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ABSTRACT

Interaction Between Monetary Policy and Regulatory Capital Requirements*

Banks' behaviour can be influenced by both monetary policy and regulatory capital requirements. This paper explores the interaction between these two policy tools in promoting better lending decisions by banks. We develop and calibrate a model of bank lending to examine what an optimal combination of monetary policy and regulatory capital requirements might look like. We find that as prudential standards strengthen globally in the aftermath of the financial crises, it is likely that the that equilibrium level of central bank policy rates should be lower than they had been prior to the crisis.

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1 Introduction

There has a been a move towards tougher standards in macro-prudential and micro-prudential regulation for banks in the aftermath of the financial crisis. For bank capital requirements, this process may have some way still to go (see Admati and Hellwig (2013) and Miles et al (2013)). At the same time, the interest rates set by many central banks remain at historically low levels. But whereas new prudential requirements, and specifically higher capital requirements, are likely here to stay, central bank rates will need to rise at some point. The question is to what level. Do policy rates need to return to around the pre-crisis average? We suggest that part of the answer lies in the interaction between monetary policy and regulatory capital requirements, in particular with respect to the effects these policy instruments may have on the lending behaviour of banks.

There is an existing body of literature focused on the role of bank capital in monetary policy transmission. For instance, Van den Heuvel (2002) found that the effect of monetary policy on bank lending depends on the capital adequacy of the banking sector; in particular, lending by banks with low capital has a delayed and then amplified reaction to interest rate shocks, relative to well-capitalized banks. Others, such as Kashyap and Stein (1994) and Tanaka (2002), suggested that a binding capital constraint would attenuate the effect of (expansionary) monetary policy. Ehrmann et al (2001) argued that whether the degree of capitalisation matters for banks' reaction to interest rate changes depends on the extent of informational asymmetries in the sector. Borio and Zhu (2012) provides a more comprehensive survey of the existing literature. Our paper seeks to examine a separate set of questions, namely: what are the trade-offs involved in using monetary policy and capital requirements in influencing banks' lending behaviour; and what might an optimal combination of these policy tools look like.

The model we use to analyse these questions is an extension of the theoretical framework developed in Bernanke-Gertler's (1990) paper on financial fragility and economic performance. Their model described an economy with two types of risk-neutral agents: entrepreneurs that have access to risky investment projects; and households from whom the entrepreneurs must borrow in order to fund their investment project. Entrepreneurs must undertake costly screening before they can find out the probability of success of their project, so some with low endowments (and

thus high funding requirements) may choose to forego the investment opportunity even before the screening stage. Those that do undertake screening are incentivised to take excessive risks. This is because entrepreneurs enjoy limited liability and cannot credibly reveal their probability of success to their creditors. The extent to which an individual entrepreneur is prone to excessive risk-taking is also exacerbated by lower initial endowments. Bernanke and Gertler concluded that the dependence of aggregate investment on the initial distribution of endowment introduces 'financial fragility'. A 'financially fragile' economy, defined as one with a sizeable proportion of its entrepreneurs operating at the threshold level of endowment between screening and not screening, may experience a dramatic collapse in investment if subjected to a negative shock to endowments.

We make a number of changes to the Bernanke-Gertler (1990) model:

1. *Banks*: First we re-interpret 'entrepreneurs' as 'banks'; 'risky investments' as 'risky bank lending projects'; and 'initial endowment' as 'initial bank capital'. Banks need to screen potential loans and to raise funds from households in order to engage in bank lending. Adapting the Bernanke-Gertler model in this way provides us with the basis of a model where aggregate economic outcomes depend on the extent of leverage in banks.

2. *Debt and Equity*: We distinguish between two types of funding contracts: banks in our model can raise funds from households either in the form of debt or in the form of equity. Debt contracts promise to pay the creditor a fixed sum; whereas equity contracts promise a share of the investment return (net of any debt obligations). There may be a non-zero pay-off for the risky project in the event it fails. Providers of debt have priority claim over this 'liquidation value' of the risky project.

3. *Policy tools*: The policy maker has two levers: monetary policy and regulatory capital requirements. Monetary policy sets the safe rate paid on the outside option (i.e. a risk-free deposit facility) that is available to both entrepreneurs and households. So a tightening of monetary policy can be seen as an increase in the risk-free rate, or a decline in the premium offered by the risky bank lending project. This safe rate is paid by the central bank, and recouped through ex post lump sum taxes. Regulatory capital requirements prohibit banks with initial endowments less than the regulatory minimum from proceeding with the risky

lending project, unless they raise the additional equity from households. Banks for which the capital regulation is binding will be financed by a mixture of debt and external equity.

We find that in the context of this model monetary policy and prudential capital requirements operate as imperfect substitutes. A tightening of either instruments can improve 'prudence' (by disincentivising the banks against undertaking projects with low probability of success); but only at the cost of decreased 'participation' (where decreased 'participation' means that more banks will choose to forego the investment opportunity even before they discover its probability of success through costly screening). The substitutability between the policy instruments, and this trade-off between 'prudence' and 'participation', implies that the optimal level of the central bank policy rate falls as prudential capital requirements are tightened. The intuition is simple. We should encourage greater participation in bank lending with looser monetary policy, provided that we can rely on prudential policy to ensure banks that do participate are undertaking their lending activities in a prudent fashion.

The rest of the paper is organised as follows. Section 2 lays out the basic theoretical model for analysing bank lending decisions, and calculates first-best outcomes. Section 3 presents the model mechanics under asymmetric information. The two policy instruments are introduced in Section 4 where we also describe the main propositions of the model. Section 5 provides some quantitative results for numerical calibrations of the model. Section 6 concludes with discussions on policy implications. Proofs are presented in the Annex.

2 Model mechanics

The analytical framework we use is a simple three-period model at the centre of which is the lending decision of banks. Banks need external financing in order to proceed with their lending/investment projects. They have private information on the likelihood of success on the projects they finance, and limited liability in the event of failure. This asymmetry relative to the external provider of funds give rise to moral hazard and a departure from the Modigliani-Miller theorem on capital structure irrelevance.

There are two types of risk-neutral agents in the economy: households with access to a risk-free deposit facility; and banks with access to both the risk-free facility and a risky lending/investment technology. The risk-free deposit facility remunerates any amount deposited at the risk-free rate $(1 + r)$. The risky lending technology always require 1 unit of input and pays out $y_h > 1+r$ when it succeeds (with probability p), and $y_l < 1+r$ when it fails (with probability $(1 - p)$). All banks have initial endowment $\omega \leq 1$, so the vast majority need to obtain external financing from households in order to lend[1]. In the event that the risky lending project fails, banks and any external equity providers are protected by limited liability. Debt providers have priority claim on y_l, the liquidation value of the project. We normalise the size of the aggregate population of agents (banks and households) to 1 and denote the proportion of banks and households in the population by μ and $(1 - \mu)$ respectively.

Almost all of the structural parameters of the model are common knowledge across all agents. The exception is the probability of success for individual lending project, p, which is only revealed privately to banks that undertake costly screening.

The probability of success on lending projects, p, is a random variable following a uniform distribution bounded between $[0, 1]$; with probability density function $h(p)$ and cumulative density function $H(p)$. Each bank draws a realisation of p when it undertakes the project. A bank can find out its own realisation, p_i, by incurring a fixed screening cost of $C < 1$. While the distribution of p is common knowledge, we assume there is no way for any bank to credibly convey its realisation of p_i to other agents. By construction, the lending project is not worthwhile in the absence of screening: $y_l + E[p](y_h - y_l) < 1 + r$. This implies that banks which do not screen will not want to lend.

ω is the level of initial endowments for banks. Endowments are uniformly distributed between $[\omega_{lb}, \omega_{ub}]$, where $0 \leq \omega_{lb} < \omega_{ub} \leq 1$. The p.d.f. and c.d.f. of ω are denoted by $f(\omega)$ and

[1] One way to see the distinction between banks and households is that only banks have the ability to screen each risky project. This screening technology allows banks to discover the probablity of success on the risky project for a fixed cost C<1, before having to commit a full unit of input into the project. By constructution, the risky project is not worthwhile in the absence of screening. This ensures households would not be interested in conducting these bank lending activites directly even if they had access to this risky technology.

$F(\omega)$ respectively. Both the distribution of ω and each bank's realisation of ω_i are publically observed. Houesholds have an endowment of $\omega_{h.}$ The average endowment (across both banks ω and households ω_h) is normalised to 1: $E[\omega]\mu + E[\omega_h](1-\mu) = 1$, such that there will always be enough funds in the economy for every lending project to be undertaken. y_h, y_l and C, are

constants and assumed to be publically observed.

The timing of the model is divided into three stages (see Figure 1)

- In stage 1, each individual bank draws a realisation of their initial endowments, ω_i, from the common distribution for endowments.
- Given the realisation of ω_i, each bank decides in stage 2 whether or not to undertake costly screening to discover the success rate of their risky lending project, p_i, (again drawing a realisation of the success rate from the common distribution). Banks that do not screen deposit the entirety of their endowments in the risk-free facility.
- In stage 3, those banks that did screen choose whether to proceed with their project based on its probability of success p_i. When banks proceed with the project, they commit the entirety of their initial endowment [2], and decide whether to fund the shortfall (relative to the 1 unit of input required for the project) through debt, additional equity, or a combination of the two. Any external equity injections take place before the bank seeks debt. By assumption, households can observe the capital of banks (composed of a bank's initial endowment plus any subsequent equity injections) but not the probability of their success.

An equity contract takes the form $\left\{\frac{\tilde{\omega}}{\omega+\tilde{\omega}},\ \tilde{\omega}\right\}$, where $\tilde{\omega}$ is the size of the equity injection, ω is the bank's own endowment, and $\frac{\tilde{\omega}}{\omega+\tilde{\omega}}$ is the share of output promised to the external equity providers. A debt contract takes the form $\{R(\omega+\tilde{\omega}, r), (1-\omega-\tilde{\omega})\}$, where $\omega+\tilde{\omega}$ is the total capital of the bank and $(1-\omega-\tilde{\omega})$ is the amount of debt . $R(\omega+\tilde{\omega}, r)$ denotes the gross amount (principal plus interest) that is promised on debt. A bank that funds its shortfall only through

[2] Banks would only proceed with the project if its expected return exceeds the certain return from the deposit facility (recall that all banks are assumed to be risk neutral). Therefore, banks would prefer to commit the entirety of their endowments rather than to place a portion in the safe deposit facility.

debt sets $\tilde{\omega} = 0$; whereas a bank that uses only equity sets $\tilde{\omega} = 1 - \omega$. All intermediate cases are permitted.

Two key thresholds drive the mechanics of the model:

1. The threshold level of a bank's initial endowment, $\hat{\omega}$, at which point it is indifferent between screening and simply depositing its endowments in stage 2 to earn the safe rate; and
2. The threshold level of the success probability, $\hat{p}$, at which point the banks are indifferent between proceeding after screening and depositing at the safe rate in stage 3.

The level of bank endowment matters for the screening decision because a lower initial endowment implies a larger funding gap and higher cost of funding. A bank with initial endowment less than $\hat{\omega}$ would find that the funding cost, plus the fixed cost of screening, exceed the expected return from undertaking the risky lending. The endowment threshold $\hat{\omega}$ is defined formally in Section 3.2 **.**

The realisation of the success probability p_i determines whether the bank proceeds with the project after screening. A bank with $p_i < \hat{p}$ finds that it can achieve a higher expected return by using the risk-free deposit facility, so will forego the investment opportunity. We define $\hat{p}$ more formally in Section 3.1 .

For the rest of the paper, we will refer to $\hat{\omega}$ as the '**participation threshold**' and $\hat{p}$ as the '**prudence threshold**'. $\hat{\omega}$ is described as the 'participation threshold' because banks with initial endowment below this level do not 'participates' in bank lending. $\hat{p}$ is described as the 'prudence threshold' because the success rate banks are willing to accept is an indicator of how prudent they are in handling funds from households. Imprudent banks will tend to take excessive risks to take advantage of their private information and limited liability.

2.1 First-Best outcomes - the model under perfect information

In the first-best, a social planner will ensure that banks only proceed with the risky lending project post screening if its expected return at least matches that from the risk-free deposit facility. That is: $p_i (y_h - y_l) + y_l \geq (1 + r)$. So the 'prudence threshold' under the first-best is

given by:

$$\hat{p}_{fb} = \frac{1 + r - y_l}{(y_h - y_l)} \tag{1}$$

Therefore, as long as the cost of screening is less than the expected value-added from the project when all banks act prudently, it would be optimal for the social planner to screen every project. Specifically, we need $C < V\left(\hat{p}_{fb}\right) = \int_{\hat{p}_{fb}}^{1} \left[p\left(y_h - y_l\right) + y_l - (1 + r)\right] h\left(p\right) dp$. We assume that this condition holds. This implies the participation threshold under the first-best is zero:

$$\hat{\omega}_{fb} = 0 \tag{2}$$

The aggregate amount of bank lending/investment in the first best is then given by:

$$I_{fb} = \mu(1 - H(\hat{p}_{fb})) \tag{3}$$

and the aggregate output of the economy (composed of returns from both bank lending and risk-free deposits) is given by:

$$Q_{fb} = (1 + r) + \mu \int_{\hat{p}_{fb}}^{1} \left[p\left(y_h - y_l\right) + y_l - (1 + r)\right] h(p)dp - \mu C \tag{4}$$

3 The model with asymmetry of information

Frictions in the model arise because of the interaction between asymmetric information and limited liability. This moral hazard problem manifests in a sub-optimal level of screening for the risky project; and for those banks that do screen, a lower level of prudence in deciding whether to undertake the project.

3.1 The Prudence Threshold

Working backwards, we will start by examining the prudence threshold (at Stage 2 of the game) for those banks that have discovered their individual realisation of p_i through screening. The

prudence threshold for banks, $\hat{p}$, is defined as the success rate required to make a bank indifferent between undertaking the risky lending project and simply depositing its endowments.

Specifically, for a bank with initial endowment ω_i, and which is looking for $\tilde{\omega}_i$ in additional equity and $1-(\omega_i+\tilde{\omega}_i)$ in debt from households, its prudence threshold $\hat{p}_i = \hat{p}(\omega_i+\tilde{\omega}_i, r)$ and the terms of its funding contract are jointly determined by the following three equations[3]:

1. The condition where the bank is indifferent between lending and depositing:

$$\frac{\omega}{\omega+\tilde{\omega}}\left\{\hat{p}\left(y_h - R\left(\omega+\tilde{\omega},r\right)\right) + \left(1-\hat{p}\right)\max\left[0, y_l - R\left(\omega+\tilde{\omega},r\right)\right]\right\} = \omega(1+r) \qquad \text{or}$$

$$\hat{p}\left(y_h - R\left(\omega+\tilde{\omega},r\right)\right) + \left(1-\hat{p}\right)\max\left[0, y_l - R\left(\omega+\tilde{\omega},r\right)\right] = \left(\omega+\tilde{\omega}\right)\left(1+r\right) \tag{5}$$

   where $\tilde{\omega}$ is the size of the equity injection from households; $\omega+\tilde{\omega}$ is the total capital of the bank; and $\frac{\omega}{\omega+\tilde{\omega}}$ is the share of the return retained by the bank. We assume that external providers of equity will always receive $\frac{\tilde{\omega}}{\omega+\tilde{\omega}}$ - a fair share of the proceeds. A bank with capital of $\omega+\tilde{\omega}$ needs to finance the remainder of the project $(1-\omega-\tilde{\omega})$ through debt. $R(\omega+\tilde{\omega},r)$ denotes the gross amount (principal plus interest) that is promised on debt.

2. The condition where the households is indifferent between providing debt and depositing:

$$A\left(\hat{p}\right)R\left(\omega+\tilde{\omega},r\right) + \left(1-A\left(\hat{p}\right)\right)\left(\min\left[y_l, R\left(\omega+\tilde{\omega},r\right)\right]\right) = \left(1-\omega-\tilde{\omega}\right)\left(1+r\right) \tag{6}$$

   where $A\left(\hat{p}\right) \equiv E[p \mid p > \hat{p}] \equiv \frac{1}{1-H(\hat{p})}\int_{\hat{p}}^{1} ph\left(p\right)dp$ denotes the conditional expectation of p given that $p \geq \hat{p}$.

3. The condition where for the households providing additional equity to the bank, their expected return from the capital injection is at least equal to the safe rate of return:

$$\frac{\tilde{\omega}}{\omega+\tilde{\omega}}\left\{A\left(\hat{p}\right)\left(y_h - R\left(\omega+\tilde{\omega},r\right)\right) + \left(1-A\left(\hat{p}\right)\right)\left(\max\left[0, y_l - R\left(\omega+\tilde{\omega},r\right)\right]\right)\right\} \geq \tilde{\omega}\left(1+r\right) \qquad \text{or}$$

$$A\left(\hat{p}\right)\left(y_h - R\left(\omega+\tilde{\omega},r\right)\right) + \left(1-A\left(\hat{p}\right)\right)\left(\max\left[0, y_l - R\left(\omega+\tilde{\omega},r\right)\right]\right) \geq \left(\omega+\tilde{\omega}\right)\left(1+r\right) \tag{7}$$

   This last condition is always satisfied given equation 5 and the fact that $A\left(\hat{p}\right) \geq \hat{p}$.

