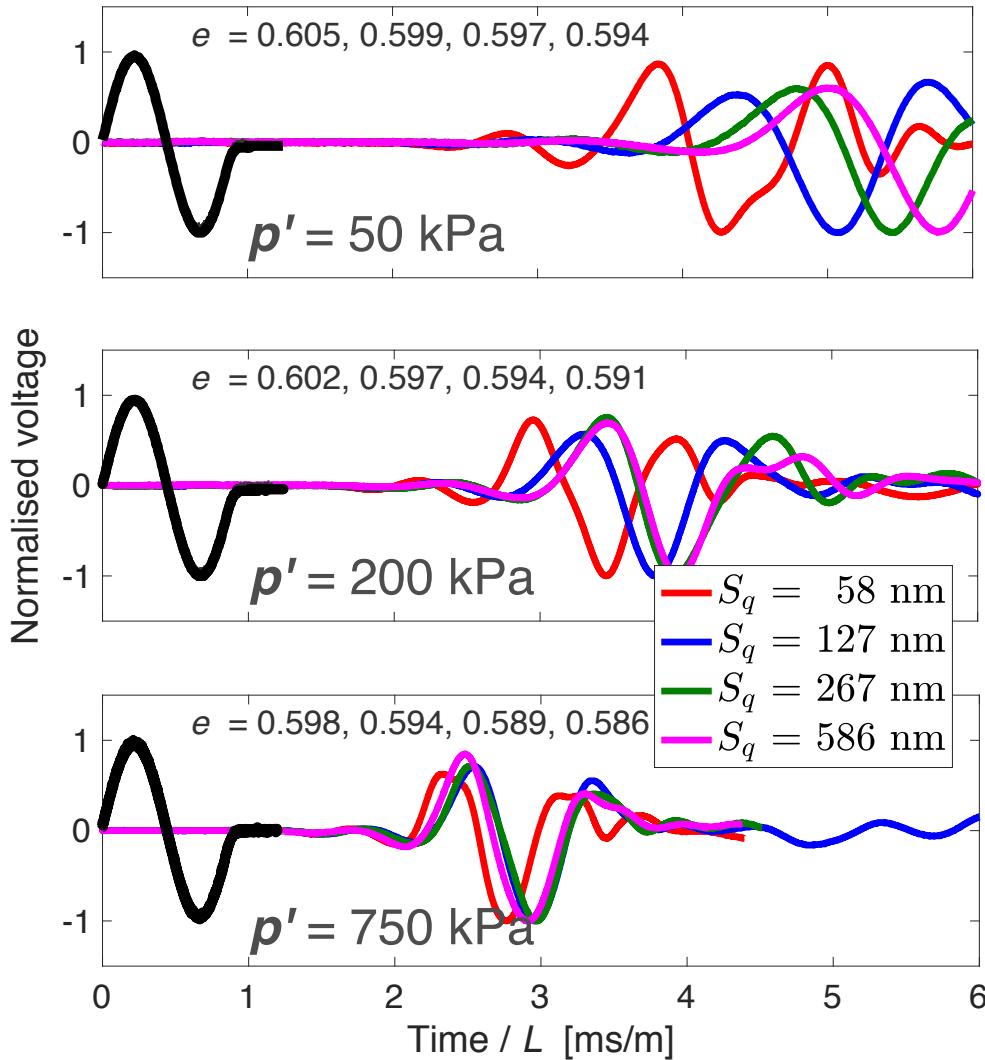
# Roughness-dependant contact behaviour

DEM Contact Model

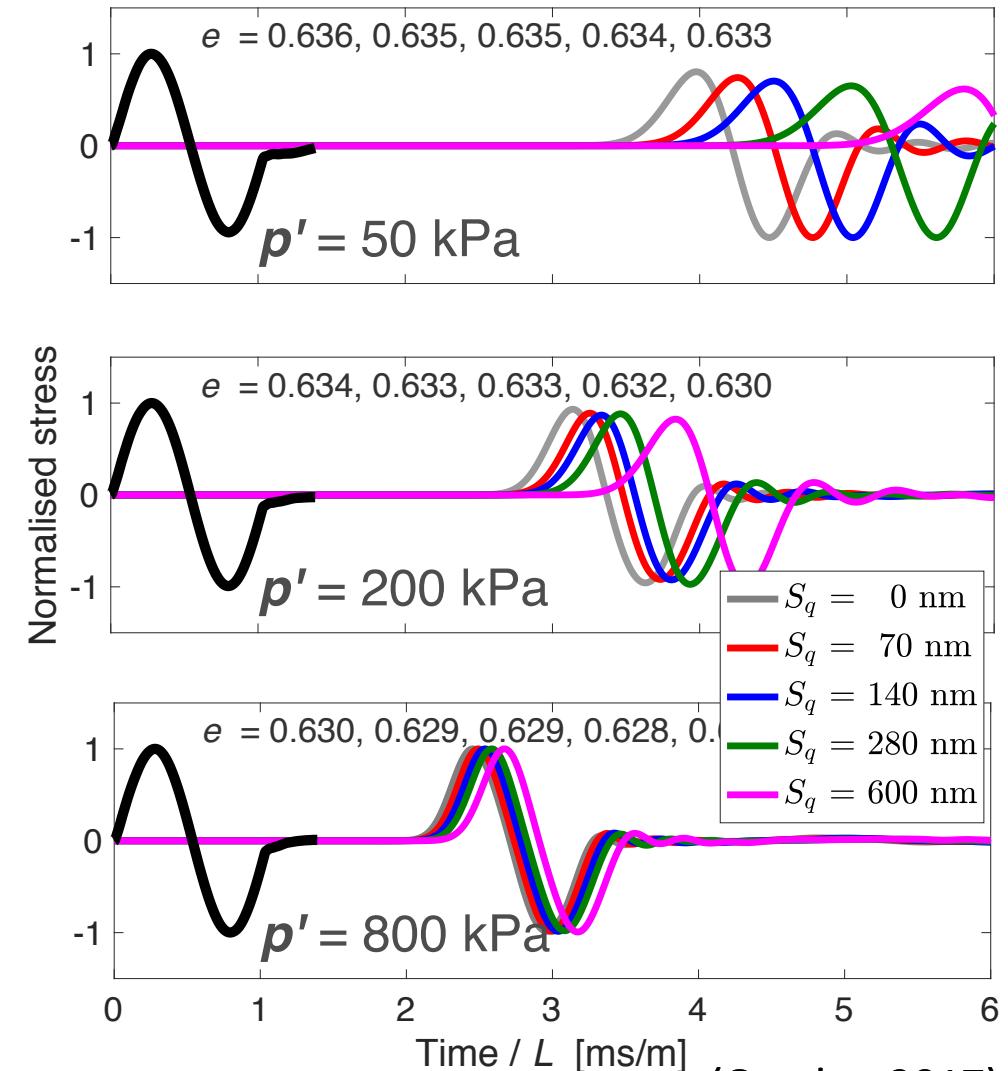


# Surface Roughness and Stiffness

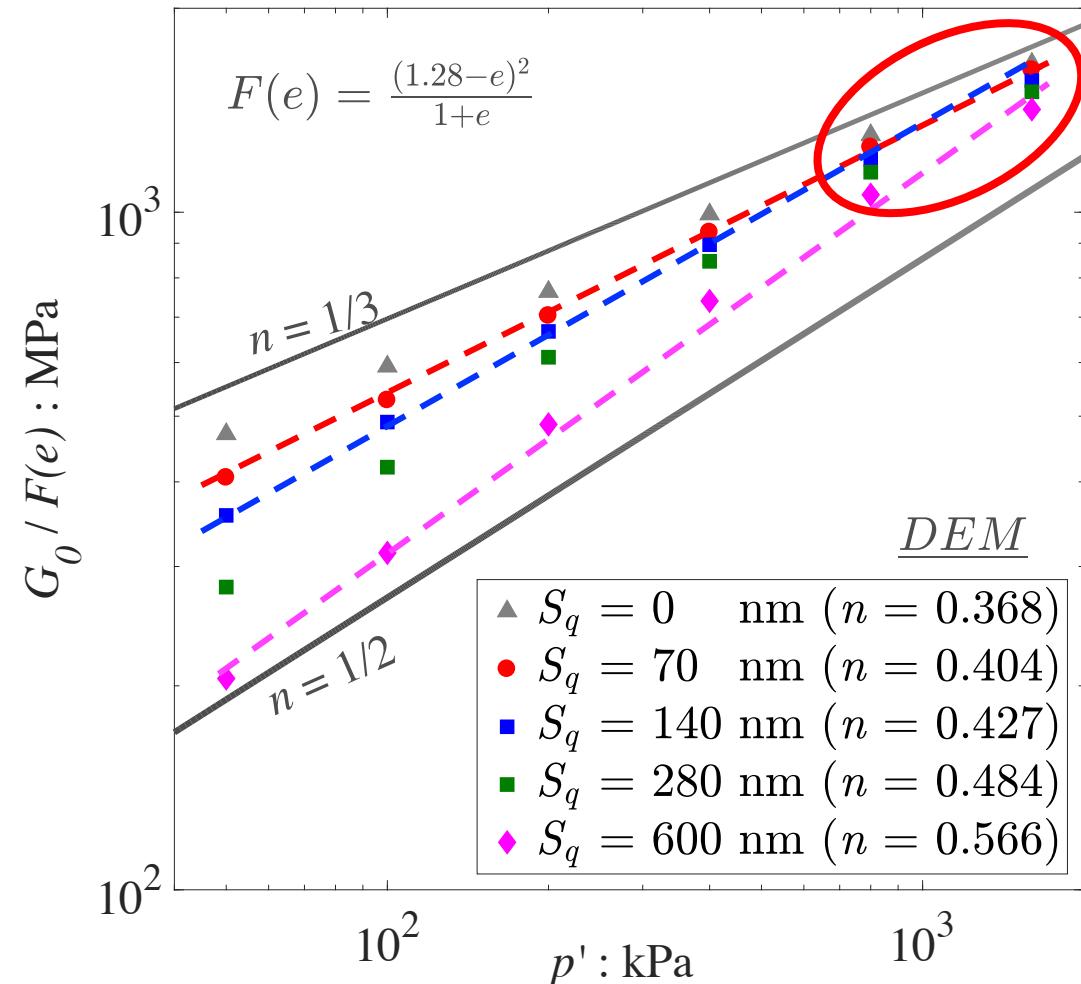
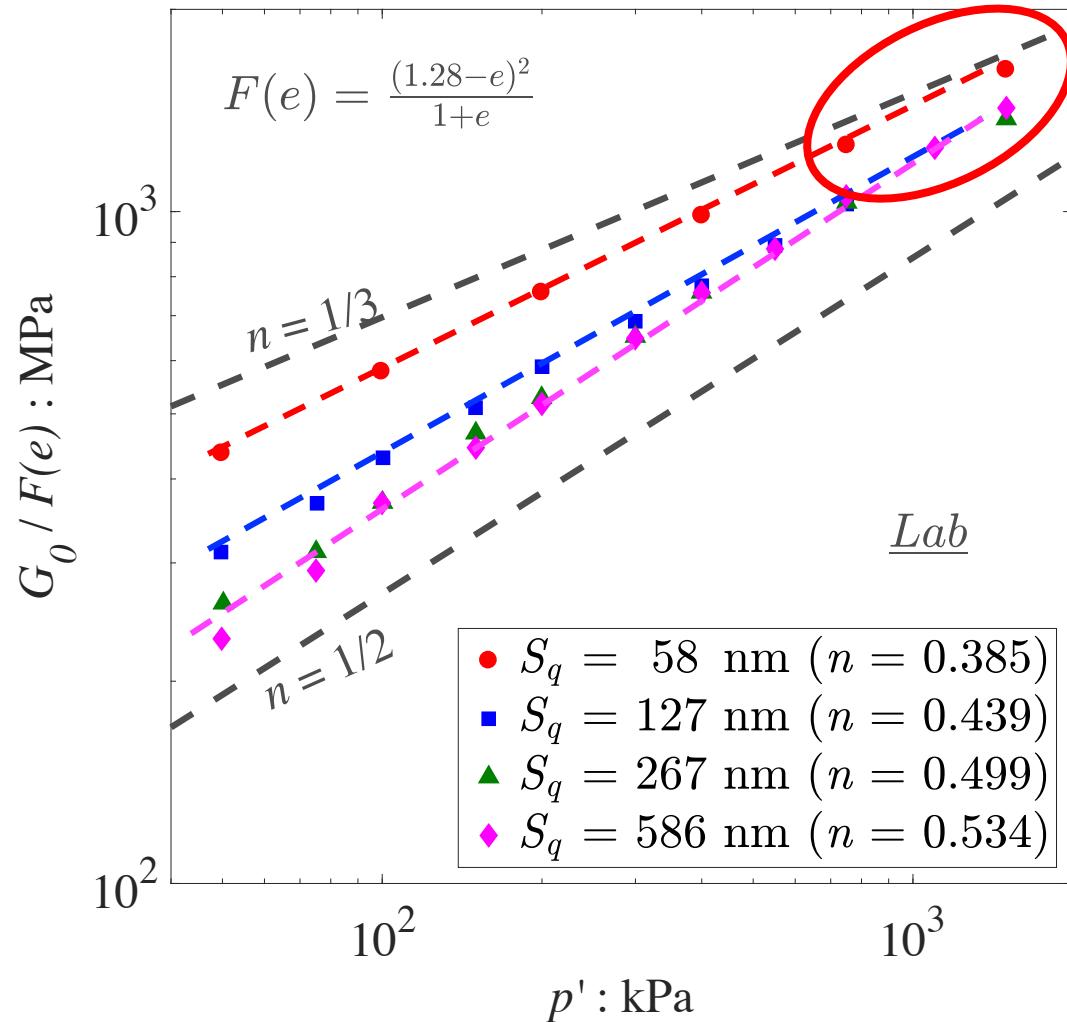
## Experiments



