

Developing robust DEM models to simulate element tests

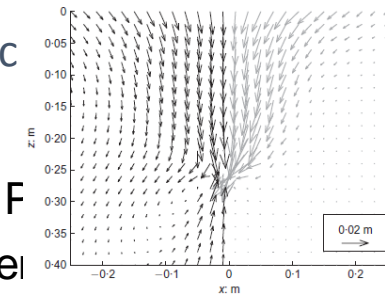
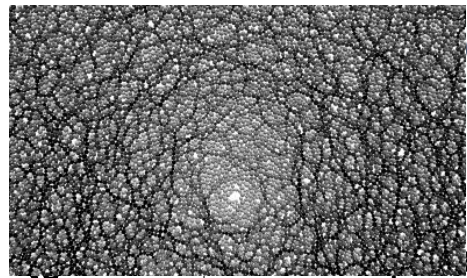
Catherine O'Sullivan

Talk overview

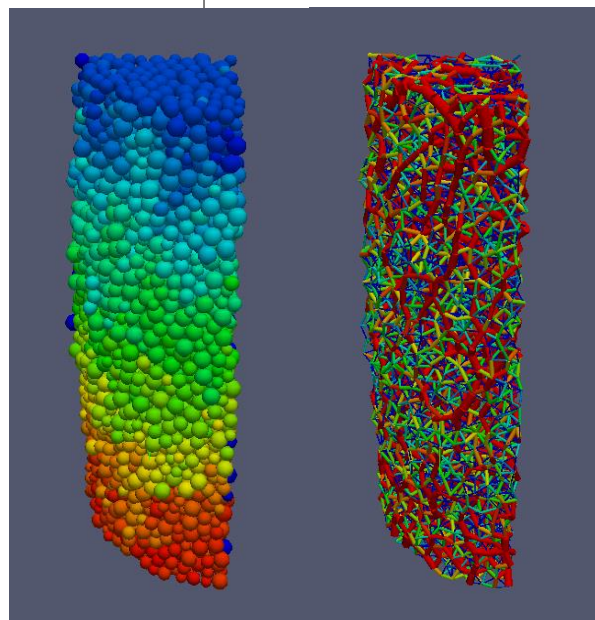
- DEM in geomechanics
- Choice of DEM code
- Code validation
- Time step
- Boundary conditions
- Specimen generation + sample size
- Input parameters
- Contact model
- Running the simulation
- Post processing – data interpretation
- Conclusions

DEM in geomechanics

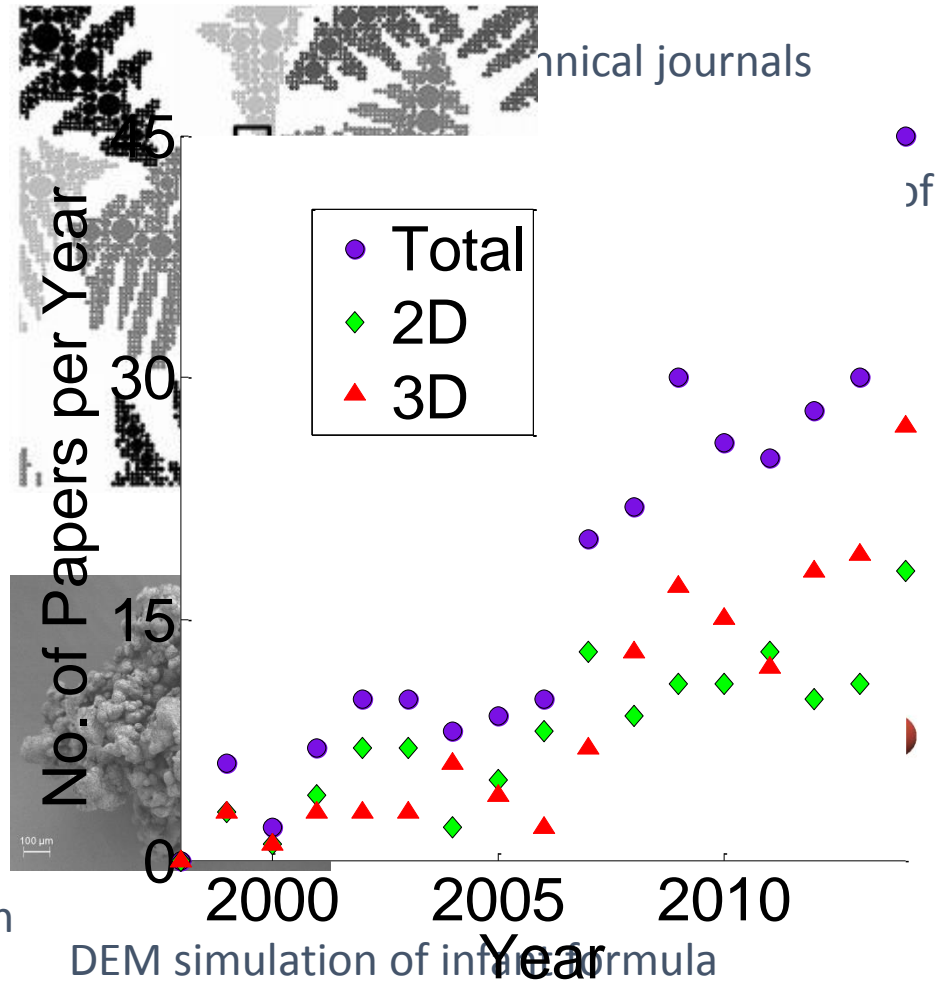
Use of DEM – No of Publications per Year



1000
800
per Year
Tunnelling induced displacements
Meym et al. (2013) Géotechnique



2000 2010
Deformation in
triaxial cell
Cui et al. (2007)
Géotechnique



DEM simulation of infill formula
Hanley et al (2012)

Choice of DEM code

TRUBAL

- Original DEM code developed by Cundall and Strack – code listing + validation given in 1979 report to the US National Science Foundation
- Written in Fortran to run in serial mode
- Derivatives of this code:
 - Aston DEM code used by Colin Thornton (e.g. Thornton (2000) *Géotechnique*)
 - ELLIPSE3D (Ng, e.g. Ng (2006) *Géotechnique*)
 - Modified Trubal code used by O’Sullivan/Barreto/Cui

LAMMPS

- LAMMPS (<http://lammps.sandia.gov/>)
- Classical molecular dynamics code that can be used for DEM
- Uses spatial decomposition and MPI (message passage interface)
- Can run large high performance computers (hpc) with distributed memory
- Researchers at IC (Marketos, Hanley, Shire, Huang, Otsubo) have been working to modify the granular LAMMPS package

LIGGGHTS

- Derivative of LAMMPS – developed by Christof Kloss
- No longer compatible with main LAMMPS codebase
- Specifically developed for granular materials
- Commercial and public versions of code exist
- Limited experience at IC with this code

PFC

- Commercial software – developed by Itasca.
- Peter Cundall close connection to Itasca.
- Flexible
- Used by O’Sullivan / Cheung / Ciantia
- Coupled version used by Kawano

Other codes

- Oval
- YADE
- EDEM

Conclusions

- PFC is the most popular DEM code based on review of DEM studies published in geomechanics / geotechnical /soil mechanics journals
- Now a good few DEM codes available open source
- Tendency to exploit parallelism.
- YADE / PFC Version 5 – shared memory
- LIGGGHTS / LAMMPS – distributed memory

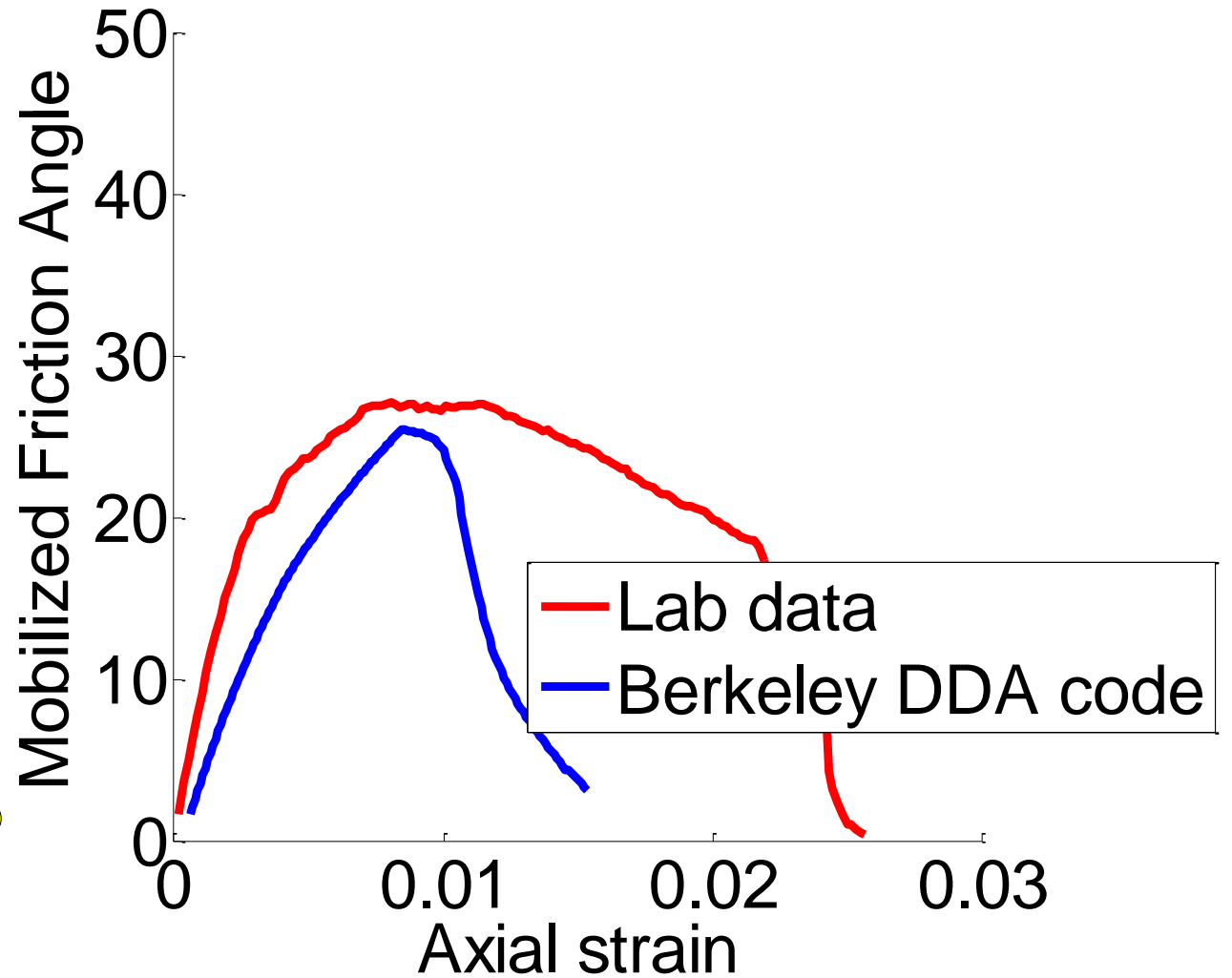
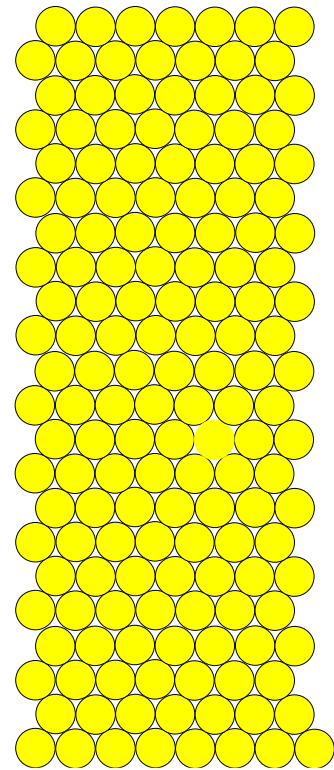
Code validation

Code validation

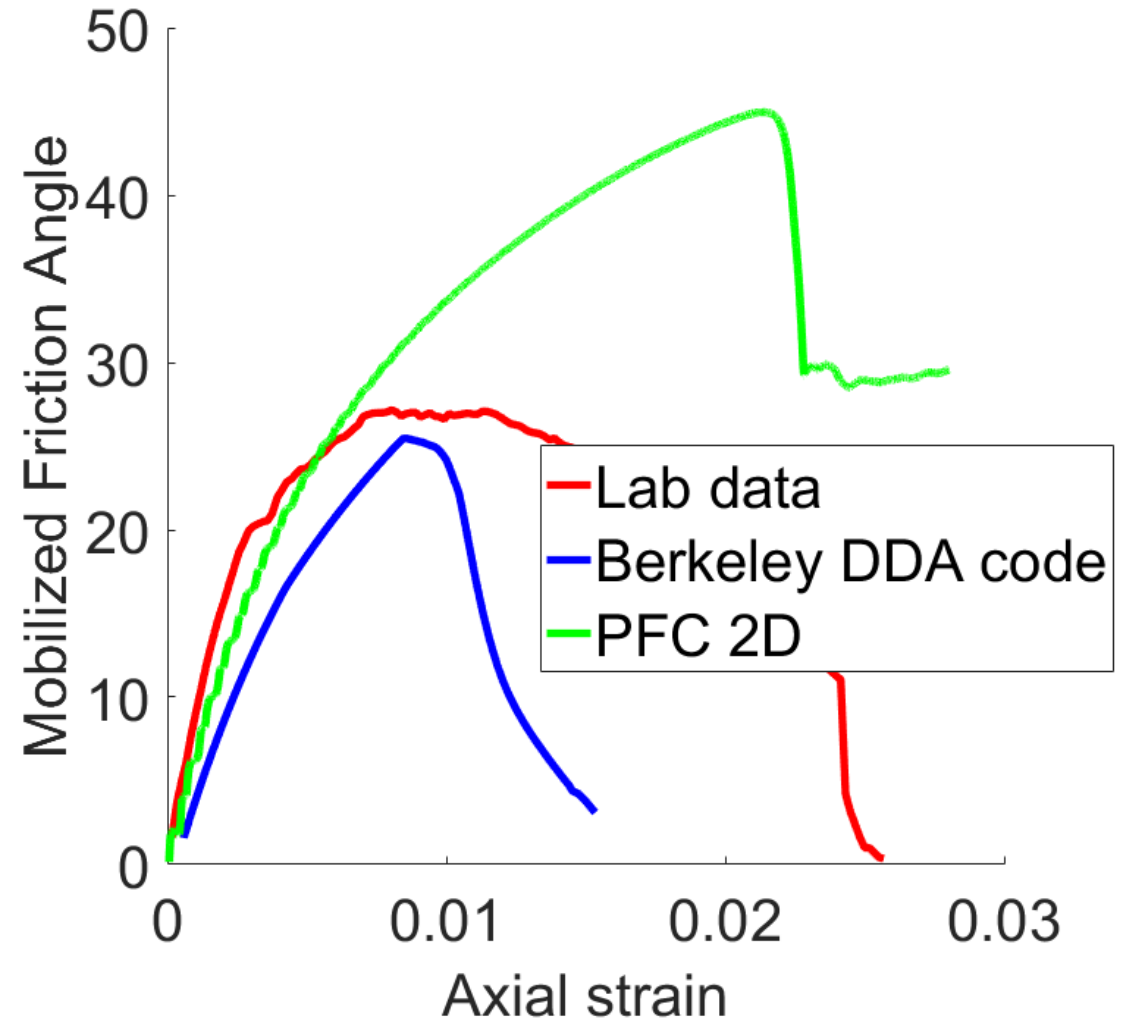
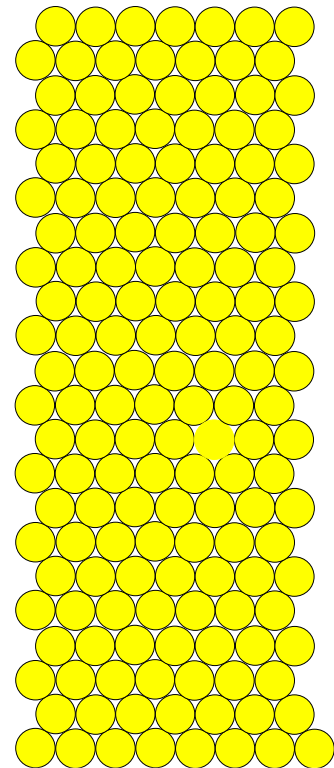
Calibration → tuning code to match experimental data – optimization problem

Validation → verification / confirmation that code correctly and accurately captures the intended physics / mechanics

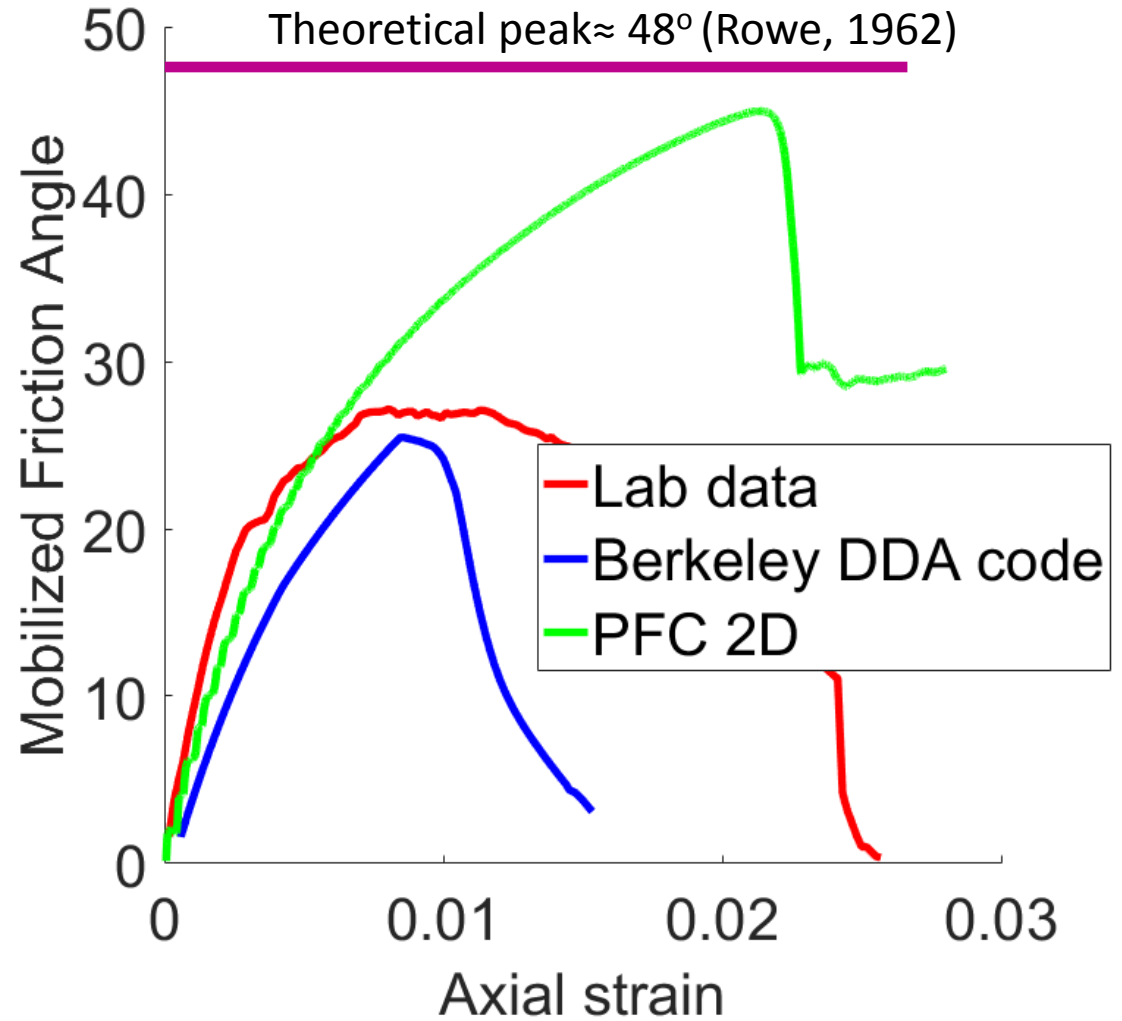
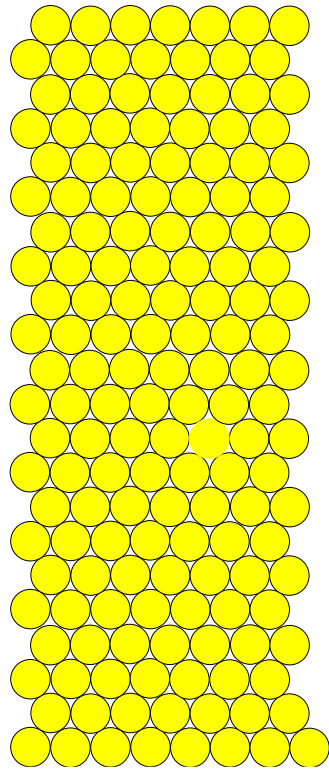
2D DEM validation with glass rods



2D DEM validation with glass rods



2D DEM validation with glass rods

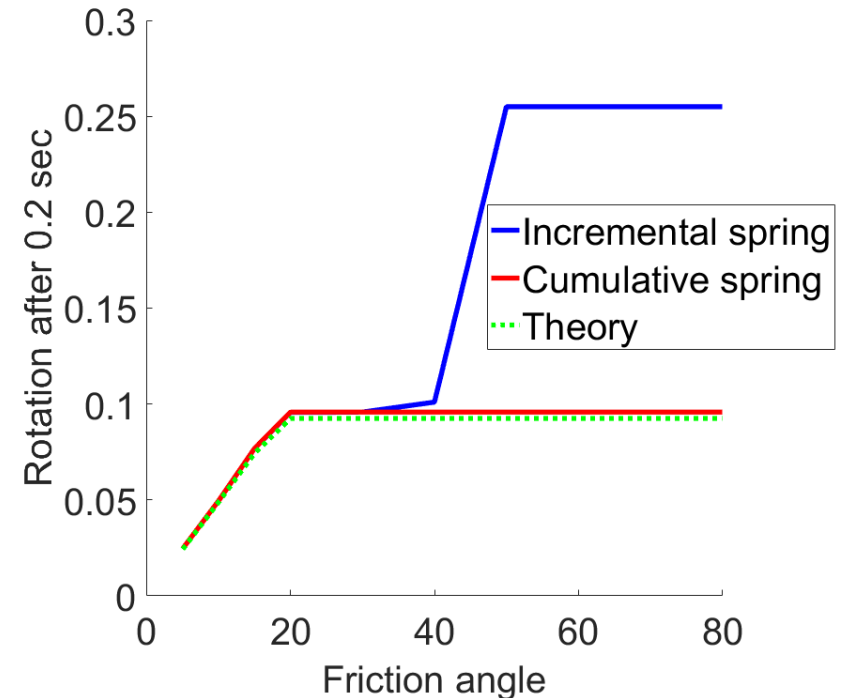
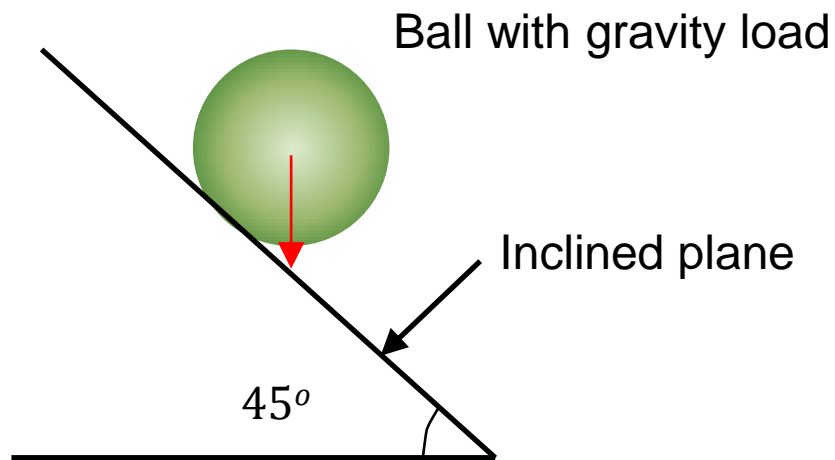


2D DEM validation with glass rods

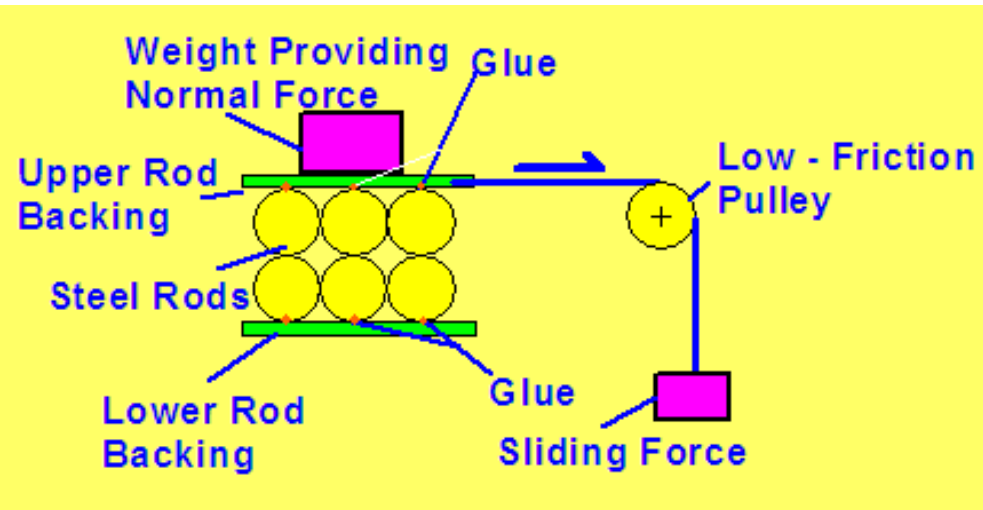
- Original DDAD formulation: incremental shear spring
- DEM formulation in PFC: cumulative shear spring

$$F_s = k_s \Delta u_s$$

$$F_s = \sum k_s \Delta u_s$$



Repeat tests and simulations



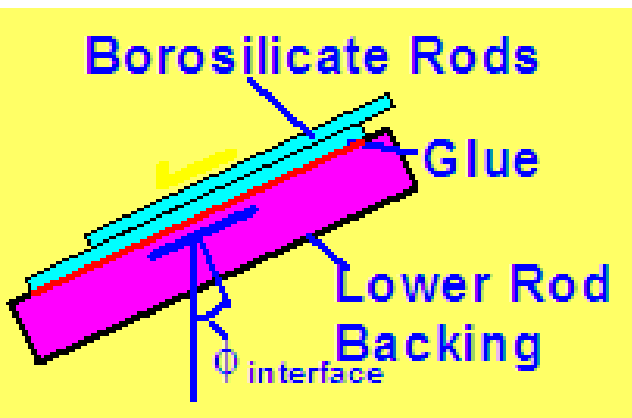
Steel Rods:

Size: 5.3 - 6.3 mm Diameter

Tolerance 1/400 mm

Friction Mean 12.1 °,
 Std. Dev. 3.0 °

***New tests +
simulations***



Borosilicate Rods

Size: 4,5 and 6 mm Diameter

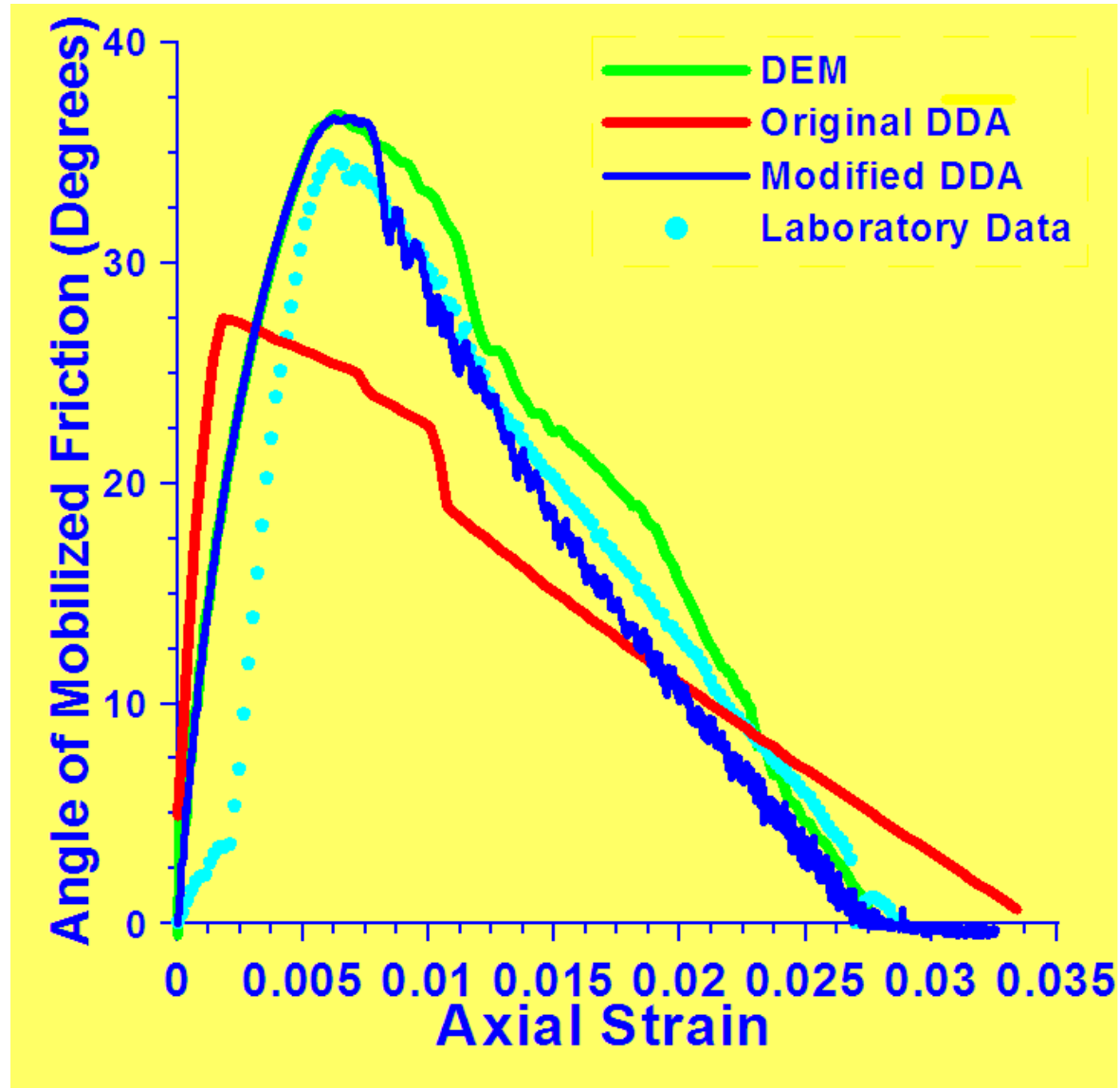
Standard Deviation 0.5% - 1.5%

Aspect ratio 1.002-1.016

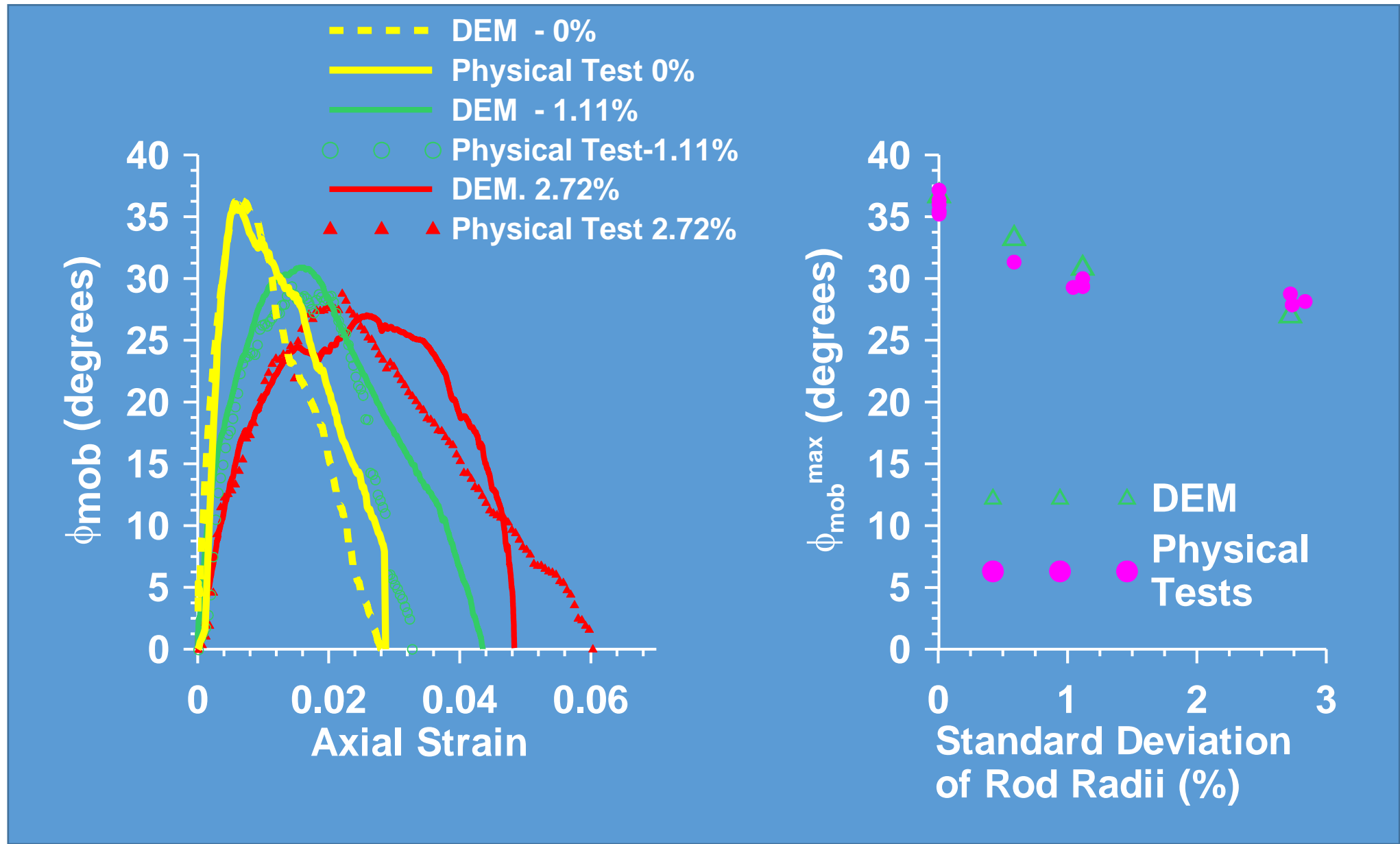
Friction Mean 18°,
 Std. Dev. 2.15°

New simulations

Repeat tests and simulations: steel rods

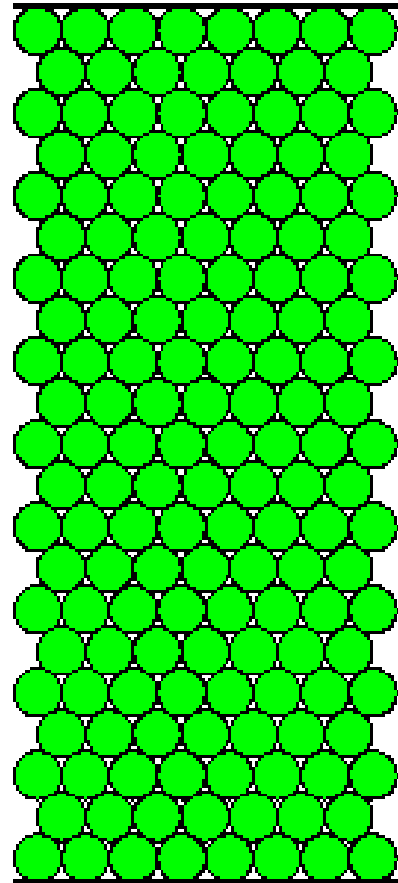


Repeat tests and simulations: steel rods

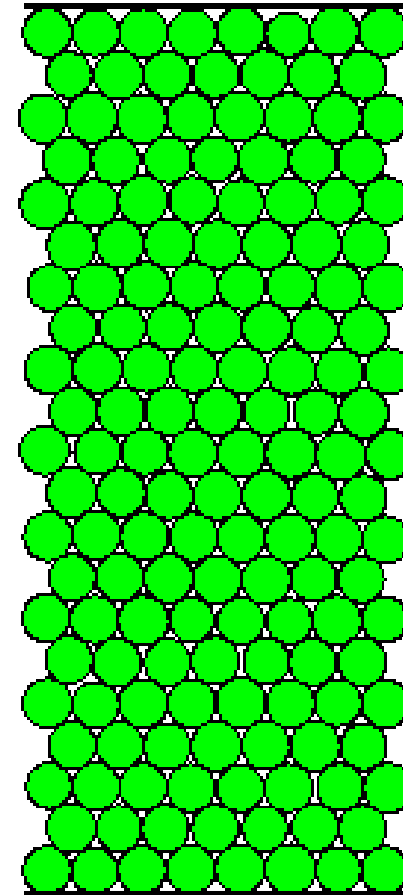


Repeat tests and simulations: steel rods

Uniform Specimen

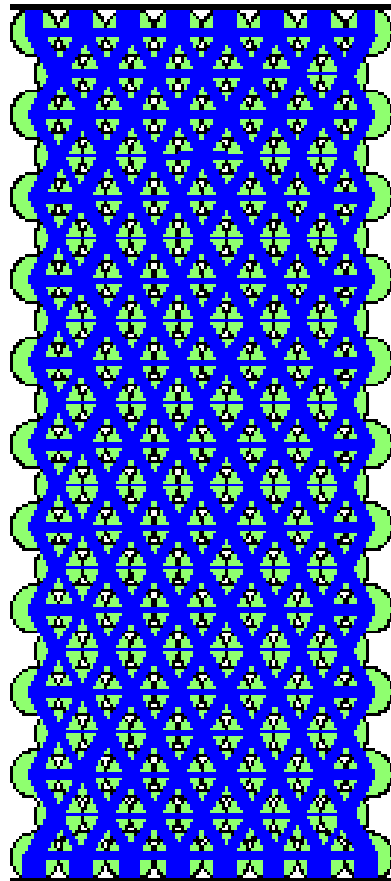


Std. Dev. 2.7%

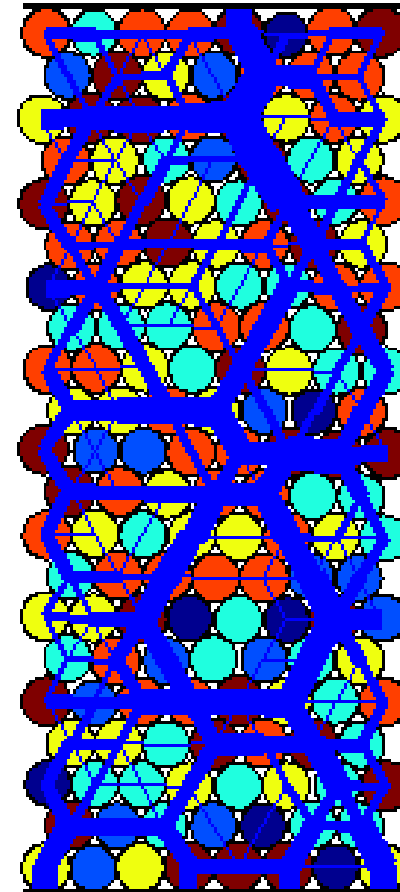


Repeat tests and simulations: steel rods

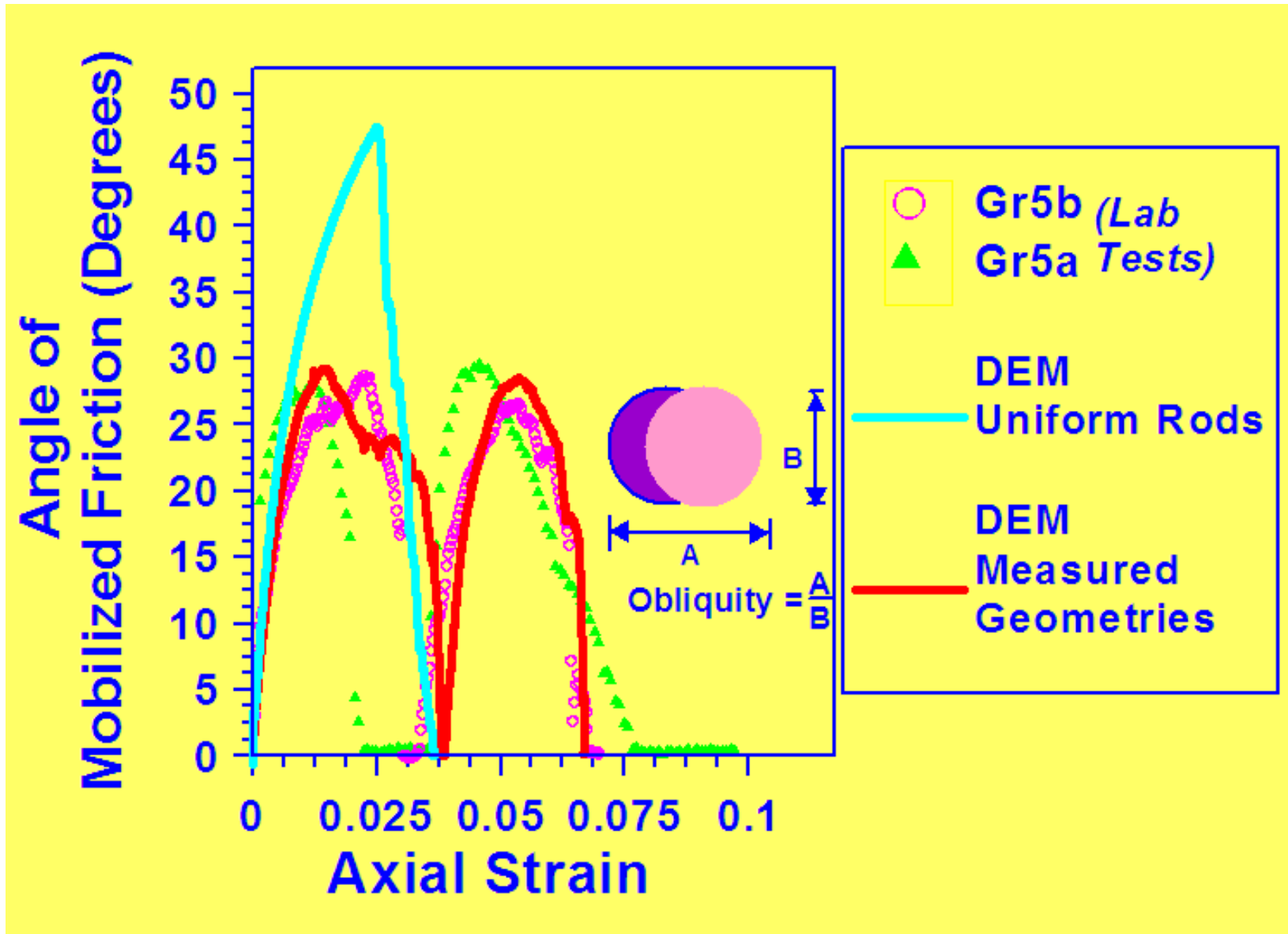
Uniform Specimen



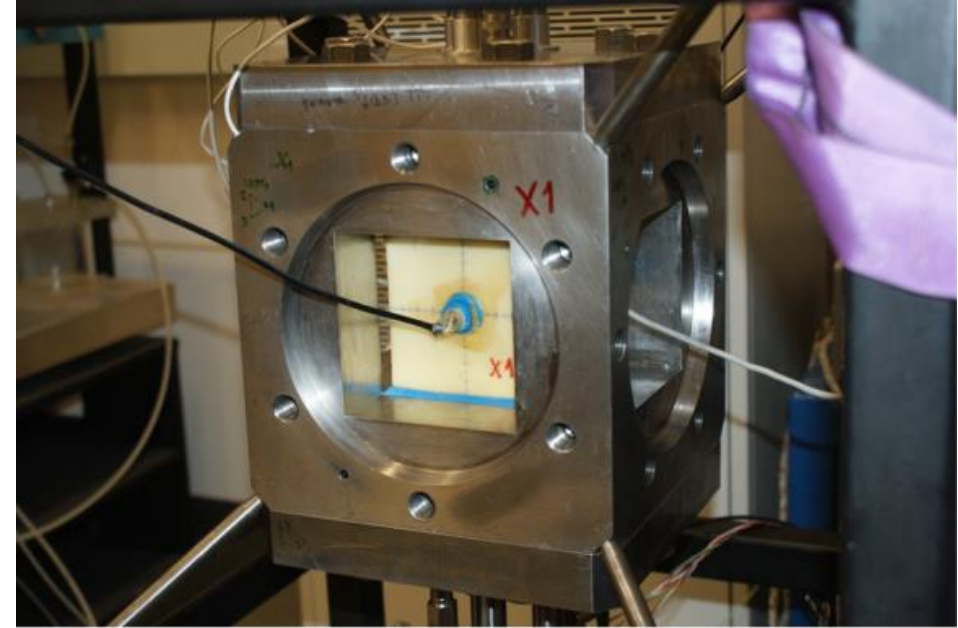
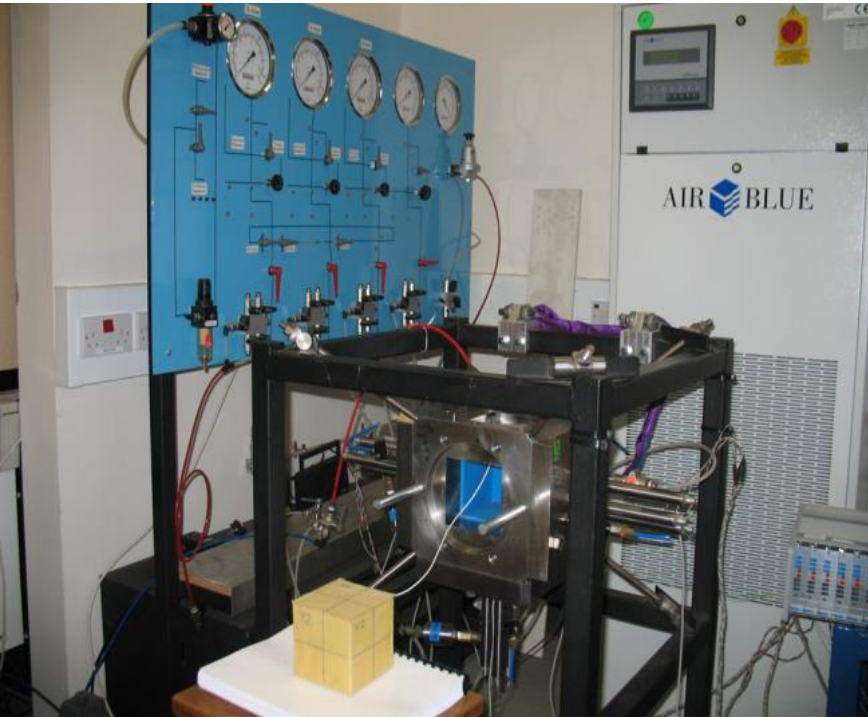
Std. Dev. 2.7%



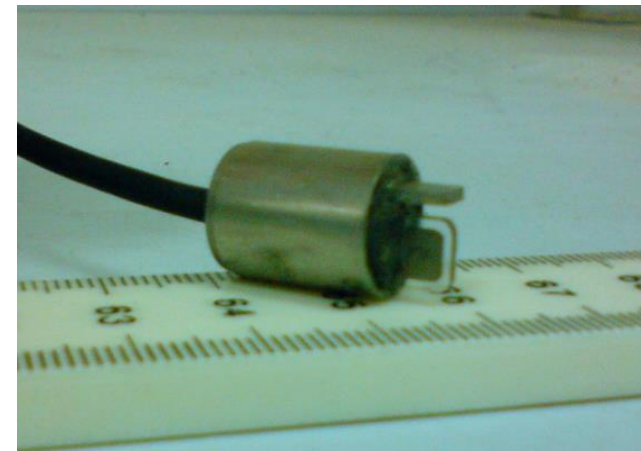
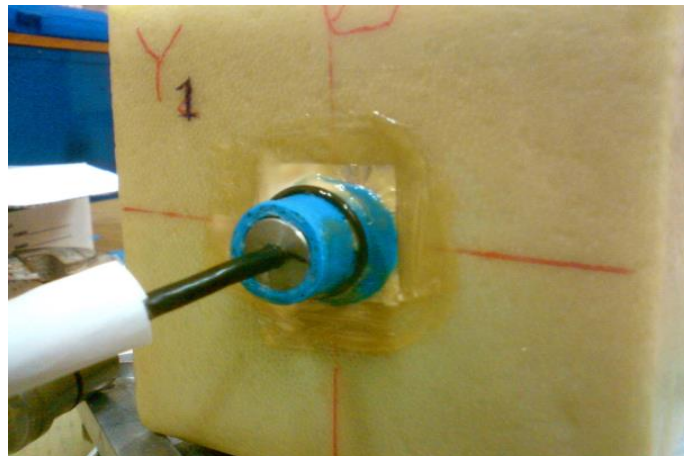
Repeat simulations: glass rods



Validation of simulation approach



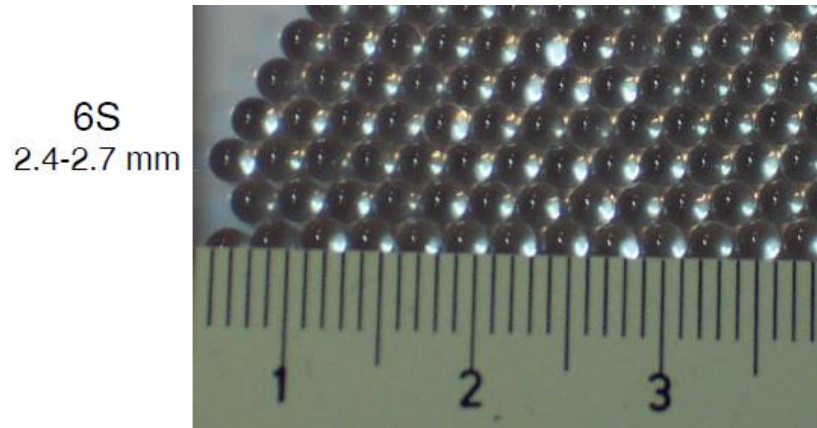
University of Bristol Cubical Cell



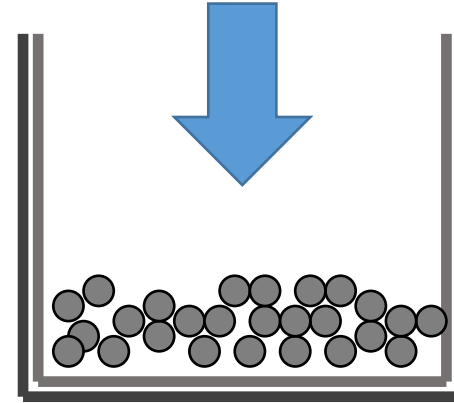
O'Donovan et al. (2016)

Validation of simulation approach

Borosilicate glass ballotini
Diameter 2.4 mm – 2.7 mm
Cavarretta et al. (2012)

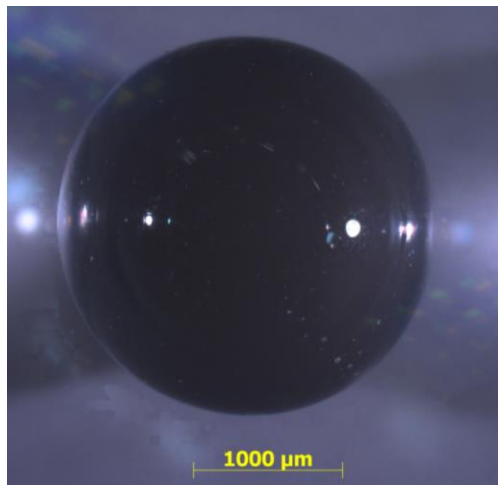
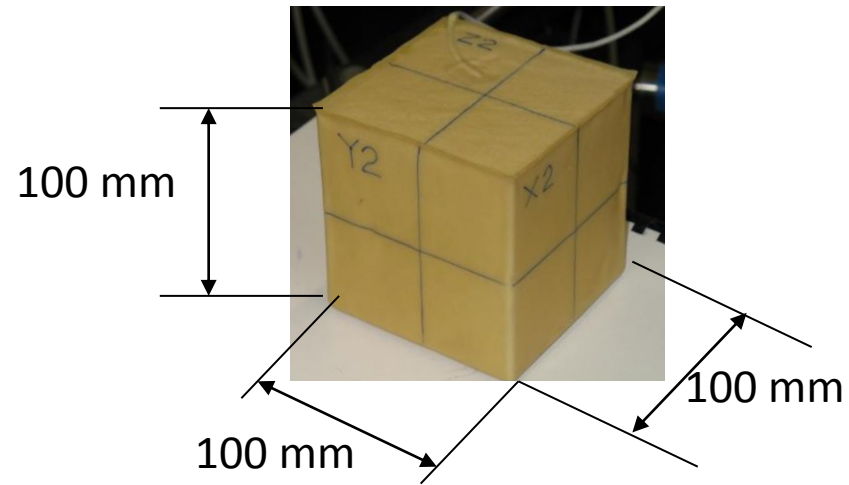


Controlled pluviation into
membrane lined mould



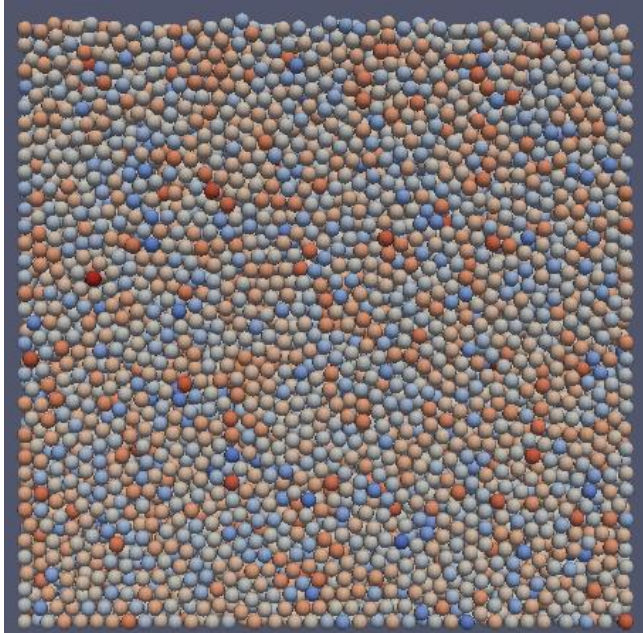
Void ratio: $e_{lab}=0.60$

50 kPa vacuum applied to
place sample in apparatus

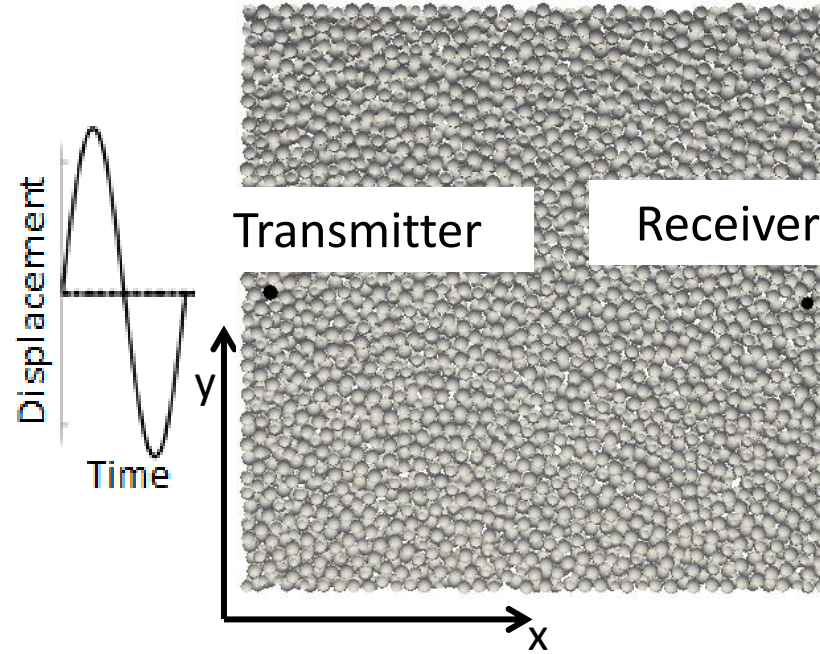


O'Donovan et
al. (2016)

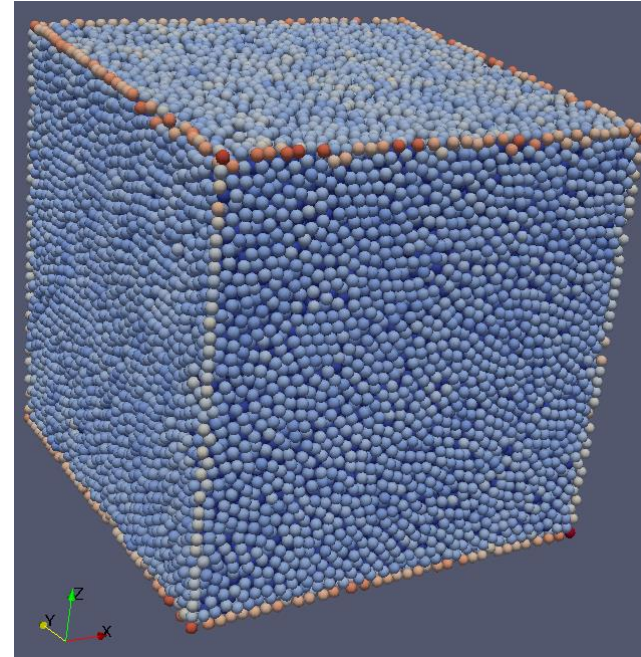
Validation of simulation approach



62,445 particles
Pluviated using Granular
Lammps
Ave. coord. number: 5.185
Cross-anisotropic fabric



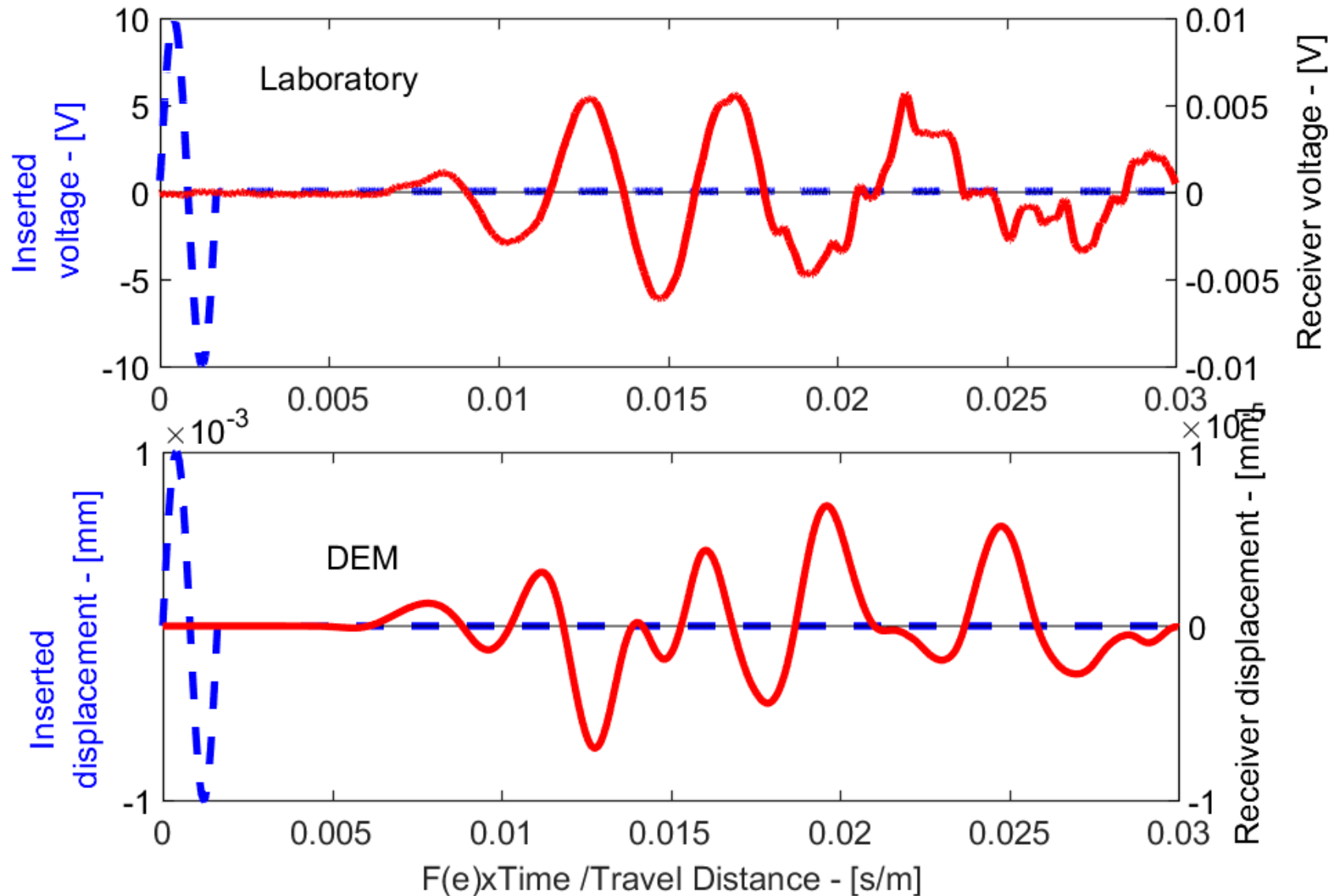
Simulated using PFC 3D
Flexible membrane boundaries
Simplified Hertz-Mindlin contact model
Bender/extender elements modelled as point sources



Void ratio: $e_{DEM}=0.67$

O'Donovan et
al. (2016)

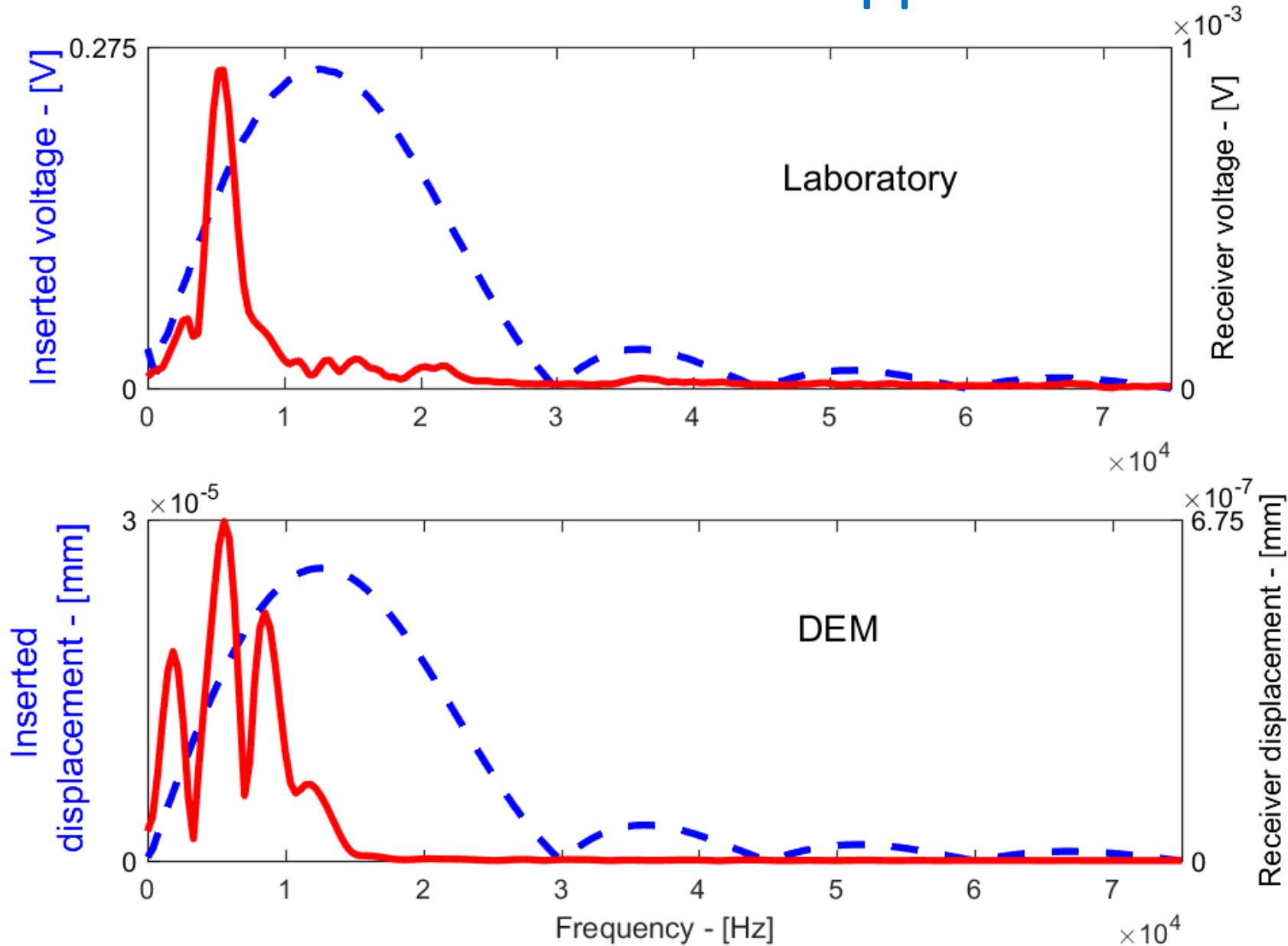
Validation of simulation approach



Time domain comparison

- Void ratio correction:
 $F(e) = 2.9 - e$
- Adjusted for differences in travel distance

Validation of simulation approach

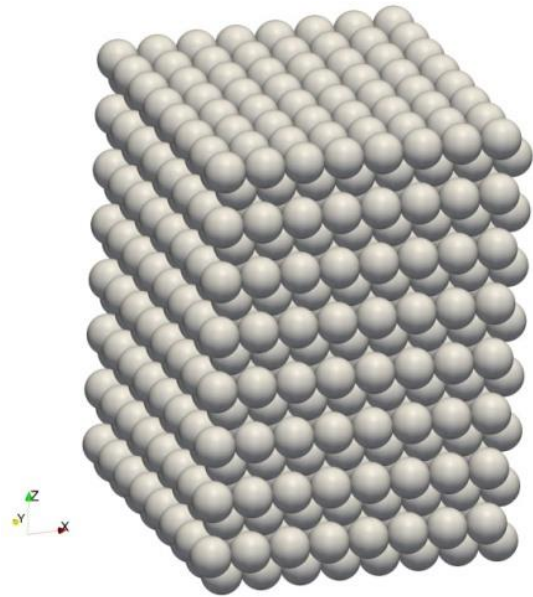


Frequency domain comparison

Analytical validation

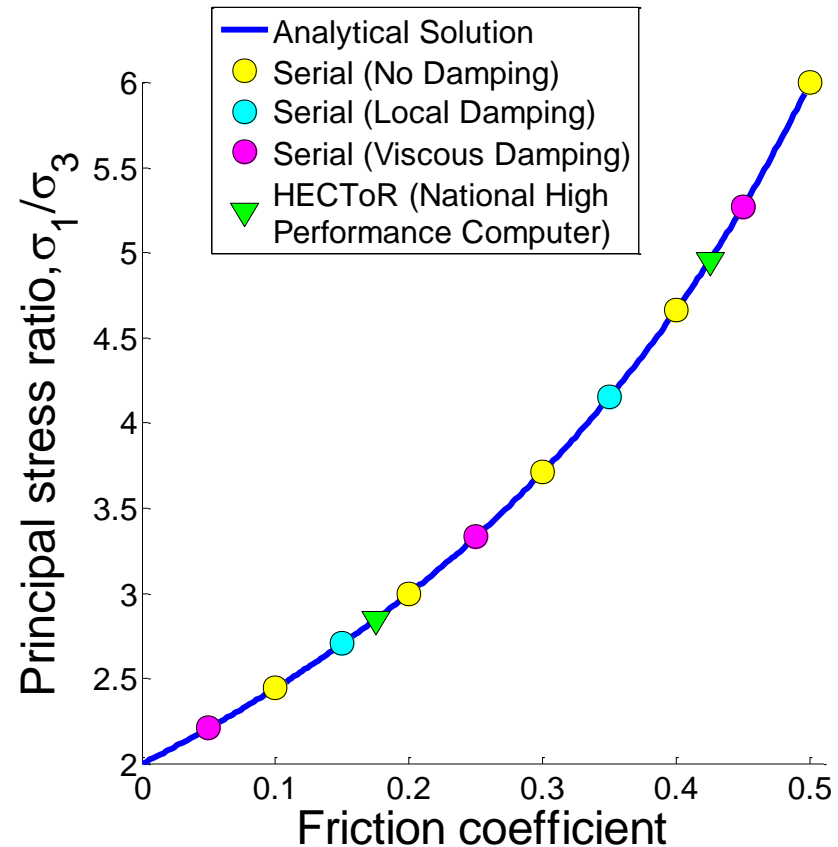
- Response of single ball systems worth checking
- Assemblies of contacting grains form statically indeterminate systems
- Lattice packings - can exploit symmetric to get analytical expressions for strength
- Thornton (1979) Géotechnique contribution – cited in PFC manual – very useful

Validation of granular LAMMPS



Compression of face-centred-cubic array of spheres.

