Imperial College London

The history and role of the cubic law for fluid flow in fractured rocks

Robert W. Zimmerman
Imperial College
London, UK

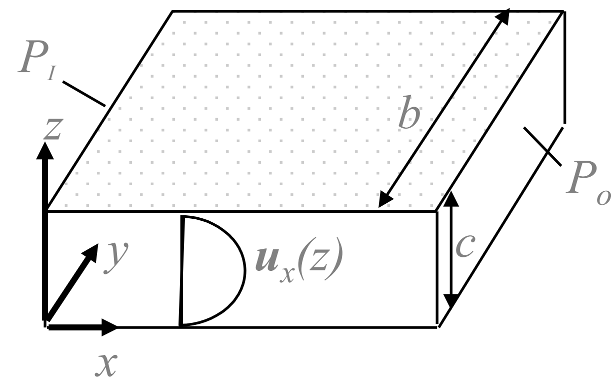
Session H071:
Dynamics of Fluids and Transport in Fractured Porous Media

3 December 2012

AGU FALL MEETING
San Francisco | 3–7 December 2012

Flow between two smooth parallel walls

J. Boussinesq, Mémoire sur l'influence des frottements dans les mouvements réguliers des fluides. (Study of the effect of friction on the laminar flow of fluids.) *Journal de Mathématiques Pures et Appliquées*, 2e série, tome 13 (1868), pp. 377-424.



$$\mathrm{V} = \frac{\mathrm{L}}{2}\,\frac{b^2c^2}{b^2+c^2}\left\{1-\frac{y^2}{b^2}-\frac{z^2}{c^2}+2\sum_{k'=0}^{k'=\infty}\frac{\pm 1}{k}\left(1-\frac{2}{k^2}\right)\left[\frac{e^{k\frac{z}{b}}+e^{-k\frac{z}{b}}}{e^{k\frac{c}{b}}+e^{-k\frac{c}{b}}}\cos k\frac{y}{b}\right.\right.$$

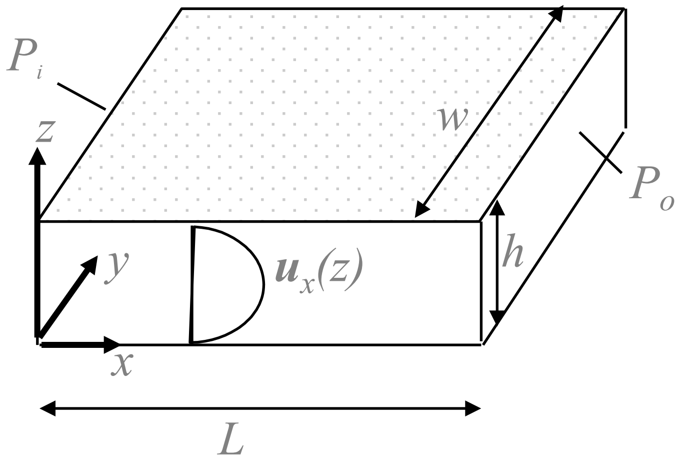
$$\left.\left.+\frac{e^{k\frac{y}{c}}+e^{-k\frac{y}{c}}}{e^{k\frac{b}{c}}+e^{-k\frac{b}{c}}}\cos k\right.\right.$$

On aura donc, pour $\frac{b}{c}=\infty$,

$$\alpha = \frac{1}{12} = 0{,}0833.$$

Imperial College
London

"Cubic Law"



$$u_x(z) = \frac{|\nabla P|}{2\mu}[z^2 - (h/2)^2]$$

$$Q_x = \int_{-h/2}^{-h/2} w\bar{u}_x dz = -\frac{wh^3}{12\mu}\nabla P \quad \Rightarrow \quad T = \frac{wh^3}{12}$$

Key features of "cubic law":

- Transmissivity is proportional to cube of (mean?) aperture
- Flowrate is directly proportional to pressure gradient

Research on single-fracture flow in the 1950s and 1960s

Lomize, G. M., *Flow in Fractured Rocks* (in Russian), 27 pp. Gosenergoizdat, Moscow, 1951.

Romm, E. S., *Flow Characteristics of Fractured Rocks* (in Russian), 283 pp., Nedra, Moscow, 1966.

Louis C., *A study of groundwater flow in jointed rock and its influence on the stability of rock masses*, Rock Mechanics Research Report 10, 90 pp., Imperial College, London, 1969 (English version of Louis's Ph.D. 1967 thesis at Karlsruhe, under supervision of Prof. W. Wittke).

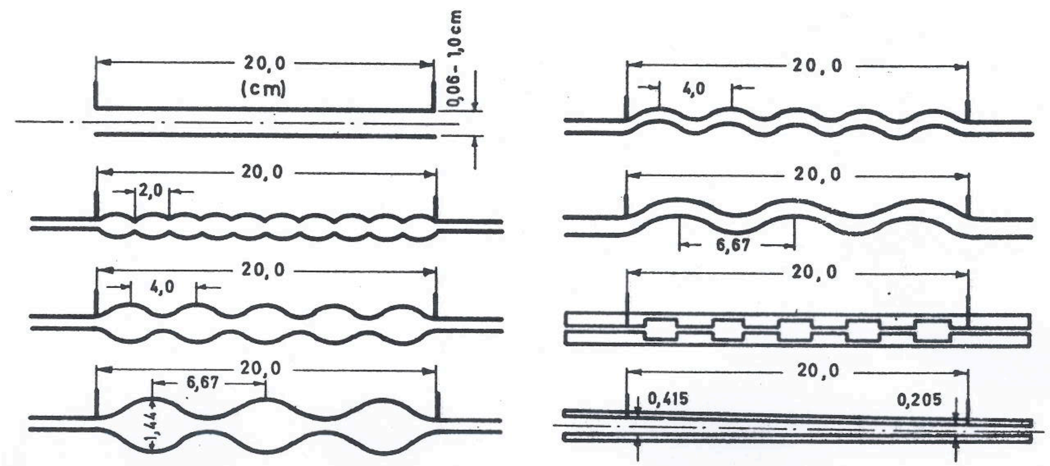
These researchers generally did *not* work with rock fractures (instead, glass plates or concrete slabs), but did identified some key issues, such as

- Effect of small scale roughness
- Effect of larger-scale aperture variation
- Nonlinearity at high Reynolds number

Imperial College London

Research on single-fracture flow in the 1950s and 1960s

Lomize, G. M., *Flow in Fractured Rocks* (in Russian), 27 pp. Gosenergoizdat, Moscow, 1951.

