Capital requirements, monetary policy and the fundamental problem of bank risk taking☆

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Abstract

We ask what should be the role of limits on bank capital structure and monetary policy, the latter aimed at influencing the required return on bank liabilities, in countering a tendency for banks to take excessive risks because of asymmetric information and limited liability. We find that the two policy instruments operate as imperfect substitutes. A tightening of either instrument can improve "prudence", by disincentivising banks against investing in excessively risky assets with low probability of success; but only at the cost of decreased "participation", whereby more banks will forego the opportunity to invest. Our numerical simulations show that, in general, capital requirements should be the main policy lever in combating excessive risk taking by banks. But there are situations where monetary policy should also be used, and some where it should be used on its own.

Keywords: capital requirements, macroprudential policy, monetary policy

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1. Introduction

In this paper we ask what should be the role of limits on bank capital structure and monetary policy, the latter aimed at influencing the required return on bank liabilities, in countering a tendency for banks to take excessive risks because of asymmetric information and limited liability. We show that banks take too many risks when they know more about the probability of success of loans than do those who provide debt funding. There are only imperfect tools available to bank regulators and central banks to counter this tendency for banks to take too many risks. One tool is to force banks to use more equity funding than they would choose themselves. Another tool is to raise the cost of debt to banks and counter the tendency to excessive lending by using monetary policy – setting a slightly higher policy rate than would otherwise be appropriate. Both are rather blunt instruments and both have costs. We show under what conditions each tool is more effective and when both might be used.

Much of the literature which explores whether monetary policy should play a role in reducing problems in the financial sector assumes that interest rates can be used to control leverage and that - for reasons not modeled - leverage is sometimes too high. An exception is Stein (2012) who is precise about the source of market failure and we follow that route. Not being clear about exactly why financial outcomes are problematic means that policy conclusions are not robust. The fundamental source of market failure in our model is precisely that which provides the most compelling rationale for the existence of banks - namely their specialisation in collecting and interpreting information on types of assets whose risks are hard to assess and which make their financing through the issuance of securities in capital markets problematic.

There is a growing literature on the impact of capital requirements on bank lending; there is also a much larger (and older) literature on the effects of monetary policy on banks. But there is less research on the interaction of monetary policy with bank capital requirements in offsetting inefficient lending - which is the focus of this paper.

The broad consensus in the existing literature is that stricter capital requirements can curb excessive risk-taking and lending (see for instance Furlong and Keeley (1989); and Gersbach and Rochet (2017)); and a tightening of monetary policy could achieve similar effects, albeit potentially at a cost to other macroeconomic considerations (Farhi and Tirole (2009) and Nicolò et al. (2010)). A separate body of work exists on the role of bank capital in the transmission of monetary policy (such as Kashyap and Stein (1994), and Bolton and Freixas (2006))[1].

More recently, Angeloni and Faia (2013) compared the performance of different Taylor Rules under four banking regimes. They concluded that the optimal monetary policy rule should incorporate some response to financial conditions. Angelini et al. (2014) analysed the impact of monetary policy and capital requirements on macroeconomic performance and stability, and discussed the need for cooperative arrangements between the authorities responsible for each instrument.

Woodford (2012) asks to what extent one might tilt monetary policy away from a 'pure' inflation targeting stance because of its potential impact on financial stability. His way of posing the policy issue is one which matches the approach we take. He says:

> The real issue, I would argue, should not be one of controlling the possible mispricing of assets in the marketplace - where the central bank has good reason to doubt whether its own judgments should be more reliable than those of market participants - but rather one of seeking to deter extreme levels of leverage and of maturity transformation in the financial sector. Once the problem is recast in this way, the relevance of interest-rate policy decisions - whether to exacerbate the problem or to mitigate it - is more obvious. Even modest changes in short-term rates can have a significant effect on firms' incentives to seek high degrees of leverage or excessively short-term sources of funding. Again, this is something that we need to understand better than we currently do; acceptance that monetary policy deliberations should take account of the consequences of the policy decision for financial stability will require a sustained research effort, to develop the quantitative models that will be needed as a basis for such a discussion. But there is certainly no ground, on the basis of current economic knowledge, to assert that interest rate policy is likely to be irrelevant.

Our paper develops a simple theoretical framework to model the trade-offs involved in using monetary policy and capital requirements to influence banks' risk taking decisions. We analyse what an optimal combination of capital requirements and monetary policy might look like.

[1] See Borio and Zhu (2012) for a survey of this literature

The model we use is an extension of the framework developed in Bernanke and Gertler (1990) , which focused on financial fragility and economic performance. Their model described an economy with two types of risk-neutral agents: entrepreneurs that have access to risky investment projects; and households from whom the entrepreneurs must borrow in order to fund their investment project. Entrepreneurs must undertake costly screening before they can find out the probability of success of their project, so some with low endowments (and thus high funding requirements) may choose to forego the lending opportunity even before the screening stage. Those that do undertake screening are incentivised to take excessive risks. This is because entrepreneurs enjoy limited liability and cannot credibly reveal their probability of success to their creditors. The extent to which an individual entrepreneur is prone to excessive risk-taking is also exacerbated by lower initial endowments. Bernanke and Gertler concluded that the dependence of aggregate investment on the initial distribution of endowment introduces 'financial fragility'. A 'financially fragile' economy, defined as one with a sizeable proportion of its entrepreneurs operating around the threshold level of endowment between screening and not screening, may experience a dramatic collapse in investment if subjected to a negative shock to endowments.

We make a number of changes to the Bernanke and Gertler (1990) model:

1. *Banks*: We introduce bank intermediation by distinguishing between two types of Bernanke and Gertler 'entrepreneurs': those that can credibly communicate the quality of their assets to creditors and access funding directly (we call these 'firms'); and those that cannot and thus rely on banks. We assume that banks fund projects where it is hard for individual investors to assess their risks. Banks effectively come to own these projects and finance them from internal equity and from debt (and possibly equity) raised from outside investors. We therefore have two types of corporations that invest: (1) 'firms' who fund risky assets that are easy to assess and monitor by creditors; and (2) 'banks' who fund assets that are hard to value and where information is not shared with other investors.
2. *Debt and Equity*: We distinguish between two types of funding contracts: banks in our model can raise funds from households either in the form of debt or in the form of equity. Debt contracts promise to pay the creditor a fixed sum; equity contracts promise a share of the return on the risky project (net of any debt obligations). There may be a non-zero

pay-off for the risky project in the event it fails. Providers of debt have priority claim over this liquidation value of the risky project.

3. *Policy tools*: The policy maker has two levers - monetary policy and regulatory capital requirements - with which to ameliorate the effects of the asymmetry of information in the banking sector that combine with limited liability to make market outcomes inefficient. Monetary policy sets the safe rate paid on the outside option (i.e. a risk-free deposit facility) that is available to all agents. So a tightening of monetary policy can be seen as an increase in the risk-free rate, or a decline in the premium offered by risky bank lending. This safe rate is paid by the central bank, and recouped through taxes set by the fiscal authority. So what we call monetary policy (the setting of a rate of remuneration on deposits at the central bank) has clear fiscal implications. (In fact what we call the monetary policy instrument is equivalent to a decision by the fiscal authority to offer savings accounts to households with a guaranteed interest rate the cost of which are financed out of taxation). Regulatory capital requirements prohibit banks with initial endowments less than the regulatory minimum from proceeding with the risky lending project, unless they raise the additional equity from households. Banks for which the capital regulation is binding will be financed by a mixture of debt and external equity.

We find that in the context of this model monetary policy and prudential capital requirements operate as imperfect substitutes. A tightening of either instruments can improve 'prudence' (by disincentivising the banks against acquiring excessively risky assets with sub-optimally low probability of success); but only at the cost of decreased 'participation' (where decreased 'participation' means that more banks will choose to forego the risky asset even before they discover its probability of success through costly screening). The substitutability between the policy instruments, and this trade-off between 'prudence' and 'participation', implies that the optimal level of the central bank policy rate falls as prudential capital requirements are tightened. We find that, across a broad range of calibrations, using capital requirements is more efficient as a way to get the level of risk-taking right in the economy than using monetary policy (interest rates). But there are situations where using both tools is optimal and some where using only monetary policy is optimal.

The rest of the paper is organised as follows. Section 2 lays out the basic theoretical model for analysing bank risk taking decisions. Section 3 calculates the outcome under the first-best scenario. Section 4 solves the model under asymmetric information. The two policy instruments are introduced in Section 5 where we also describe the main propositions of the model. Section 6 provides quantitative results for numerical simulations of the model. Section 7 concludes with a discussion of policy implications.

2. The Model

The analytical framework we use is a two-sector, two-period, model at the centre of which is the decision of banks to acquire risky assets. Banks are intermediaries that screen risky projects, and then finance these risky assets by raising funds from households. By undertaking costly screening, banks receive private information on the value of the assets they acquire; and by borrowing from households, banks can maximise their returns through leverage. In the down-state, banks that borrow benefit from limited liability. The information asymmetry in the model give rise to moral hazard and the Modigliani-Miller theorem does not hold.

2.1. Agents: The banking sector and the non-financial corporate sector

2.1.1. Banks

On the asset side, the banks in our model have access to a risky asset and a risk-free deposit facility at the central bank.

The risk-free deposit facility remunerates any amount deposited at the risk-free rate $(1 + r)$. This is a purely real model so we should think of r as a safe real rate offered by the central bank and potentially requiring fiscal backing to make good on the promise[2]. This risk-free asset is common to all banks.

Each bank also has access to a risky asset that is unique to them[3]. (We will use the terms "risky asset" and "project" interchangeably). The risky project requires 1 unit of input in the first period. In the second period, it pays out $y_h > 1 + r$ when it succeeds (with probability p),

[2] One could equally think of the policy rate as being directly chosen by the fiscal authorities who offer real government debt to households and finance the real interest costs by levying taxes. In this sense our labeling the use of the policy rate r as monetary policy, rather than debt management or fiscal policy, is somewhat semantic.

[3] We do not think of the risky asset as literally being one single asset. Rather we interpret it as a portfolio of assets which pays out y_h in a good state, and y_l in a bad state. Risky assets differ only in their probability of success p.

and $y_l < 1 + r$ when it fails (with asset-specific probability $(1 - p)$). The probability of success for each project, p, is independently and identically distributed between $[0, 1]$ with probability density function $h(p)$ and cumulative density function $H(p)$.

A key feature of banks is that they can undertake costly screening (fixed cost $C \ll 1$) to discover the realisation of p for their own projects in period 1, before having to commit a full unit of input. We assume that the risky project is not worthwhile in the absence of screening, so $y_l + E[p](y_h - y_l) < 1 + r$.

We allow for a continuum of banks in our model that are heterogeneous in two dimensions. First, they differ in the realisation of their probability of success (p_i) for their risky projects. Second each bank makes a draw of initial endowment ω_i (i.e. initial bank equity). The p.d.f. and c.d.f. of ω are denoted by $f(\omega)$ and $F(\omega)$ respectively, with support $[\omega_{lb}, \omega_{ub}]$, where $0 \leq \omega_{lb} < \omega_{ub} \ll 1$.

Since $\omega_{ub} \ll 1$, all banks must seek funding from households in order to proceed with their project. This external funding can take the form of additional equity $(\tilde{\omega})$ or debt (i.e. deposits from the general public).

Both the distribution of ω and each bank's realisation of ω_i are publicly observed. But whilst the ex ante distribution of p is also common knowledge, each bank's individual realisation of p_i is assumed to be private information. We assume p and ω to be independently distributed.

The assumption that the probability of success of a bank's assets p_i is the private information of the bank is crucial. This captures the idea that banks have assets (projects) where the information gathered in screening is hard to share and where the probability of success cannot be credibly announced. Households depositing in banks know that they don't know much about the quality of the assets.

2.1.2. Firms

The second sector in our model is composed of a continuum of firms, who are identical to banks except in that they do not accept deposits. Thus firms also have access to both the risk-free deposit facility and the risky asset/project. But critically, firms are assumed to be sufficiently transparent that investors know as much about the chances of success (p_i) as the firms themselves know after screening. The fraction of "firms" and "banks" in the population are given by μ and $(1 - \mu)$ respectively.

The existence of asymmetry of information in the banking sector, and the lack of it in the non-financial corporate sector, lies at the core of our model and results. The model captures the idea that banks specialise in acquiring information on the probabilities of good and bad outcomes for the value of the assets they acquire - assets where the assessment of outcomes is time consuming and tricky and where continued monitoring may be required. In many ways that is the fundamental reason why banks, who largely hold non-marketable assets, exist (see Diamond (1984); and Gale and Hellwig (1985)).

2.1.3. Households

Households provide funding to both banks and firms. For simplicity[4], we do not model the households explicitly except that they demand an expected return of at least $(1 + r)$.

All agents in the model are assumed to be risk neutral.

2.2. Timing

The model has two periods. The timing in the first period is sub-divided into three stages:

1. In the first stage of the first period, each firm and bank draws a realisation of their initial endowments, ω_i, from the common distribution for endowments. Given the realisation of ω_i, each then decides whether or not to undertake costly screening to discover the success rate of the risky assets that they can acquire, p_i. Those that do not screen put the entirety of their endowments in the risk-free facility.
2. In the second stage, those firms or banks that did screen discover their probability of success, p_i, and decide whether to proceed with the the acquisition of risky assets. When firms/banks do proceed, they commit the entirety of their initial endowment [5] and begin to seek external funding
3. In the third stage, those firms/banks that are proceeding with their projects decide whether to fund the shortfall through debt, additional equity, or a combination of the two. Any external equity injections take place before the firm/bank seeks debt. By assumption,

[4]Our framework can be extended to a general equilibrium setting where households maximise utility across two periods, and the risk-free interest rate on the deposit facility r adjust endogenously to clear the market for household funds. Such an extension does not affect our key results regarding the behaviour of banks.

[5]Corporations would only proceed with the project if its expected return exceeds the certain return from the deposit facility (recall that all agents are assumed to be risk neutral). Therefore, corporations would prefer to commit the entirety of their endowment rather than to place a portion in the safe deposit facility.

households can observe the capital of the firm/bank ($\omega+\tilde{\omega}$, composed of the initial endowment plus any subsequent equity injections). Firms can also credibly communicate their probability of success p, whereas banks cannot.