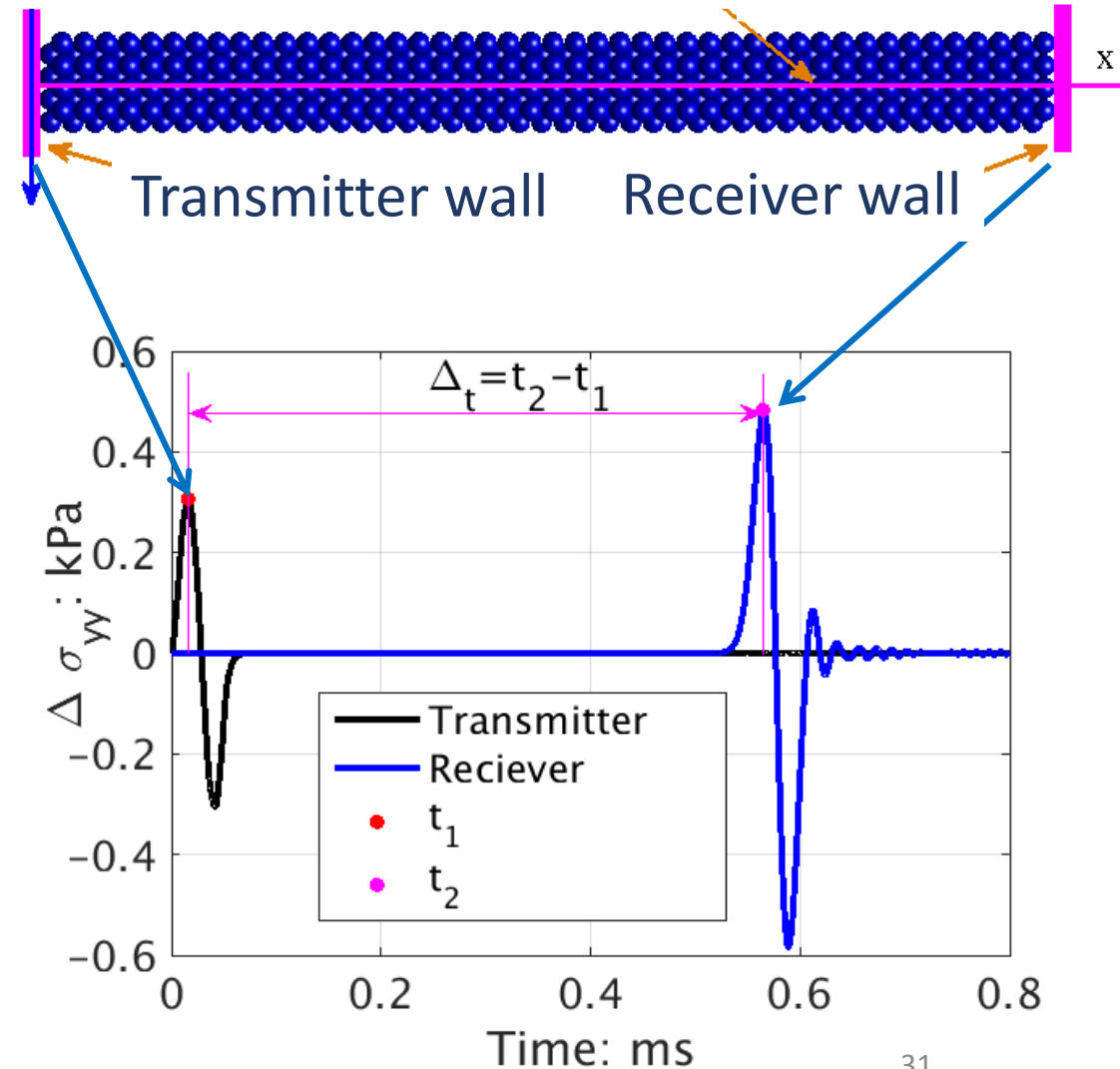
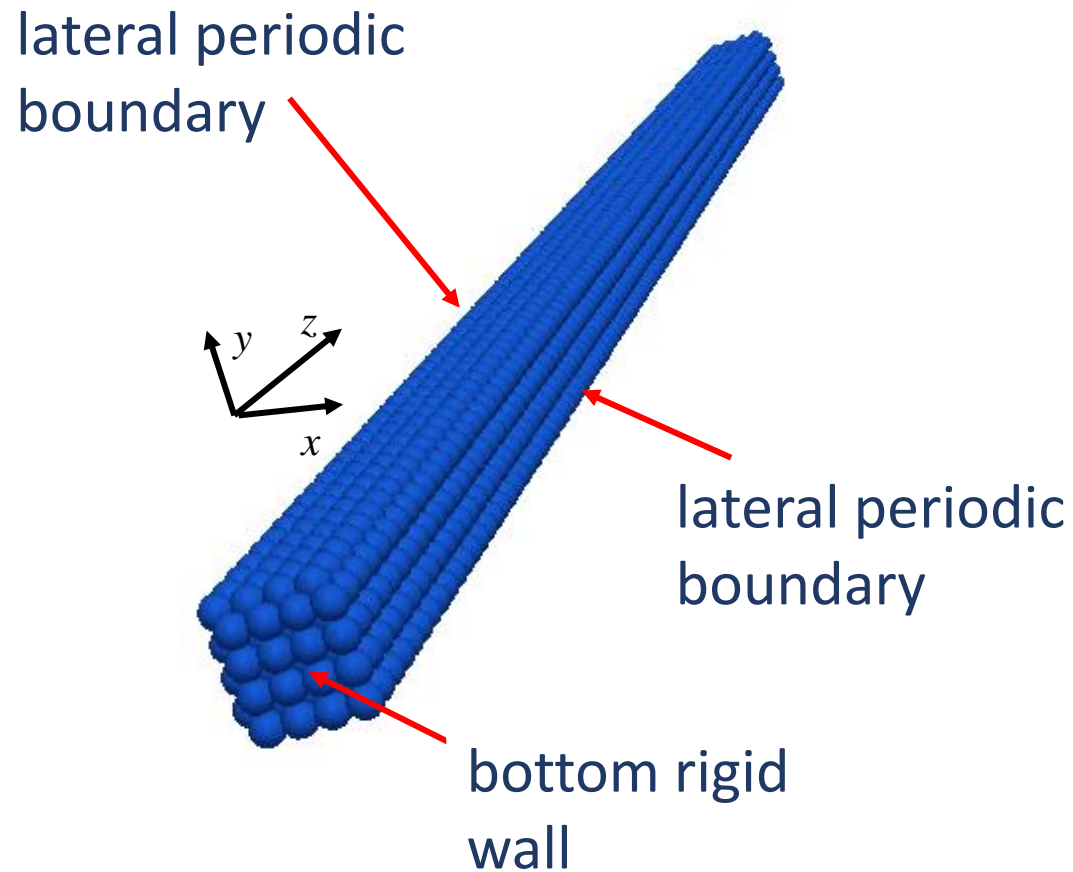
Analytical solution documented in Thornton (1979) Géotechnique



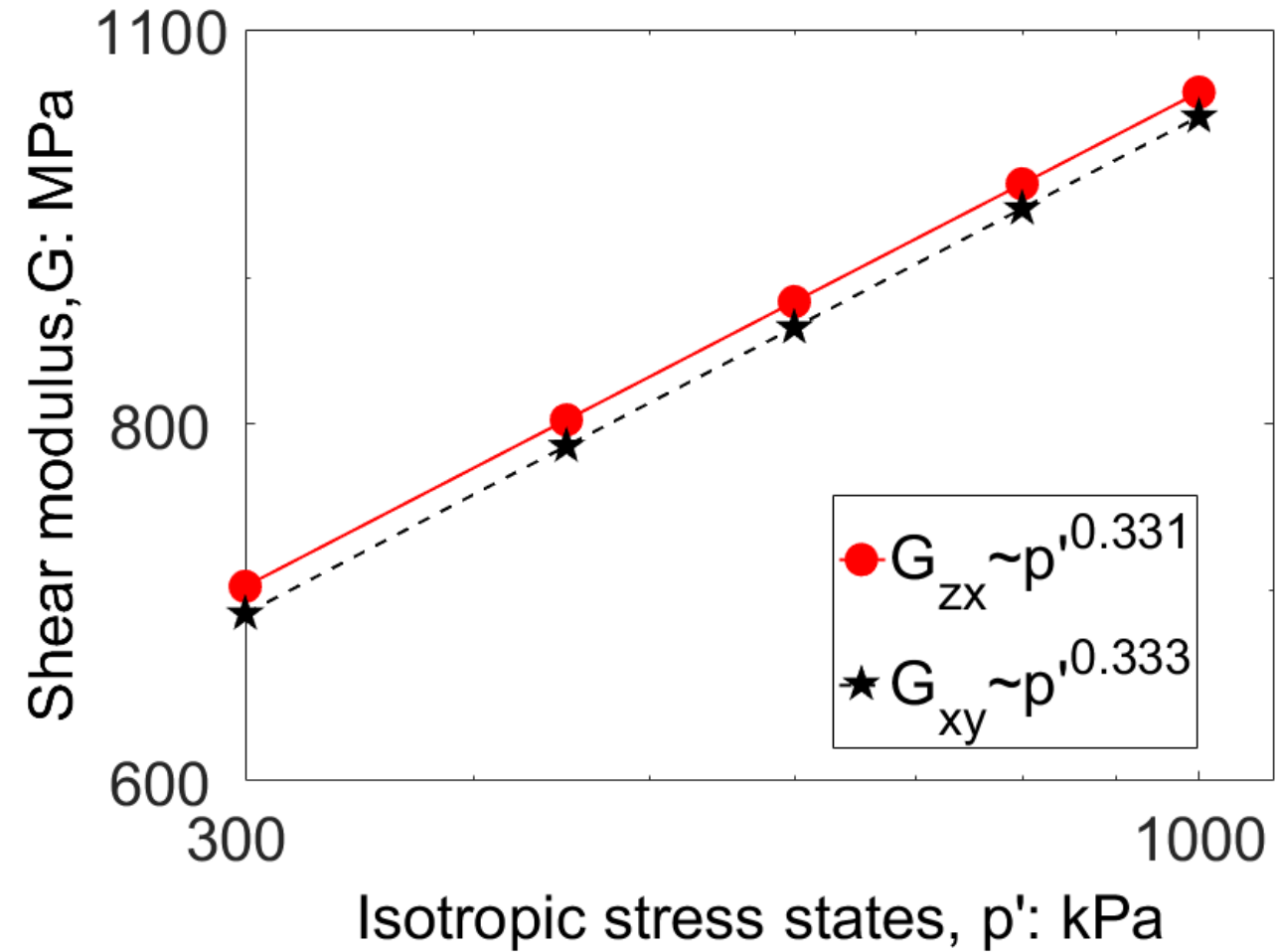
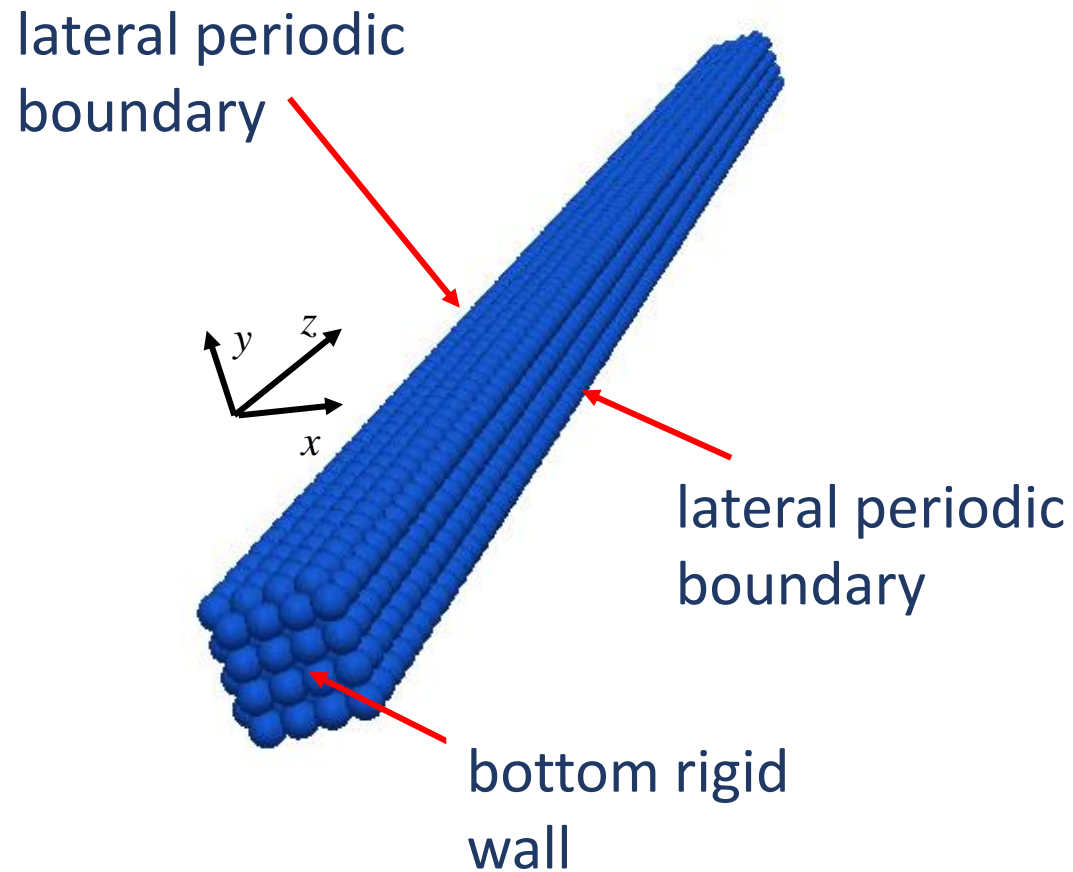
Validations by K. Hanley and X. Huang

O'Sullivan (2014)

Validation - granular LAMMPS – small strain / dynamic response



Validation - granular LAMMPS – small strain / dynamic response



Conclusions

- Experimental validation is useful/essential, but not straightforward
- Analytical approaches using regular packing v important
- Benchmarking on random systems is helpful – cross comparison of codes (more later)

Time step

Time integration in DEM

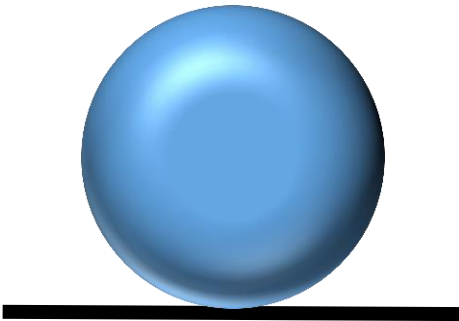
- The displacement of each particle is calculated using the explicit, conditionally stable, Verlet time integration approach.

$$\Delta u_i^t = (v_i^{t-\Delta t/2} + (F_i^t/M_i) \times (\Delta t)) \times \Delta t$$

- Δu_i^t = incremental displacement from time t to time $t+\Delta t$ for particle i
 - $v_i^{t-\Delta t/2}$ = velocity at time $t-\Delta t/2$ for particle i
 - F_i^t = total force acting on particle i at time t
 - M_i = mass of particle i
- Very similar to finite difference method

Calculating critical time increment

Single ball
on a
boundary



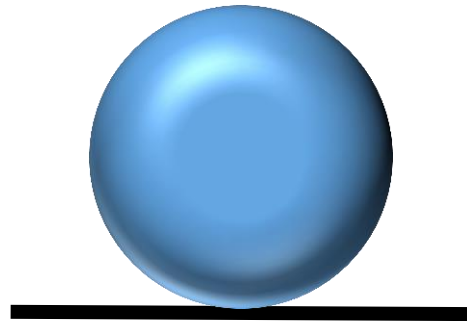
Single
degree of
freedom
system



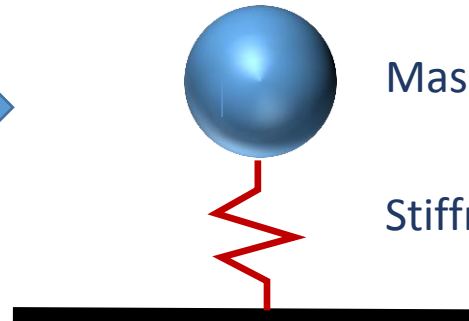
$$\Delta t_{crit,SDOF} = 2\sqrt{m/K}$$

Calculating critical time increment

Single ball
on a
boundary



Single
degree of
freedom
system



Mass, m

Stiffness, K

Hart et al. (1988):

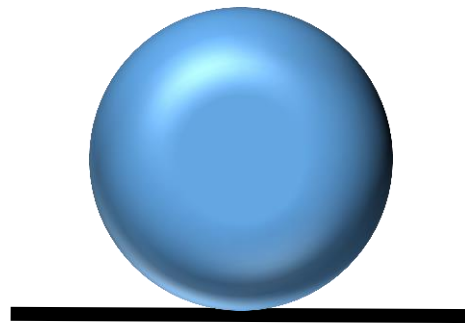
$$\Delta t_{crit,DEM} = 2\alpha \sqrt{m_{min} / 2K_{max}}$$

If $\alpha = 0.1$:

$$\Delta t_{crit,DEM} = 0.14 \sqrt{m_{min} / K_{max}}$$

Calculating critical time increment

Single ball
on a
boundary



Single
degree of
freedom
system



Tsuji et al. (1993):

Parametric study

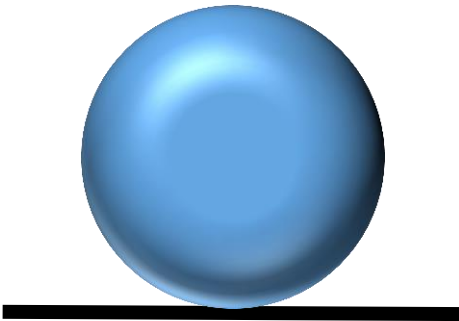
Monodisperse spheres

$$\Delta t_{crit,DEM} = \frac{\pi}{5} \sqrt{m/K}$$

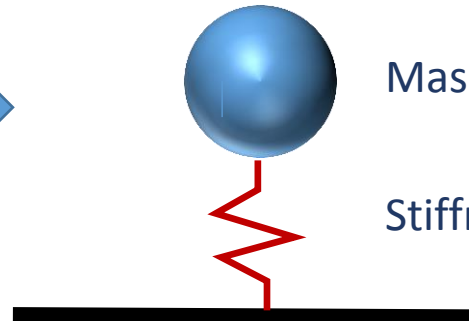
$$\Delta t_{crit,DEM} = 0.63 \sqrt{m/K}$$

Calculating critical time increment

Single ball
on a
boundary



Single
degree of
freedom
system



Mass, m

Stiffness, K

Tu and Andrade (2008)

Rotation critical

$$\Delta t_{crit,DEM} = 1.2 \sqrt{m_{min} / K_T}$$

K_T is the tangential spring stiffness

Calculating critical time increment: Itasca Approach

- Based on estimating eigenvalues of system
- For each particle assume rotational and translational degrees of freedom are uncoupled
- Sum contributions from all contacts to get translational and rotational stiffnesses (k_{trans} , k_{rotate})
- Look at minimum of ratios $\sqrt{m/k_{trans}}$, $\sqrt{I/k_{rot}}$, I moment of inertia

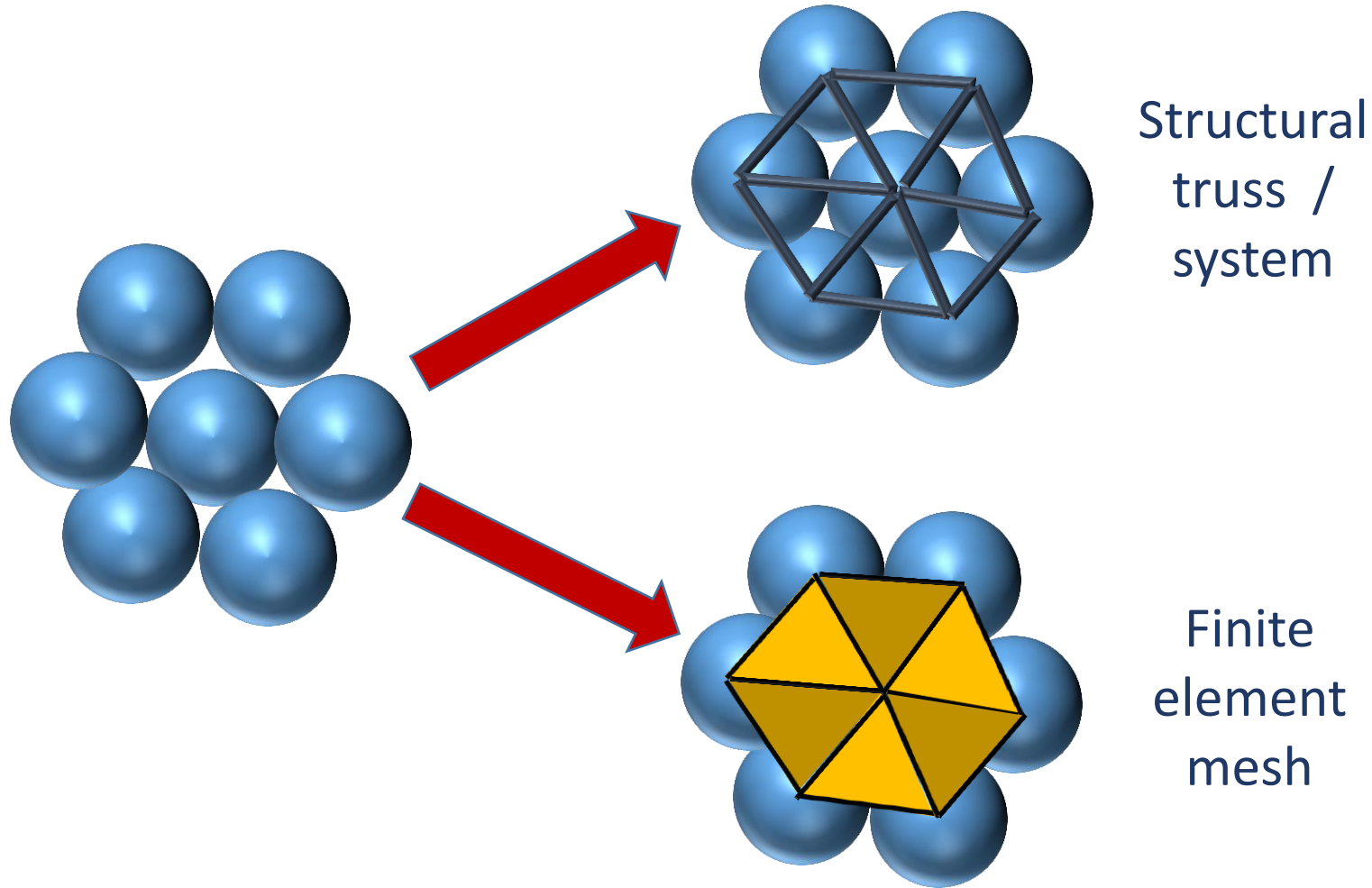
Calculating critical time increment: Rayleigh wave speed approach

- Considers time for a Rayleigh wave to pass a sphere in a single time step

$$T_R = \frac{\pi R \sqrt{\frac{\rho}{G}}}{0.1631\nu + 0.8766}$$

- ρ is the particle density, ν is the particle Poisson's ratio, G is the particle shear modulus, R is the particle radius
- Expression given in Kremmer and Favier (2001), Li et al. (2005) amongst others
- Some authors focus on minimum particle radius (R_{min}) others on average radius (R_{ave})

Calculating critical time increment: eigenvalue approach

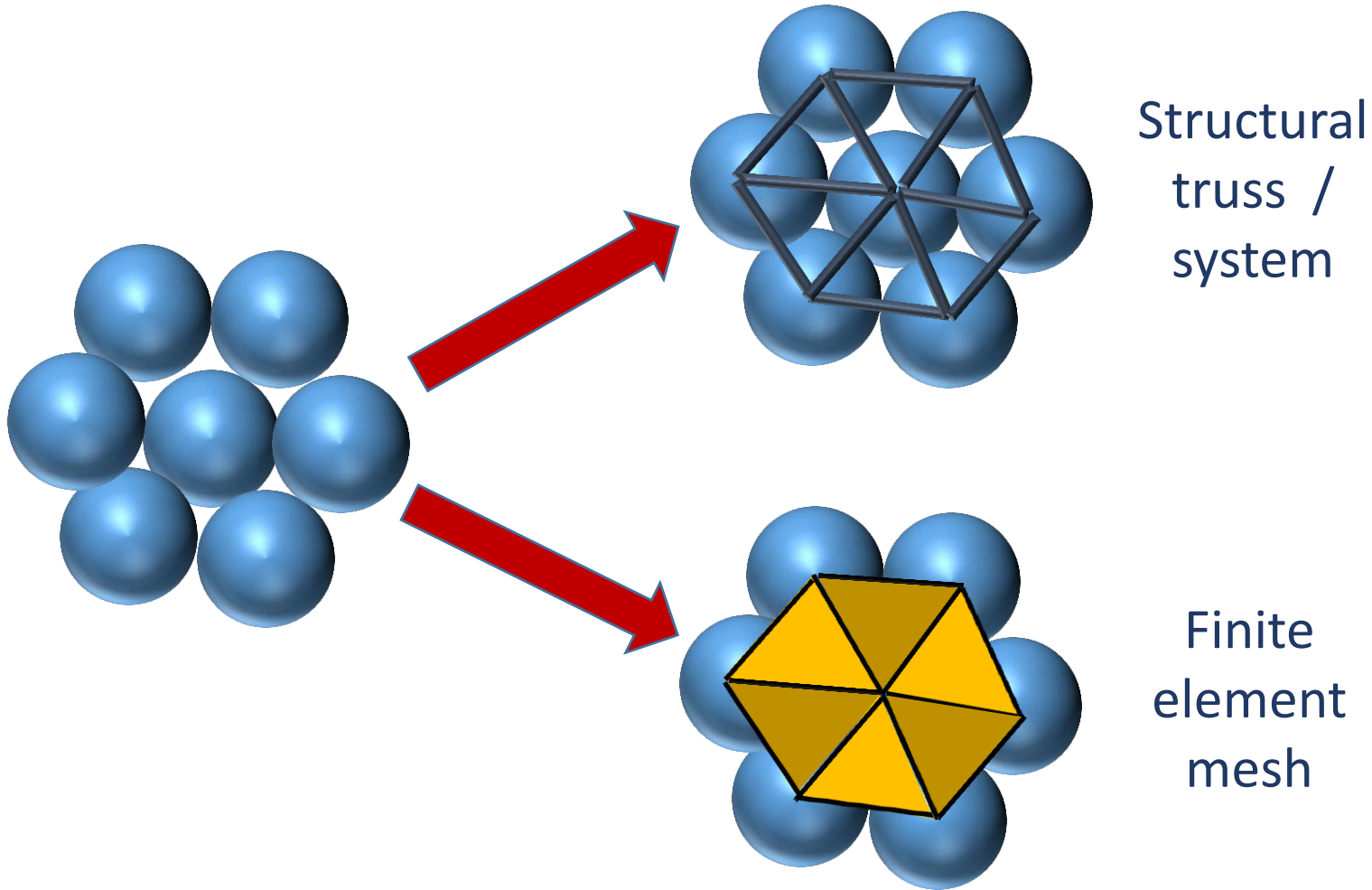


$$\Delta t_{crit} = \frac{2}{\omega_{max}} \leq \min_{ele} \frac{2}{\omega_{ele}}$$

ω_{max} = max frequency of linearized system

ω_{ele} = frequency of element ele

Calculating critical time increment: eigenvalue approach



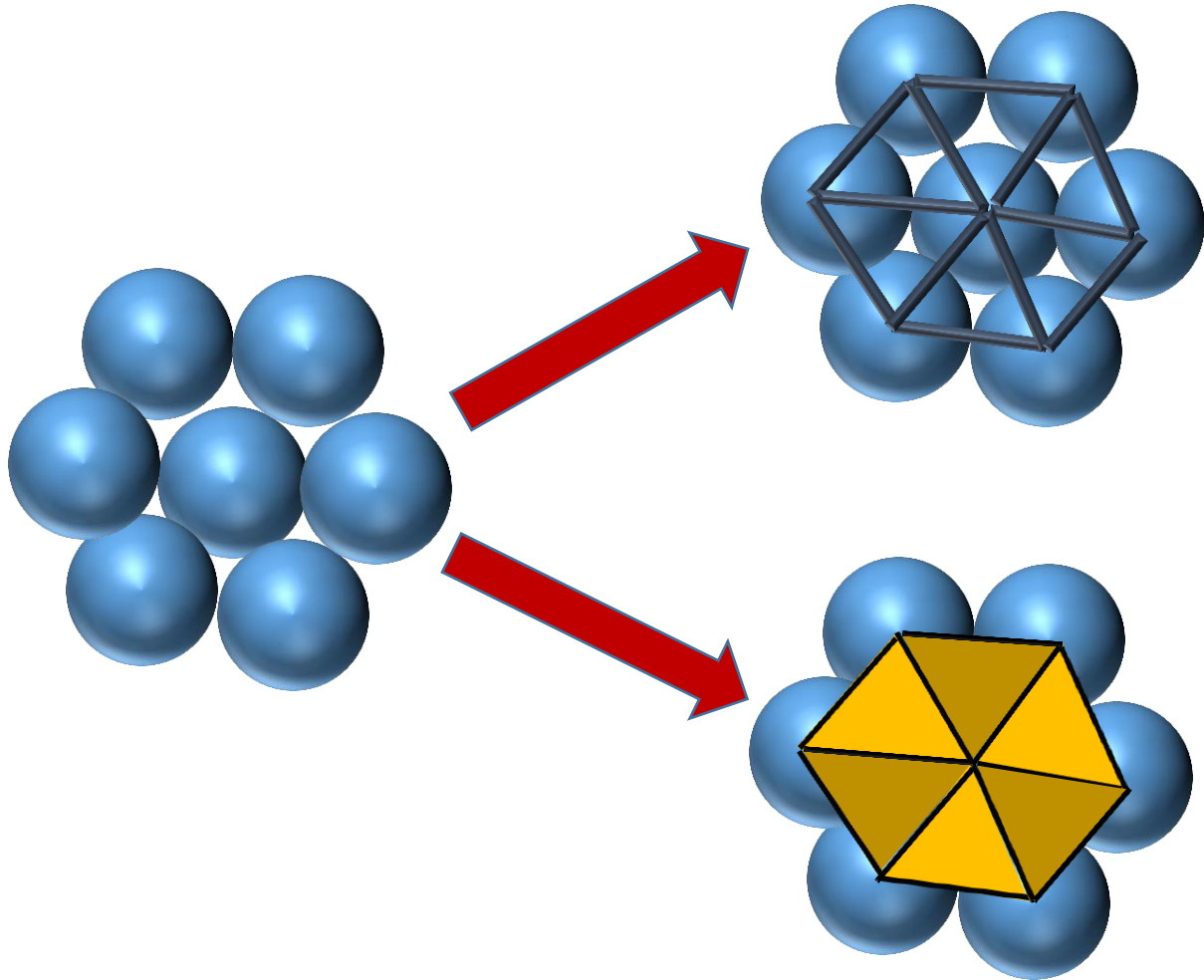
Structural
truss /
system

Finite
element
mesh

$$\Delta t_{crit} = \frac{2}{\omega_{max}} \leq \min_{ele} \frac{2}{\omega_{ele}}$$

- $\omega_{max} = \sqrt{\lambda_{max}}$
- λ_{max} = max eigenvalue of $\mathbf{M}^{-1}\mathbf{K}$
- \mathbf{M} global mass matrix
- \mathbf{K} global stiffness matrix

Calculating critical time increment: eigenvalue approach



Structural
truss /
system

$$\Delta t_{crit} = \frac{2}{\omega_{max}} \leq \min_{ele} \frac{2}{\omega_{ele}}$$

- $\omega_{max} = \sqrt{\lambda_{max}}$

- $\lambda_{max} = \text{max eigenvalue of } \mathbf{M}^{-1}\mathbf{K}$

Finite
element
mesh

- $\lambda_{max} \leq \lambda_{max}^{ele}$

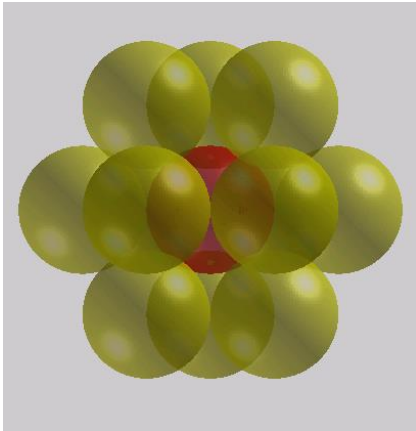
- λ_{max}^{ele} max eigenvalue of $\mathbf{M}^{ele-1}\mathbf{K}^{ele}$

Calculating critical time increment: eigenvalue approach

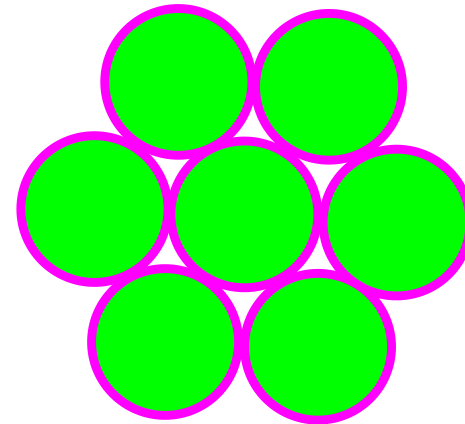
O'Sullivan and Bray (2004): Analytical study