[3] In the interest of brevity, we suppress the i subscript henceforth where possible.

Proposition 1 *Banks that are not 'fully capitalised' will take excessive risks*

1. *Let ω^* denote the level of capital such that $y_l = R(\omega^*, r)$, then $\omega^* = 1 - \frac{y_l}{(1+r)}$ and banks with $\omega_i + \tilde{\omega}_i \geq \omega^*$ can borrow at the risk-free rate.*

   We refer to any bank with $\omega_i + \tilde{\omega}_i \geq \omega^*$ as a "fully capitalised' bank, because such a bank can pay its debt in full - even in the bad state[4].

2. $\hat{p}(\omega + \tilde{\omega}, r) \leq \frac{(1+r)-y_l}{(y_h - y_l)} = \hat{p}_{fb}$, *with equality only when $\omega + \tilde{\omega} \geq \omega^*$.*

   Banks that are not 'fully capitalised' $(\omega_i + \tilde{\omega}_i < \omega^*)$ will take excessive risks.

3. $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega} = \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \tilde{\omega}} \geq 0$, *with equality only when $\omega + \tilde{\omega} \geq \omega^*$.*

   Banks become more prudent as they increase their capital, up to a cap when they become 'fully capitalised'. An increase in initial endowment has the same marginal impact on prudence as an increase in external equity. Banks lending behaviour is influenced by the size of their equity rather than its source.

Proof. *We give an outline here. A full proof is in the annex.*

In the first part of the proposition, ω^* is defined as the level of endowments such that $y_l = R(\omega^*, r)$, so $\omega^* = 1 - \frac{y_l}{(1+r)}$ follows directly from re-arranging equation 6. Banks with capital above this level can borrow at the risk free rate because the liquidation value of the lending project is sufficient to cover the bank's debt obligations (again from re-arranging equation 6 after setting $R(\omega^*, r) = y_l$).

The second part of the proposition says that banks that are not fully capitalised will take excessive risks. Limited liability encourages banks to take excessive risks when the liquidation value of the project is less than the bank's debt obligations. Households can anticipate this, so will increase the cost of debt $\frac{R(\omega+\tilde{\omega},r)}{(1-\omega-\tilde{\omega})}$ for banks with small capital (and thus high funding needs). The fact that banks cannot credibly communicate their success rate (p_i) to households exacerbates this issue as the debt contract can only be formulated as a function of the bank's observable capital $(\omega + \tilde{\omega})$. This means two banks with the same level of capital will face the same borrowing costs regardless of the success probabilities on their projects - the bank with the better project is effectively subsidising the borrowing costs of the one with the poorer project. Taken together,

[4] These banks can still 'fail' in the sense that their equity holders may be wiped out in the bad state.

limited liabilities and this cross-subsidisation towards less attractive projects give rise to moral hazard in bank lending.

The last part of the proposition follows from the proof for the second part (see annex). The fact that prudence is an increasing function of capital reflects the observation that the moral hazard issue becomes less pronounced when banks have more 'skin in the game'. ■

Proposition 2 *After screening:*

1. *banks invest in the risky lending project if and only if* $p \geq \hat{p}(\omega + \tilde{\omega}, r)$*; and*
2. *when banks do undertake the project they choose to fund with external debt rather than external equity (i.e. banks will choose to set* $\tilde{\omega} = 0$*).*

Proof. *Part (1) follows from the definition of* $\hat{p}(\omega + \tilde{\omega}, r)$*: banks with success probability* $p < \hat{p}(\omega + \tilde{\omega}, r)$ *would receive a higher expected return from depositing their endowment than from investing with debt finance.*

Part (2) holds because banks expect to pay out a higher rate of return to external equity providers than to debt providers. We show this formally in the annex. Qualitatively, equity is more costly in this model because debt is provided at a zero-profit basis by the households: they charge a rate which makes them indifferent (in expectation) between providing the debt and using the safe deposit facility. Equity, on the other hand, allows the household to share the surplus from bank lending, and therefore delivers a rate of return for households that is greater than the risk-free facility. [5] *The original owners of bank equity have the value of their claims diluted by raising new equity from outsiders. Consequently banks stick with debt in the absence of any policy intervention, because this is the cheapest way for them to raise funds at the margin.* ■

Corollary 1 *In the absence of capital regulations, banks will not seek external equity when they do choose to undertake risky lending.*

[5] We have assumed equity always provide a fair-share of the risky return in the interest of mathematical simplicity. If we instead assumed that competition amongst households also drove down the expected return from equity to the safe rate, then households would be content to receive a less-than-fair share in the bank.

However, banks in reality are very far from fully capitalised; in contrast they tend to be highly leveraged. We reflect this stylised observation in the analytical and numerical analyses of the paper by restricting the support of the ω distribution to $[\omega_{lb}, \omega_{ub}]$, where ω_{ub} is a number that is (significantly) lower than 1.

It is (/should be) possible to show that for banks with very low capital stocks, debt is preferred to equity. [Poorly capitalised banks are more incentivised to take risks. Their downside risks are low (capped by limited liability), but their upside gains are considerable. So they prefer debt rather than equity, because the latter would take a share of their upside.] - Need to illustrate formally.

3.2 Participation Thresholds

Having established the 'prudence threshold' of banks, we continue to work backwards to find banks' 'participation threshold'. Upon discovering its realisation of ω_i at Stage 1 of the game, a bank decides whether to engage in costly screening. The 'participation threshold' of banks is defined as the level of initial endowment that equates the expected value of screening (V) to the fixed cost of screening (C). The costly screening process can be interpreted as buying an option in the risky lending opportunity. The value of the option will depend on the funding structure of the bank, in particular how much additional equity it is trying to attract from households.

Formally, V is given by:

$$V(\omega, \tilde{\omega}, r) \equiv E_p \left[\max \left(\begin{array}{c} 0\ , \\ \frac{\omega}{\omega+\tilde{\omega}} \left\{ \begin{array}{c} p\left(y_h - R\left(\omega+\tilde{\omega}, r\right)\right) + \\ (1-p)\max\left[0, y_l - R\left(\omega+\tilde{\omega}, r\right)\right] \end{array} \ldots \right\} \\ - \ (\omega+\tilde{\omega})(1+r) \end{array} \right) \right] \tag{8}$$

$$= \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - (1+r)\right] h(p)dp \tag{9}$$

where the last line follows because the debt is provided by households at the risk-free rate (more detail can be found in the annex).

Proposition 3 $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial \omega} \begin{cases} > 0 \text{ for } \omega < \omega^* \\ = 0 \text{ for } \omega \geq \omega^* \end{cases}$ *. In the absence of capital regulations (and* $\tilde{\omega} = 0$*), the expected value of screening is increasing in the bank's initial endowments (up to a cap when the bank becomes fully well capitalised). (see proof in the annex)*

Definition 1 *The participation threshold for banks,* $\hat{\omega}\left(\tilde{\omega}, r\right)$*, is implicitly defined by:*

$$V(\hat{\omega}, \tilde{\omega}, r) = \frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}} \int_{\hat{p}(\hat{\omega}+\tilde{\omega},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - (1+r)\right] h(p)dp = C \tag{10}$$

where C is the fixed cost of screening.

Since we have established through corollary 1 that banks will only use debt funding in the absence of capital regulations, for now we can only concern ourselves with the case where $\tilde{\omega} = 0$.

3.3 Summary and Implications

The presence of moral hazard in the model give rise to sub-optimal outcomes compared to the first-best scenario. Banks have no incentives to top-up their capital. Consequently, the hurdle for participation, $\hat{\omega}\left(\tilde{\omega} = 0, r\right) > 0$, is too high; and the level of prudence, $\hat{p}(\omega, r) \leq \frac{(1+r)-y_l}{(y_h-y_l)} = \hat{p}_{fb}$ is too low. Figure 2 and 3 illustrate these distortions. Aggregate output of the economy will suffer as a result.

4 Policy tools and their transmission

Monetary policy and capital regulations can be used to drive outcomes closer to the first-best level. Both policy tools function through their effects on the participation and prudence thresholds.

4.1 Monetary Policy

We introduce monetary policy by allowing the central bank to affect the risk-free rate of return on the deposit facility - r. So the stance of monetary policy can affect the attractiveness of risky bank lending relative to the outside option. The central bank remunerates all resources placed in its deposit facilities and recoups the cost through lump sum taxation on all agents (denoted by τ):

$$(1 - I)r = \tau \tag{11}$$

where I is the aggregate amount of bank lending undertaken.

The monetary policy stance (r) is revealed to all agents in Stage 1, as soon as banks find out their realisation of ω_i.

We normalise $r = 0$ as the neutral stance of monetary policy. In the first-best scenario, there is no need for monetary policy intervention, so $r = 0$ and the first-best level of prudence and the first-best level of output become:

$$\hat{p}_{fb} = \frac{1 - y_l}{(y_h - y_l)} \tag{12}$$

and

$$Q_{fb} = 1 + \mu \int_{\hat{p}_{fb}}^{1} \left[p\left(y_h - y_l\right) + y_l - 1 \right] h(p) dp - \mu C \tag{13}$$

respectively. The first-best level of investment I_{fb} and participation threshold $\hat{\omega}_{fb}$ are the same as before (see equation 3 and 2).

Note that the possibility of monetary policy intervention introduces a wedge between the socially optimal outcome and what is privately optimal in the absence of asymmetric information. In particular, the privately optimal level of prudence is given by

$$\hat{p}^*\left(r\right) = \frac{1 + r - y_l}{(y_h - y_l)} \tag{14}$$

which will be different to $\hat{p}_{fb}$ for non-zero r.

The effect of monetary policy is summarised in the proposition below.

Proposition 4 *Monetary policy tightening improves 'prudence' at the cost of decreasing 'participation'. Monetary policy loosening achieves the converse. In other words:*

1. $\frac{\partial \hat{p}}{\partial r} > 0$*; and*
2. $\frac{\partial \hat{\omega}}{\partial r} > 0$

Proof. *We present informal justifications for the proposition here; formal proof is in the annex.*

1. *Monetary policy tightening improves prudence for all banks by increasing the attractiveness of the outside option. A quick way to illustrate the result* $\frac{\partial \hat{p}}{\partial r} > 0$ *is by appealing to proposition 1:* $\hat{p}(\omega + \tilde{\omega}, r) \leq \hat{p}_{fb}$*. In the case where neutral monetary policy implies* $r = 0$*,* $\hat{p}_{fb} = \frac{1-y_l}{(y_h - y_l)}$ *and by Proposition 1* $\hat{p}(\omega + \tilde{\omega}, r) \leq \frac{1+r-y_l}{(y_h - y_l)} = \hat{p}^*\left(r\right)$*. So by increasing* r*, a central bank can increase* $\hat{p}^*\left(r\right)$ *and push* $\hat{p}(\omega + \tilde{\omega}, r)$ *closer to* $\hat{p}_{fb}$ *for all* ω*.*

2. *Money policy tightening decreases the level of participation because it increases the relative attractiveness of the outside option. Recall that* $\hat{\omega}(\tilde{\omega}, r)$ *is implicitly defined by* $V(\hat{\omega}, \tilde{\omega}, r) = \frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}} \int_{\hat{p}(\hat{\omega}+\tilde{\omega},r)}^{1} [p(y_h - y_l) + y_l - (1+r)] h(p)dp = C$*. Increasing* r *reduces the value of screening, so for a given fixed cost of screening, banks need a higher amount of initial capital to make the screening process worth-while.*

■

4.2 Prudential Policy

We model capital regulations in the simple form of a leverage ratio. Regulators impose a minimum capital requirement ω_{reg} such that banks with endowments $\omega_i < \omega_{reg}$ must seek outside equity of at least $(\omega_{reg} - \omega_i)$ or are barred from lending. ω_{reg} is announced as soon as banks find out their individual realisations of ω_i. A strengthening of capital requirements achieves the same qualitative effect as a tightening of monetary policy, albeit through slightly different means.

Proposition 5 *Capital regulations improve prudence by forcing some banks to seek additional equity funding, but do so at the cost of decreased participation:*

1. $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$*. Only banks that are constrained by the capital regulations will seek equity injections from households. And when they do, they will only top-up to the minimum capital standard.*

2. $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} \geq 0$ *with inequality as long as* $\hat{\omega}(0, r) < \omega_{reg} < \omega^*$*. A capital requirement will improve the prudence level of banks for which it binds, so long as they are not already "fully capitalised'.*

3. $\hat{\omega}(\tilde{\omega} = \omega_{reg} - \omega, r) > \hat{\omega}(\tilde{\omega} = 0, r)$ *for all* ω_{reg} *such that* $\omega_{reg} > \hat{\omega}(0, r)$*. The participation threshold is higher for banks that require external equity injections (i.e. banks that falls short of the capital requirement), than for banks that are not bound by the capital requirement.*

4. $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} > 0$ *. A tightening of capital standards increases the threshold for participation.*

Proof. *We outline the proof here, details are in the annex.*

1. *$\tilde{\omega} = \max[0, \omega_{reg} - \omega]$ follows directly from the observation that banks weakly prefer debt to equity (proposition 2). Consequently, banks for which the regulations are binding will only top-up their capital to the regulatory minimum, and seek the remaining funds in the form of debt.*

2. *$\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} \geq 0$ is a corollary of proposition 1 ($\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \tilde{\omega}} \geq 0$) and the previous observation that $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$. In particular, $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} = 0$ for banks with $\omega_i > \omega_{reg}$ or $\omega_i \geq \omega^*$. In the former case, banks with $\omega_i > \omega_{reg}$ are not in violation of the capital requirement, and thus will remain unaffected by a marginal increase in ω_{reg}. In the latter case, banks with $\omega_i \geq \omega^*$ already operate at the optimal level of prudence, so pushing ω_{reg} above ω^* does not bring any further gains in prudence.*

3. *$\hat{\omega}(\omega_{reg} - \omega, r) > \hat{\omega}(0, r)$ holds because equity funding is more costly than debt funding due to the dilution effect (proposition 2). Therefore, the value of screening is lower for banks that are constrained by the capital requirement. (ω_{reg} is announced in Stage 1, so banks can anticipate capital needs in advance of screening.) Consequently, the presence of binding capital regulations increases the participation threshold for a given fixed cost of screening.*

4. *$\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$ follows from part 1 and 3. Higher capital requirement increases the size of equity injection required; and thus reduces the value of screening and increases the threshold for participation.*

■

4.3 Summary of Policy Implications

A tightening of monetary policy – an increase in the risk-free interest rate - increases the opportunity cost of risky investment. As Figure 4 illustrates, this pushes up prudence for all banks at all levels of initial endowments. The prudence curve shift upwards, so poorly endowed (or highly leveraged banks) becomes more prudent, but very well capitalised banks may become too cautious[6]. A strengthening of capital standards forces more banks to seek out more external equity. This pushes banks along the curve – forcing them to behave as if they are a better

[6] This side effect falls away if we restrict the upper support of ω to a value significantly below 1.

endowed bank. Unlike monetary policy, capital regulations will not lead to some banks being too cautious, but it only affects banks for which the regulation is binding.

Figure 5 demonstrates the effect of the two policy instruments on participation. Because external equity is more costly than debt, tougher capital requirements makes the break-even level of initial endowment higher for a given fixed cost of screening. So more banks will drop out from screening due to a low level of initial endowments. This is the disadvantage of using capital regulations. A tightening of monetary policy shares the same drawback. A higher r raises the participation threshold for all banks.

So the trade-off between 'prudence' and 'participation' is common across both monetary policy and capital regulations. But there are two important differences. First, monetary policy distorts incentives for all banks; whereas capital regulations only affect those banks for which it is binding (i.e. poorly capitalised banks). Second, a loosening in policy rates can increase participation beyond the level possible in the absence of any intervention (and thus gets closer to the state of universal participation under the first-best). In contrast, capital regulations can be at best non-binding. This means capital requirements alone can never address the participation distortions relative to the first best.

4.4 Aggregate Economic Variables

4.4.1 The (expected) level of bank lending[7]: I

In the presence of a binding regulatory regime, (where $\omega_{reg} > \hat{\omega}(\omega_{reg} - \omega, r) > \hat{\omega}(0, r)$), the expected amount of bank lending (at the start of the first period) is given by:

$$I(r, \omega_{reg}) = \mu \begin{bmatrix} \int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]} (1 - H(\hat{p}(\omega_{reg}, r)) dF(\omega) + \\ \int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}} (1 - H(\hat{p}(\omega, r))) dF(\omega) \end{bmatrix} \text{ if } \omega_{reg} > \hat{\omega}(0, r) \quad (15)$$

where

- ω_{ub} is the upper bound for the distribution of ω;

[7] We look at the expected level of bank lending at the start of the game (prior to stage 1), before realisations of ω_i and p_i are drawn.