- An equity contract takes the form $\left\{\tilde{\omega}, \frac{\tilde{\omega}}{\omega+\tilde{\omega}}\right\}$, where $\tilde{\omega}$ is the size of the equity injection, ω is the firm/bank's own endowment, and $\frac{\tilde{\omega}}{\omega+\tilde{\omega}}$ is the share of net return promised to the external equity providers.
- A debt contract between a household and a bank takes the form $\{R(\omega+\tilde{\omega}, r), (1-\omega-\tilde{\omega})\}$, where $\omega+\tilde{\omega}$ is the total capital of the bank and $(1-\omega-\tilde{\omega})$ is the amount of debt sought. $R(\omega+\tilde{\omega}, r)$ denotes the gross amount (principal plus interest) that is promised on debt. A bank that funds its shortfall only through debt sets $\tilde{\omega}=0$; whereas a bank that uses only equity sets $\tilde{\omega}=1-\omega$. All intermediate cases are permitted.
- A debt contract between a household and a firm takes a similar form, except the terms of the contract is also condition on p.
- All corporations (firms or banks) are protected by limited liability, so when the project fails the period 2 return to debtors is $\min\{y_l, R\}$.

In the second period, the returns from the risky projects are realised and deposits in the safe facility mature, contracts are fulfilled (or defaulted).

2.3. Mechanics

Since there is no asymmetry of information between households and firms, their analysis is relatively straight-forward. Inefficiencies arise in the relationship between households and banks.

Two key thresholds drive the mechanics of the model for banks:

1. The threshold level of a banks' initial endowment, $\hat{\omega}$, at which point it is indifferent between screening and simply depositing its endowments in stage 1 to earn the safe rate; and
2. The threshold level of the success probability of projects, $\hat{p}$, at which point a bank in stage 2 is indifferent between proceeding after screening and depositing at the safe-rate .

The level of bank endowment matters for the screening decision because a lower initial endowment implies a larger funding gap and higher cost of funding. A bank with initial endowment (which is bank equity) less than $\hat{\omega}$ would find that the funding cost, plus the fixed cost of screening, exceed the expected return from acquiring risky assets. The endowment threshold $\hat{\omega}$ is defined formally in Section 3.2**.**

The realisation of a bank's success probability p_i determines whether it proceeds with the project after screening. A bank with $p_i < \hat{p}$ finds that it can achieve a higher expected return by using the risk-free deposit facility, so will forego the lending opportunity. We define $\hat{p}$ more formally in Section 3.1.

For the rest of the paper, we will refer to $\hat{\omega}$ as the '**participation threshold**' of a bank and $\hat{p}$ as the '**prudence threshold**' of a bank. $\hat{\omega}$ is described as the 'participation threshold' because banks with initial endowment (or internal equity) below this level do not 'participate' in acquiring risky assets. $\hat{p}$ is described as the 'prudence threshold' because the success rate banks are willing to accept is an indicator of how prudent they are in handling funds from households. Imprudent banks will tend to take excessive risks to take advantage of their private information and limited liability.

3. The First-Best - the model under perfect information

Under the first-best, households can observe a bank's probability of success, p_i, as well as its level of capitalisation $(\omega_i + \tilde{\omega}_i)$. Therefore there is no information asymmetry, and no distinction between firms and banks. Thus for the rest of this section, we will use the "firms" to encapsulate the behaviour of both firms and banks.

3.1. The Prudence Threshold - First-Best Contracts

Working backwards, we will start by examining the prudence threshold (at Stage 2 of the game) for those firms that have discovered their individual realisation of p_i through screening. The prudence threshold, $\hat{p}$, is defined as the success rate required to make a firm indifferent between undertaking the risky project and simply depositing its endowments in the risk-free facility.

Specifically, for a firm with initial endowment ω_i, and which is looking for $\tilde{\omega}_i$ in additional equity and $1 - (\omega_i + \tilde{\omega}_i)$ in debt from households, its prudence threshold $\hat{p}_i = \hat{p}(\omega_i + \tilde{\omega}_i)$ and the terms of its funding contract are jointly determined by the following three equations[6]:

1. The condition where the firm is indifferent between the risky asset and the safe deposit facility:

$$\frac{\omega}{\omega+\tilde{\omega}}\left(\hat{p}\left(y_h-R\right)+(1-\hat{p})\max\left[0,y_l-R\right]\right)=\omega(1+r);\ \text{or}$$
$$\hat{p}\left(y_h-R\right)+(1-\hat{p})\max\left[0,y_l-R\right]=\left(\omega+\tilde{\omega}\right)(1+r) \tag{3.1}$$

   where $\tilde{\omega}$ is the size of the equity injection from households; $\omega+\tilde{\omega}$ is the total capital of the firm; and $\frac{\omega}{\omega+\tilde{\omega}}$ is the share of the return retained by the firm. A firm with capital of $\omega+\tilde{\omega}$ needs to finance the remainder of the project $(1-\omega-\tilde{\omega})$ through debt. $R(\omega+\tilde{\omega},p)$ denotes the gross amount (principal plus interest) that is promised on that debt.

2. The condition where the households are receiving, in expectation, their required return from providing debt funding to a firm:

$$pR+(1-p)\min\left[y_l,R\right]=\left(1-\left(\omega+\tilde{\omega}\right)\right)(1+r) \tag{3.2}$$

3. The condition where for the households providing additional equity to the firm their expected return from the capital injection is at least equal to the expected return on debt/deposits:

$$\frac{\tilde{\omega}}{\omega+\tilde{\omega}}\left(p\left(y_h-R\right)+(1-p)\max\left[0,y_l-R\right]\right)\geq\tilde{\omega}(1+r)$$
$$\text{or } p\left(y_h-R\right)+(1-p)\max\left[0,y_l-R\right]\geq\left(\omega+\tilde{\omega}\right)(1+r) \tag{3.3}$$

   This last condition is always satisfied given equation 3.1, and the fact that corporations will only proceed with $p\geq\hat{p}_{fb}$, where $\hat{p}_{fb}$ is the first-best level of prudence that reflects risk neutrality of all agents.

Proposition 1. *When p is common knowledge between all agents, firms will not take excessive*

[6]In the interest of brevity, we suppress the i subscript henceforth where possible.

risks, regardless of the level of their capitalisation.

$$\hat{p}(\omega_i + \tilde{\omega}_i) = \frac{(1+r) - y_l}{(y_h - y_l)} \equiv \hat{p}_{fb} \; for \; \forall \left(\omega_i + \tilde{\omega}_i\right) \tag{3.4}$$

Proof. The intuition of the proof is simple. Creditors to the firm can perfectly observe the risks the firm is taking, so they can structure their contracts such that there are no incentives for the firm to engage in excessive risk-taking. More formally: Let $\omega^* (p)$ denote the level of capital $(\omega + \tilde{\omega})$ such that $y_l = R(p, \omega^*)$, so for a firm with $\{p, \omega^* (p)\}$, we can re-arrange equation 3.2 and 3.1 to give:

- $\omega^* (p) = \omega^* = 1 - \frac{y_l}{(1+r)}$ [so ω^* is independent of p]; and
- $R(p, \omega^*) = R(\omega^*) = (1 - \omega^*)(1+r)$ [so $R(\omega^*)$ is independent of p]; and
- $\hat{p}(\omega^*) = \frac{(1+r) - y_l}{(y_h - y_l)} = \hat{p}_{fb}$

We will refer to any firm with $\omega + \tilde{\omega} \geq \omega^*$ as a 'fully capitalised' firm, because such a firm can pay its debt in full - even in the bad state[7]. Consider the case where $\min[y_l, R(p, \omega + \tilde{\omega})] = R(p, \omega + \tilde{\omega})$ (the debt obligation is less than the liquidation value), re-arranging equations 3.1 and 3.2 gives:

$$\begin{aligned} R(p, \omega + \tilde{\omega} \mid y_l > R) &= (1 - (\omega + \tilde{\omega}))(1+r) \\ \hat{p}(\omega_i + \tilde{\omega}_i \mid y_l &> R) = \frac{(1+r) - y_l}{(y_h - y_l)} \end{aligned}$$

Consider the case where $\min[y_l, R(p, \omega + \tilde{\omega})] = y_l$ (the debt obligation is greater than the liquidation value): So from equation 3.2 we have $pR + (1-p) y_l = (1 - (\omega + \tilde{\omega}))(1+r)$, or

$$R(p, \omega + \tilde{\omega} \mid y_l < R) = \frac{(1 - (\omega + \tilde{\omega}))(1+r) - (1-p) y_l}{p}$$

Substitute $R(\hat{p}, \omega + \tilde{\omega} \mid y_l < R)$ into equation 3.1 to give $\hat{p}\left(y_h - \frac{(1-(\omega+\tilde{\omega}))(1+r)-(1-\hat{p})y_l}{\hat{p}}\right) = (\omega + \tilde{\omega})(1+r)$, which simplifies to

$$\hat{p}(\omega + \tilde{\omega} | \; y_l < R) = \frac{(1+r) - y_l}{(y_h - y_l)}$$

[7] These firms can still 'fail' in the sense that their equity holders may be wiped out in the bad state.

Since it is easy to see that R is a decreasing function of $(\omega + \tilde{\omega})$, from the definition of ω^* we can conclude that $y_l < R$ whenever $\omega + \tilde{\omega} < \omega^*$, and $y_l > R$ whenever $\omega + \tilde{\omega} > \omega^*$. So $\hat{p}\left((\omega + \tilde{\omega}) > \omega^*\right) = \hat{p}\left((\omega + \tilde{\omega}) < \omega^*\right) = \hat{p}\left(\omega^*\right) = \frac{(1+r)-y_l}{(y_h-y_l)}$. We define this first-best level of prudence as $\hat{p}_{fb} \equiv \frac{(1+r)-y_l}{(y_h-y_l)}$. □

Proposition 2. *After screening in the first-best:*

1. firms invest in the risky asset if and only if $p \geq \hat{p}(\omega + \tilde{\omega}, r)$; and
2. 'fully-capitalised' firms weakly prefer debt funding to equity funding. 'Not-fully-capitalised' firms are indifferent between using all debt, or using a mixture of debt and additional equity (provided that the additional equity injection does not make the firm 'fully-capitalised'). For simplicity, we assume that when firms are indifferent between debt and equity, they will proceed with the project using debt finance only.

Proof. Part (1) follows from the definition of $\hat{p}(\omega + \tilde{\omega})$: firms with success probability $p < \hat{p}(\omega + \tilde{\omega}, r)$ would receive a higher expected return from the risk-free facility than the risky asset. Part (2) For 'fully-capitalised' firms $(\omega + \tilde{\omega} > \omega^*)$, debt can be obtained at the risk-free rate [see the proof for proposition 1], but equity comes at the cost of sharing the proportion of the expected return - and that expected return exceeds the expected return on debt. We know that a firm will proceed with the project if and only if the expected return is weakly greater than the safe-rate of return (from part (1) and equation 3.1). Thus equity is more expensive than debt for a 'fully-capitalised' firm. For 'not-fully capitalised' firms, households with perfect information set debt contracts as a function of both p and ω. Additional equity injections lowers the cost on the bank's entire stock of debt, so equity becomes just as attractive as debt [the Modigliani-Miller Theorem holds]. We offer a more formal proof of this proposition in the Annex. □

Corollary 1. *In the absence of capital regulations, firms will not seek external equity injections when they undertake risky lending.*

3.2. The Participation Threshold - First Best Case

Having established the 'prudence threshold' of firms, we continue to work backwards to find firms' 'participation threshold'. Upon discovering its realisation of ω_i at Stage 1 of the game, a firm decides whether to engage in costly screening. The 'participation threshold' of firms is defined as the level of initial endowment that equates the expected value of screening (V) to the fixed cost of screening (C). The costly screening process can be interpreted as buying an option on the risky asset. The value of the option will depend on the funding structure of the firm, in particular how much additional equity it is trying to attract from households.

Formally, V is given by:

$$
\begin{aligned}
& V(\omega, \tilde{\omega}, r) \\
& \equiv E_p \left[\max \left\{ 0 \ , \ \frac{\omega}{\omega + \tilde{\omega}} \begin{pmatrix} p\,(y_h - R) + \\ (1-p) \max\,[0, y_l - R] \end{pmatrix} - \ \omega(1+r) \right\} \right] \\
& = \frac{\omega}{\omega + \tilde{\omega}} \int_{\hat{p}}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p) dp \qquad (3.5)
\end{aligned}
$$

where the last line follows because the debt is provided by households at an expected return of $1 + r$ (more detail can be found in the annex).

Definition. The participation threshold for firms, $\hat{\omega}$, is implicitly defined by:

$$
V(\hat{\omega}, \tilde{\omega}, r) = \frac{\hat{\omega}}{\hat{\omega} + \tilde{\omega}} \int_{\hat{p}}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p) dp = C \qquad (3.6)
$$

where C is the fixed cost of screening.

We know that in the absence of any capital requirements ($\omega_{reg} = 0$), $\tilde{\omega} = 0$; and in the first-best $\hat{p}(\omega + \tilde{\omega}) = \hat{p}_{fb} = \frac{1+r-y_l}{(y_h - y_l)}$, so

$$
V_{fb}(\omega_{reg} = 0, r) = \int_{\hat{p}_{fb}}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p) dp \qquad (3.7)
$$

We impose by assumption that: $\int_{\hat{p}_{fb}}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p) dp > C$, so that

$$
\hat{\omega}_{fb}\,(\omega_{reg} = 0, r) = 0 \qquad (3.8)
$$

In the first-best, with no asymmetry of information between households and firms, every firm screens in the absence of any policy intervention. The aggregate value-added of firms (and banks) is given by:

$$V_{fb} = \int_{\hat{p}_{fb}}^{1} \left[y_l + p\left(y_h - y_l\right) - (1+r)\right] h\left(p\right) dp - C \tag{3.9}$$

3.3. Summary

In the first-best households can observe the probability of success (p_i). This is sufficient to ensure that firms (and banks) operate in the most appropriate and prudent manner, regardless of the level of their capitalisation/leverage. Specifically, every firm sets the prudence threshold at $\hat{p}_{fb} = \frac{(1+r)-y_l}{(y_h-y_l)}$, the level which ensures that every risky project is expected to deliver at least the risk-free return. In addition, with the participation threshold ($\hat{\omega}$) equal to zero, all firms screen so no profitable lending opportunities are wasted. In such a state, there is no need for any policy intervention.

4. The model with asymmetry of information

Frictions in the model arise because of the interaction between asymmetric information and limited liability. When households cannot observe banks' probability of success (p_i), moral hazard becomes prevalent in the banking sector. This problem manifests itself in a sub-optimal level of screening for the risky asset; and for those banks that do screen, a lower level of prudence in deciding whether to invest in the risky asset.

Analytically, the structure of the model remains unchanged under asymmetric information (the majority of the equations are identical). The main distinction is that for banks $\hat{p}$ becomes a function of ω, and therefore can no longer be evaluated outside of the integral over ω. The analysis for firms is identical to the first-best case.