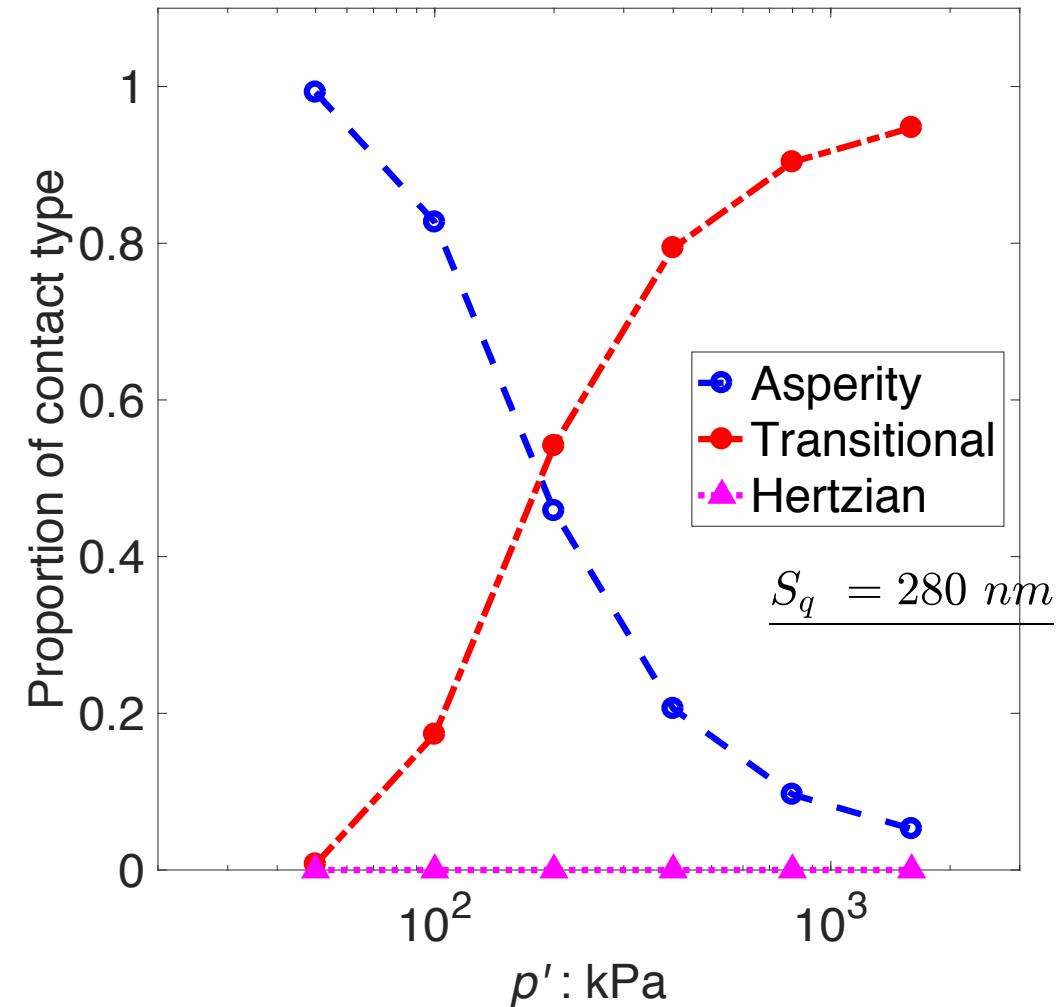
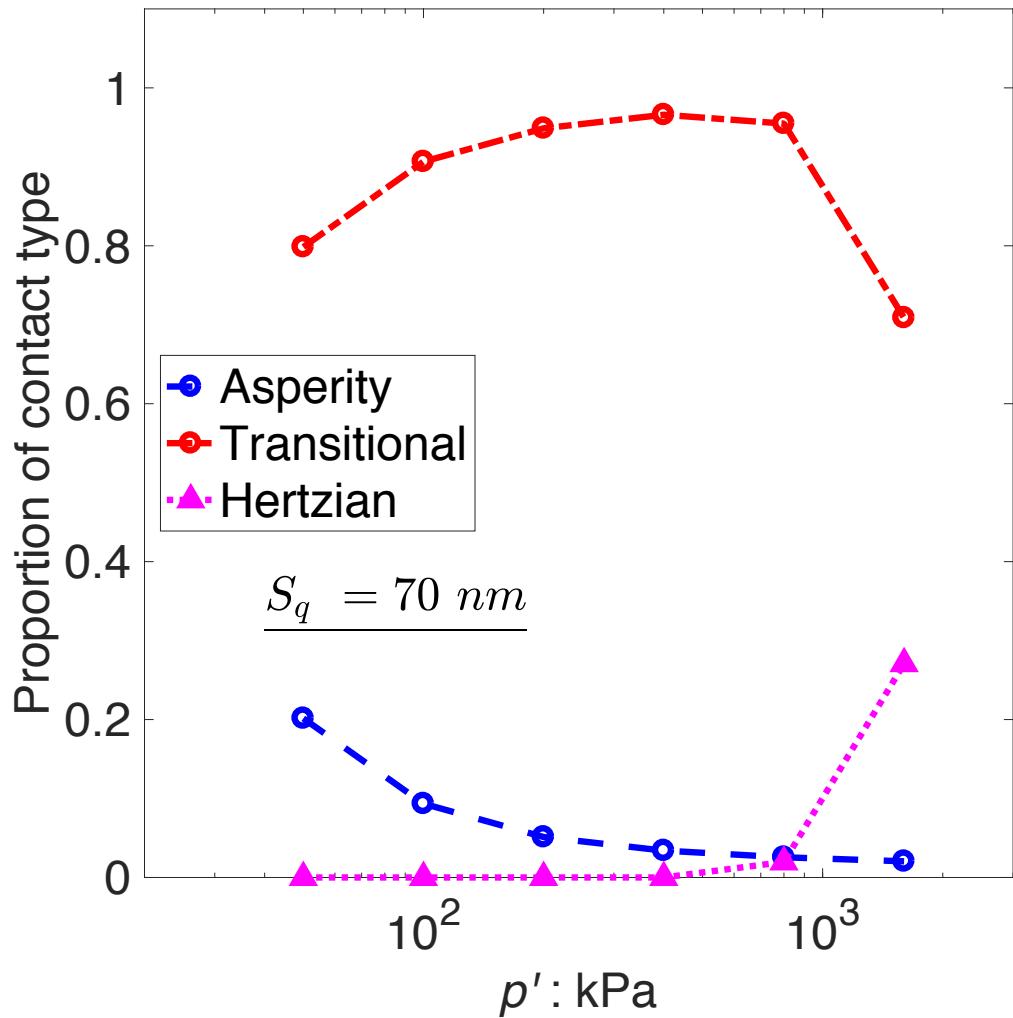
## DEM simulations



# Surface Roughness and Stiffness



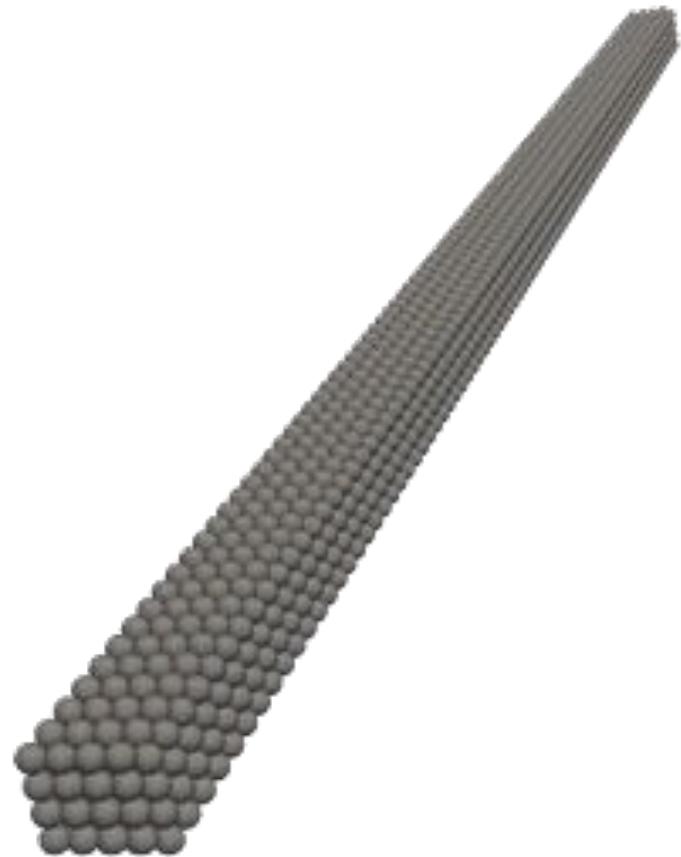
# DEM data on Contact Evolution



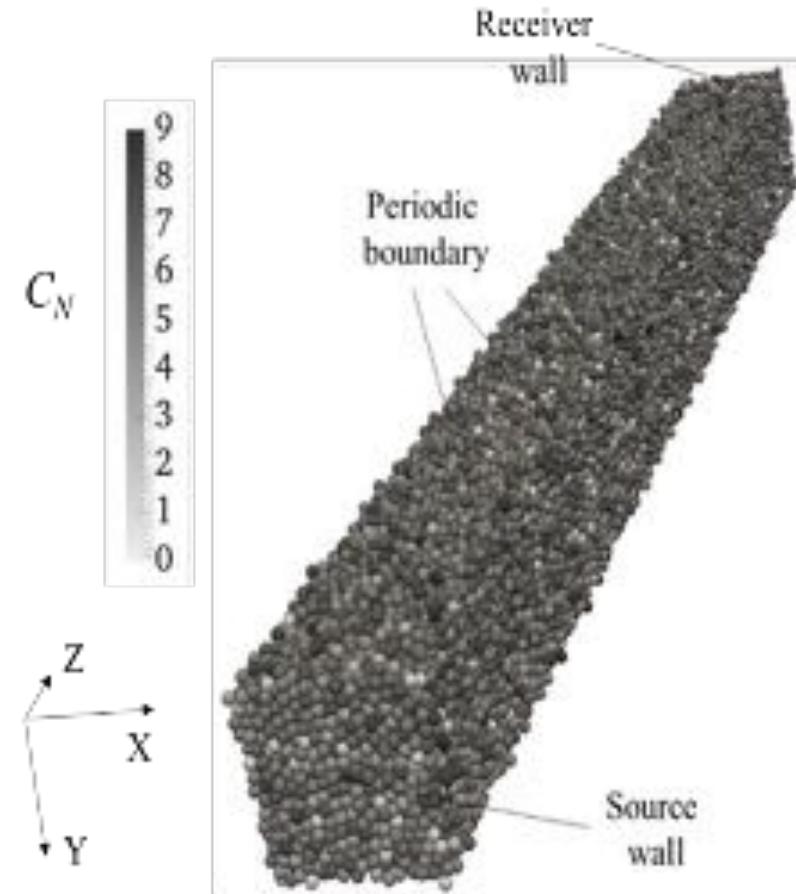
# Micromechanics of stress wave propagation.

1. Can DEM simulations reproduce experimental observations of stress wave propagation?
2. How reliable are empirical approaches to interpret laboratory geophysics tests?
3. Can DEM simulations inform the design of laboratory geophysics experiments?
4. How does surface roughness effect stiffness?
5. **What can we learn about the material fabric from the frequency domain response?**
6. What opportunities does the BRISS project present?

