

# Should central banks provide reserves via repos or outright bond purchases?

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**Abstract:** In the wake of the financial crisis banks are likely to wish to hold far more highly liquid assets than before. Some of those liquid assets are likely to be held in the form of reserves at the central bank. We ask whether the central bank should provide these reserves by purchasing nominal, fixed-rate government bonds outright, or by repo-ing them in for a limited period. The key difference between these options is that with repos, the private sector retains the price risk associated with bonds, whereas this risk rests with the central bank if it purchases these bonds outright. There is a significant, practical policy issue for central banks here: should those central banks (most notably the Fed and the Bank of England) who built up a large stock of bonds during the QE operations, which were financed by creating reserves for commercial banks, expect to sell those bonds in due course or continue to hold a high proportion of them for a long period since the demand for reserves will be permanently higher? We develop and calibrate a simple OLG model in which risk-averse households hold money and bonds to insure against risk. We find that the composition of the central bank's assets should depend on how fiscal policy is conducted; but in general it has only a small impact on welfare.

**Key words:** Liquidity provision; Central bank balance sheet

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# 1 Introduction

The financial crisis, during which the interbank markets in many countries became wholly dysfunctional, is likely to mean that banks in future will want to hold far more highly liquid assets than they did. Some of those liquid assets are likely to be held in the form of reserves at the central bank. Assuming that the central bank accommodates changes in the demand for its reserves, we investigate what assets it should hold to back these reserves. Specifically, we ask whether the central bank should back its liabilities by purchasing nominal, fixed-rate government bonds outright, or by repo-ing them in for a limited period. There is a significant, practical policy issue for central banks here: should those central banks (most notably the Fed and the Bank of England) who built up a large stock of bonds during the QE operations, which were financed by creating reserves for commercial banks, expect to sell those bonds in due course or continue to hold a high proportion of them for a long period since the demand for reserves will be permanently higher?

The key difference between these options is that with repos, the private sector retains the price risk associated with fixed-rate government bonds, whereas this risk rests with the central bank if it purchases these bonds outright. We abstract from financial intermediation by banks and investigate directly how households allocate their wealth between central bank reserves (money) and fixed-rate nominal government bonds. Households need money for transaction purposes. But for savings purposes, they might prefer to hold some government bonds. This could be the case if the yield on bonds is higher than that of money, or if changes in prices of bonds are positively correlated with consumer price inflation, such that bonds help households to insure against unexpected inflation. In this situation welfare might be higher if the central bank offered repos rather than purchased bonds outright. A repo facility would allow an individual household to separate the problem of choosing an optimal portfolio for savings purposes from that of choosing an optimal portfolio for transaction purposes.

There is a separate reason for why repos may be the preferred option for providing reserves. Changing official interest rates affects households' decisions not only contem-

poraneously but also in future periods. When interest rates are set, future realisations of shocks are unknown, such that it is uncertain whether the level of today's interest rates is optimal given future realisations of shocks. We show that when reserves are provided via repos, monetary policy affects contemporaneous decisions to a greater extent, and future decisions to a lesser extent, than when reserves are provided via outright purchases of bonds. The intuition is that the value of households' portfolio today depends on today's interest rates only when households own bonds: the value of money today is independent of today's interest rate.

We determine the welfare-maximising (linear) interest rate rule for each regime for providing money for transaction purposes – repo and outright bond purchases – and compare welfare achieved in each regime, given that the central bank follows the respective optimal interest rate rule. We find that whether reserves are provided via repos or outright purchases has only a small impact on welfare. Repos are generally optimal, *unless* the central bank is unable to control the interest rate precisely. When interest control is poor, the market price of bonds become very volatile. If households own bonds, this increases the volatility of the price level because households rely on selling their bonds to finance consumption when old. They dislike this uncertainty because they are risk-averse. In contrast, the central bank holds any bonds it purchases outright until maturity and can therefore look through variations in their market price. We also find that the central bank's welfare-maximising linear interest rate rule only differs somewhat between the two regimes for providing money for transaction purposes.

Our framework is an OLG model in which monetary policy affects real variables not because of frictions in product or labor markets, but because it leads to intergenerational transfers of wealth. We assume that fiscal policy does not offset these intergenerational transfers.<sup>1</sup> An aggregate productivity shock creates risk that cannot be traded in financial markets. Households have to invest in money (which can be thought of as a short-term bond) and long-term bonds to partially insure against risk. These bonds earn different returns in equilibrium. This is because households are risk averse, and short-term and

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<sup>1</sup>See Sterk and Tenreyro (2013) for a recent paper in which monetary policy works through similar channels.

long-term financial assets allow them to respond to risks in different ways. Variations in the central bank's holdings of government bonds can – depending on how fiscal policy is set – affect a household's wealth, and therefore its real decisions.

Because a larger transaction friction clearly favours reserve provision via repos – being able to separate the problem of choosing an optimal portfolio for savings purposes from that of choosing an optimal portfolio for transaction purposes only matters with strictly positive transaction frictions – we focus in our numerical results on the case in which these transaction frictions are zero. Even in this case,] we find that providing reserves via repos is generally preferred, although the reserve provisioning regime has relatively little influence on welfare. Getting the reserves provisioning regime wrong results in a welfare loss of about 0.03% of GDP per year in our base calibration, equivalent to 1.5% of GDP when cumulated and discounted at a real rate of 2%. Holding the nominal policy interest rate constant instead of allowing it to vary optimally with productivity leads to a welfare loss five times as large, even when the reserve provisioning regime is chosen optimally.

In the context of our model, the problem of choosing the best way of providing reserves to the private sector is equivalent to that of choosing the optimal maturity of government debt. A situation in which the central bank provides reserves via outright purchases of government bonds is equivalent to one in which the debt management office reduces the maturity of the bonds it issues.

There is a well-established literature on the impact of government debt maturity and the central bank balance sheet. Much of the literature finds that switches in the composition of assets by the central bank are irrelevant. For example, Wallace (1981) showed that the path of the government's stock of liabilities – that is the composition of its portfolio for a given fiscal policy – is irrelevant in a model with complete markets. In this case open market operations conducted by a central bank that purchases government bonds of any maturity in exchange for other liabilities with different maturities have no impact on real outcomes. Chamley and Polemarchakis (1984) showed that the neutrality result could also hold in a world with incomplete markets but only so long as the central bank purchased real assets. Sargent and Smith (1987) showed that a neutrality, or ineffectiveness,

result for government financial policies (which include the kind of central bank purchases of assets we consider in this paper) could hold in a world in which government issued fiat currency is dominated in rate of return. But as in Chamley and Polemarchakis, the result holds for open market operations where physical capital is exchanged for currency (and where there are simultaneously alterations in lump sum taxes and transfers). In Woodford and Eggertson (2003) an infinitely lived, representative household maximizes utility in a world with complete markets and faces no limit on borrowing against future income. It is clear that with these assumptions central bank purchases - which are essentially swaps of assets with the representative agent - can do nothing because that single representative agent owns the balance sheet of the central bank and such swaps do not change its opportunity set. But their results do not apply in more realistic settings, in which households are not homogenous and markets are not complete, and in which fiscal policy cannot be set to offset the distributional impact of monetary policy. Our model allows for all those features which mean that these neutrality results do not hold. So the question as to the best way for the central bank to fund its liabilities - largely reserves held by commercial banks - is not made irrelevant by assumption.

In the first part of this paper we describe the model. Section 2 describes the model in non-technical terms; Section 3 presents the formal structure. Section 4 contains the results. We conclude in Section 5.

## 2 Model Overview

The model has the same structure as Miles and Schanz (2014). Following Wallace (1981), Sargent and Smith (1987) and others, we model the impact of central bank purchases of government bonds within the framework of an overlapping generations model. There are two assets in the economy: money, and government bonds. Households live for two periods. When bonds are issued they also have a maturity of two periods. Each generation is born without an endowment but the ability to transform their own labor into a perishable consumption good. Production uses labor only; there is no capital

accumulation. The production technology has stochastic productivity.

Each young household decides how much labor to supply to produce the consumption good, and how much of this to consume. They sell the remainder to old households, in exchange for money, and decide how many government bonds to buy. Because the consumption good perishes unless consumed, young households can only transfer wealth to when they are old by holding money or government bonds. Neither young nor old households can borrow.

Money is remunerated at the policy rate set by the central bank. Money could be thought of as Treasury bills, or bills issued by the central bank. But we could just as well think that there are 100% reserve backed commercial banks that are intermediaries between households, who hold deposits, and the central bank, which pays interest on reserves. Either way, we can think of money in this model as deposits (reserves) of the central bank that are its liability and which pay a rate of interest equal to the central bank's policy rate. All money is interest bearing as long as the central bank sets a non-zero interest rate. We make this assumption because in developed economies non-interest bearing notes and coins are very much smaller than interest bearing accounts which can be easily used to finance transactions.

The Treasury issues bonds at a discount; bonds do not pay coupons. The amount issued is constant in each period. Bonds have a maturity of two periods at issuance. This fiscal rule is a very simple one which keeps the face value of debt constant (the market value of government debt depends on real shocks to productivity). Taxes, which are lump sum, are varied to satisfy the fiscal rule. We abstract from credit risk of government bonds.

Households pay state-dependent nominal lump sum taxes to the Treasury. The government is able to levy different taxes on the young and old. Old households have simple decisions to make: they have no bequest motive so simply liquidate all their assets to finance purchasing the consumption good which they buy from young households. We assume old households do not supply labor.

What distinguishes money and bonds in our model? Money in this model is the only

asset that can be used to buy goods. Bonds must be sold for money to purchase the consumption good. The central bank takes deposits, which we could think of as reserves that a commercial banking sector holds against deposits held by households. Reserves (“money”) pay a 1 period interest rate set by the central bank. The central bank will use a policy rule to set the rate, which will be some function of the state variables in the model. The two financial assets – money and bonds – differ because money has a known nominal value 1 period ahead (one plus the interest rate set by the central bank today) while newly issued bonds have a value one period ahead which depends on the interest rate that the central bank will set in the next period – which is not known today. Bonds with one period left to maturity are perfect substitutes for money because both assets have a known nominal value one period ahead. This means that the price of a one period bond is tied to the interest rate set by the central bank. The financial assets in the model are dramatically simple – in fact as simple as is possible while allowing longer dated government bonds to be imperfect substitutes for shorter dated financial assets.

The central bank balance sheet is straightforward: it holds 1 period bonds and either 2 period bonds (regime 1) or repos (regime 2) as assets which it acquires in exchange for issuing money (its liability). Repos are repurchase agreements between the central bank and a household. In a repo, a household sells a 2 period bond to the central bank against central bank reserves and contracts to purchase it back a period later for the sales price times the central bank’s (gross) policy rate. The central bank remunerates the reserves created by the policy rate: so overall, the repo is costless for both the central bank and households. The repo only serves to replace bonds with money in households’ portfolio during a period during which households might have to transact using money. Any profits (or losses) made by the central bank from its portfolio of assets and liabilities is passed to the Treasury and taxes are raised or lowered accordingly so as to ensure that the Treasury can continue to issue an unchanged quantity of new bonds to replace those that mature.