4.1. The Prudence Threshold

As with the first-best, we start by examining the prudence threshold (at Stage 2 of the game) for those banks that have discovered their individual realisation of p_i through screening. Under asymmetric information, the analogues of equations 3.1-3.4 are:

1. The condition that a bank with initial capital ω and additional equity injection $\tilde{\omega}$ is indif-

ferent between the risky asset and the risk-free asset:

$$\frac{\omega}{\omega+\tilde{\omega}}\left\{\hat{p}\left(y_h-R\right)+\left(1-\hat{p}\right)\max\left[0,y_l-R\right]\right\}=\omega(1+r)$$

or:

$$\hat{p}\left(y_h-R\right)+\left(1-\hat{p}\right)\max\left[0,y_l-R\right]=\left(\omega+\tilde{\omega}\right)\left(1+r\right) \tag{4.1}$$

this is identical to equation 3.1.

2. The condition that the households are indifferent between providing debt and the risk-free facility:

$$A\left(\hat{p}\right)R+\left(1-A\left(\hat{p}\right)\right)\left(\min\left[y_l,R\right]\right)=\left(1-\omega-\tilde{\omega}\right)\left(1+r\right) \tag{4.2}$$

where $A\left(\hat{p}\right)\equiv E[p\mid p>\hat{p}]\equiv\frac{1}{1-H(\hat{p})}\int_{\hat{p}}^{1}ph\left(p\right)dp$ denotes the conditional expectation of p given that $p\geq\hat{p}$.

Note that the key difference here (relative to equation 3.2) is that instead of observing the p directly, households now need to form a conditional expectation of p.

3. The condition that for households providing additional equity to the bank their expected return from the capital injection is at least equal to the safe rate of return:

$$\frac{\tilde{\omega}}{\omega+\tilde{\omega}}\left\{A\left(\hat{p}\right)\left(y_h-R\right)+\left(1-A\left(\hat{p}\right)\right)\left(\max\left[0,y_l-R\right]\right)\right\}\geq\tilde{\omega}\left(1+r\right)$$

or:

$$A\left(\hat{p}\right)\left(y_h-R\right)+\left(1-A\left(\hat{p}\right)\right)\left(\max\left[0,y_l-R\right]\right)\geq\left(\omega+\tilde{\omega}\right)\left(1+r\right) \tag{4.3}$$

This last condition is always satisfied given equation 4.1 and the fact that $A\left(\hat{p}\right)\geq\hat{p}$.

Proposition 3. *With asymmetric information, banks that are not 'fully capitalised' will take excessive risks:*

1. *As before, let $\omega^*=\omega+\tilde{\omega}$ denote the level of capital such that $R\left(\omega^*\right)=y_l$, so $\omega^*=1-\frac{y_l}{(1+r)}$. We refer to any bank with $\omega_i+\tilde{\omega}_i\geq\omega^*$ as a "fully capitalised" bank.*
2. *Banks become more prudent as they increase their capital, up to a cap when they become fully capitalised.*
   (a) *For $\omega+\tilde{\omega}\geq\omega^*$, we have $\hat{p}=\frac{(1+r)-y_l}{(y_h-y_l)}$; $R=\left(1-\left(\omega+\tilde{\omega}\right)\right)\left(1+r\right)$;*

(b) *For* $\omega+\tilde{\omega}<\omega^*$*, we have* $\frac{\partial\hat{p}}{\partial\omega}=\frac{(A(\hat{p})-\hat{p})(1+r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)}$*; and* $\hat{p}\leq\frac{(1+r)-y_l}{(y_h-y_l)}=\hat{p}_{fb}$.

The function for $\hat{p}(\omega+\tilde{\omega})$ *kinks at* ω^**, with right-hand derivative given by* $\partial_+\hat{p}(\omega^*)=0$ *and left-hand derivative given by* $\partial_-\hat{p}(\omega^*)=\frac{(A(\hat{p})-\hat{p})(1+r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)}>0$.

An increase in initial endowment has the same marginal impact on prudence as an increase in external equity. Banks behaviour is influenced by the size of their equity rather than its source.

Proof. We give an outline here. A full proof is in the annex. In the first part of the proposition, ω^* is defined as the level of endowments such that $y_l=R(\omega^*,r)$, so $\omega^*=1-\frac{y_l}{(1+r)}$ follows directly from re-arranging equation 4.2. Banks with capital above this level can borrow at the risk free rate because the liquidation value of their risky assets is sufficient to cover the bank's debt obligations (again from re-arranging equation 4.2 after setting $R(\omega^*)=y_l$). The second part of the proposition says that banks that are not fully capitalised will take excessive risks. Limited liability encourages banks to take excessive risks when the liquidation value of the assets is less than the bank's debt obligations. Households can anticipate this, so will increase the cost of debt $\frac{R(\omega+\tilde{\omega},r)}{(1-\omega-\tilde{\omega})}$ for banks with small capital (and thus high funding needs). The fact that banks cannot credibly communicate their success rate (p_i) to households exacerbates this issue as the debt contract can only be formulated as a function of the bank's observable capital ($\omega+\tilde{\omega}$). This means two banks with the same level of capital will face the same borrowing costs regardless of the success probabilities on their assets - the bank with the better project is effectively subsidising the borrowing costs of the one with the worse asset. Taken together, limited liability and this cross-subsidisation towards less attractive assets give rise to moral hazard in bank lending. The fact that prudence is an increasing function of capital reflects the observation that the moral hazard issue becomes less pronounced when banks have more 'skin in the game'. □

Proposition 4. *Preference for Debt*

1. *After screening banks invest in the risky asset if and only if* $p\geq\hat{p}(\omega+\tilde{\omega},r)$*; and*
2. *when banks acquire risky assets they prefer to use as much debt finance as possible (i.e. banks will choose to set* $\tilde{\omega}=0$*)* .

Proof. Part (1) follows from the definition of $\hat{p}(\omega+\tilde{\omega})$: banks with success probability $p<\hat{p}(\omega+\tilde{\omega})$ would receive a higher expected return from investing in the safe asset. Part (2) holds because banks expect to pay out a higher rate of return to external equity providers than to debt providers. We show this formally in the annex. Qualitatively, equity is more costly in this model because debt is available at a rate which makes households indifferent (in expectation) between providing the debt and using the safe deposit facility. Equity, on the other hand, allows the household to share the surplus from banks acquiring risky assets, and therefore delivers a rate of return for households that is greater than the risk-free facility. The original owners of bank equity have the value of their claims diluted by raising new equity from outsiders. Consequently banks stick with debt in the absence of any policy intervention, because this is the cheapest way for them to raise funds at the margin. This reflects an important assumption that banks with access to the potential to screen risky assets and find out their chance of success have a valuable option which is not competed away by free entry of new banks. □

Corollary 2. *In the absence of capital regulations, banks will not seek external equity injections when they acquire risky assets.*

4.2. The Participation Threshold

As in the first-best case, the value of screening for banks is still given by equation 3.5:

$$
\begin{aligned}
& V(\omega, \tilde{\omega}, r+\delta_r) \\
& \equiv E_p\left[\max\left\{0\ ,\ \frac{\omega}{\omega+\tilde{\omega}}\left(\begin{array}{c} p\left(y_h-R\right)+ \\ (1-p)\max\left[0, y_l-R\right] \end{array}\right)-\ \omega(1+r)\right\}\right] \\
& =\frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}(\omega+\tilde{\omega})}^{1}\left[p\left(y_h-y_l\right)+y_l-(1+r)\right]h(p)dp
\end{aligned}
\tag{4.4}
$$

And the participation threshold $\hat{\omega}$ is still given implicitly by equation 3.6:

$$
\frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}}\int_{\hat{p}(\omega+\tilde{\omega})}^{1}\left[p\left(y_h-y_l\right)+y_l-(1+r)\right]h(p)dp=C \tag{4.5}
$$

with the only difference that $\hat{p}$ is now a function of $\omega+\tilde{\omega}$. Since $\hat{p}\left(\omega+\tilde{\omega}\right)\leq\hat{p}_{fb}$, this means that even when $\tilde{\omega}=0$, the participation threshold under asymmetric information will in general be a positive number.

Proposition 5. $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial\omega}\begin{cases} >0 \text{ for } \omega<\omega^* \\ =0 \text{ for } \omega\geq\omega^* \end{cases}$. *In the absence of capital regulations (and* $\tilde{\omega}=$

0), the expected value of screening is increasing in the bank's initial endowments (up to a cap when the bank becomes fully well capitalised). (see proof in the annex)

Since we have established through corollary 2 that banks will only use debt funding in the absence of capital regulations, for now we can only concern ourselves with the case where $\tilde{\omega} = 0$.

The equations for the rest of the model (market for funds, households and bank optimisation) are unchanged from the generalised forms presented in the first-best case.

4.3. Summary and Implications

The presence of moral hazard in the model give rise to sub-optimal outcomes compared to the first-best scenario. Banks have no incentives to top-up their capital. Consequently, the hurdle for participation, $\hat{\omega} > 0$, is too high; and the level of prudence, $\hat{p}(\omega, r) \leq \frac{(1+r)-y_l}{(y_h-y_l)} = \hat{p}_{fb}$ is too low. As a result aggregate value added of the banking sector is lower than it is in the first-best (where we assumed that p is common knowledge).

5. Policy tools and their transmission

Monetary policy and capital regulations can be used to drive outcomes closer to the first-best level in the banking sector. Both policy tools function through their effects on the participation and prudence thresholds.

5.1. Monetary Policy

We introduce monetary policy by allowing the central bank set the risk-free rate of return on the deposit facility (r). The stance of monetary policy therefore affects the attractiveness of the risky asset relative to the risk-free deposit. The central bank remunerates all resources placed in its deposit facilities at the rate r, the cost of which is covered by lump sum taxation of households. The monetary policy stance (r) is revealed to all agents at the very beginning of the game.

We normalise $r = 0$ as the neutral stance of monetary policy. In the first-best scenario, there is no need for monetary policy intervention, so $r = 0$ and the first-best level of prudence becomes:

$$\hat{p}_{fb} = \frac{1 - y_l}{(y_h - y_l)} \tag{5.1}$$

The first-best level of the participation threshold $\hat{\omega}_{fb}(\omega_{reg} = 0) = 0$ remains the same as before (see equation 3.8)

Note that the possibility of monetary policy intervention introduces a wedge between the socially optimal outcome and what is privately optimal in the absence of asymmetric information. In particular, the privately optimal level of prudence is given by

$$\hat{p}^*(r) = \frac{1 + r - y_l}{(y_h - y_l)} \tag{5.2}$$

which will be different to $\hat{p}_{fb}$ (the socially optimal level of prudence) for non-zero r. So using monetary policy to combat the inefficiencies of the banking sector will lead to distortions in the non-bank sector (i.e. the risk-taking behaviour of "firms").

The effect of monetary policy on banks' behaviour is summarised in the proposition below.

Proposition 6. *Monetary policy tightening improves 'prudence' at the cost of decreasing 'participation' for banks. Monetary policy loosening achieves the converse:*

1. $\frac{\partial \hat{p}}{\partial r} > 0$*; and*
2. $\frac{\partial \hat{\omega}}{\partial r} > 0$

Proof. We describe the intuition for the proposition here; formal proof is in the annex. Monetary policy tightening improves prudence for all firms and banks by increasing the attractiveness of the outside option. A quick way to illustrate the result $\frac{\partial \hat{p}}{\partial r} > 0$ is by appealing to proposition 3: $\hat{p}(\omega + \tilde{\omega}) \leq \hat{p}_{fb}$. Recall from equation 5.1: $\hat{p}_{fb} = \frac{1-y_l}{(y_h-y_l)}$; and from Proposition 3: $\hat{p}(\omega + \tilde{\omega}, r) \leq \frac{1+r-y_l}{(y_h-y_l)} = \hat{p}^*(r)$. So by increasing r, a central bank can increase $\hat{p}^*(r)$ and thus push $\hat{p}(\omega + \tilde{\omega}, r)$ closer to $\hat{p}_{fb}$ for all ω. Monetary policy tightening decreases the level of participation because it increases the relative attractiveness of the outside option. Recall that $\hat{\omega}(\tilde{\omega}, r)$ is implicitly defined by

$$V(\hat{\omega}, \tilde{\omega}, r) = \frac{\hat{\omega}}{\hat{\omega} + \tilde{\omega}} \int_{\hat{p}(\hat{\omega}+\tilde{\omega},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - (1 + r)\right] h(p)dp = C$$

Increasing r reduces the value of screening, so for a given fixed cost of screening C, banks need a higher amount of initial capital to make the screening process worthwhile. □

5.2. Prudential Policy

We model capital regulations in the simple form of a leverage ratio. Regulators impose a minimum capital requirement ω_{reg} such that banks with endowments $\omega_i < \omega_{reg}$ must seek outside equity of at least $(\omega_{reg} - \omega_i)$ or are barred from investing in the risky asset. We rule out the possibility that the regulator can set a capital requirement that depends on the riskiness of bank assets by assuming the asset specific chance of success p_i remains private bank information.

The capital requirement ω_{reg} is announced as soon as banks find out their individual realisations of ω_i. A strengthening of capital requirements achieves the same qualitative effect as a tightening of monetary policy on banks' decisions, albeit through different means. Critically, capital regulations can be imposed on banks alone, without affecting the behaviour of firms. This is an important difference between the blunt instrument of monetary policy (one "that gets in all the cracks") and the targeted instrument of bank capital requirements.

Proposition 7. *Capital regulations improve prudence by forcing some banks to seek additional equity funding, but do so at the cost of decreased participation:*

1. $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$. *Only banks that are constrained by the capital regulations will seek equity injections from households. And when they do, they will only top-up to the minimum capital standard.*
2. $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} \geq 0$ *with inequality as long as* $\hat{\omega}(\tilde{\omega}=0,r) < \omega_{reg} < \omega^*$. *A capital requirement will improve the prudence level of banks for which it binds, so long as they are not already 'fully capitalised'.*
3. $\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r) > \hat{\omega}(\tilde{\omega}=0,r)$ *for all* ω_{reg} *such that* $\omega_{reg} > \hat{\omega}(\tilde{\omega}=0,r)$. *The participation threshold is higher for banks that require external equity injections (i.e. banks that falls short of the capital requirement), than for banks that are not bound by the capital requirement.*
4. $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$. *A tightening of capital standards increases the threshold for participation.*

Proof. We outline the proof here, details are in the annex.

1. $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$ follows directly from the observation that banks weakly prefer debt to equity (proposition 4). Consequently, banks for which the regulations are binding will

only top-up their capital to the regulatory minimum, and seek the remaining funds in the form of debt.