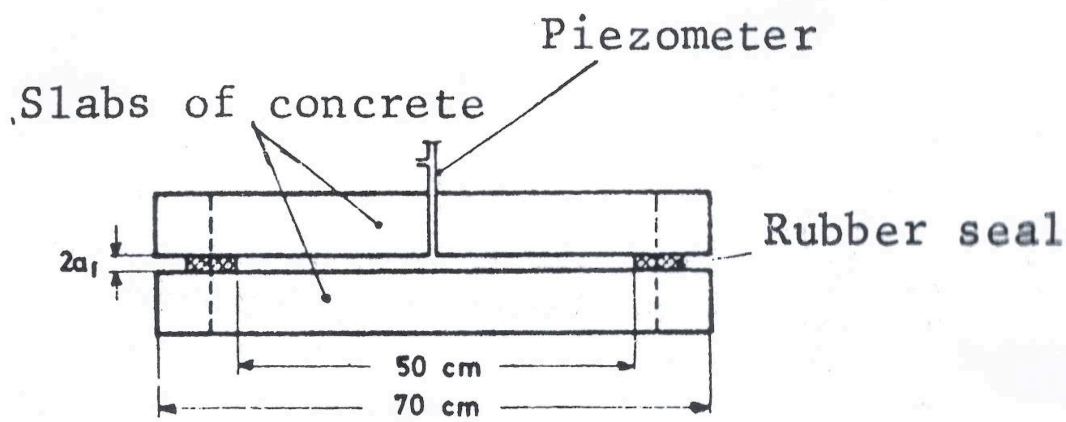


$$T = \frac{h^3}{12}\left[1 + 17\left(\frac{\varepsilon}{h}\right)^{1.5}\right]^{-1}$$

Imperial College London

Research on single-fracture flow in the 1950s and 1960s

Louis C., *A study of groundwater flow in jointed rock and its influence on the stability of rock masses*, Rock Mechanics Research Report 10, 90 pp., Imperial College, London, 1969.



$$T = \frac{h^3}{12}\left[1 + 8.8\left(\frac{\varepsilon}{h}\right)^{1.5}\right]^{-1}$$

Witherspoon, Wang, Iwai and Gale (LBL 1979 / WRR 1980)

Submitted for publication to Water Resources Research.

LBL-9557
SAC-23
UC-70

VALIDITY OF CUBIC LAW FOR FLUID FLOW

IN A DEFORMABLE ROCK FRACTURE

P. A. Witherspoon, J. S. Y. Wang, K. Iwai[1], and J. E. Gale[2]

Department of Materials Science and Mineral Engineering
University of California, Berkeley

and

Lawrence Berkeley Laboratory
University of California
Berkeley, California

October 1979

[1] Now with Nakano Corporation, Niigatashi, Japan.

[2] Now at the Department of Earth Sciences, University of Waterloo, Waterloo, Ontario, Canada.

Imperial College London

Witherspoon, Wang, Iwai and Gale (LBL 1979 / WRR 1980)

"Validity of cubic law for fluid-flow in a deformable rock fracture", P.A. Witherspoon, J.S.Y. Wang, K. Iwai, and J.E. Gale, *Water Resour. Res.*, 1980;16:1016-1024.

Arguably the most influential paper on the topic of "fluid flow in a single rock fracture":

394 citations in Web of Knowledge database

631 citations on Google Scholar

(as of 30 November 2012)

Identified and investigated several key issues:

1. Effect of roughness on the transmissivity
2. Effect of normal stress on the transmissivity
3. Validity of $T \sim h_m^3$ as the fracture closes under stress
4. Nonlinearity in T *vs*. ΔP relationship at higher Re numbers

Effect of roughness on the transmissivity

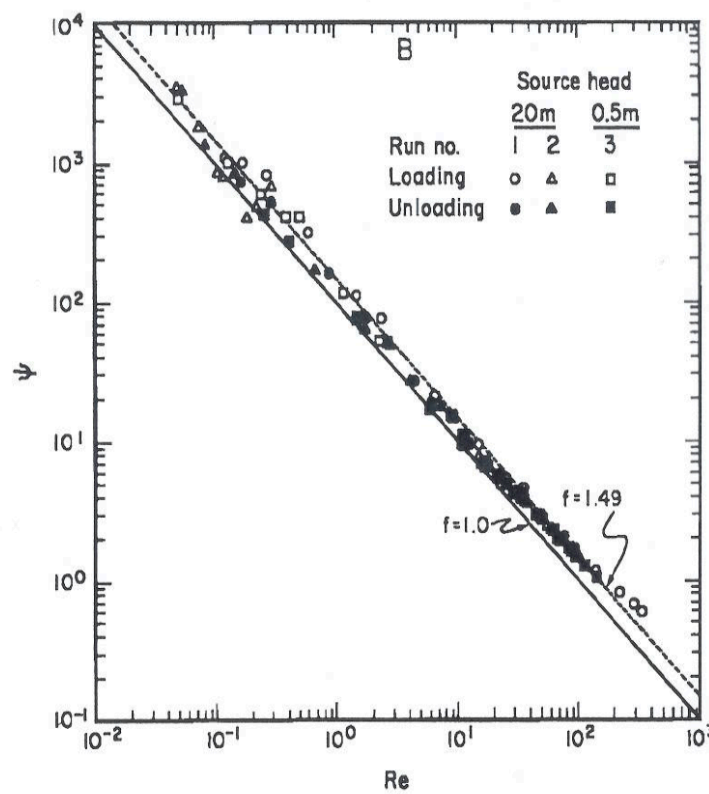
"Validity of cubic law for fluid-flow in a deformable rock fracture", P.A. Witherspoon, J.S.Y. Wang, K. Iwai, and J.E. Gale, *Water Resour. Res.*, 1980;16:1016-1024.

Data from a fracture in a greyish-white, medium-grained Cretaceous granite from Raymond, Calif:

They accounted for roughness by introducing a multiplicative factor in the denominator:

$$T = \frac{h^3}{12f}$$

Depending on rock type and loading cycle, they found values of *f* ranging from 1.04 to 1.65.



Imperial College London

Effect of roughness on the transmissivity

Numerous researchers have investigated the influence of fracture roughness on *f*, usually using the “local cubic law”, *i.e.,* the Reynolds lubrication approximation.

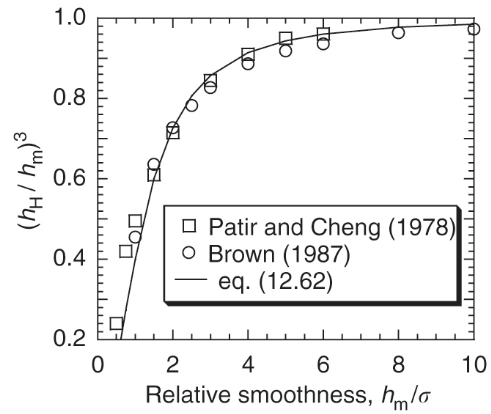
Many different analytical approaches (Elrod, *J. Lubr. Tech*., 1979; Landau and Lifschitz, *Electrodynamics*, 1960; Zimmerman *et al., IJRM*, 1991) yield:

$$T = \frac{\langle h \rangle^3}{12}\left[1 - 1.5\sigma_h^2/\langle h \rangle^2 + \ldots\right]$$

Renshaw (*JGR*, 1995) modified this result to avoid *f* becoming infinite:

$$T = \frac{\langle h \rangle^3}{12}\left[1 + \sigma_h^2/\langle h \rangle^2\right]^{-1.5}$$

This expression is in good agreement with numerical solutions of Reynolds equation:

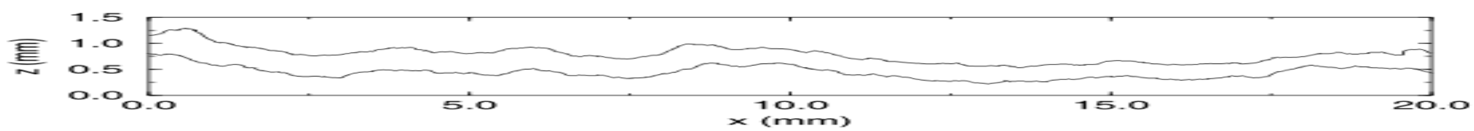


Imperial College London

Effect of roughness on the transmissivity

However, there is some evidence that by ignoring the out-of-plane flow components, the local cubic law may be overestimating the transmissivity.