Regular (lattice packings) – linear spring, stiffness k

2D and 3D



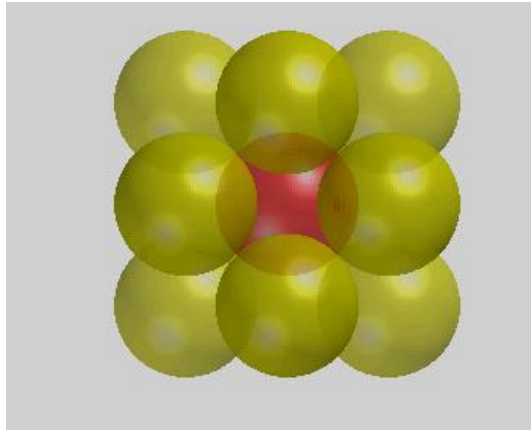
$$\Delta t_{crit} = 0.22 \sqrt{m/k}$$



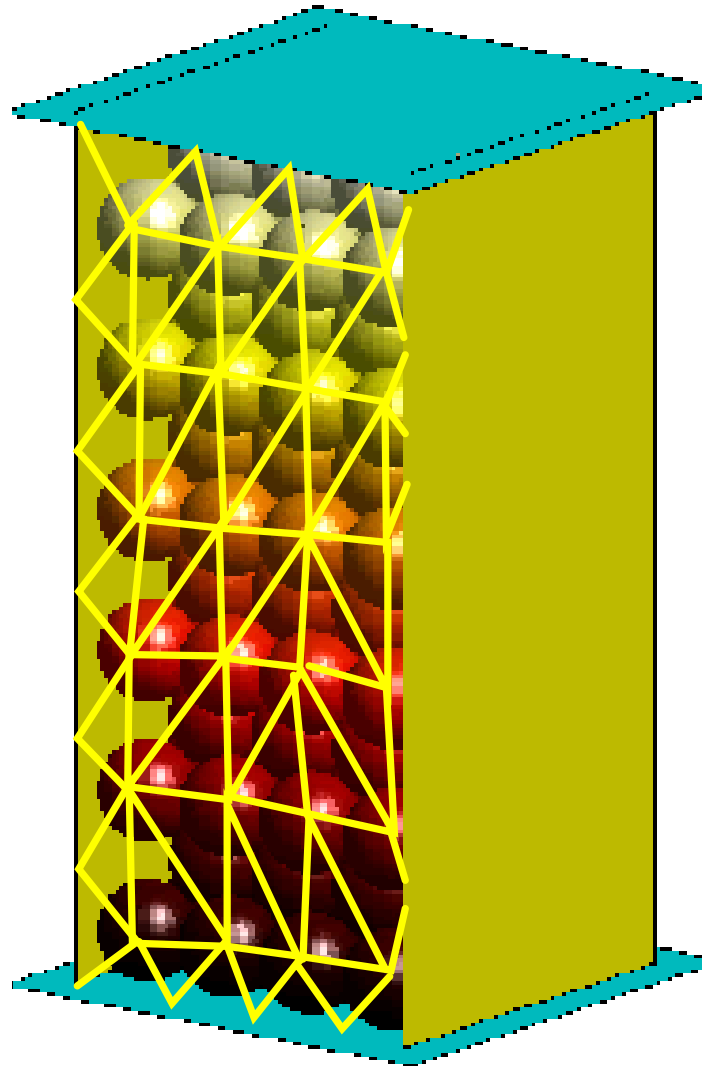
$$\Delta t_{crit} = 0.41 \sqrt{m/k}$$

Calculating critical time increment: eigenvalue approach

O'Sullivan and Bray (2004): Analytical study



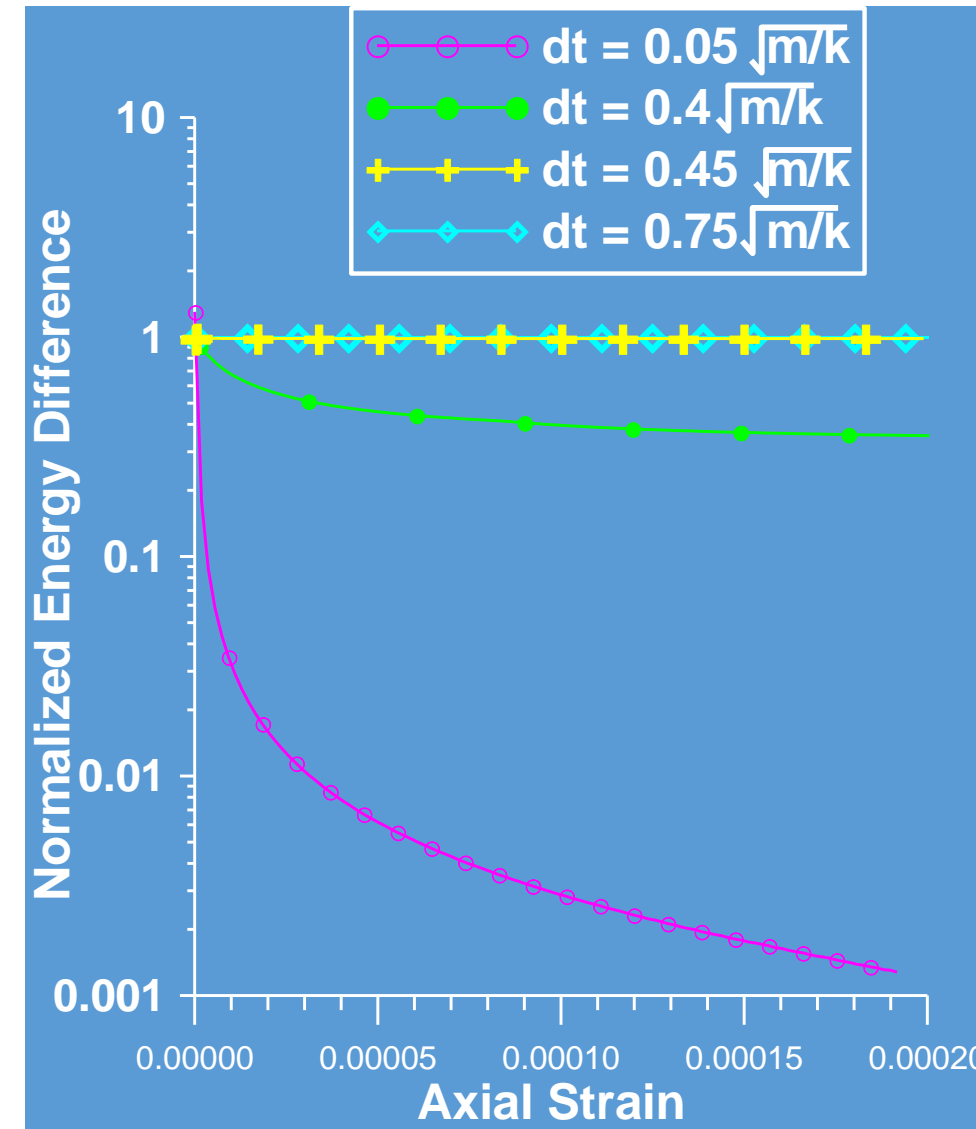
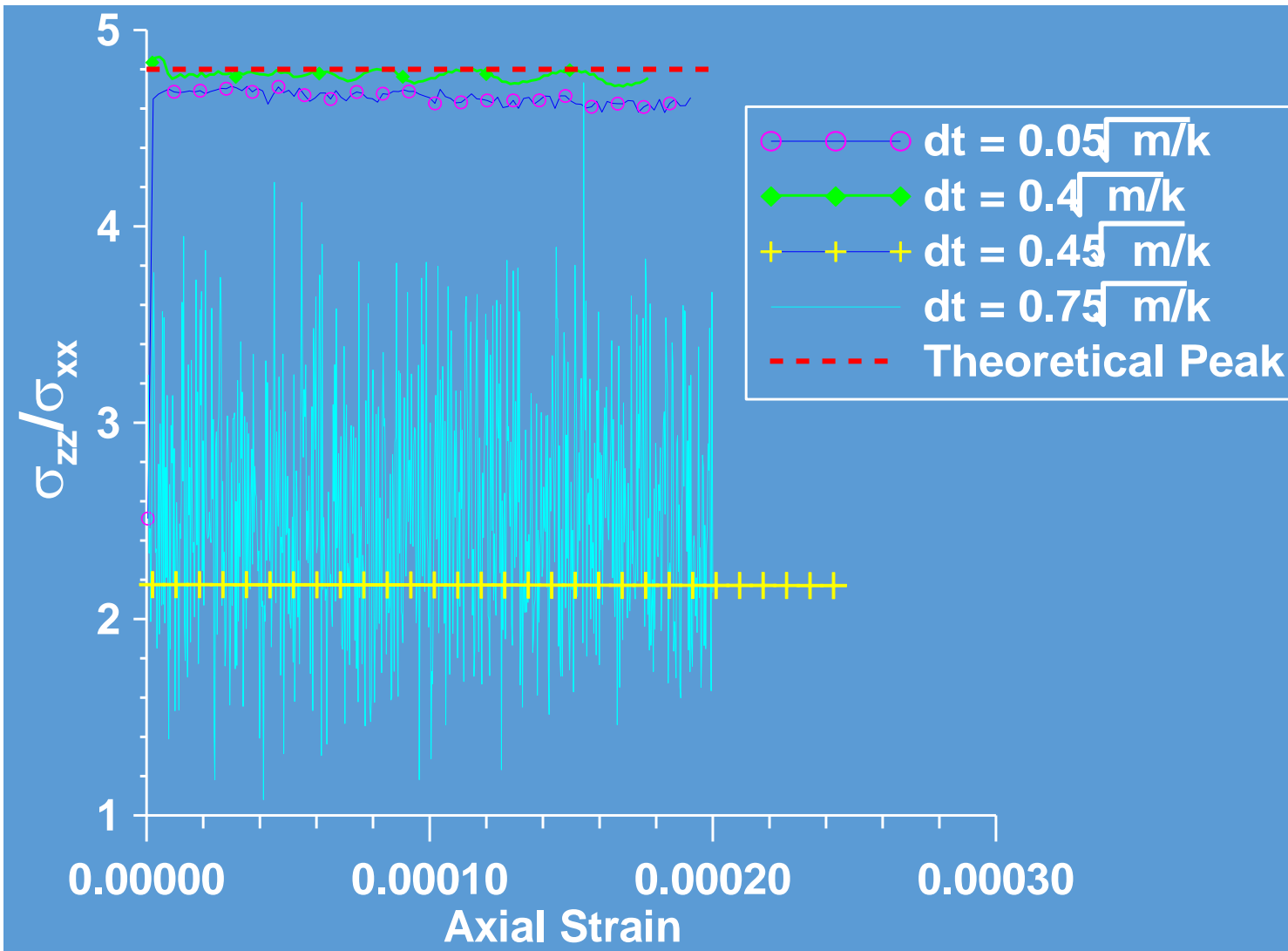
Face-Centered-Cubic



3-D Plane Strain
Compression of 150 spheres

Calculating critical time increment: eigenvalue approach

O'Sullivan and Bray (2004): Analytical study



Calculating critical time increment: eigenvalue approach

O'Sullivan and Bray (2004): Analytical study

- Highlighted dependency of critical time increment on no of contacts per particle
- Consideration of rotation reduced critical time increment
- Study limited to lattice packings of uniform spheres
- Contact model linear springs

Conclusions

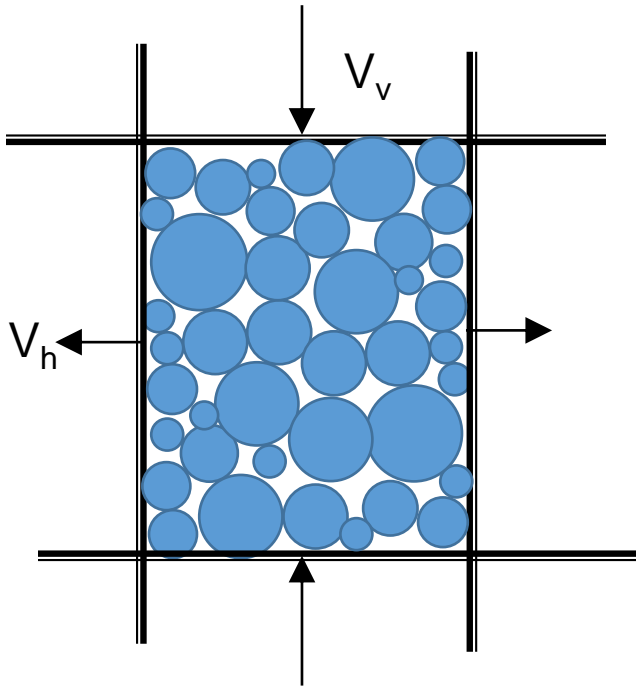
- Conditionally stable method
- Essential to be within stability limit, but should not be over conservative

From eigenvalue approach:

- Critical time step depends on contact stiffness (which depends on confining pressure for Hertz-Mindlin model), particle mass and packing density (coordination number)
- Method based on Rayleigh wave speed not always conservative

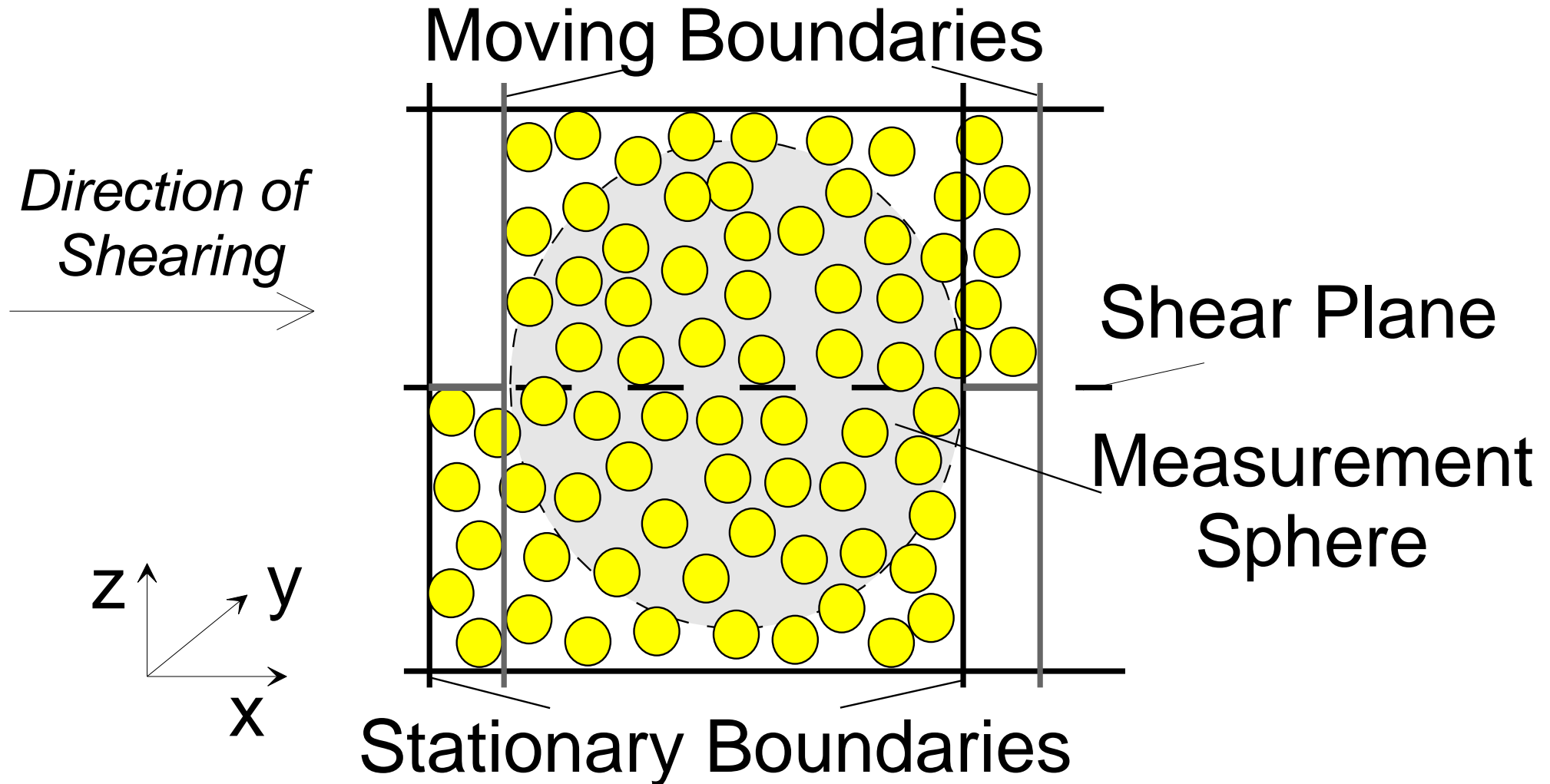
Boundary conditions

Rigid wall boundaries

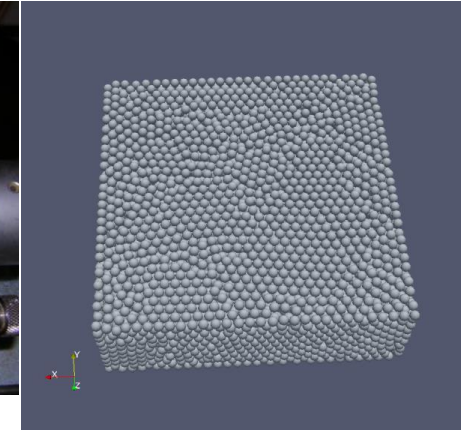
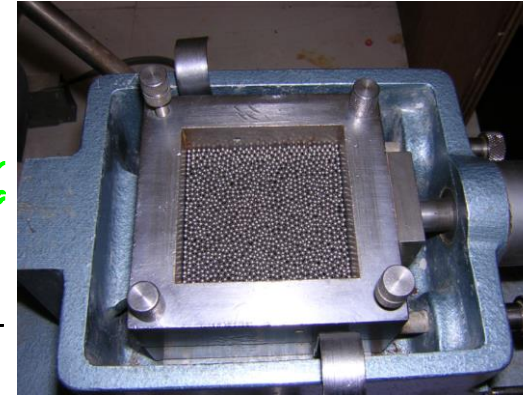
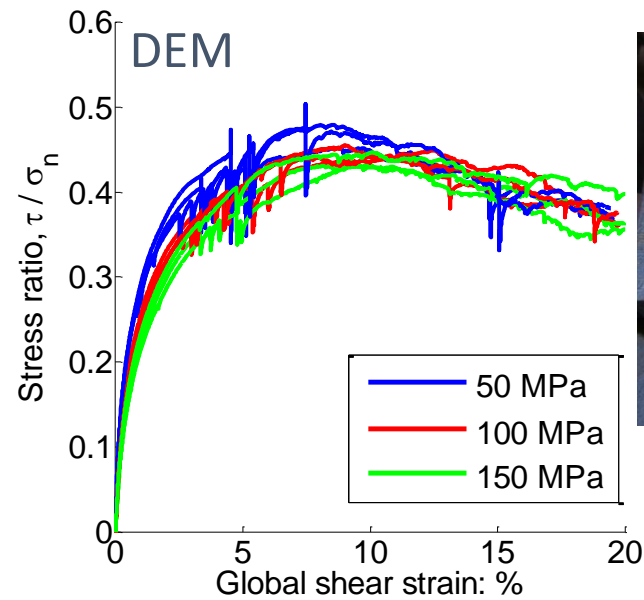
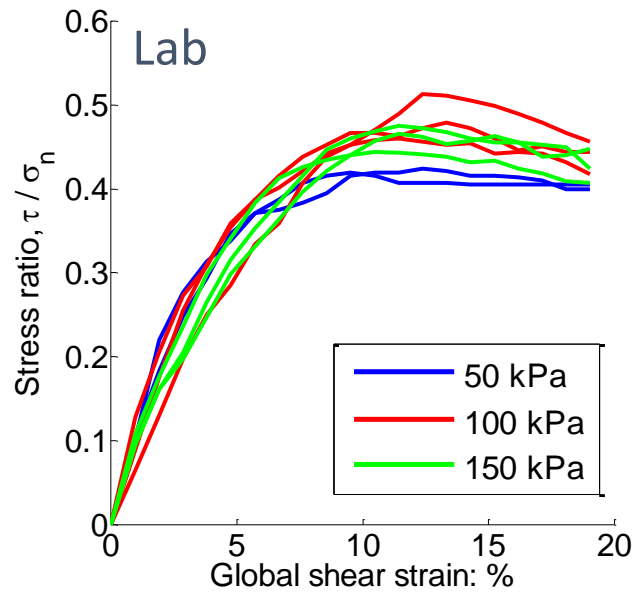


- Planar / cylindrical / mesh of triangles
- Can move to control stress - servo control
- Can control stress on boundary or in specified internal region

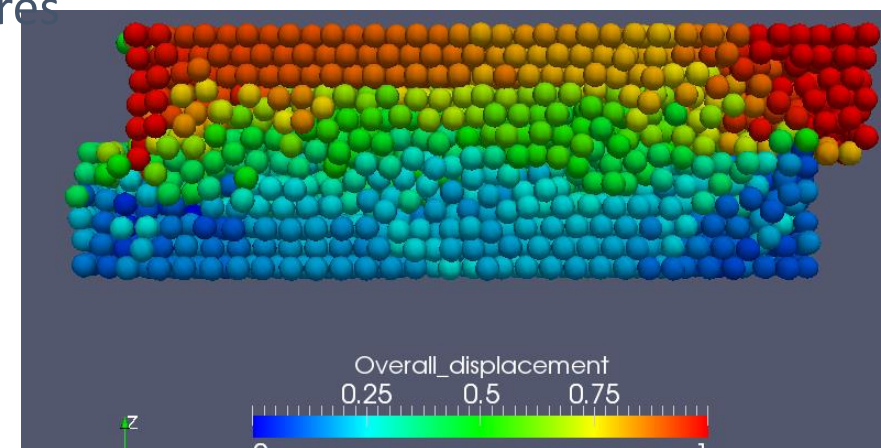
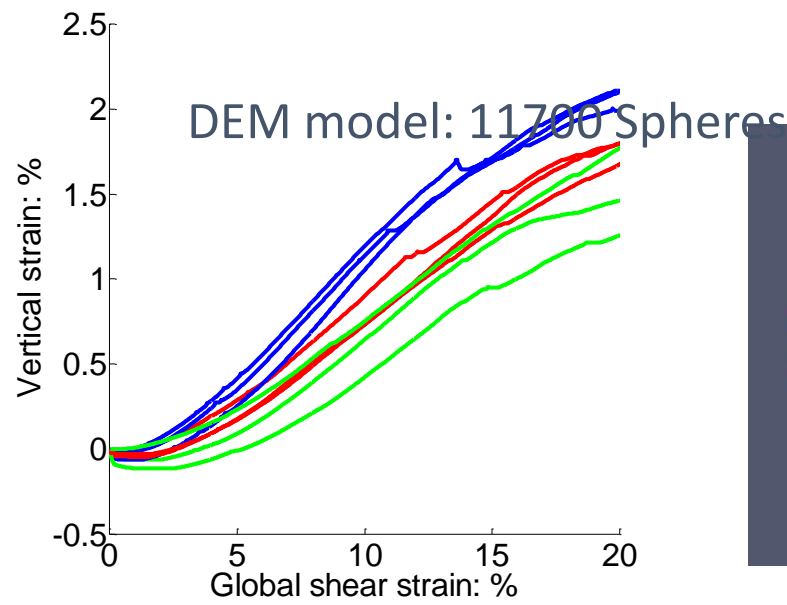
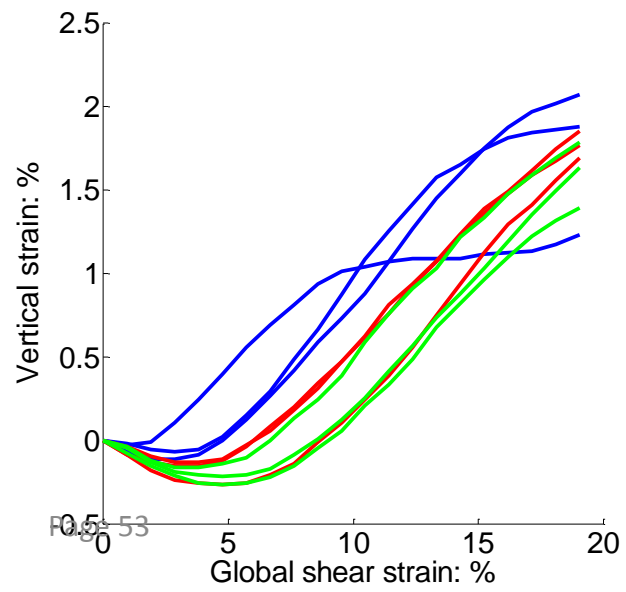
Rigid wall boundaries – simulation of direct shear test



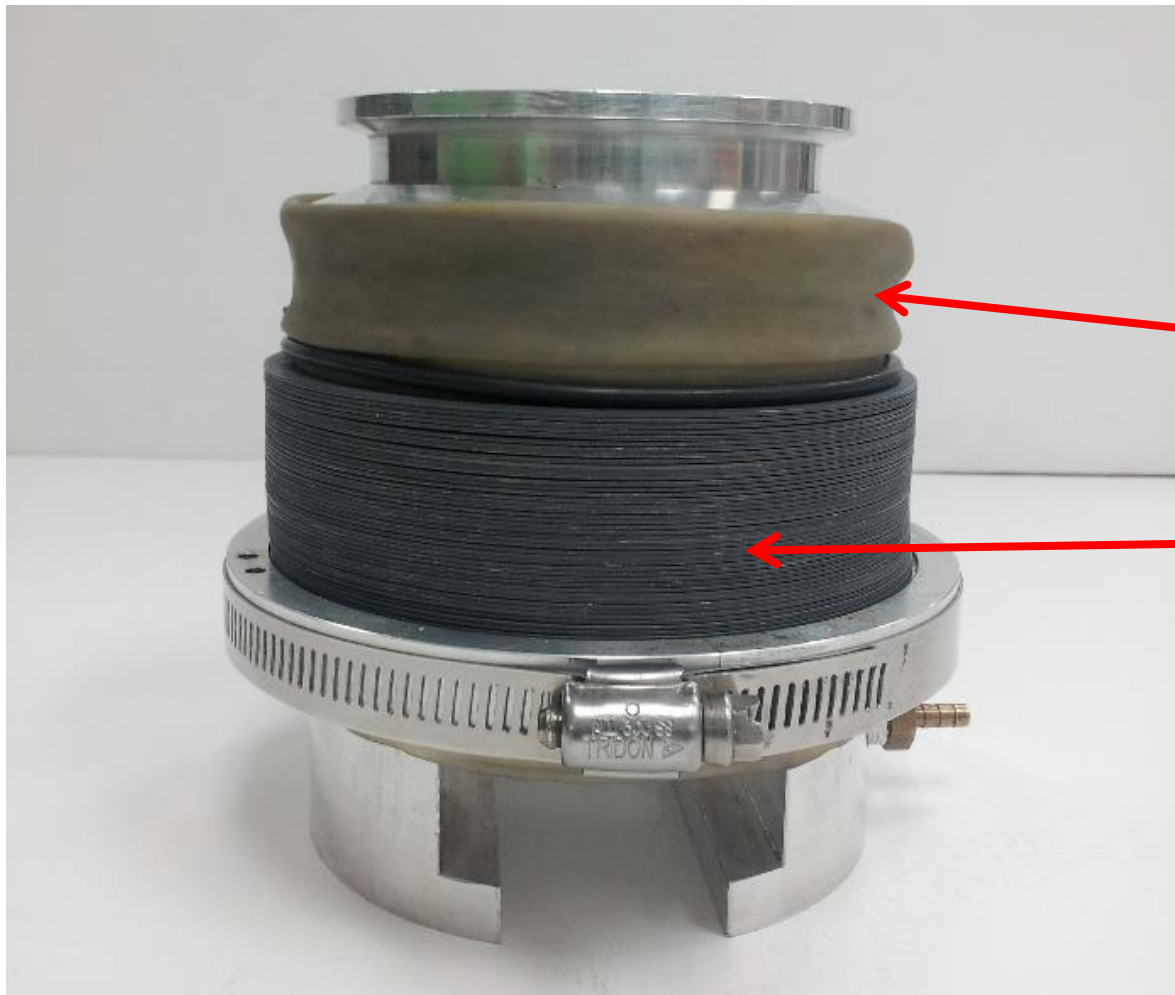
Rigid wall boundaries – simulation of direct shear test