In each period  $t$ :

1. The stochastic productivity of young households becomes known. (This is an ag-

gregate shock.) Young households decide how much labor to supply to produce the consumption good. The central bank sets the interest rate for the remuneration of money from  $t$  to  $t + 1$ .

2. Old households receive interest on their money holdings from  $t - 1$  to  $t$ . If they entered a repo with the central bank in  $t - 1$ , this repo is now unwound; old households then sell these bonds (which now have a remaining maturity of one period) to the central bank in exchange for money. (The central bank has to accept all 1-period bonds sold by old households at the price implied by its choice for the policy rate. One can think of these transactions as open market operations conducted by the central bank to implement a particular decision over the policy rate.) Old households use their money to purchase some of the young households' newly-produced consumption good. Old households die.
3. The Treasury issues new 2-period bonds to young households and collects taxes to balance the budget.
4. The central bank either offers young households to purchase newly issued bonds outright (regime 1) at the market price, or to repo them in (regime 2). Young households are assumed to need cash to exploit subsequent, unmodelled, trade opportunities.

Figure 1 is a schematic overview of the timing. One-period bonds are perfect substitutes to money in our model because their price is known at the start of period  $t$  (it is determined by the interest rate prevailing from  $t$  to  $t + 1$ ); the assumption that the central bank purchases these bonds is therefore inconsequential. In contrast, the central bank's choice between (1) purchasing bonds with a remaining maturity of two periods, and (2) repo-ing in two-period bonds, matters because it affects the return characteristics of households' portfolio.

We do not explicitly model why young households might need to transact using cash with each other. Explicitly modeling these transactions would complicate the model without affecting the impact that altering households' portfolio composition has on their



behavior. But one might think of the following justification, which would leave the model's first order conditions unchanged. Imagine that once households have sold a share of their production to the old, each young household may, with some probability, find that the good they retained for consumption has perished. This risk is idiosyncratic and can therefore be perfectly insured. Insurance is implemented as follows. There is a single insurance company which compensates in cash households who have lost their good at the end of each period. This payout is financed from contributions by households whose good has not perished. Suppose households need to consume before the end of the period. Then a young household whose good has perished needs to finance purchases of the consumption good while it is waiting for the insurance payout. If other households accept only cash, the household much prefers to own cash rather than bonds for the purpose of these transactions. A costless repo with the central bank allows the household to raise this cash without changing the return distributions of his savings.

How does monetary policy affect real variables in this context? Variations in nominal interest rates affect the actions of both current and future generations of households. When today's interest rate is raised, today's young expect to earn a higher nominal return on their portfolio. The associated positive wealth effect means that the young work and save less in real terms. Their real consumption increases. In addition, if old households hold bonds in their portfolio, their nominal wealth falls. The price level falls as well, and young households have less incentive to sell their good. So young households work and save even less. This means that varying the policy rate has a stronger impact on households' contemporaneous behaviour when bonds are held by households rather than by the central bank.

An increase in today's interest rate also affects real variables in the next period because the wealth of tomorrow's old increases, raising the price level tomorrow, thereby encouraging tomorrow's young to produce more.

When bonds are owned by households, the impact of monetary policy on today's households is stronger, and the volatility of the price level tomorrow is smaller because households own less money, and the value of their bonds tomorrow does not depend on

today's interest rate.

### 3 Model: Detailed Specification

#### 3.1 Households

Our notation is as follows. We index individual households by  $j$ . Labor supply of a young household born in period  $t$  is  $h_{j,t}$ . Using this labor input, the household produces  $y_{j,t} = \omega_t h_{j,t}^\alpha$  real units of the consumption good.  $\omega_t$  is an aggregate productivity shock, distributed independently across periods according to a log-normal distribution with mean  $\mu_\omega$  and standard deviation  $\sigma_\omega$ . A young household's real consumption in period  $t$  is  $c_{j,t}^Y$ . Its nominal money holdings are  $m_{j,t}^Y$ , and its holdings of bonds issued in  $t$  and maturing in  $t+2$  are denoted by  $g_{j,t}^Y$ . The market-clearing price of these bonds is  $P_{t,t+2}^g$ , and that of bonds with a remaining maturity of one period is  $P_{t,t+1}^g$ . The price of the consumption good is  $P_t^c$ . The lump-sum tax is denoted  $\tau_t^Y$  if levied on the young, and  $\tau_t^O$  if levied on the old. We do not indicate the dependence of the endogenous variables on the state variables of our model but it should be taken as read.

Each household's period utility is of the CRRA variety:

$$u(c, h) = \frac{(c^{1-\rho}(1-h)^\rho)^{1-\sigma_c} - 1}{1-\sigma_c} \quad (1)$$

and lifetime utility (recalling no-one can work when they are old) is:

$$U(c_{j,t}^Y, c_{j,t+1}^O, h_{j,t}) = u(c_{j,t}^Y, h_{j,t}) + \beta \mathbb{E} [u(c_{j,t+1}^O, 0) | s_t] \quad (2)$$

where the expectation is taken over future states given the household's information in period  $t$ , summarized by the model's state variables  $s_t$ . Each household maximizes its lifetime utility over  $\{h_{j,t}, m_{j,t}, g_{j,t}^Y\}$  subject to the budget constraints

$$P_t^c (y_{j,t} - c_{j,t}^Y) = m_{j,t}^Y + P_{t,t+2}^g g_{j,t}^Y + \tau_t^Y \quad (3)$$

$$m_{j,t}^Y (1 + i_t) + P_{t+1,t+2}^g g_{j,t}^Y - \tau_{t+1}^O = P_{t+1}^c c_{j,t+1}^O \quad (4)$$

and  $m_{j,t}^Y, g_{j,t}^Y \geq 0$ . The left-hand side of (3) is the proceeds from selling the consumption good, and the right-hand side is the young household's use of the proceeds: it holds some of it in money, uses some to purchase newly issued bonds, and pays some lump-sum taxes. Note that the young do not buy 1 period bonds, which are perfect substitutes for money. We assume that the central bank stands ready to swap one period bonds for money – these are open market operations required to establish a particular 1 period interest rate.

The left-hand side of (4) is the nominal wealth of the old after taxes,  $w_{j,t+1}^O$ : this is the sum of remunerated money holdings and the receipts from selling bonds (now 1-period) to the central bank, minus tax payments. (Notice that one could equally write this problem as one of choosing any other three of the period- $t$  decision variables  $\{c_{j,t}^Y, h_{j,t}, m_{j,t}, g_{j,t}^Y\}$ ; it is clearly optimal for the old households to spend their entire nominal wealth on the consumption good in the absence of a bequest motive.)

### 3.2 Monetary policy

The central bank's policy instrument is the nominal interest rate at which it remunerates money ('Bank Rate'). We compare welfare under two regimes of supplying money to households for transaction purposes. In the first, the central bank repoes in two-period government bonds against money. Repos are unwound in the following period. In the second, the central bank purchases two-period government bonds outright from the private sector and holds them until maturity. Because we are interested in the decision of how best to structure the central bank's assets in the long run rather than in using bond purchases as a conjunctural instrument, we assume that the amount of bonds purchased or repo-ed in is constant.

We make the inconsequential assumption that the central bank buys all bonds with a remaining maturity of one period from old households. The central bank's assets therefore comprise all government bonds with a remaining maturity of one period, and either two-period bonds purchased outright (regime 1) or repo-ed in (regime 2). Its liabilities consist of money which we can think of as issued directly to households or else as held as reserves by commercial banks who issue deposits to households of exactly equivalent value.

We assume that the policy rule for the interest rate is a linear function of the productivity shock,  $\omega_t$ , the random innovation to the level of Bank Rate (a monetary policy shock),  $\varepsilon_t$ , and old households' money holdings at the start of period  $t$ ,  $l_{t-1} = m_{t-1}^Y (1 + i_{t-1})$ , before they convert their bond holdings into money. We show below that the model can be written in terms of these three variables as state variables. We also assume that the policy interest rate is subject to a zero lower bound. Within this bound, the policy rule takes the form

$$i_t = a_1 + a_2 (\omega_t - \mu_\omega) + a_3 (l_{t-1} - \mu_l) + \varepsilon_t \quad (5)$$

where  $a_1$ ,  $a_2$ , and  $a_3$  are scalars. The amount of bonds that the central bank either purchases or repos in,  $g^{CB}$ , is limited by the Treasury's (constant) issuance of bonds,  $\gamma$ . The central bank cannot issue any liabilities other than money ( $g^{CB} \in [0, \gamma]$ ).

### 3.3 Fiscal policy

Fiscal policy only involves setting the size of the nominal lump-sum taxes to levy on households. There are no government expenditures other than transfers to the central bank on maturity of the bonds, and those required to indemnify the central bank for any losses it may make. Government revenues consist only of taxes levied on households and of profits that the central bank may make.

We assume that the government balances its budget in each period by levying an appropriate amount of lump-sum taxes. The amount of bonds issued is assumed to be constant in each period. (We set this quantity at  $\gamma = 1$  so that at any time there are bonds with aggregate face value of 2 outstanding). The costs of servicing the outstanding zero-coupon debt are booked on an accrual basis. In each period, the tax is then equal to the nominal return that that period's old earned on their portfolio:

$$\tau_t^Y + \tau_t^O = i_{t-1} m_{t-1}^Y + (P_{t,t+1}^g - P_{t-1,t+1}^g) (\gamma - g^{CB})$$

The government may be able to levy non-zero lump sum taxes at different rates on the

young and old alive at the same time. But tax policy may be more constrained. We will consider two more constrained scenarios: either the young are taxed ( $\tau_t^O = 0$ ), or the old ( $\tau_t^Y = 0$ ). If the old are taxed, each generation in equilibrium pays as tax the exact amount they earned on their financial assets during their lifetime. This should be interpreted as a fiscal policy that does not attempt to redistribute wealth across generations. It may therefore not come as a surprise that the composition of the central bank's assets has no impact under this tax rule: any impact of asset purchases on the return on households' portfolio is neutralized by this type of fiscal policy. We show this formally in our companion paper, Miles and Schanz (2014). In contrast, if the young are taxed, each generation in equilibrium pays as tax the amount that the previous generation earned on its financial assets during its lifetime. In this case, central bank asset purchases change the composition of the youngs' portfolio, transfer wealth across generations, and has the potential to affect their real decisions. We therefore focus on a fiscal policy which taxes exclusively the young.