# Analysis of frequency domain response – DEM samples

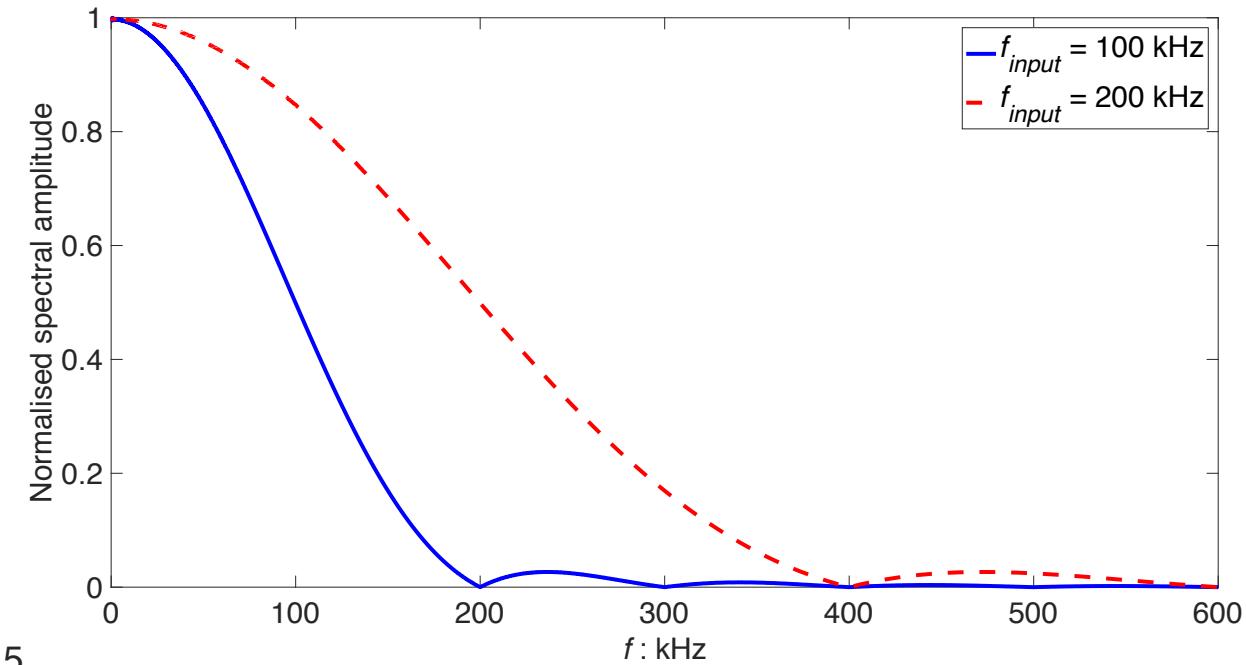
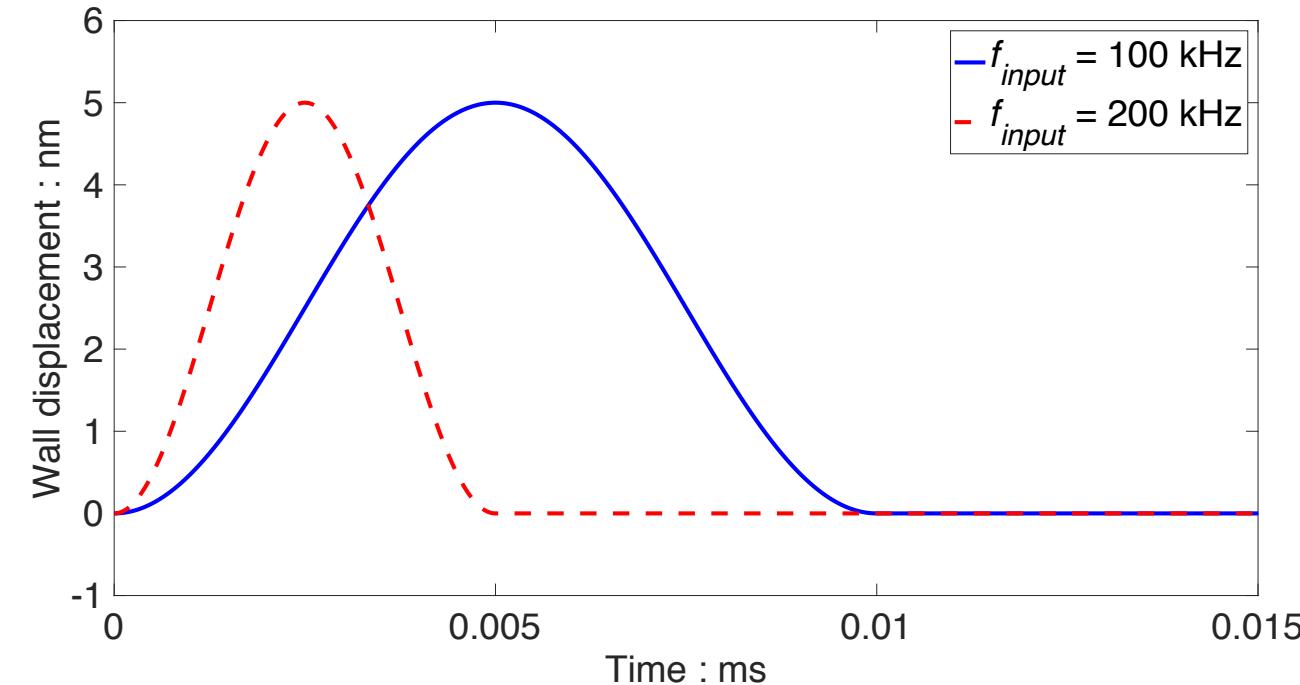


FCC Sample: 3,200 particles (4x4x200 layers)

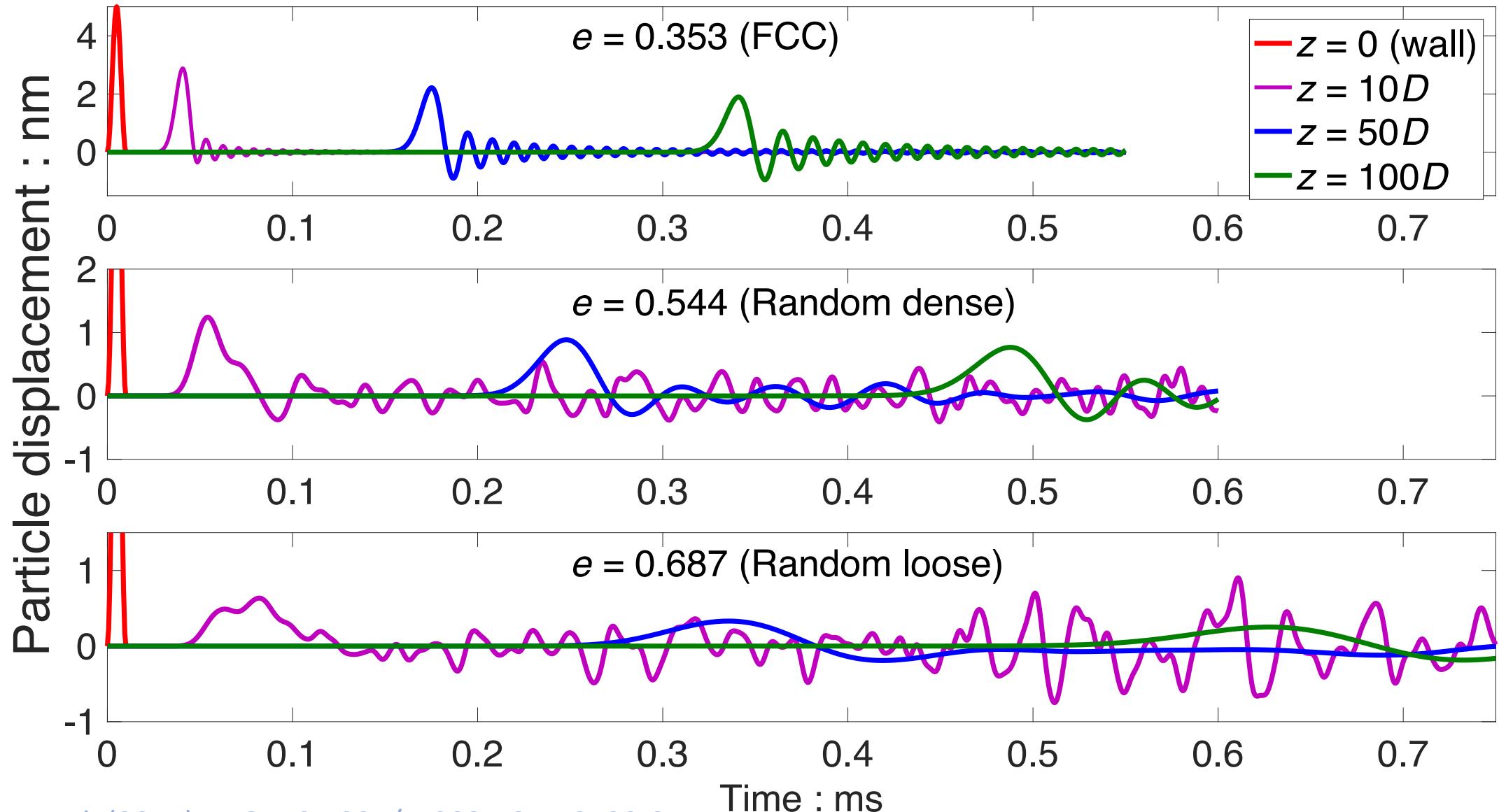


Random monodisperse sample:  
35,201 particles  
sample lengths (L) 141D to 144D

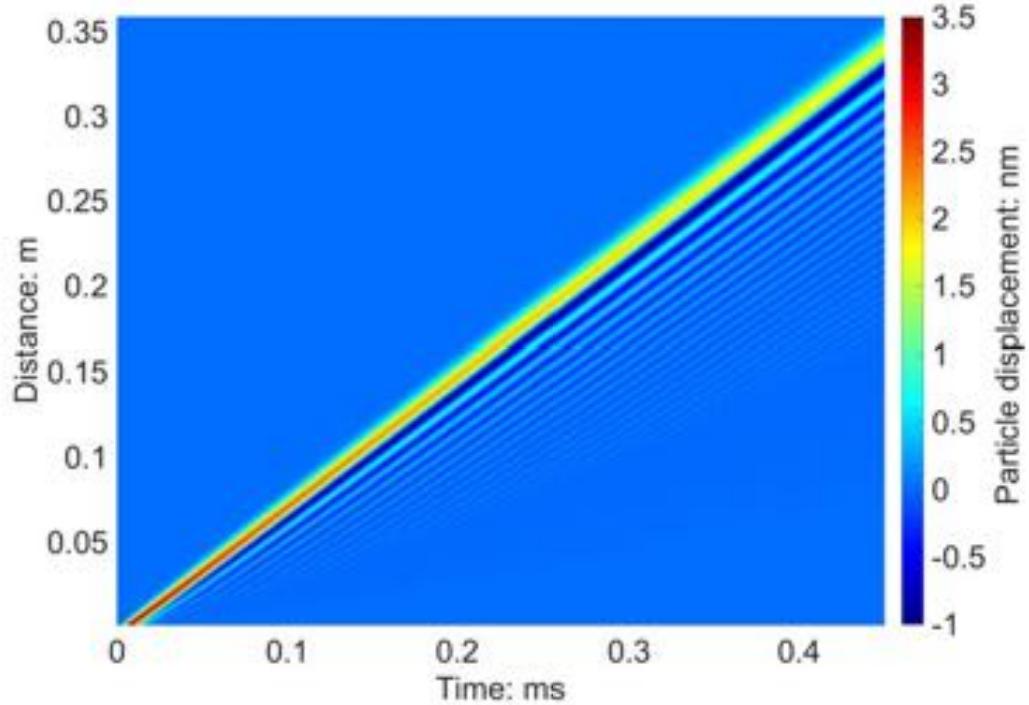
# Input signals: Time and frequency domains



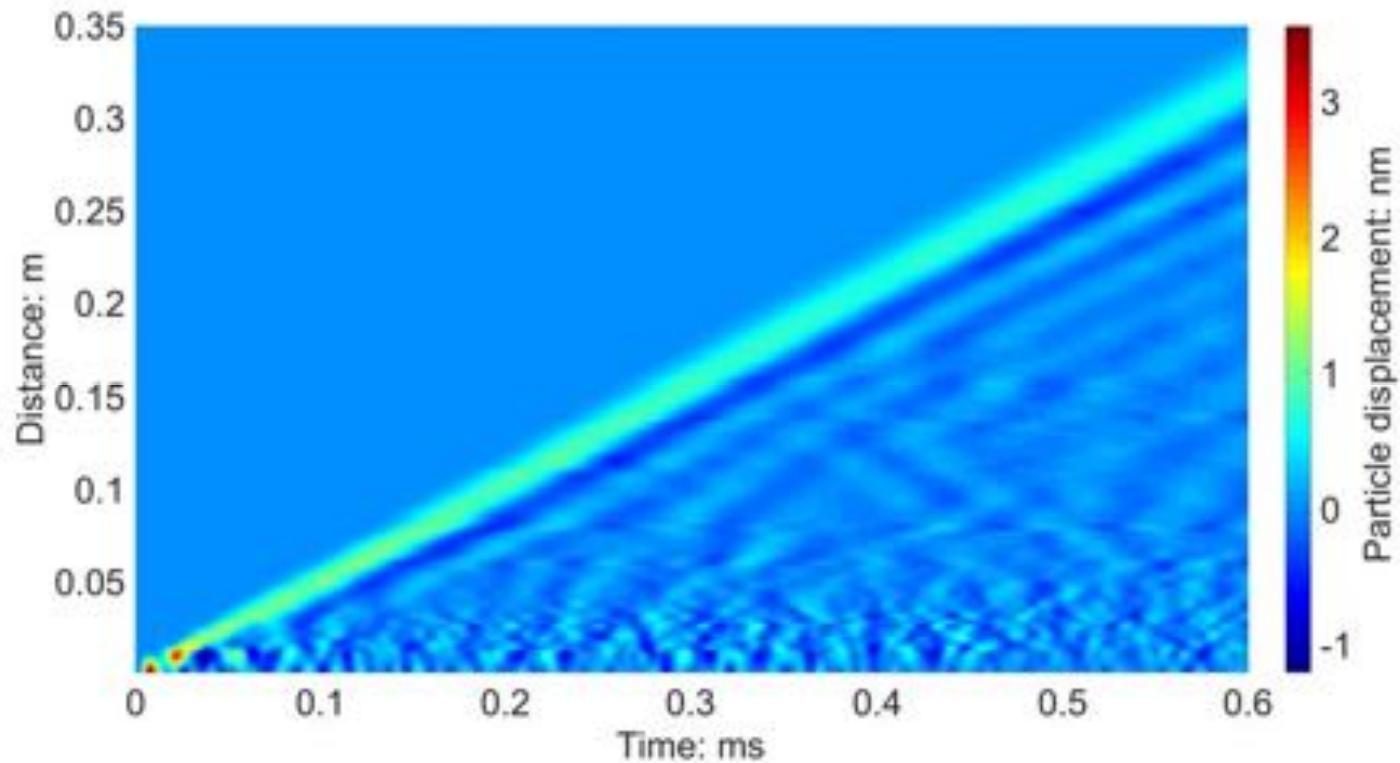
# Variation in signal with position – time domain