Grade 25 steel precision balls
Diameter 0.992 mm



Simple shear simulations – Laminar (NGI) Device

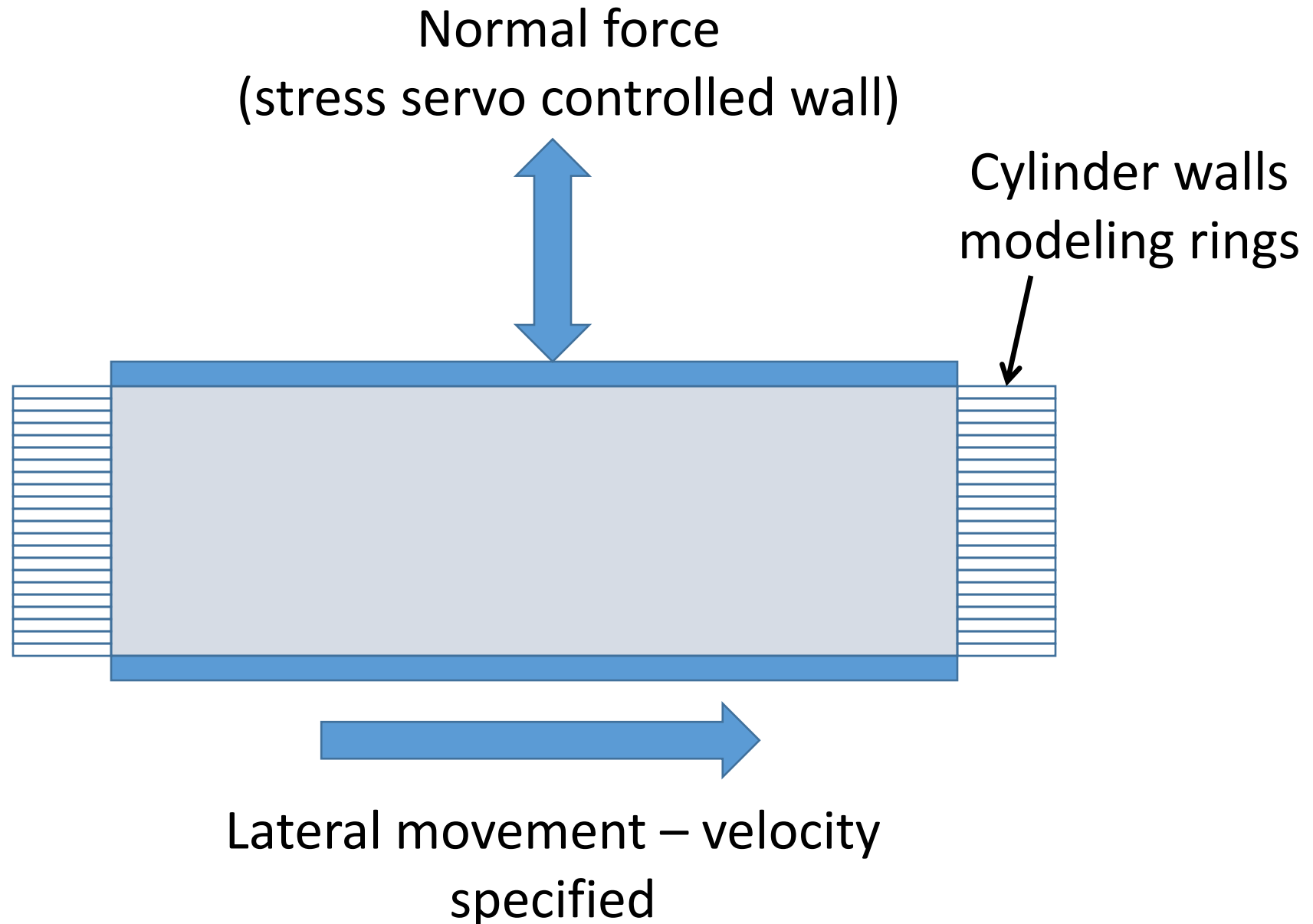


Membrane

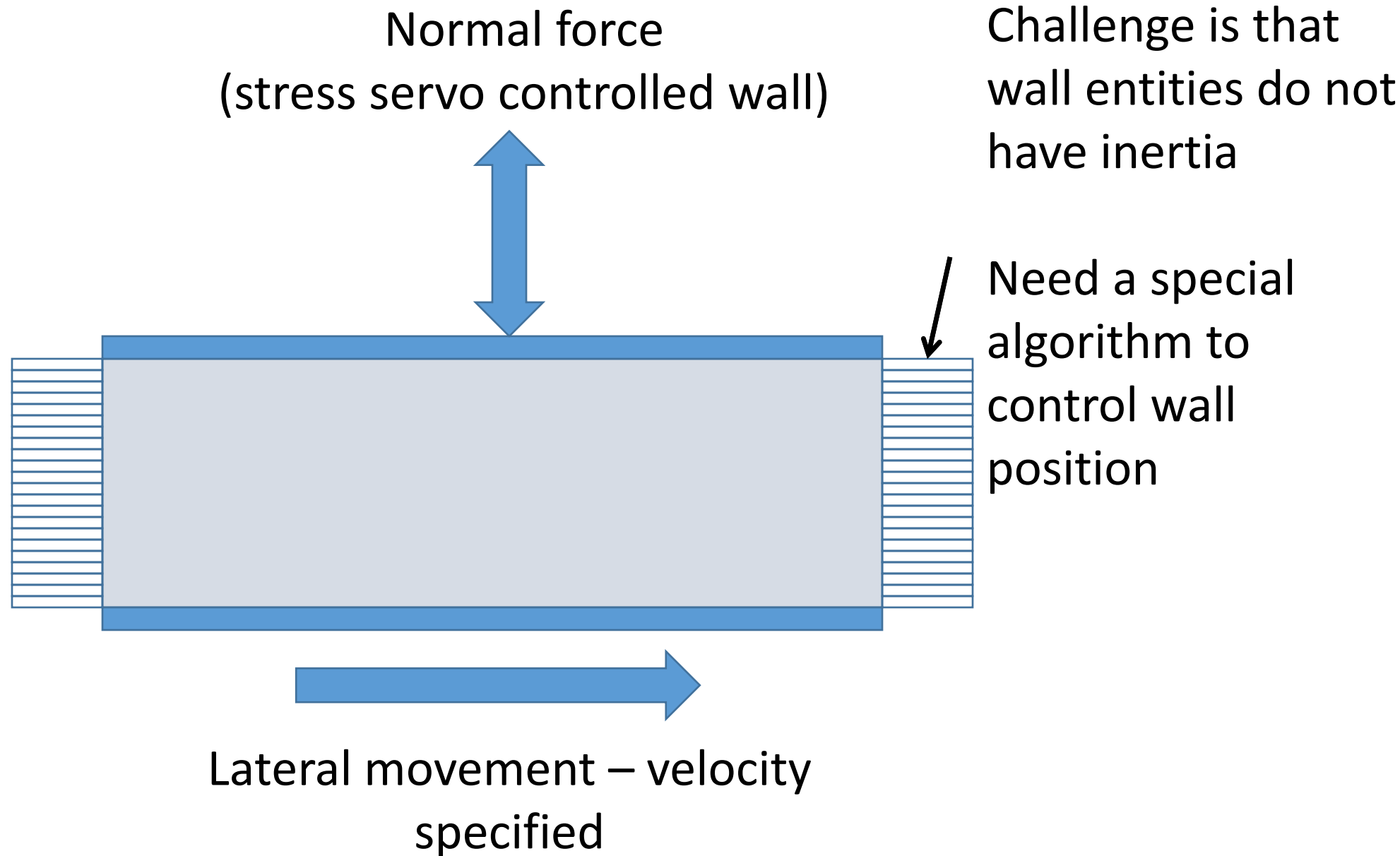
Stacked ring configuration



Simple shear simulations – Laminar (NGI) Device

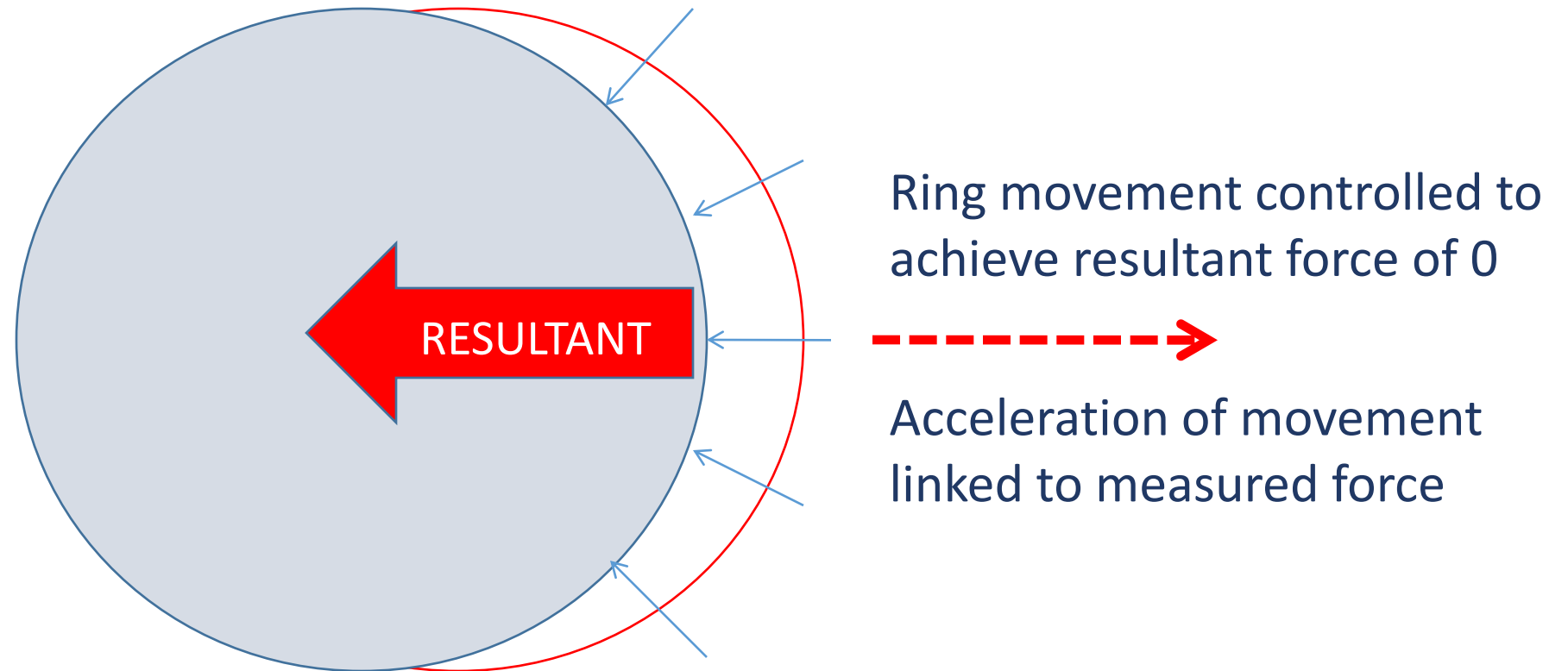


Simple shear simulations – Laminar (NGI) Device



Simple shear simulations – Laminar (NGI) Device

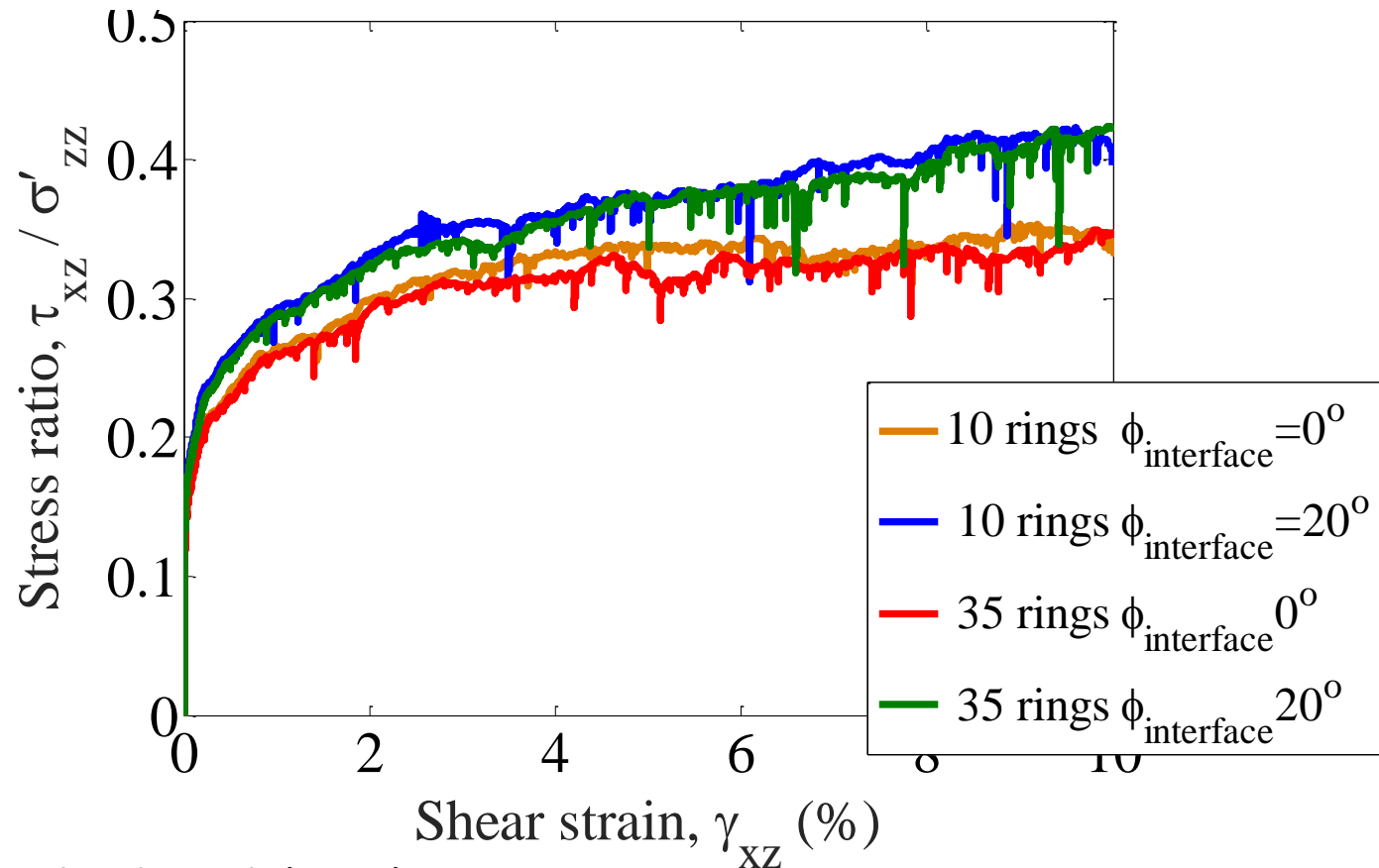
- Used cylindrical wall boundaries
- Controlled wall motion by using forces acting on wall – aim to have resultant force is 0.0



Simple shear simulations – Laminar (NGI) Device



10 confining rings

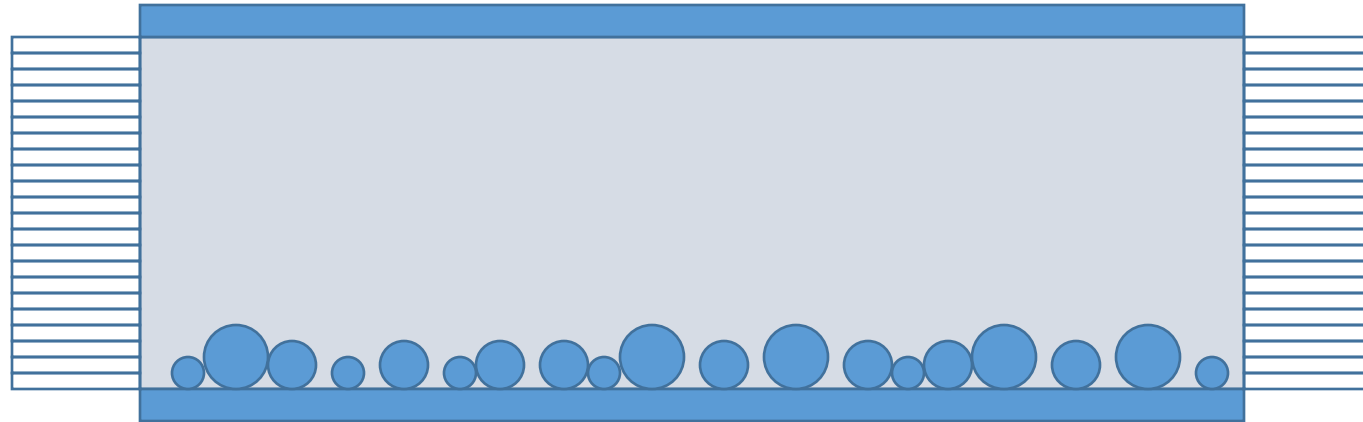


- Load-deformation responses equivalent for 10 and 35 rings
- Considered assumption of smooth interface
- 20o is friction measured between membrane and rings

Simple shear simulations – Laminar (NGI) Device

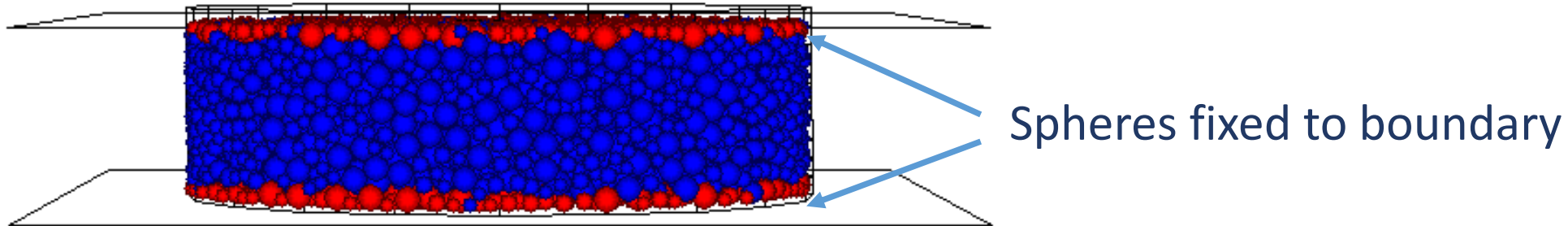
Horizontal boundaries

Option 1 – Flat friction → localization close to boundaries)



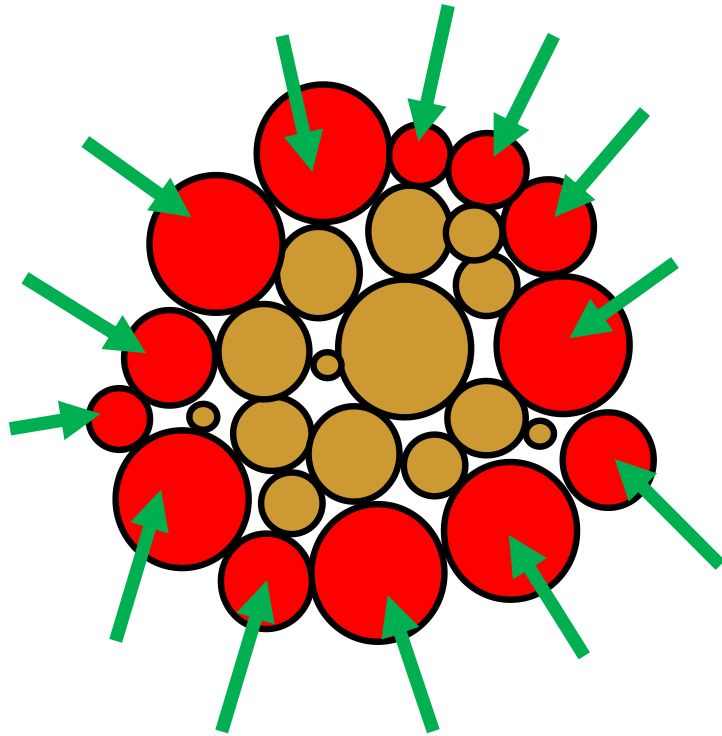
Option 2 – Rough surface achieved by gluing particles to boundary walls

Simple shear simulations – Laminar (NGI) Device

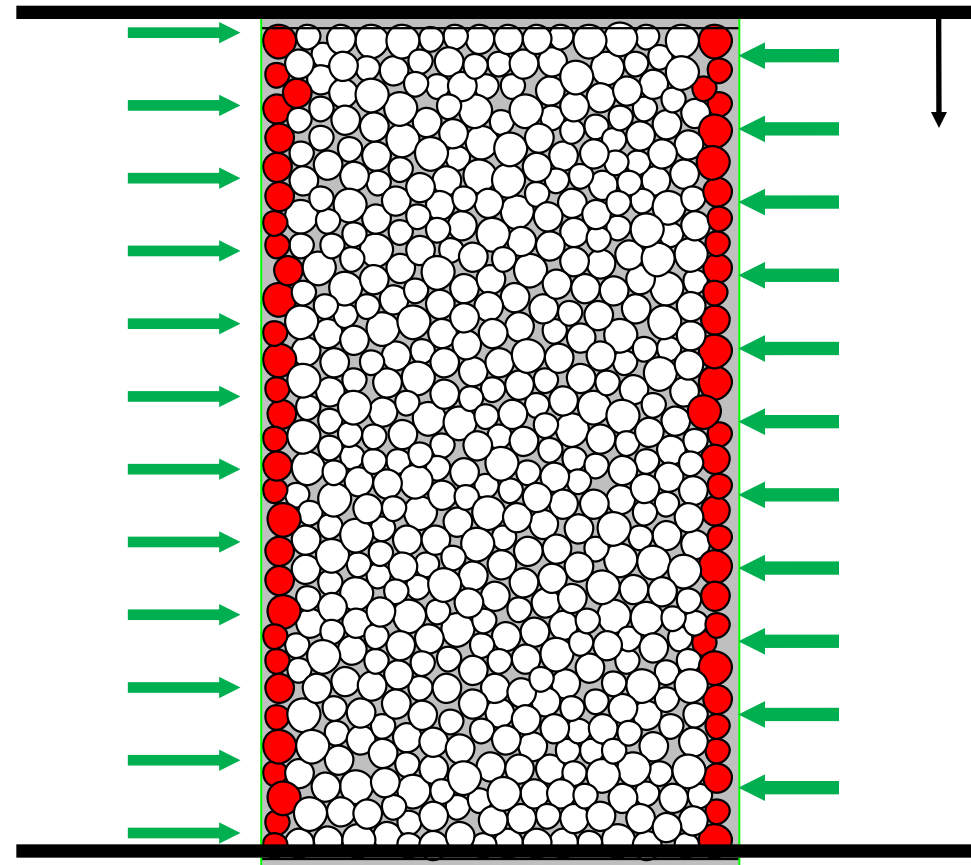


- Vertical stress applied by considering the sum of the out of balance forces on the group of "glued" particles and the force on the wall coming from contact with these particles.
- Contact forces with the wall remain constant once the particles are glued.
- Additional forces are given by out of balance forces on the individual particles.

Stress controlled boundaries / membranes

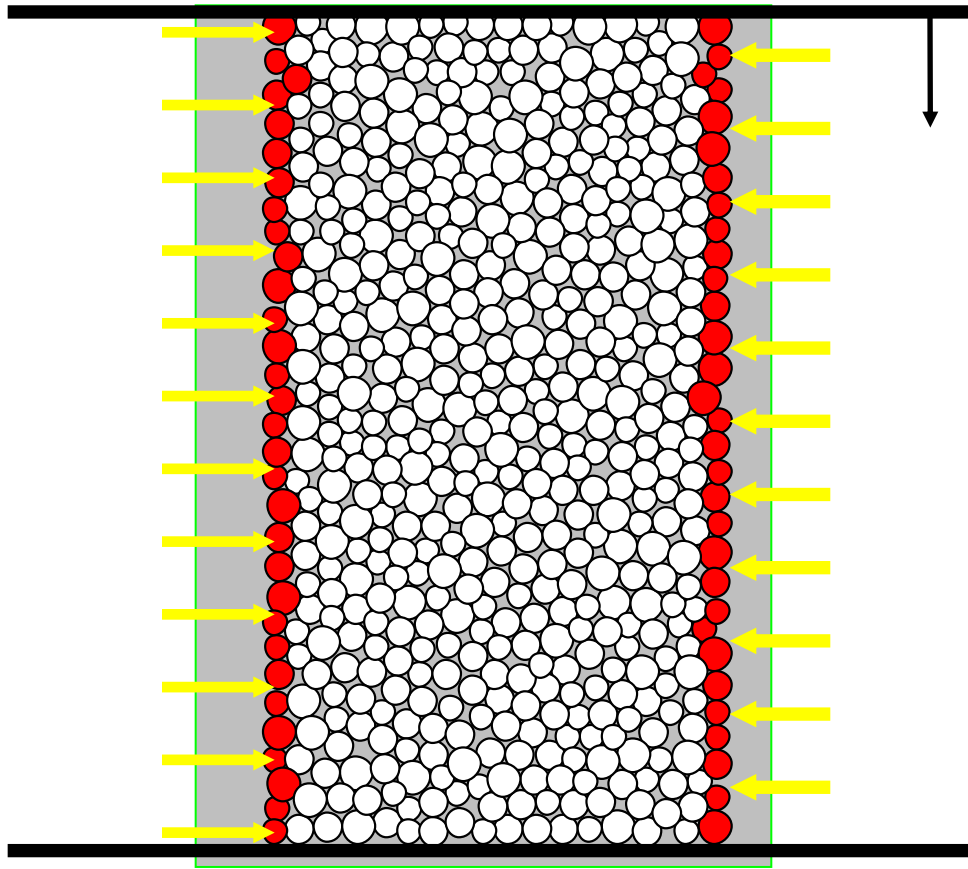


Discrete forces applied to boundary particles



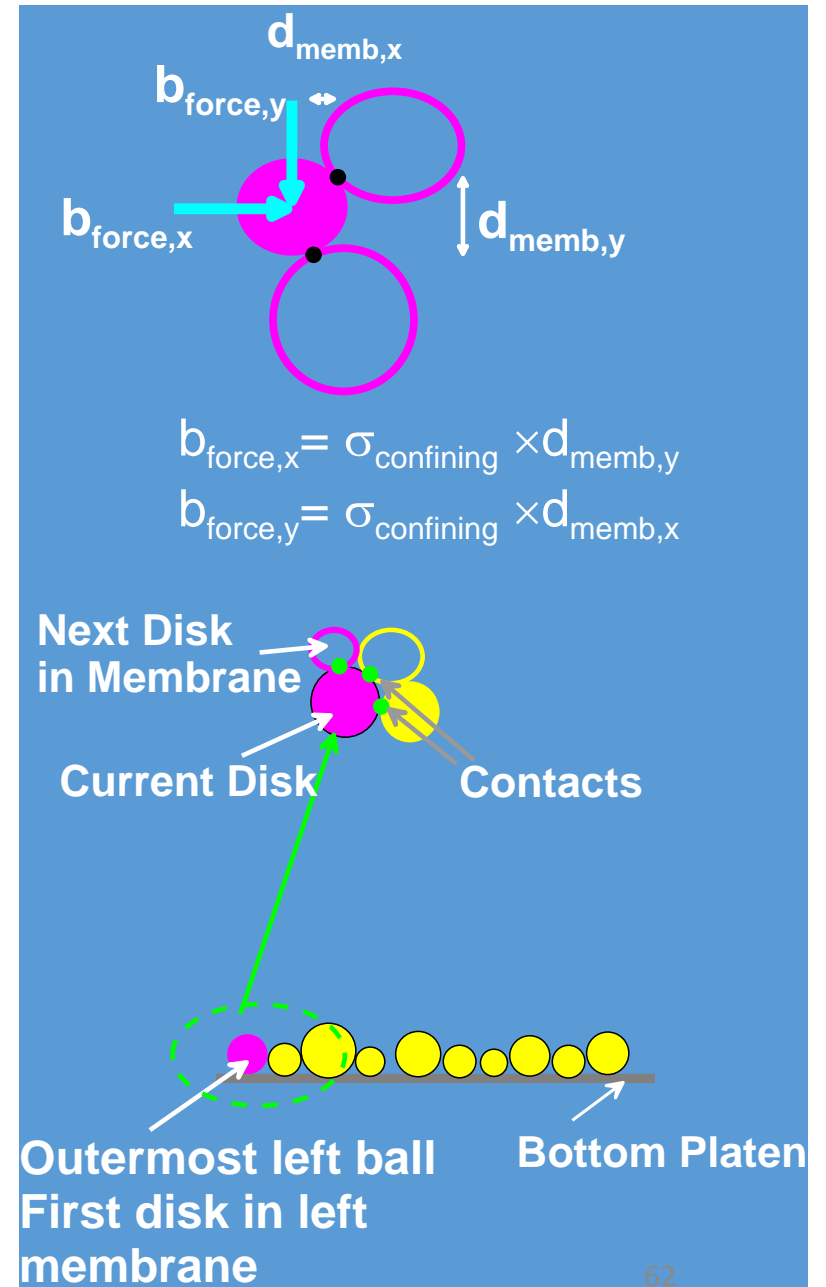
Cheung and O'Sullivan (2014)

2D Membrane

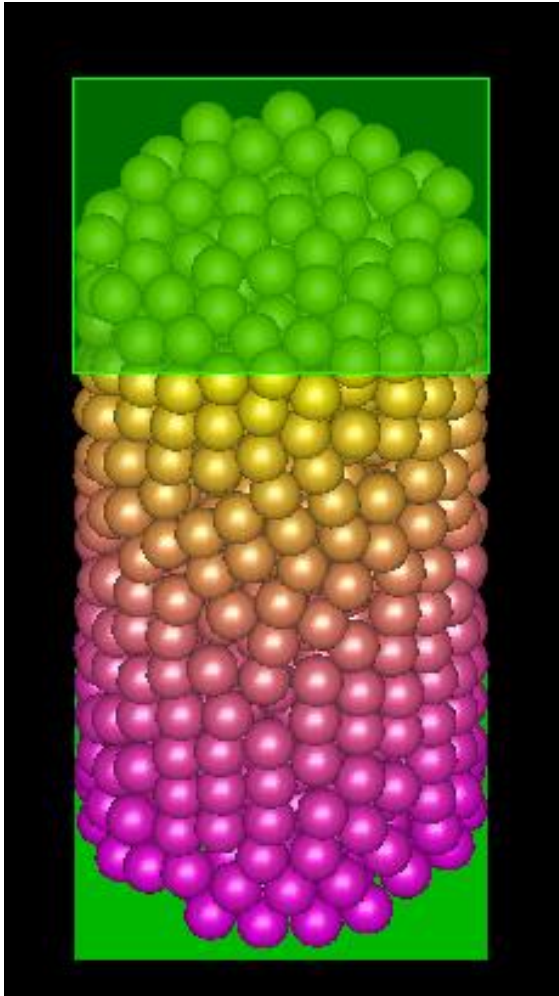


Contact based
identification most
effective

Cheung and
O'Sullivan (2014)

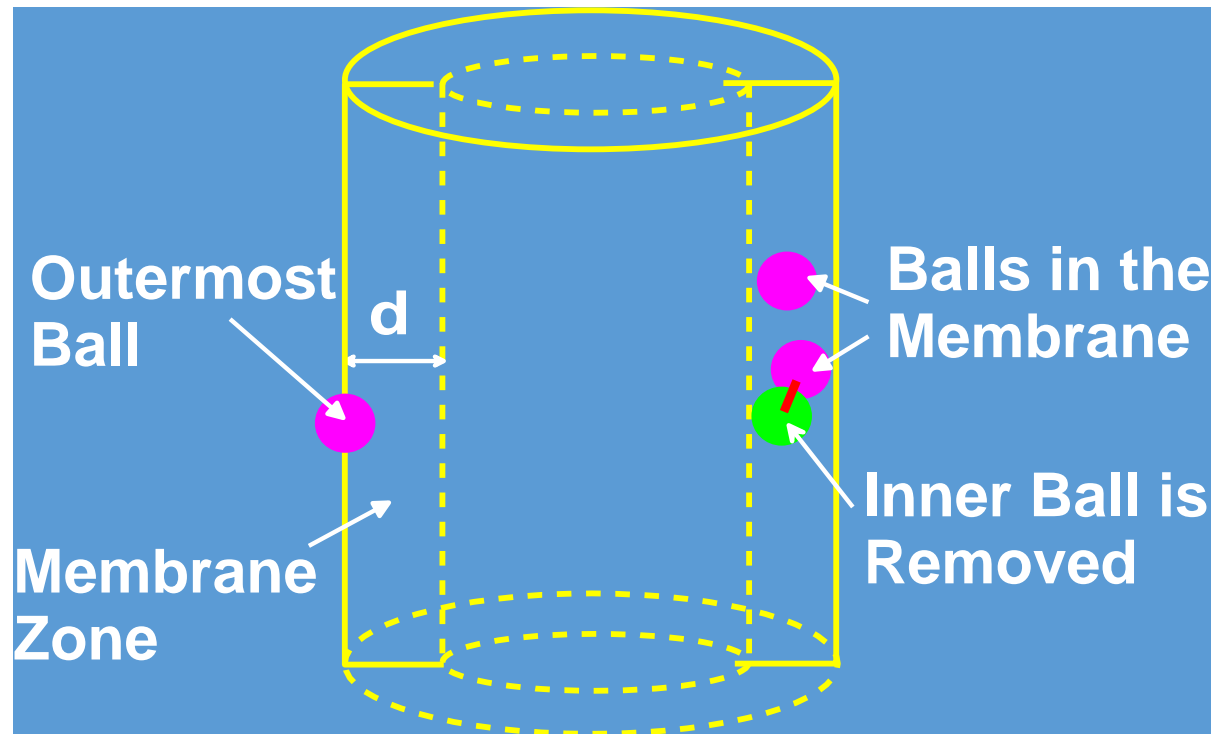


3D Membrane



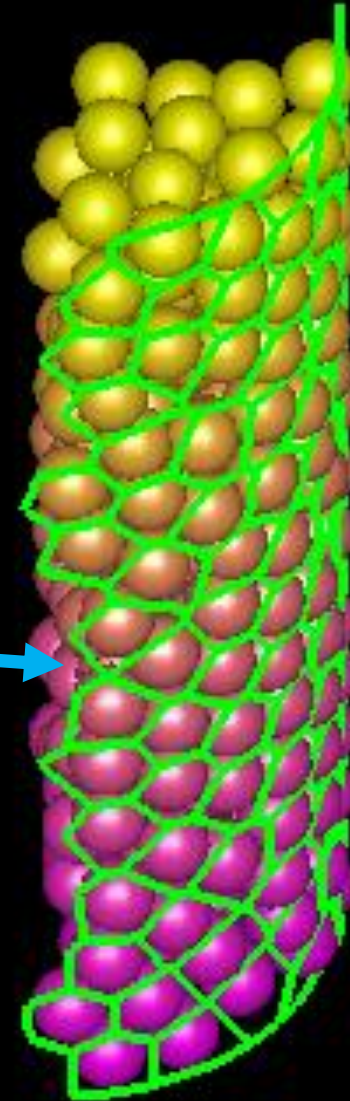
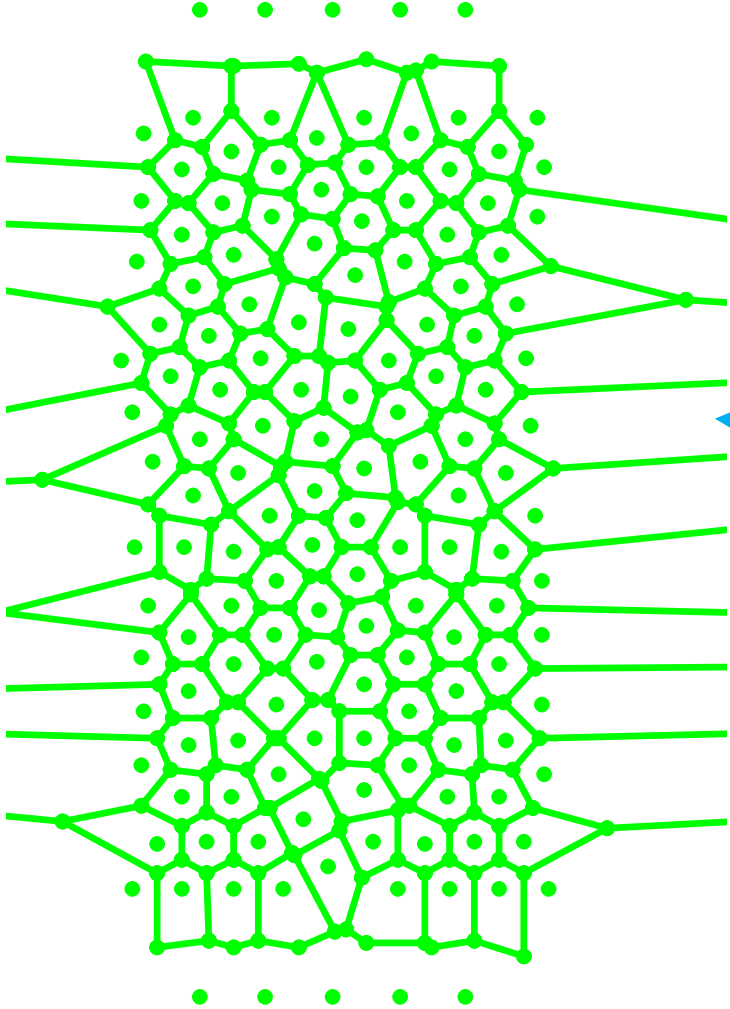
Cheung and
O'Sullivan (2014)