### 3.4 Equilibrium

This section characterizes the equilibrium of the model. Let  $x_{j,t} \in \{c_{j,t}^Y, c_{j,t}^O, h_{j,t}, y_{j,t}, m_{j,t}^Y, g_{j,t}^Y\}$  denote the vector of household  $j$ 's choices. A symmetric rational expectations equilibrium, that is, one where all agents within one generation take the same decisions, is a set of contingent plans  $\{c_{j,t}^Y, c_{j,t}^O, h_{j,t}, y_{j,t}, m_{j,t}^Y, g_{j,t}^Y\}$ , prices, taxes, a nominal interest rate and bond purchases by the central bank,  $\{P_t^c, P_{t,t+1}^g, P_{t,t+2}^g, \tau_t^Y, \tau_t^O, i_t, g^{CB}\}$ , and exogenous

processes  $\{\omega_t, \varepsilon_t\}$ , satisfying at all dates  $t$ , for all households  $j$ , and at all states:

$$\frac{\partial U}{\partial m_{j,t}^Y} = 0 \quad (6)$$

$$\frac{\partial U}{\partial g_{j,t}^Y} = 0 \quad (7)$$

$$\frac{\partial U}{\partial h_{j,t}} = 0 \quad (8)$$

$$P_t^c c_{j,t}^O = m_{j,t}^Y + P_{t,t+2}^g g_{j,t}^Y + \tau_t^Y \quad (9)$$

$$P_t^c c_{j,t}^O = m_{j,t-1}^Y (1 + i_{t-1}) + P_{t,t+1}^g g_{j,t-1}^Y \quad (10)$$

$$y_{j,t} = \omega_t h_{j,t}^\alpha \quad (11)$$

$$1/P_{t,t+1}^g = 1 + i_t \quad (12)$$

$$g_t^Y = \gamma - g^{CB} \quad (13)$$

$$c_t^Y + c_t^O = y_t \quad (14)$$

$$\tau_t^Y = m_{t-1}^Y i_{t-1} + (P_{t,t+1}^g - P_{t-1,t+1}^g) g^Y \quad (15)$$

$$i_t = a_1 + a_2 (\omega_t - \mu_\omega) + a_3 (l_{t-1} - \mu_l) + \varepsilon_t \quad (16)$$

$$l_{t-1} = m_{t-1}^Y (1 + i_{t-1}) \quad (17)$$

In equilibrium, all households of a given cohort make the same decisions. (6) is household  $j$ 's first-order condition with respect to the young's money holdings; (7) the first-order condition with respect to bond holdings; and (8) the first-order condition with respect to labor supply. The first-order conditions for money and bonds can be expressed in the typical Euler equation form. Substituting the budget constraints for a young household's consumption into the lifetime utility yields

$$\begin{aligned} & U \quad (18) \\ = & \frac{1}{1 - \sigma_c} \left( \left( (\omega_t h_{j,t}^\alpha - (\tau_t^Y + m_{j,t}^Y + P_{t,t+2}^g g_{j,t}^Y) / P_t^c)^{1-\rho} (1 - h_{j,t})^\rho \right)^{1-\sigma_c} - 1 \right) \\ & + \beta \mathbb{E}_t \left[ \frac{1}{1 - \sigma_c} \left( \left( (m_{j,t}^Y (1 + i_t) + P_{t+1,t+2}^g g_{j,t}^Y - \tau_t^O) / P_{t+1}^c \right)^{(1-\rho)(1-\sigma_c)} - 1 \right) \right] \end{aligned}$$

The optimal solution has first order conditions

- with respect to money holdings,  $m_{j,t}$ :

$$\frac{\partial U}{\partial m_{j,t}^Y} = u'_c(c_{j,t}^Y, h_{j,t}) \left( -\frac{1}{P_t^c} \right) + \beta \mathbb{E}_t \left[ u'_c(c_{j,t+1}^O, 0) \frac{1+i_t}{P_{t+1}^c} \right] = 0 \quad (19)$$

where we denote the marginal utility with respect to consumption as

$$u'_c(c, h) = (1 - \rho) c^{(1-\sigma_c)(1-\rho)-1} (1 - h)^{\rho(1-\sigma_c)} \quad (20)$$

- with respect to holdings of newly issued bonds,  $g_{j,t}^Y$ :

$$\frac{\partial U}{\partial g_{j,t}^Y} = u'_c(c_{j,t}^Y, h_{j,t}) \left( -\frac{P_{t,t+2}^g}{P_t^c} \right) + \beta \mathbb{E}_t \left[ u'_c(c_{j,t+1}^O, 0) \frac{P_{t+1,t+2}^g}{P_{t+1}^c} \right] = 0 \quad (21)$$

The first-order conditions for money and bonds can therefore be written as

$$u'_c(c_{j,t}^Y, h_{j,t}) = \beta \mathbb{E}_t \left[ (1+i_t) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{j,t+1}^O, 0) \right] \quad (22)$$

$$u'_c(c_{j,t}^Y, h_{j,t}) = \beta \mathbb{E}_t \left[ \frac{P_{t+1,t+2}^g}{P_{t,t+2}^g} \frac{P_t^c}{P_{t+1}^c} u'_c(c_{j,t+1}^O, 0) \right] \quad (23)$$

$(1+i_t)(P_t^c/P_{t+1}^c)$  is the real gross return on money;  $(P_{t+1,t+2}^g/P_{t,t+2}^g)(P_t^c/P_{t+1}^c)$  is the real gross return on bonds.

(9) is the budget constraint of the young: the revenues from selling their production (net of own consumption) equals their nominal savings and tax payments. (10) is the budget constraint of the old: they consume their entire savings. (11) is the production function. (12) says that the gross return of bonds with a remaining maturity of one period must equal that on money: this is because the nominal return of these two assets is known at  $t$ . (13) and (14) are the market clearing conditions for bonds and the consumption good, respectively. (13) states that per-person purchases of newly issued bonds must equal the net per-person supply of bonds: the difference between issuance,  $\gamma$ , and the amount of newly issued bonds that the central bank buys (per person),  $g^{CB}$ . (14) states that in equilibrium, the period  $t$  per-person consumption of the old and the young must equal per-person production in  $t$ ,  $y_t$ .

(15) is the fiscal policy rule: taxes are set equal to the government sector's payments to the household sector: this is a balanced budget constraint. (16) is the monetary policy rule.

The budget constraints (9) and (10) imply a condition for equilibrium in the market for money:

$$m_{t-1}^Y (1 + i_{t-1}) + P_{t,t+1}^g g_{j,t-1}^Y = m_t^Y + P_{t,t+2}^g g_{j,t}^Y + \tau_t^Y$$

The left-hand side is the nominal wealth of the old, which the old use to pay for the consumption good; the right-hand side shows what the young do with the money earned. The tax rule implies that the nominal wealth of the young post taxes,  $w_t^Y$ , is constant and so is state-independent. Using (9), and (15),

$$\begin{aligned} w_t^Y &= m_t^Y + P_{t,t+2}^g g_{j,t}^Y = P_t^c c_t^O - \tau_t^Y \\ &= P_t^c c_t^O - (m_{t-1}^Y i_{t-1} + (P_{t,t+1}^g - P_{t-1,t+1}^g) g_{t-1}^Y) \\ &= (m_{t-1}^Y (1 + i_{t-1}) + P_{t,t+1}^g g_{t-1}^Y) - (m_{t-1}^Y i_{t-1} + (P_{t,t+1}^g - P_{t-1,t+1}^g) g_{t-1}^Y) \\ &= m_{t-1}^Y + P_{t-1,t+1}^g g_{t-1}^Y = w_{t-1}^Y \end{aligned}$$

We impose the initial condition that  $w_{-1}^Y = w^Y$ .

Equations (6) - (17) show that the endogenous variables in period  $t$  depend on history via  $m_{t-1}^Y$ ,  $i_{t-1}$ ,  $P_{t-1,t+1}^g$ , and  $g_{t-1}^Y$ . Because we assume that the central bank's purchases of government bonds are constant,  $g_{t-1}^Y = \gamma - g^{CB}$ . We also know that when the government taxes the young, their post-tax nominal wealth is constant. So

$$m_{t-1}^Y + P_{t-1,t+1}^g g_{t-1}^Y = m_{t-1}^Y + P_{t-1,t+1}^g (\gamma - g^{CB}) = w^Y$$

such that  $P_{t-1,t+1}^g$  is a linear function of  $m_{t-1}^Y$ . Finally, notice that the balanced budget rule requires the tax to be equal to the nominal return on the previous period's young households' savings. So we can rewrite the tax as

$$\tau_t^Y = l_{t-1} + (\gamma - g^{CB}) / (1 + i_t) - w^Y$$



where  $l_{t-1} = m_{t-1}^Y (1 + i_{t-1})$ . This means that the model, after imposing symmetry, can be written such that it only depends on  $l_{t-1}$ ,  $\omega_t$ , and  $\varepsilon_t$ . We choose these three variables as the state variables. The remaining seven endogenous variables are  $c_t^Y, c_t^O, h_t, i_t, m_t^Y, P_t^c$  and  $P_{t,t+2}^g$ . They are determined by the following seven equations:

$$u'_c(c_t^Y, h_t) = \beta \mathbb{E}_t \left[ (1 + i_t) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{t+1}^O, 0) \right] \quad (24)$$

$$u'_c(c_t^Y, h_t) = \beta \mathbb{E}_t \left[ \frac{1/(1 + i_{t+1})}{P_{t,t+2}^g} \frac{P_t^c}{P_{t+1}^c} u'_c(c_{t+1}^O, 0) \right] \quad (25)$$

$$1 - h_t = \rho / (1 - \rho) c_t^Y \quad (26)$$

$$m_t^Y = w^Y - P_{t,t+2}^g (\gamma - g^{CB}) \quad (27)$$

$$P_t^c = (l_{t-1} + (\gamma - g^{CB}) / (1 + i_t)) / c_t^O \quad (28)$$

$$c_t^O = \omega_t h_t^\alpha - c_t^Y \quad (29)$$

$$i_t = a_1 + a_2 (\omega_t - \mu_\omega) + a_3 (l_{t-1} - \mu_l) + \varepsilon_t \quad (30)$$

## 4 Results

It is useful to keep in mind that the value of having nominal assets is that they can lift welfare above the decentralised solution that would arise if these assets did not exist. If households had no access to nominal assets, they would be unable to save. In the unique decentralised equilibrium, young households would consume their entire production, the old would starve, and the real interest rate would be  $-100\%$ , because the consumption good is perishable.

It is well known how to improve welfare above what is achieved in the decentralised solution. One possibility is a scheme that resembles unfunded social security: transfers between young and old households that provide old households with just the right amount of nominal wealth that allows them to consume precisely the first-best amount at given market prices. This does not rely on households having access to nominal assets for savings purposes: instead, the first best is implemented by a simple transfer between contemporaneous generations, where the size of the transfer depends on productivity. The first best can therefore be implemented while keeping the fiscal budget in balance.