2. $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} \geq 0$ is a corollary of proposition 3 ($\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \tilde{\omega}} \geq 0$) and the previous observation that $\tilde{\omega} = \max\left[0, \omega_{reg} - \omega\right]$. In particular, $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} = 0$ for banks with $\omega_i > \omega_{reg}$ or $\omega_i \geq \omega^*$. In the former case, banks with $\omega_i > \omega_{reg}$ are not in violation of the capital requirement, and thus will remain unaffected by a marginal increase in ω_{reg}. In the latter case, banks with $\omega_i \geq \omega^*$ already operate at the optimal level of prudence, so pushing ω_{reg} above ω^* does not bring any further gains in prudence.

3. $\hat{\omega}\left(\omega_{reg} - \omega, r\right) > \hat{\omega}\left(0, r\right)$ holds because equity funding is more costly than debt funding due to the dilution effect (proposition 4). Therefore, the value of screening is lower for banks that are constrained by the capital requirement. (ω_{reg} is announced in Stage 1, so banks can anticipate capital needs in advance of screening.) Consequently, the presence of binding capital regulations increases the participation threshold for a given fixed cost of screening.

4. $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$ follows from part 1 and 3. Higher capital requirements increase the size of equity injection required, and thus reduce the value of screening and increase the threshold for participation.

□

5.3. Summary of Policy Implications

A tightening of monetary policy – an increase in the risk-free interest rate - increases the opportunity cost of funding corporations' acquisition of risky assets. This pushes up prudence for all banks at all levels of initial endowments. So poorly endowed (or highly leveraged) banks become more prudent, but firms which do not suffer asymmetric information will become too cautious[8]. A strengthening of bank capital standards forces more banks to seek out more external equity, inducing them to behave as if they are a better endowed bank.

The two policy instruments will also affect participation. Because external equity is more costly than debt, tougher capital requirements make the break-even level of initial endowment

[8]Very well capitalised banks will also become too cautious. But this side effect falls away if we restrict the upper support of ω to a value significantly below 1.

for banks higher for a given fixed cost of screening. So more banks will drop out from screening due to a low level of initial endowments. A tightening of monetary policy shares the same drawback.

So the trade-off between 'prudence' and 'participation' is common across both monetary policy and capital regulations. But there are two important differences. First, monetary policy distorts incentives for all firms and banks; whereas capital regulations only affect those banks for which it is binding (i.e. poorly capitalised banks). Second, a loosening in the central bank policy rate (i.e. an negative r) can increase participation of banks beyond the level possible in the absence of any intervention (and thus gets closer to the state of universal participation under the first-best). In contrast, capital regulations can be at best non-binding. This means capital requirements alone can never correct the participation distortions relative to the first best.

5.4. Aggregate Economic Variables

5.4.1. Expected bank investment

In the presence of a binding regulatory regime (where $\omega_{reg} > \hat{\omega}(\omega_{reg} r) > \hat{\omega}(0, r)$), the expected amount of investment by banks in the risky asset is given by:

$$I(r, \omega_{reg}) = (1-\mu)\left[\begin{array}{c} \int_{\hat{\omega}(\omega_{reg}, r)}^{\min[\omega_{reg}, \omega_{ub}]} (1 - H(\hat{p}(\omega_{reg}, r)))\, dF(\omega) + \\ \int_{\min[\omega_{reg}, \omega_{ub}]}^{\omega_{ub}} (1 - H(\hat{p}(\omega, r)))\, dF(\omega) \end{array}\right] \tag{5.3}$$

where:

- ω_{ub} is the upper bound for the distribution of ω
- $\int_{\hat{\omega}(\omega_{reg}, r)}^{\min[\omega_{reg}, \omega_{ub}]} (1 - H(\hat{p}(\omega_{reg}, r)))\, dF(\omega)$ is the amount of lending expected from banks whose initial endowment fell short of the capital regulations (and hence need to raise additional equity); and
- $\int_{\min[\omega_{reg}, \omega_{ub}]}^{\omega_{ub}} (1 - H(\hat{p}(\omega, r)))\, dF(\omega)$ is the amount of lending expected from banks that are not affected by the capital regulations (and hence can finance itself with debt and initial endowments only).

In the absence of prudential regulation (or if the regulation is completely non-binding: $\omega_{reg} \leq \hat{\omega}(0,r)$), the expected level of bank investment is given by:

$$I(r) = (1-\mu)\int_{\hat{\omega}(0,r)}^{\omega_{ub}} (1 - H(\hat{p}(\omega, r)))\, dF(\omega) \tag{5.4}$$

The expected level of bank investment in the first-best is:

$$I_{fb} = (1-\mu)(1 - H(\hat{p}_{fb})) \quad \text{where } \hat{p}_{fb} = \frac{1-y_l}{y_h - y_l} \tag{5.5}$$

A comparison of these equations shows that whilst both monetary policy and capital requirements can be used to push up the prudence threshold of banks to a level that is closer to the first best, this can only be achieved with a decline in the funding of risky assets. Part of the fall in the investment in risky assets is socially desirable - banks with low p_i should rightly give up their risky asset after screening. What is not desirable is the decline in bank acquisition of risky assets due to falling participation. When capital requirements and the stance of monetary policy are strict, more banks may choose to pass up the chance to fund risky investments even before the screening stage. This fall in participation is sub-optimal because ω and p are assumed to be independently distributed. Banks with low initial endowments (low ω_i) might be disincentivised from screening by the regulatory environment, even though they might discover very good assets (with high p_i) if they screen.

5.4.2. Expected value added

Under a binding regulatory regime, where $\omega_{reg} > \hat{\omega}(\omega_{reg} r) > \hat{\omega}(0,r)$), the expected amount of value added from firms and banks is given by:

$$\begin{aligned} V(\omega_{reg}, r) \;=\; & \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} \left[\int_{p^*(r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) - C \right] dF(\omega) \\ & + (1-\mu) \int_{\hat{\omega}(\omega_{reg},r)}^{\min[\omega_{reg},\omega_{ub}]} \left[\int_{\hat{p}(\omega,r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) \right] dF(\omega) \\ & + (1-\mu) \int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}} \left[\int_{\hat{p}(\omega,r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) \right] dF(\omega) \\ & - (1-\mu)\, C \left[\int_{\hat{\omega}(\omega_{reg},r)}^{\omega_{ub}} dF(\omega) \right] \end{aligned} \tag{5.6}$$

where $V(\omega_{reg}, r)$ is composed of:

- The value-added of firms $\mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} \left[\int_{p^*(r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) - C\right] dF(\omega)$, where $\hat{p}^*(r) = \frac{1+r-y_l}{(y_h-y_l)}$.

- The value-added of banks that are constrained by the capital requirement:

$$(1-\mu)\int_{\hat{\omega}(\omega_{reg},r)}^{\min[\omega_{reg},\omega_{ub}]} \left[\int_{\hat{p}(\omega,r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p)\right] dF(\omega)$$

- The value-added of banks that are not constrained by the capital requirement:

$$(1-\mu)\int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}} \left[\int_{\hat{p}(\omega,r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p)\right] dF(\omega)$$

- Minus the cost of screening by all banks: $(1-\mu)\, C \left[\int_{\hat{\omega}(\omega_{reg},r)}^{\omega_{ub}} dF(\omega)\right]$

In the absence of capital regulations (or when it is entirely non-binding), the expected aggregate value-added is given by:

$$\begin{aligned} V(r) &= \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} \left[\int_{p^*(r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) - C\right] dF(\omega) \\ &\quad + (1-\mu)\int_{\hat{\omega}(0,r)}^{\omega_{ub}} \left[\int_{\hat{p}(\omega,r)}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) - C\right] dF(\omega) \end{aligned}$$

Lastly, recall that the aggregate value added in the first-best is given by:

$$V_{fb} = \int_{\hat{p}_{fb}}^{1} [y_l + p(y_h - y_l) - 1]\, dH(p) - C \qquad (5.7)$$

where $\hat{p}_{fb} = \frac{1-y_l}{y_h-y_l}$.

In the following section, we treat the expected level of aggregate value added $V(\omega_{reg}, r)$ as the objective function for policymakers (which makes sense given the assumption of risk neutrality). This allows us to employ numerical techniques to calculate the optimal setting of monetary policy and capital requirements under illustrative parameterisations of the model.

6. Numerical Simulations

We use numerical simulations to illustrate the trade-off between monetary policy and capital requirements. The model we have outlined above is necessarily a significant simplification of the financial intermediation process. A precise calibration of the model is not straight-forward because there are no direct empirical counterparts to some of the key parameters of the model, specifically the pair of binary outcomes(y_h, y_l) for the risky asset. We chose a wide range of parameter combinations which cover high and low risk investment environments. And we rely on robustness tests to demonstrate that our main qualitative results are sound, namely that: (1) monetary policy and capital regulations function as imperfect substitutes; (2) these policy interventions can trade-off between 'prudence' and 'participation' of banks; and (3) across a wide, plausible, range of calibrations, the optimum setting of policy is one where capital regulations plays the main role in addressing the asymmetric information distortions in the banking sector.

We allow the parameters (y_h, y_l) to vary between permissible values in the model[9], whilst making some assumptions on the rest. Specifically, we assume that the probability of success, p, is uniformly distributed between $[0, 1]$; initial endowments ω are uniformly distributed between $[0, 0.1]$[10]; and the cost of screening C is 1%. In our model, μ represents the proportion of initial endowments in the economy that are available to firms rather than banks. We set μ to 0.85, to reflect the fact that financial companies account for roughly 15% of the capitalisation of major stock indices in the UK and US.

[9]As part of the theoretical model, we impose two conditions on the permissible levels for (y_h, y_l):

1. The risky asset must not be so attractive that banks will always proceed without screening:

$$y_l + E[p](y_h - y_l) < 1$$

2. The risky asset must be attractive enough such that all banks will screen in the first-best:

$$\int_{\hat{p}_{fb}}^{1} [p(y_h - y_l) + y_l - 1]\, h(p)\, dp > C$$

where $\hat{p}_{fb} = \frac{1-y_l}{(y_h-y_l)}$.

In the numerical simulation, we also required that the risky asset must be attractive enough such that at least some banks will screen under asymmetric information when there is no policy intervention: $\hat{\omega}(r = 0, \omega_{reg} = 0) < \omega^{ub}$. This additional restriction ruled out the combination $(y_h = 1.1, y_l = 0.6)$ in table 6.1.

[10]Note that because the prudence threshold of firms is independent of ω, and that firms are essentially indifferent between debt and equity funding, the results of the model is invariant to the distribution of firm's initial endowments. So for simplicity, we assume that banks and firms share the same distribution for ω. In the Appendix, we show that when we extend the model to allow for taxes (that have debt deductions), the distribution of endowments for firms does matter.

The range of combinations of y_h and y_l that we consider runs from very low to very high risk worlds. A world in which y_h is as low as 1.1 and y_l as high as 0.8 is one in which at the first best losses on those investments which fail are only 3% of total investment. But the expected return on investments are also quite low at 5%. At the other extreme when y_h is 1.9 and $y_l = 0$ optimal investment implies that losses from projects failing are around 24% of total investment, but the expected return is over 45%.

Table 6.1 presents the optimal policy combinations when we allow (y_h, y_l) to vary. The figures in curved brackets for each combination are the optimal interest rate and the capital requirement. The figure in square brackets is the value added (as per equation 5.6, which can also be interpreted as a weighted average rate of return on all savings invested in firms, banks and the safe deposit facility). The results here shows that, across these wide-ranging parameterisations, capital regulations are generally the main tool policy makers should use in addressing banking inefficiencies. Only in the safest investment environment ($y_h = 1.1;\ y_l = 0.8$) should monetary policy be used on its own, and then interest rates should be set to 0.5 percentage points above a neutral level. In most cases capital requirements are substantial - ranging from around 10% to up to 30%. In the most risky environment ($y_h = 1.9;\ y_l = 0.9$), where incentives for banks to raise too much debt and invest too much are at their greatest, both monetary policy and capital requirements are very tight; interest rates are 2.5% above neutral and capital should be 30% of total bank assets.

Table 6.1: Optimal Policy Setting across (y_h, y_l) pairs

Optimal Policy (r %, ω_{reg}%); [VA%]		Good state payoff (y_h)				
		1.1	1.2	1.3	1.4	1.5
Bad state payoff (y_l)	0.0	-	(0.0, 0.0); [0.57]	(0.0, 15.0); [2.12]	(0.5, 20.0); [4.20]	(0.5, 0.25); [6.71]
	0.1	-	(0.0, 0.0); [0.70]	(0.0, 15.0); [2.39]	(0.5, 20.0); [4.65]	(0.5, 0.25); [7.33]
	0.2	-	(0.0, 0.0); [0.85]	(0.5, 15.0); [2.73]	(0.5, 20.0); [5.18]	(0.5, 0.25); [8.05]
	0.3	-	(0.0, 10.0); [1.04]	(0.0, 17.5); [3.14]	(0.5, 20.0); [5.81]	(0.5, 0.25); [8.89]
	0.4	-	(0.0, 12.5); [1.30]	(0.0, 17.5); [3.65]	(0.5, 20.0); [6.57]	(0.5, 0.225); [9.88]
	0.5	-	(0.0, 12.5); [1.64]	(0. 0,17.5); [4.30]	(0.5, 17.5); [7.50]	-
	0.6	-	(0.0, 12.5); [2.12]	(0.0, 15.0); [5.14]	-	-
	0.7	(0.0, 0.0); [0.21]	(0.0, 12.5); [2.81]	-	-	-
	0.8	(0.5, 0.0); [0.60]	-	-	-	-

Optimal Policy (r %, ω_{reg}%); [VA%]		Good state payoff (y_h)				
		1.6	1.7	1.8	1.9	2.0
Bad state payoff (y_l)	0.0	(0.5, 30.0); [9.55]	(1.5, 30.0); [12.65]	(2.0, 30.0); [15.96]	(2.5, 30.0); [19.46]	-
	0.1	(0.5, 30.0); [10.33]	(1.0, 30.0); [13.60]	(1.5, 30.0); [17.07]	(2.0, 30.0); [20.71]	-
	0.2	(0.5, 27.5); [11.23]	(1.0, 30.0); [14.67]	-	-	-
	0.3	(0.5, 27.5); [12.27]	-	-	-	-
	0.4	-	-	-	-	-
	0.5	-	-	-	-	-
	0.6	-	-	-	-	-
	0.7	-	-	-	-	-
	0.8	-	-	-	-	-

In order to examine the trade-offs between monetary policy and capital regulations, and to demonstrate the more detailed implications of the model, we take a specific illustrative parameterisation (see Table 6.2) and present the associated results. We have chosen the pair $(y_h, y_l) = (1.4, 0.5)$ for this illustrative example. This combination generates first-best outcomes where about 20% of those assets acquired (after screening) become non-performing (and in this low state, the loss of value is such that the residual worth of the asset is only one half of the investment). This scale of risk and of losses is not implausible for loans to finance corporate activity - particularly of small businesses that rely on bank intermediation. Given the losses on sub-prime mortgage lending that was behind the financial crisis of 2007-08, this scale of risk may not be excessive even for lending backed by residential property which used to be considered relatively safe. Our aim in choosing these values is not however an attempt to closely match returns on past investment, but rather to provide a numerical illustration of outcomes that are broadly indicative of the levels of risk.