"Effect of shear displacement on the aperture and permeability of a rock fracture", I.W. Yeo, M.H. de Freitas, R.W. Zimmerman, *Int. J. Rock Mech.*, 1998;35:1051-1070:



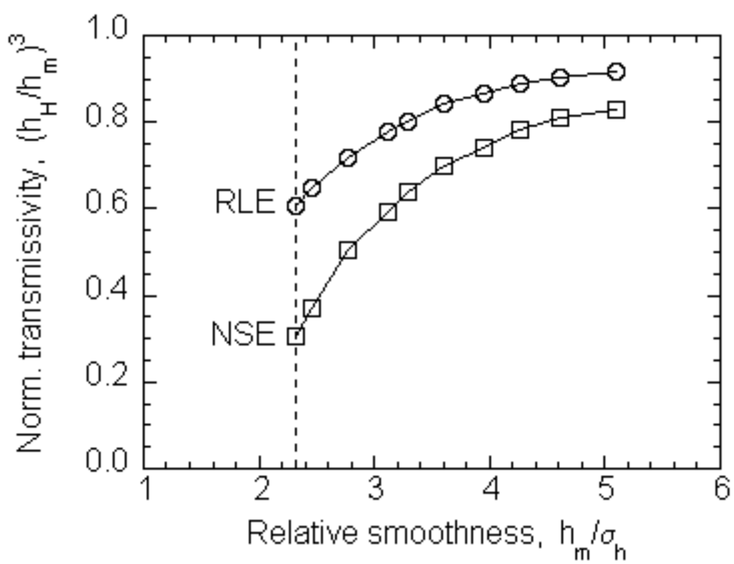
Approach	T (10^{-12} m^3)
Experimental	8.0
Numerical solution of RLE	17.3
Homogenization of RLE $T = \langle h \rangle^3 [1 - 1.5(\sigma / \langle h \rangle)^2]/12$	16.7

Effect of roughness on the transmissivity

Numerical solutions of the Navier-Stokes equations yield closer agreement with experimentally measured transmissivities.

"Nonlinear regimes of fluid flow in rock fractures", R.W. Zimmerman, A.H. Al-Yaarubi, C.C. Pain, and C.A. Grattoni, *Int. J. Rock Mech.*, 2005;41:paper 1A27:

Sample	h_m (μm)	σ (μm)	T_{exp} (10^{-15} m^3)	T_{NS} (10^{-15} m^3)	T_{RLE} (10^{-15} m^3)
1	130	72	29.8	27.4	35.2
2	149	56	133.5	147.6	191.7



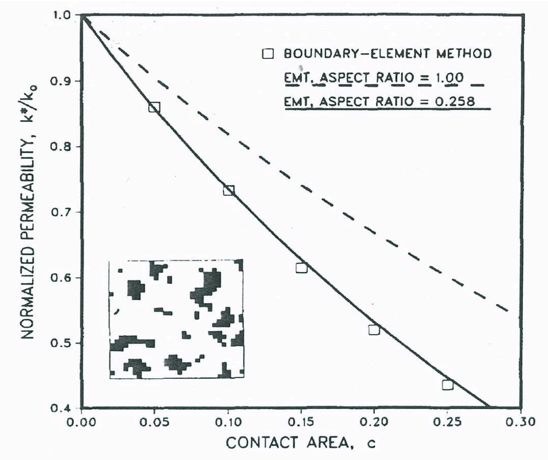
Effect of contact area on the transmissivity

As the fractional contact area increases, the *f* factor increases, and hence the transmissivity decreases.

"The effect of contact area on the permeability of fractures", R.W. Zimmerman, D.W. Chen, and N.G.W. Cook, *J. Hydrol.*, 1992;139:79-96:

$$f = \frac{1+\beta c}{1-\beta c}, \quad \beta = \frac{(1+\alpha)^2}{4\alpha},$$

α = aspect ratio of elliptical contact area, c = total fractional contact area



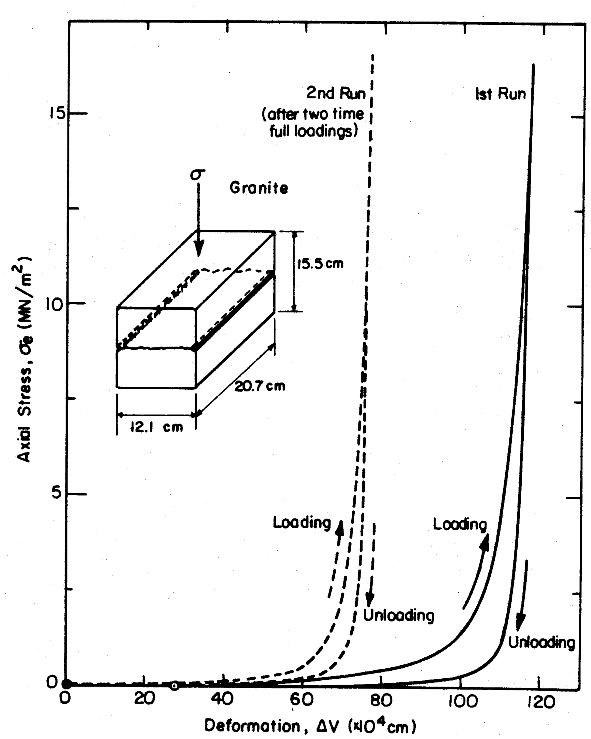
see also "Analytical models for flow through obstructed domains", A.R. Piggott and D. Elsworth, *J. Geophys. Res.,* 1992;B97:2085-2093.

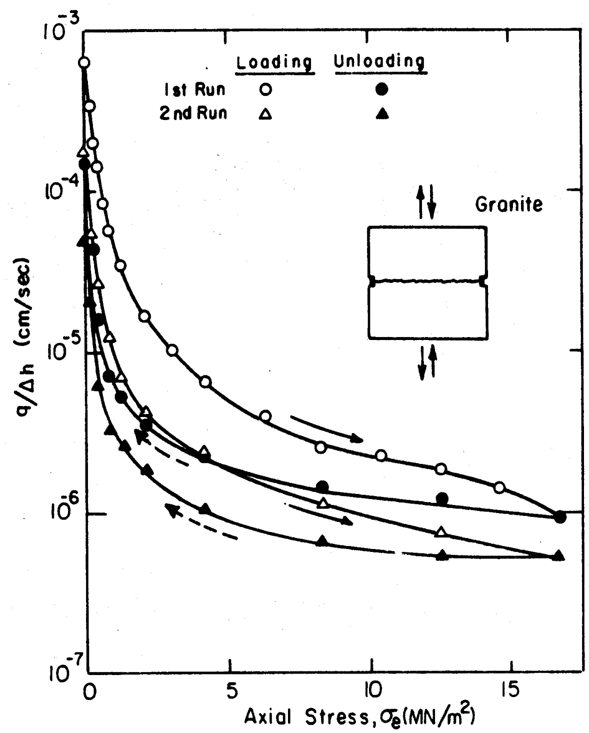
Imperial College London

Effect of Normal Stress / Stiffness on Transmissivity

"Validity of cubic law for fluid-flow in a deformable rock fracture", P.A. Witherspoon, J.S.Y. Wang, K. Iwai, and J.E. Gale, *Water Resour. Res.*, 1980;16:1016-1024.