# Particle velocities



FCC Sample

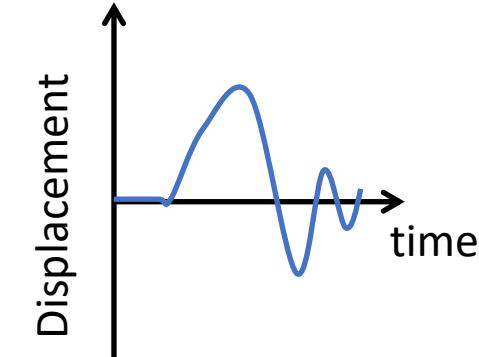
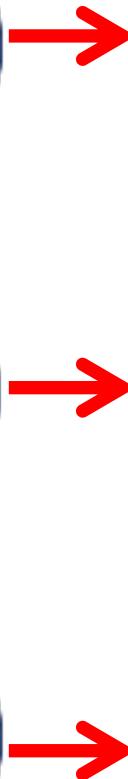
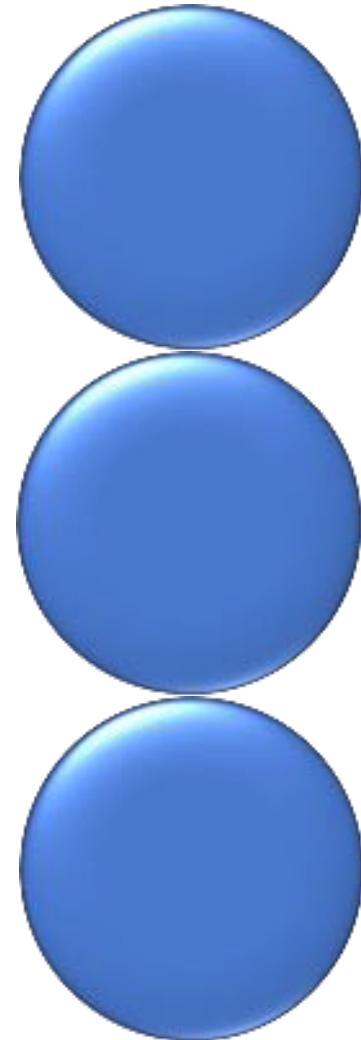


Random Dense Sample

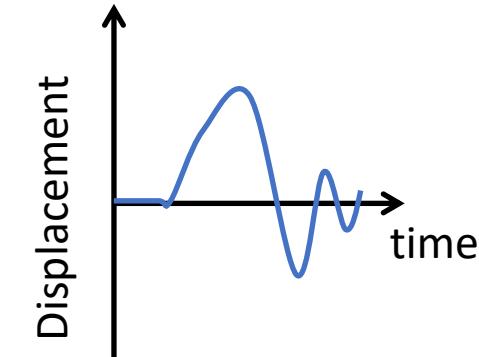
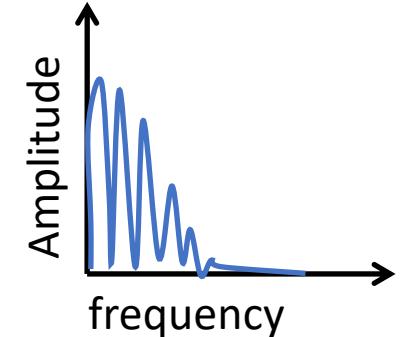
# Analysis of propagating wave: Frequency Domain

Central  
column of  
particles  
between  
transmitter  
and receiver

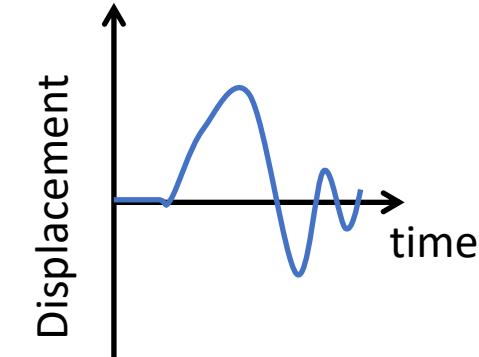
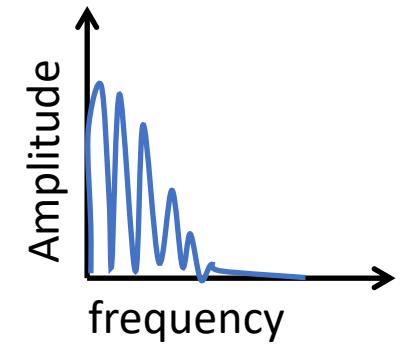
Mouraille, O.  
University of Twente



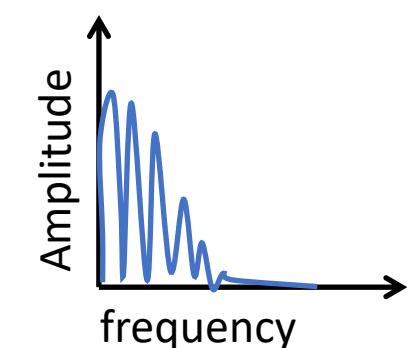
FFT

A red arrow points from the top time-domain plot to the top frequency-domain plot, indicating the result of the Fourier Transform.

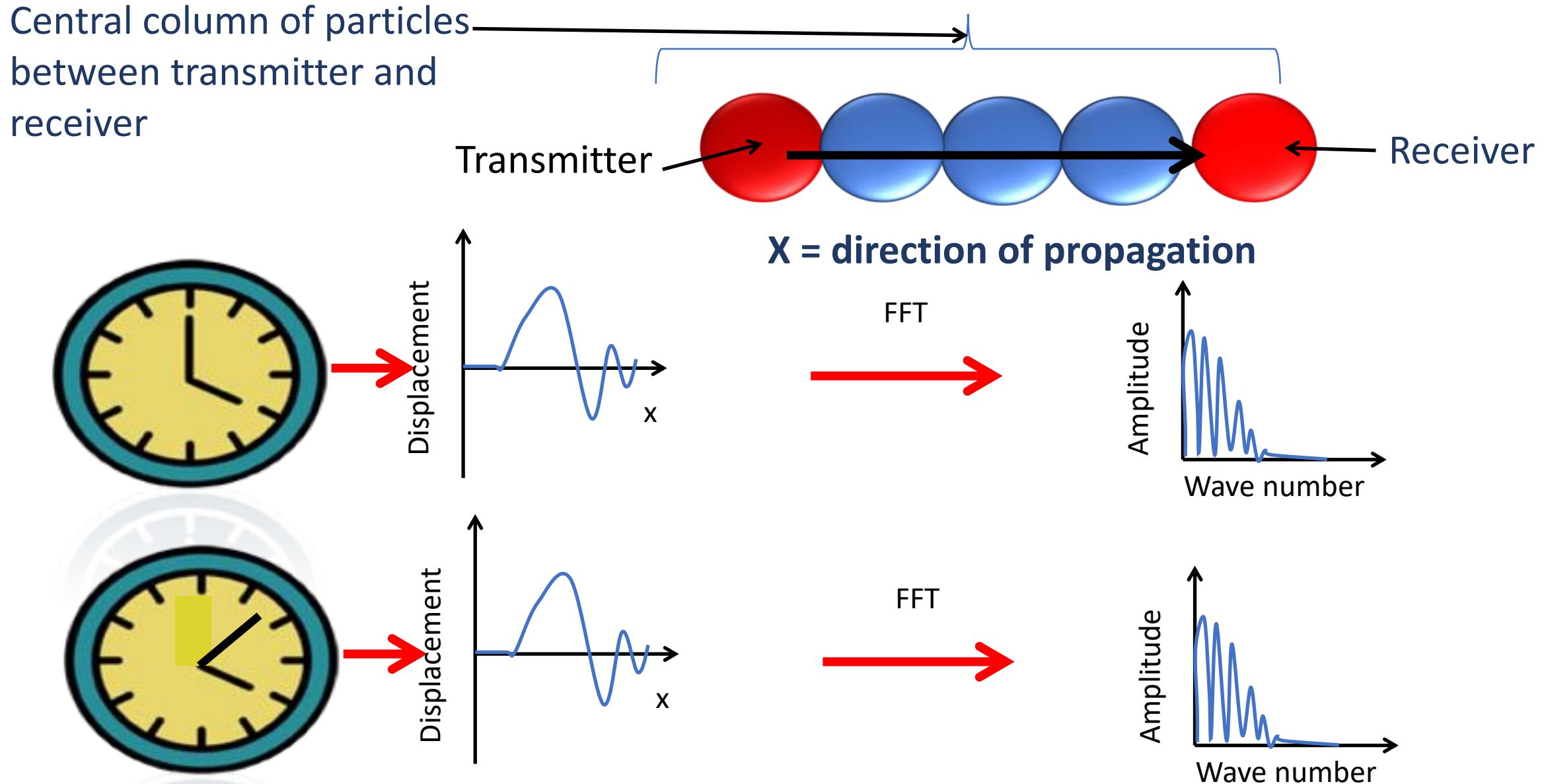
FFT

A red arrow points from the middle time-domain plot to the middle frequency-domain plot, indicating the result of the Fourier Transform.

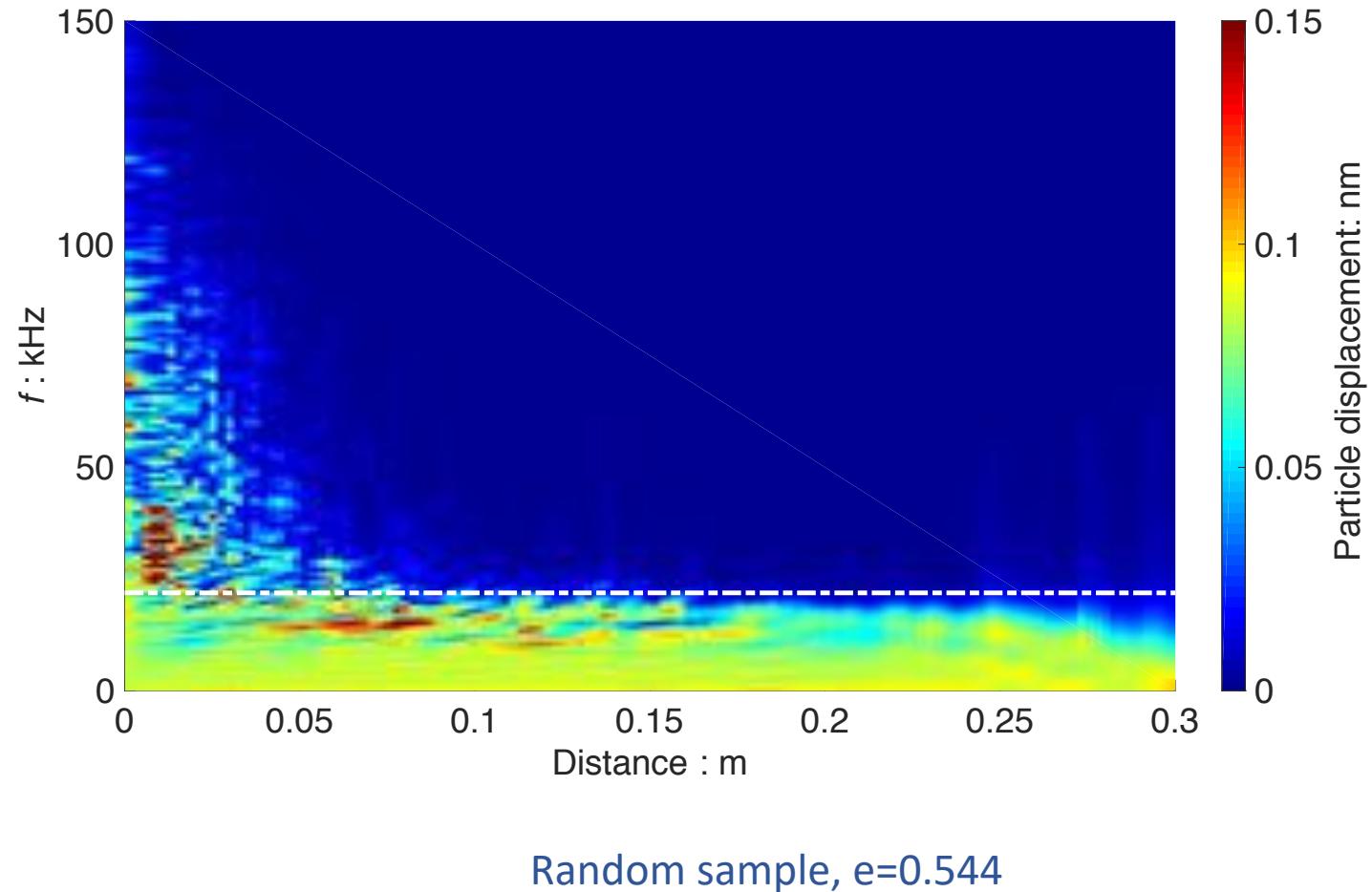
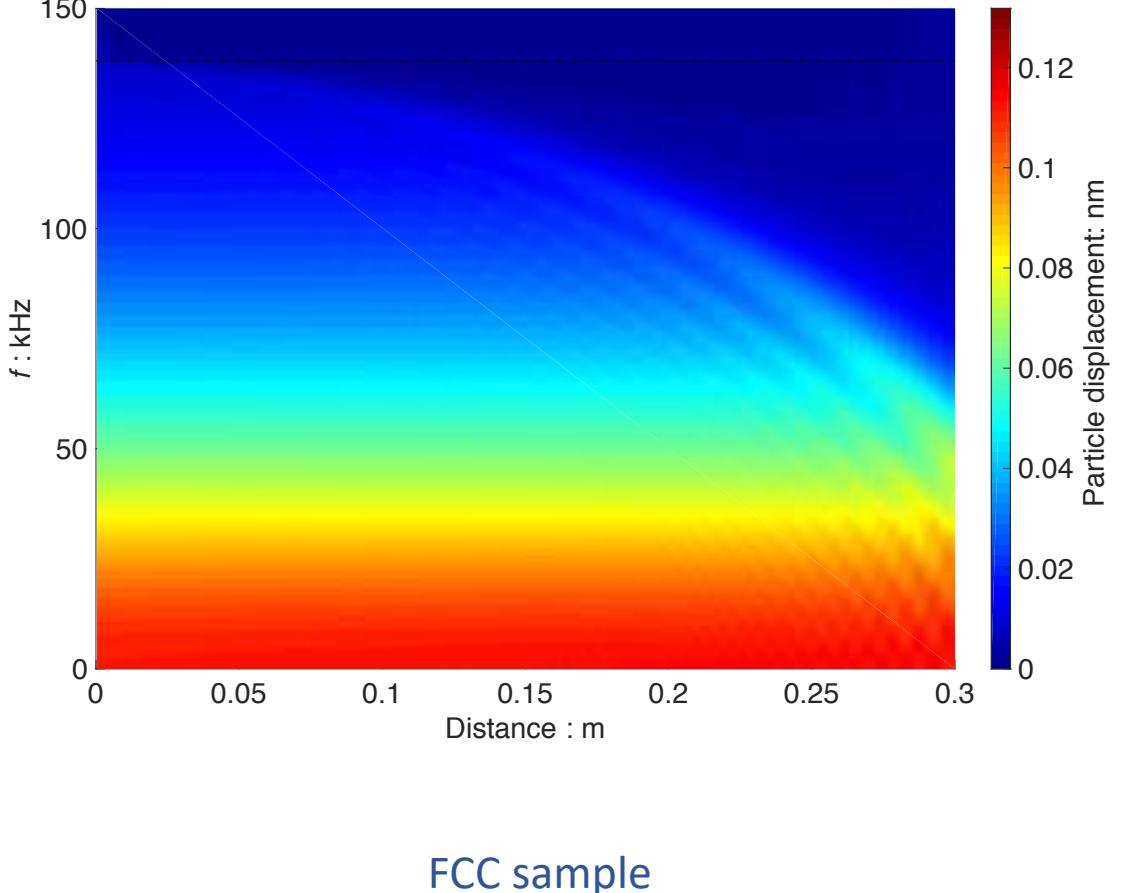
FFT