- $\int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]}(1-H(\hat{p}(\omega_{reg},r))dF(\omega)$ is the amount of lending expected from banks whose initial endowment fell short of the capital regulations (and hence need to raise additional equity); and
- $\int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}}(1-H(\hat{p}(\omega,r)))dF(\omega)$ is the amount of lending expected from banks that are not affected by the capital regulations (and hence can finance itself with debt and initial endowments only).

In the absence of prudential regulation [or if the regulation is completely non-binding: $\omega_{reg} \leq \hat{\omega}(0,r)$], the expected level of bank lending is given by:

$$I(r) = \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} (1-H(\hat{p}(\omega,r)))dF(\omega) \tag{16}$$

The expected level of bank lending under the first best is given by:

$$I_{fb} = \mu(1-H(\hat{p}_{fb})) \text{ where } \hat{p}_{fb} = \frac{1-y_l}{(y_h-y_l)} \tag{17}$$

A comparison of these equations shows that whilst both monetary policy and capital requirements can be used to push up the prudence threshold of banks to a level that is closer to the first-best, this can only be achieved with a decline in the volume of lending. Part of the fall in bank lending is socially desirable - banks with low p_i should rightly give up their lending project after screening. What is not desirable is the decline in bank lending due to falling participation. When capital requirements and the stance of monetary policy are tough, more banks may choose to give up on their projects even before the screening stage. This fall in participation is sub-optimal because ω and p are assumed to be independently distributed. Banks with low initial endowments (low ω_i) may be disincentivised from screening by the regulatory environment, even though they might discover a very good projects (with high p_i) if they did.

This trade-off between prudence and participation can lead to changes in the expected level of aggregate output.

4.4.2 The (expected) level of aggregate output: Q

Under a binding regulatory regime, $\omega_{reg} > \hat{\omega}(\omega_{reg} - \omega, r) > \hat{\omega}(0, r)$, the expected level of aggregate output is given by:

$$\begin{aligned} Q(\omega_{reg}, r) &= 1 + \mu \int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]} \left[\int_{\hat{p}(\omega_{reg},r)}^{1} \{p(y_h - y_l) + y_l - 1)\} h(p) dp \right] dF(\omega) \\ & + \mu \int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}} \left[\int_{\hat{p}(\omega,r)}^{1} [p(y_h - y_l) + y_l - 1] h(p) dp \right] dF(\omega) \\ & - \mu C \left[\int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\omega_{ub}} f(\omega) d\omega \right] \end{aligned} \quad (18)$$

where $Q(\omega_{reg}, r)$ is composed of:

- The (base) return from the risk-free deposit facility[8]: 1
- plus the surplus return from bank lending projects financed by a mixture of debt and external equity:
  $\mu \int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]} \left[\int_{\hat{p}(\omega_{reg},r)}^{1} \{p(y_h - y_l) + y_l - 1)\} h(p) dp \right] dF(\omega)$
- plus the surplus return from bank lending projects financed by debt:
  $\mu \int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}} \left[\int_{\hat{p}(\omega,r)}^{1} [p(y_h - y_l) + y_l - 1] h(p) dp \right] dF(\omega)$; and
- minus the cost of screening by banks $\mu C \left[\int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\omega_{ub}} f(\omega) d\omega \right]$.

In the absence of capital regulations (or when it is entirely non-binding), the expected level of aggregate output is given by:

$$Q(r) = 1 + \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} \left[\int_{\hat{p}(\omega,r)}^{1} [p(y_h - y_l) + y_l - 1] h(p) dp \right] dF(\omega) - \mu C \left[\int_{\hat{\omega}(0,r)}^{\omega_{ub}} f(\omega) d\omega \right] \quad (19)$$

And lastly, the aggregate output in the first best (where every bank screens, and only proceeds if $p_i \geq \hat{p}_{fb} = \frac{1-y_l}{(y_h - y_l)}$) is given by:

[8] Note that the rate of remuneration on the safe deposits r, is irrelevant for aggregate output considerations becaus the central bank need to recoup its costs through lump sum taxation ex post. So r only affects aggregate output through its effect on the prudence and participation threhsold ($\hat{p}$ and $\hat{\omega}$ respectively).

$$Q_{fb} = 1 + \mu \int_{\hat{p}_{fb}}^{1} \left[p\left(y_h - y_l\right) + y_l - 1 \right] h(p) dp - \mu C \tag{20}$$

4.5 Main Results

Proposition 6 $\frac{\partial Q(\omega_{reg}, r)}{\partial \omega_{reg}} > 0$, *when* $r = 0$ *and* $\omega_{reg} = \hat{\omega}(0, r)$. *Capital regulation (when employed alone, in the absence of monetary policy) can increase the expected level of aggregate output.*

Proof. *The full proof is shown in the appendix. To see why this result hold, we can decompose the impact of binding prudential regulation into two parts:*

1. *The gains in expected surplus from lending due to increased prudence in banks that are constrained by the regulation.*

   ... minus

2. *The net costs of falling participation (foregone surplus from banks with* $\omega_i \in [\hat{\omega}(0, r), \hat{\omega}(\omega_{reg} - \omega_i, r))$ *that drops out of participation, net of the screening costs saved).*

If we start from the position of no policy interventions, the bank that is operating at the margin of participation, with $\omega_i = \hat{\omega}(\tilde{\omega} = 0, r = 0)$, *is expected to deliver a surplus of exactly* C *(by the definition of* $\hat{\omega}$*). Therefore the net cost of having bank i fall out of participation is exactly zero, and the introduction of capital regulations can deliver an improved outcome.* ∎

Proposition 7 $\frac{\partial Q(\omega_{reg}, r)}{\partial r} > 0$ *when* $r = 0$, *and* $\omega_{reg} = \hat{\omega}(0, r)$. *A tightening of monetary policy (when employed alone, in the absence of binding capital regulations) can increase the expected level of aggregate output.*

Proof. *The full proof of this proposition is shown in the annex; but the broad intuition is the same as in the previous proposition: starting from the position of no policy interventions, the cost of falling participation at the margin is exactly offset by the saving in the fixed cost of saving. Aggregate output is increased as a result of the improvement in prudence.* ∎

Proposition 8 $\frac{dr^*(\omega_{reg})}{d\omega_{reg}} < 0$, *and* $\frac{d\omega^*_{reg}(r)}{dr} < 0$. *The optimal level of central bank rate (capital requirement) falls as capital requirement (central bank rate) is tightened.*

Proof. *[Still need to complete the formal proof]*

$r^*(\omega_{reg})$ *is defined (implicitly) by:* $\frac{\partial Q(\omega_{reg},r)}{\partial r} = 0$

$\omega^*_{reg}(r)$ *is defined (implicitly) by:* $\frac{\partial Q(\omega_{reg},r)}{\partial \omega_{reg}} = 0$

So by the implicit function theorem: $\frac{dr^*(\omega_{reg})}{d\omega_{reg}} = -\frac{\left(\frac{\partial^2 Q(\omega_{reg},r)}{\partial r \partial \omega_{reg}}\right)}{\left(\frac{\partial^2 Q(\omega_{reg},r)}{\partial r^2}\right)}$; $\frac{d\omega^*_{reg}(r)}{dr} = -\frac{\left(\frac{\partial^2 Q(\omega_{reg},r)}{\partial \omega_{reg} \partial r}\right)}{\left(\frac{\partial^2 Q(\omega_{reg},r)}{\partial \omega^2_{reg}}\right)}$

It is therefore sufficient to show that: a) each policy instrument exhibit diminishing return; and b) the marginal impact of any given instrument declines as the other instrument is tightened (i.e. substitutability between instruments). ■

5 Numerical Simulation

Numerical simulations can help to illustrate the degree of trade-off between monetary policy and capital requirements as predicted by the model. Nevertheless, it is important to note that the model we have outlined above is necessarily a significant simplification of the financial intermediation process. A precise calibration of the model is not straight-forward because there are no direct empirical counterparts to the key parameters of the model. For the benchmark calibrations illustrated here we chose parameter values that we felt were broadly reflective of the UK banking sector.

The key parameters of the model and our benchmark calibrations can be summarised as follows:

- **The fundamentals of the bank lending project**: This include: (i) the payoff of the risky lending project in the good state and in the bad state, y_h and y_l respectively; (ii) the random variable for the probability of success of the project p; and (iii) the fixed cost of screening C (for banks to find out its realisation of p_i). As described in the previous sections, the model imposes a few restrictions on the value these parameters can take. First, the unconditional expected return from the project must be less than the risk-free return, $E(p)(y_h - y_l) + y_l < 1 + r$, such that it would not be optimal for banks to undertake the

project in the absence of screening. Second, the expected value-added from the project, conditional on all banks acting prudently, needs to exceed the cost of screening: $C < V(\hat{p}_{fb}) = \int_{\hat{p}_{fb}}^{1} [p(y_h - y_l) + y_l - (1+r)]\, h(p)\, dp$. This second condition ensures that it would be socially optimal for every lending project to be screened.

- **The significance of the banking sector to the overall economy**: This is partly determined by the fundamentals of the bank lending project as set out above and partly by the parameter μ (see equation 15 and 18 for aggregate lending and output respectively). The literal definition of μ from the theoretical set-up in the previous sections is the proportion of banks relative to households (or the proportion of agents with access to the screening/lending technology). But because μ does not affect the behavioural equations (on 'prudence' and 'participation'), and only enters as a scalar multiplier in the equations for aggregate lending and output, we can treat it as a proxy for the size of the banking sector when it comes to numerical calibrations.

  In the benchmark calibration that follows (see Table 1 below), we choose a combination of $\{y_h\ ,\ y_l\ ,\ p,\ \mu\}$ that delivers a banking sector that adds around 5% to aggregate output, a figure that broadly corresponds to the average GDP weight of the UK banking sector between 1997 and 2013[9].

- **The distribution of banks' initial endowments** ω: Specifically, ω represents the distribution of banks' equity capital in the counterfactual scenario where there are no capital regulations at all. We assume for the benchmark calibration that ω is distributed uniformly between 0 and 0.1; in other words, in the absence of any regulatory requirements, the most well-capitalised bank would hold equity that equate to 10% of its total assets (or be leveraged 10 times).

- **Frictions in financial intermediation and policy intervention**: It is costly for banks to undertake screening, and for a central bank to adopt a super-aggressive monetary policy stance purely for the sake of reining in excessive risk-taking in financial intermediation. For the benchmark calibration, we set the fixed cost of screening C to 0.01, or 1% of the funds

[9] See the Annual GDP weights publication from the Office of National Statistics, series KZZ8: Financial service activities, except insurance & pension funding.

Table 1: Benchmark Calibrations

Parameters	**Benchmark Calibrations**
The payoff from risky bank lending in the good state (y_h)	1.4
The payoff from risky bank lending in the bad state (y_l)	0.5
The probability of success on bank lending projects (p)	p ˜ $U[0,1]$
The proportion of banks relative to households - a proxy for the significance of the banking sector to the overall economy (μ)	1
The intial endowment of banks (ω)	ω ˜ $U[0,0.1]$
The cost of screening for banks (c)	0.01
The cost of using monetary policy to address financial stability concerns (α)	2.5%

lent. In the theoretical set-up of the model, we assumed that the central bank could affect banks' lending behaviour through influencing the attractiveness of the outside option (by changing the remuneration rate r on the safe deposit facility), and then recoup its costs through lump sum taxation across all agents. But if the central bank were to raise revenue in reality, the effect is likely to have some distortionary impacts. So for the calibration we introduce a small deadweight cost to using monetary policy, which takes the form of

$$\alpha r^2 (1 - I) \tag{21}$$

where $(1 - I)$ is the total amount of funds deposited with the central bank in the safe facility, and r is the policy rate (or the rate of remuneration on these deposits). The deadweight cost associated with monetary policy action is quadratic in the policy rate used, and we set α to 2.5% in the benchmark case. This ensures that the cost of using monetary policy is negligible when r is close to the 'neutral stance' of monetary policy (when $r = 0$), but increase progressively when r deviates from its neutral level. The aim of introducing this deadweight cost is to penalise scenarios where r is set to extreme levels (e.g. 6-7% above the neutral level), because such monetary policy settings will likely have knock on impact on other sectors of the economy.

Table 2: Outcomes under selected policy combinations

Aggregate output under different policy choices		Capital Requirement							
		0%	3%	5%		10%		**18%**	20%
Monetary Policy Stance (Deviation from neutral)	**0.25%**	1.037560	1.039474	1.041941		1.048553		1.053035	1.052996
	0.50%	1.038424	1.040186	1.042547		1.048902		1.053068	1.052968
	0.75%	1.039268	1.040878	1.043132		1.049226		**1.053072**	1.052910
	..								
	3.25%	1.046354	1.046566	1.047670		1.050971		1.051366	1.050540
	3.50%	1.046903	1.046989	1.047974		1.050978		1.051006	1.050108
	3.75%	1.047419	1.047382	1.048246		1.050952		1.050608	1.049640
	..								
	5.50%	1.049998	1.049200	1.049203		1.049757		1.046718	1.045226
	5.75%	1.050205	1.049313	1.049192		1.049431		1.045994	1.044425
	6.00%	1.050368	1.049386	1.049141		1.049063		1.045226	1.043578
	6.25%	1.050486	1.049418	1.049049		1.048653		1.044412	1.042686
	6.50%	1.050557	1.049409	1.048915		1.048200		1.043553	1.041747
	6.75%	1.050582	1.049357	1.048738		1.047701		1.042646	1.040759

5.1 Main results under benchmark calibration

With a numerically calibrated model we can show the effect on aggregate output of different combinations of monetary policy stance and regulatory capital requirements. Figure 6 and Table A1 in the annex illustrate the full spectrum of results for ω_{reg} between 0% and 23% (step size of 1 percentage point), and r between -0.5% and +8% (step size of 25 basis points). For brevity, Table 2 shows the outcomes under a selection of these possible policy combinations.

Table 2 shows that the optimal stance of monetary policy loosens as regulatory capital standard strengthens[10]. In particular, under this benchmark calibration, the central bank would need to set interest rates at 6.75 percentage points above the 'neutral stance' ($r = 0$) if there are no capital requirement for banks. The optimal stance of monetary policy falls to $r = 3.5\%$ when capital requirement increases to 10% of total assets, which given ω is assumed to be distributed between 0 and 0.1, means a leverage ratio that is binding for all but the most capitalised bank in the population. The globally optimal outcome is achieved when capital requirement is set to 18% of total assets and monetary policy stance is 0.75 percentage points above the neutral level. For no monetary policy interventions to be (locally) optimal, capital requirement need to increase to 21%.

[10]Note to Table 2: Highlighted cells (in yellow) shows the aggregate output that result from the optimal monetary policy stance for a given level of capital requirement (i.e. the highlighted cell is the highest number in each column). Red and bold font indicates the globally optimal level of capital requirement and monetary policy stance, and the associated level of aggregate outcome.

Table 3: Outcomes under different scenarios

Scenarios	**Participation Threshold** ($\hat{\omega}$)	**Prudence Threshold** ($\hat{p}$)	**Probability of failure for projects undertaken** $(1 - A(\hat{p}))$	**Aggregate Lending** (I)	**Aggregate Output** (Q)
Optimal Policy (r = 0.75% and ω_{reg} = 18%)	0.0239	0.41	0.30	0.45	1.0531
First Best	0	0.56	0.22	0.44	1.0789
No Policy Intervention	0.0037	Varies with ω_i; average = 0.25	0.38	0.71	1.0367

The model also allows us to describe banks' behaviour under each set of policy setting. Under the globally optimal combination of policy ($r = 0.75\%$ and $\omega_{reg} = 18\%$), capital regulation becomes binding for all banks in the population. The participation threshold $\hat{\omega}$ equals 0.0239, meaning that 23.9% of banks with access to the risky lending opportunity and the screening technology would choose not to screen and forego the prospect of 'actually becoming a bank'. For banks that do screen, they will proceed with the lending project if and only if the probability of success exceeds the prudence threshold of $\hat{p} = 0.41$. The conditional expectation of success is therefore $E(p|p > \hat{p} = 0.41) = 0.705$, such that around 30% of the projects that are taken forward are expected to fail. If one were to take the '1 project per bank' interpretation of the model literally, this would imply that in a perfectly competitive banking sector (with lots of small but identical banks), 30% of banks are expected to fail when monetary policy and capital regulations take their optimal value. Aggregate lending in this case is around 0.45 and aggregate output is around 1.053.

Table 3 compares the outcomes under optimal policy to those under the first-best scenario and the case of no policy-intervention.