However, the taxes and transfers associated with the implementation of the first best solution require fiscal policy to be flexible to a degree that appears unrealistic. In practice, workers contribute a larger share to the Treasury's income than pensioners, for example via taxes on wages and salaries and on firms' profits. We therefore constrain taxes to fall only on the working generation (young households) levied in order to pay the returns on money and bonds issued by the Treasury and the central bank. This constraint, in combination with the requirement that the fiscal budget needs to be balanced in each period, renders fiscal policy completely passive. Taxes do not lead to a distribution of wealth that achieves the first best. In this situation, nominal assets - reserves and bonds - can help improve welfare because they allow households to transfer wealth into retirement, and the reserve provisioning regime matters to the extent that it affects the returns that households earn on these assets.

We first characterize the first best allocation assuming that the policymaker can choose households' consumption and labour supply directly and show how it can be implemented via transfers when fiscal policy is unconstrained (Section 4.1). The more interesting case is a monetary economy - that is one in which money and bonds are willingly held - in which fiscal policy is constrained to taxing only the young to balance the budget in each period (Section 4.2).

## 4.1 First best allocation and implementation when fiscal policy is only constrained to achieve a balanced budget

We define welfare as the unconditional expected utility of all households alive at some point in time:

$$W = \mathbb{E}_{s_t} [u(c_t^Y, h_t) + u(c_t^O, 0)]$$

where the expectation is taken over all realizations of the state variables  $s_t = (l_{t-1}, \omega_t, \varepsilon_t)$ , that is, before the realisation of the shocks is known.<sup>2</sup> To determine the first-best allocation with respect to this definition of welfare, we assume that the social planner chooses

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<sup>2</sup>This is a timeless measure of welfare; it is not conditioned on  $l_{t-1}$ . When evaluating the expectation numerically, we first calculate the distribution of  $l_{t-1}$  and then draw from this distribution.

labour supply and consumption levels directly. He solves

$$\max_{\{c_{j,t}^Y, h_{j,t}, c_{j,t}^O\}} W$$

for all households  $j$ , subject to the condition that aggregate consumption must not exceed aggregate production:  $c_t^Y + c_t^O \leq y_t$ . Proposition 1 states the intuitive result that in the welfare maximizing allocation, the entire production of the perishable good is consumed in each period, and that the effect of increasing the consumption of the young, (31), and of increasing their labour supply, (32), on the sum of the utilities of the young and the old must equal zero.

**Proposition 1** *The first-best allocation is given by the solution to*

$$u'_c(c_t^Y, h_t) - u'_c(c_t^O, 0) = 0 \quad (31)$$

$$u'_h(c_t^Y, h_t) + u'_c(c_t^O, 0) y'_h(\omega_t, h_t) = 0 \quad (32)$$

for all  $t$ .

**Proof.** The first best allocation has the property that all young households at a given point in time produce and consume the same, and all old households at a given point in time consume the same, because the utility function is strictly concave in both consumption and leisure. We therefore restrict attention to allocations that have the following symmetry properties: for all households  $i, j$ ,

$$c_{i,t}^Y = c_{j,t}^Y \quad (33)$$

$$h_{i,t} = h_{j,t} \quad (34)$$

$$c_{i,t}^O = c_{j,t}^O \quad (35)$$

We omit individual-specific subscripts in the following. Since  $c_t^O = y_t - c_t^Y$ , we can write the welfare maximization problem as

$$\max_{\{c_t^Y, h_t\}} E [u(c_t^Y, h_t) + u(y(\omega_t, h_t) - c_t^Y, 0)] \quad (36)$$

The first-order conditions of this problem are, for all  $t$ , given by (31) and (32). ■

We solve equations (31) and (32) for the specific utility and productions functions (1) and (11) in the annex; see Proposition 3. Labour supply is constant in the first best allocation, while consumption and production are proportional to the productivity shock.

Proposition 2 states that the first-best allocation can be implemented under a simple and intuitive combination of tax and interest rate rules. Equation (37) states that the interest rate is constant, and equal to the inverse of the discount factor. Equation (38) says that taxes on the old are set such that the old retain exactly the amount of wealth necessary to purchase the first-best amount of the good,  $c_t^{O*}$ . ( $c_t^{O*}$  can be calculated from the equations determining the first best allocation, (31) and (32) in Proposition 1 independently of taxes;  $w^Y$  is the post-tax nominal wealth of the young, a constant.) Equation (39) states that taxes on the young are set to balance the government's budget in each period.

**Proposition 2** *Let  $c_t^{O*}$  denote the first-best consumption level of old households. The policymaker can implement the first-best allocation by setting*

$$1 + i = 1/\beta \quad (37)$$

$$\tau_t^O = w^Y (1 + i) - w^Y u'_c(c_t^{O*}, 0) c_t^{O*} \quad (38)$$

$$\tau_t^Y + \tau_t^O = iw^Y \quad (39)$$

The first best can be implemented even though the government's budget is required to be balanced in each period: Taxes on young households ensure that the government's budget is balanced in each period (eq. (39)). The nominal interest rate is held constant at the inverse of the discount factor (eq. (37)). Setting the nominal interest rate at this

level encourages a young household to save at the welfare maximizing level. The proof is in the annex.

With a constant interest rate, there is no role for bonds separate from that of money in our decentralized economy. Bond purchases by the central bank are completely neutral, and welfare is the same independently of how the central bank provides reserves for transaction purposes. As discussed above, neutrality results of this sort have been derived elsewhere in the literature, under the assumption that fiscal policy can offset the distributional effects of monetary policy. But there are good reasons to assume that this assumption is too strong. The tax rule that implements the first best in our model is, effectively, a tax that depends only on the age of the household. In the following section, we consider a tax rule that is more restrictive but probably more realistic: where taxes are levied only on workers (the young).

## **4.2 Impact of central bank asset purchases when the first best cannot be implemented**

We now consider the impact of central bank asset purchases when the Treasury can tax only young households. This makes sense for practical purposes, as most of the government's tax revenue results from taxing workers rather than pensioners.<sup>3</sup> By varying the nominal interest rate, monetary policy can then play a role in improving welfare by creating transfers between generations which fiscal policy, constrained to taxing only young households, is unable to achieve by itself.

We turn to numerical optimisation to solve for optimal policy. Section 4.2.1 explains how we calibrate the model. Section 4.2.2 shows the central bank's balance sheet for the base calibration. Section 4.2.3 presents our results for the optimal reserve provision regime, and optimal linear interest rate rules in these regimes.

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<sup>3</sup>We investigated the opposite assumption, where only the old are taxed, in Miles and Schanz (2014). We showed that in this case, even though bonds are not perfect substitutes for money (the term premium is generally non-zero), central bank asset purchases are neutral. Their proof carries through to the model presented here.

### 4.2.1 Calibration

Table 1 summarizes the parameters of the model in the base calibration. There is inevitably a tension between wanting the model to be simple (so using two period lives) and realism. Two period lives means periods must be long. That stretches the nature of the monetary policy decision uncomfortably, because we want the policy rate to be set for one period. But for our purposes what really matters is that we have one asset (a bond) with a life which is significantly longer than the period for which the interest rate set by the central bank can be known with some certainty. Correspondingly, the key characteristic of ‘money’ in our model is not its maturity but the absence of interest rate risk.

We should think of a period as about half an adult life – so of the order of 25 years. We set the discount factor,  $\beta$ , to 0.66, implying a discount rate of 0.5 (or 50%). With a 25 year period that corresponds to a discount rate of about 2% a year.

For the utility function, we set the exponent  $\rho$  on consumption and leisure such that in equilibrium hours worked are about a fifth of maximum labor supply, corresponding to the idea that people on average work about 8 hours for 220 days per year, that is  $8 \cdot 220 / (24 \cdot 365) = 20\%$  of their total time. This is approximately the case for  $\rho = .85$ . For the CRRA risk aversion parameter,  $\sigma_c$ , we use a value of 4 for our base case but also present results for lower risk aversion.

We assume that the production function is linear in labor. This is a natural assumption in a model that covers the long run but, at the same time, omits capital as an explicit production factor: what we call labor input should best be thought of as a composite capital and labor input. Assuming constant returns to scale then seems plausible.

We assume that the labour productivity shock is log-normally distributed with parameters  $(0, 0.2)$ . This translates into a standard deviation of 20% and a mean of just above 1. The standard deviation of labor productivity relative to its mean is  $SD[y/h]/E[y/h] = SD[\omega h/h]/E[\omega h/h] = 20\%$ . This corresponds approximately to the standard deviation of detrended labor productivity in the UK and the US since 1855 over non-overlapping 25-year periods. (Using data since 1889 provided by Carter et al

(2006), that of the US appears to be a little lower.) For the base calibration, we assume that the standard deviation of the policy innovation is zero. We also show results for positive standard deviations, mainly to be able to compute impulse response functions for interest rate shocks.

Nominal post-tax wealth of the young,  $w^Y$ , and the face value of bonds issued in each period,  $\gamma$ , are selected such that the central bank's assets (bonds) and its money liabilities are approximately equal in steady state for a policy rule under which the central bank purchases all newly issued bonds. In this case, the central bank owns bonds with a market value of  $\gamma (P_{t,t+1}^g + P_{t,t+2}^g)$ . Normalising  $\gamma = 1$ , we set  $w^Y = (1/\beta + 1/\beta^2) = 1.11$ .

#### 4.2.2 The central bank's balance sheet

Table 2 shows how the central bank's balance sheet depends on the realisation of the productivity shock (in columns) for the base parameterisation of the model, in which it is optimal to provide reserves via repos and to lower the interest rate when productivity is high. The central bank's assets consist of repos and bonds. In the case illustrated here, the central bank does not purchase any newly issued (ie, two-period) bonds outright. Instead, households purchase these bonds and repo them out to the central bank. The 'repo' position is therefore equal to the market value of newly issued bonds. Because the interest rate falls in the aggregate productivity shock, the market value of these bonds increases as productivity rises.

The position 'Bonds with remaining maturity of one period' reflects our assumption that the central bank purchases all bonds with a remaining maturity of one period. This assumption is inconsequential because the price of these bonds is certain once the central bank has set today's interest rate, so these bonds are equivalent to money.

The central bank's liabilities consist of money. Some has been created from remunerating reserve accounts; this amount is independent of contemporaneous productivity because reserves were remunerated at the previous period's interest rate. Money is also created when the central bank purchases bonds outright or when it repos them in.

Money is absorbed when new bonds are issued and when young households are taxed.

Because taxes are set to balance the budget, they keep constant the value of claims on the government (money and bonds). One-period bonds are exchanged against money, so the central bank's liabilities remain constant when the central bank provides reserves via repos.