A red arrow points from the bottom time-domain plot to the bottom frequency-domain plot, indicating the result of the Fourier Transform.

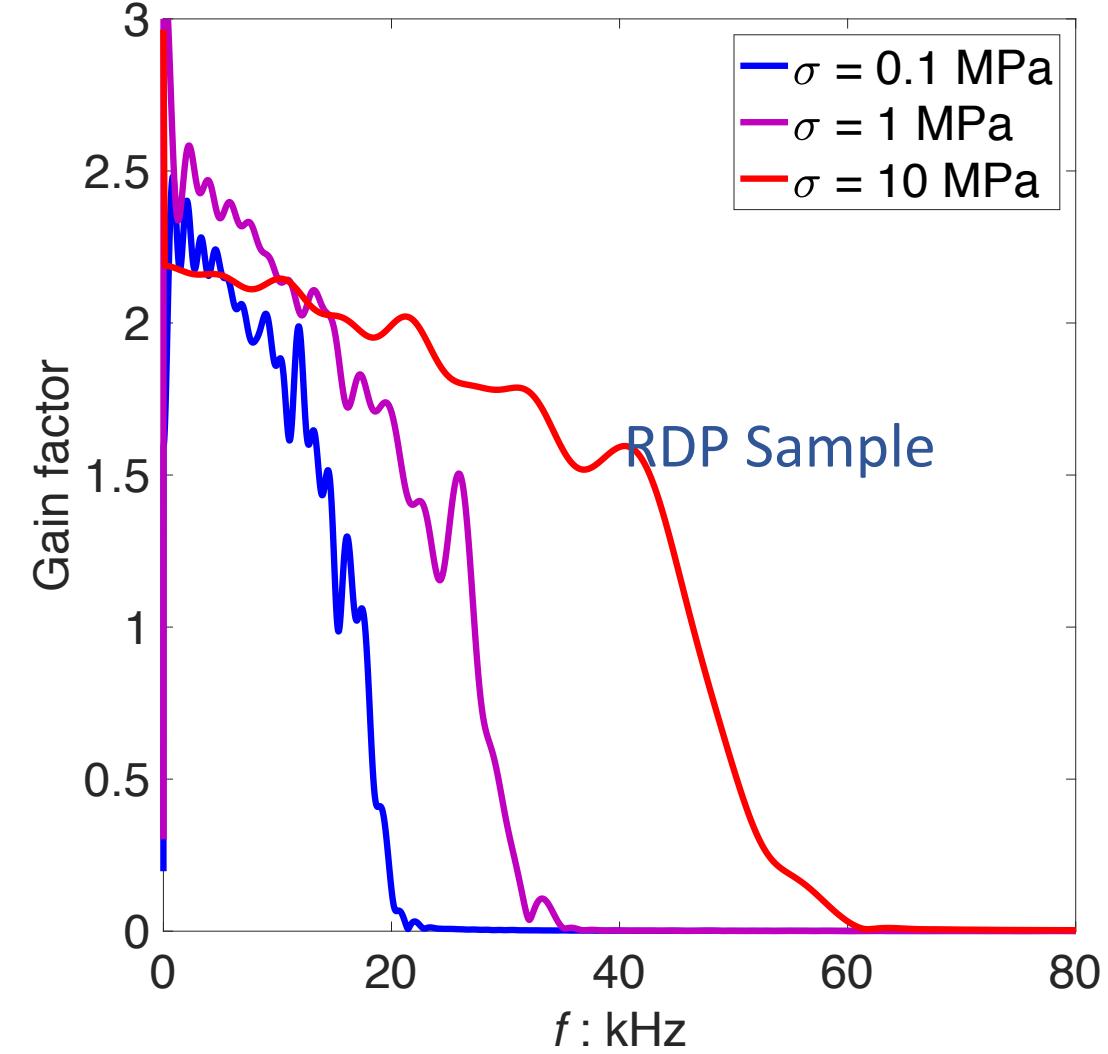
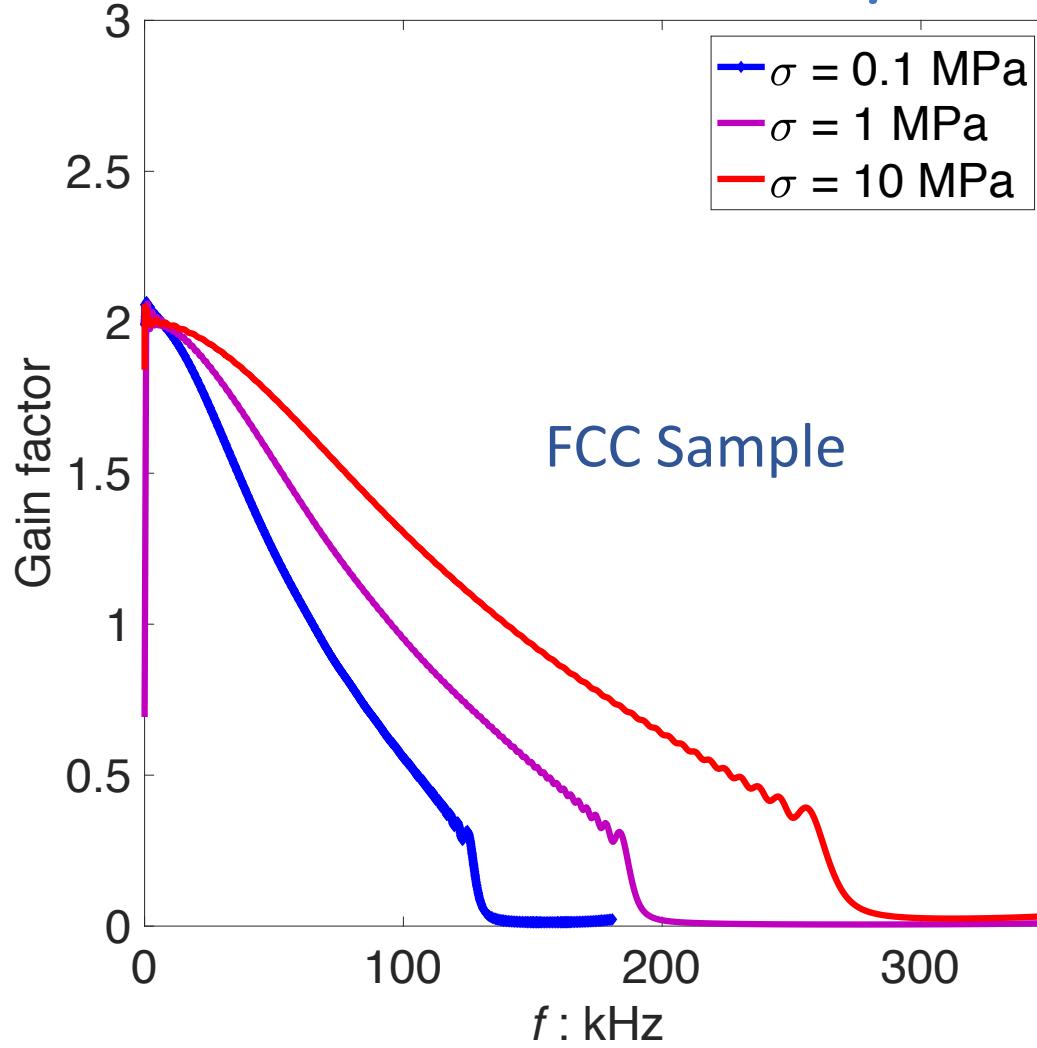
# Analysis of propagating wave: Frequency Domain