Note that although the banks in our model are heterogeneous at the start (they take independent draws from the distribution of ω), having a minimum capital requirement that exceeds the upper bound of the ω distribution (i.e. $\omega_{reg} \geq \omega_{ub}$) imposes homogeneity amongst those banks that do proceed with the lending project. All banks that proceed will top-up

their equity capital to the minimum level required ($\omega_{reg} = 18\%$ in the optimal case) and fund the remainder through debt. These banks therefore share a common prudence threshold: e.g. $\hat{p}(\omega + \tilde{\omega} = 0.18, r = 0.0075) = 0.41$. In contrast, if ω_{reg} was lower than ω_{ub} (e.g. $\omega_{reg} = 5\%$), then we would observe a cluster of banks with capital equal to the regulatory minimum, as well as some banks with a surplus of capital for regulatory purposes (having started with a better endowment). Since the prudence threshold $\hat{p}$ is a function of capital ω, banks would no longer behave in an homogeneous fashion.

Another interesting observation from Table 2 is that the policy combination of no capital regulation but optimally set monetary policy $\{r = 6.75\%, \omega_{reg} = 0\}$ delivers a better outcome than cases where monetary policy responds optimally to very low capital requirements (e.g. $\{r = 6.25\%, \omega_{reg} = 3\%\}$ or $\{r = 5.5\%, \omega_{reg} = 5\%\}$. In fact, if monetary policy always responds optimally to the level of capital requirements, we need $\omega_{reg} \geq 10\%$ under the benchmark calibration for aggregate output to exceed the case of no capital regulations.

That is not to say that low-to-modest capital requirements are worse than useless. When the official interest rates is *not* set optimally to address the excessive risk-taking behaviour in the banking sector (e.g. when a loose policy stance is required to stimulate aggregate demand) then a low capital requirement can be an improvement to the case of no capital requirements at all. For instance, if $r = 0.5\%$ then moving from $\omega_{reg} = 0$ to $\omega_{reg} = 3\%$ can increase aggregate output. It is also possible that we have underestimated the distortionary costs of using monetary policy under the benchmark calibration. If we had instead took a value for α greater than 2.5%, then the optimal interest rate in the absence of capital regulations falls below 6.75%, and small increases in capital requirements makes a bigger impact. A higher α gives a bigger role to capital requirements.

The exact numerical result will differ across alternative calibrations, but the qualitative results remain quite robust, namely that: monetary policy and capital regulations function as imperfect substitutes; these policy interventions can trade-off between prudence and participation of banks; and as long as there is a non-negligible cost to using monetary policy in this fashion, the global optimum is one where r is close to neutral, and capital regulation is binding for the vast majority of the banking population (ω_{reg} close to or above 10%).

6 Concluding Remarks

We find that the appropriate equilibrium level of central bank policy rate falls as capital regulation is tightened. Numerical simulations of our model can help to illustrate, and to broadly quantify, the magnitude of this interaction between monetary policy and capital regulation.

Nevertheless, it is important to note that the (imperfect) substitutability of monetary policy and capital regulations does not imply that there will never be a case where it is optimal to raise both interest rates and capital requirements at the same time. If the economy starts from a point far away from its 'bliss point', with both sub-optimally low capital requirements and ultra loose monetary policy, then aggregate welfare can be improved by tightening both. This is consistent with the consensus that central banks should increase policy rates from their post-crisis lows as economies return to normality, even if prudential requirements strengthens at the same time. Our key result implies that, ceteris paribus, monetary policy should return to a normal that is lower than what was appropriate before the crisis, when the regulatory standards for banks were less stringent.

References

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A Annex

A.1 Proof to Proposition 1 - Prudence Threshold

A.1.1 Proposition 1.1

The proof for $\omega^* = 1 - \frac{y_l}{(1+r)}$ follows directly from re-arranging equation 6, which also shows that banks with $\omega_i + \tilde{\omega}_i = \omega^*$ can borrow at the risk-free rate $1+r$. For banks with $\omega_i + \tilde{\omega}_i > \omega^*$, their funding shortfall and thus their gross debt obligation is lower: $R(\omega_i + \tilde{\omega}_i, r) < R(\omega^*, r) = y_l$ (since $\frac{\partial R(\omega_i + \tilde{\omega}_i, r)}{\partial \omega} < 0$, a result we will show in the proof to part 2 and 3 of the proposition below). Though the rate at which the debt is provided is still floored at $1 + r$, again from re-arranging equation 6.

A.1.2 Proposition 1.2 and 1.3

1. **Step 1: Assert (for now) that** $\frac{\partial R(\omega_i + \tilde{\omega}_i, r)}{\partial \omega} < 0$ **for all** $\omega_i + \tilde{\omega}_i \in [0, 1]$[11]. Since ω^* is defined as the level of endowments such that $y_l = R(\omega^*, r)$, $\frac{\partial R(\omega_i + \tilde{\omega}_i, r)}{\partial \omega} < 0$ means $y_l > R(\omega_i + \tilde{\omega}_i, r)$ for all $\omega_i + \tilde{\omega}_i > \omega^*$.

2. **Step 2: Given** $\frac{\partial R(\omega_i + \tilde{\omega}_i, r)}{\partial \omega} < 0$**, show that for all** $\omega_i + \tilde{\omega}_i \geq \omega^*$**:** $\hat{p}(\omega_i + \tilde{\omega}_i, r) = \frac{(1+r) - y_l}{(y_h - y_l)} = \hat{p}_{fb}$ [Banks are prudent when they are 'fully capitalised' and bear all the downside risks]

   For banks with $\omega_i + \tilde{\omega}_i \geq \omega^*$, $\frac{\partial R(\omega_i + \tilde{\omega}_i, r)}{\partial \omega} < 0$ implies liquidation value exceeds debt obligations $y_l \geq R(\omega_i + \tilde{\omega}_i, r)$. So equation 5 and equation 6 becomes:

   $\hat{p}(y_h - R(\omega + \tilde{\omega}, r)) + (1 - \hat{p})(y_l - R(\omega + \tilde{\omega}, r)) = (\omega + \tilde{\omega})(1 + r)$, or $\hat{p}(y_h - y_l) + y_l - R(\omega + \tilde{\omega}, r) = (\omega + \tilde{\omega})(1 + r)$; and

   $A(\hat{p}) R(\omega + \tilde{\omega}, r) + (1 - A(\hat{p})) R(\omega + \tilde{\omega}, r) = (1 - \omega - \tilde{\omega})(1 + r)$, or $R(\omega + \tilde{\omega}, r) = (1 - \omega - \tilde{\omega})(1 + r)$ [households are willing to provide debt at the risk free rate because the downside risks are borne by the equity holders].

   Note that $\frac{\partial R(\omega + \tilde{\omega}, r)}{\partial \omega} = \frac{\partial R(\omega + \tilde{\omega}, r)}{\partial \tilde{\omega}} = -(1 + r) < 0$ (for banks with $\omega_i + \tilde{\omega}_i > \omega^*$).

[11][Intuitively: the gross amount of debt obligations (principal plus interest) falls when the bank has more endowments and need to borrow less.

Substituting equation 5 into equation 6 then gives: $\hat{p}\left(y_h - y_l\right)+y_l = (1+r)$, or $\hat{p}(\omega+\tilde{\omega},r) = \frac{(1+r)-y_l}{(y_h-y_l)}$ (for banks with $\omega + \tilde{\omega} \geq \omega^*$). The optimal degree of prudence is achieved when banks are 'fully capitalised' and limited liability is irrelevant.

3. **Step 3: Given** $\frac{\partial R(\omega_i+\tilde{\omega}_i,r)}{\partial \omega} < 0$**, show that for** $\omega_i + \tilde{\omega}_i < \omega^*$**:** $\frac{\partial \hat{p}(\omega_i+\tilde{\omega}_i,r)}{\partial \omega} > 0$. [Banks are less prudent when they have less skin-in-the-game]

   For banks with $\omega_i + \tilde{\omega}_i < \omega^*$, $\frac{\partial R(\omega_i+\tilde{\omega}_i,r)}{\partial \omega} < 0$ implies debt obligation exceeds liquidation value $y_l < R\left(\omega_i + \tilde{\omega}_i, r\right)$. Therefore equation 5 and equation 6 becomes:

   $\hat{p}\left(y_h - R\left(\omega + \tilde{\omega}, r\right)\right) = \left(\omega + \tilde{\omega}\right)\left(1 + r\right)$; and

   $A\left(\hat{p}\right) R\left(\omega + \tilde{\omega}, r\right) + \left(1 - A\left(\hat{p}\right)\right) y_l = (1 - \omega - \tilde{\omega})(1 + r)$

   Partially differentiate both equations wrt ω gives:

   $\frac{\partial \hat{p}}{\partial \omega} = \frac{(A(\hat{p})-\hat{p})(1+r)}{A(\hat{p})(y_h-R)+\hat{p}A'(\hat{p})(R-y_l)} > 0$ (for banks with $\omega < \omega^*$), since $\left(A\left(\hat{p}\right) - \hat{p}\right) > 0$, $A'\left(\hat{p}\right) > 0$ (both by definition of $A\left(\hat{p}\right)$ as the conditional expectation of p for $p > \hat{p}$) and $y_h > R > y_l$.

   $\frac{\partial R}{\partial \omega} = -\frac{\left[A'(\hat{p})(R-y_l)+(y_h-R)\right](1+r)}{A(\hat{p})(y_h-R)+\hat{p}A'(\hat{p})(R-y_l)} < 0$ (for banks with $\omega < \omega^*$).

4. **Step 4: complete the proof by showing that** $\frac{\partial R(\omega_i+\tilde{\omega}_i,r)}{\partial \omega} < 0$ **for all** $\omega_i + \tilde{\omega}_i \in [0, 1]$**.** (Note that 1 is the maximum value ω_{ub} can take).

   Proof by contradiction:

   Suppose that $\frac{\partial R(\omega_i+\tilde{\omega}_i,r)}{\partial \omega} \geq 0$ for some $\omega_i + \tilde{\omega}_i \in [0, 1]$.

   Suppose further that for this $\omega_i + \tilde{\omega}_i$, $y_l \geq R\left(\omega_i + \tilde{\omega}_i, r\right)$, then as per step 2 we have shown that $\frac{\partial R(\omega_i,r)}{\partial \omega} = -(1 + r) < 0$; contradiction.

   Suppose instead that for this $\omega_i + \tilde{\omega}_i$, $y_l \leq R\left(\omega_i + \tilde{\omega}_i, r\right)$, then as per step 3 we have shown that $\frac{\partial R(\omega_i,r)}{\partial \omega} = -\frac{\left[A'(\hat{p}_d)(R-y_l)+(y_h-R)\right](1+r)}{A(\hat{p}_d)(y_h-R)+\hat{p}_d A'(\hat{p}_d)(R-y_l)} < 0$; contradiction.

   Therefore $\frac{\partial R(\omega_i+\tilde{\omega}_i,r)}{\partial \omega} < 0$ for all $\omega_i + \tilde{\omega}_i \in [0, 1]$. Q.E.D.

A.2 Proof to Proposition 2 - Preference for Debt.

Part 1 of the proposition follows directly from the definition of $\hat{p}(\omega + \tilde{\omega}, r)$ as the 'prudence threshold' for banks (see discussion in main body).

For part 2 of the proposition, we show that for any given level of capital (initial endowment plus equity injection) banks weakly prefer to fund the remainder of the project through debt rather than additional equity :

1. For banks with capital $\omega+\tilde{\omega} \geq \omega^*$, a weak preference for debt requires $(1-\omega-\tilde{\omega})\left[p\left(y_h-y_l\right)+y_l\right] \geq R\left(\omega+\tilde{\omega}, r\right)$ [banks expect to pay out more to new equity providers than to debt providers]. This condition holds because from Proposition 1 we have $R\left(\omega+\tilde{\omega}, r\right)=(1-\omega-\tilde{\omega})(1+r)$, and $\hat{p}(\omega+\tilde{\omega}, r)=\frac{(1+r)-y_l}{(y_h-y_l)}$, when $\omega+\tilde{\omega}>\omega^*$; and because from part 1 of Proposition 2 banks that choose to invest have $p \geq \hat{p}(\omega+\tilde{\omega}, r)$. [Such that $(1-\omega-\tilde{\omega})\left[p\left(y_h-y_l\right)+y_l\right] \geq (1-\omega-\tilde{\omega})\left[\hat{p}(\omega+\tilde{\omega}, r)\left(y_h-y_l\right)+y_l\right]=(1-\omega-\tilde{\omega})(1+r)=R\left(\omega+\tilde{\omega}, r\right)$].

2. For banks with capital $\omega+\tilde{\omega}<\omega^*$, a weak preference for debt requires $p\left(y_h-R\left(\omega+\tilde{\omega}, r\right)\right) \geq \frac{\omega+\tilde{\omega}}{1}\left[y_l+p\left(y_h-y_l\right)\right]$ for any $p \geq \hat{p}(\omega+\tilde{\omega}, r)$ [i.e. the bank expects a higher return from using debt funding than using equity funding]. We show that this condition holds in two steps:

   (a) Step 1: Show that $p\left(y_h-R\left(\omega+\tilde{\omega}, r\right)\right)>(\omega+\tilde{\omega})\left[y_l+p\left(y_h-y_l\right)\right]$ for any $p \in\left[\hat{p}(\omega+\tilde{\omega}, r), \hat{p}_{fb}\right]$ when $\omega+\tilde{\omega}<\omega^*$.
   Recall from proposition 1 that $\hat{p}(\omega+\tilde{\omega}, r)<\hat{p}_{fb}=\frac{(1+r)-y_l}{(y_h-y_l)}$ when $\omega+\tilde{\omega}<\omega^*$; and from the bank's indifference condition when using debt (equation 5): $\hat{p}(\omega+\tilde{\omega}, r)\left(y_h-R\left(\omega+\tilde{\omega}, r\right)\right)=(\omega+\tilde{\omega})(1+r)$. On the hand, if the bank decided to top-up the remainder using additional equity, then the entirety of its project would becomes fully capitalised, so we have $\hat{p}(1, r)=\hat{p}_{fb}=\frac{(1+r)-y_l}{(y_h-y_l)}$, such that $(\omega+\tilde{\omega})\left[y_l+\hat{p}_{fb}\left(y_h-y_l\right)\right]=(\omega+\tilde{\omega})(1+r)$. Combining the two indifference conditions we have $\hat{p}(\omega+\tilde{\omega}, r)\left(y_h-R\left(\omega+\tilde{\omega}, r\right)\right)=(\omega+\tilde{\omega})\left[y_l+\hat{p}_{fb}\left(y_h-y_l\right)\right]$. So for any $p \in\left[\hat{p}(\omega+\tilde{\omega}, r), \hat{p}_{fb}\right]$, we have $p\left(y_h-R\left(\omega+\tilde{\omega}, r\right)\right)>(\omega+\tilde{\omega})\left[y_l+p\left(y_h-y_l\right)\right]$.

   (b) Step 2: Show that as p increases, the return from debt financed investments increases faster than that for equity financed investments. In other words $\frac{\partial p(y_h-R(\omega+\tilde{\omega}, r))}{\partial p}>\frac{\partial(\omega+\tilde{\omega})[y_l+p(y_h-y_l)]}{\partial p}$, or $y_h-R\left(\omega+\tilde{\omega}, r\right)>(\omega+\tilde{\omega})\left(y_h-y_l\right)$, for any p.
   Recall from equation 5, when $\omega<\omega^*$, $y_h-R\left(\omega+\tilde{\omega}, r\right)=\frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}(\omega+\tilde{\omega}, r)}$. Substitute in $\hat{p}(\omega+\tilde{\omega}, r)<\frac{(1+r)-y_l}{(y_h-y_l)}$ from proposition 1 gives: $y_h-R\left(\omega+\tilde{\omega}, r\right)>\frac{(\omega+\tilde{\omega})(1+r)}{\left[\frac{(1+r)-y_l}{(y_h-y_l)}\right]}=$

$(\omega + \tilde{\omega}) (y_h - y_l) \frac{(1+r)}{(1+r)-y_l}$. Since by construction $1+r > y_l > 0$, we have $\frac{(1+r)}{(1+r)-y_l} > 1$ and therefore $y_h - R(\omega + \tilde{\omega}, r) > (\omega + \tilde{\omega}) (y_h - y_l)$.

Since for any given level of capital (initial endowment plus equity injection) banks weakly prefer to fund the remainder of the project through debt rather than additional equity, then by backward induction banks would prefer to not use any equity injections at all (i.e. set $\tilde{\omega} = 0$, and the fund the shortfall relative to initial endowment $(1 - \omega)$ using debt only). Q.E.D.