Data from a greyish-white, medium-grained Cretaceous granite from Raymond, Calif:





Imperial College London

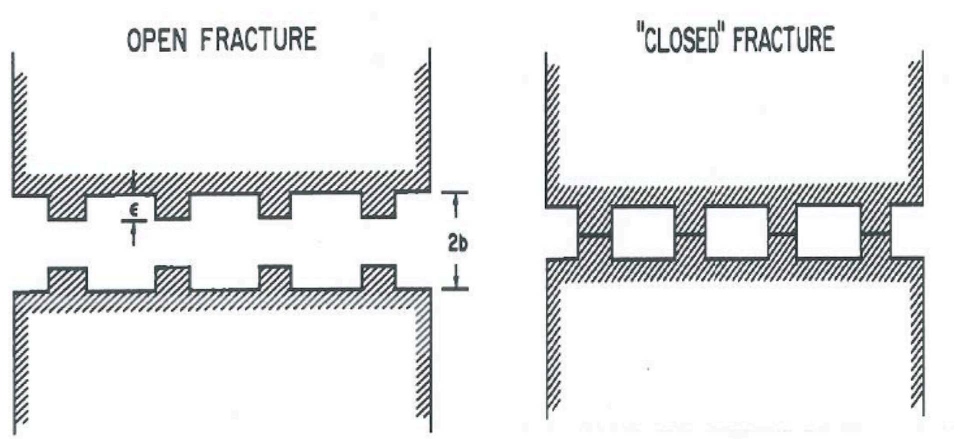
Effect of Normal Stress / Stiffness on Transmissivity

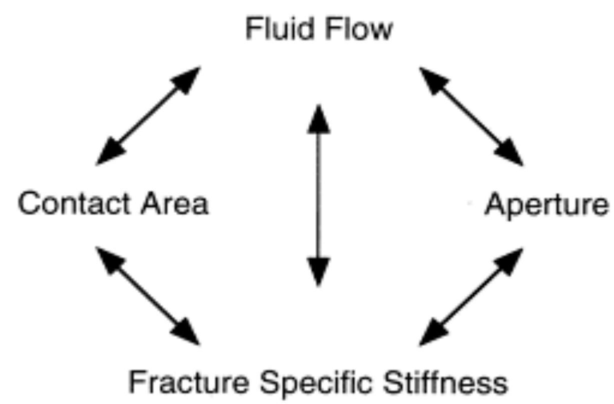
Fracture consists of regions of contact between the two surfaces, separated by open regions that have a variable aperture

The open regions provide pathways for flow, and also provide mechanical compliance

→ Normal stiffness and transmissivity both depend on contact area, and aperture, so we expect that they should somehow be related

Conceptual model proposed by Witherspoon *et al.* (*WRR*,1980):

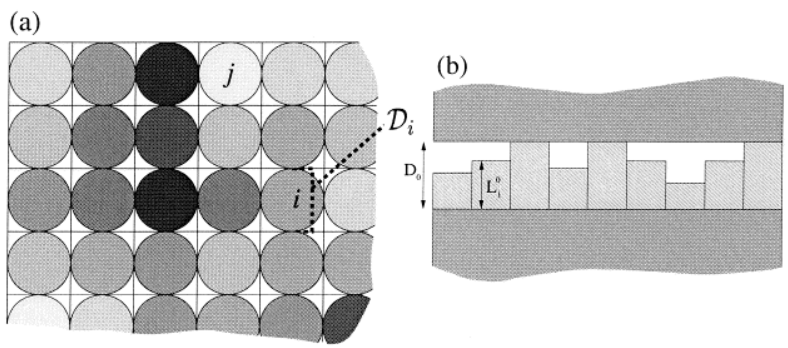


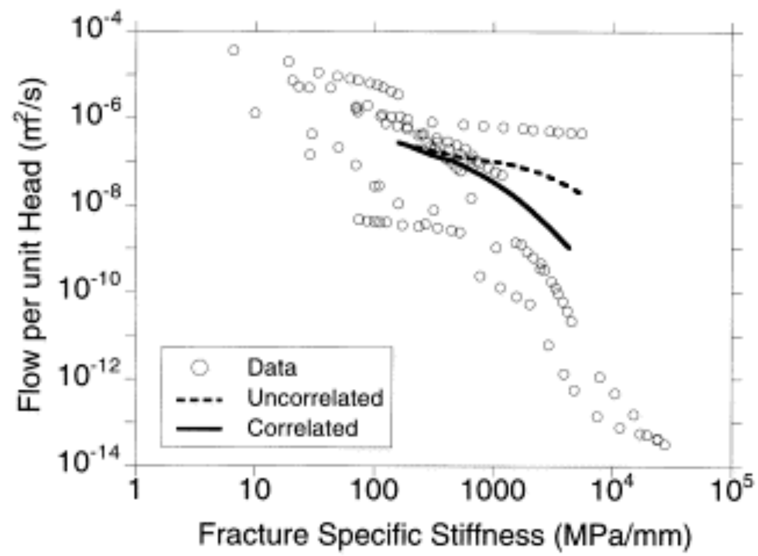


Pyrak-Nolte & Morris (*IJRM*, 2000)

Effect of Normal Stress / Stiffness on Transmissivity

"Single fractures under normal stress: The relation between fracture specific stiffness and fluid flow", L.J. Pyrak-Nolte and J.P. Morris, *Int. J. Rock Mech.,* 2000; 37: 245-262.



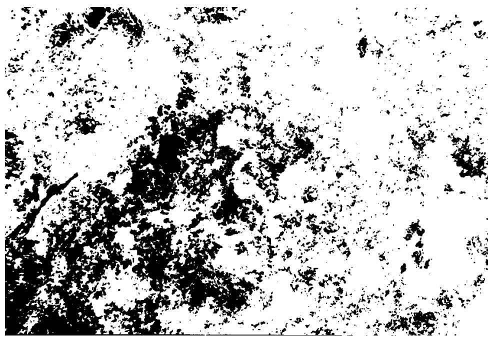


$$w_{\frac{1}{2}} = f\frac{4\left(1-\nu^2\right)}{\pi^2 E a}\left(\frac{r}{a}\right) I_2\left(\frac{r}{a}\right),$$

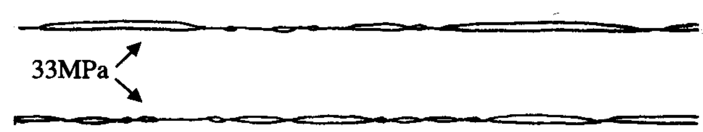
$$I_2(s) = \int_0^{\frac{\pi}{2}} \sqrt{1-(1/s^2)\ \sin^2\theta}\,d\theta - \left[1-\frac{1}{s^2}\right]\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-(1/s^2)\ \sin^2\theta}}$$

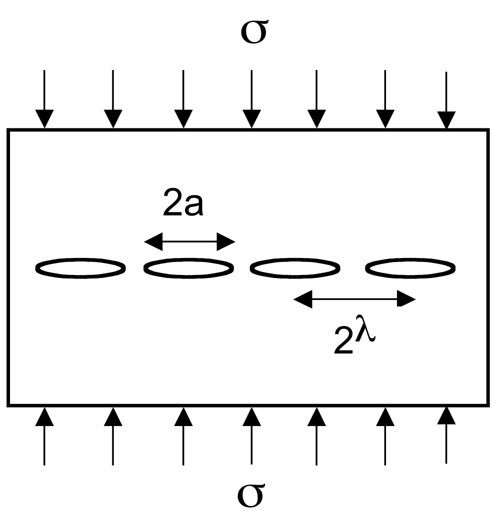
Effect of Normal Stress / Stiffness on Transmissivity