# Variation in frequency content with position

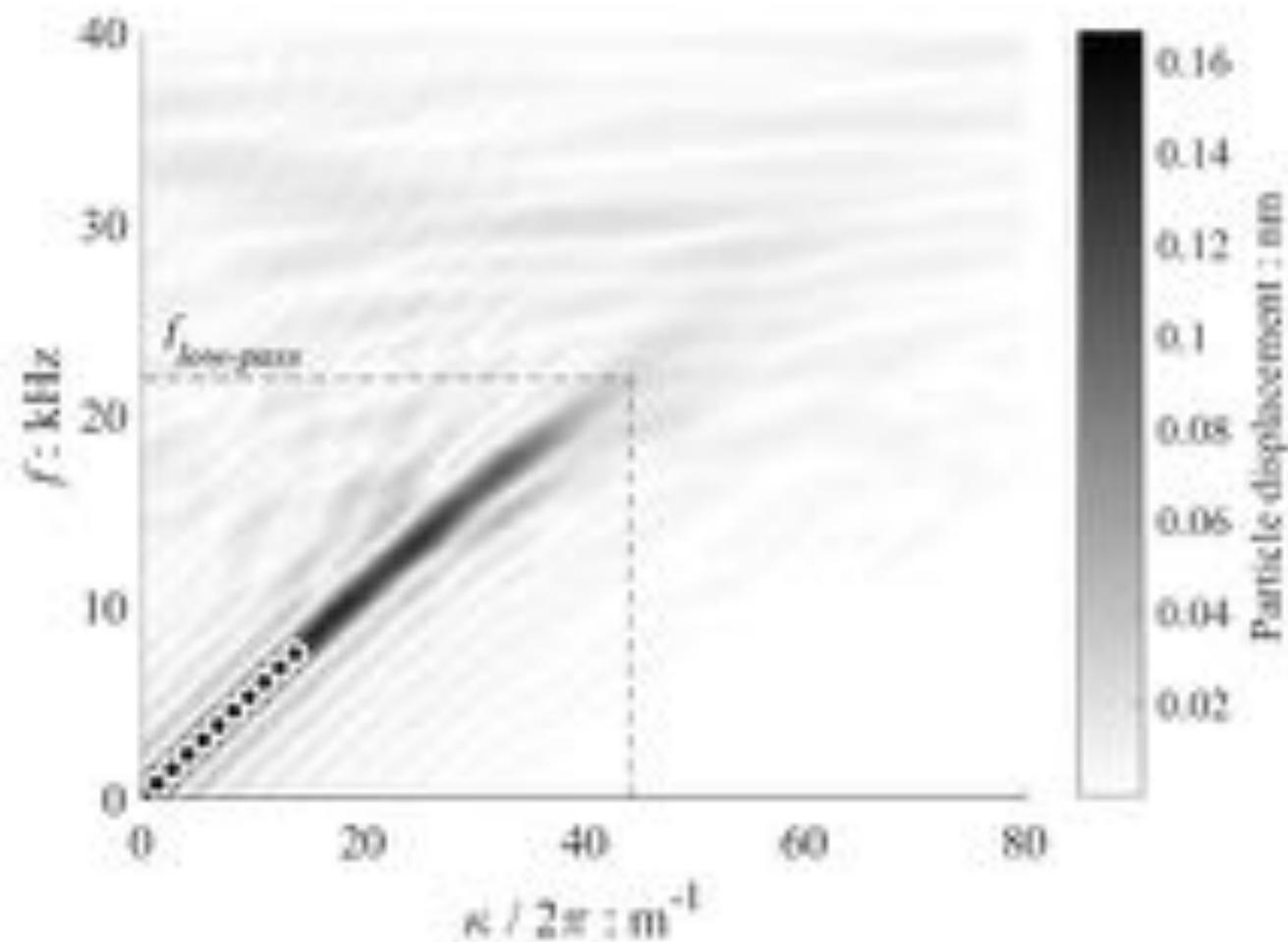
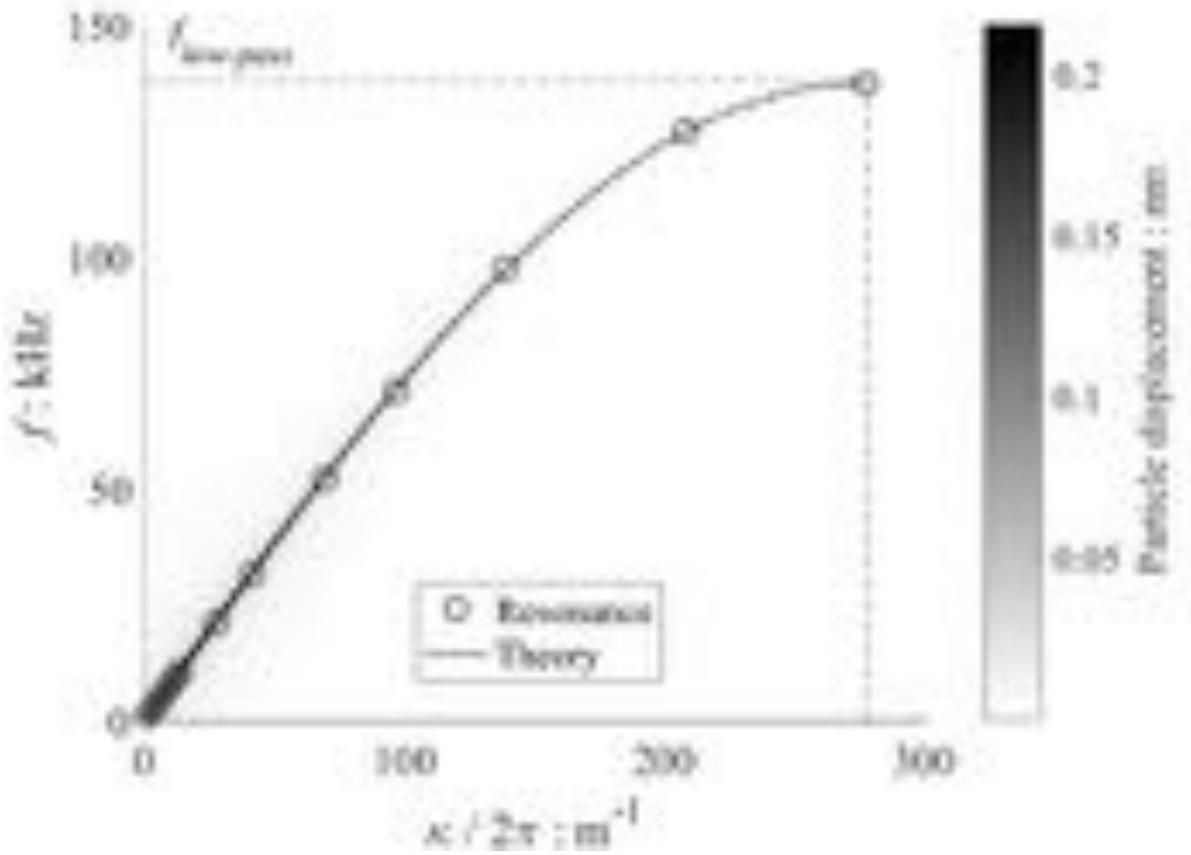


# Ratio of receiver data / source data

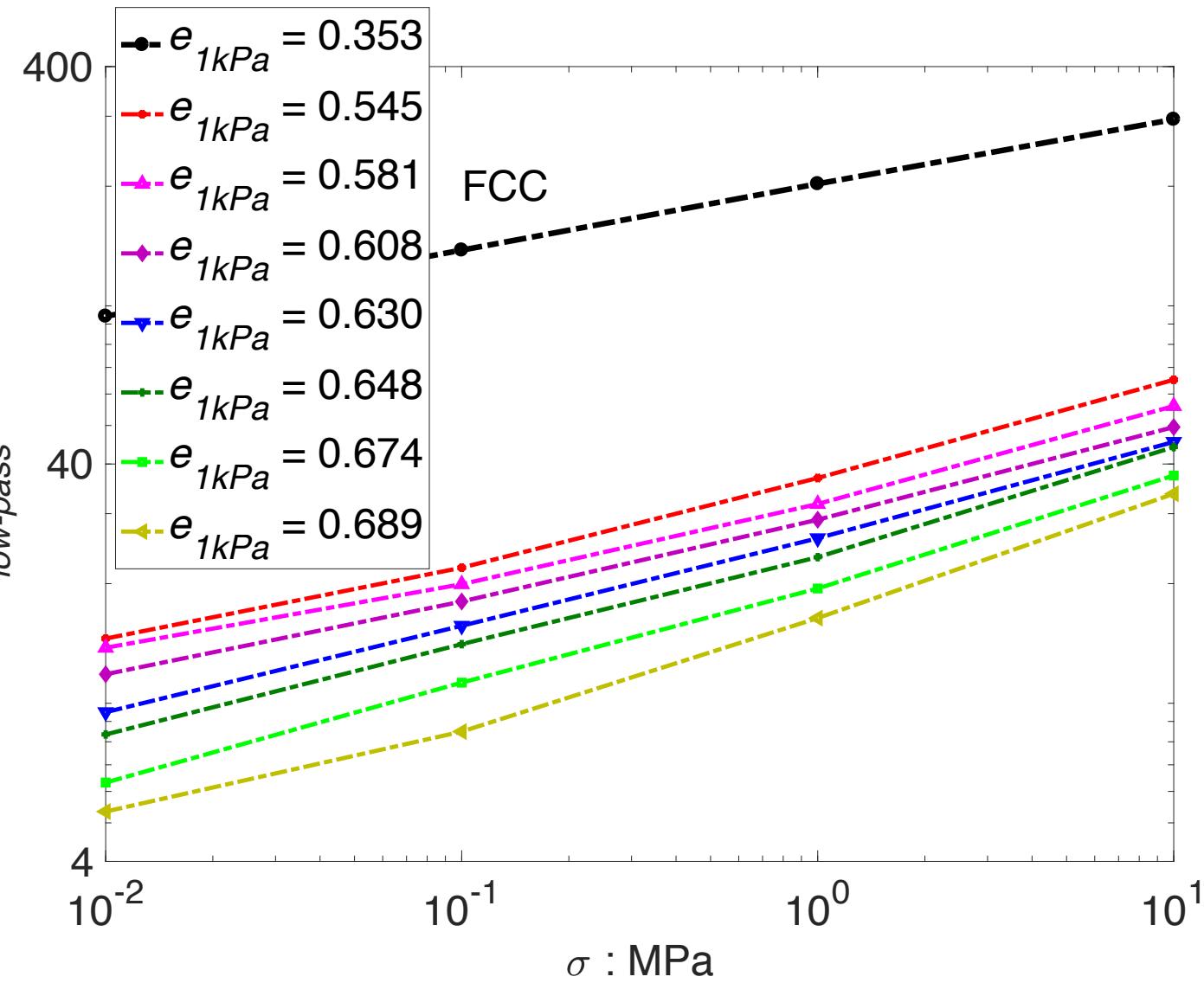
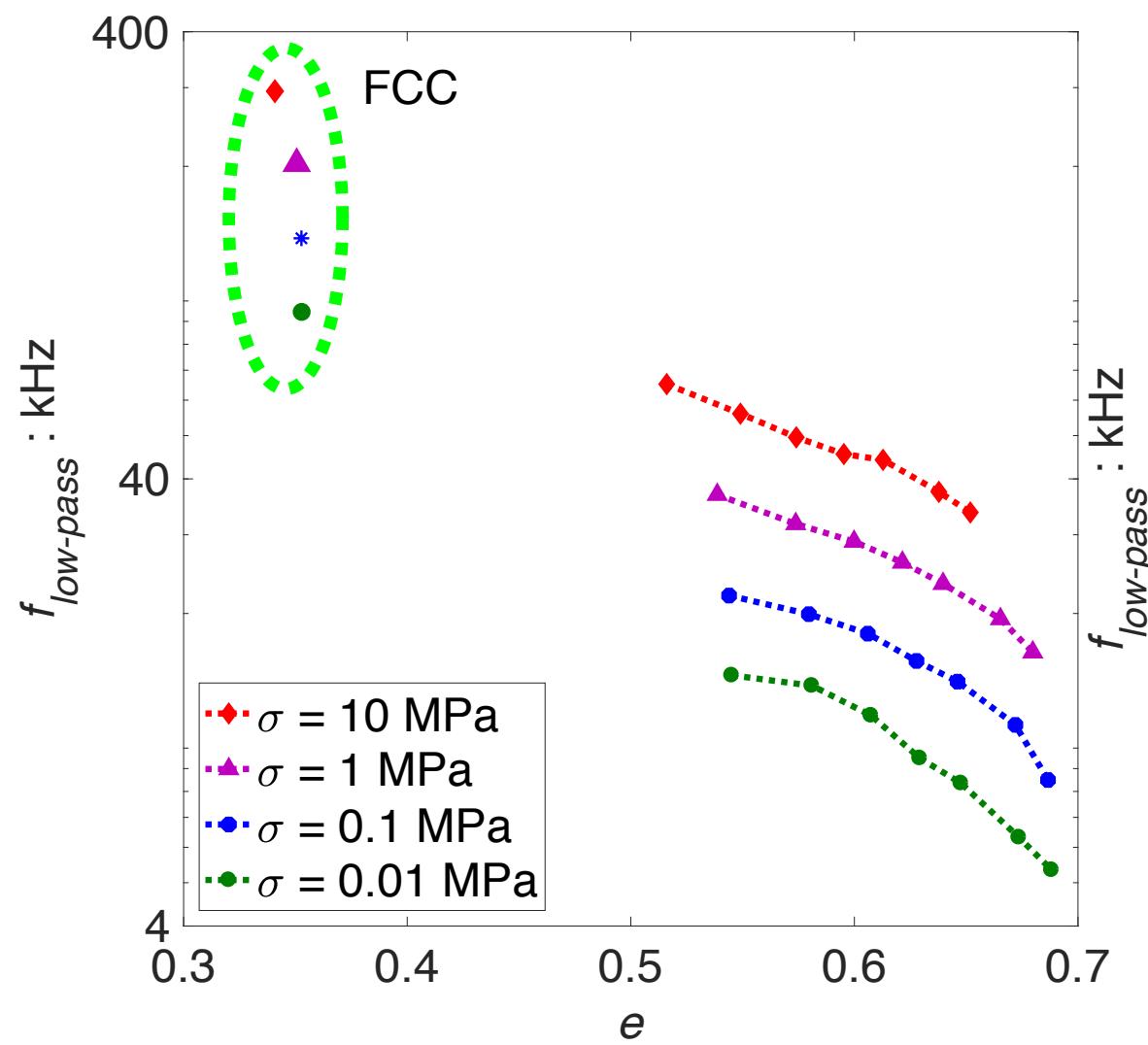


gain factor: the ratio of the frequency spectra

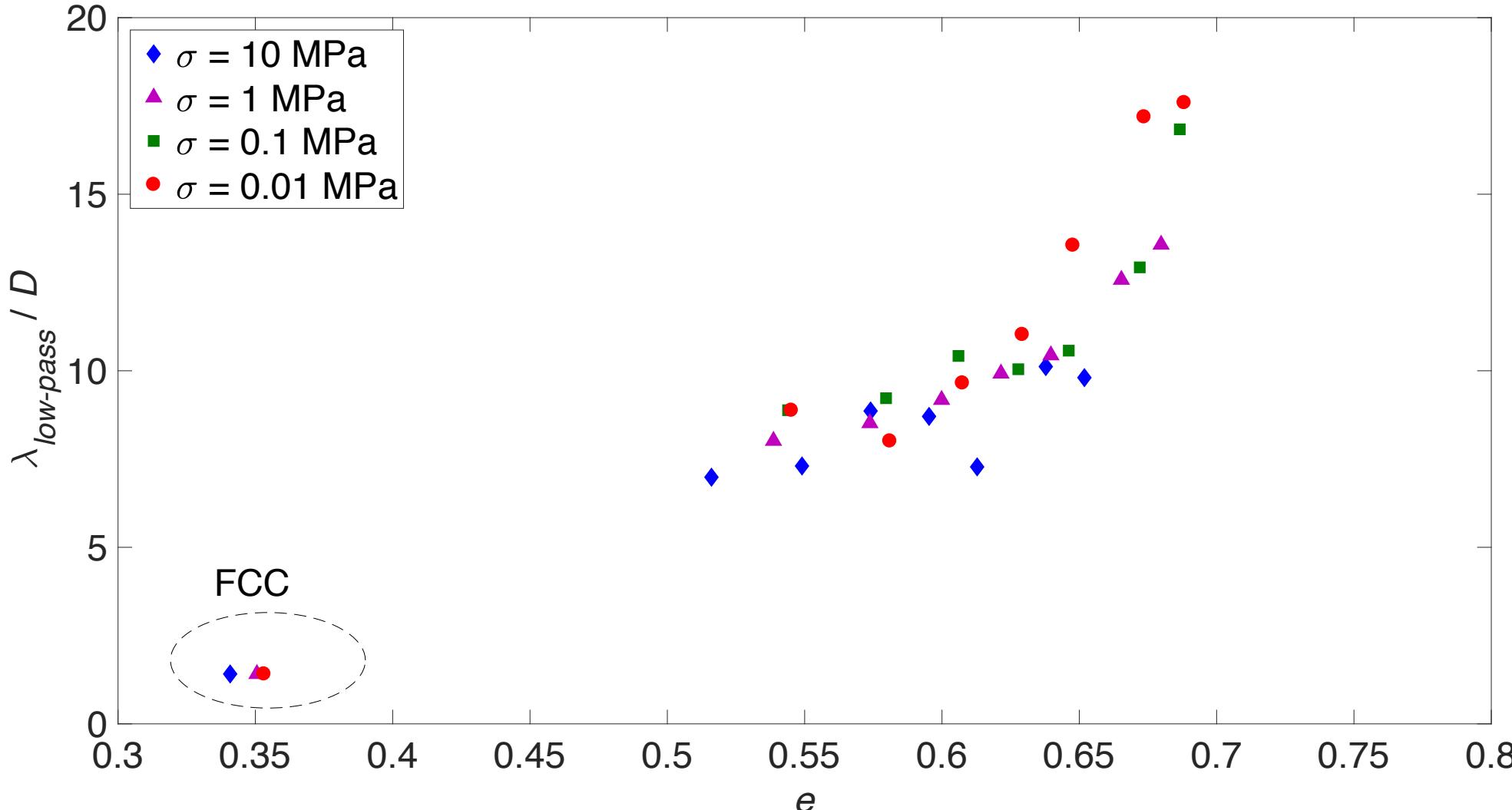
# Influence of contact plasticity on overall behaviour