While taxes ensure that the central bank's liabilities remain constant, its assets vary with productivity because the value of newly issued bonds depends on the prevailing interest rate. When productivity is low, interest rates are high, and newly issued bonds are worth little, so the central bank's equity is negative. (If it held bonds outright, it would also book a loss on its holdings of bonds with a remaining maturity of one period.) The opposite is true when productivity is high.

### **4.2.3 Welfare and optimal interest rates for different regimes for providing reserves**

When the Treasury can levy state-dependent taxes on both the old and the young, the interest rate that implements the first best is constant and equal to the inverse of the discount factor, while taxes vary with productivity (see Proposition 2). When the Treasury can only tax the young, the optimal interest rate shows some state dependence. This is because the central bank can transfer wealth between generations through one-off variations of the policy rate. Monetary policy aims to offset the fact that taxes are restricted to fall on the young only.

But monetary policy cannot fully substitute for flexible taxation. In particular, varying today's interest rate shifts nominal wealth between young and old households next period. Next period's transfers depend on today's productivity. This is not a feature of the first-best rule, where next period's transfers only depend on next period's realisation of productivity. In this sense, an activist monetary policy risks introducing noise in the distribution of wealth of future generations. So it may not surprise that the interest rules that are optimal for the parameterisations we consider respond only weakly to productivity shocks.

When interest rates are not very volatile, money and bonds are relatively close sub-



stitutes in equilibrium, and changes in the central bank's balance sheet structure matter little for welfare. Our results suggest that the cumulated discounted welfare gain from getting the reserves provisioning regime right lies in the order of 1% of the level of aggregate production.

Section 4.2.3 describes the impulse responses to productivity and interest rate shocks. Section 4.2.3 focuses on the interaction of optimal reserve provisioning and associated optimal interest rate rule.

**Impulse responses** Table 3 shows impulse responses for parameters that are identical to the base calibration (Table 1) but allows for modest policy innovations ( $\sigma_\varepsilon = 10\%$ ) to be able to trace the impact of an interest rate shock. Wealth effects dominate substitution effects throughout in this calibration. This is also the case in the other calibrations that we consider.

Panel 3.1 holds the interest rate constant in order to isolate the effects of a productivity shock of +20%. Production rises, but by less than productivity as young households respond to the positive wealth of higher productivity effect by consuming more and by reducing their labour supply. This reduces the price level, increases expected inflation, and lowers the real interest rate. Young households respond to the negative wealth effect of a lower real interest rate by increasing their savings ratio.

Panel 3.2 shows impulse responses to an interest rate shock of +10pp. A one-off increase in the nominal interest rate increases the nominal wealth of the old tomorrow. The associated increase in tomorrow's price level is dampened: at higher prices, tomorrow's young have a greater incentive to sell their good. On balance, the real interest rate rises. The young today respond to the associated positive wealth effect by consuming more and saving less. If old households hold bonds, a higher interest rate reduces their nominal wealth, lowers the price level, and reduces today's young households' incentive to sell their good. (Monetary policy does not lead to a wealth transfer between contemporaneous generations because taxes are set to keep the young's post-tax wealth constant.)

**Optimal reserve provisioning and interest rate rules** Table 4 shows the optimal reserve provisioning regime (col. 5) and the associated interest rate rule (cols. 6-7), steady state, and welfare (cols. 8-13) for a variety of parameterisations. We set the coefficient  $a_3$  on the predetermined state variable  $l_{t-1}$  to zero: varying this parameter affected welfare so little that we were often unable to find an optimum that was robust to the different sets of realisations of the shocks.<sup>4</sup>

The optimal reserve provisioning regime turns out to be a corner solution for all parameter combinations that we investigated, with the exception of the case in which the interest rate was, for all practical purposes, independent of the productivity shock (row 7). Repos appear to be generally preferable (col. 5), but the welfare differences are small. This is illustrated in columns 12 and 13. In column 12, welfare is expressed in consumption equivalents relative to first best. For example, a value of 98.9% means that if, starting from the first best allocation, the consumption levels of all households were reduced to 98.9% of their first-best levels, then welfare would be just as high as in the decentralised solution. In column 13, we assume that reserves are provided not by the optimal regime (repos, in the base case), but by the worse method (outright purchases, in the base case). Column 13 shows by how many percentage points the welfare measure of column 12 declines in this case when monetary policy is allowed to adjust optimally to the changed provision of reserves. For example, the value 0.03 means that the reduction in welfare under the 'wrong' reserve provisioning regime is equal to 0.03% of aggregate consumption for as long as the regime is in place. This number is relatively small. Discounted over all future periods the welfare gain of choosing the reserve provisioning regime correctly is, if we discounted the future by a 2% real rate, equal to  $0.03\%/2\% = 1.5\%$  of annual GDP. This is the same order of magnitude as in the other calibrations.

Outright purchases of all bonds are optimal when there is volatility in the prices of bonds that is unrelated to changes in productivity: that is, when innovations to the central bank's policy rule have a high variance (rows 5-6). This appears intuitive: If the central bank holds these bonds, the volatility of bond prices has no impact on any

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<sup>4</sup>In most cases, the algorithm found values of  $a_3$  between zero and 0.3 to be optimal.

variables in the model, in particular not the price level, because taxes do not change when the market price of bonds changes. In contrast, if they are held in private sector, the market value of the portfolio of the old is subject to shocks due to fluctuations in the interest rate unrelated to productivity. With risk-averse households, welfare improves if these random fluctuations are avoided.

Rows 1-4 and 7-8 show that when the central bank can control the interest rate precisely, repos are the optimal form of providing reserves. As explained before, this reflects that when bonds are owned by households, the desired impact of monetary policy on today's households is stronger, and the costs - in terms of volatility of the price level tomorrow - is smaller because households own less money, and the value of their bonds tomorrow does not depend on today's interest rate. Repos leave the price risk associated with bond holdings with households, and are therefore generally preferred. The same reasoning also suggests that policy should be more activist when bonds are owned by households. This is indeed suggested by the simulations, which show that the response of interest rates to productivity shocks,  $a_2^*$ , has a larger absolute value when reserves are provided via repos.

Rows 5-6 consider scenarios in which outright purchases are preferred. For these parameterisations, the central bank's interest rate rule contains noise (the standard deviation of the 'implementation error',  $\sigma_\varepsilon$ , is positive). If bonds are held by households, this increases the volatility of their nominal wealth for reasons unrelated to productivity. If, in contrast, the central bank owns these bonds, variations in the market value of the bonds have no impact on taxes, nor on any other nominal nor real variables in the model. Hence the result that if interest rates are set with some noise, reserves should be provided via outright purchases.

For most parameterisations, it is optimal to lower interest rates when productivity is high. This means that bonds are worth less when the price level is high (and the price level is low, see Corollary 1 of Proposition 2) and to raise rates when productivity is low (and the price level is high). Bonds therefore do not provide insurance against inflation shocks. But households nevertheless hold them because they earn a (small) positive term

premium (col. 6). If the central bank purchases bonds, this premium falls somewhat (not shown). This term premium turns negative in the parameterisation in row 8, for which it is optimal to raise the interest rate when productivity is high, because bonds in this case do provide insurance against unexpected inflation. The term premium is small because interest rates do not vary much with productivity: for example, in the first parameterisation, the interest rate falls by  $0.26 \cdot 0.2 = 0.05$ , or *5pp*, from its steady state level of 48.4% in response to a one standard deviation increase in productivity. The term premium tends to increase with the degree of risk aversion (rows 2-4).

## 5 Conclusions

We have developed a simple and highly stylized model of the economy to assess whether shifts in the balance sheet of the central bank have a significant impact on real variables. Given the recent financial crisis banks are likely to demand a considerably higher stock of liquid assets than before. Some of those liquid assets are likely to be held in the form of reserves at the central bank. Assuming that the central bank accommodates changes in the demand for its reserves, it would have to decide how to provide the reserves. We investigate two options: outright purchases of nominal, fixed rate government bonds, versus repos, which leave the price risk of bonds with the private sector. Analytical solutions to that model are not, in general, available. So we turn to simulations of a calibrated version of this OLG model. We find that across a fairly wide set of environments – with different rules for the setting of interest rates and different ways in which fiscal policy is conducted – the impact of changing the way in which the central bank provides reserves to the private sector working through a portfolio re-balancing channel is weak or absent. This suggests that central banks that bought large quantities of bonds, financed by creating reserves, in their monetary policy operations (QE) need not feel any urgency about selling them should commercial bank demand for reserves stay high. This is because our results suggest that the benefits of the central bank switching from holding bonds against reserves to providing reserves through repo are not likely to be large. If there is signif-

icant noise in implementing monetary policy, it is possible that central banks holding significant stocks of long-dated bonds is sensible.

## 6 Annex

### 6.1 Computational solution strategy

We reduce the model to two equations in  $(c_t^Y, P_{t+2}^g)$  by entering the production function (11), the budget constraints (9) and (10), and the market clearing conditions for the consumption good and bonds, (14) and (13), into the first-order conditions for money and bonds, (22) and (23). This yields the equilibrium condition

$$u'_c(c_t^Y, h_t) = \beta E_t \left[ (1 + i_t) \frac{m_t^O + (\gamma - g^{CB}) / (1 + i_t)}{m_{t+1}^O + (\gamma - g^{CB}) / (1 + i_{t+1})} \frac{c_{t+1}^O}{c_t^O} u'_c(\omega_{t+1} h_{t+1}^\alpha - c_{t+1}^Y, 0) \right] \quad (40)$$

We also exploit that in equilibrium, optimal labour supply only depends on contemporaneous consumption of the young:  $1 - h_t = \rho / (1 - \rho) c_t^Y$ . This yields

$$u'_c(c_t^Y, h_t) = (1 - \rho) \left( \frac{1 - h_t}{c_t^Y} \right)^\rho \frac{1}{\left( (c_t^Y)^{1-\rho} (1 - h_t)^\rho \right)^\sigma} \quad (41)$$

$$= (1 - \rho) \left( \frac{\rho}{1 - \rho} \right)^{-\rho(\sigma-1)} (c_t^Y)^{-\sigma} \quad (42)$$

$$u'_c(c_{t+1}^O, 0) = \frac{\partial}{\partial c} \left( \frac{c^{(1-\rho)(1-\sigma_c)} - 1}{1 - \sigma_c} \right) = (1 - \rho) c^{-\rho - \sigma_c(1-\rho)} \quad (43)$$

and

$$\begin{aligned} c_{t+1}^O u'_c(c_{t+1}^O, 0) &= c_{t+1}^O (1 - \rho) (c_{t+1}^O)^{-\rho - \sigma_c(1-\rho)} \\ &= (1 - \rho) (c_{t+1}^O)^{-(1-\rho)(\sigma_c-1)} \end{aligned} \quad (44)$$

$$= (1 - \rho) (\omega_{t+1} h_{t+1} - c_{t+1}^Y)^{-(1-\rho)(\sigma-1)} \quad (45)$$

$$= (1 - \rho) \left( \omega_{t+1} \left( 1 - \frac{\rho}{1 - \rho} c_{t+1}^Y \right) - c_{t+1}^Y \right)^{-(1-\rho)(\sigma-1)} \quad (46)$$