Table 6.2: Illustrative Baseline Parameterisation

Parameters	Illus. Calibration
Good state payoff on risky asset (y_h)	1.4
Bad state payoff on risky asset (y_l)	0.5
Success prob. of risky asset (p)	$p \sim U\,[0,1]$
Cost of screening (C)	0.01
Proportion of firms relative to banks (μ)	0.85
Initial endowments for banks and firms (ω)	$\omega \sim U\,[0,0.1]$

With the set of parameters in Table 6.2, we compute the aggregate value-added of firms and banks for different settings of monetary policy and regulatory capital requirements. Table 6.3 present the results for ω_{reg} between 0% and 30% (step size of 2.5 percentage points), and r between -2% and 10% (step size of 50 basis points).

Table 6.3: Aggregate value added under selected policy combinations

VA (%)		ω_{reg} (%)												
		0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0
	-2.0	7.13	7.16	7.23	7.29	7.36	7.41	7.44	7.46	7.47	*7.48*	7.47	7.46	7.45
	-1.5	7.16	7.20	7.26	7.32	7.38	7.43	7.46	7.48	7.49	*7.49*	7.48	7.47	7.46
	-1.0	7.20	7.23	7.28	7.34	7.40	7.44	7.47	7.49	*7.49*	7.49	7.48	**7.47**	**7.46**
	-0.5	7.23	7.25	7.30	7.36	7.42	7.46	7.48	7.50	*7.50*	**7.49**	**7.49**	7.47	7.45
	0.0	7.26	7.28	7.33	7.38	7.43	7.47	7.49	7.50	*7.50*	7.49	7.48	7.47	7.45
	0.5	7.28	7.30	7.34	7.39	7.44	7.47	7.49	***7.50***	7.50	7.49	7.48	7.46	7.44
	1.0	7.30	7.32	7.36	7.40	7.45	7.48	**7.49**	*7.50*	7.50	7.49	7.47	7.45	7.43
	1.5	7.32	7.33	7.37	7.41	7.45	**7.48**	7.49	*7.49*	7.49	7.48	7.46	7.44	7.42
	2.0	7.34	7.35	7.38	7.41	**7.45**	7.48	*7.49*	7.49	7.48	7.46	7.45	7.42	7.40
	2.5	7.35	7.36	7.38	**7.42**	7.45	7.47	*7.48*	7.47	7.46	7.45	7.43	7.40	7.38
	3.0	7.36	7.36	**7.39**	7.42	7.44	7.46	*7.47*	7.46	7.45	7.43	7.41	7.38	7.35
	3.5	7.37	7.37	7.39	7.41	7.44	7.45	*7.45*	7.44	7.43	7.41	7.38	7.36	7.33
r (%)	4.0	**7.37**	**7.37**	7.38	7.40	7.42	*7.43*	7.43	7.42	7.41	7.38	7.36	7.33	7.29
	4.5	7.37	7.37	7.38	7.39	7.41	*7.42*	7.41	7.40	7.38	7.35	7.33	7.29	7.26
	5.0	7.37	7.36	7.37	7.38	7.39	*7.39*	7.39	7.37	7.35	7.32	7.29	7.26	7.22
	5.5	7.36	7.35	7.35	7.36	7.37	*7.37*	7.36	7.34	7.32	7.29	7.26	7.22	7.18
	6.0	*7.35*	7.34	7.34	7.34	7.35	7.34	7.33	7.31	7.28	7.25	7.22	7.18	7.14
	6.5	*7.33*	7.32	7.32	7.32	7.32	7.31	7.29	7.27	7.24	7.21	7.17	7.13	7.09
	7.0	*7.31*	7.30	7.29	7.29	7.29	7.27	7.26	7.23	7.20	7.16	7.12	7.08	7.04
	7.5	*7.29*	7.28	7.27	7.26	7.25	7.24	7.21	7.18	7.15	7.11	7.07	7.03	6.99
	8.0	*7.27*	7.25	7.23	7.22	7.21	7.19	7.17	7.14	7.10	7.06	7.02	6.98	6.93
	8.5	*7.23*	7.22	7.20	7.19	7.17	7.15	7.12	7.09	7.05	7.01	6.96	6.92	6.87
	9.0	*7.20*	7.19	7.16	7.14	7.12	7.10	7.07	7.03	6.99	6.95	6.90	6.85	6.80
	9.5	*7.16*	7.15	7.12	7.10	7.07	7.05	7.01	6.97	6.93	6.88	6.84	6.79	6.74
	10.0	*7.12*	7.11	7.07	7.05	7.02	6.99	6.95	6.91	6.86	6.82	6.77	6.72	6.66

Each column in Table 6.3 illustrates how expected aggregate value added changes as we vary the stance of monetary policy, for a given level of capital requirements. This allows us to trace a path for the optimal stance of monetary policy conditioned on the level of capital requirement (bold, green fill). Table 6.3 thus shows that the optimal stance of monetary policy loosens as regulatory capital standard strengthens.

In particular, under this illustrative calibration, the central bank would need to set interest rate at 4 percentage points above the neutral stance ($r = 0.04$) if there are no capital requirements for banks. The optimal stance of monetary policy falls to $r = 0.02$ when capital requirements increase to 10% of total assets, which given ω is assumed to be distributed between 0 and 0.1, means a leverage ratio that is binding for almost all bank in the population. For no monetary intervention to be (locally) optimal, capital requirements need to increase to 20%.

We could alternatively read Table 6.3 on a row-by-row basis. This would allow us to trace out a path for the optimal level of bank capital requirements for given setting of monetary policy (italic, blue fill). We find that the optimal level of bank capital requirements falls as monetary policy tightens. Under this specific set of parameters, the globally optimal outcome is achieved when capital requirement is set to 17.5% and the monetary policy stance is 0.5 percentage points above the neutral level.

The model also allows us to describe the banks' behaviour under each policy setting (see Table 6.4). Under the globally optimal combination of $r = 0.5\%$ and $\omega_{reg} = 17.5\%$, the capital regulation is binding for all banks in the population. The participation threshold equals 0.023, meaning that 23% of banks would choose not to screen the risky asset. For banks that do screen, they will invest if and only if the probability of success exceeds the prudence threshold of 0.41 (the first-best level is 0.56). The conditional expectation of success is therefore $E\left[p|p \geq \hat{p}\left(\omega_{reg}\right) = 0.41\right] \approx 0.70$, which implies that around 30% of the risky investments made are expected to underperform. In aggregate about 46% of banks invest in risky assets.

Table 6.4 also shows that across a broad range of measures - including aggregate value added, investment in the risky asset, and probability of failure - the outcomes under optimal policy are between first-best levels (where there is no asymmetry of information, no capital requirements and monetary policy is neutral) and levels under asymmetry information but no policy intervention. The model shows that investment in the risky asset is significantly lower in the first-best case and under optimal policy than compared to the case of no-intervention; free market outcomes with asymmetric information generate too much bank funding of risky assets.

Note that although the banks in our model are heterogeneous at the start (they take inde-

Table 6.4: Behaviour of banks under illustrative baseline parameterisation

Scenarios	Participation Threshold	Prudence Threshold	Avg. prob. of failure on risky investments made	Investment in risky assets	Value Added (%)
First-Best	0	0.56	0.22	0.44	7.89
Optimal Policy Combination	0.0230	0.41	0.30	0.46	7.50
No Policy Intervention	0.0037	Varies with ω, avg.=0.27	0.37	0.71	7.26

pendent draws from the distribution of ω), having a minimum capital requirement that exceeds the upper bound of the ω distribution (i.e. $\omega_{reg} \geq \omega_{ub}$) imposes homogeneity of leverage amongst those banks that do proceed with investing in the risky asset. All banks that proceed will top-up their equity to the minimum level required ($\omega = 17.5\%$ in the optimal case) and fund the remainder through debt. These banks therefore share a common prudence threshold: $\hat{p}\left(\omega_{reg}\right) = 0.41$. In contrast, if ω_{reg} was lower than ω_{ub} (say $\omega_{reg} = 5\%$), then we would observe a cluster of banks with capital equal to the regulatory minimum, as well as some banks with a surplus of capital for regulatory purposes (having started with a better endowment). Since the prudence threshold $\hat{p}$ is a function of capital ω, banks would no longer behave in a homogeneous fashion.

7. Concluding Remarks

We find that monetary policy and prudential capital requirements operate as imperfect substitutes. A tightening of either instruments can improve 'prudence' (by dis-incentivising banks against investing in risky assets with low probability of success); but only at the cost of decreased 'participation' (as more banks will choose to forego the opportunity to acquire the risky asset even before they discover its probability of success through costly screening).

Numerical simulations of our model help to illustrate, and to broadly quantify, the magnitude of this interaction between monetary policy and capital regulation. We find that the trade-off between 'prudence' and 'participation' is such that the optimal level of the central bank policy rate falls as prudential capital requirements are tightened. Moreover, in general the global optimum is one where interest rates should be slightly higher than a neutral setting and capital requirements should be substantial and binding.

The (limited) substitutability between monetary policy and prudential capital requirements illustrated here does not imply that there will never be a case where it is optimal to raise both interest rates and capital requirements at the same time. If the economy starts from a point far away from the optimal policy setting, with both sub-optimally low capital requirements and too loose monetary policy, then aggregate welfare can be improved by tightening both. It may be that monetary policy has been set at a very loose level for reasons that are not captured in our stylised model which abstracts from fluctuations that require counter-cyclical monetary policy. So our results are consistent with the view that central banks should increase policy rates from the lows they had fallen to after the 2007-08 financial crisis as economies return to normal, even if prudential requirements strengthen at the same time.

There are several interesting ways one could extend the baseline analytical framework presented here. One of these is to introduce corporate taxation and explore whether using the tax rate can help offset a tendency to over-invest in risky assets. We present one version of this extension in the appendix and discuss preliminary results.

We assumed here that banking regulators have no more information on the riskiness of bank assets than outside investors. If they have some - but less than complete - information on risks p_i then risk based capital requirements become feasible. That could mean that optimal risk weighted bank capital requirements are on average lower than the leverage ratios we have calculated. It is not clear however that regulators having very imperfect information on bank risk p_i should mean that average leverage requirements would be very substantially lower. If they are not - so that capital requirements of 10% or more of bank assets are indeed optimal - then this suggests that current rules on bank leverage that allow capital to be as low as 3% or so of total assets are likely to be far too lax.

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Appendix

A.1 Proof of Proposition 2 - Weak preference for debt in the First-Best

1. Case A: For a firm with $\omega \geq \omega^*$, and thus $y_l > R(\omega)$, we show that the firm has at least a weak preference for external finance using debt, through proof by contradiction:
   (a) Statement A: suppose that the firm with $\omega \geq \omega^*$ strictly prefers using some amount of external equity as opposed to only using debt:

   $p(y_h - R(\omega)) + (1-p)(y_l - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - R(\omega+\tilde{\omega})) + (1-p)(y_l - R(\omega+\tilde{\omega}))]$

   for some $\tilde{\omega} \in [0, 1-\omega]$.

   Then this implies:

   $p(y_h - y_l) + (y_l - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l)(y_l - R(\omega+\tilde{\omega}))]$

   From proposition 1: $R(\omega+\tilde{\omega}) = (1-(\omega+\tilde{\omega}))(1+r)$, so:

   $p(y_h - y_l) + (y_l - (1-\omega)(1+r)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l) + (y_l - (1-(\omega+\tilde{\omega}))(1+r))]$

   $\tilde{\omega}[p(y_h - y_l) + y_l] < -\omega(1-(\omega+\tilde{\omega}))(1+r) + (\omega+\tilde{\omega})(1-\omega)(1+r)$

   $\tilde{\omega}[p(y_h - y_l) + y_l] < [(\omega+\tilde{\omega}) - \omega(\omega+\tilde{\omega}) - \omega + \omega(\omega+\tilde{\omega})](1+r)$

   $p(y_h - y_l) + y_l < (1+r)$

   $p < \frac{(1+r)-y_l}{y_h-y_l} = \hat{p}_{fb}$. Contradiction - since we know from the definition of the prudence threshold $\hat{p}_{fb}$, that firms will only proceed with lending if $p \geq \hat{p}_{fb}$.

2. Case B: Consider firms with $\omega < \omega^*$, and thus $y_l < R(\omega)$. As before, we show weak preference for debt through proof by contradiction.
   (a) Statement B1: firms strictly prefer to use some external equity when the equity injection means they are still not 'fully capitalised':

   $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - R(\omega+\tilde{\omega}))]$ for some $\tilde{\omega} \in [0, \omega^* - \omega)$

   (b) Statement B2: firms strictly prefer to use some external equity when the equity injection means they become 'fully capitalised':

   $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - R(\omega+\tilde{\omega})) + (1-p)(y_l - R(\omega+\tilde{\omega}))]$ for some $\tilde{\omega} \in [\omega^* - \omega, 1-\omega]$

   Consider statement B1, which simplifies to

   $y_h - R(\omega) < \frac{\omega}{\omega+\tilde{\omega}}[(y_h - R(\omega+\tilde{\omega}))]$

   From equation 3.1 and proposition 1, we have: $\hat{p}_{fb}(y_h - R(\omega)) = \omega(1+r) = \frac{\omega}{\omega+\tilde{\omega}}[\hat{p}_{fb}(y_h - R(\omega+\tilde{\omega})$

   So $(y_h - R(\omega)) = \frac{\omega}{\omega+\tilde{\omega}}(y_h - R(\omega+\tilde{\omega}))$. Contradiction. In fact, the same proof shows that a 'not-fully-capitalised firm' would be indifferent between using all debt, or using a mixture of debt and additional equity (provided that the additional equity injection does not make the firm 'fully-capitalised').

Consider statement B2, which simplifies to:

$p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l) + y_l - R(\omega+\tilde{\omega})]$,

where $R(\omega < \omega^*) = \frac{(1-\omega)(1+r)-(1-p)y_l}{p}$, and

$R(\omega + \tilde{\omega} > \omega^*) = (1-(\omega+\tilde{\omega}))(1+r)$.

