Big and Small: Challenges Associated with Granular Mixtures

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Motivation

Minas Gerais – January 2019 – tailings dam failure in Brazil
157 fatalities, 182 people unaccounted for
Motivation

Liquefaction in Maria District, San Francisco 1989 Loma Prieta Earthquake

Earthquake-induced liquefaction at Onahama Port, Japan, 2011

Shortland Street in the suburb of Aranui, New Zealand, 2011
Motivation

https://www.windpowerengineering.com/comparing-offshore-wind-turbine-foundations/
Sand behaviour

Stiffness

Strength

Seepage
Soil Mechanics Element Testing

Laboratory element tests ➔ How sand behaves
Particulate soil mechanics

Particle scale simulation and observation ➔ Why sand behaves the way it does
Size matters in soil classification

- Clay grains smaller than 2 μm (0.002 mm)
- Ratio of surface area to volume is large
- Surface charges influence behaviour

Figure 1.2 Knappett and Craig (2012)
Size matters in soil classification

For particles larger than about 100 micron we can neglect surface charge and model using the combination of normal and tangential springs as per Cundall and Strack DEM.

Fines / finer grains in this presentation are >100 micron

Figure 1.2 Knappett and Craig (2012)
Challenge 1: Describing the PSD
Observation of size range: Particle size distribution

Figure 1.13 Knappett and Craig (2012)
Typical particle size distributions

Continuous grading

Figure 1.13 Knappett and Craig (2012)

Gap-graded

Figure 1.19 Knappett and Craig (2012)
Quantifying the PSD: Coefficient of uniformity, $C_u$

Cumulative distribution by volume / mass

10% of particles by mass are smaller than $D_{10}$

60% of particles by mass are smaller than $D_{60}$

$$C_u = \frac{D_{60}}{D_{10}}$$
Quantifying the PSD: Coefficient of uniformity, $C_u$

$C_u = \frac{D_{60}}{D_{10}}$

10% of particles by mass are smaller than $D_{10}$

60% of particles by mass are smaller than $D_{60}$
Quantifying the PSD: Coefficient of curvature, $C_Z$

Cumulative distribution by volume / mass

$C_Z = \frac{D_{30}^2}{D_{60}D_{10}}$

30% of particles by mass are smaller than $D_{30}$
Quantifying the PSD pf gap-graded materials

$C_u$ and $C_z$ can’t discriminate effectively between different gap graded samples

Tend to consider

- Fines content ($F_c$ or $F_{\text{finer}}$)
- Size ratio (SR)

- SR must be carefully considered
- $SR \leq 6$ a single fine grain might fill a void

SR = \( \frac{D_{50,\text{coarse}}}{D_{50,\text{finer}}} \)
Bimodal gradings

% Passing by mass vs Diameter: mm

SR 3.7
SR 8.4
SR 18.1

SR 3.7 \( F_c \) 10%
SR 3.7 \( F_c \) 20%
SR 3.7 \( F_c \) 35%
SR 3.7 \( F_c \) 50%
SR 8.4 \( F_c \) 10%
SR 8.4 \( F_c \) 20%
SR 8.4 \( F_c \) 35%
SR 8.4 \( F_c \) 50%
SR 13.0 \( F_c \) 10%
SR 13.0 \( F_c \) 20%
SR 13.0 \( F_c \) 35%
SR 13.0 \( F_c \) 50%
SR 18.1 \( F_c \) 10%
SR 18.1 \( F_c \) 20%
SR 18.1 \( F_c \) 35%
SR 18.1 \( F_c \) 50%
IG – alternative measure of polydispersity

\[ I_G = \exp \left( \sqrt{\sum_i \frac{w_i}{w_{tot}} (\ln s_i - \ln \bar{s})^2} \right) \]

\[ \bar{s} = \exp \left[ \sum_i \frac{w_i}{w_{tot}} \ln \left( \frac{s_i}{\bar{s}} \right) \right] \]

\( w_i = \) weight retained at sieve opening \( d_i \)

\( s_i = \frac{d_i}{d_{\text{max}}} \)

Guida et al. (2019) Int. J. Solids and Structures
IG – alternative measure of polydispersity

Liu, Carraro, O’Sullivan, Géotechnique (2022) AOP
Challenge 2: Quantifying state
Packing density

Void ratio $e$:

$$e = \frac{V_v}{V_{sol}}$$

$V_v$ volume of voids

$V_{sol}$ volume of solid soil grains

Solids fraction:

$$\phi = \frac{V_{sol}}{V} = \frac{V_{sol}}{V_v + V_{sol}} = \frac{eV_v}{V_v + eV_v} = \frac{e}{1 + e}$$

$$e = \frac{V_v}{V_{sol}} = \frac{V - V_{sol}}{V_{sol}} = \frac{V - \phi V}{\phi V} = \frac{1 - \phi}{\phi}$$