Identification of "membrane" balls



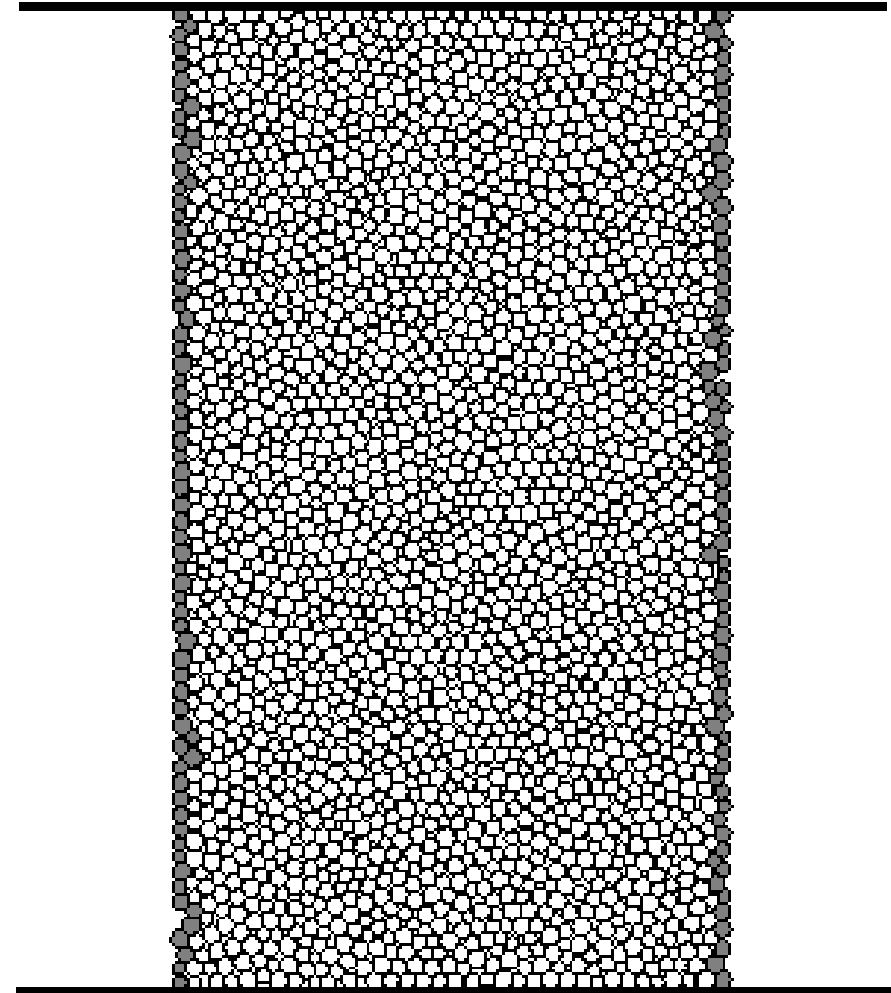
3D Membrane

Force Boundary – Voronoi Diagram

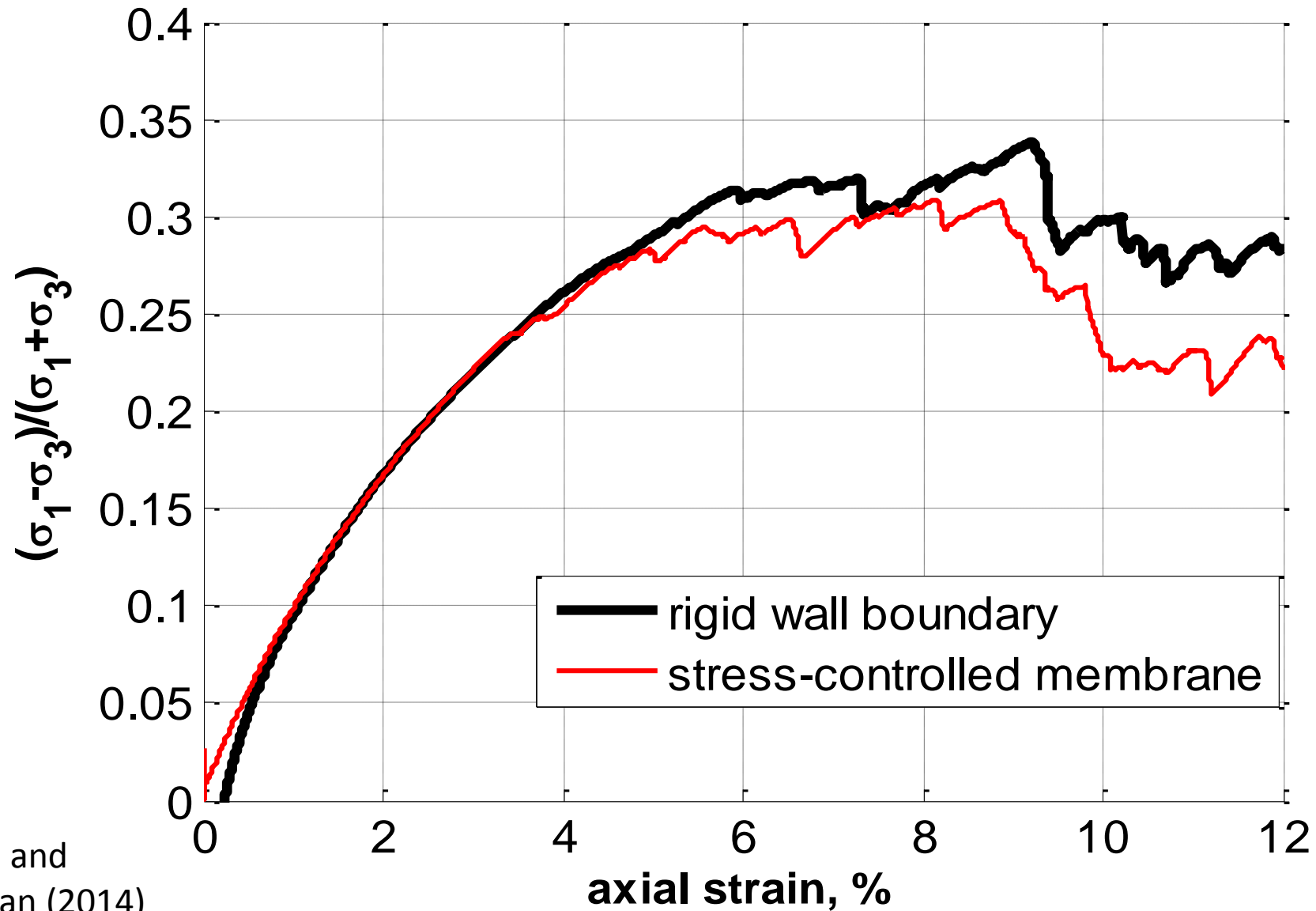


Influence of boundaries on response: 2D

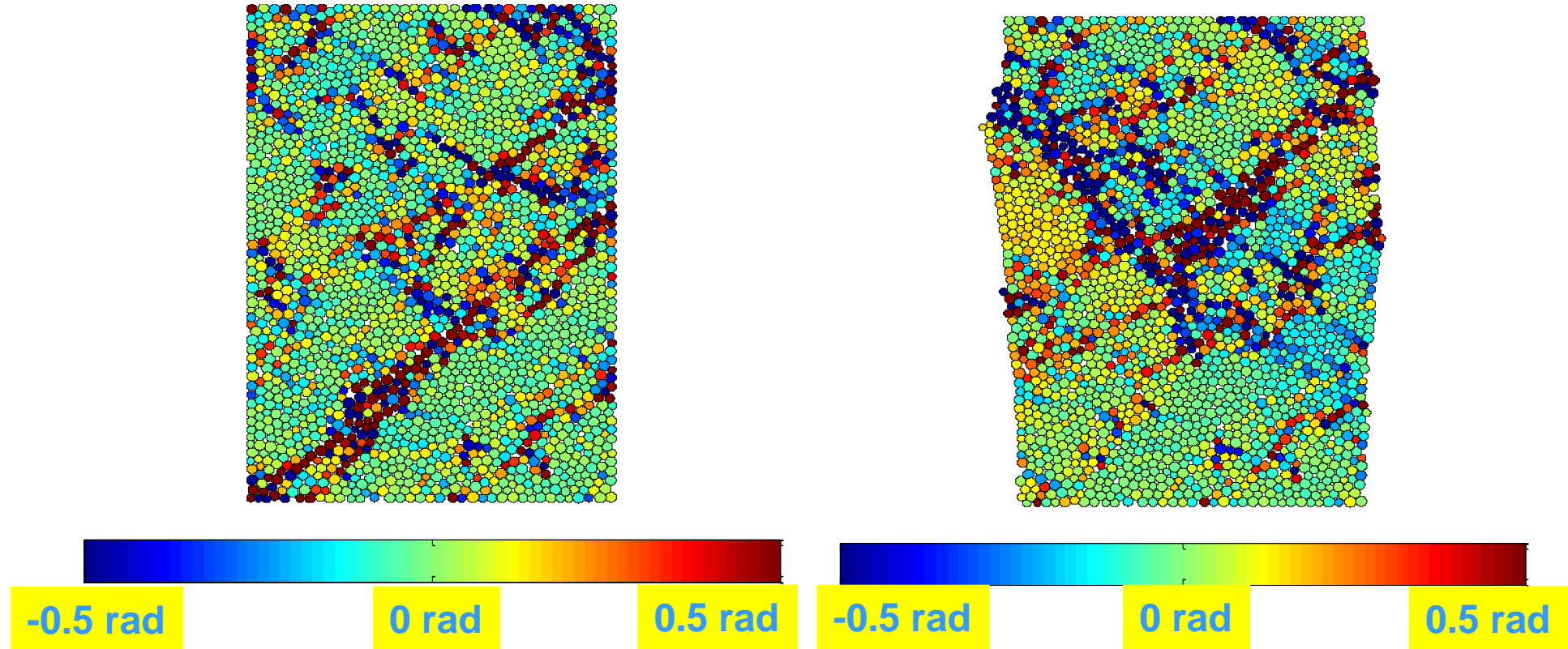
- Specimen size:
40 mm x 80 mm
- 2377 disks
- Radii uniformly distributed between
0.48 mm and 0.72 mm
- $m = 0.5$
- No bonding



Influence of boundaries on response: 2D

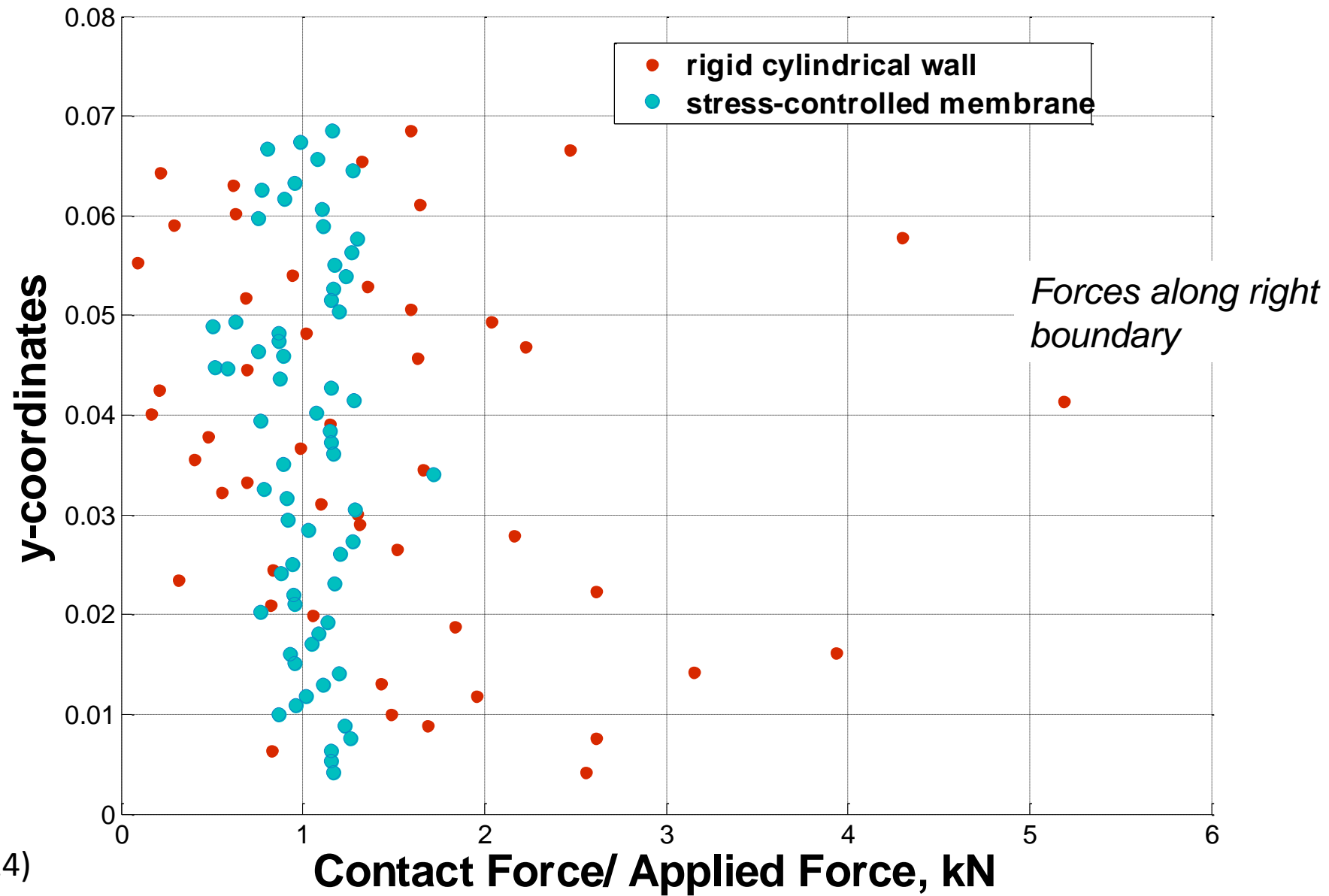


Influence of boundaries on response: 2D



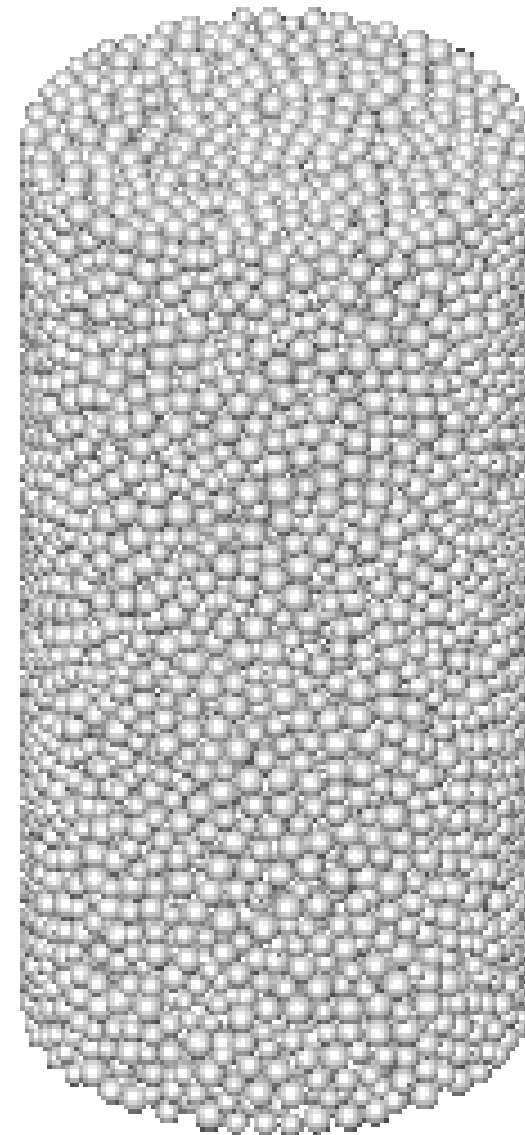
Cheung and
O'Sullivan (2014)

Influence of boundaries on response: 2D

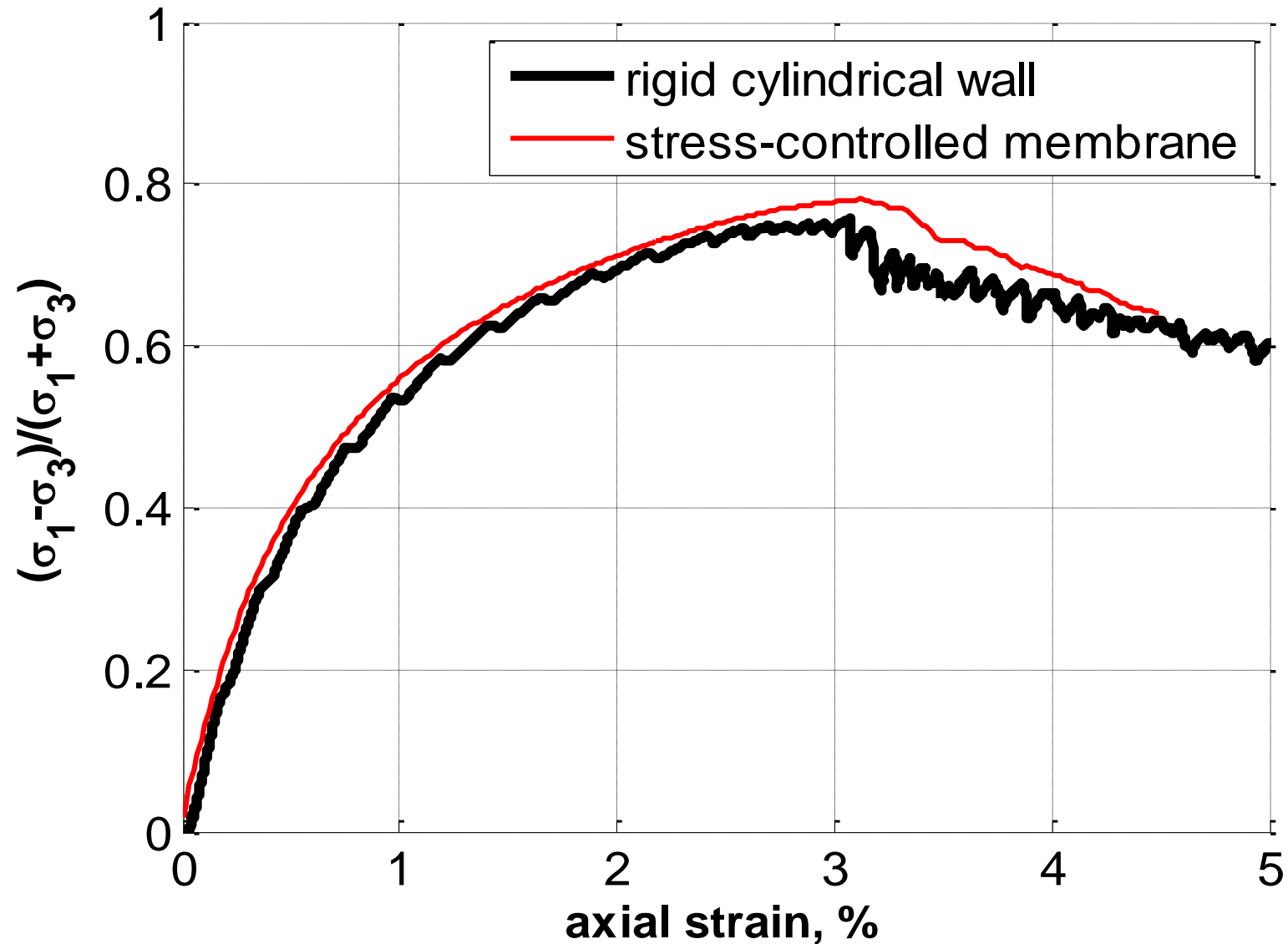


Influence of boundaries on response: 3D

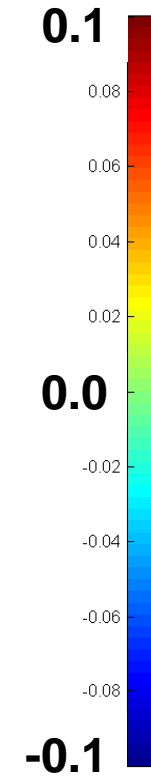
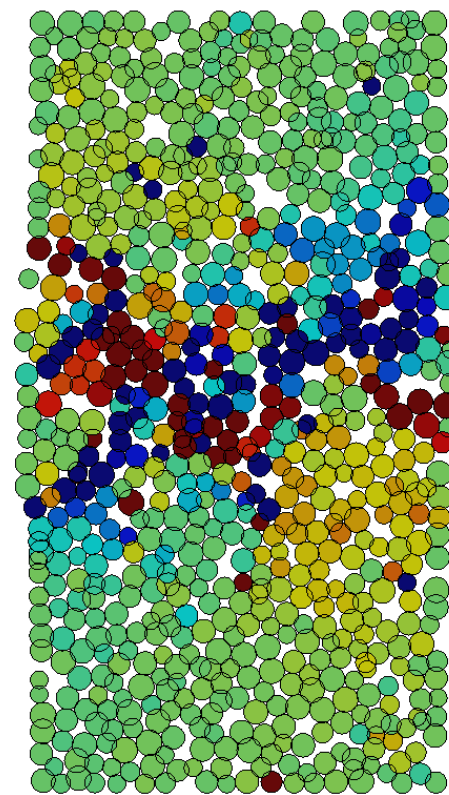
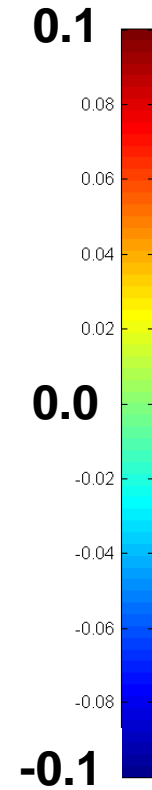
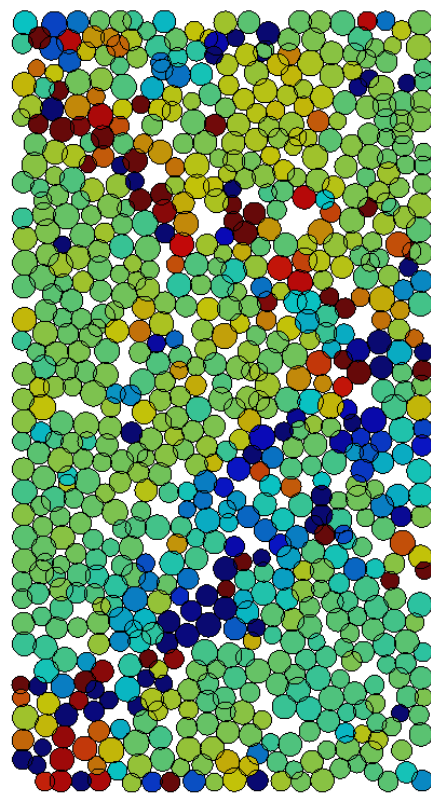
- Specimen Size:
40 mm (diameter) x 80 mm
- 12,622 spheres
- Sphere radii uniformly distributed between 0.88mm and 1.32mm
- A parallel bond used to model cementation



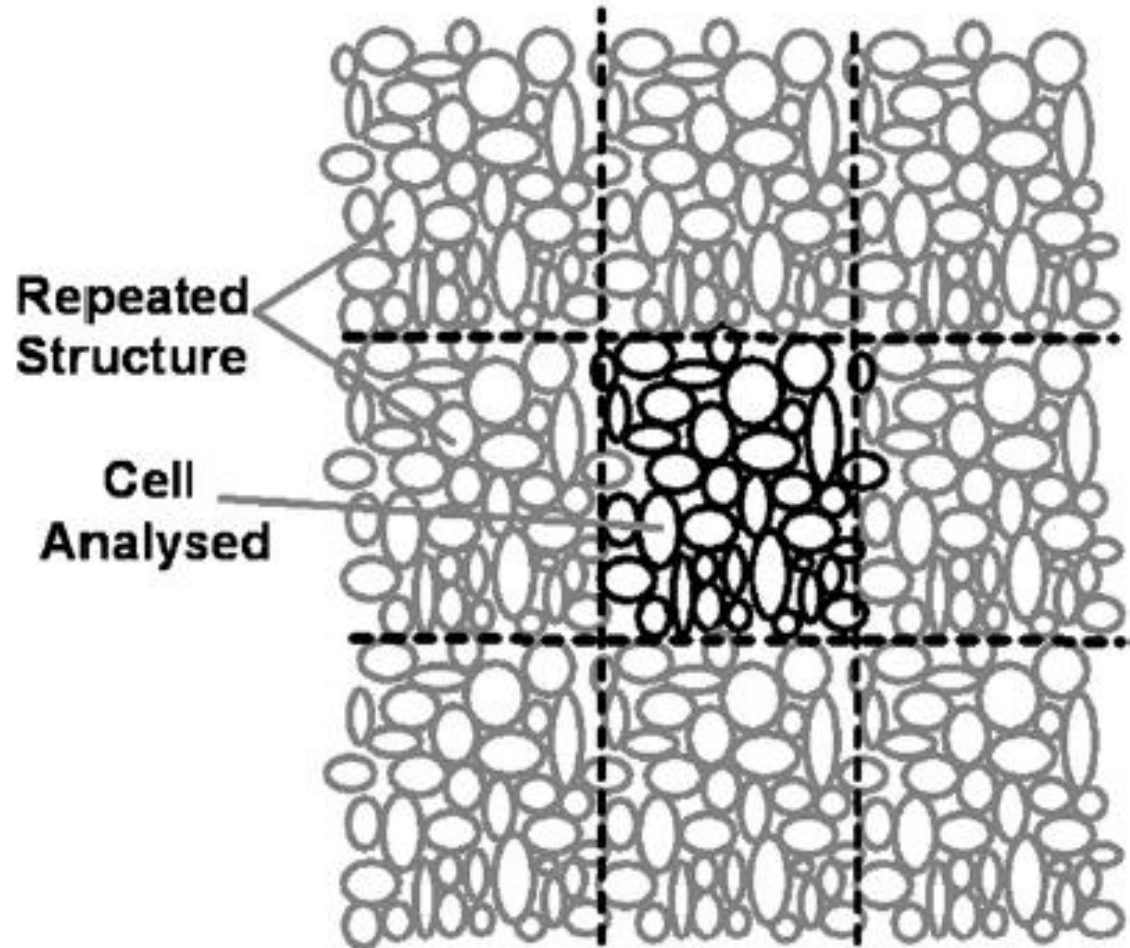
Influence of boundaries on response: 2D



Influence of boundaries on response: 3D

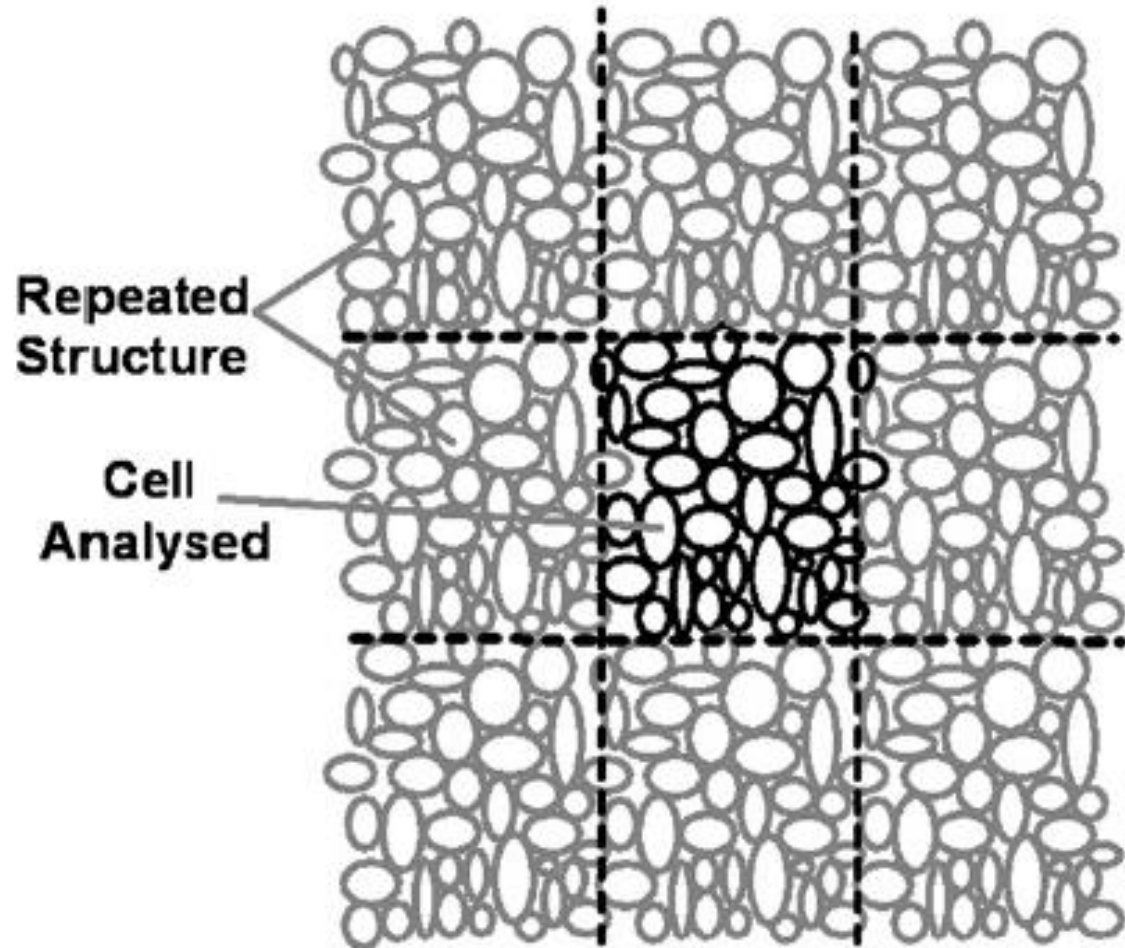


Periodic boundaries



- Contacts are detected across periodic boundaries
- Less sensitive to sample size
- Homogeneous

Periodic boundaries



- Deformation achieved by moving both boundaries and particles according to global strain rate
- Need to account for movement of particles in shear component of contact model
- Good explanation given in Thornton (2000)

$$\Delta x_i = \dot{\epsilon}_{ij} x_j \Delta t$$

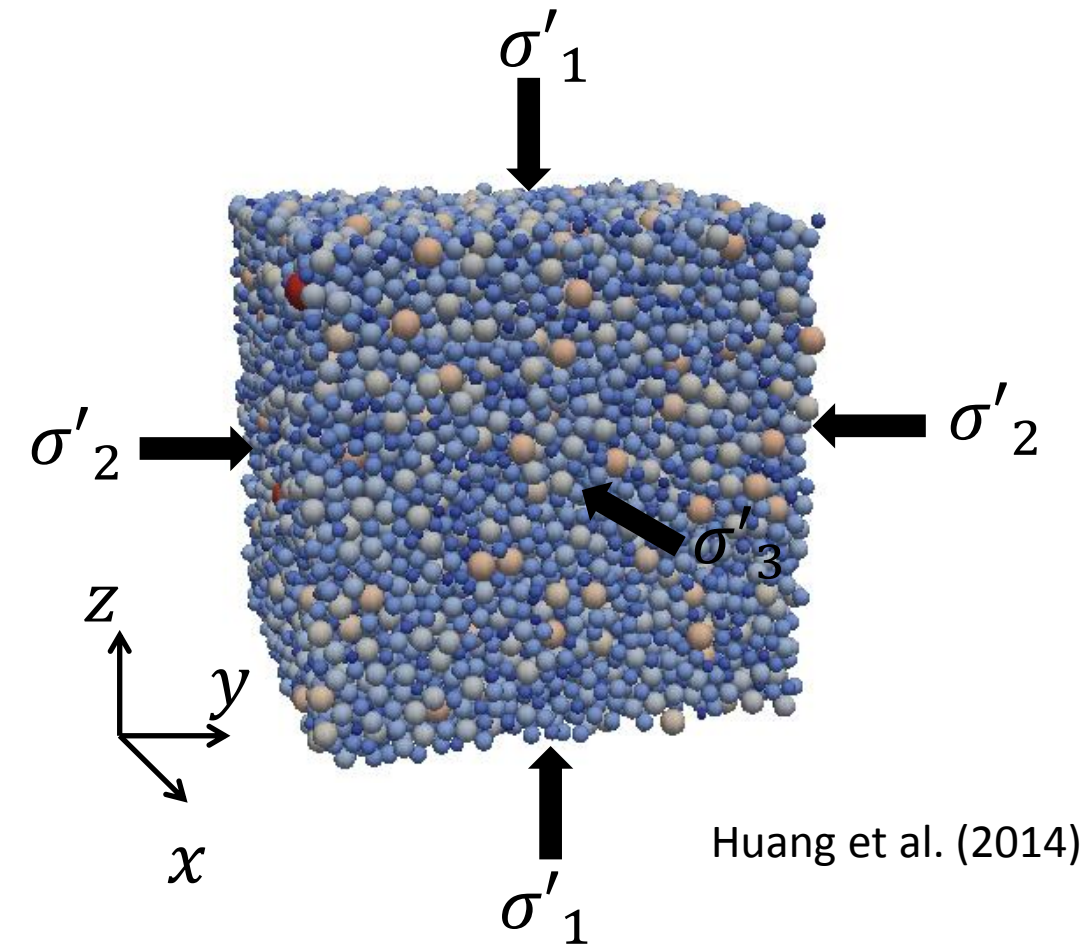
$$\Delta x_i \quad \bullet \quad \text{Increment in displacement}$$

$$\dot{\epsilon}_{ij} \quad \bullet \quad \text{Strain rate}$$

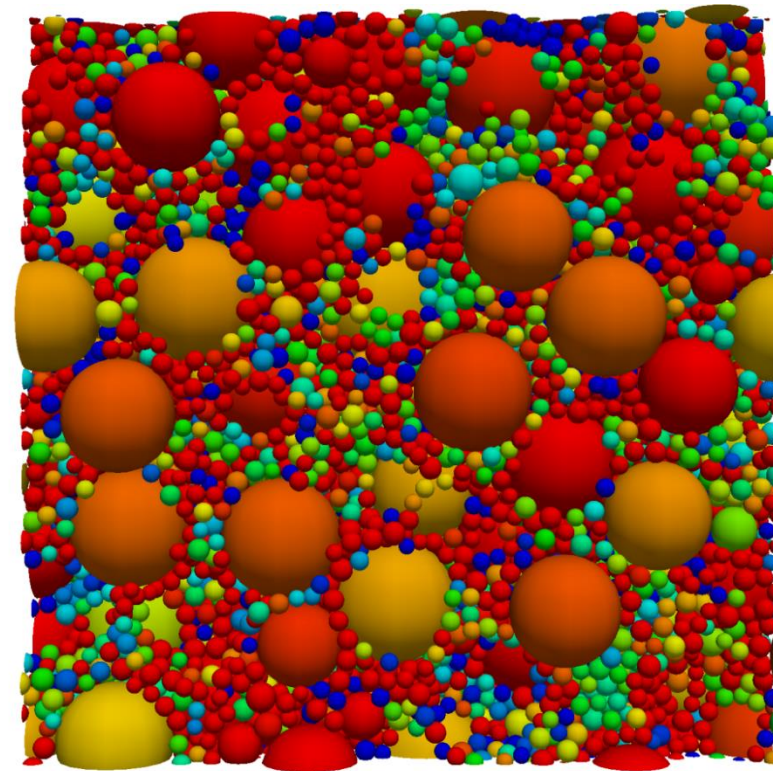
$$x_j \quad \bullet \quad \text{Position}$$

Periodic boundaries

Effective to achieve a REV



20,164 particles

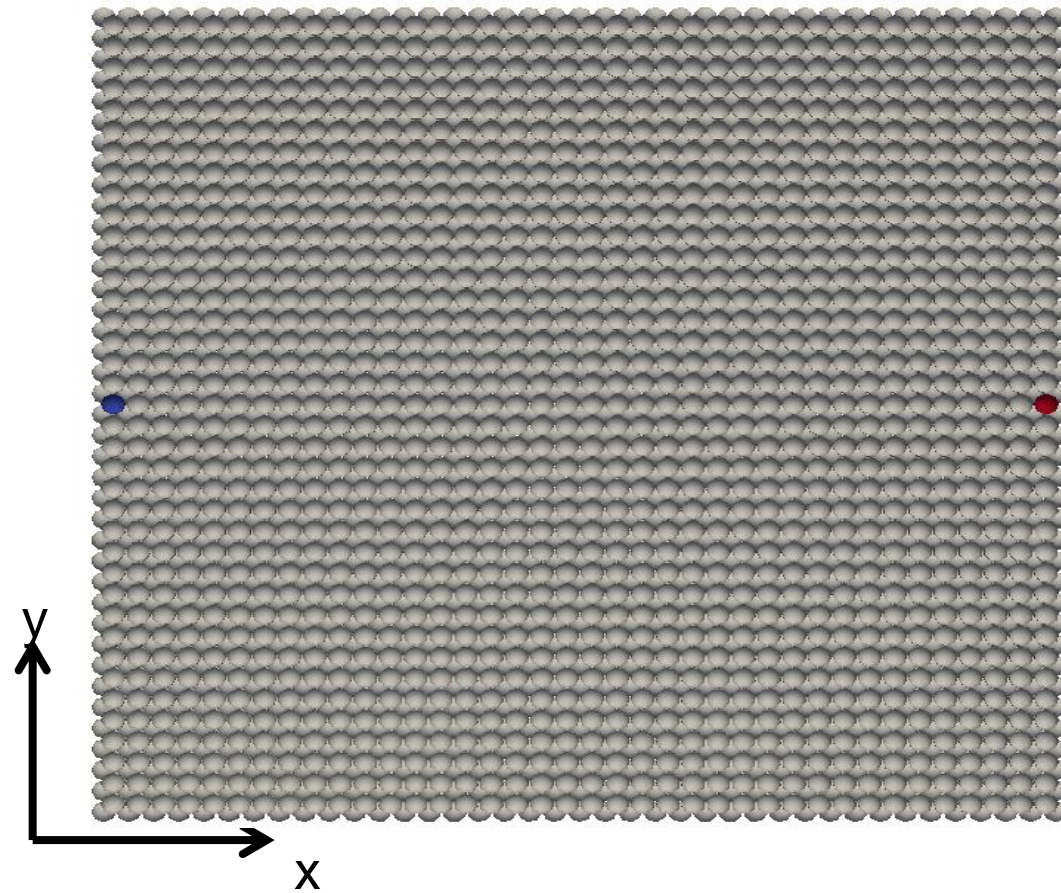
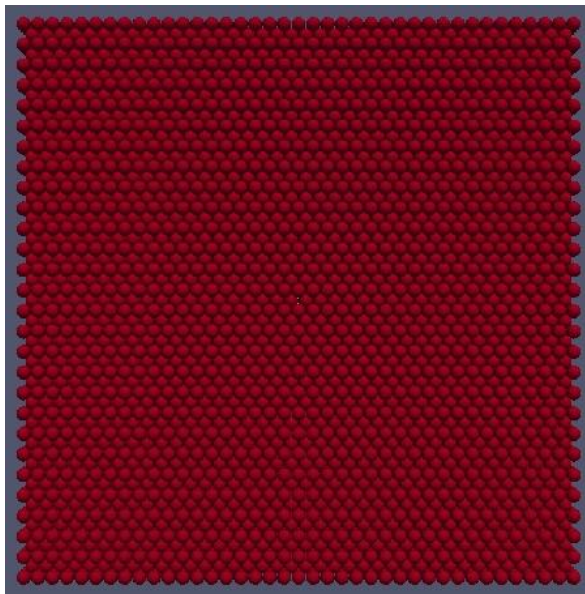