Entering (42) and (46) into the equilibrium condition (41) yields

$$c_t^O u'_c(c_t^Y, h_t) = \beta (1 + i_t) (m_t^O + (\gamma - g^{CB}) / (1 + i_t)) \mathbb{E}_t \left[ \frac{c_{t+1}^O u'_c(\omega_{t+1} h_{t+1} - c_{t+1}^Y, 0)}{m_{t+1}^O + (\gamma - g^{CB}) / (1 + i_{t+1})} \right]$$

equivalently

$$\begin{aligned} & \left( \omega_t \left( 1 - \frac{\rho}{1-\rho} c_t^Y \right) - c_t^Y \right) (1 - \rho) \left( \frac{\rho}{1-\rho} \right)^{-\rho(\sigma-1)} (c_t^Y)^{-\sigma} \\ = & \beta ((1 + i_t) m_t^O + (\gamma - g^{CB})) \mathbb{E}_t \left[ \frac{(1 - \rho) \left( \omega_{t+1} \left( 1 - \frac{\rho}{1-\rho} c_{t+1}^Y \right) - c_{t+1}^Y \right)^{-(1-\rho)(\sigma-1)}}{m_{t+1}^O + (\gamma - g^{CB}) / (1 + i_{t+1})} \right] \end{aligned}$$

equivalently,

$$\begin{aligned} & \left( \omega_t \left( 1 - \frac{\rho}{1-\rho} c_t^Y \right) - c_t^Y \right) \left( \frac{\rho}{1-\rho} \right)^{-\rho(\sigma-1)} (c_t^Y)^{-\sigma} \\ = & \beta (1 + i_t) (l_{t-1} + (\gamma - g^{CB}) / (1 + i_t)) \\ & \cdot \mathbb{E}_t \left[ \frac{\left( \omega_{t+1} \left( 1 - \frac{\rho}{1-\rho} c_{t+1}^Y \right) - c_{t+1}^Y \right)^{-(1-\rho)(\sigma-1)}}{l_t + (\gamma - g^{CB}) / (1 + i_{t+1})} \right] \end{aligned} \quad (47)$$

where the expectation is over the shocks,  $\omega_t$  and  $\varepsilon_t$ . We solve (47) numerically for some initial guess for each node in the grid of state variables.  $i_t$  and  $i_{t+1}$  are computed from (16) for some initial value of  $\mu_m$ . We solve the expectation using Gauss-Hermite integration (we report results for at least 9 nodes in each dimension). Notice that  $\omega_{t+1}$  is log-normally distributed, so we define  $w^{(k)} = \exp \{ n_w^{(k)} \}$  where  $n_w^{(k)}$  are the nodes pertaining to the normally distributed random variable. That is,  $w^{(k)}$  is log-normally distributed with

$$\begin{aligned} m &= \exp(\mu + \sigma^2/2) \\ s &= (\exp(\sigma^2) - 1) \exp(2\mu + \sigma^2) \end{aligned}$$

We compute the expectation as

$$\left(\frac{1}{2\pi}\right)^2 \sum_k \sum_l w_\omega^{(k)} w_\varepsilon^{(l)} \left( \frac{\left( e^{n_\omega^{(k)}} \left( 1 - \frac{\rho}{1-\rho} c_{t+1}^Y \right) - c_{t+1}^Y \right)^{-(1-\rho)(\sigma-1)}}{m_t^Y \left( 1 + i_t^{(k,l)} \right) + (\gamma - g^{CB}) / \left( 1 + i_t^{(k,l)} \right)} \right)$$

where  $w_\omega^{(k)}$ ,  $w_\varepsilon^{(l)}$  are the weights in the Gauss-Hermite integration pertaining to node  $(k, l)$ .

This yields a grid of equilibrium outcomes for  $c_{t+1}^Y$  for each combination of nodes. We evaluate the expectation in (47) using this grid, and compute revised guesses for  $c_t^Y$  in (47), until the revisions become small. The other endogenous variables can then be computed explicitly.

Welfare is defined as the unconditional expected utility of all households alive at some point in time:

$$W = \mathbb{E}_{s_t} [u(c_t^Y, h_t) + u(c_{t+1}^O, 0)] \quad (48)$$

where the expectation is running over all state variables  $s_t = (l_{t-1}, \omega_t, \varepsilon_t)$ . The distribution of  $\omega_t$  and  $\varepsilon_t$  is exogenous. We determine the stationary distribution of the lagged endogenous variable by simulating the model forward. We then compute (48) conditional on  $l_{t-1}$  using Gauss-Hermite integration and average over realizations of  $l_{t-1}$ .

When holding  $h$  constant at its first-best value, we solve

$$c_t^O u'_c(c_t^Y, h_t) = \beta (1 + i_t) (m_t^O + (\gamma - g^{CB}) / (1 + i_t)) \mathbb{E}_t \left[ \frac{c_{t+1}^O u'_c(\omega_{t+1} h^* - c_{t+1}^Y, 0)}{m_{t+1}^O + (\gamma - g^{CB}) / (1 + i_{t+1})} \right]$$

where

$$\begin{aligned} c_{t+1}^O u'_c(c_{t+1}^O, 0) &= c_{t+1}^O (1 - \rho) (c_{t+1}^O)^{-\rho - \sigma_c(1-\rho)} \\ &= (1 - \rho) (c_{t+1}^O)^{-(1-\rho)(\sigma_c-1)} \\ &= (1 - \rho) (\omega_{t+1} h^* - c_{t+1}^Y)^{-(1-\rho)(\sigma-1)} \end{aligned}$$

and

$$u'_c(c_t^Y, h_t) = (1 - \rho) \left( \frac{1 - h^*}{c_t^Y} \right)^\rho \frac{1}{\left( (c_t^Y)^{1-\rho} (1 - h^*)^\rho \right)^\sigma} \quad (49)$$

$$= (1 - \rho) (1 - h^*)^{\rho - \rho\sigma} (c_t^Y)^{-\rho - \sigma(1-\rho)} \quad (50)$$

and

$$\begin{aligned} c_t^O u'_c(c_t^Y, h_t) &= (\omega_t h^* - c_t^Y) u'_c(c_t^Y, h^*) \\ &= (\omega_t h^* - c_t^Y) (1 - \rho) (1 - h^*)^{\rho - \rho\sigma} (c_t^Y)^{-\rho - \sigma(1-\rho)} \end{aligned}$$

equivalently

$$\begin{aligned} &(\omega_t h^* - c_t^Y) (1 - \rho) (1 - h^*)^{\rho - \rho\sigma} (c_t^Y)^{-\rho - \sigma(1-\rho)} \\ &= \beta \left( (1 + i_t) m_t^O + (\gamma - g^{CB}) \right) \mathbb{E}_t \left[ \frac{(1 - \rho) (\omega_{t+1} h^* - c_{t+1}^Y)^{-(1-\rho)(\sigma-1)}}{m_{t+1}^O + (\gamma - g^{CB}) / (1 + i_{t+1})} \right] \end{aligned}$$

## 6.2 First best allocation

### 6.2.1 Implementation of the first best - proof of Proposition 2

**Proof.** To prove Proposition 2, we evaluate the first derivatives of utility with respect to a young household's labour supply, and its holdings of money and bonds, at the first best allocation  $(c_t^{Y*}, h_t^*, c_t^{O*})$  and show that these derivatives are zero under the policy rules (37)-(39). Using the production function  $y(\omega_t, h_t) = \omega_t h_t^\alpha$ , the first derivative with respect to labour supply is given by

$$U'_h(c_{j,t}^Y, h_{j,t}) = u_h(c_{j,t}^Y, h_{j,t}) + \beta E \left[ \frac{\partial c_{j,t+1}^O}{\partial h_{j,t}} u'_c(c_{j,t+1}^O, 0) \mid s_t \right]$$

Using the budget constraints (3) and (4),  $c_{j,t+1}^O$  can be expressed as

$$\begin{aligned} c_{j,t+1}^O &= (m_{j,t}^Y (1 + i_t) + P_{t+1,t+2}^g g_{j,t,t+2}^Y - \tau_{t+1}^O) / P_{t+1}^c \\ &= \left( (P_t^c (\omega_t h_{j,t}^\alpha - c_{j,t}^Y) - P_{t,t+2}^g g_{j,t,t+2}^Y - \tau_t^Y) (1 + i_t) + P_{t+1,t+2}^g g_{j,t,t+2}^Y - \tau_{t+1}^O \right) / P_{t+1}^c \end{aligned}$$



In equilibrium, for a linear production function ( $\alpha = 1$ ),  $\partial c_{j,t+1}^O / \partial h_{j,t} = \omega_t (1 + i_t) P_t^c / P_{t+1}^c$  and so the derivative of a young household's lifetime utility with respect to labour supply is

$$U'_h(c_{j,t}^Y, h_{j,t}) = u'_h(c_{j,t}^Y, h_{j,t}) + \omega_t \beta E \left[ (1 + i_t) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{j,t+1}^O, 0) | s_t \right] \quad (51)$$

The first derivative of a young household's lifetime utility with respect to money savings is

$$U'_m(c_{j,t}^Y, h_{j,t}) = -u'_c(c_{j,t}^Y, h_{j,t}) + \beta E \left[ (1 + i_t) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{j,t+1}^O, 0) | s_t \right] \quad (52)$$

and the first derivative with respect to bond savings is

$$U'_b(c_{j,t}^Y, h_{j,t}) = -u'_c(c_{j,t}^Y, h_{j,t}) + \beta E \left[ \frac{P_{t+1,t+2}^g}{P_{t,t+2}^g} \frac{P_t^c}{P_{t+1}^c} u'_c(c_{j,t+1}^O, 0) | s_t \right] \quad (53)$$

Denote the first best allocation by  $(c_t^{Y*}, h_t^*, c_t^{O*})$ . The first-order conditions of the first-best allocation problem, (31) and (32), imply

$$\begin{aligned} u'_c(c_t^{Y*}, h_t^*) &= u'_c(c_t^{O*}, 0) \\ -u'_h(c_t^{Y*}, h_t^*) &= \omega_t u'_c(c_t^{O*}, 0) \end{aligned}$$

because  $u'_h(c_t^O, 0) = y'_h(\omega_t, h_t) u'_c(c_t^O, 0)$ . Evaluating (51) - (53) at the first-best allocation yields

$$U'_h(c_{j,t}^Y, h_{j,t}) = -\omega_t u'_c(c_t^{O*}, 0) + \omega_t \beta E \left[ (1 + i_t) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{t+1}^{O*}, 0) | s_t \right] \quad (54)$$

$$U'_m(c_{j,t}^Y, h_{j,t}) = -u'_c(c_t^{O*}, 0) + \beta E \left[ (1 + i_t) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{t+1}^{O*}, 0) | s_t \right] \quad (55)$$

$$U'_b(c_{j,t}^Y, h_{j,t}) = -u'_c(c_t^{O*}, 0) + \beta E \left[ \frac{P_{t+1,t+2}^g}{P_{t,t+2}^g} \frac{P_t^c}{P_{t+1}^c} u'_c(c_{t+1}^{O*}, 0) | s_t \right] \quad (56)$$