$e \uparrow \iff \phi \downarrow$
State parameter framework

\[ q = \sigma'_1 - \sigma'_3 \]

\[ p' = \frac{1}{3} (\sigma'_1 + 2\sigma'_3) \]  
(triaxial case)

- \( q \): deviatoric stress
- \( p' \): mean effective stress
- \( \sigma' \): effective stress
- \( \sigma \): total stress
- \( u \): pore water pressure
- \( \varepsilon_a \): void ratio

Loose: \( \Psi > 0 \)
Dense: \( \Psi < 0 \)

Been and Jefferies (1985) - Géotechnique
Stiffness

\[ G_0 = A F(e) (p')^n \]

\[ F(e) = \frac{(c - e)^2}{1 + e} \]

A, c empirical constants

Initial slope small strain or elastic stiffness
Gap-graded soils or mixtures

Underfilled – coarse fraction dominates

Threshold fines content often used to distinguish coarse-dominated samples from fines dominated samples

Overfilled – finer fraction dominates

How do we calculate a voids ratio when fine particles are in the voids and not stress-transmitting?
Classifications by Thevanayagam et al. (2002)

FC< Threshold fines content: Inter coarse grain contact dominant

**Case 1**
- Fines fully contained within void
- Grain contact density index
  \[ e_c = \frac{e + F_c}{1 - F_c} \]

**Case 2**
- Fines are confined and partially in contact with coarse grains
- Grain contact density index
  \[ e_{eq} = \frac{e + (1 - b)F_c}{e - (1 - b)F_c} \]

**Case 3**
- Confined and separator of coarse grain
- Grain contact density index
  \[ e_{eq} = \frac{e + (1 - b)f_c}{e - (1 - b)f_c} \]

\( F_c \) = fines content = proportion of fines by mass

\( b \) = proportion of fine grains that contribute to intergrain contacts
Classifications by Thevanayagam et al. (2002)

FC > Threshold fines content: Inter fine grain contact dominant

Case 4 - (i)
Fully dispersed

Grain contact density index

\[ e_c = \frac{e}{F_c} \]

Case 4 - (ii)
Partially dispersed reinforcing elements

\[ e_{eq} = \frac{e}{F_c - (1 - F_c)/SR^m} \]
Variation in void ratio with Fc

Collation of published experimental studies

Zuo and Baudet (2015)
Soils and Foundations

Otsubo et al. (2022) Géotechnique 72(7)
https://doi.org/10.1680/jgeot.19.P.334
Packing density

Mechanical void ratio $e_m$:

$$e_m = \frac{V_v + V_{sol}^{inactive}}{V_{sol}^{active}}$$

$V_v \rightarrow$ volume of voids

$V_{sol}^{inactive} \rightarrow$ grains with 1 or no contacts

$V_{sol}^{active} \rightarrow$ volume of solid soil grains with at least 2 contacts

$e_m$ is a measure of density of stress-transmitting grains

Figure 1.6 Knappett and Craig (2012)
DEM simulation approach

Create a cloud of spheres

Isotropically compress to 500 kPa

- DEM code granular LAMMPS
- Periodic boundaries
- Control packing density using friction
- Up to > 1 million particles
- Spheres
Void ratio measures—linear + bimodal gradings

Liu et al. (2021) ASCE JGGE Vol 147 No. 3
Distribution of stress amongst different sized grains

Liu et al. (2022) Géotechnique (2022)
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Guida et al. (2019) Int. J. Solids and Structures
Distribution of stress amongst different sized grains

![Graph showing the distribution of stress amongst different sized grains.](image)

- **PSD**
- **PSD_active (D)**
- **Stress contribution (D)**
- **PSD_active (L)**
- **Stress contribution (L)**

Liu et al. (2022) Géotechnique (2022)
Quantifying Stiffness in Laboratory

\[ G_{\text{dyn}} = \rho V_s^2 \]

(Otsubo, 2017)
Influence of FC on wave propagation velocity

Otsubo et al. (2022) Géotechnique 72(7)
https://doi.org/10.1680/jgeot.19.P.334
Influence of FC on low-pass frequency

Gain factor = Amplitude received signal
             Amplitude inserted signal

Otsubo et al. (2022) Géotechnique 72(7)
https://doi.org/10.1680/jgeot.19.P.334
Relating $F_c$ to $f_{lp}$ and $V_s$

Otsubo et al. (2022) Géotechnique 72(7)
https://doi.org/10.1680/jgeot.19.P.334
Linking the ratio $G_{sta}/G_{dyn}$ to $e/e_m$

- $G_{sta}$ obtained via quasi-static probe

- $G_{dyn} = \rho V_s^2$

Liu et al. (2022) ASCE JGGE - accepted
Challenge 3: Seepage behaviour
Flood embankments

Design aim:
- Reduce downstream hydraulic gradient, $i$
- Critical case $i = 1$

Susceptibility of gap graded material to internal instability

Skempton and Brogan (1994) Géotechnique
Susceptibility of gap graded material to internal instability