Wood's metal cast (Pyrak-Nolte *et al.*, *ISRM*, 1987):



Transect (Myer, *IJRM*, 2000):





"A simple model for coupling between the normal stiffness and the Hydraulic transmissivity of a fracture", R.W. Zimmerman, ARMA, 2008:

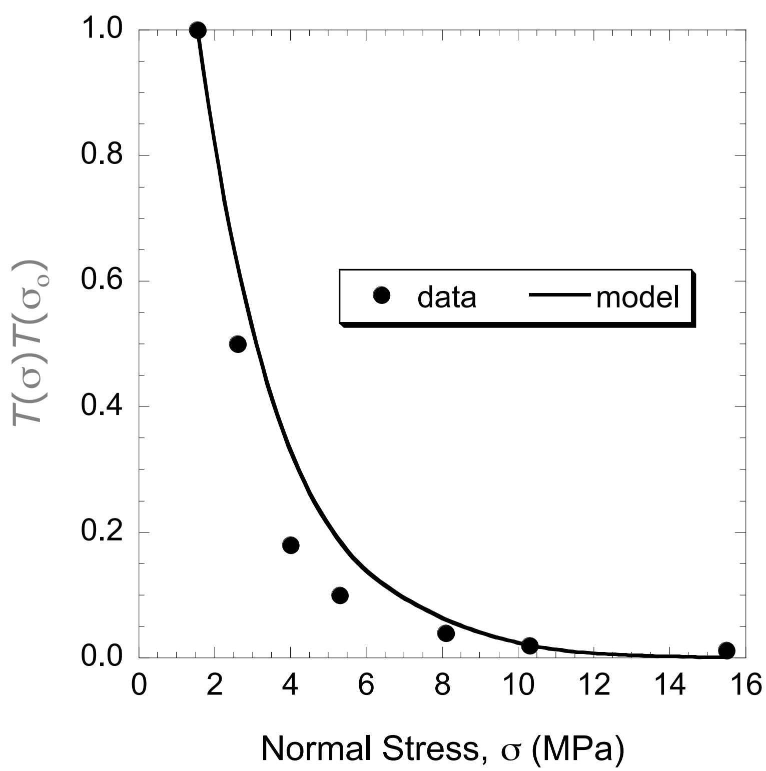
$$T(\sigma) = \frac{2(1-\nu^2)^2 a^2}{E^2} \int_{\sigma}^{\infty} (\sigma' - \sigma)^3 \frac{d\beta}{d\sigma'} d\sigma'$$

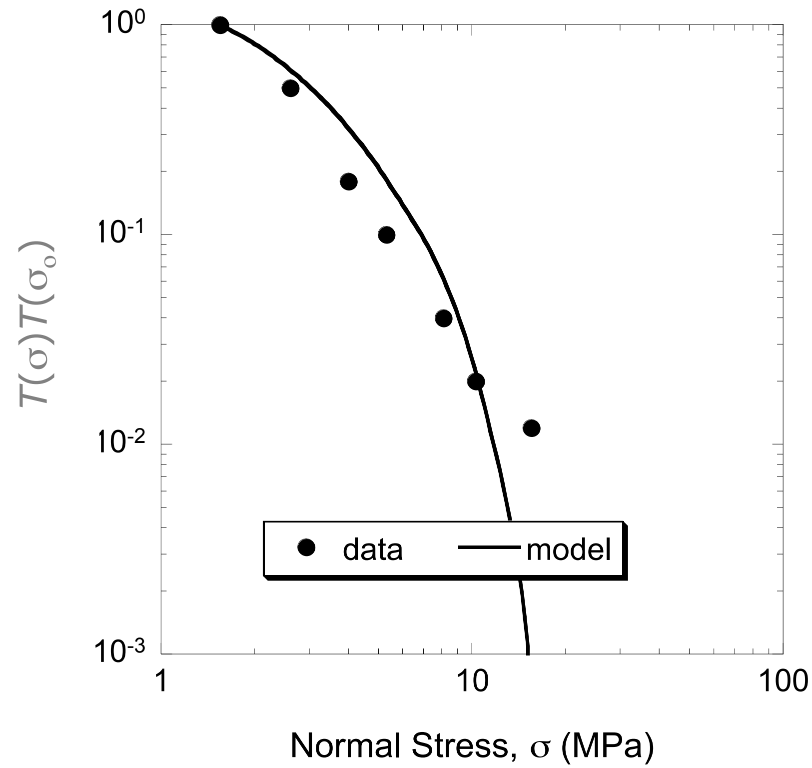
$$\text{compliance } \beta \equiv \frac{1}{\kappa_n} = \frac{1}{\text{normal stiffness}}$$

Imperial College London

Effect of Normal Stress / Stiffness on Transmissivity

Results of applying this model to data from Witherspoon *et al.* (*WRR*, 1980):

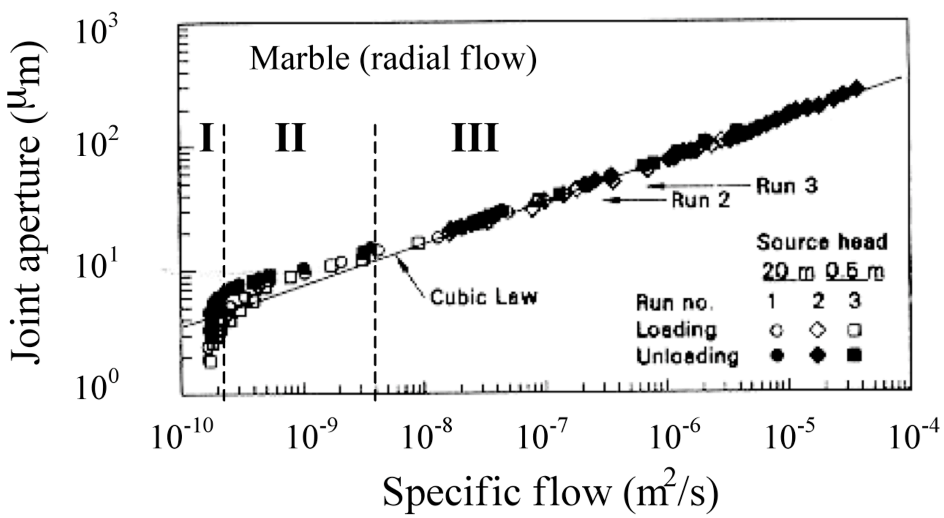




Imperial College London

Validity of "Cubic Law" as Normal Stress Increases

Data on a fracture in marble from Witherspoon *et al.* (*WRR*, 1980):