A.3 Value of Screening (equation 9)

We present the proof for the last line of equation 9:

$$\begin{aligned} V(\omega, \tilde{\omega}, r) &\equiv E_p \left[\max \left(\begin{array}{c} 0 \,, \\ \frac{\omega}{\omega+\tilde{\omega}} \left\{ \begin{array}{c} p \left(y_h - R(\omega+\tilde{\omega}, r)\right) + \\ (1-p) \max\left[0, y_l - R(\omega+\tilde{\omega}, r)\right] \end{array} \right\} - (\omega+\tilde{\omega})(1+r) \end{array} \right) \right] \\ &= \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p (y_h - y_l) + y_l - (1+r)\right] h(p) dp \end{aligned}$$

The last equality follows because household provide debt at the risk-free rate:

1. For fully capitalised banks with $\omega + \tilde{\omega} \geq \omega^* = 1 - \frac{y_l}{(1+r)}$:

   $V(\omega, \tilde{\omega}, r) = E_p \left[\max \left(0 \,, \frac{\omega}{\omega+\tilde{\omega}} \left\{ p\left(y_h - R(\omega+\tilde{\omega}, r)\right) + (1-p)\left(y_l - R(\omega+\tilde{\omega}, r)\right) \right\} - (\omega+\tilde{\omega})(1+r) \right) \right]$

   So from equation 5 we have:

   $V(\omega, \tilde{\omega}, r) = \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[\frac{\omega}{\omega+\tilde{\omega}} \left\{ p\left(y_h - R(\omega+\tilde{\omega}, r)\right) + (1-p)\left(y_l - R(\omega+\tilde{\omega}, r)\right) \right\} - (\omega+\tilde{\omega})(1+r) \right] h(p)dp$

   Which can be simplified to:

   $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p (y_h - y_l) + y_l - R(\omega+\tilde{\omega}, r) - (\omega+\tilde{\omega})(1+r)\right] h(p)dp$

   But when $\omega + \tilde{\omega} \geq \omega$, we know from proposition 1 that $R(\omega+\tilde{\omega}, r) = (1 - \omega - \tilde{\omega})(1+r)$

   So $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p (y_h - y_l) + y_l - (1+r)\right] h(p)dp$.

2. For banks with $\omega + \tilde{\omega} < \omega^{*}$:

$V(\omega, \tilde{\omega}, r) = E_p \left[\max \left(0 \, , \, \frac{\omega}{\omega+\tilde{\omega}} p \left(y_h - R \left(\omega + \tilde{\omega}, r \right) \right) - \left(\omega + \tilde{\omega} \right) \left(1 + r \right) \right) \right]$

Which can be simplified again to: $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p \left(y_h - R \left(\omega + \tilde{\omega}, r \right) \right) - \left(\omega + \tilde{\omega} \right) \left(1 + r \right) \right] h(p) dp$

Recall from equation 6 $A \left(\hat{p} \right) R \left(\omega + \tilde{\omega}, r \right) + \left(1 - A \left(\hat{p} \right) \right) y_l = \left(1 - \omega - \tilde{\omega} \right) \left(1 + r \right)$,

so $A \left(\hat{p} \right) \left(R \left(\omega + \tilde{\omega}, r \right) - y_l \right) + y_l = \left(1 - \omega - \tilde{\omega} \right) \left(1 + r \right)$. [Debt is provided by households at a zero-profit-basis].

By the definition of $A \left(. \right)$: $\left[\frac{1}{1-H(\hat{p})} \int_{\hat{p}}^{1} ph \left(p \right) dp \right] \left(R \left(\omega + \tilde{\omega}, r \right) - y_l \right) + y_l = \left(1 - \omega - \tilde{\omega} \right) \left(1 + r \right)$

Re-arranging gives: $\int_{\hat{p}}^{1} p \left(R \left(\omega + \tilde{\omega}, r \right) - y_l \right) h \left(p \right) dp = \left[\left(1 - \omega - \tilde{\omega} \right) \left(1 + r \right) - y_l \right] \left[\int_{\hat{p}}^{1} h \left(p \right) dp \right]$

because $R \left(. \right)$ is not a function of p, and ω and p are independently distributed.

So $\int_{\hat{p}}^{1} p \left(R \left(\omega + \tilde{\omega}, r \right) \right) h \left(p \right) dp = \int_{\hat{p}_d}^{1} \left[py_l - y_l + (1 - \omega - \tilde{\omega})(1 + r) \right] h \left(p \right) dp$

And $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p \left(y_h - y_l \right) + y_l - \left(1 + r \right) \right] h(p) dp$

3. Note that when $\tilde{\omega} = 0$ (i.e. in the absence of capital regulation):

$V(\omega, 0, r) = \int_{\hat{p}(\omega,r)}^{1} \left[p \left(y_h - y_l \right) + y_l - \left(1 + r \right) \right] h(p) dp.$

Note further that when $\tilde{\omega} = \omega_{reg} - \omega$ (i.e. when banks top-up to the regulatory minimum in the presence of capital regulation - see proposition 5), $V(\omega, \tilde{\omega} = \omega_{reg} - \omega, r) = \frac{\omega}{\omega_{reg}} \int_{\hat{p}(\omega_{reg},r)}^{1} \left[p \left(y_h - y_l \right) + y_l - \left(1 + r \right) \right] h(p) dp = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg} \, , \tilde{\omega} = 0 \, , r)$. So the value of screening for a bank which falls short of the regulatory capital requirement ($\omega_i < \omega_{reg}$) is a $\frac{\omega_i}{\omega_{reg}}$ fraction of the value of screening for a bank that is just meeting the regulatory requirement:

$$V(\omega, \tilde{\omega} = \omega_{reg} - \omega, r) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0, r) \tag{22}$$

and $\hat{\omega} \left(\tilde{\omega} = \omega_{reg} - \omega, r \right) = \frac{C \omega_{reg}}{V(\omega = \omega_{reg}, \tilde{\omega} = 0, r)}$.

A.4 Proof to Proposition 3.2 - Participation Threshold

Recall: $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p \left(y_h - y_l \right) + y_l - \left(1 + r \right) \right] h(p) dp$

Using the Product Rule and the Leibniz Integral Rule[12] we get:

[12] Leibniz Integral Rule:
$\frac{d}{dx} \int_{y_0}^{y_1} f(x, y) dy = \int_{y_0}^{y_1} f_x(x, y) dy + f(x, y_1) \frac{dy_1}{dx} - f(x, y_0) \frac{dy_0}{dx}$ provided f and f_x are both continuous over a region in the form $[x_0, x_1] \times [y_0, y_1]$

$\frac{\partial V(\omega,\tilde{\omega},r)}{\partial \omega} = \frac{\tilde{\omega}}{(\omega+\tilde{\omega})^2} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - (1+r)\right] h(p)dp - \frac{\omega}{\omega+\tilde{\omega}} \left[\hat{p}\left(y_h - y_l\right) + y_l - (1+r)\right] h(\hat{p}) \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega}$ ■

or

$\frac{\partial V(\omega,\tilde{\omega},r)}{\partial \omega} = \frac{1}{(\omega+\tilde{\omega})} \frac{\tilde{\omega}}{\omega} V(\omega,\tilde{\omega},r) - \frac{\omega}{\omega+\tilde{\omega}} \left[\hat{p}\left(y_h - y_l\right) + y_l - (1+r)\right] h(\hat{p}) \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega}$

So when $\tilde{\omega} = 0$, $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial \omega} = -\left[\hat{p}\left(y_h - y_l\right) + y_l - (1+r)\right] h(\hat{p}) \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega} \geq 0$

where the inequality holds because $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega} \geq 0$ with equality when $(\omega+\tilde{\omega}) \geq \omega^*$; and $\hat{p}\left(y_h - y_l\right) + y_l - (1+r) \leq 0$ with equality when $(\omega+\tilde{\omega}) \geq \omega^*$.

When $\tilde{\omega} = \omega_{reg} - \omega$ (see proposition 5), equation 22 shows $V(\omega, \tilde{\omega} = \omega_{reg} - \omega, r) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0, r)$, so $\frac{\partial V(\omega,\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial \omega} = \frac{1}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0, r) > 0$.

A.5 Proof of Proposition 4 - Transmission of Monetary Policy

A.5.1 Proposition 4.1: $\frac{\partial \hat{p}}{\partial r} > 0$

1. For banks with $\omega + \tilde{\omega} \geq \omega^*$, recall that $\hat{p}\left(\omega+\tilde{\omega},r\right) = \frac{(1+r)-y_l}{(y_h-y_l)}$, so $\frac{\partial \hat{p}}{\partial r} > 0$.

2. For banks with $\omega + \tilde{\omega} < \omega^*$ (and thus $y_l < R\left(\omega,r\right)$) we have from equation 5 and equation 6:

   $\hat{p}\left(y_h - R\left(\omega+\tilde{\omega},r\right)\right) = \left(\omega+\tilde{\omega}\right)(1+r)$; and

   $A\left(\hat{p}\right) R\left(\omega+\tilde{\omega},r\right) + \left(1 - A\left(\hat{p}\right)\right) y_l = (1-\omega-\tilde{\omega})(1+r)$.

   Partially differentiate both equations with respect to r gives:

   $\left(y_h - R\left(\omega+\tilde{\omega},r\right)\right) \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial r} - \hat{p}\left(\omega+\tilde{\omega},r\right) \frac{\partial R(\omega+\tilde{\omega},r)}{\partial r} = (\omega+\tilde{\omega})$; and

   $A'\left(\hat{p}\right) R\left(\omega+\tilde{\omega},r\right) \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial r} + A\left(\hat{p}\right) \frac{\partial R(\omega+\tilde{\omega},r)}{\partial r} - A'\left(\hat{p}\right) y_l \frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial r} = (1-\omega-\tilde{\omega})$

   So: $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial r} = \frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(\omega+\tilde{\omega},r)(1-\omega-\tilde{\omega})}{(y_h-R(\omega+\tilde{\omega},r))A(\hat{p})+\hat{p}(\omega+\tilde{\omega},r)A'(\hat{p})(R(\omega+\tilde{\omega},r)-y_l)} > 0$.

Q.E.D.

A.5.2 Proposition 4.2: $\frac{\partial \hat{\omega}}{\partial r} > 0$

Recall that $\hat{\omega}\left(\tilde{\omega},r\right)$ is implicitly defined by $V(\hat{\omega},\tilde{\omega},r) = \frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}} \int_{\hat{p}(\hat{\omega}+\tilde{\omega},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - (1+r)\right] h(p)dp = C$.

We know also from proposition 3.2 that $\frac{\partial V(\omega,\tilde{\omega},r)}{\partial \omega}\begin{cases} >0 \text{ for } \omega+\tilde{\omega}<\omega^* \\ =0 \text{ for } \omega+\tilde{\omega}\geq\omega^* \end{cases}$. So to show $\frac{\partial\hat{\omega}}{\partial r}>0$ [for $\hat{\omega}<\omega^{*13}$], it would be sufficient to show that $\frac{\partial V(\omega,\tilde{\omega},r)}{\partial r}<0$ (by the implicit function theorem).

Using the Leibniz Integral Rule we get;

$\frac{\partial V(\omega,\tilde{\omega},r)}{\partial r}(\hat{\omega},r)=-\frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}}\left[1-H\left(\hat{p}(\hat{\omega},r)\right)\right]+\frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}}\left[(1+r)-y_l-\hat{p}(\hat{\omega},r)\left(y_h-y_l\right)\right]h(\hat{p}(\hat{\omega},r))\frac{\partial\hat{p}(\hat{\omega},r)}{\partial r}$.

So $\frac{\partial V(\omega,\tilde{\omega},r)}{\partial r}<0$ iff $\frac{\partial\hat{p}(\hat{\omega},r)}{\partial r}<\frac{[1-H(\hat{p}(\hat{\omega},r))]}{[(1+r)-y_l-\hat{p}(\hat{\omega},r)(y_h-y_l)]h(\hat{p}(\hat{\omega},r))}$[14]

For $\hat{\omega}<\omega^*$, recall from the first part of this proposition that:

$\frac{\partial\hat{p}(\omega+\tilde{\omega},r)}{\partial r}=\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(\omega+\tilde{\omega},r)(1-\omega-\tilde{\omega})}{(y_h-R(\omega+\tilde{\omega},r))A(\hat{p})+\hat{p}(\omega+\tilde{\omega},r)A'(\hat{p})(R(\omega+\tilde{\omega},r)-y_l)}$,

substituting into the above means we need to show: $\frac{A(\hat{p})(\hat{\omega})+\hat{p}(\hat{\omega},r)(1-\hat{\omega})}{(y_h-R(\hat{\omega},r))A(\hat{p})+\hat{p}(\hat{\omega},r)A'(\hat{p})(R(\hat{\omega},r)-y_l)}<\frac{[1-H(\hat{p}(\hat{\omega},r))]}{[(1+r)-y_l-\hat{p}(\hat{\omega},r)(y_h-y_l)]h(\hat{p}(\hat{\omega},r))}$

Re-arranging equations 5 and 6 (when $\hat{\omega}=\omega+\tilde{\omega}<\omega^*$) gives:

$y_h-R\left(\hat{\omega},r\right)=\frac{\hat{\omega}(1+r)}{\hat{p}}$; $R\left(\hat{\omega},r\right)=\frac{(1-\hat{\omega})(1+r)-y_l}{A(\hat{p})}+y_l$; and $(y_h-y_l)=\frac{\hat{\omega}(1+r)}{\hat{p}}+\frac{(1-\hat{\omega})(1+r)-y_l}{A(\hat{p})}$.

It can also be shown generically that $\frac{[1-H(\hat{p})]}{h(\hat{p})}\equiv\frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ (to be shown in annex A.6 below).

So substituting into the inequality above we get:

$$\frac{A(\hat{p})(\hat{\omega})+\hat{p}(1-\hat{\omega})}{\hat{\omega}(1+r)\frac{A(\hat{p})}{\hat{p}}+\hat{p}\frac{A'(\hat{p})}{A(\hat{p})}[(1-\hat{\omega})(1+r)-y_l]}<\frac{1}{\left[(1+r)-y_l-\hat{p}\left(\frac{\hat{\omega}(1+r)}{\hat{p}}+\frac{(1-\hat{\omega})(1+r)-y_l}{A(\hat{p})}\right)\right]}\frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$$

Re-arrange and simplify the RHS to give:

$$\frac{A(\hat{p})(\hat{\omega})+\hat{p}(1-\hat{\omega})}{\hat{\omega}(1+r)\frac{A(\hat{p})}{\hat{p}}+\hat{p}\frac{A'(\hat{p})}{A(\hat{p})}[(1-\hat{\omega})(1+r)-y_l]}<\frac{1}{[(1-\hat{\omega})(1+r)-y_l]}\frac{A(\hat{p})}{A'(\hat{p})}$$

So:

$$\left[A\left(\hat{p}\right)\left(\hat{\omega}\right)+\hat{p}(1-\hat{\omega})\right]\left[(1-\hat{\omega})(1+r)-y_l\right]<\left(\hat{\omega}(1+r)\frac{A(\hat{p})}{\hat{p}}+\hat{p}\frac{A'(\hat{p})}{A(\hat{p})}\left[(1-\hat{\omega})(1+r)-y_l\right]\right)\frac{A(\hat{p})}{A'(\hat{p})}$$

... which can be further simplified to:

$$A\left(\hat{p}\right)(1-\hat{\omega})(1+r)-A\left(\hat{p}\right)y_l-\hat{p}\left[(1-\hat{\omega})(1+r)-y_l\right]<A\left(\hat{p}\right)\frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}(1+r)$$

Recall from equation 6: $(1-\hat{\omega})(1+r)-y_l=A\left(\hat{p}\right)\left(R-y_l\right)>0$, so for the above inequality to hold it is sufficient to show that $A\left(\hat{p}\right)(1-\hat{\omega})(1+r)<A\left(\hat{p}\right)\frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}(1+r)$ or $(1-\hat{\omega})<\frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}$.

For $p\tilde{\ }U\left[p_{lb},p_{ub}\right]$: $\frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}=\frac{p_{ub}}{\hat{p}}+1>1>(1-\hat{\omega})$. Therefore $\frac{\partial V(\omega,\tilde{\omega},r)}{\partial r}<0$ and $\frac{\partial\hat{\omega}}{\partial r}>0$ [for $\hat{\omega}<\omega^*$]. Q.E.D.

[13] $\frac{\partial\hat{\omega}}{\partial r}$ for $\hat{\omega}>\omega^*$ is not well defined. $V\left(\hat{\omega},r\right)=C$ is not continuously differentiable.

[14] We are looking at cases where $\hat{\omega}<\omega^*$, so $\left[(1+r)-y_l-\hat{p}\left(\hat{\omega},r\right)\left(y_h-y_l\right)\right]>0$.