From equation 3.1 and proposition 1, we have:

$\hat{p}_{fb}(y_h - R(\omega)) = \omega(1+r) = \frac{\omega}{\omega+\tilde{\omega}}[\hat{p}_{fb}(y_h - R(\omega+\tilde{\omega})) + (1-\hat{p}_{fb})(y_l - R(\omega+\tilde{\omega}))]$, so when $p = \hat{p}_{fb}$, the statement is false.

For the remainder of the proof we just need to show that the statement is also false for $p > \hat{p}_{fb}$.

The derivative of the LHS of statement B2 w.r.t. p is: $(y_h - R(\omega)) - p\frac{\partial R(\omega)}{\partial p} = (y_h - R(\omega)) - p\left(-\frac{R(\omega)}{p} + \frac{y_l}{p}\right) = (y_h - y_l)$

The derivative of the RHS of statement B2 w.r.t. of p is $\frac{\omega}{\omega+\tilde{\omega}}(y_h - y_l) < (y_h - y_l)$. The LHS increases faster than the RHS when p increases from $\hat{p}_{fb}$.

Therefore $p(y_h - R(\omega)) \geq \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l) + y_l - R(\omega+\tilde{\omega})]$ for any $p \geq \hat{p}_{fb}$. Contradiction reached.

A.2 Proof of Proposition 3 - Prudence Threshold under Asymmetric Information

Proposition 3.1

The proof for $\omega^* = 1 - \frac{y_l}{(1+r+\delta_r)}$ follows directly from re-arranging equation 4.2, which also shows that banks with $\omega_i + \tilde{\omega}_i = \omega^*$ can borrow at the risk-free rate $1+r$. For banks with $\omega_i + \tilde{\omega}_i > \omega^*$, their funding shortfall is smaller and thus their gross debt obligation is lower: $R(\omega_i + \tilde{\omega}_i) < R(\omega^*) = y_l$ (since $\frac{\partial R(\omega_i+\tilde{\omega}_i, r)}{\partial \omega} < 0$, a result we will show in the proof to part 2 of the proposition below). The rate at which the debt is provided is still floored at $1+r$, again from re-arranging equation 4.2.

Proposition 3.2

1. Consider case A where $y_l \geq R$.

   Re-arrange the banks' and the households' indifference equations to give: $R = (1-(\omega+\tilde{\omega}))(1+r)$ and $\hat{p} = \frac{(1+r)-y_l}{(y_h-y_l)}$.

Note that under this case: $\frac{\partial R}{\partial \omega} < 0$ so we can deduce that $y_l > R$ holds whenever we have $(\omega + \tilde{\omega}) > \omega^*$.

2. Consider case B where $y_l \leq R$.

   Re-arranging the indifference equations gives:

   $\hat{p}\left[y_h - R\right] = (\omega + \tilde{\omega})(1+r)$; and $A(\hat{p})R + (1 - A(\hat{p}))y_l = (1 - (\omega + \tilde{\omega}))(1+r)$.

   Partially differentiate both w.r.t. ω, [or $\tilde{\omega}$], and solve the system of linear equations to give:

$$\begin{bmatrix} \frac{\partial \hat{p}}{\partial \omega} \\ \frac{\partial R}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \frac{(A(\hat{p})-\hat{p})(1+r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} \\ -\frac{[A'(\hat{p})(R-y_l)+(y_h-R)](1+r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} \end{bmatrix}$$

   Since $(A(\hat{p}) - \hat{p}) > 0$ and $A'(\hat{p}) > 0$ unless $\hat{p} = 1$ (i.e. the degenerate case where no banks lend - which we will rule out by parameterisation), and $y_h \geq R \geq y_l$, we can deduce that $\frac{\partial \hat{p}}{\partial \omega} > 0$ and $\frac{\partial R}{\partial \omega} < 0$. Also, $y_l < R$ whenever we have $(\omega + \tilde{\omega}) < \omega^*$.

3. To summarise: whenever $(\omega + \tilde{\omega}) \geq \omega^*$, we have $\hat{p}(\omega + \tilde{\omega}) = \frac{(1+r)-y_l}{(y_h-y_l)}$; and whenever $(\omega + \tilde{\omega}) < \omega^*$, we have $\frac{\partial \hat{p}}{\partial \omega} > 0$. Therefore $\hat{p} \leq \frac{(1+r)-y_l}{(y_h-y_l)}$ for all $(\omega + \tilde{\omega})$.

A.3 Proof of Proposition 4 - Preference for Debt under Asymmetric Information.

Part 1 of the proposition follows directly from the definition of $\hat{p}(\omega + \tilde{\omega}, r)$ as the 'prudence threshold' for banks (see discussion in main body).

For part 2 of the proposition, we show that for any given level of initial endowment banks weakly prefer to fund the remainder of the project through debt rather than through any other funding arrangements:

- Case A: consider a bank with $\omega \geq \omega^*$, and thus $y_l \geq R$, we show that the bank has a weak preference to top-up the remainder using debt through proof by contradiction:

  Statement A: suppose that the bank with $\omega \geq \omega^*$ strictly prefers using some amount of external equity as opposed to only using debt:

  $p(y_h - R(\omega)) + (1-p)(y_l - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}\left(p(y_h - R(\omega + \tilde{\omega})) + (1-p)(y_l - R(\omega + \tilde{\omega}))\right)$

  for some $0 < \tilde{\omega} \leq 1 - (\omega + \tilde{\omega})$.

  Then this implies $y_l - R(\omega) + p(y_h - y_l) < \frac{\omega}{\omega+\tilde{\omega}}\left[y_l - R(\omega + \tilde{\omega}) + p(y_h - y_l)\right]$

  Recall from Proposition 3, for $\omega \geq \omega^*$: $R(\omega) = (1-\omega)(1+r)$; $R(\omega + \tilde{\omega}) = (1 - (\omega + \tilde{\omega}))(1+r)$

  So $\left(1 - \frac{\omega}{\omega+\tilde{\omega}}\right)(y_l + p(y_h - y_l)) < R(\omega) - \frac{\omega}{\omega+\tilde{\omega}}R(\omega + \tilde{\omega})$

$\frac{\tilde{\omega}}{\omega+\tilde{\omega}}(y_l + p(y_h - y_l)) < (1-\omega)(1+r) - \frac{\omega}{\omega+\tilde{\omega}}(1-(\omega+\tilde{\omega}))(1+r)$

$\tilde{\omega}(y_l + p(y_h - y_l)) < (\omega+\tilde{\omega})(1-\omega)(1+r) - \omega(1-(\omega+\tilde{\omega}))(1+r)$

$\tilde{\omega}(y_l + p(y_h - y_l)) < [(\omega+\tilde{\omega})(1-\omega) - \omega(1-(\omega+\tilde{\omega}))](1+r)$

$\tilde{\omega}(y_l + p(y_h - y_l)) < [(\omega+\tilde{\omega}) - \omega(\omega+\tilde{\omega}) - \omega + \omega(\omega+\tilde{\omega})](1+r)$

$\tilde{\omega}(y_l + p(y_h - y_l)) < \tilde{\omega}(1+r)$

$y_l + p(y_h - y_l) < (1+r)$

$p < \frac{(1+r)-y_l}{(y_h-y_l)}$

But recall from Proposition 1 that for any bank with $\omega \geq \omega^*$, $\hat{p} = \frac{(1+r)-y_l}{(y_h-y_l)}$.

Therefore, given banks will proceed with the project if and only if $p > \hat{p} = \frac{(1+r)-y_l}{(y_h-y_l)}$, statement A cannot be true, and the bank will weakly prefer to use debt for any $\tilde{\omega} \in [0, 1-(\omega+\tilde{\omega})]$. [For simplicity, we assume that when banks are indifferent between debt and equity, they will proceed with the project using debt finance].

- Case B: consider a bank with $\omega < \omega^*$, and thus $y_l < R(\omega)$, we show a weak preference for debt through proof by contradiction:

  1. Statement B1: the bank strictly prefers to use some external equity:
     $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}} p(y_h - R(\omega+\tilde{\omega}))$ for some $\tilde{\omega} \in [0, \omega^* - \omega)$; and
  2. Statement B2: the bank strictly prefers to use some external equity:
     $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}_2}[p(y_h - R(\omega+\tilde{\omega}_2)) + (1-p)(y_l - R(\omega+\tilde{\omega}_2))]$ for some $\tilde{\omega}_2 \in [\omega^* - \omega, 1-\omega]$

Consider Statement B1, which simplifies to $y_h - R(\omega) < \frac{\omega}{\omega+\tilde{\omega}}(y_h - R(\omega+\tilde{\omega}))$.

Let $w = \omega + \tilde{\omega}$ denote total capital. Recall from the bank's indifference condition $\hat{p}(w)[y_h - R(w)] = w(1+r)$; so $y_h - R(w) = \frac{w(1+r)}{\hat{p}(w)}$;

So Statement B1 becomes $\frac{\omega(1+r)}{\hat{p}(\omega)} < \frac{\omega}{\omega+\tilde{\omega}} \frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}(\omega+\tilde{\omega})}$; or $\hat{p}(\omega+\tilde{\omega}) < \hat{p}(\omega)$ when $\omega+\tilde{\omega} < \omega^*$

i.e. a better capitalised bank is less prudent. Contradiction [see Proposition 3].

Consider Statement B2, which simplifies to: $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}_2}[(y_l - R(\omega+\tilde{\omega}_2)) + p(y_h - y_l)]$, for some $\tilde{\omega}_2 \in [\omega^* - \omega, 1-\omega]$, where $R(\omega+\tilde{\omega}_2) = (\omega+\tilde{\omega}_2)(1+r)$

But from bank's indifference condition we know:

$\hat{p}(\omega)[y_h - R(\omega)] = \omega(1+r)$; and $\frac{\omega}{\omega+\tilde{\omega}_2}((y_l - R(\omega+\tilde{\omega}_2)) + \hat{p}(\omega+\tilde{\omega}_2)(y_h - y_l)) = \omega(1+r)$

So $\hat{p}(\omega)[y_h - R(\omega)] = \frac{\omega}{\omega+\tilde{\omega}_2}((y_l - R(\omega+\tilde{\omega}_2)) + \hat{p}(\omega+\tilde{\omega}_2)(y_h - y_l))$

Since $\omega < \omega^* \leq \omega + \tilde{\omega}_2$ by assumption, we know from proposition 3 that $\hat{p}(\omega) < \hat{p}(\omega+\tilde{\omega}_2)$

Therefore we have $p(y_h - R(\omega)) \geq \frac{\omega}{\omega+\tilde{\omega}_2}[(y_l - R(\omega+\tilde{\omega}_2)) + p(y_h - y_l)]$ for any $p \in [\hat{p}(\omega), \hat{p}(\omega+\tilde{\omega}_2)$

Contradiction.

For the last part of the proof by contradiction, just need to show that as p increases, the return from debt financed investments increases faster than that from mixed financed investments:

i.e.$(y_h - R(\omega)) \geq \frac{\omega}{\omega+\tilde{\omega}_2}(y_h - y_l)$. $\begin{bmatrix} \frac{\partial \hat{p}}{\partial \omega} \\ \frac{\partial R}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \frac{(A(\hat{p})-\hat{p})(1+r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} \\ -\frac{[A'(\hat{p})(R-y_l)+(y_h-R)](1+r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} \end{bmatrix}$

Recall again $y_h - R(\omega) = \frac{\omega(1+r)}{\hat{p}(\omega)}$, and $\hat{p}(\omega) < \frac{(1+r)-y_l}{(y_h-y_l)}$, we have $y_h - R(\omega) > \omega(1+r)\frac{(y_h-y_l)}{(1+r)-y_l} = \omega(y_h - y_l)\frac{(1+r)}{(1+r)-y_l}$

so to complete the contradiction to statement B2, just need to show $\omega(y_h - y_l)\frac{(1+r)}{(1+r)-y_l} \geq \frac{\omega}{\omega+\tilde{\omega}_2}(y_h - y_l)$; or $\frac{(1+r)}{(1+r)-y_l} \geq \frac{1}{\omega+\tilde{\omega}_2}$

Recall $\omega^* = 1 - \frac{y_l}{(1+r)} = \frac{(1+r)-y_l}{(1+r)}$, so $\frac{1}{\omega^*} = \frac{(1+r)}{(1+r)-y_l}$. And given $\omega + \tilde{\omega}_2 \geq \omega^*$, we have $\frac{1}{\omega^*} = \frac{(1+r)}{(1+r)-y_l} \geq \frac{1}{\omega+\tilde{\omega}_2}$. QED.

A.4 Value of Screening (equation 3.5)

We present the proof for the last line of equation 3.5:

$$\begin{aligned} & V(\omega, \tilde{\omega}, r) \\ & \equiv E_p\left[\max\left\{0\ ,\ \frac{\omega}{\omega+\tilde{\omega}}\begin{pmatrix} p(y_h - R) + \\ (1-p)\max[0, y_l - R] \end{pmatrix} - \omega(1+r)\right\}\right] \\ & = \frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}}^{1}[p(y_h - y_l) + y_l - (1+r)]\,h(p)dp \end{aligned}$$

The last equality follows because household provide debt at the risk-free rate:

1. For fully capitalised banks with $\omega + \tilde{\omega} \geq \omega^* = 1 - \frac{y_l}{(1+r)}$:

$$V(\omega, \tilde{\omega}, r) = E_p \left[\max \left(0 \ , \ \frac{\omega}{\omega + \tilde{\omega}} \left\{ \begin{array}{c} p\,(y_h - R) + \\ (1-p)\,(y_l - R) \end{array} \right\} - \omega(1+r) \right) \right]$$

So from equation 4.1 we have:

$$V(\omega, \tilde{\omega}, r) = \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[\frac{\omega}{\omega + \tilde{\omega}} \left\{ \begin{array}{c} p\,(y_h - R) + \\ (1-p)\,(y_l - R) \end{array} \right\} - \omega(1+r) \right] h(p)dp$$

Which can be simplified to:

$V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}}^{1} \left[p\,(y_h - y_l) + y_l - R\,(\omega + \tilde{\omega}) - (\omega + \tilde{\omega})\,(1+r) \right] h(p)dp$

But when $\omega + \tilde{\omega} \geq \omega$, we know from proposition 3 that $R\,(\omega + \tilde{\omega}) = (1 - (\omega + \tilde{\omega}))\,(1+r)$

So $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p)dp.$

2. For banks with $\omega + \tilde{\omega} < \omega^*$:

$V(\omega, \tilde{\omega}, r) = E_p \left[\max \left(0 \ , \ \frac{\omega}{\omega+\tilde{\omega}} p\,(y_h - R\,(\omega + \tilde{\omega})) - \omega(1+r) \right) \right]$

Which can be simplified again to:

$V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}}^{1} \left[p\,(y_h - R\,(\omega + \tilde{\omega}, r)) - (\omega + \tilde{\omega})\,(1+r) \right] h(p)dp$

Recall from equation 4.2 $\; A\,(\hat{p})\,R\,(\omega + \tilde{\omega}) + (1 - A\,(\hat{p}))\,y_l = (1 - \omega - \tilde{\omega})\,(1+r)$,

so $A\,(\hat{p})\,(R\,(\omega + \tilde{\omega}, r) - y_l) + y_l = (1 - \omega - \tilde{\omega})\,(1+r)$.