Shire et al. (2014)

Up to 300,000 particles

Influence of boundaries on response: Wave propagation

Face-centred cubic (FCC)
packing
Ave. coord. no. = 11.54



81, 576 particles

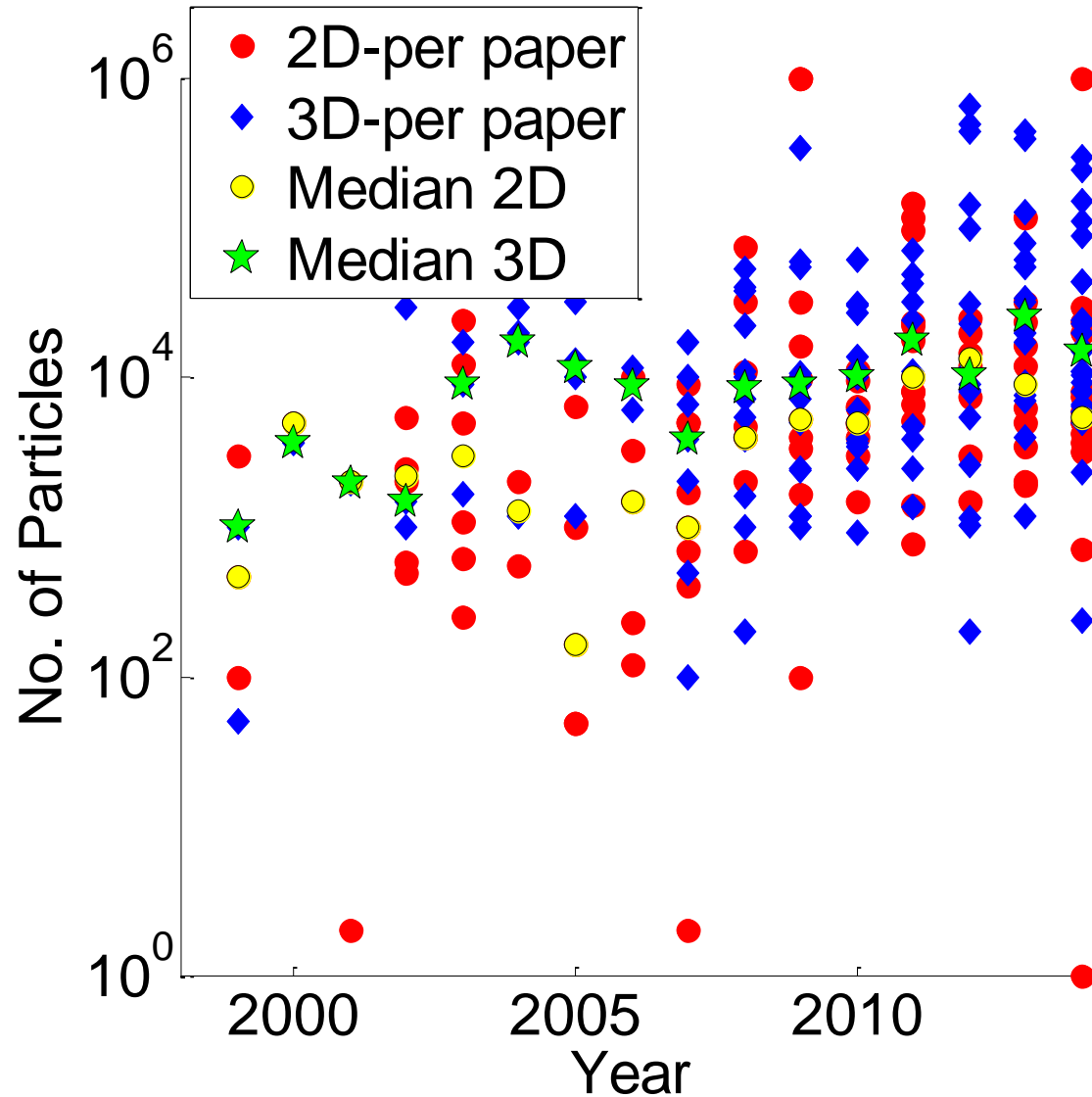
$f_{trans} = 30 \text{ kHz}$
 $R_d = 5.63$ and $\lambda/d_{50} =$
6.40

Conclusions

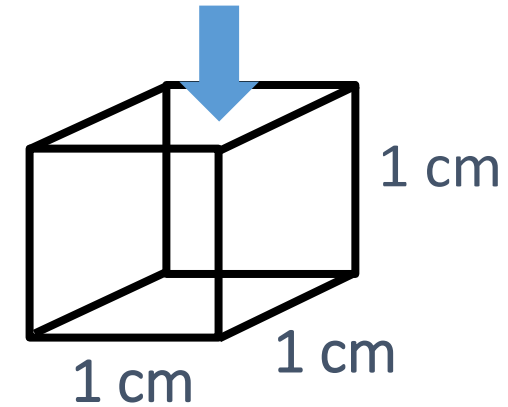
- Largely three types of boundaries: rigid wall, force, periodic
- Servo control can be used to capture complex conditions
- Use of computationally expensive force / membrane boundaries may not always be justifiable (small strain)
- Post-peak deformation patterns may be more sensitive

Sample size

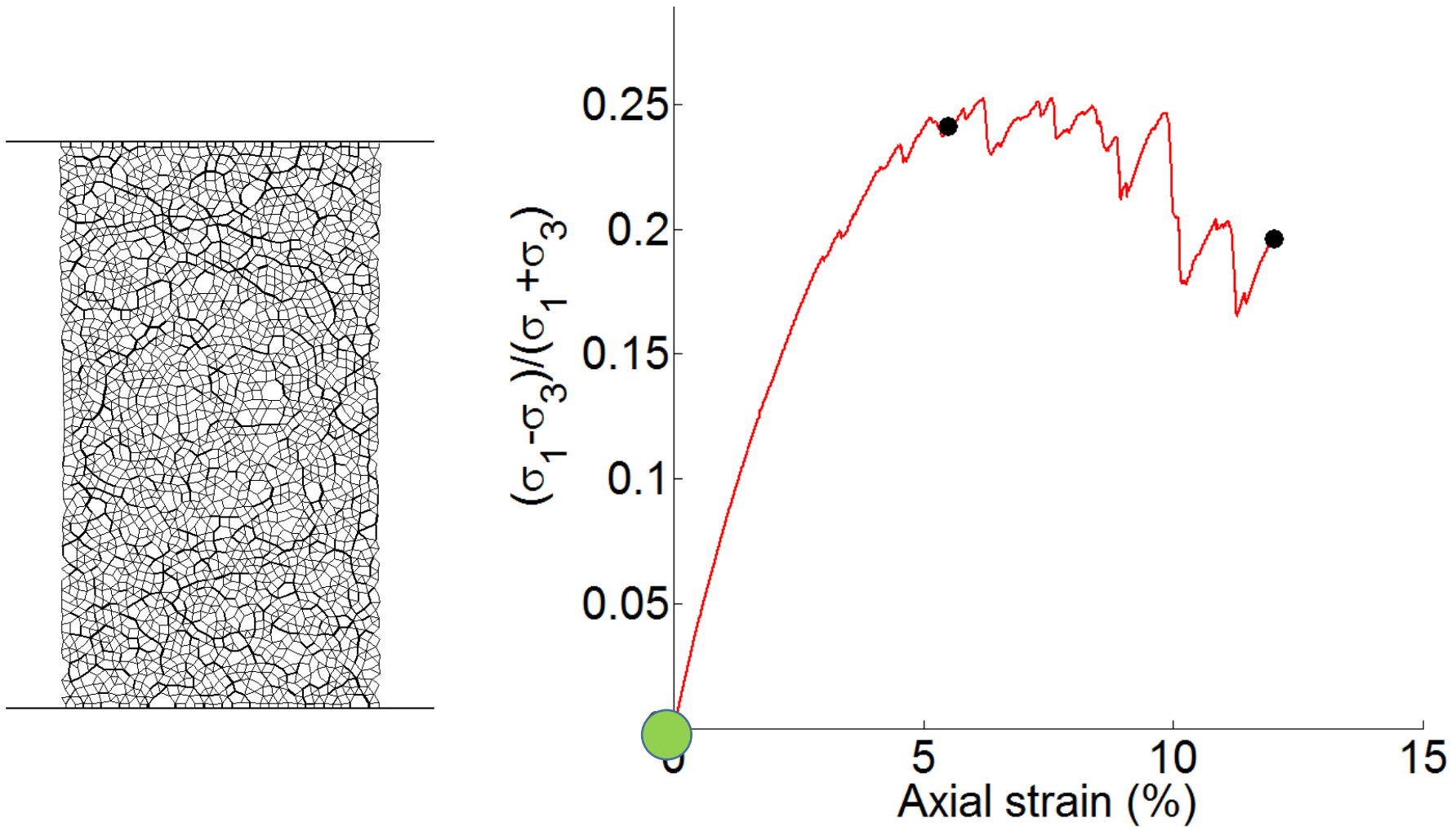
No of particles in DEM related publications



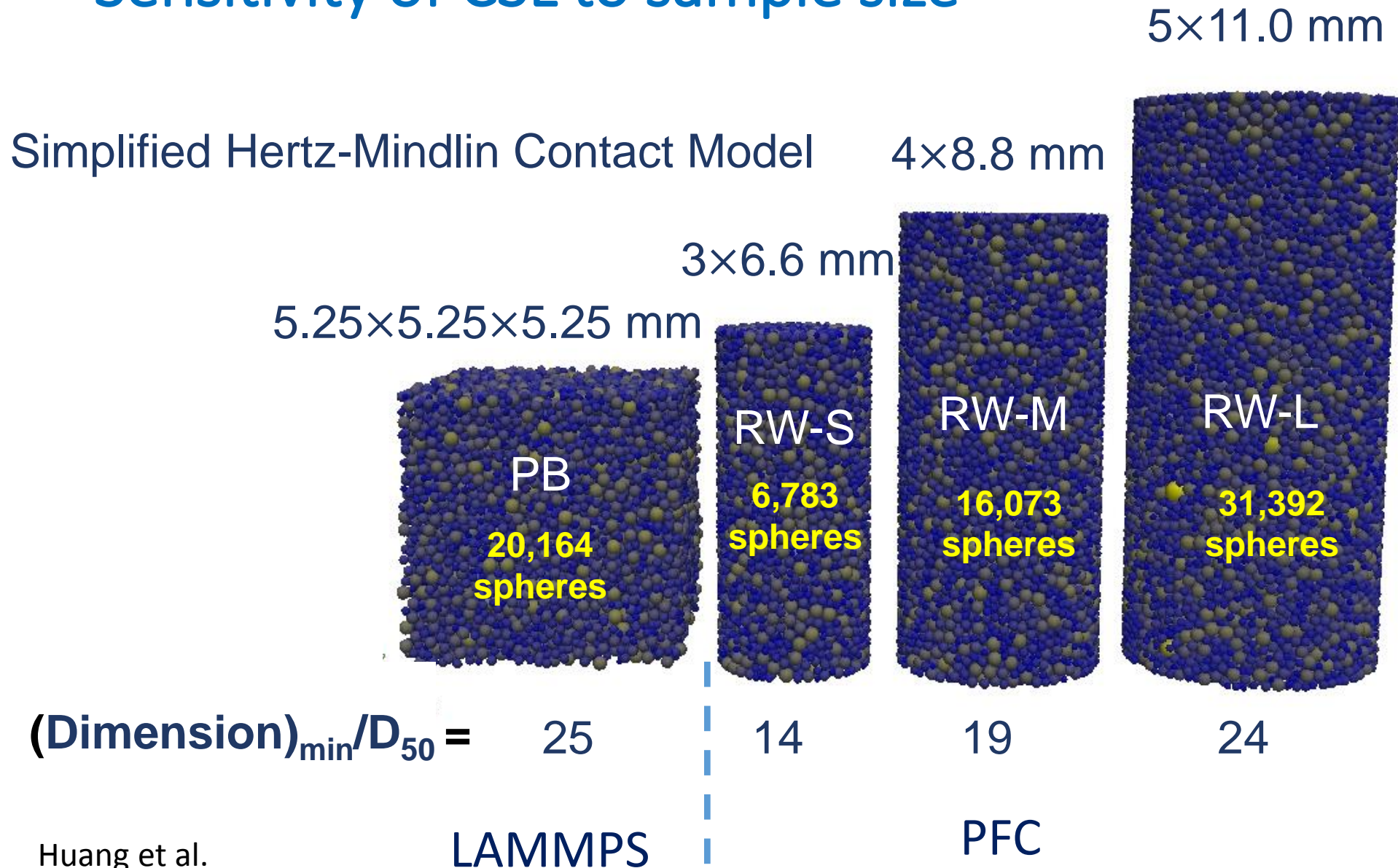
$D_{50}=200$ micron
>150,000 particles



Very small samples – erratic load:deformation response



Sensitivity of CSL to sample size

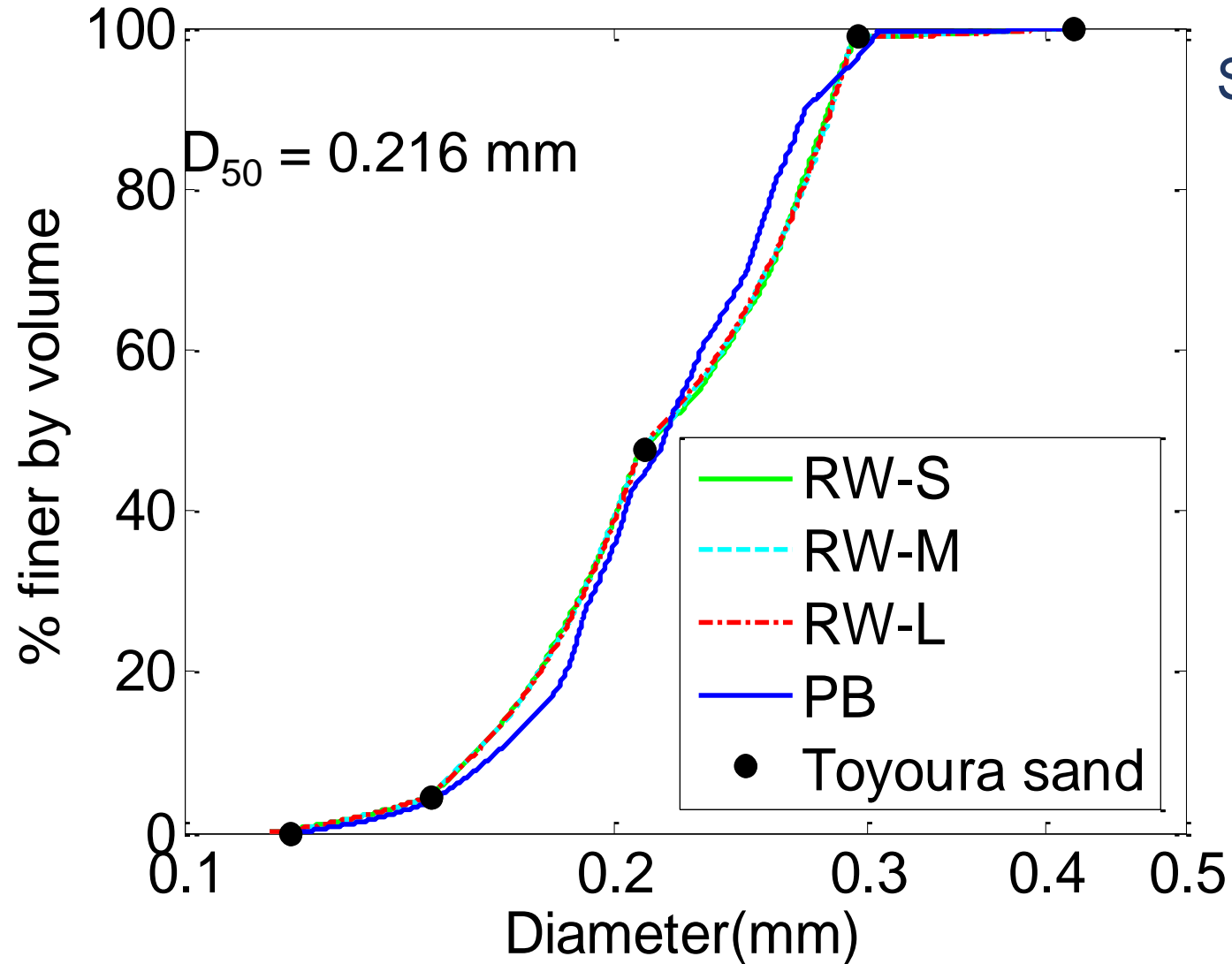


Huang et al.
(2014)

LAMMPS

PFC

Sensitivity of CSL to sample size

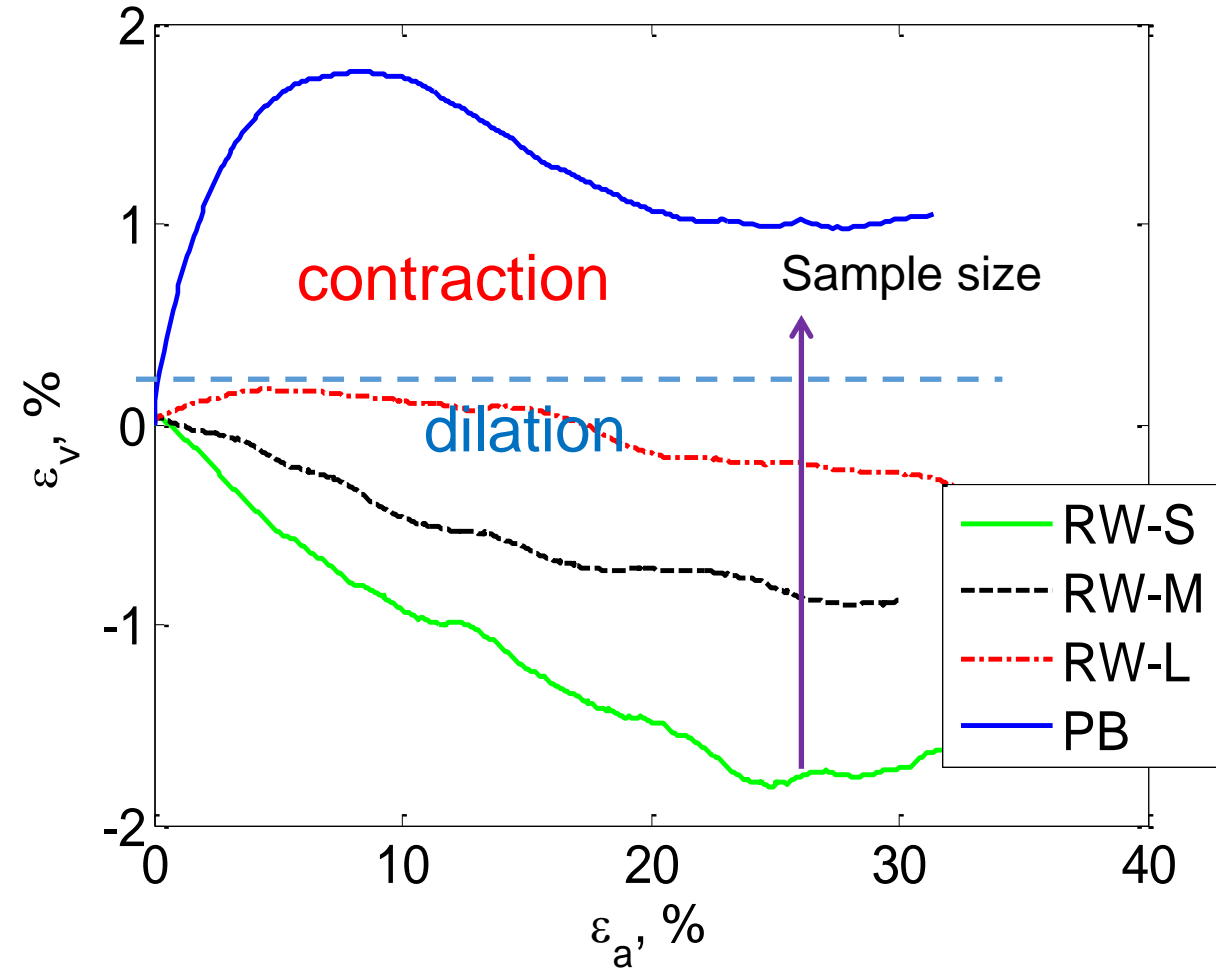
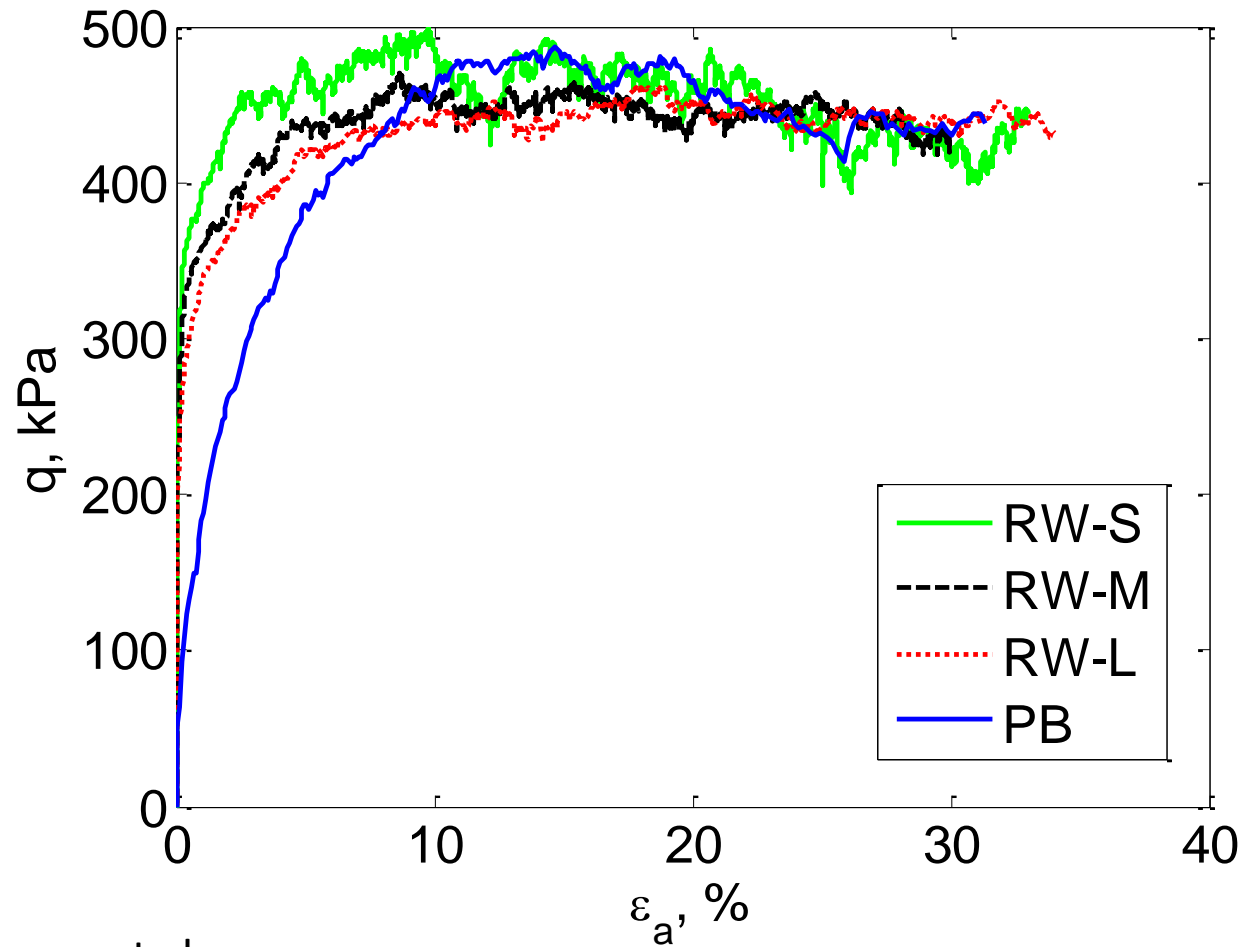


Simplified Hertz-Mindlin Contact Model

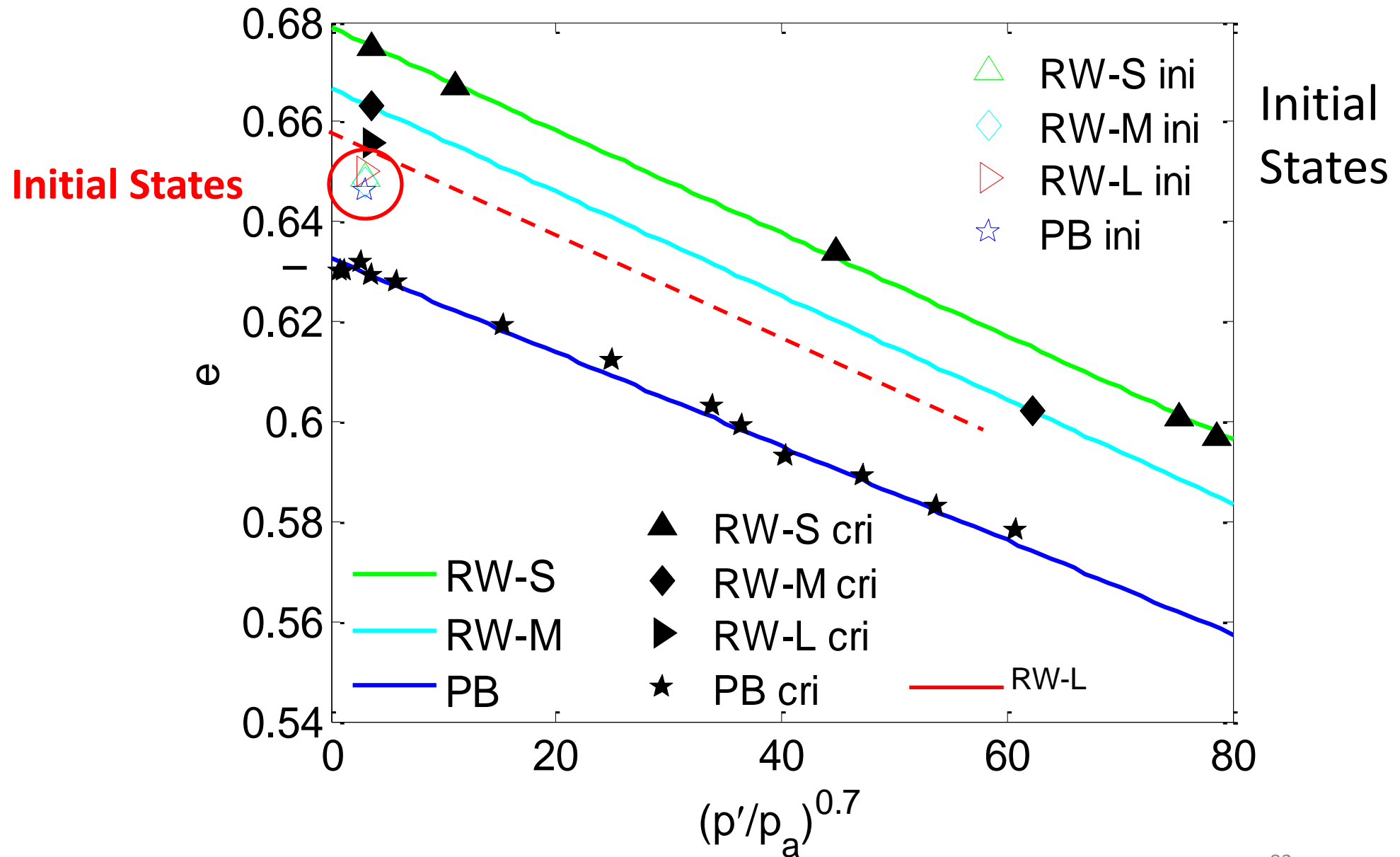
G	Shear modulus	29 GPa
ν	Poisson's ratio	0.12
μ	Inter-particle friction	0.25

Sensitivity of CSL to sample size

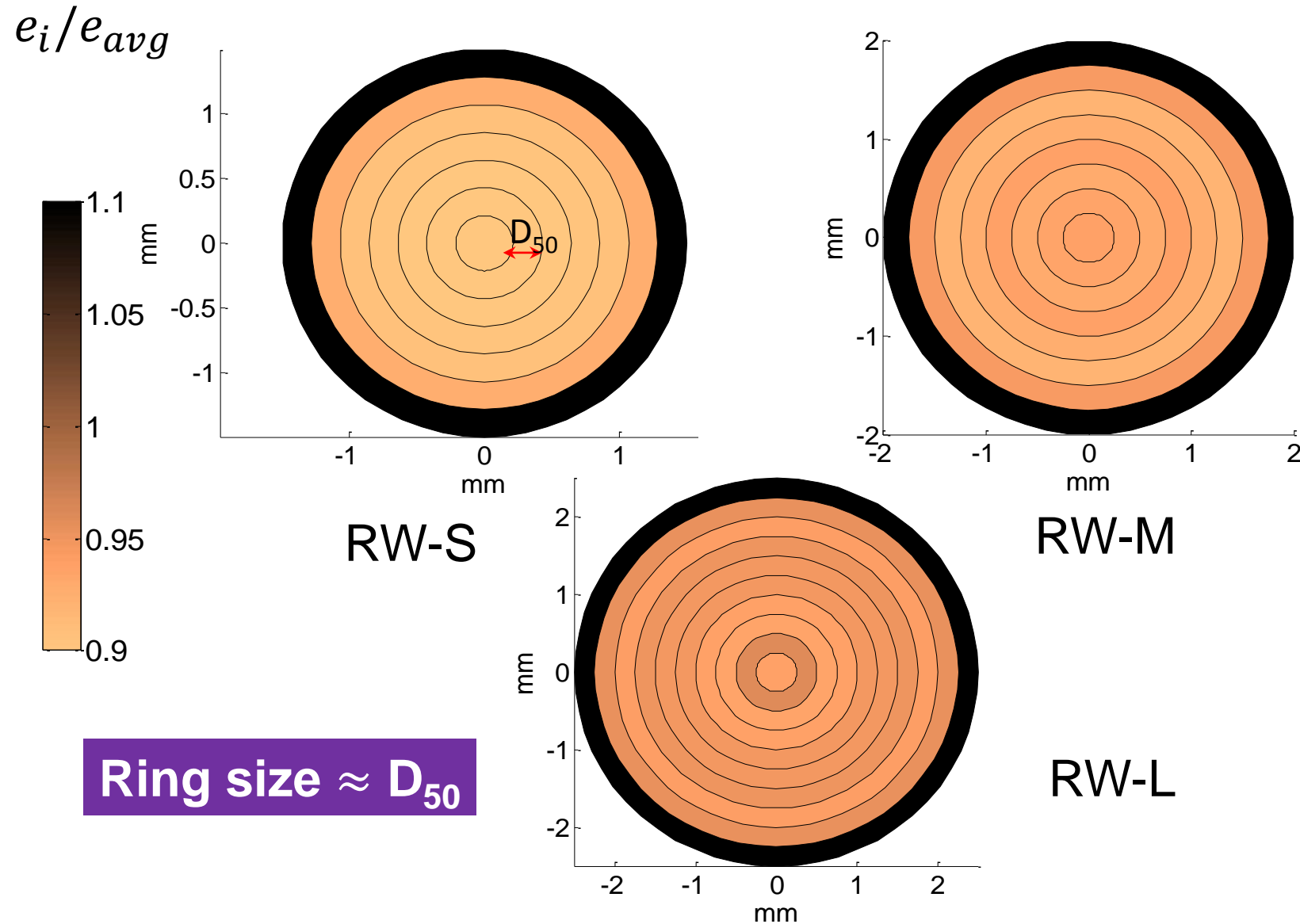
$$e_0 = 0.646, \sigma'_3 = 500 \text{ kPa}; q = \sigma'_1 - \sigma'_3$$