A.6 Relationship between conditional expectation and probability density function

Proposition 9 *$\frac{[1-H(\hat{p})]}{h(\hat{p})} \equiv \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$, where $A(\hat{p}) \equiv \frac{1}{[1-H(\hat{p})]}\int_{\hat{p}}^{p_{ub}} ph(p)\,dp$ and p_{ub} is the upper bound of the p distribution with pdf $h(.)$ and cdf $H(.)$.*

Proof. *Let $g(p) = \int ph(p)\,dp$, then $A(\hat{p}) = [1-H(\hat{p})]^{-1}[g(p_{ub}) - g(\hat{p})]$*

and $A'(\hat{p}) = h(\hat{p})[1-H(\hat{p})]^{-2}[g(p_{ub}) - g(\hat{p})] - [1-H(\hat{p})]^{-1}[g'(\hat{p})] = A(\hat{p})\frac{h(\hat{p})}{[1-H(\hat{p})]} - \frac{1}{[1-H(\hat{p})]}[\hat{p}h(\hat{p})]$

So $A'(\hat{p}) = \frac{h(\hat{p})}{[1-H(\hat{p})]}[A(\hat{p}) - \hat{p}]$ and $\frac{[1-H(\hat{p})]}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$. ■

A.7 Proof to Proposition 5

Part 1 and 2 of the proposition are proved in the main text (these are corollaries of previous propositions). We start with the proof for part 4 of the proposition here, and use the result $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}} > 0$ to prove part 3 of the proposition.

A.7.1 Proof to Proposition 5.4: $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}} \geq 0$

Given equation 22, we have $\hat{\omega}(\tilde{\omega} = \omega_{reg} - \omega, r) = \frac{C\omega_{reg}}{V(\omega=\omega_{reg},\tilde{\omega}=0,r)}$

So $\frac{\partial\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial\omega_{reg}} = C[V(\omega=\omega_{reg},\tilde{\omega}=0,r)]^{-1} - C\omega_{reg}[V(\omega=\omega_{reg},\tilde{\omega}=0,r)]^{-2}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}} = C\left\{\frac{V(\omega=\omega_{reg},\tilde{\omega}=0,r)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}}}{V(\omega=\omega_{reg},\tilde{\omega}=0,r)^2}\right\}$

implying that for non-zero C and $V(\omega=\omega_{reg},\tilde{\omega}=0,r)$, $\frac{\partial\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial\omega_{reg}} \geq 0$ iff $V(\omega=\omega_{reg},\tilde{\omega}=0,r) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}} \geq 0$.

Recall that $V(\omega,\tilde{\omega},r) \geq 0$ by definition (V is the value of an option in the risky lending project), so when $\omega_{reg=0}$, $V(\omega=\omega_{reg},\tilde{\omega}=0,r) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}} = V(0,0,r) \geq 0$. All that remains is to show $V(\omega=\omega_{reg},\tilde{\omega}=0,r) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}}$ is non-decreasing for $\omega_{reg} \in [0,1]$.

Differentiating with respect to ω_{reg} gives: $\frac{\partial\left[V(\omega=\omega_{reg},\tilde{\omega}=0,r)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}}\right]}{\partial\omega_{reg}} = \frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}} - \frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}} - \omega_{reg}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}^2} = -\omega_{reg}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}^2}$. So we need to show that $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}^2} \leq 0$.

Recall from the proof to proposition 3.2: $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial\omega} = -[\hat{p}(y_h - y_l) + y_l - (1+r)]\,h(\hat{p})\frac{\partial\hat{p}(\omega,r)}{\partial\omega}$

So $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}^2} = -\left\{ \begin{array}{c} h(\hat{p}(\omega_{reg},r))\frac{\partial\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}}\left[\frac{\partial\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}}(y_h-y_l)\right]+... \\ h'(\hat{p}(\omega_{reg},r))\frac{\partial\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}}\left[\hat{p}(\omega_{reg},r)(y_h-y_l)+y_l-(1+r)\right]\frac{\partial\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}}+... \\ \left[\hat{p}(\omega_{reg},r)(y_h-y_l)+y_l-(1+r)\right]h(\hat{p}(\omega_{reg},r))\frac{\partial^2\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}^2} \end{array} \right\}$

For uniformly distributed p, $h'(.) = 0$. So $\frac{\partial^2\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}^2} \leq 0$[15] is sufficient for $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}^2} \leq 0$. This in turn implies that $\frac{\partial\left[V(\omega=\omega_{reg},\tilde{\omega}=0,r)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0,r)}{\partial\omega_{reg}}\right]}{\partial\omega_{reg}} \geq 0$ and finally $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}} \geq 0$.

A.7.2 Proof to Proposition 5 .3: $\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r) > \hat{\omega}(\tilde{\omega}=0,r)$ for all $\omega_{reg} > \hat{\omega}(0,r)$

Recall from the (implicit) definition of $\hat{\omega}(\tilde{\omega},r)$:

$V(\hat{\omega}(0,r),\tilde{\omega}=0,r) = C$; and

$V(\hat{\omega}(\omega_{reg}-\omega,r),\tilde{\omega}=\omega_{reg}-\hat{\omega}(\omega_{reg}-\omega,r),r) = C$.

We also know from equation 22 that $V(\omega,\tilde{\omega}=\omega_{reg}-\omega,r) = \frac{\omega}{\omega_{reg}}V(\omega=\omega_{reg},\tilde{\omega}=0,r)$.

Taken together, the above means $\frac{\hat{\omega}(\omega_{reg}-\omega,r)}{\omega_{reg}}V(\omega=\omega_{reg},\tilde{\omega}=0,r) = V(\hat{\omega}(0,r),\tilde{\omega}=0,r)$; and therefore $\hat{\omega}(\omega_{reg}-\omega,r) = \hat{\omega}(0,r)$ if and only if $\omega_{reg} = \hat{\omega}(0,r)$. [In other words, when the minimum capital requirement is set at the level of the participation threshold which prevails in the absence of the requirement, then the capital requirement is has no impact on participation). We can define a 'binding regulatory regime' as one where $\omega_{reg} > \min[\hat{\omega}(\omega_{reg}-\omega,r),\hat{\omega}(0,r)]$, because if the capital requirement is less than or equal to the lower of the two participation thresholds then no banks would be bound by it should they decide to participate in the game.

The rest of the proof is shown by contradiction:

Suppose $\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r) \leq \hat{\omega}(\tilde{\omega}=0,r)$ and the regulatory regime is binding $\omega_{reg} > \min[\hat{\omega}(\omega_{reg}-\omega,r),\hat{\omega}(0,r)]$. We can then examine the participation thresholds for banks with $\omega_i < \omega_{reg}$ (banks for which the regulatory regime applies), and show that a contradiction must arise:

[15] Recall for $\omega+\tilde{\omega}<\omega^*$: $\frac{\partial\hat{p}}{\partial\omega} = \frac{(A(\hat{p})-\hat{p})(1+r)}{A(\hat{p})(y_h-R)+\hat{p}A'(\hat{p})(R-y_l)} > 0$. For uniformly distributed p, we have: $h(p) = \frac{1}{p_{ub}-p_{lb}}$, $h'(p) = 0$ and $A(\hat{p}) = \frac{1}{2}(\hat{p}+p_{ub})$.

So $\frac{\partial\hat{p}}{\partial\omega} = \frac{\left(\frac{1}{2}(\hat{p}+p_{ub})-\hat{p}\right)(1+r)}{\frac{1}{2}(\hat{p}+p_{ub})(y_h-R)+\hat{p}\frac{1}{2}(R-y_l)} = \frac{(p_{ub}-\hat{p})(1+r)}{(\hat{p}+p_{ub})(y_h-R)+\hat{p}(R-y_l)} = \frac{(p_{ub}-\hat{p})(1+r)}{p_{ub}(y_h-R)+\hat{p}(y_h-y_l)} > 0$

$\frac{\partial^2\hat{p}}{\partial\omega^2} = \frac{-\frac{\partial\hat{p}}{\partial\omega}(1+r)[p_{ub}(y_h-R)+\hat{p}(y_h-y_l)]-(p_{ub}-\hat{p})(1+r)\left[-p_{ub}\frac{\partial R}{\partial\omega}+(y_h-y_l)\frac{\partial\hat{p}}{\partial\omega}\right]}{[p_{ub}(y_h-R)+\hat{p}_d(y_h-y_l)]^2} \leq 0$ since $\frac{\partial\hat{p}}{\partial\omega} \geq 0$ and $\frac{\partial R}{\partial\omega} \leq 0$.

1. First consider regulatory regime (i): $\hat{\omega}(\tilde{\omega}=0,r) \geq \omega_{reg} > \hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)$

   Under this regime $\omega_{reg} > \hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)$. So for $\frac{\hat{\omega}(\omega_{reg}-\omega,r)}{\omega_{reg}} V(\omega=\omega_{reg},\tilde{\omega}=0,r) = V(\hat{\omega}(0,r),\tilde{\omega}=0,r)$ (the condition shown at the start of the proof) to hold we need $V(\omega=\omega_{reg},\tilde{\omega}=0,r) > V(\hat{\omega}(0,r),\tilde{\omega}=0,r)$, which given $\frac{\partial V(\omega,\tilde{\omega},r)}{\partial\omega} \geq 0$ (proposition 3.2) means $\omega_{reg} > \hat{\omega}(0,r)$. Contradiction.

2. Now consider regime (ii): $\omega_{reg} > \hat{\omega}(\tilde{\omega}=0,r) \geq \hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)$

   We have already shown that $\hat{\omega}(\omega_{reg}-\omega,r) = \hat{\omega}(0,r)$ if and only if $\omega_{reg} = \hat{\omega}(0,r)$.

   So given $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}} \geq 0$, when $\omega_{reg} > \hat{\omega}(0,r)$, $\hat{\omega}(\omega_{reg}-\omega,r) > \hat{\omega}(0,r)$. Contradiction.

Therefore $\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r) > \hat{\omega}(\tilde{\omega}=0,r)$ for all $\omega_{reg} > \hat{\omega}(0,r)$. Q.E.D.

A.8 Proof to Proposition 6: $\frac{\partial Q(\omega_{reg},r)}{\partial\omega_{reg}} > 0$, when $r=0$ and $\omega_{reg}=\hat{\omega}(0,r)$

Let $g(\omega_{reg},\omega) = \left[\int_{\hat{p}(\omega_{reg},r)}^{1}\{p(y_h-y_l)+y_l-1)\}h(p)dp\right] f(\omega)$, and re-write equation 18 as:

$Q(\omega_{reg},r) = 1+\mu\int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]}[g(\omega_{reg},\omega)]\,d\omega+\mu\int_{\min[\omega_{reg},\omega_{ub}]}^{\omega_{ub}}\left[\int_{\hat{p}(\omega,r)}^{1}[p(y_h-y_l)+y_l-1]\,h(p)dp\right]dF(\omega)- \mu C\left[\int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\omega_{ub}} f(\omega)\,d\omega\right]$

Apply the Leibniz Integral Rule (with variable limits):

$$\frac{\partial Q(\omega_{reg},r)}{\partial\omega_{reg}} = \mu\left(\begin{array}{c}\int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]}\frac{\partial g(\omega_{reg},\omega)}{\partial\omega_{reg}}d\omega + \begin{cases} g(\omega_{reg},\omega=\omega_{reg}) & \text{if } \omega_{reg}<\omega_{ub} \\ 0 & \text{otherwise}\end{cases} \ldots \\ -g(\omega_{reg},\omega=\hat{\omega}(\omega_{reg}-\omega,r))\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}}\end{array}\right)\ldots$$

$$-\begin{cases}\mu f(\omega_{reg})\int_{\hat{p}(\omega_{reg},r)}^{1}[p(y_h-y_l)+y_l-1]\,h(p)dp & \text{if } \omega_{reg}<\omega_{ub} \\ 0 & \text{otherwise}\end{cases} -\mu C\left[-f(\hat{\omega}(\omega_{reg}-\omega,r))\right]\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}};$$

where $\frac{\partial g(\omega_{reg},\omega)}{\partial\omega_{reg}} = -\{\hat{p}(\omega_{reg},r)(y_h-y_l)+y_l-1)\}h(\hat{p}(\omega_{reg},r))\frac{\partial\hat{p}(\omega_{reg},r)}{\partial\omega_{reg}}f(\omega)$.

Recall that $\frac{\hat{\omega}(\omega_{reg}-\omega,r)}{\omega_{reg}}V(\omega=\omega_{reg},\tilde{\omega}=0,r) = \frac{\hat{\omega}(\omega_{reg}-\omega,r)}{\omega_{reg}}\int_{\hat{p}(\omega_{reg},r)}^{1}[p(y_h-y_l)+y_l-(1+r)]\,h(p)dp = C$ (for $\omega < \omega_{reg}$)

So when $r=0$, and $\omega_{reg}=\hat{\omega}(0,r)=\hat{\omega}(\omega_{reg}-\omega,r)$: $C = \frac{g(\omega_{reg},\omega=\hat{\omega}(\omega_{reg}-\omega,r))}{f(\hat{\omega}(\omega_{reg}-\omega,r))}$

So $\frac{\partial Q(\omega_{reg},r)}{\partial \omega_{reg}} = \mu \left(\begin{array}{c} \int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]} \frac{\partial g(\omega_{reg},\omega)}{\partial \omega_{reg}} d\omega + \begin{cases} g(\omega_{reg}, \omega = \omega_{reg}) & \text{if } \omega_{reg} < \omega_{ub} \\ 0 & \text{otherwise} \end{cases} \ldots \\ -g(\omega_{reg}, \omega = \hat{\omega}(\omega_{reg} - \omega, r)) \frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \end{array} \right) \ldots$

$- \begin{cases} \mu g(\omega_{reg}, \omega = \omega_{reg}) & \text{if } \omega_{reg} < \omega_{ub} \\ 0 & \text{otherwise} \end{cases} + \mu g(\omega_{reg}, \omega = \hat{\omega}(\omega_{reg} - \omega, r)) \frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}}$

which simplifies to: $\frac{\partial Q(\omega_{reg},r)}{\partial \omega_{reg}} = \mu \int_{\hat{\omega}(\omega_{reg}-\omega,r)}^{\min[\omega_{reg},\omega_{ub}]} \frac{\partial g(\omega_{reg},\omega)}{\partial \omega_{reg}} d\omega.$

Recall $\frac{\partial g(\omega_{reg},\omega)}{\partial \omega_{reg}} = -\{\hat{p}(\omega_{reg}, r)(y_h - y_l) + y_l - 1)\} h(\hat{p}(\omega_{reg}, r)) \frac{\partial \hat{p}(\omega_{reg},r)}{\partial \omega_{reg}} f(\omega)$, and $\hat{p}(\omega, r) \leq \frac{(1+r)-y_l}{(y_h-y_l)}$. So when $r = 0$, $\frac{\partial Q(\omega_{reg},r)}{\partial \omega_{reg}} \geq 0$, with equality only when $\omega_{reg} = \hat{\omega}(\tilde{\omega} = 0, r = 0) = \omega^*(r = 0)$. In other words binding a capital requirement can improve output; unless in the absence of which banks only screen if they are already 'fully capitalised' - in which case introducing binding prudential regulation would have no impact at the margin.

A.9 Proof to Proposition 7: $\frac{\partial Q(\omega_{reg},r)}{\partial r} > 0$ when $r = 0$, and $\omega_{reg} = \hat{\omega}(0, r)$

Re-write equation 19 (aggregate output when capital requirement is non-binding) as:

$Q(r) = 1 + \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} [A(\hat{p}(\omega, r))(y_h - y_l) + y_l - 1)][1 - H(\hat{p}(\omega, r))] dF(\omega) - \mu C [1 - F(\hat{\omega}(0, r))]$

Let $g(r, \omega) = [A(\hat{p}(\omega, r))(y_h - y_l) + y_l - 1)][1 - H(\hat{p}(\omega, r))] f(\omega)$;

so $Q(r) = 1 + \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} g(r, \omega) d\omega - \mu C [1 - F(\hat{\omega}(0, r))]$

Apply the Leibniz Integral Rule (with variable limits):

$\frac{\partial Q(\omega_{reg},r)}{\partial r} = \mu \left\{ \int_{\hat{\omega}(0,r)}^{\omega_{ub}} g_r(r, \omega) d\omega - g(r, \hat{\omega}(0, r)) \frac{d\hat{\omega}(0,r)}{dr} \right\} + \mu C f(\hat{\omega}(0, r)) \frac{d\hat{\omega}(0,r)}{dr}$

$\frac{\partial Q(\omega_{reg},r)}{\partial r} = \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} \{g_r(r, \omega)\} d\omega - \mu \{g(r, \hat{\omega}(0, r)) - C f(\hat{\omega}(0, r))\} \frac{d\hat{\omega}(0,r)}{dr}$

where $g_r(r, \omega) = -[(\hat{p}(\omega, r)(y_h - y_l) + y_l - 1) h(\hat{p}(\omega, r))] \frac{\partial \hat{p}(\omega,r)}{\partial r} f(\omega)$

Recall that $V(\hat{\omega}(0, r), 0, r) = \int_{\hat{p}(\hat{\omega}(0,r),r)}^{1} [p(y_h - y_l) + y_l - (1 + r)] h(p) dp = C$.