Skempton and Brogan (1994) Géotechnique

Significant increase in permeability

Clear that complete failure is happening at a low hydraulic gradient

Variations in permeability will cause preferential flow

i = hydraulic gradient
Expect \( v = ki \)

\( i = 0.2: \)
- “strong general piping of fines throughout”
- permeability twice initial value

\[ v \text{ cm/s} \]

Sample A

0.6

0.2

0.2

0.1

0.3

0.1

0.2

Inflow

Piezometers

Rigid wall transparent cylinder

139 mm

Screen
Susceptibility of gap graded material to internal instability

MSc student photo of internal instability
Skempton and Brogan Sample A: comparison of $\alpha$ values

<table>
<thead>
<tr>
<th>Density</th>
<th>$\alpha_{DEM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose</td>
<td>0.15</td>
</tr>
<tr>
<td>Medium</td>
<td>0.06</td>
</tr>
<tr>
<td>Dense</td>
<td>0.04</td>
</tr>
</tbody>
</table>

$\alpha_{experiment} = 0.18$

Experimental sample placed moist with no densification

Shire et al. (2014) ASCE JGGE
DEM simulations to investigate instability

- Range of gap graded materials
- Density varied for all samples using inter particle friction ($\mu$)
- Isotropic compression in periodic cell to $p′=50$ kPa

Shire et al. (2014) ASCE JGGE
https://doi.org/10.1061/(ASCE)GT.1943-5606.0001184
Relationship between Fc and stress in finer grains

Shire et al. (2014) ASCE JGGE
https://doi.org/10.1061/(ASCE)GT.1943-5606.0001184
Challenge 4: Predicting load:deformation behaviour
Correlating elastic stiffness and state

The best fit straight line

Fitting curve for all of 8
Correlating elastic stiffness and state

(a)

G_0 e^{\frac{Zm}{1+e^m}}p

\(c = 1.057\)
\(A = 146.2\)
\(n = 0.33\)
\(R^2 = 0.45\)

G_0 = 1.707
A = 61.19
n = 0.33
R^2 = 0.90
Effect of Fc on Shearing response

Undrained stress-strain behaviour
(Modified from Yang and Wei, 2012)

- Toyoura sand (coarser grains)
- Crushed silica (finer grains)

\[ q = \sigma_1 - \sigma_3 \]

Increasing Fc leads to an increase in undrained strain-softening

Critical state locus
(Modified from Yang and Wei, 2012)

\[ p' = \frac{\sigma'_1 + 2\sigma'_3}{3} \]

Increase in Fc leads to a downward shift to the critical state line (CSL) (in e-logp' plane)
Heterogeneity in gap graded materials

G2 Middle
\[ D_{15}^{\text{coarse}}/d_{85}^{\text{fine}} = 4.29 \]

G1 Bottom
\[ D_{15}^{\text{coarse}}/d_{85}^{\text{fine}} = 3.30 \]
Influence of fabric on stress transmitting grains

3,300-4,750 coarse grains per sample

Up to 3.29 million particles

Density varied for all samples

Considered mean stress of $p' = 100$ kPa

Constant $p'$ shearing

Sufian et al. (2021) ASCE JGGE Vol 147 No. 5
https://doi.org/10.1061/(ASCE)GT.1943-5606.0002487
Shearing response – dense samples

Sufian et al. (2021) ASCE JGGE Vol 147 No. 5
https://doi.org/10.1061/(ASCE)GT.1943-5606.0002487
Evolution in stress in finer fraction during shearing

Start of shearing

Axial strain of 5%

Sufian et al. (2021) ASCE JGGE Vol 147 No. 5
https://doi.org/10.1061/(ASCE)GT.1943-5606.0002487
Evolution in stress in finer fraction during shearing

C: Coarse dominated

TC: Transitional coarse dominated

TF: Transitional fines-dominated

F: Fines-dominated

Sufian et al. (2021) ASCE JGGE Vol 147 No. 5
https://doi.org/10.1061/(ASCE)GT.1943-5606.0002487
Challenge 5: Obtaining a REV
Modelling challenge

Number of particles


GN, FC=30%
GW, FC=30%

GN: SR=3.4
GW: SR=8.8
Imaging challenge

X-Ray source + typical sample for scanning

(H. Taylor, former PhD student)
Imaging challenge

(a) UF18-Dense
(b) TR30-Dense
(c) Gap35Medium
(d) OF45-Dense

Fine-Coarse contacts
0 1 2 3
Challenge 6: Sand-clay or sand-silt mixtures
Morston till

Norfolk UK

Image by Dr. Simon Carr
Questions

• How should we describe polydispersity?

• Does the heterogeneous nature of stress distribution complicate the meaning of state?

• Can laboratory geophysics shed light on stress transmission?

• Is fabric more important for mixtures?

• Coordination number determines stiffness – routine measurement?

• How do we deal with sand – clay mixtures?
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PARTICLE-SCALE INVESTIGATION OF SEEPAGE INDUCED GEOTECHNICAL INSTABILITY
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