The final step of the proof shows that under the rules (37)-(39), these derivatives are all zero, implying that the decentralized optimal allocation coincides with the first best allocation. Notice first that (54) and (55) just differ by the factor  $\omega_t$ : so if (55) is zero,

then (54) is zero as well. With a constant interest rate, (55) and (56) simplify to

$$U'_m(c_{j,t}^Y, h_{j,t}) = -u'_c(c_t^{O*}, 0) + \beta E \left[ (1+i) \frac{P_t^c}{P_{t+1}^c} u'_c(c_{t+1}^{O*}, 0) | s_t \right] \quad (57)$$

$$U'_b(c_{j,t}^Y, h_{j,t}) = -u'_c(c_t^{O*}, 0) + \beta E \left[ \frac{1/(1+i) P_t^c}{P_{t,t+2}^g P_{t+1}^c} u'_c(c_{t+1}^{O*}, 0) | s_t \right] \quad (58)$$

Thus, if (57) is zero and  $P_{t,t+2}^g = 1/(1+i)^2$ , then (58) is zero as well. Intuitively, without uncertainty about interest rates, the term premium is zero, and there is no role for bonds separate from that of money. Taxes are set such that the ratio of marginal utilities of consumption when young today and when old tomorrow is equal to that of the price of the consumption good today and tomorrow in each state of the world:

$$\begin{aligned} \frac{P_t^c}{P_{t+1}^c} &= \frac{w_t^{O*}/c_t^{O*}}{w_{t+1}^{O*}/c_{t+1}^{O*}} = \frac{(w^Y(1+i) - \tau_t^O)/c_t^{O*}}{(w^Y(1+i) - \tau_{t+1}^O)/c_{t+1}^{O*}} = \frac{w^Y c_t^{O*} u'_c(c_t^{O*}, 0)/c_t^{O*}}{w^Y c_{t+1}^{O*} u'_c(c_{t+1}^{O*}, 0)/c_{t+1}^{O*}} = \frac{u'_c(c_t^{O*}, 0)}{u'_c(c_{t+1}^{O*}, 0)} \\ &= \frac{u'_c(c_t^{Y*}, h_t^*)}{u'_c(c_{t+1}^{O*}, 0)} \end{aligned}$$

Then the marginal utility of holding money is equal to zero in the decentralised solution:

$$\begin{aligned} U'_m(c_{j,t}^Y, h_{j,t})/u'_c(c_t^{O*}, 0) &= -1 + \beta(1+i) \mathbb{E} \left[ \frac{u'_c(c_{t+1}^{O*}, 0)}{u'_c(c_t^{O*}, 0)} \frac{P_t^c}{P_{t+1}^c} | s_t \right] = -1 + \beta(1+i) \\ &= 0 \end{aligned}$$

■

The following two corollaries state comparative static properties which help interpret optimal monetary policy rules.

**Corollary 1** *The price level is strictly decreasing in the productivity shock: It is decreasing in the first-best consumption of the old because*

$$P_t^c = w_t^{O*}/c_t^{O*} = (w^Y(1+i) - \tau_t^O)/c_t^{O*} = w^Y u'_c(c_t^{O*}, 0)$$

and  $c_t^{O*}$  is proportional to the productivity shock (see Proposition 3).

**Corollary 2** *Expected inflation is strictly increasing in the productivity shock:*

$$\frac{\mathbb{E}_t [P_{t+1}^c]}{P_t^c} = \frac{\mathbb{E}_t [u'_c(c_{t+1}^{O*}, 0)]}{u'_c(c_t^{O*}, 0)}$$

and

$$\frac{\partial}{\partial \omega_t} \left( \frac{\mathbb{E}_t [u'_c(c_{t+1}^{O*}, 0)]}{u'_c(c_t^{O*}, 0)} \right) = \mathbb{E}_t [u'_c(c_{t+1}^{O*}, 0)] \left( \frac{\partial}{\partial \omega_t} \frac{1}{u'_c(c_t^{O*}, 0)} \right) > 0$$

### 6.2.2 Properties of the first-best allocation with CRRA utility

Proposition 3 states that using the CRRA utility function (1) and the production function (11), labour supply is constant in the first best allocation, while consumption of the young and the old are proportional to the productivity shock,  $\omega_t$ .

**Proposition 3** *The first best allocation is given by*

$$h^* : \left( \frac{1}{1-h^*} \right)^\sigma = \left( \frac{\rho}{(1-\rho)\alpha} h^* - (1-h^*) \right)^{(1-\rho)(1-\sigma)-1} \quad (59)$$

$$c_t^{Y*} = \omega_t \frac{1-\rho}{\rho} (\alpha (h^*)^{\alpha-1}) (1-h^*) \quad (60)$$

$$y_t^* = \omega_t (h^*)^\alpha \quad (61)$$

$$c_t^{O*} = y_t^* - c_t^{Y*} \quad (62)$$

**Proof.** Entering the production function (11) and the utility function (1) into the first-order constraint of the planner's problem, (31)-(32), yields<sup>5</sup>

$$\begin{aligned} & \frac{\partial u(c_t^Y, h_t)}{\partial c_t^Y} + \frac{\partial u(c_t^O, 0)}{\partial c_t^Y} = \frac{\partial u(c_t^Y, h_t)}{\partial c_t^Y} + \frac{\partial u(\omega h^\alpha - c_t^Y, 0)}{\partial c_t^Y} \\ &= u'_c(c_t^Y, h_t) - u'_c(c_t^O, 0) \\ &= \frac{\partial}{\partial c} \left( \frac{(c^{1-\rho} (1-h)^\rho)^{1-\sigma} - 1}{1-\sigma} \right) \Big|_{c=c_t^Y} - \frac{\partial}{\partial c} \left( \frac{c^{(1-\rho)(1-\sigma)} - 1}{1-\sigma} \right) \Big|_{c=c_t^O} \\ &= \frac{(1-\rho)(1-h)^\rho}{(c_t^Y)^\rho \left( (c_t^Y)^{1-\rho} (1-h)^\rho \right)^\sigma} - (1-\rho) (h^\alpha \omega - c_t^Y)^{\sigma\rho-\rho-\sigma} = 0 \end{aligned} \quad (63)$$

<sup>5</sup>We write  $c$  for  $c_t^Y$  and omit time subscripts on other variables in the following equations to make them easier to read.

and

$$\frac{\partial u(c_t^Y, h_t)}{\partial h_t} + \frac{\partial u(c_t^O, 0)}{\partial h_t} = u'_h(c_t^Y, h_t) + \frac{\partial u(c_t^O, 0)}{\partial c_t^O} \frac{\partial c_t^O}{\partial h_t} \quad (64)$$

$$= u'_h(c_t^Y, h_t) + u'_c(c_t^O, 0) y'_h(\omega_t, h_t) \quad (65)$$

$$= \frac{\partial}{\partial h} \left( \frac{(c^{1-\rho}(1-h)^\rho)^{1-\sigma} - 1}{1-\sigma} \right) \Big|_{c=c_t^Y} + \left( \frac{\partial}{\partial c} \left( \frac{c^{(1-\rho)(1-\sigma)} - 1}{1-\sigma} \right) \Big|_{c=c_t^O} \right) (h^{\alpha-1} \alpha \omega) \quad (66)$$

$$= -\frac{\rho (c_t^Y / (1-h))^{1-\rho}}{\left( (c_t^Y)^{1-\rho} (1-h)^\rho \right)^\sigma} + h^{\alpha-1} \alpha \omega (1-\rho) (h^\alpha \omega - c_t^Y)^{\sigma\rho - \rho - \sigma} \quad (67)$$

Replacing  $(1-\rho)(h^\alpha \omega - c)^{\sigma\rho - \rho - \sigma}$  in (67) by the first term in (63) yields

$$\begin{aligned} & u'_h(c_t^Y, h_t) + y'_h(\omega_t, h_t) u'_c(c_t^O, 0) \\ &= -\frac{\rho (c_t^Y / (1-h_t))^{1-\rho}}{\left( (c_t^Y)^{1-\rho} (1-h_t)^\rho \right)^\sigma} + h^{\alpha-1} \alpha \omega \frac{(1-\rho)(1-h_t)^\rho}{(c_t^Y)^\rho \left( (c_t^Y)^{1-\rho} (1-h_t)^\rho \right)^\sigma} \\ &= \frac{(c_t^Y / (1-h_t))^{-\rho}}{\left( (c_t^Y)^{1-\rho} (1-h_t)^\rho \right)^\sigma} (-\rho (c_t^Y / (1-h_t)) + h_t^{\alpha-1} \alpha \omega (1-\rho)) \end{aligned}$$

This is equal to zero if (60) holds. The following equivalent ways of expressing (60) will be useful below:

$$\omega h^\alpha - c = \omega h^{\alpha-1} \left( h - (1-h) \frac{1-\rho}{\rho} \alpha \right) \quad (68)$$

$$\frac{c}{1-h} = \omega \frac{1-\rho}{\rho} \alpha h^{\alpha-1} \quad (69)$$

$$c^{1-\rho} (1-h)^\rho = \left( \frac{1-\rho}{\rho} \alpha h^{\alpha-1} \omega \right)^{1-\rho} (1-h) \quad (70)$$

Using (68)-(70) to substitute  $c$  out of (67) yields

$$\begin{aligned}
& -\frac{\rho \left( \omega \frac{1-\rho}{\rho} \alpha h^{\alpha-1} \right)^{1-\rho}}{\left( \left( \frac{1-\rho}{\rho} \alpha h^{\alpha-1} \omega \right)^{1-\rho} (1-h) \right)^{\sigma}} + h^{\alpha-1} \alpha \omega (1-\rho) \left( \omega h^{\alpha-1} \left( h - (1-h) \frac{1-\rho}{\rho} \alpha \right) \right)^{\sigma\rho-\rho-\sigma} \\
= & -(\omega h^{\alpha-1})^{1-\rho-(1-\rho)\sigma} \frac{\rho \left( (\alpha) \frac{1-\rho}{\rho} \right)^{1-\rho-(1-\rho)\sigma}}{(1-h)^{\sigma}} \\
& + \alpha (\omega h^{\alpha-1})^{1+\sigma\rho-\rho-\sigma} (1-\rho) \left( h - (1-h) \frac{1-\rho}{\rho} \alpha \right)^{\sigma\rho-\rho-\sigma}
\end{aligned} \tag{71}$$

The exponents on  $\omega h^{\alpha-1}$  on each term in (71),  $1-\rho-(1-\rho)\sigma$  on the first and  $1+\sigma\rho-\rho-\sigma$  on the second, are equal, implying that the optimal labour supply is independent of the productivity shock:

$$\begin{aligned}
& u'_c(c_t^Y, h_t) + u'_c(y_t - c_t^Y, 0) \\
= & (\omega h^{\alpha-1})^{(1-\rho)(1-\sigma)} \frac{1-\rho}{\alpha} \left( \alpha \frac{1-\rho}{\rho} \right)^{(1-\rho)(1-\sigma)-1} \\
& \cdot \left( -\frac{1}{(1-h)^{\sigma}} + \left( \frac{\rho}{(1-\rho)\alpha} h - (1-h) \right)^{(1-\rho)(1-\sigma)-1} \right)
\end{aligned} \tag{72}$$

Setting (72) to zero yields (59). ■

## 7 References

Carter, S et al (2006): "Historical statistics of the United States: earliest times to the present", Cambridge University Press.