Substituting in the definition of $A\,(.)$ as the conditional expectation:

$\left[\frac{1}{1-H(\hat{p})} \int_{\hat{p}}^{1} ph\,(p)\,dp \right] (R\,(\omega + \tilde{\omega}) - y_l) + y_l = (1 - (\omega + \tilde{\omega}))\,(1+r)$

Re-arrange to give:

$\int_{\hat{p}}^{1} p\,(R\,(\omega + \tilde{\omega}) - y_l)\,h\,(p)\,dp = \left[(1 - (\omega + \tilde{\omega}))\,(1+r) - y_l \right] \left[\int_{\hat{p}}^{1} h\,(p)\,dp \right]$

... because $R\,(.)$ is not a function of p, and ω and p are independently distributed.

So $\int_{\hat{p}}^{1} pR\,(\omega + \tilde{\omega})\,h\,(p)\,dp = \int_{\hat{p}}^{1} \left[py_l - y_l + (1 - \omega - \tilde{\omega})\,(1+r) \right] h\,(p)\,dp$

And $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p)dp$ Q.E.D.

3. Note that when $\tilde{\omega} = 0$ (i.e. in the absence of capital regulation):

$V(\omega, 0, r) = \int_{\hat{p}}^{1} \left[p\,(y_h - y_l) + y_l - (1+r) \right] h(p)dp.$

Note further that when $\tilde{\omega} = \omega_{reg} - \omega$ (i.e. when a bank tops-up to the regulatory minimum - see proposition 7):

$V(\omega, \tilde{\omega} = \omega_{reg} - \omega, r) = \frac{\omega}{\omega_{reg}} \int_{\hat{p}(\omega_{reg},r)}^{1} [p(y_h - y_l) + y_l - (1+r)] h(p)dp = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}\ ,$
$\tilde{\omega} = 0\ ,\ r)$.

So the value of screening for a bank which falls short of the regulatory capital requirement ($\omega_i < \omega_{reg}$) is a $\frac{\omega_i}{\omega_{reg}}$ fraction of the value of screening for a bank that is just meeting the regulatory requirement:

$$V(\omega, \tilde{\omega} = \omega_{reg} - \omega) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0) \tag{7.1}$$

and the participation threshold for a capital constrained bank is given by:

$$\hat{\omega}\left(\tilde{\omega} = \omega_{reg} - \omega\right) = \frac{\omega_{reg} C}{V(\omega = \omega_{reg}, \tilde{\omega} = 0)} \tag{7.2}$$

(see definition 3.6).

A.5 Proof of Proposition 5 - Participation Threshold

Recall: $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega + \tilde{\omega}} \int_{\hat{p}}^{1} [p(y_h - y_l) + y_l - (1+r)] h(p)dp$

Using the Product Rule and the Leibniz Integral Rule[11] we get:

$$\begin{aligned} \frac{\partial V(\omega, \tilde{\omega})}{\partial \omega} &= \frac{\tilde{\omega}}{(\omega + \tilde{\omega})^2} \int_{\hat{p}}^{1} [p(y_h - y_l) + y_l - (1+r)] h(p)dp \\ &\quad - \frac{\omega}{\omega + \tilde{\omega}} [\hat{p}(y_h - y_l) + y_l - (1+r)] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega} \end{aligned}$$

or $\frac{\partial V(\omega,\tilde{\omega})}{\partial \omega} = \frac{1}{(\omega+\tilde{\omega})} \frac{\tilde{\omega}}{\omega} V - \frac{\omega}{\omega+\tilde{\omega}} [\hat{p}(y_h - y_l) + y_l - (1+r)] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega}$.

Recall $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$ (see proposition 7):

- So when $\tilde{\omega} = 0$, $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial \omega} = -[\hat{p}(y_h - y_l) + y_l - (1+r)] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega} \geq 0$

  where the inequality holds because $\frac{\partial \hat{p}}{\partial \omega} \geq 0$ with equality when $(\omega + \tilde{\omega}) \geq \omega^*$; and $\hat{p}(y_h - y_l) + y_l - (1+r) \leq 0$ with equality when $(\omega + \tilde{\omega}) \geq \omega^*$.

[11] Leibniz Integral Rule:
$\frac{d}{dx} \int_{y_0}^{y_1} f(x,y)dy = \int_{y_0}^{y_1} f_x(x,y)dy + f(x,y_1)\frac{dy_1}{dx} - f(x,y_0)\frac{dy_0}{dx}$ provided f and f_x are both continuous over a region in the form $[x_0, x_1] \times [y_0, y_1]$

- When $\tilde{\omega} = \omega_{reg} - \omega$, equation 7.1 shows $V(\omega, \tilde{\omega} = \omega_{reg} - \omega) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0)$, so $\frac{\partial V(\omega, \tilde{\omega} = \omega_{reg} - \omega)}{\partial \omega} = \frac{1}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0) \geq 0$.

A.6 Proof of Proposition 6 - Transmission of Monetary Policy

Proposition 6.1: $\frac{\partial \hat{p}}{\partial r} > 0$

1. For banks with $\omega + \tilde{\omega} \geq \omega^*$, recall that $\hat{p}(\omega + \tilde{\omega}) = \frac{(1+r) - y_l}{(y_h - y_l)}$, so $\frac{\partial \hat{p}}{\partial r} > 0$.
2. For banks with $\omega + \tilde{\omega} < \omega^*$ (and thus $y_l < R(\omega + \tilde{\omega})$) we have from equation 4.1 and equation 4.2:

   $\hat{p}(y_h - R(\omega + \tilde{\omega})) = (\omega + \tilde{\omega})(1 + r)$; and

   $A(\hat{p}) R(\omega + \tilde{\omega}) + (1 - A(\hat{p})) y_l = (1 - \omega - \tilde{\omega})(1 + r)$.

   Partially differentiate both equations with respect to r, and solving the resulting system of linear equations to give:

   $\frac{\partial \hat{p}(\omega + \tilde{\omega}, r)}{\partial r} = \frac{A(\hat{p})(\omega + \tilde{\omega}) + \hat{p}(1 - \omega - \tilde{\omega})}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} > 0$. Q.E.D.

Proposition 6.2: $\frac{\partial \hat{\omega}}{\partial r} > 0$

Recall that $\hat{\omega}(\tilde{\omega}, r)$ is implicitly defined by:

$$V(\hat{\omega}, \tilde{\omega}, r) = \frac{\hat{\omega}}{\hat{\omega} + \tilde{\omega}} \int_{\hat{p}}^{1} [p(y_h - y_l) + y_l - (1 + r)] h(p) dp = C$$

We know also from proposition 5 that $\frac{\partial V(\omega, \tilde{\omega}, r)}{\partial \omega} \begin{cases} > 0 & \text{for} \omega + \tilde{\omega} ¡ \omega^* \\ = 0 & \text{for } \omega + \tilde{\omega} \geq \omega^* \end{cases}$. So to show $\frac{\partial \hat{\omega}}{\partial r} > 0$ for $\hat{\omega} < \omega^{*}$[12], it would be sufficient to show that $\frac{\partial V(\omega, \tilde{\omega}, r)}{\partial r} < 0$ for $\omega + \tilde{\omega} < \omega^*$ (by the implicit function theorem).

Using the Leibniz Integral Rule we get:

$$\frac{\partial V(\omega, \tilde{\omega}, r)}{\partial r} = -\frac{\omega}{\omega + \tilde{\omega}} [1 - H(\hat{p})] + \frac{\omega}{\omega + \tilde{\omega}} [(1 + r) - y_l - \hat{p}(y_h - y_l)] h(\hat{p}) \frac{\partial \hat{p}}{\partial r}$$

So $\frac{\partial V}{\partial r} < 0$ for $\omega + \tilde{\omega} < \omega^*$ iff $\frac{\partial \hat{p}}{\partial r}(\omega + \tilde{\omega} < \omega^*) < \frac{1 - H(\hat{p})}{[(1+r) - y_l - \hat{p}(y_h - y_l)] h(\hat{p})}$ [13]

[12] $\frac{\partial \hat{\omega}}{\partial r}$ for $\omega + \tilde{\omega} \geq \omega^*$ is not well defined.

[13] We are looking at cases where $\omega + \tilde{\omega} < \omega^*$, so $[(1 + r + \delta_r) - y_l - \hat{p}(y_h - y_l)] > 0$.

For $\omega + \tilde{\omega} < \omega^*$, recall from the first part of this proposition that:

$\frac{\partial \hat{p}}{\partial r}\left(\omega + \tilde{\omega} < \omega^*\right) = \frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)}$

Substituting this into the inequality above means we need to show:

$\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} < \frac{1-H(\hat{p})}{[(1+r)-y_l-\hat{p}(y_h-y_l)]h(\hat{p})}$

Re-arranging equation 4.1 and 4.2 (when $\omega + \tilde{\omega} < \omega^*$) gives:

$y_h - R = \frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}}$; $R = \frac{(1-(\omega+\tilde{\omega}))(1+r)-y_l}{A(\hat{p})} + y_l$; and $(y_h - y_l) = \frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}} + \frac{(1-(\omega+\tilde{\omega}))(1+r)-y_l}{A(\hat{p})}$.

It can also be shown generically that $\frac{1-H(\hat{p})}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ (to be shown in Annex # below).

So substituting these results into the inequality above we get:

$\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{\left(\frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}}\right)A(\hat{p})+\hat{p}A'(\hat{p})\left(\frac{(1-(\omega+\tilde{\omega}))(1+r)-y_l}{A(\hat{p})}\right)} < \frac{1}{\left[(1+r)-y_l-\hat{p}\left(\frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}}+\frac{(1-(\omega+\tilde{\omega}))(1+r)-y_l}{A(\hat{p})}\right)\right]}\frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$

Re-arrange and simplify the RHS to give:

$\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{\left(\frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}}\right)A(\hat{p})+\hat{p}A'(\hat{p})\left(\frac{(1-(\omega+\tilde{\omega}))(1+r)-y_l}{A(\hat{p})}\right)} < \frac{1}{[(1-(\omega+\tilde{\omega}))(1+r)-y_l]}\frac{A(\hat{p})}{A'(\hat{p})}$

So:

$$[A(\hat{p})(\omega+\tilde{\omega}) + \hat{p}(1-(\omega+\tilde{\omega}))]\left[(1-(\omega+\tilde{\omega}))(1+r) - y_l\right] < \left\{ \begin{array}{c} \left(\frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}}\right) A(\hat{p}) \frac{A(\hat{p})}{A'(\hat{p})} + \\ \hat{p}\left((1-(\omega+\tilde{\omega}))(1+r) - y_l\right) \end{array} \right\}$$

... which can be further simplified to:

$A(\hat{p})(1-(\omega+\tilde{\omega}))(1+r) - A(\hat{p})y_l - \hat{p}\left[(1-(\omega+\tilde{\omega}))(1+r) - y_l\right] < \frac{A(\hat{p})}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}(1+r)$.

Recall from equation 4.2 : $(1-(\omega+\tilde{\omega}))(1+r) - y_l = A(\hat{p})(R-y_l) > 0$, so for the above inequality to hold it is sufficient to show that:

$A(\hat{p})(1-(\omega+\tilde{\omega}))(1+r) < \frac{A(\hat{p})}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}(1+r)$; or $(1-(\omega+\tilde{\omega})) < \frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}$.

For $p \sim U[0,1]$, $\frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})} = \frac{(\hat{p}+1)}{\hat{p}} > 1 > (1-(\omega+\tilde{\omega}))$. Therefore, $\frac{\partial V}{\partial r} < 0$ for $\omega + \tilde{\omega} < \omega^*$, and $\frac{\partial \hat{\omega}}{\partial r} > 0$ for $\hat{\omega} < \omega^*$. Q.E.D.

A.7 Relationship between conditional expectation and probability density function

Proposition 8. $\frac{1-H(\hat{p})}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ *, where* $A(\hat{p}) \equiv \frac{1}{[1-H(\hat{p})]}\int_{\hat{p}}^{p_{ub}} ph(p)\,dp$ *and* p_{ub} *is the upper bound of the* p *distribution with pdf* $h(.)$ *and cdf* $H(.)$.

Proof. Let $g(p) = \int ph(p)\,dp$, then $A(\hat{p}) = \frac{1}{[1-H(\hat{p})]}[g(p_{ub}) - g(\hat{p})]$, and

$A'(\hat{p}) = \frac{h(\hat{p})}{[1-H(\hat{p})]^2}[g(p_{ub}) - g(\hat{p})] + \frac{1}{[1-H(\hat{p})]}g'(\hat{p}) = A(\hat{p})\frac{h(\hat{p})}{[1-H(\hat{p})]} + \frac{1}{[1-H(\hat{p})]}\hat{p}h(\hat{p})$

So $A'(\hat{p}) = (A(\hat{p}) - \hat{p})\frac{h(\hat{p})}{[1-H(\hat{p})]}$, and $\frac{1-H(\hat{p})}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ □

A.8 Proof of Proposition 7

Part 1 and 2 of the proposition are proved in the main text (these are corollaries of previous propositions). We start with the proof for part 4 of the proposition here, and use the result $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} > 0$ to prove part 3 of the proposition.

Proof of Proposition 7.4: $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$

Given equation 7.2 in Annex 7, we have $\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega\right) = \frac{\omega_{reg}C}{V(\omega=\omega_{reg},\tilde{\omega}=0)}$, so
$\frac{\partial \hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial \omega_{reg}} = \frac{C}{V(\omega=\omega_{reg},\tilde{\omega}=0)} - \frac{\omega_{reg}C}{[V(\omega=\omega_{reg},\tilde{\omega}=0)]^2}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}} = C\left\{\frac{V(\omega=\omega_{reg},\tilde{\omega}=0)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}}}{[V(\omega=\omega_{reg},\tilde{\omega}=0)]^2}\right\}$
implying that for non-zero C and $V(\omega=\omega_{reg},\tilde{\omega}=0)$, $\frac{\partial \hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$ iff $V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}} \geq 0$.