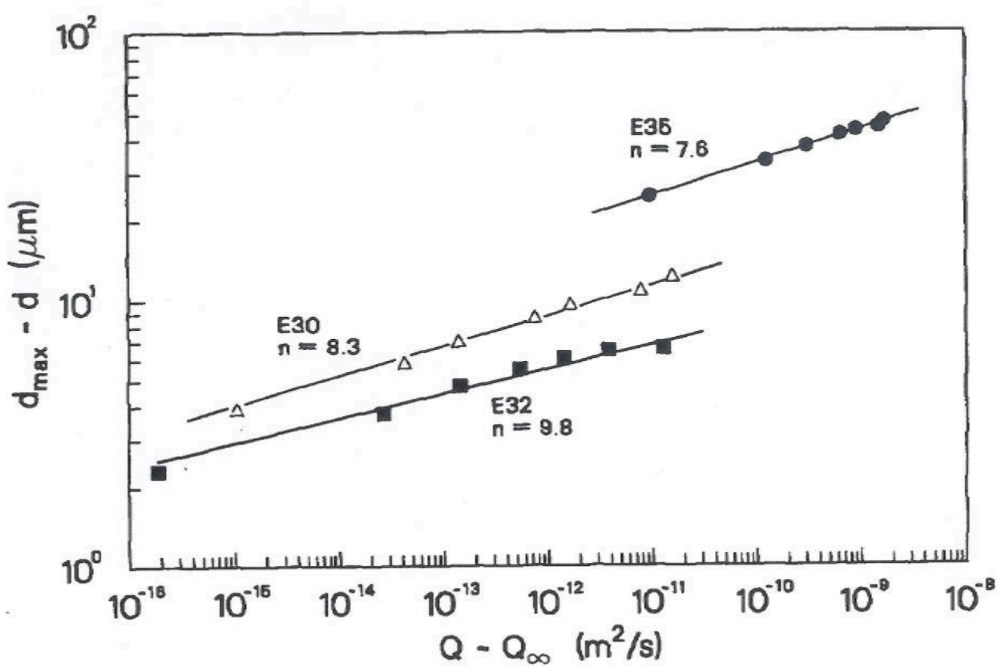
As the normal stress increases (read graph from right to left):

- Initially (III), the transmissivity decreases as the cube of the mean aperture
- In the next regime (II), transmissivity decreases according to a power > 3
- In the high stress regime (I), transmissivity levels off to some constant value

Imperial College London

Validity of "Cubic Law" as Normal Stress Increases

"The fractal geometry of flow paths in natural fractures in rock and the approach to percolation", D.D. Nolte, L.J. Pyrak-Nolte, and N.G.W. Cook, *PAGEOPH*, 1989;131:111-138.

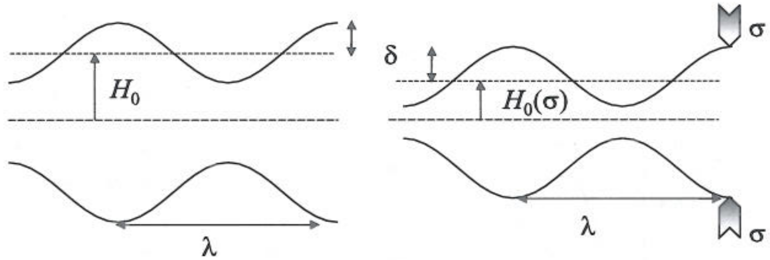


Models the change in exponent as being due to increased *in-plane* tortuosity due to an increase of contact area with stress, using percolation theory.

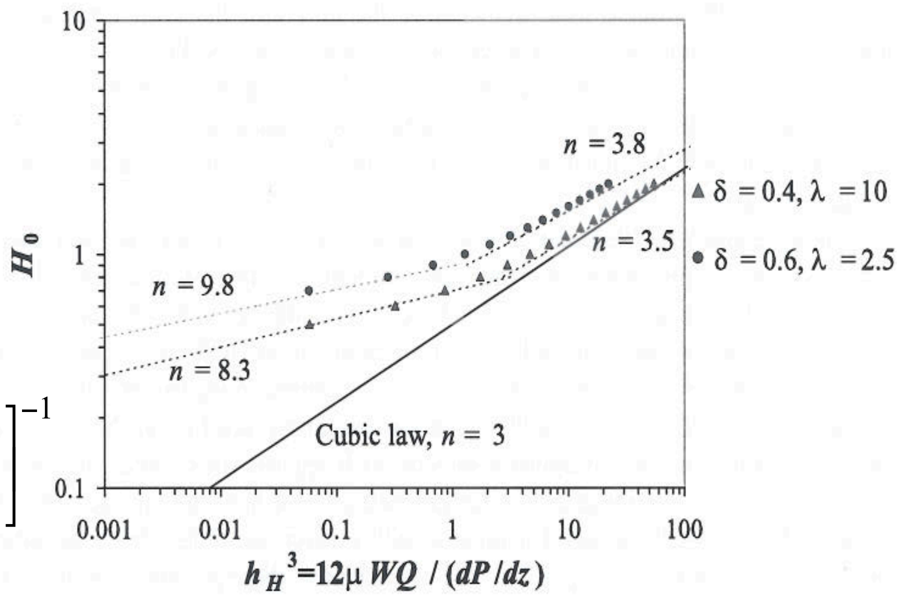
Imperial College London

Validity of "Cubic Law" as Normal Stress Increases

"A simple model for deviations from the cubic law for a fracture undergoing dilation or closure", S. Sisavath, A. Al-Yaarubi, C.C. Pain, and R.W. Zimmerman, *PAGEOPH*, 2003;160:1009–1022.



$$h_H^3 = \frac{(2H_0)^3(1-\hat{\delta}^2)^{5/2}}{1+(\hat{\delta}^2/2)}\left[1+\frac{\hat{\delta}^2}{\hat{\lambda}^2}\left(\frac{36}{15}\right)\frac{\pi^2(1-\hat{\delta}^2)}{1+(\hat{\delta}^2/2)}\right]^{-1}$$

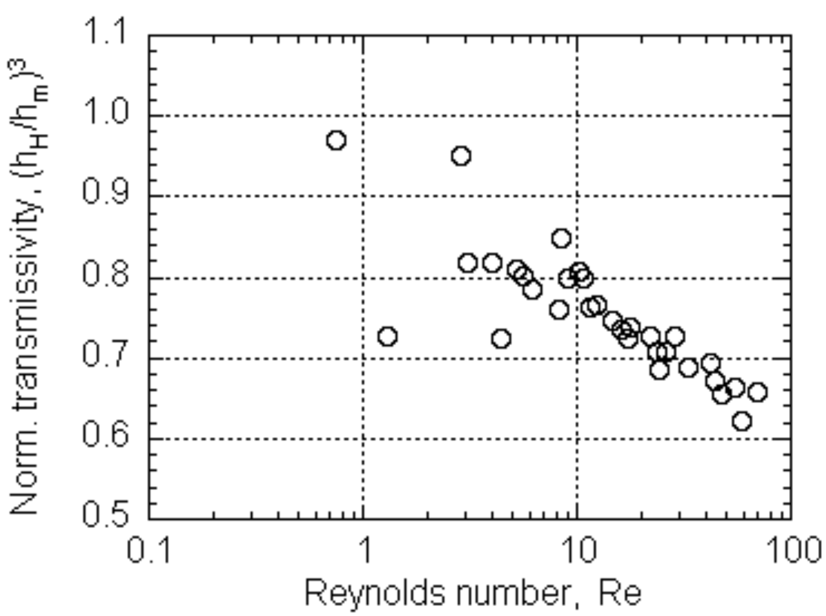


Models the change in exponent as being due to increased *out-of-plane* tortuosity due to an increase of relative roughness with stress, using perturbation theory.