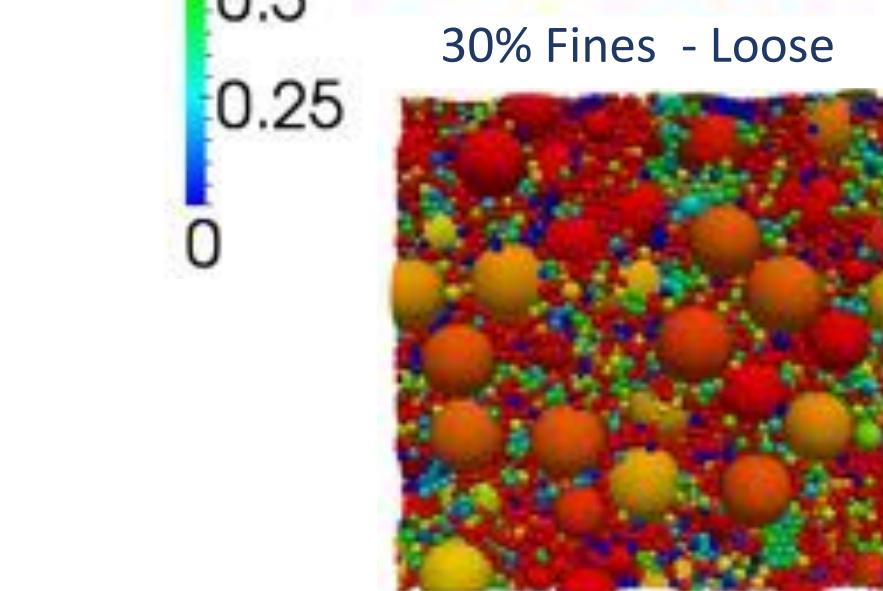
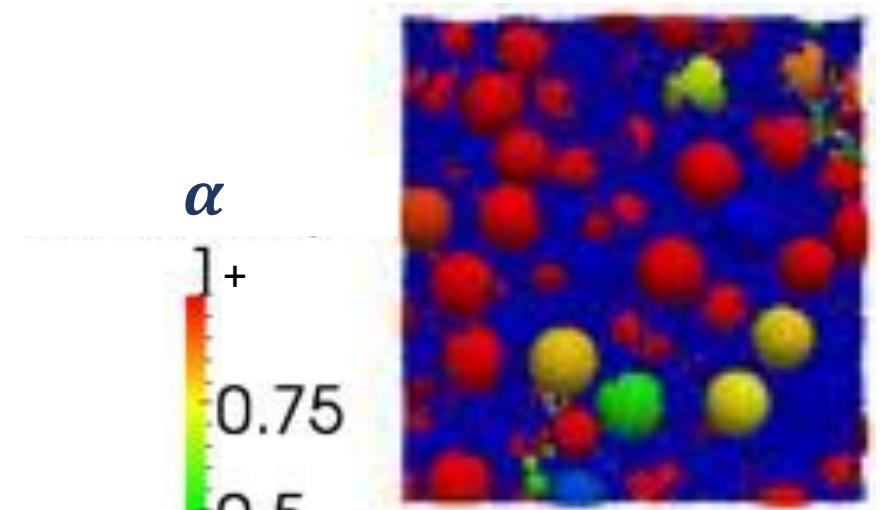
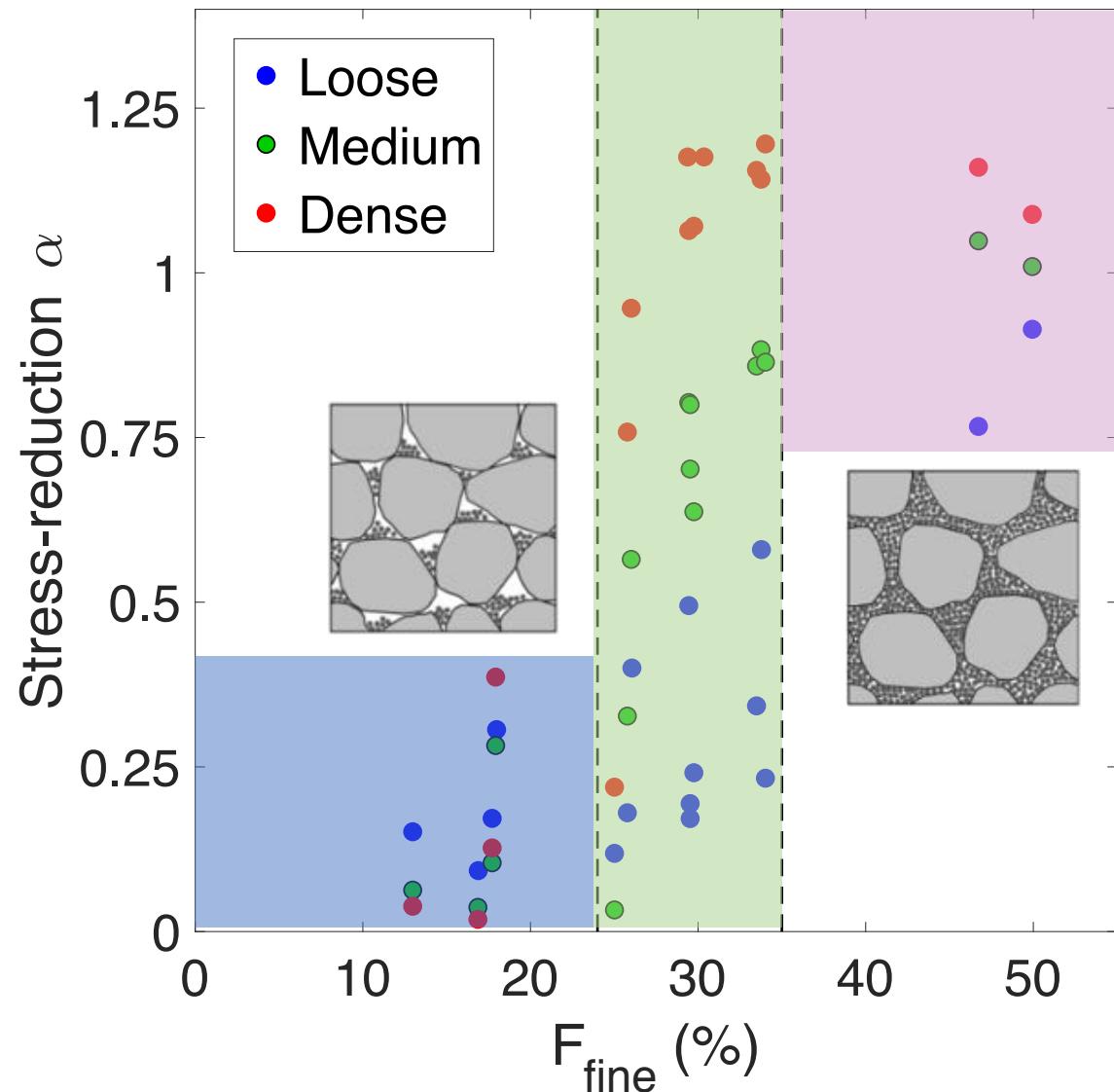
# Influence of packing density and stress on $f_{\text{low-pass}}$



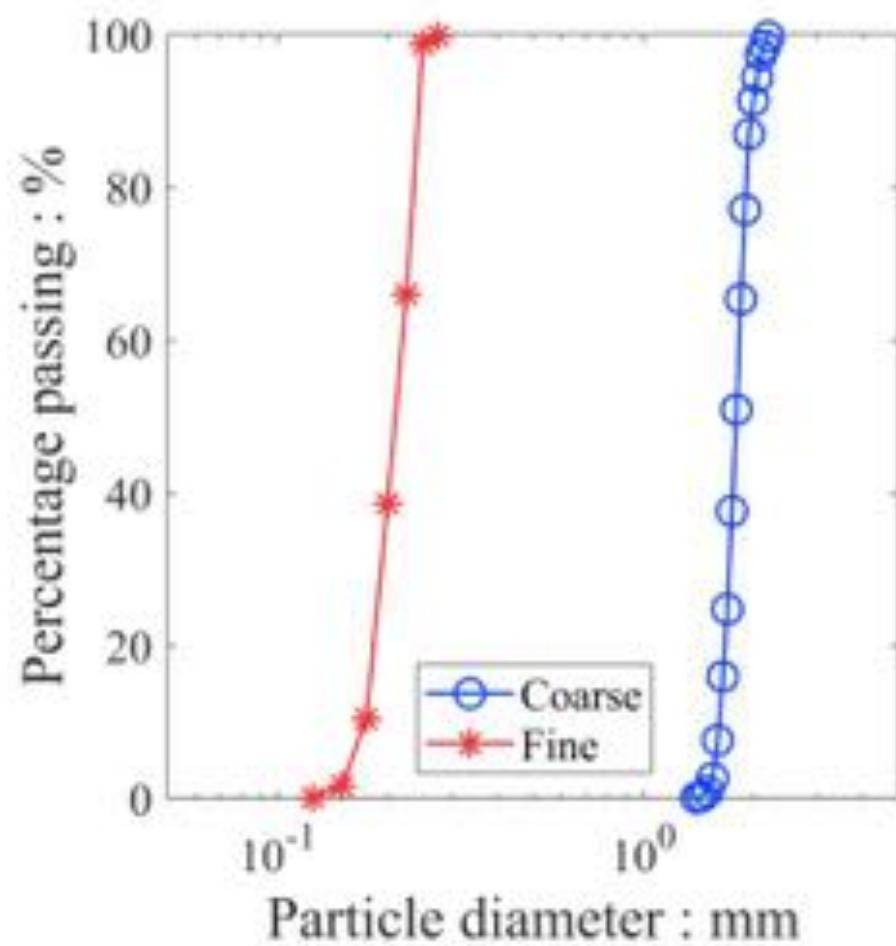
# Low pass wavelength versus void ratio



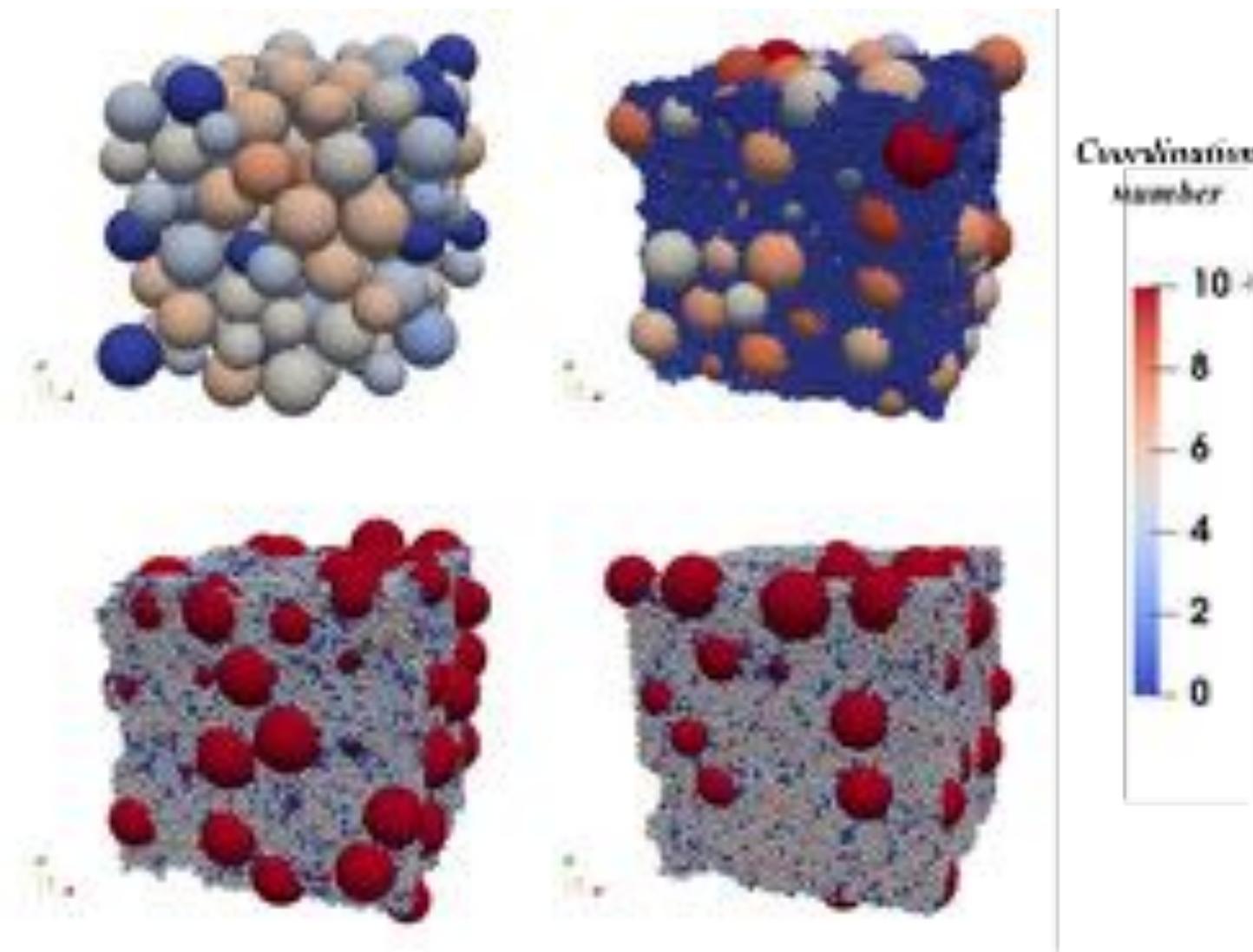
# Stress transmission in gap-graded soils