So$[A(\hat{p}(\hat{\omega}(0, r), r))(y_h - y_l) + y_l - (1 + r)][1 - H(\hat{p}(\hat{\omega}(0, r), r))] = \frac{g(r,\hat{\omega}(0,r))}{f(\hat{\omega}(0,r))} = C$

So$\frac{\partial Q(\omega_{reg},r)}{\partial r} = \mu \int_{\hat{\omega}(0,r)}^{\omega_{ub}} \{g_r(r, \omega)\} d\omega$

Recall: $\hat{p}(\omega, r) \leq \frac{(1+r)-y_l}{(y_h-y_l)} = p^*(r)$, so $\hat{p}(\omega, 0) \leq \frac{1-y_l}{(y_h-y_l)}$. Therefore $g_r(r, \omega) \geq 0$ (given

$\frac{\partial \hat{p}(\omega,r)}{\partial r} \geq 0$); and $\frac{\partial Q(\omega_{reg},r)}{\partial r} \geq 0$. Q.E.D.

In the absence of prudential policy, a tightening of monetary policy (relative to the equilibrium level / neutral stance) can improve aggregate output.

Figure 1: Model Timing

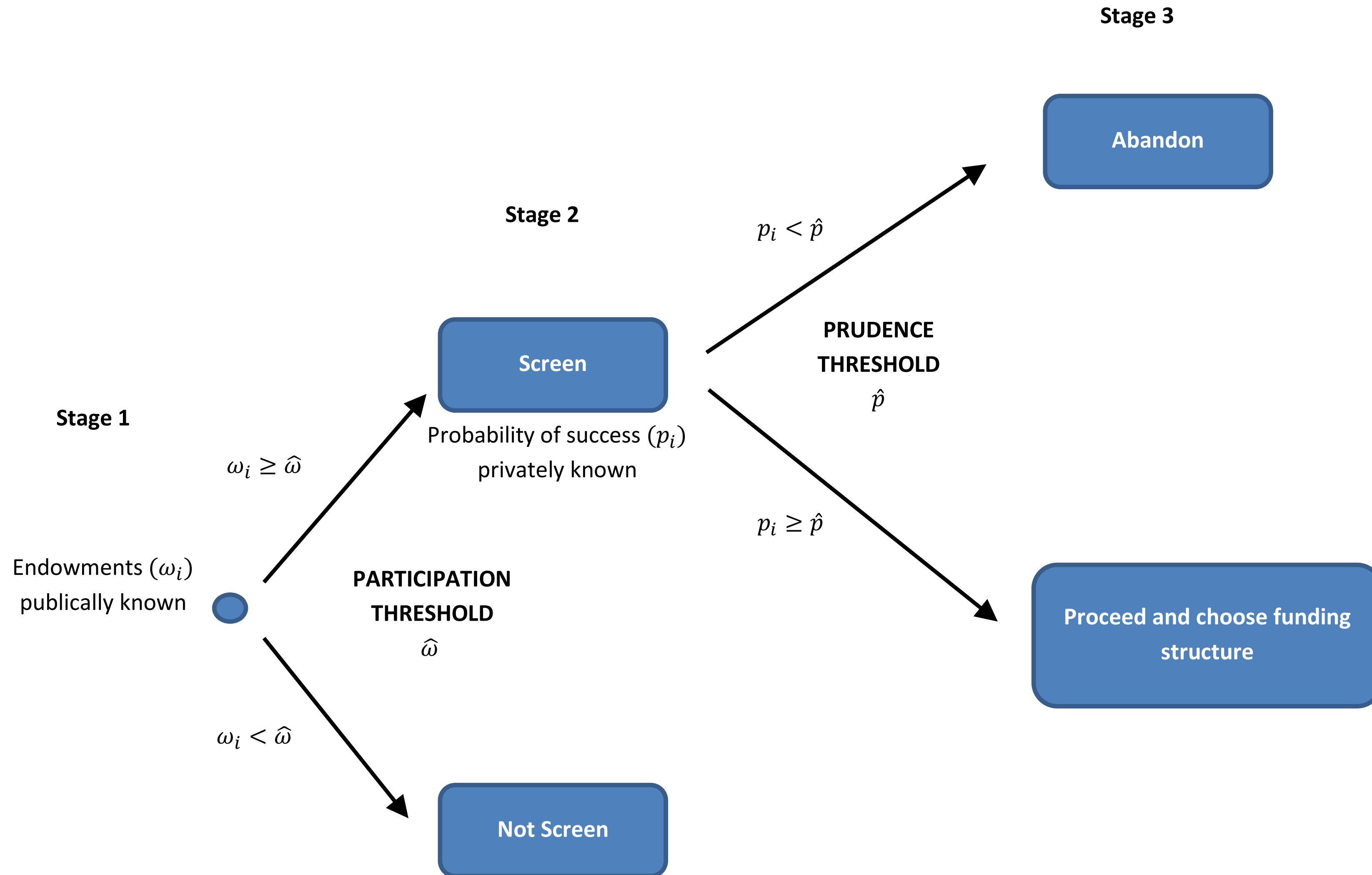
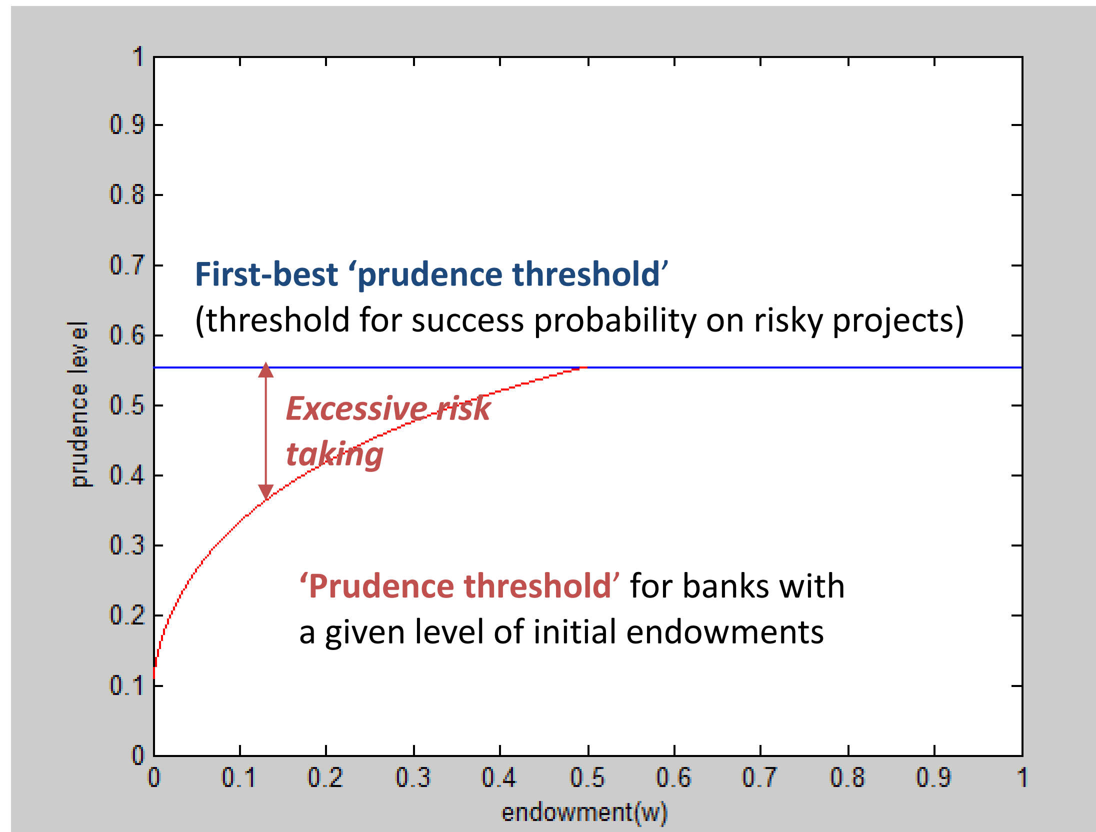


Figure 2: Excessive risk taking

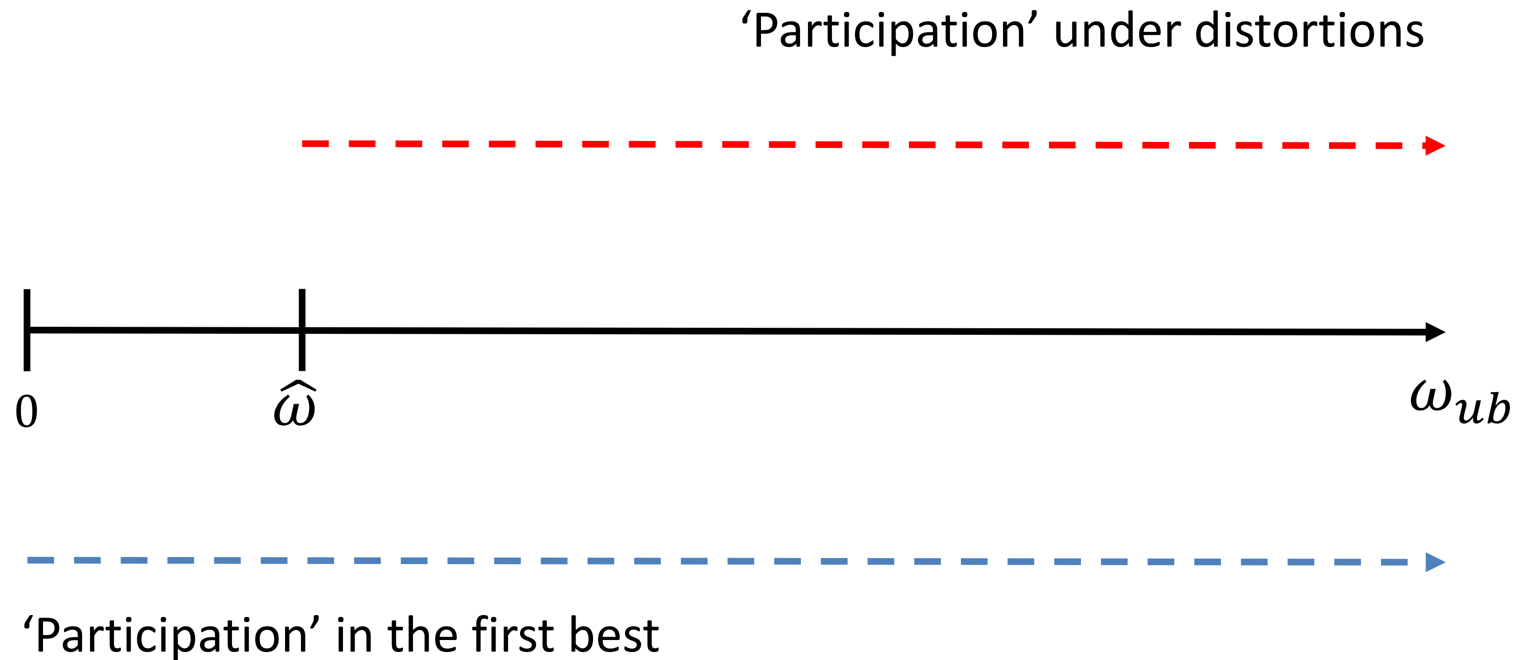


In the first best, banks should only proceed with the project after screening if the probability of success exceeds the first-best 'prudence threshold' $(\hat{p}_{fb})$ - shown in the blue line.

But in the presence of asymmetric information and limited liability, banks partly funded by debt are willing to accept projects with a lower threshold $\hat{p}(\omega + \widetilde{\omega}, r)$ – shown in the red line.

This gives rise to excessive risk taking. And the problem is more pronounced for banks with low initial equity capital.

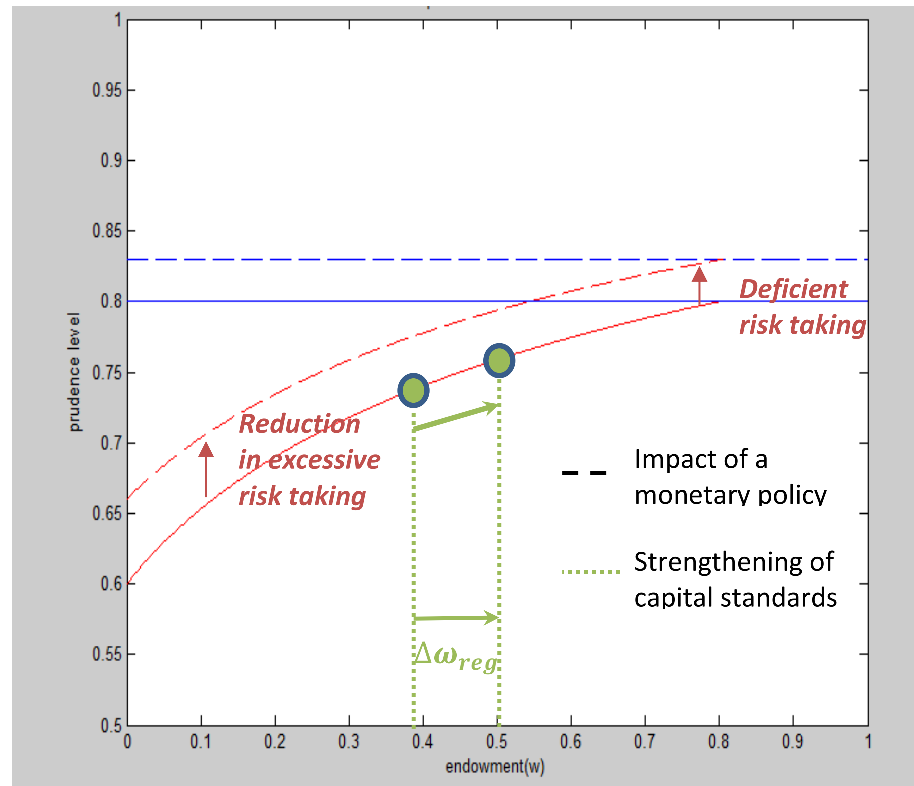
Figure 3: Lower than optimal participation



Under the first best – every bank screens (and thus participates in the game). This is illustrated by the blue dotted line.

But in the presence of asymmetric information and limited liability, banks with low endowments ($\omega_i < \widehat{\omega}$) anticipate large funding costs and calculate that the expected gain from screening is not sufficient to justify the fixed costs of screening. These banks therefore drop out of the game at Stage 1 (see Figure 1). This leads to lower than optimal participation in screening – and therefore bank lending - (as illustrated by the red dotted line above).

Figure 4: Policy Transmission – Prudence Threshold

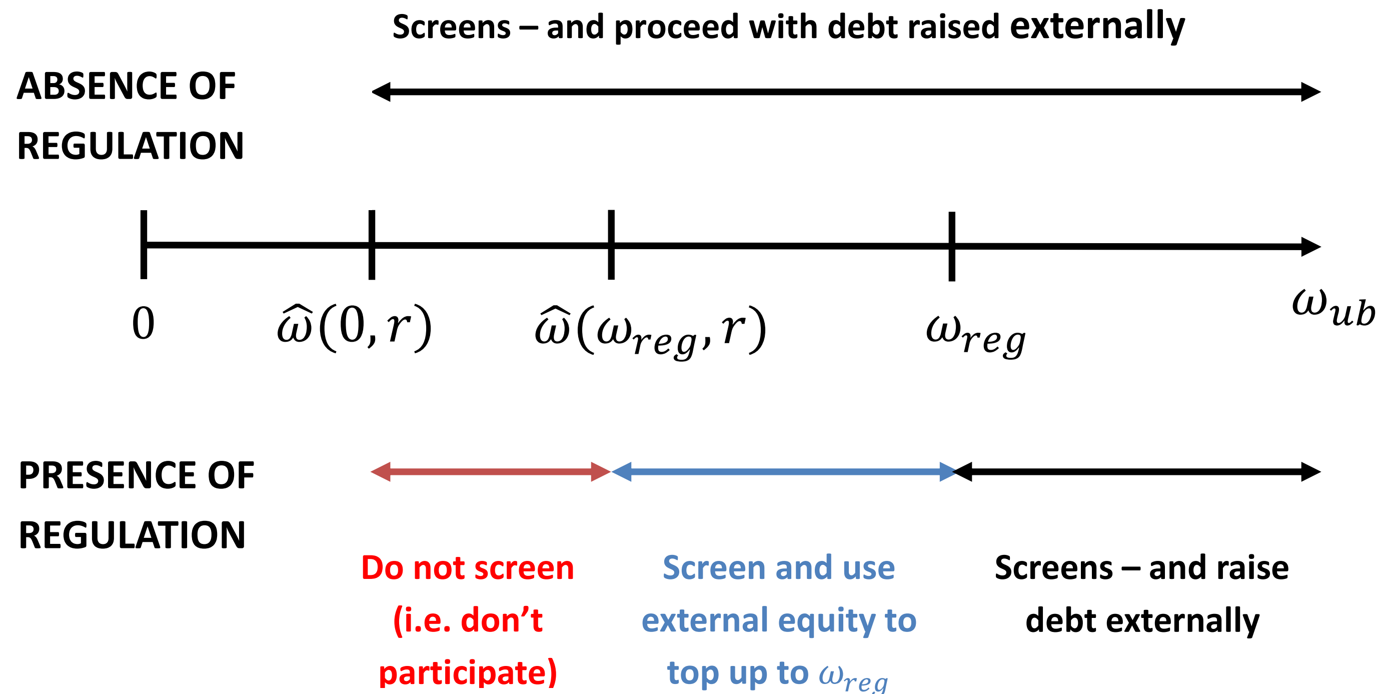


A tightening of monetary policy – an increase in the risk-free interest rate - increases the opportunity cost of risky lending. This pushes up prudence for all banks at all levels of initial endowments. So poorly endowed (or highly leveraged banks) becomes more prudent, but very well capitalised banks may become too cautious.