Chamley, C and H Polemarchakis (1984): "Assets, general equilibrium, and the neutrality of money", Review of Economic Studies 51, pages 129-138.

Durre, A and Pill, H (2012): "Central Bank Balance Sheets as Policy Tools", Bank for International Settlement papers no 66.

Eggertson, G. and Woodford, M. (2003): "The Zero Bound on Interest Rate and Optimal Monetary Policy", Brookings Papers on Economic Activity 34(1), pages 139-211.

Gagnon, J; Raskin, M; Remache, J; and Sack, B (2010): "Large Scale Asset Purchases by the Federal Reserve: Did they Work?", FRB New York Staff Report no 441.

Joyce, M; Lasasosa, A; Stevens, I and Tong, M (2010): "The Financial Market Impact of Quantitative Easing", Bank of England Working paper no. 393.

Joyce, M; Miles, D; Scott, A; and Vayanos, D (2012): "Quantitative Easing and Unconventional Monetary Policy", *The Economic Journal* 122 (November), pages 271-289.

Miles, D and Schanz, J (2014): "The relevance or otherwise of the central bank's balance sheet", *Journal of International Economics* 92, pages S103-S116.

Sargent, T J and B D Smith (1987): "Irrelevance of open market operations in some economies with government currency being dominated in rate of return", *American Economic Review* 77(1), pages 78-92.

Sims, C (2013), "Paper Money", *American Economic Review* 103(2), pages 563-84.

Sterk, V and Tenreyro, S (2013), "The Transmission of Monetary Policy Operations through Redistributions and Durable Purchases", CFM Discussion Paper Series No. CFM-DP2013-5.

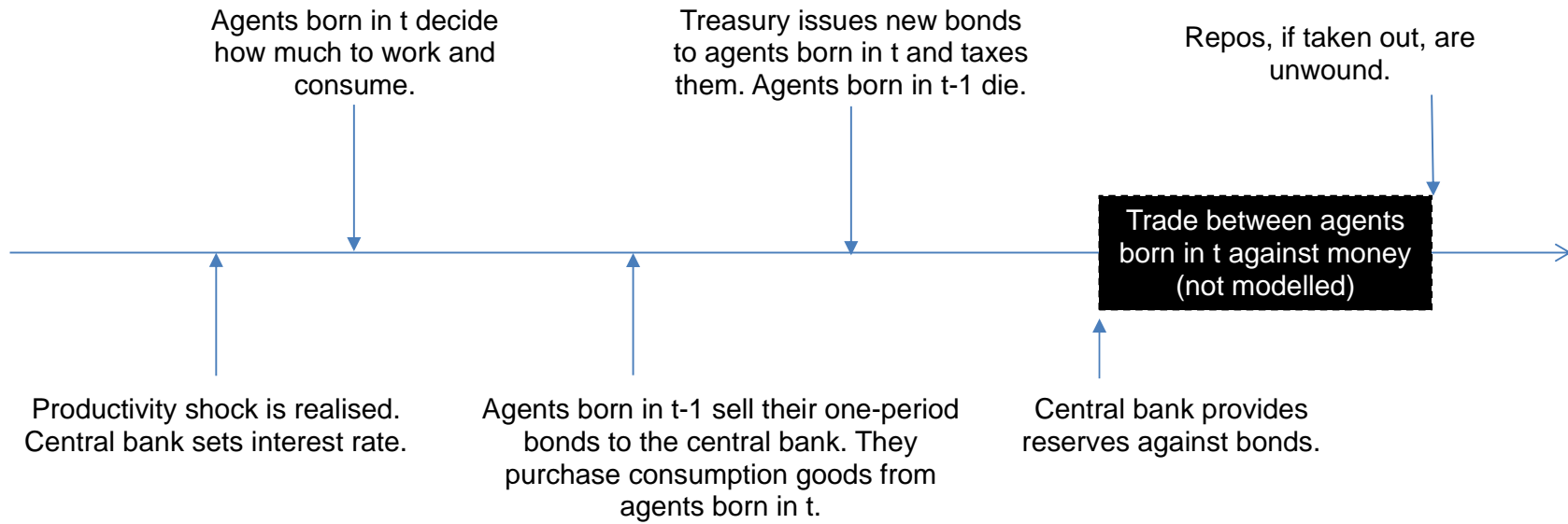
Vayanos, D and J-L Vila (2009): "A preferred-habitat model of the term structure of interest rates", NBER Working Paper No 15487.

Wallace, N (1981), "A Modigliani-Miller theorem for open-market operations", *American Economic Review* 71(3), pages 267-274.

Weill, P (2008), "Overlapping Generations: The First Jubilee", *Journal of Economic Perspectives* 22(4), pages 115-134.

Zampolli, F (2012), "Sovereign Debt Management as an Instrument of Monetary Policy: An Overview", Bank for International Settlements Paper 65.

**Figure 1: Timeline**



**Table 1: Calibration**

<b>Parameters</b>		<b>Base calibration</b>
$\beta$	Discount factor	2/3
$\rho$	Exponent of leisure in utility function	0.85
$\sigma_c$	CRRA coefficient	4
$\alpha$	Exponent of labour in production function	1
$\gamma$	Amount of bonds issued in each period	1
$w^y$	Nominal wealth of young HHs net of taxes when only young households are taxed	1.11
<b>Shocks</b>		
$\mu_\omega$	mean productivity shock	1
$\sigma_\omega$	SD of productivity shock	0.2
$\sigma_\varepsilon$	SD of CB interest rate innovation	0



**2. Central bank balance sheet when liquidity is provided by repoing in all two-period bonds**

	$\omega_t = 0.8$	Steady State	$\omega_t = 1.2$
Repos	0.44	0.45	0.47
Bonds with remaining maturity of one period	0.65	0.67	0.70
Newly issued bonds (remaining maturity of two periods)	0	0	0
<b>Total assets</b>	<b>1.09</b>	<b>1.13</b>	<b>1.17</b>
Money created by remunerating previous period money holdings	0.98	0.98	0.98
Money created by purchasing bonds with remaining maturity of one period	0.65	0.67	0.70
Money created by purchasing newly issued bonds	0	0	0
Money created by repos	0.44	0.45	0.47
Money absorbed by issuing new bonds	-0.44	-0.45	-0.47
Money absorbed by taxation	-0.52	-0.54	-0.56
<b>Total liabilities</b>	<b>1.11</b>	<b>1.11</b>	<b>1.11</b>
<b>Equity</b>	<b>-0.02</b>	<b>0.02</b>	<b>0.06</b>

**Table 3: Impulse responses****1. Productivity shock (+ 1SD, corresponding to 20%), holding the interest rate constant**

	SS	impact	+1	+2	+3
Productivity	1.0	1.2	1.0	1.0	1.0
CB innovation on policy rate	0	0	0	0	0
CB policy rate	51%	51%	51%	51%	51%
Wealth of the old	1.68	1.68	1.68	1.68	1.68
Labour supply	0.22	0.20	0.22	0.22	0.22
Production	0.23	0.24	0.23	0.23	0.23
Consumption of the young	0.137	0.142	0.137	0.137	0.137
Savings ratio	39%	42%	39%	39%	39%
Price of consumption good	19	17	19	19	19
Issuance price of bonds	44%	44%	44%	44%	44%
Expected nominal portfolio return	51%	51%	51%	51%	51%
Expected inflation	4%	17%	4%	4%	4%

**2. Interest rate shock (+ 1SD, corresponding to 10pp)**

	SS	impact	+1	+2	+3
Productivity	1.0	1.0	1.0	1.0	1.0
CB innovation on policy rate	0	0	0	0	0
CB policy rate	51%	61%	51%	51%	51%
Production	1.68	1.64	1.79	1.68	1.68
Consumption of the young	0.222	0.220	0.226	0.222	0.222
Savings ratio	0.227	0.224	0.230	0.227	0.227
Expected wealth of old households	0.137	0.138	0.137	0.137	0.137
Issuance price of bonds	39.5%	38.6%	40.7%	39.5%	39.5%
Price of consumption good	18.8	18.9	19.1	18.8	18.8
Expected nominal return on bonds	43.7%	41.0%	43.7%	43.7%	43.7%
Expected nominal portfolio return	51.56%	61.55%	51.56%	51.56%	51.56%
Expected inflation	3.3%	4.4%	1.5%	3.3%	3.3%

**Table 4: Impact of alternative parameterisations on steady state**

#	Parameters			Optimal to provide reserves via...	Optimal interest rate rule s.t. $a_3=0$		Steady state				Welfare	
	CRRRA coefficient	SD of innovations to CB interest rates	Exponent on leisure in utility function	b*	Constant (a1*)	Sensitivity to productivity (a2*)	Labour supply	Savings ratio	Nominal term premium (pp)	Expected real portfolio return	Welfare (consumption equivalent of first best)	Loss if reserves are provided by worse method (pp)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	4	0	0.85	repos	48.4%	-0.26	0.22	39%	0.48	48%	98.9%	0.03
2	2	0	0.85	repos	48.4%	-0.40	0.24	45%	0.34	48%	98.8%	0.08
3	1	0	0.85	repos	48.2%	-0.42	0.26	50%	0.22	48%	98.6%	0.09
4	0.5	0	0.85	repos	48.1%	-0.40	0.27	53%	0.17	48%	98.5%	0.07
5	4	0.1	0.85	outright	49.8%	-0.16	0.22	39%	0.3	49%	98.8%	0.02
6	4	0.2	0.85	outright	53.9%	-0.18	0.22	40%	0.4	53%	98.5%	0.08
7	4	0	0.75	indifferent	48.9%	0.01	0.34	37%	0.0	48%	99.0%	0.00
8	4	0	0.66	repos	49.0%	0.28	0.44	37%	-1.2	48%	99.0%	0.02