Recall that $V(\omega,\tilde{\omega}) \geq 0$ by definition (V is the value of an option in risky bank lending), so when $\omega_{reg}=0$: $V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}} = V(0,0) \geq 0$. All that remains is to show $V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}}$ is non-decreasing for $\omega_{reg} \in [0,1]$.

Differentiating with respect to ω_{reg} gives:

$$\begin{aligned}\frac{\partial\left\{V(\omega=\omega_{reg},\tilde{\omega}=0)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}}\right\}}{\partial \omega_{reg}} &= \left\{\begin{array}{c}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}} + ... \\ -\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}} - \omega_{reg}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2}\end{array}\right\} \\ &= -\omega_{reg}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2}\end{aligned}$$

So we need to show that $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2} \leq 0$.

Recall from the proof to proposition 5 that:

$$\frac{\partial V(\omega,\tilde{\omega}=0)}{\partial \omega} = -\left[\hat{p}\left(y_h-y_l\right)+y_l-(1+r)\right]h(\hat{p})\frac{\partial \hat{p}}{\partial \omega}$$

So we have:

$$\begin{aligned}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2} &= \frac{\partial^2 V(\omega,0)}{\partial \omega^2}(\omega=\omega_{reg}) \\ &= -\left\{\begin{array}{c}\left[\frac{\partial \hat{p}(\omega_{reg})}{\partial \omega}\left(y_h-y_l\right)\right]h(\hat{p}\left(\omega_{reg}\right))\frac{\partial \hat{p}(\omega_{reg})}{\partial \omega}+ \\ h'(\hat{p}\left(\omega_{reg}\right))\left[\hat{p}\left(\omega_{reg}\right)\left(y_h-y_l\right)+y_l-(1+r)\right]\frac{\partial \hat{p}(\omega_{reg})}{\partial \omega}+ \\ \left[\hat{p}\left(\omega_{reg}\right)\left(y_h-y_l\right)+y_l-(1+r)\right]h(\hat{p}\left(\omega_{reg}\right))\frac{\partial^2 \hat{p}(\omega_{reg})}{\partial^2 \omega}\end{array}\right\}\end{aligned}$$

For uniformly distributed p, $h'(.) = 0$. So $\frac{\partial^2 \hat{p}(\omega_{reg})}{\partial^2 \omega} \leq 0$ is sufficient for $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2} \leq 0$.[14] This in turn implies that $\frac{\partial \left\{ V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg} \frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}} \right\}}{\partial \omega_{reg}} \geq 0$ and finally $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$.

Proof of Proposition 7.3: $\hat{\omega}(\tilde{\omega} = \omega_{reg} - \omega, r) > \hat{\omega}(\tilde{\omega} = 0, r)$ *for all* $\omega_{reg} > \hat{\omega}(0, r)$

From the (implicit) definition of $\hat{\omega}$, we have:

$V(\hat{\omega}(0, r), \tilde{\omega} = 0) = C$; and

$V(\hat{\omega}(\omega_{reg} - \omega, r), \tilde{\omega} = \omega_{reg} - \omega) = C$.

We also know from equation 7.1: $V(\omega, \tilde{\omega} = \omega_{reg} - \omega) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0)$ [i.e. the value of screening for a bank which falls short of the regulatory capital requirement ($\omega_i < \omega_{reg}$) is a $\frac{\omega_i}{\omega_{reg}}$ fraction of the value of screening for a bank that is just meeting the regulatory requirement].

Taken together, the above equations imply

$$V(\hat{\omega}(\omega_{reg} - \omega, r), \tilde{\omega} = \omega_{reg} - \omega) = \frac{\hat{\omega}(\omega_{reg} - \omega, r)}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0) = C$$

and therefore $\hat{\omega}(\omega_{reg} - \omega, r) = \omega_{reg}$ iff $\omega_{reg} = \hat{\omega}(0, r)$. In other words, when the minimum capital requirement is set at the level of participation threshold which prevails in the absence of the requirement, then the capital requirement has no impact on participation. We can define a 'binding regulatory regime' as one where $\omega_{reg} > min[\hat{\omega}(\omega_{reg} - \omega, r), \hat{\omega}(0, r)]$, because if the capital requirement is less than or equal to the lower of the two participation thresholds then no banks would be bound by it should their decide to engage in lending.

The rest of the proof is shown by contradiction:

Suppose $\hat{\omega}(\tilde{\omega} = \omega_{reg} - \omega, r) \leq \hat{\omega}(\tilde{\omega} = 0, r)$ and the regulatory regime is binding $\omega_{reg} > min[\hat{\omega}(\omega_{reg} - \omega, r), \hat{\omega}(0, r)]$. We can then examine the participation thresholds for banks with $\omega_i < \omega_{reg}$ (banks for which the regulatory regime is binding), and show that a contradiction must arise:

1. First consider regulatory regime (i): $\hat{\omega}(\tilde{\omega} = 0, r) \geq \omega_{reg} > \hat{\omega}(\tilde{\omega} = \omega_{reg} - \omega, r)$.

[14] Recall for $\omega + \tilde{\omega} < \omega^*$: $\frac{\partial \hat{p}}{\partial \omega} = \frac{(A(\hat{p}) - \hat{p})(1+r)}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} > 0$. For uniformly distributed p, we have $h(p) = \frac{1}{p_{ub} - p_{lb}}$, $h'(p) = 0$ and $A(\hat{p}) = \frac{1}{2}(\hat{p} + p_{ub})$.

So $\frac{\partial \hat{p}}{\partial \omega} = \frac{\left(\frac{1}{2}(\hat{p}+p_{ub}) - \hat{p}\right)(1+r)}{(y_h - R)\frac{1}{2}(\hat{p}+p_{ub}) + \hat{p}\frac{1}{2}(R - y_l)} = \frac{(p_{ub} - \hat{p})(1+r)}{p_{ub}(y_h - R) + \hat{p}(y_h - y_l)} > 0$.

$\frac{\partial^2 \hat{p}}{\partial \omega^2} = \frac{-\frac{\partial \hat{p}}{\partial \omega}(1+r)[p_{ub}(y_h - R) + \hat{p}(y_h - y_l)] - (p_{ub} - \hat{p})(1+r)\left[-p_{ub}\frac{\partial R}{\partial \omega} + \frac{\partial \hat{p}}{\partial \omega}(y_h - y_l)\right]}{[p_{ub}(y_h - R) + \hat{p}(y_h - y_l)]^2} \leq 0$ since $\frac{\partial \hat{p}}{\partial \omega} > 0$ and $\frac{\partial R}{\partial \omega} \leq 0$.

Under this regime $\omega_{reg} > \hat{\omega}\left(\tilde{\omega} = \omega_{reg} - \omega, r\right)$. So for the condition shown at the start of the proof

$$\begin{aligned} V\left(\hat{\omega}\left(0,r\right),\tilde{\omega}=0\right) &= V\left(\hat{\omega}\left(\omega_{reg}-\omega,r\right),\tilde{\omega}=\omega_{reg}-\omega\right) \\ &= \frac{\hat{\omega}\left(\omega_{reg}-\omega,r\right)}{\omega_{reg}} V(\omega=\omega_{reg},\tilde{\omega}=0) \end{aligned}$$

to hold we need $V(\omega = \omega_{reg}, \tilde{\omega} = 0) > V\left(\hat{\omega}\left(0,r\right),\tilde{\omega}=0\right)$, which given $\frac{\partial V(\omega,\tilde{\omega})}{\partial \omega} \geq 0$ (see proof to proposition 5) means $\omega_{reg} > \hat{\omega}\left(0,r\right)$. Contradiction.

2. Now consider regulatory regime (ii): $\omega_{reg} > \hat{\omega}\left(\tilde{\omega}=0,r\right) \geq \hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right)$

   We have already shown that $\hat{\omega}\left(\omega_{reg}-\omega,r\right) = \omega_{reg}$ iff $\omega_{reg} = \hat{\omega}\left(0,r\right)$; which implies $\hat{\omega}\left(\omega_{reg}-\omega,r\right) = \hat{\omega}\left(0,r\right)$ when $\omega_{reg} = \hat{\omega}\left(0,r\right)$. So given $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$ (Proposition 7.4), when $\omega_{reg} > \hat{\omega}\left(0,r\right)$, $\hat{\omega}\left(\omega_{reg}-\omega,r\right) > \hat{\omega}\left(0,r\right)$. Contradiction.

Therefore $\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right) > \hat{\omega}\left(\tilde{\omega}=0,r\right)$ for all $\omega_{reg} > \hat{\omega}\left(0,r\right)$. Q.E.D.

A.9 The Model with Corporate Taxation

Suppose the government imposes a proportional tax on corporate profits t.

The key equations for firms (and banks when there is no asymmetric information) become:

$$\hat{p}_{firm}\left(y_h - R\right) + \left(1-\hat{p}_{firm}\right)\max\left[0, y_l - R\right] = \left(\omega+\tilde{\omega}\right)\left(1+r\right) + t\hat{p}_{firm}\left(y_h - R - \left(\omega+\tilde{\omega}\right)\right) \tag{7.3}$$

$$pR + \left(1-p\right)\min\left[y_l, R\right] = \left(1-\omega-\tilde{\omega}\right)\left(1+r\right) \tag{7.4}$$

$$p\left(y_h - R\right) + \left(1-p\right)\max\left[0, y_l - R\right] \geq \left(\omega+\tilde{\omega}\right)\left(1+r\right) + tp\left(y_h - R - \left(\omega+\tilde{\omega}\right)\right) \tag{7.5}$$

So for firms that are not 'fully capitalised', we can show that:

$$\hat{p}_{firm}\left(r,t\right) = \frac{\left(1+r\right) - y_l - t\left(\left(1-\omega-\tilde{\omega}\right)\left(1+r\right) - y_l\right)}{y_h - y_l - t\left(y_h - y_l - \left(\omega+\tilde{\omega}\right)\right)} \tag{7.6}$$

The participation threshold for firms is given by:

$$V(\hat{\omega},\tilde{\omega},r) = \int_{\hat{p}_{firm}}^{1}\left[p\left(y_h - y_l\right) + y_l - \left(1+r\right) - tp\left(y_h - R_{firm} - \left(\omega+\tilde{\omega}\right)\right)\right]h(p)dp = C$$

Note that since debt payments are tax deductible in this set-up, the distribution of firms' initial endowments now matters for the equilibrium outcome. Modigiliani-Miller no longer holds.

The key equations for banks under asymmetric information (eqns 4.1, 4.2 and 4.3) become:

$$\hat{p}(y_h - R) + (1-\hat{p})\max[0, y_l - R] = (\omega + \tilde{\omega})(1+r) + t\hat{p}(y_h - R - (\omega + \tilde{\omega})) \tag{7.7}$$

$$A(\hat{p})R + (1 - A(\hat{p}))\min[y_l, R] = (1 - \omega - \tilde{\omega})(1+r) \tag{7.8}$$

$$A(\hat{p})(y_h - R) + (1 - A(\hat{p}))\max[0, y_l - R] \geq (\omega + \tilde{\omega})(1+r) + tA(\hat{p})(y_h - R - (\omega + \tilde{\omega})) \tag{7.9}$$

For banks that are not 'fully-capitalised', we can show that a sufficient condition for the prudence threshold to be increasing in t is:

$$A'(\hat{p}) < \frac{A(\hat{p})}{\hat{p}}$$

which is satisfied when p follows a uniform distribution. It is not difficult to show that banks that are not 'fully-capitalised' retains a preference for debt funding.

Next, the value of screening for banks is given by:

$$\begin{aligned}
V(\omega, \tilde{\omega}) &= E_p\left[\max\left\{0, \frac{\omega}{\omega+\tilde{\omega}}\begin{pmatrix} p(y_h - R) - tp(y_h - R - (\omega+\tilde{\omega})) + \ldots \\ (1-p)\max[0, y_l - R]\end{pmatrix} - \omega(1+r)\right\}\right] \\
&= \int_{\hat{p}(\omega+\tilde{\omega})}^{1}\left[\frac{\omega}{\omega+\tilde{\omega}}\begin{pmatrix} p(y_h - R) - tp(y_h - R - (\omega+\tilde{\omega})) + \ldots \\ (1-p)\max[0, y_l - R]\end{pmatrix} - \omega(1+r)\right] h(p)\,dp \\
&= \frac{\omega}{\omega+\tilde{\omega}}\left[\int_{\hat{p}(\omega+\tilde{\omega})}^{1}(y_l + p(y_h - y_l) - (1+r) - tp(y_h - R - (\omega+\tilde{\omega})))\,h(p)\,dp\right]
\end{aligned}$$

where the last equality can be shown by considering the two cases $\omega \geq \omega^*$ and $\omega < \omega^*$ separately.

The participation threshold $\hat{\omega}$ is implicitly defined by the equation:

$$\frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}}\left[\int_{\hat{p}(\hat{\omega}+\tilde{\omega})}^{1}(p(y_h - y_l) + y_l - (1+r) - tp(y_h - R - (\hat{\omega}+\tilde{\omega})))\,h(p)\,dp\right] = C$$

where $\tilde{\omega} = \max\{0, \omega_{reg} - \omega\}$.

Accounting for corporate taxation as an additional dimension of policy does not affect the main results in our baseline model. All three tools: monetary policy, capital regulation, and corporation tax operate as imperfect substitutes which trade-off prudence and participation. Similar to monetary policy, the corporation tax affects both the transparent 'firms' sector and the banking sector indiscriminately and therefore can lead to inefficiencies when used to tackle excessive risk taking in the banking sector. Numerical simulations based on parameterisations in table 6.2 shows that the optimal policy combination now depend on the initial distribution of endowments for firms: ω_{firm}. When leverage is just as high in the transparent firms sector as the banking sector: $\omega_{firm} \sim U[0, 0.1]$, the optimal policy response is to rely on an (prohibitively) high rate of corporate tax (>0.75) to boost prudence, with a lower than neutral (i.e. negative) interest rate to offset the costs in participation. But the level of leverage in the non-financial sector of developed economies is generally very much lower than it is for banks. When we allow for even slightly lower levels of leverage in the firm sector, say $\omega_{firm} \sim U[0, 0.2]$, the optimal policy setting becomes ($r = 0,\ \omega_{reg} = 17.5\%,\ t = 5\%$). Under more realistic assumptions, $\omega_{firm} \sim U\left[0, \omega_{firm}^{ub} \geq 0.3\right]$, we revert back to the solution in the main body of the text: ($r = 0.5\%,\ \omega_{reg} = 17.5,\ t = 0$). Thus blunt instruments, such as the interest rate, and particularly the corporate tax rate, are not the ideal tools when dealing with high leverage and excessive risk taking in the banking sector.