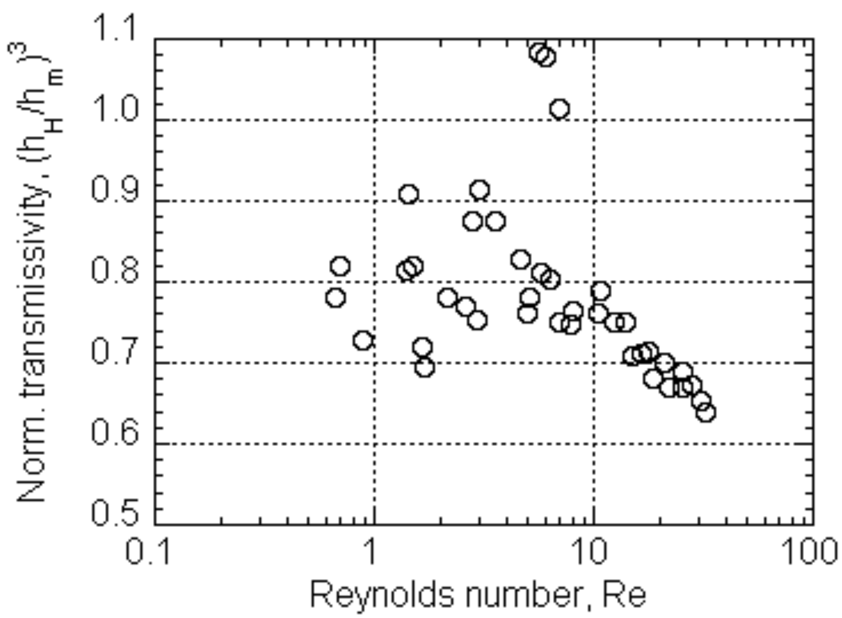
Nonlinear effects at higher Reynolds numbers

The apparent transmissivity decreases as the Reynolds number ($Re = \rho vh/\mu$) becomes greater than 1

Below: “Validity of cubic law for fluid-flow in a deformable rock fracture”, P.A. Witherspoon, J.S.Y. Wang, K. Iwai, and J.E. Gale, *Water Resour. Res.*, 1980.



Below: “Nonlinear regimes of fluid flow in rock fractures”, R.W. Zimmerman, A.H. Al-Yaarubi, C.C. Pain, and C.A. Grattoni, *Int. J. Rock Mech.*, 2005.

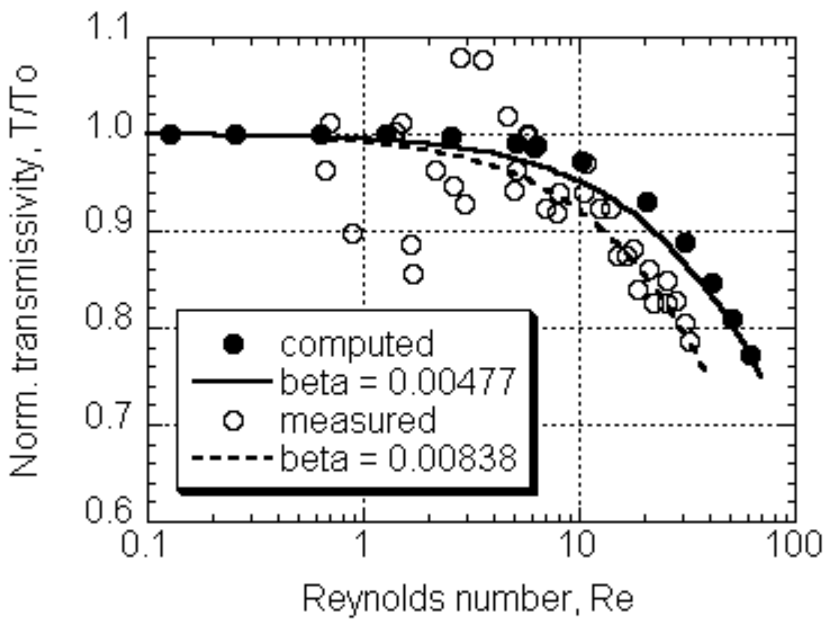


Nonlinear effects at higher Reynolds numbers

This nonlinearity can be modelled using the Forchheimer equation.

“Nonlinear regimes of fluid flow in rock fractures”, R.W. Zimmerman, A.H. Al-Yaarubi, C.C. Pain, and C.A. Grattoni, *Int. J. Rock Mech.*, 2005;41:paper 1A27:

$$\frac{T}{T_o} = \frac{1}{1 + \beta Re}$$



Areas of Current/Future Research

Effect of shear on transmissivity, and on induced anisotropy

"Effect of shear displacement on the aperture and permeability of a rock fracture", I.W. Yeo, M. de Freitas, R.W. Zimmerman, *Int. J. Rock Mech*., 1998;35:1051-1070.

Shear displacement (mm)	Hydraulic aperture (mm) (parallel to shear)	Hydraulic aperture (mm) (normal to shear)
0	446	469
1	577	664
2	740	852

"Flow channeling in a single fracture induced by shear displacement", H. Auradou, G. Drazer, A. Boschan, J.P. Hulin, J. Koplik, *Geothermics*, 2006:35:576-588.

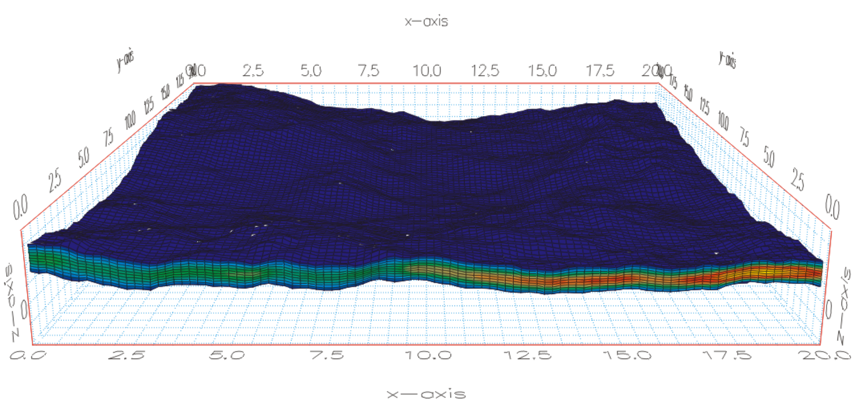
"Numerical study of flow anisotropy within a single natural rock joint", A. Giacomini, O. Buzzi, A.M. Ferrero, M. Migliazza, G.P. Giani, *Int. J. Rock Mech*., 2008;45:47-58.