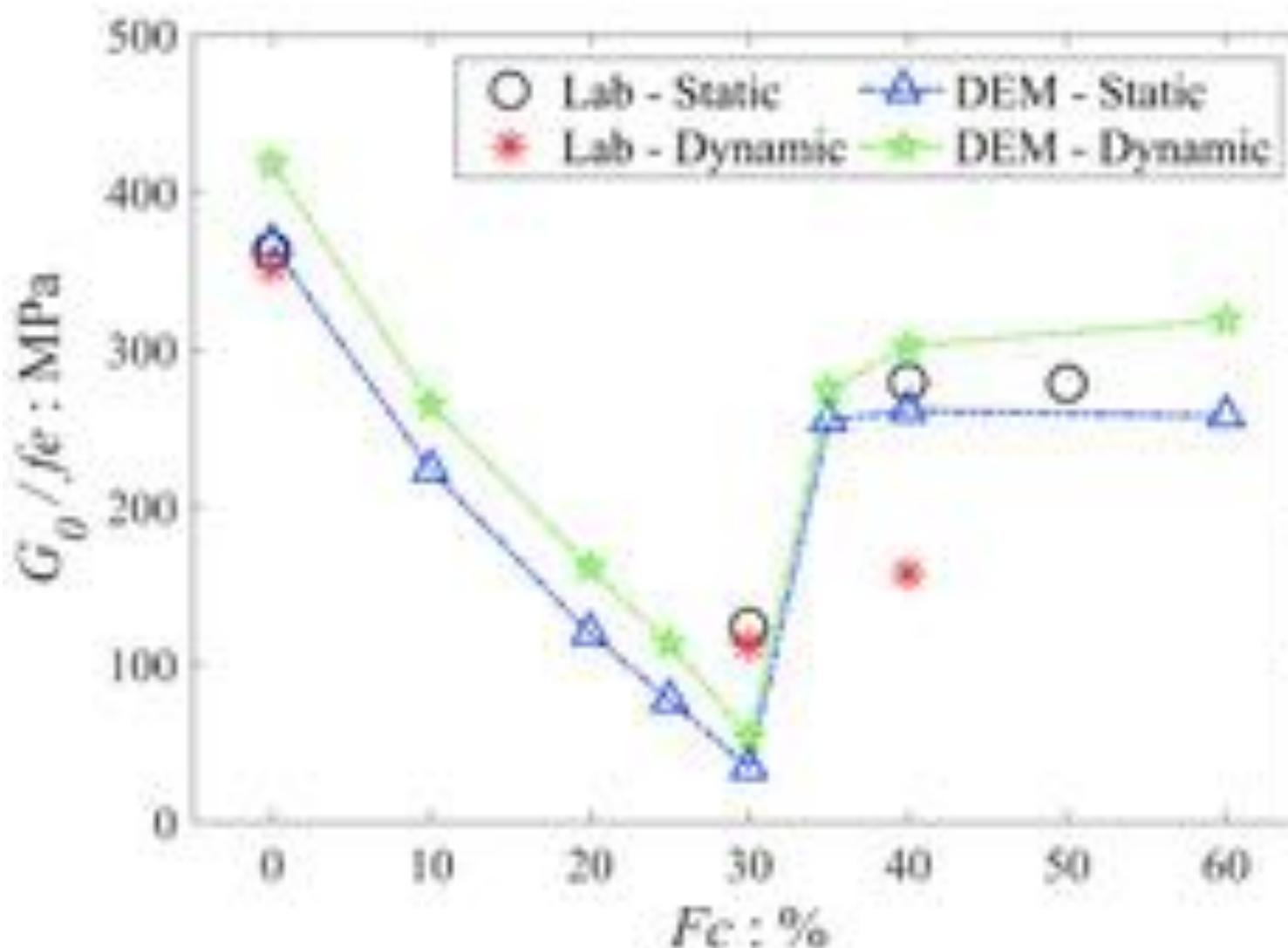
# Low pass frequency as indicator of nature of stress transmission



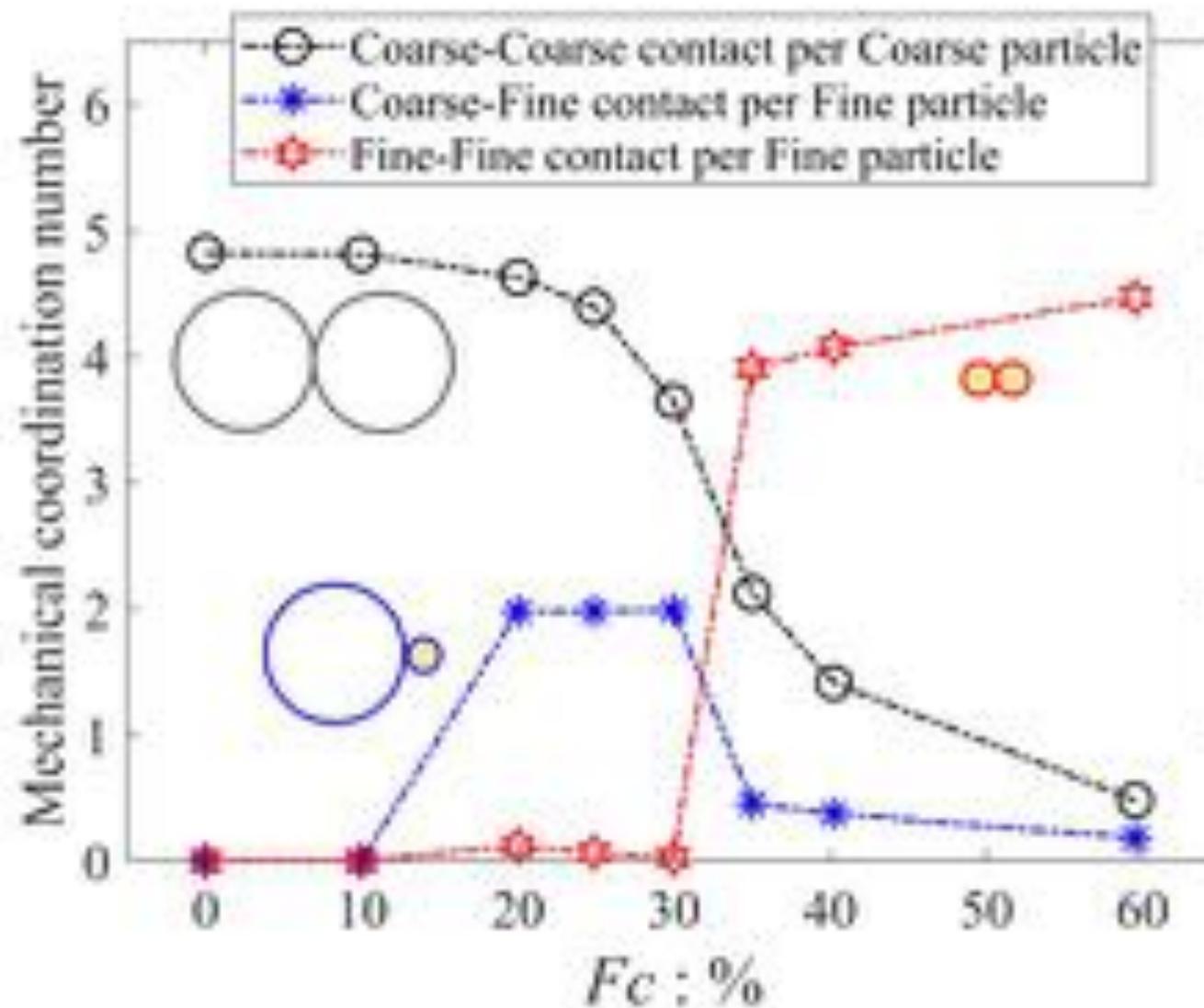
# Influence of contact plasticity on overall behaviour



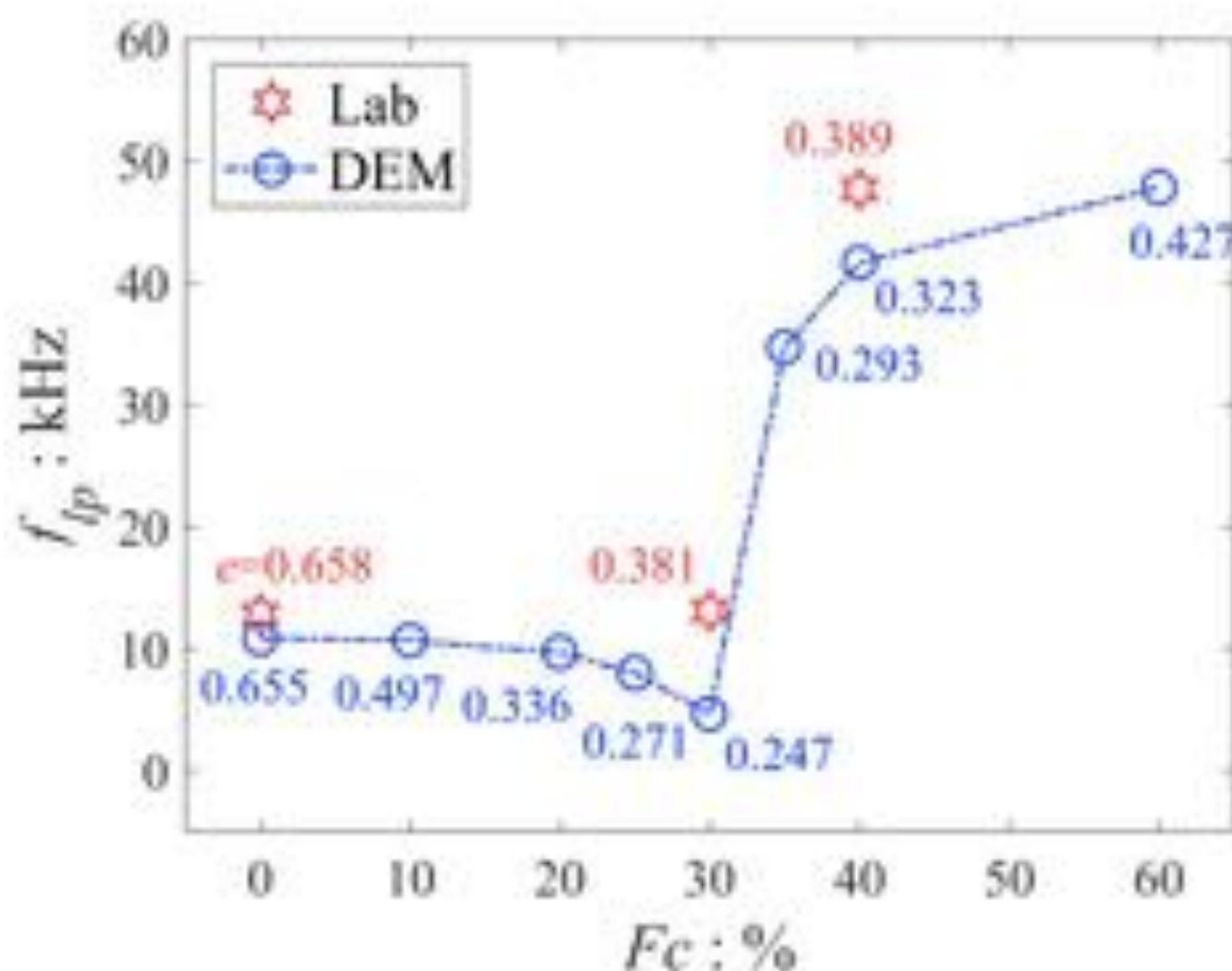
# Influence of contact plasticity on overall behaviour



# Influence of contact plasticity on overall behaviour



# Influence of contact plasticity on overall behaviour



# Acknowledgements

- Technicians at in Imperial College Soil Mechanics Laboratory
- Engineering and Physical Sciences Research Council
- JASSO
- Dixon Scholarship Imperial College
- Royal Commission for Exhibition of 1851