A strengthening of capital standards forces more banks to seek out more external equity. This pushes banks along the curve – they behave as if they are a better endowed bank. Unlike monetary policy, capital regulations will not lead to a deficient in risk taking, but it only affects banks for which the regulation is binding.

So a monetary policy tightening shifts-up the curves for every bank (as illustrated by the dashed lines). Whilst capital regulation pushes banks along the curve, but only for those for whom the regulation is binding (as illustrated by the dotted lines).

Figure 5: Policy Transmission – Participation Threshold



Because external equity is more costly than debt, tougher capital requirements makes the break-even level of initial endowment higher for a given fixed cost of screening and risk-free interest rate (the participation threshold moves from $\widehat{\omega}(0, r)$ to $\widehat{\omega}(\omega_{reg}, r)$). So more banks will drop out from screening due to a low level of initial endowment (as shown by the red arrows). Banks that have more endowments than the participation threshold but are still bounded by the regulations ($\widehat{\omega}(\omega_{reg}, r) < \omega_i < \omega_{reg}$) still choose to screen, but will need to top up to the minimum level of capital before they can lend (as shown by the blue arrows). Banks with $\omega_i \geq \omega_{reg}$ have a surplus above the regulatory minimum and are thus unaffected by the capital requirement (black arrows).

A tightening of monetary policy also leads to a reduction in participation. A higher r raises the participation threshold for all banks, irrespective of the level of capital requirements.

Figure 6: Aggregate output under different policy choices – benchmark calibration

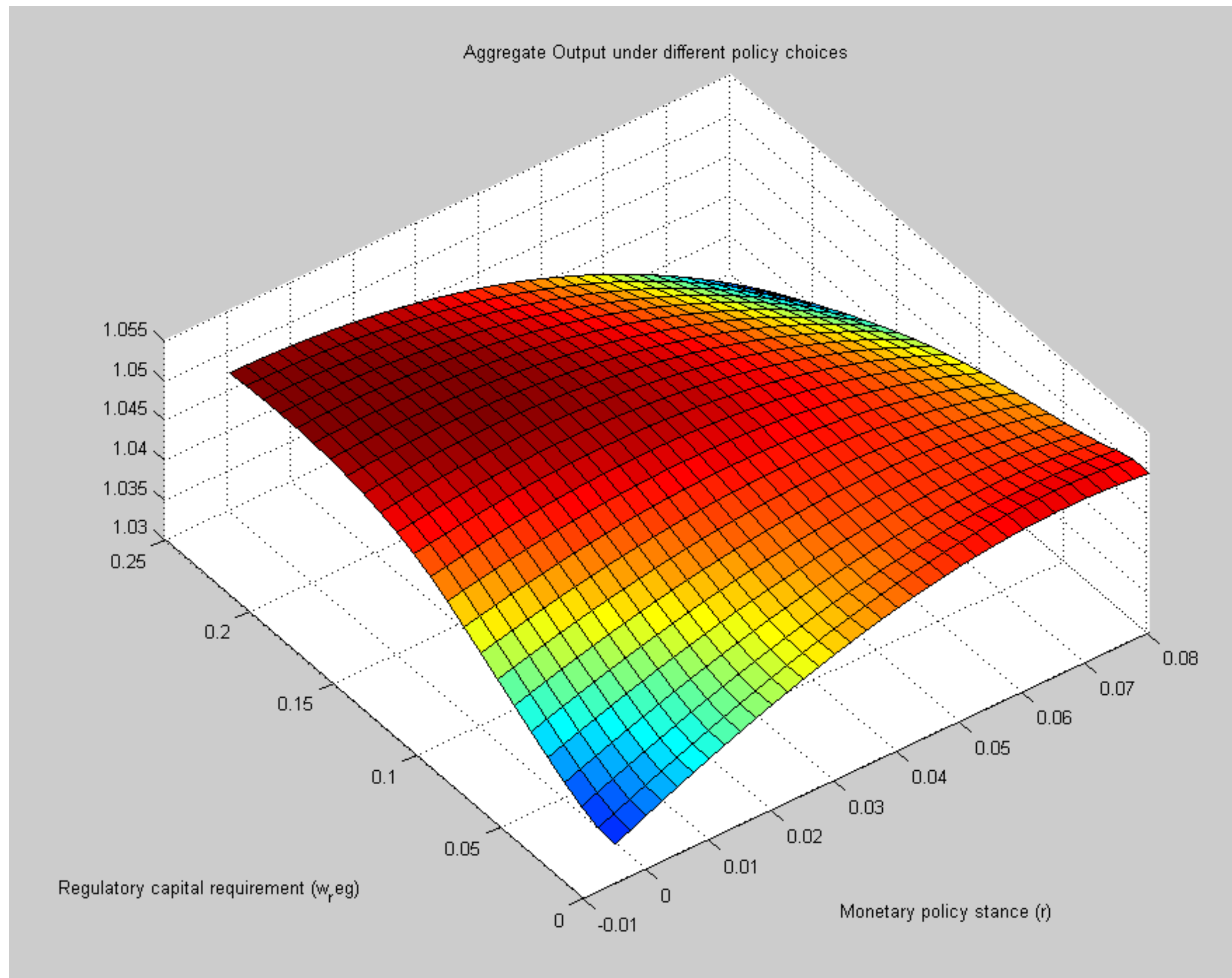


Table A1: Aggregate output under different policy choices – benchmark calibration

Aggregate output under different policy choices	Capital Requirement																						
Monetary Policy Stance (Deviation from neutral)	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%
-0.50%	1.0349	1.0351	1.0360	1.0372	1.0386	1.0400	1.0415	1.0430	1.0445	1.0459	1.0474	1.0487	1.0497	1.0506	1.0513	1.0518	1.0523	1.0526	1.0528	1.0529	1.0529	1.0529	1.0527	1.0526
-0.25%	1.0358	1.0360	1.0369	1.0380	1.0393	1.0407	1.0421	1.0435	1.0450	1.0464	1.0478	1.0490	1.0500	1.0509	1.0515	1.0521	1.0524	1.0527	1.0529	1.0530	1.0530	1.0529	1.0527	1.0525
0.00%	1.0367	1.0369	1.0377	1.0387	1.0400	1.0413	1.0427	1.0441	1.0455	1.0468	1.0482	1.0494	1.0504	1.0511	1.0518	1.0522	1.0526	1.0528	1.0530	1.0530	1.0530	1.0529	1.0527	1.0525
0.25%	1.0376	1.0377	1.0384	1.0395	1.0407	1.0419	1.0433	1.0446	1.0459	1.0473	1.0486	1.0497	1.0506	1.0514	1.0520	1.0524	1.0527	1.0529	1.0530	1.0531	1.0530	1.0529	1.0527	1.0524
0.50%	1.0384	1.0385	1.0392	1.0402	1.0413	1.0425	1.0438	1.0451	1.0464	1.0477	1.0489	1.0500	1.0509	1.0516	1.0521	1.0525	1.0528	1.0530	1.0531	1.0531	1.0530	1.0528	1.0526	1.0523
0.75%	1.0393	1.0394	1.0400	1.0409	1.0420	1.0431	1.0444	1.0456	1.0468	1.0480	1.0492	1.0503	1.0511	1.0518	1.0523	1.0526	1.0529	1.0530	**1.0531**	1.0530	1.0529	1.0527	1.0525	1.0522
1.00%	1.0401	1.0401	1.0407	1.0416	1.0426	1.0437	1.0449	1.0460	1.0472	1.0484	1.0495	1.0505	1.0513	1.0519	1.0524	1.0527	1.0529	1.0530	1.0530	1.0530	1.0528	1.0526	1.0523	1.0520
1.25%	1.0409	1.0409	1.0414	1.0422	1.0432	1.0442	1.0453	1.0465	1.0476	1.0487	1.0498	1.0507	1.0515	1.0521	1.0525	1.0528	1.0529	1.0530	1.0530	1.0529	1.0527	1.0525	1.0521	1.0518
1.50%	1.0417	1.0417	1.0421	1.0428	1.0437	1.0448	1.0458	1.0469	1.0480	1.0490	1.0500	1.0509	1.0516	1.0522	1.0525	1.0528	1.0529	1.0530	1.0529	1.0528	1.0526	1.0523	1.0519	1.0515
1.75%	1.0424	1.0424	1.0428	1.0434	1.0443	1.0452	1.0463	1.0473	1.0483	1.0493	1.0503	1.0511	1.0518	1.0522	1.0526	1.0528	1.0529	1.0529	1.0528	1.0526	1.0524	1.0521	1.0517	1.0513
2.00%	1.0431	1.0431	1.0434	1.0440	1.0448	1.0457	1.0467	1.0476	1.0486	1.0495	1.0505	1.0513	1.0519	1.0523	1.0526	1.0527	1.0528	1.0528	1.0526	1.0524	1.0521	1.0518	1.0514	1.0510
2.25%	1.0438	1.0438	1.0440	1.0446	1.0453	1.0462	1.0471	1.0480	1.0489	1.0498	1.0506	1.0514	1.0519	1.0523	1.0525	1.0527	1.0527	1.0526	1.0524	1.0522	1.0519	1.0515	1.0511	1.0506
2.50%	1.0445	1.0444	1.0446	1.0451	1.0458	1.0466	1.0474	1.0483	1.0491	1.0499	1.0508	1.0514	1.0519	1.0523	1.0525	1.0526	1.0525	1.0524	1.0522	1.0520	1.0516	1.0512	1.0508	1.0502
2.75%	1.0452	1.0451	1.0452	1.0456	1.0463	1.0470	1.0477	1.0485	1.0493	1.0501	1.0509	1.0515	1.0519	1.0522	1.0524	1.0524	1.0524	1.0522	1.0520	1.0517	1.0513	1.0509	1.0504	1.0498
3.00%	1.0458	1.0457	1.0457	1.0461	1.0467	1.0473	1.0481	1.0488	1.0495	1.0502	1.0509	1.0515	1.0519	1.0521	1.0523	1.0523	1.0522	1.0520	1.0517	1.0513	1.0509	1.0505	1.0499	1.0494
3.25%	1.0464	1.0462	1.0462	1.0466	1.0471	1.0477	1.0483	1.0490	1.0497	1.0503	1.0510	1.0515	1.0518	1.0520	1.0521	1.0521	1.0519	1.0517	1.0514	1.0510	1.0505	1.0500	1.0495	1.0489
3.50%	1.0469	1.0468	1.0467	1.0470	1.0474	1.0480	1.0486	1.0492	1.0498	1.0504	1.0510	1.0514	1.0517	1.0519	1.0519	1.0518	1.0516	1.0514	1.0510	1.0506	1.0501	1.0496	1.0490	1.0484
3.75%	1.0474	1.0473	1.0472	1.0474	1.0478	1.0482	1.0488	1.0493	1.0499	1.0504	1.0510	1.0514	1.0516	1.0517	1.0517	1.0515	1.0513	1.0510	1.0506	1.0502	1.0496	1.0491	1.0485	1.0478
4.00%	1.0479	1.0478	1.0476	1.0477	1.0481	1.0485	1.0490	1.0495	1.0500	1.0504	1.0509	1.0512	1.0514	1.0515	1.0514	1.0512	1.0510	1.0506	1.0502	1.0497	1.0491	1.0485	1.0479	1.0472
4.25%	1.0483	1.0482	1.0480	1.0481	1.0483	1.0487	1.0491	1.0495	1.0500	1.0504	1.0508	1.0511	1.0512	1.0512	1.0511	1.0509	1.0506	1.0502	1.0497	1.0492	1.0486	1.0479	1.0473	1.0465
4.50%	1.0488	1.0486	1.0483	1.0484	1.0486	1.0489	1.0492	1.0496	1.0500	1.0503	1.0507	1.0509	1.0510	1.0509	1.0508	1.0505	1.0501	1.0497	1.0492	1.0486	1.0480	1.0473	1.0466	1.0458
4.75%	1.0491	1.0490	1.0487	1.0486	1.0488	1.0490	1.0493	1.0496	1.0499	1.0502	1.0505	1.0507	1.0507	1.0506	1.0504	1.0501	1.0497	1.0492	1.0486	1.0480	1.0474	1.0467	1.0459	1.0451
5.00%	1.0495	1.0494	1.0490	1.0489	1.0489	1.0491	1.0493	1.0496	1.0498	1.0501	1.0503	1.0504	1.0504	1.0502	1.0499	1.0496	1.0491	1.0486	1.0480	1.0474	1.0467	1.0460	1.0452	1.0443
5.25%	1.0497	1.0497	1.0492	1.0490	1.0491	1.0492	1.0493	1.0495	1.0497	1.0499	1.0500	1.0501	1.0500	1.0498	1.0495	1.0491	1.0486	1.0480	1.0474	1.0467	1.0460	1.0452	1.0444	1.0435
5.50%	1.0500	1.0499	1.0494	1.0492	1.0492	1.0492	1.0493	1.0494	1.0496	1.0497	1.0498	1.0498	1.0496	1.0493	1.0490	1.0485	1.0480	1.0474	1.0467	1.0460	1.0452	1.0444	1.0436	1.0427
5.75%	1.0502	1.0502	1.0496	1.0493	1.0492	1.0492	1.0492	1.0493	1.0494	1.0494	1.0494	1.0494	1.0492	1.0488	1.0484	1.0479	1.0473	1.0467	1.0460	1.0452	1.0444	1.0436	1.0427	1.0418
6.00%	1.0504	1.0503	1.0497	1.0494	1.0492	1.0491	1.0491	1.0491	1.0491	1.0491	1.0491	1.0489	1.0487	1.0483	1.0478	1.0473	1.0467	1.0460	1.0452	1.0444	1.0436	1.0427	1.0418	1.0408
6.25%	1.0505	1.0505	1.0498	1.0494	1.0492	1.0490	1.0490	1.0489	1.0488	1.0488	1.0487	1.0485	1.0481	1.0477	1.0472	1.0466	1.0459	1.0452	1.0444	1.0436	1.0427	1.0418	1.0408	1.0398
6.50%	1.0506	1.0506	1.0499	1.0494	1.0491	1.0489	1.0488	1.0486	1.0485	1.0484	1.0482	1.0479	1.0476	1.0471	1.0465	1.0459	1.0452	1.0444	1.0436	1.0427	1.0417	1.0408	1.0398	1.0388
6.75%	1.0506	1.0506	1.0499	1.0494	1.0490	1.0487	1.0485	1.0483	1.0481	1.0479	1.0477	1.0474	1.0470	1.0464	1.0458	1.0451	1.0443	1.0435	1.0426	1.0417	1.0408	1.0398	1.0387	1.0377
7.00%	1.0506	1.0506	1.0498	1.0493	1.0488	1.0485	1.0482	1.0480	1.0477	1.0475	1.0472	1.0468	1.0463	1.0457	1.0450	1.0443	1.0435	1.0426	1.0417	1.0407	1.0397	1.0387	1.0376	1.0365
7.25%	1.0505	1.0505	1.0498	1.0491	1.0486	1.0482	1.0479	1.0476	1.0473	1.0469	1.0466	1.0461	1.0456	1.0449	1.0442	1.0434	1.0426	1.0416	1.0407	1.0397	1.0386	1.0376	1.0365	1.0353
7.50%	1.0504	1.0504	1.0496	1.0489	1.0484	1.0479	1.0475	1.0471	1.0468	1.0464	1.0459	1.0454	1.0448	1.0441	1.0433	1.0425	1.0416	1.0406	1.0396	1.0386	1.0375	1.0364	1.0352	1.0341
7.75%	1.0502	1.0502	1.0494	1.0487	1.0481	1.0476	1.0471	1.0467	1.0462	1.0457	1.0452	1.0447	1.0440	1.0433	1.0424	1.0415	1.0406	1.0396	1.0385	1.0374	1.0363	1.0352	1.0340	1.0328
8.00%	1.0499	1.0499	1.0492	1.0484	1.0477	1.0472	1.0466	1.0461	1.0456	1.0451	1.0445	1.0439	1.0432	1.0423	1.0415	1.0405	1.0395	1.0385	1.0374	1.0362	1.0351	1.0339	1.0327	1.0314

Highlighted cells (in yellow) shows the aggregate output that result from the optimal monetary policy stance for a given level of capital requirement (i.e. the highlighted cell is the highest number in each column). Red and bold font indicates the globally optimal level of capital requirement and monetary policy stance, and the associated level of aggregate outcome.