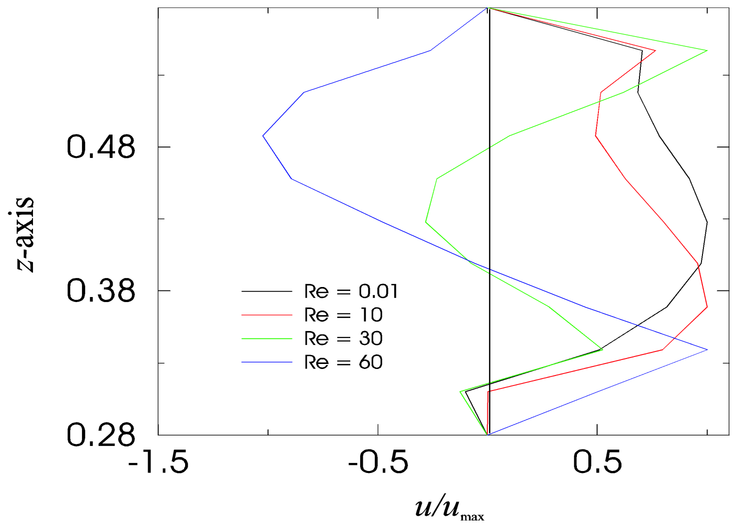
Imperial College London

Areas of Current/Future Research

Use of Navier-Stokes equations in place of the simpler Stokes or Reynolds equations to model fluid flow

"Nonlinear regimes of fluid flow in rock fractures", R.W. Zimmerman, A.H. Al-Yaarubi, C.C. Pain, and C.A. Grattoni, *Int. J. Rock Mech.*, 2005;41:paper 1A27.





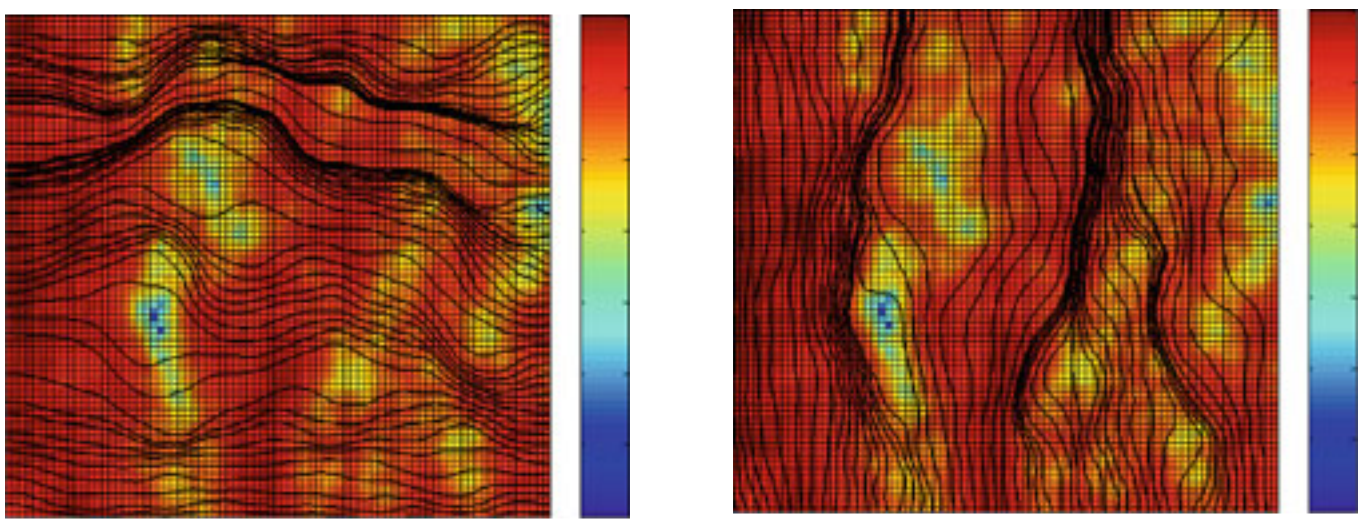
"Effects of inertia and directionality on flow and transport in a rough asymmetric fracture", M.B. Cardenas, D.T. Slottke, R.A. Ketcham, J.M. Sharp, *J. Geophys. Res.,* 2009;114:B06204.

Imperial College London

Areas of Current/Future Research

Solute transport and dispersion in rock fractures

"Shear-induced flow channels in a single rock fracture and their effect on solute transport", V. Vilarrasa, T. Koyama, I. Neretnieks, L. Jing, *Transp. Porous Media*, 2011:87:503-523:



Shear displacement 20 mm to the *left*

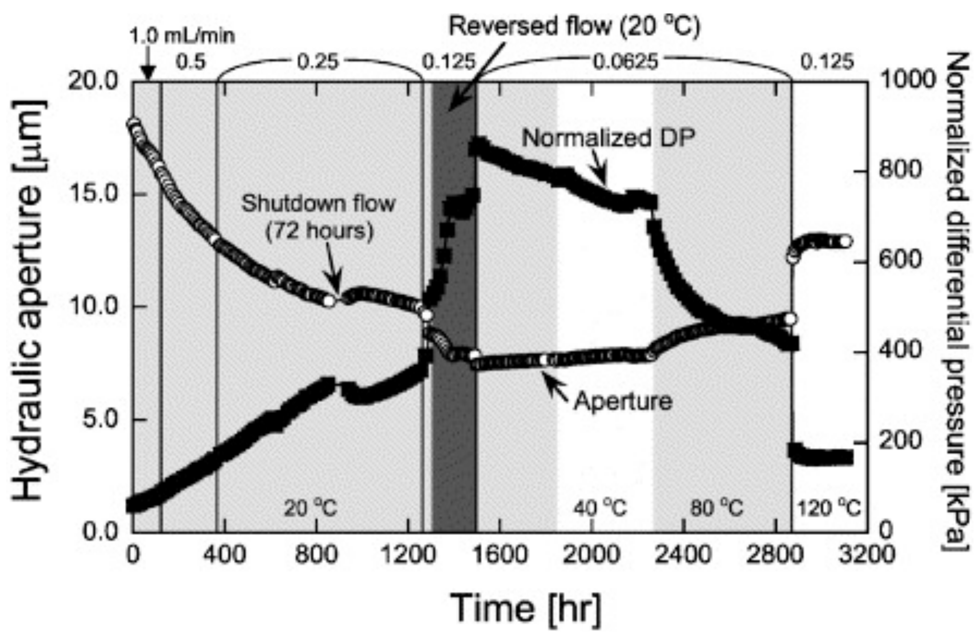
"Experimental study of the non-Darcy flow and solute transport in a channeled single fracture", Z. Chen, J.Z. Qian, H. Qin, *J. Hydrodynamics*, 2011:23:745-751.

Imperial College London

Areas of Current/Future Research

Effects of mineral dissolution and precipitation

"Evolution of fracture permeability through fluid-rock reaction under hydrothermal conditions", H. Yasuhara, A. Polak, Y. Mitani, A.S. Grader, P.M. Halleck, D. Elsworth, *Earth Planet. Sci. Letts.,* 2006:244:186-200:



"Fracture alteration by precipitation resulting from thermal gradients: Upscaled mean aperture-effective transmissivity relationship", A. Chaudhuri, H. Rajaram, H. Viswanathan, *Water Resour. Res*., 2012;48:W01601.

Imperial College London

Witherspoon, Wang, Iwai and Gale (LBL 1979 / WRR 1980)

Submitted for publication to Water Resources Research.

LBL-9557
SAC-23
UC-70

VALIDITY OF CUBIC LAW FOR FLUID FLOW IN A DEFORMABLE ROCK FRACTURE

P. A. Witherspoon, J. S. Y. Wang, K. Iwai[1], and J. E. Gale[2]

Department of Materials Science and Mineral Engineering
University of California, Berkeley

and

Lawrence Berkeley Laboratory
University of California
Berkeley, California

October 1979

[1] Now with Nakano Corporation, Niigatashi, Japan.

[2] Now at the Department of Earth Sciences, University of Waterloo, Waterloo, Ontario, Canada.