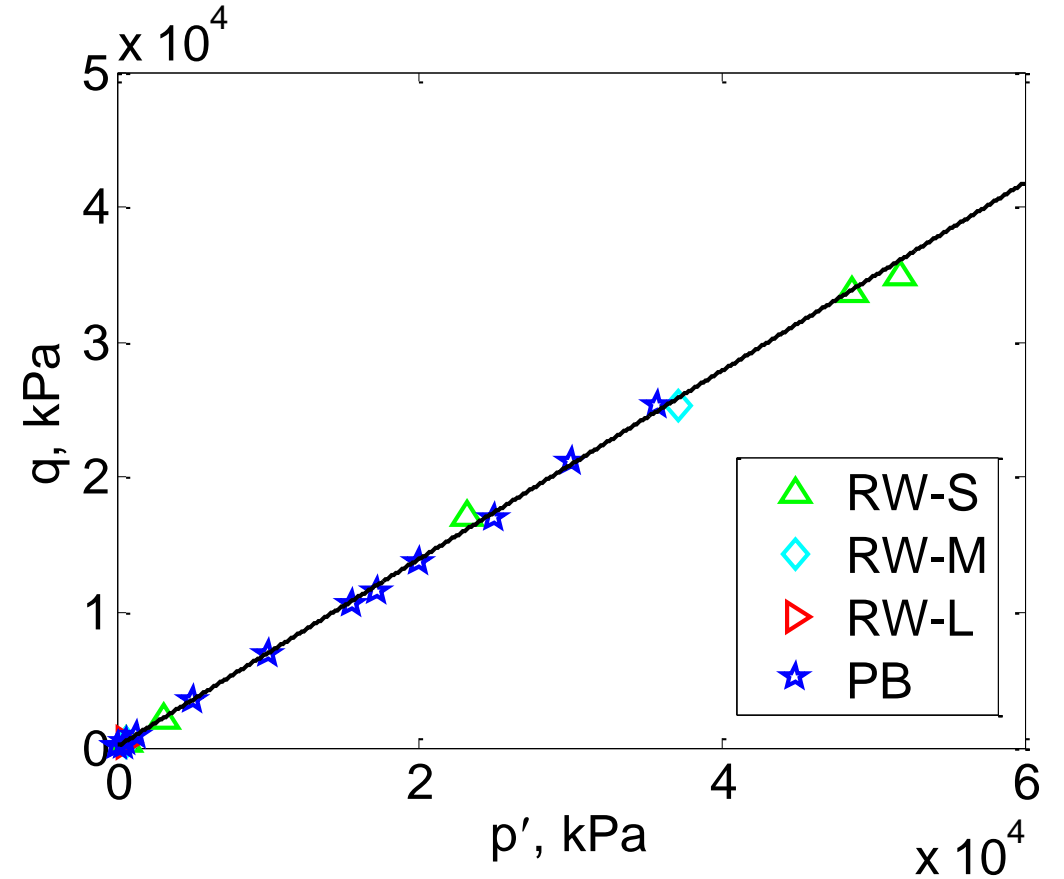
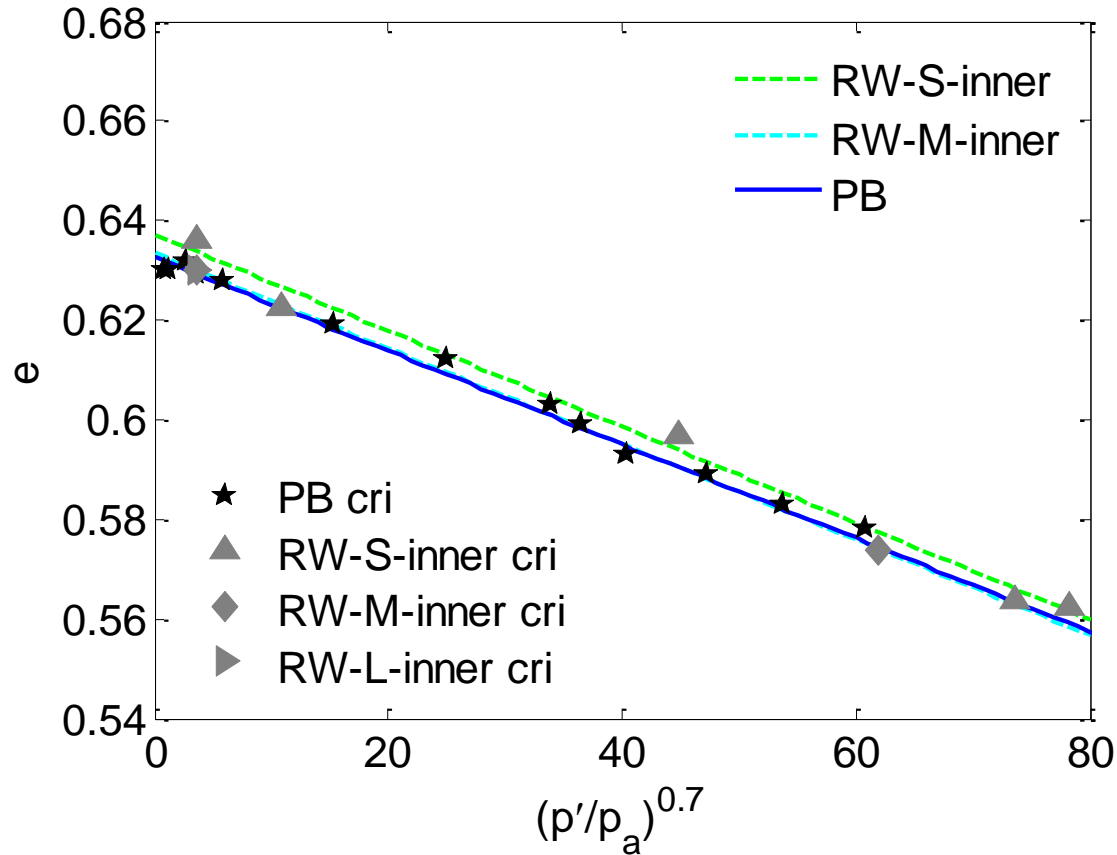
Sensitivity of CSL to sample size



Sensitivity of CSL to sample size



Sensitivity of CSL to sample size



Critical state: Interior region

Periodic cell simulations – sample size considerations

Daniel Barreto PhD Research– Simulations using Trubal

Spheres with D_{min} / D_{max} up to 1.4

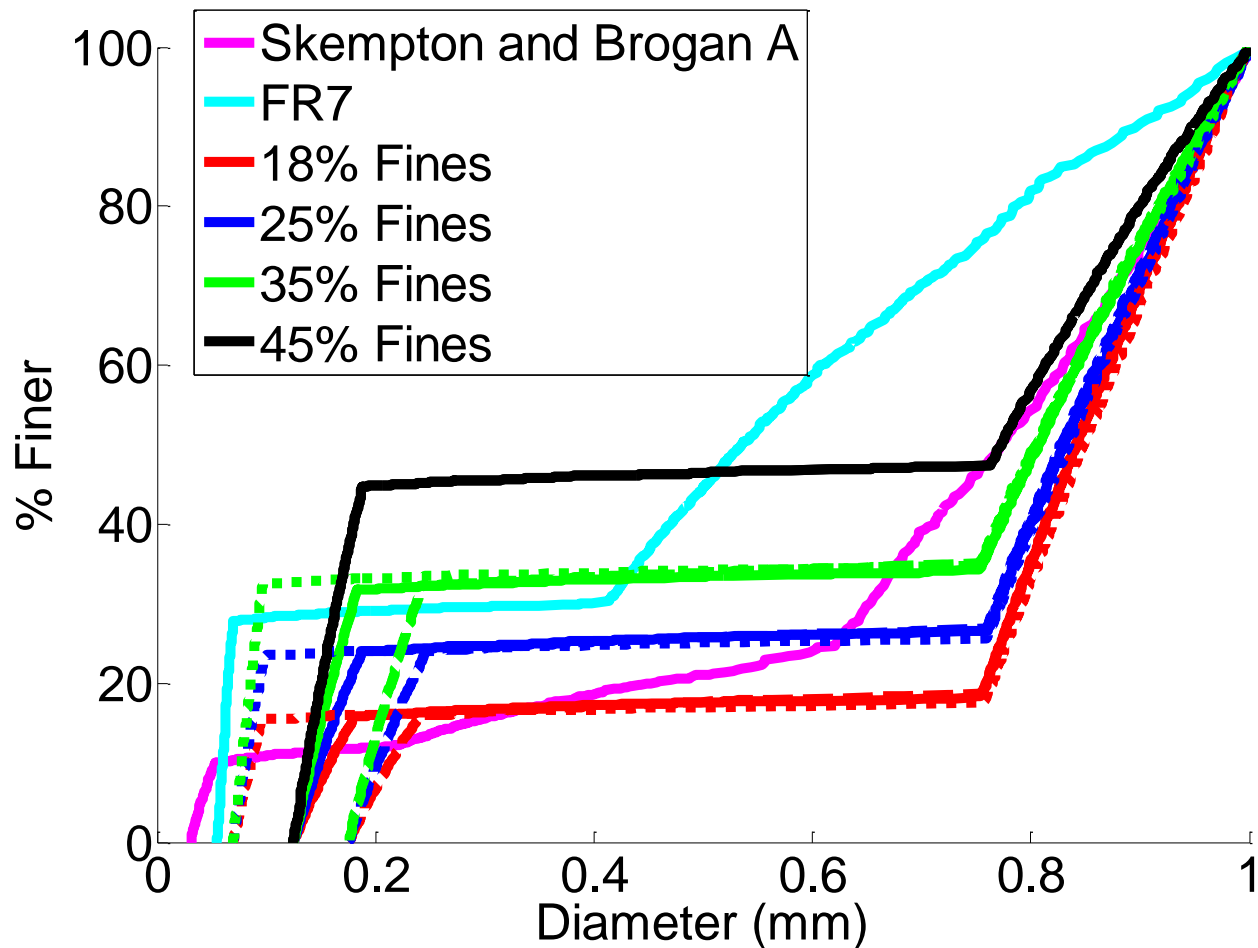
Simulations considered 2000, 4000, 8000 samples

Macroscale load:deformation responses similar

Fabric different for 2000 – considered 2nd order fabric tensor

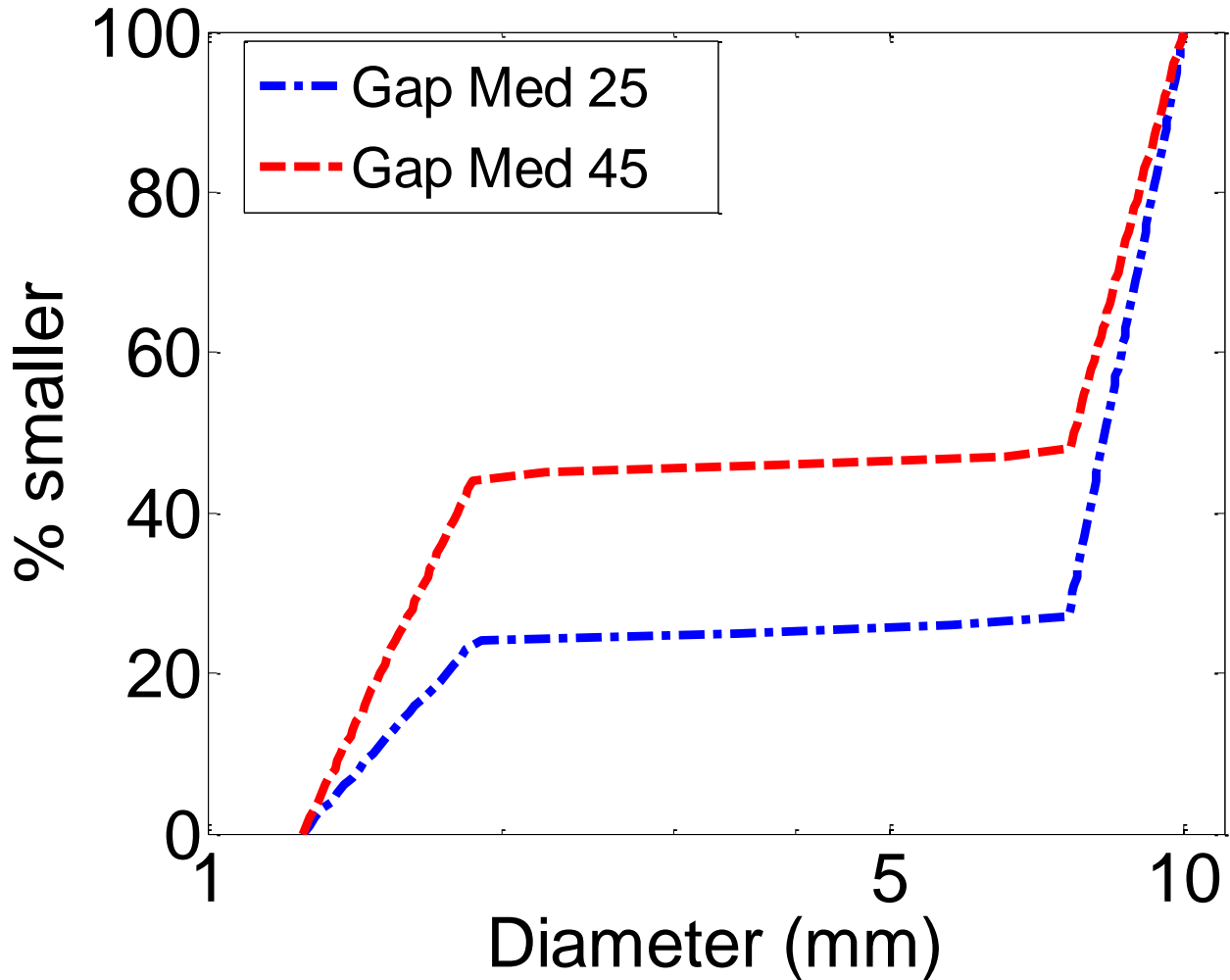
Simulations for main body of research – 4000 particles

Periodic cell simulations – sample size considerations

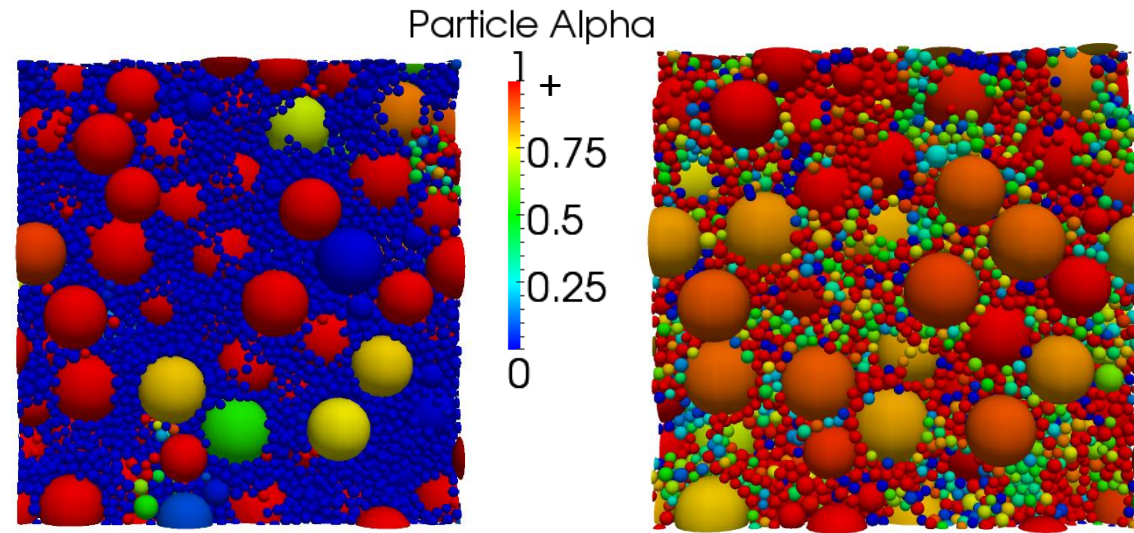


- Parametric study to assess the effect of gap ratio and fines content on fabric of gap graded soils
- If 500 coarse particles used – up to 300,000 particles per simulation
- Is this adequate?

Periodic cell simulations – sample size considerations



- Simulations on a gap-graded material
- Preliminary study to assess whether 500 coarse particles gave a satisfactory REV

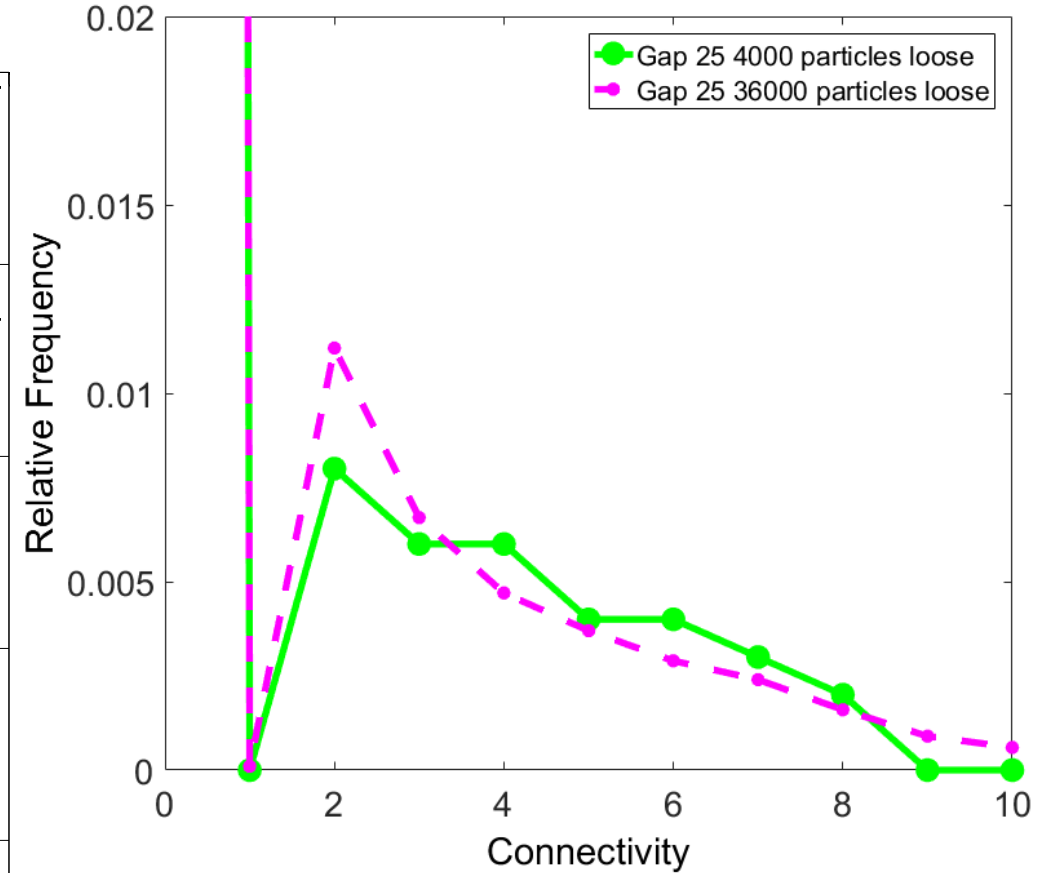
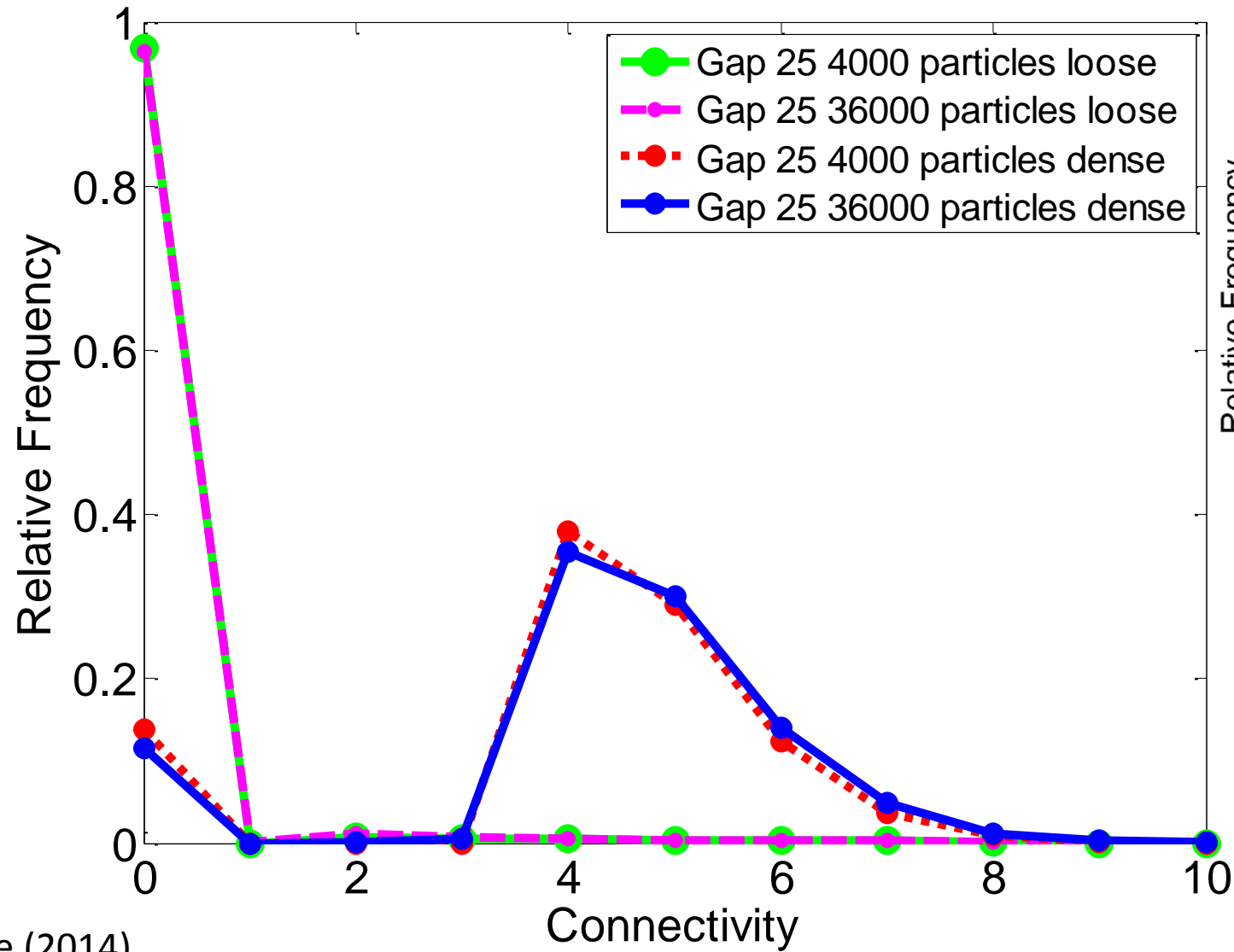


30% Fines -
Loose

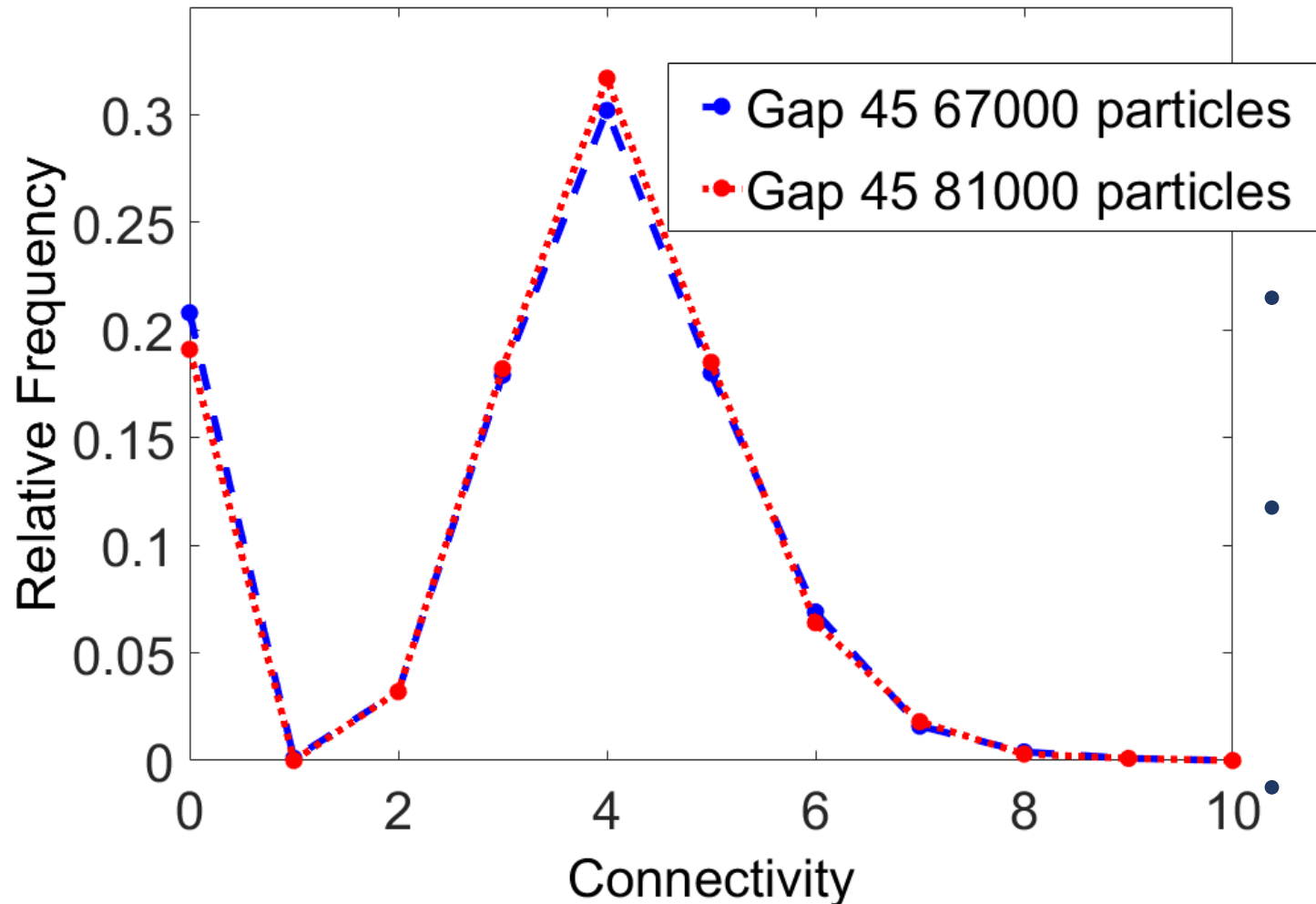
30% Fines -
Dense

88

Periodic cell simulations – sample size considerations



Periodic cell simulations – sample size considerations



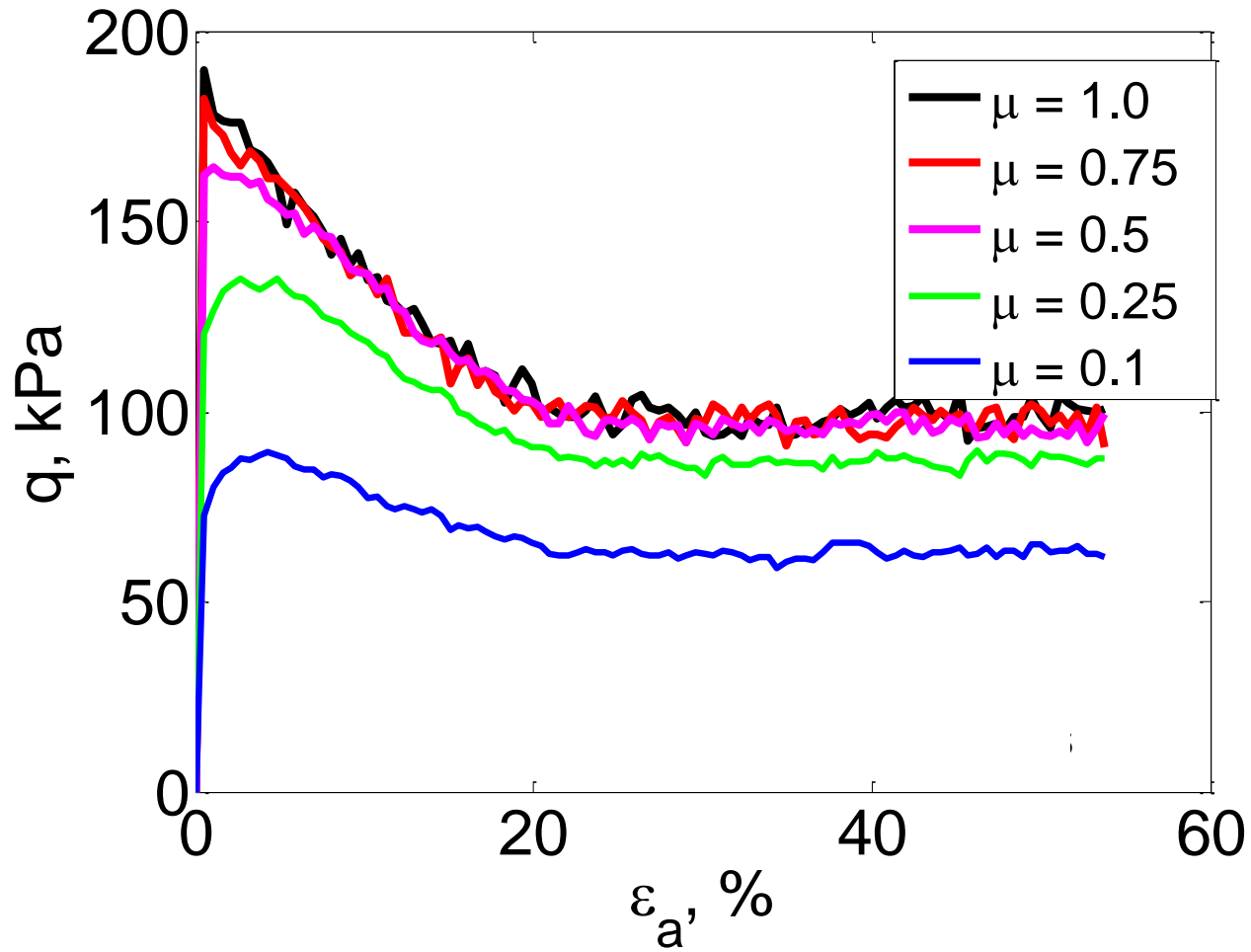
- Preliminary study to assess whether 500 coarse particles gave a satisfactory REV
- Reducing number of particles no significant effect on fabric measure of interest
- Full parametric study used 500 coarse particles for each sample

Conclusions

- V important to establish you have an REV (representative element volume)
- Critical state line position can be affected by boundary conditions – v significant implications for understanding behaviour
- Plots of local void ratio helpful to understand extent of boundary effects
- Periodic cell very useful, but you still need to check an REV is achieved

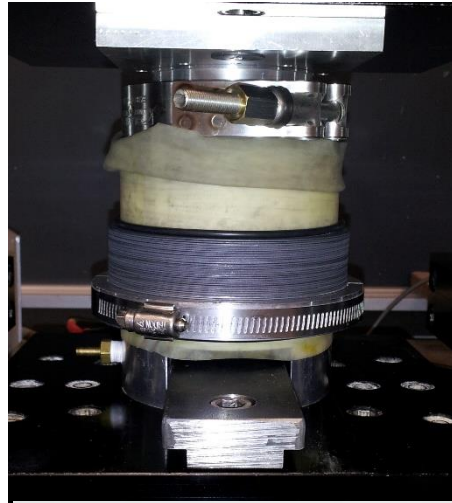
Specimen generation

Response of samples consolidated with $\mu=0.1$



- Toyoura sand PSD
- Initial void ratio, $e_0 = 0.533$
- $\sigma'_3 = 100$ kPa
- Strain controlled triaxial compression

Simple shear experiments



Experimental set up -
Texas A+M University

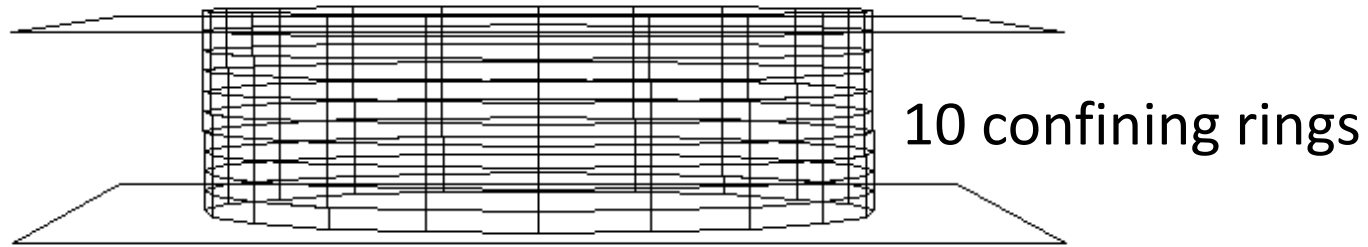
- NGI type
simple shear
- Stacked ring
configuration

Loose and dense
samples subjected to
monotonic simple
shear

Material:

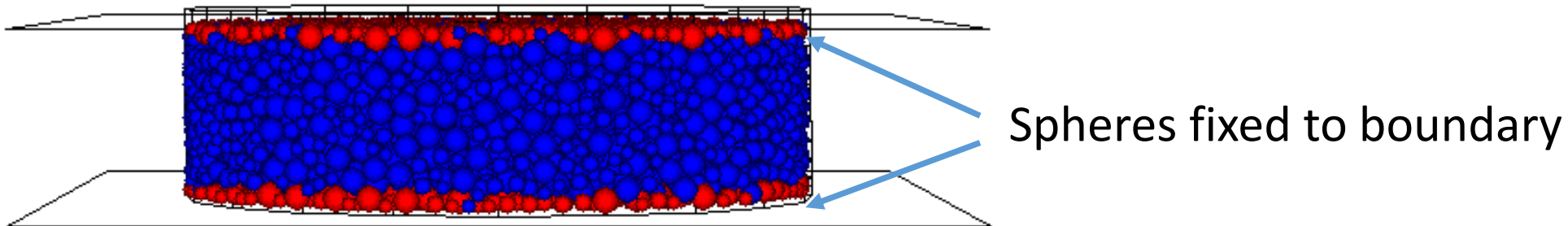
- Grade 25 chrome steel balls
- 3 sizes
- Sample 1: 2.28, 3.18, 3.97 mm, 1:1:1
Total: 7,500 spheres
- Sample 2: 1.19, 1.59, 1.98 mm, 1:1:1
Total: 60,000 spheres

Simple shear simulations

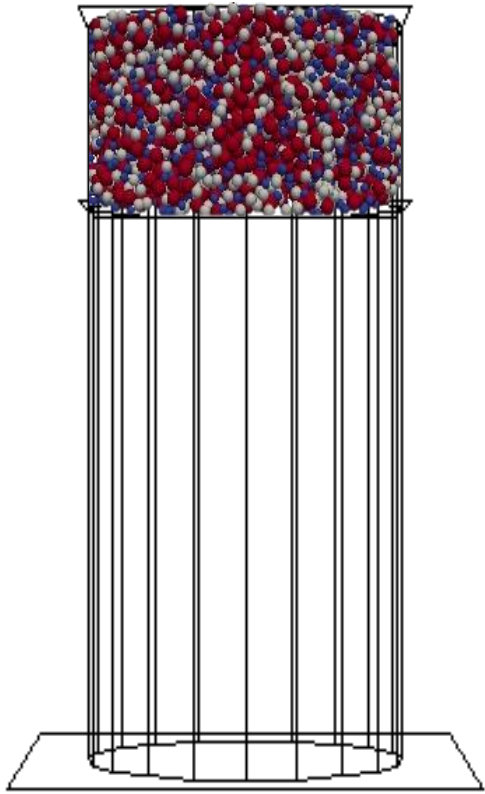


Simulation details:

- PFC3D
- Input parameters matched experiments
- $G=80\text{GPa}$, $\nu=0.3$, $\rho=7800\text{ kg/m}^3$
- $\phi_{\mu}=5.5^{\circ}$

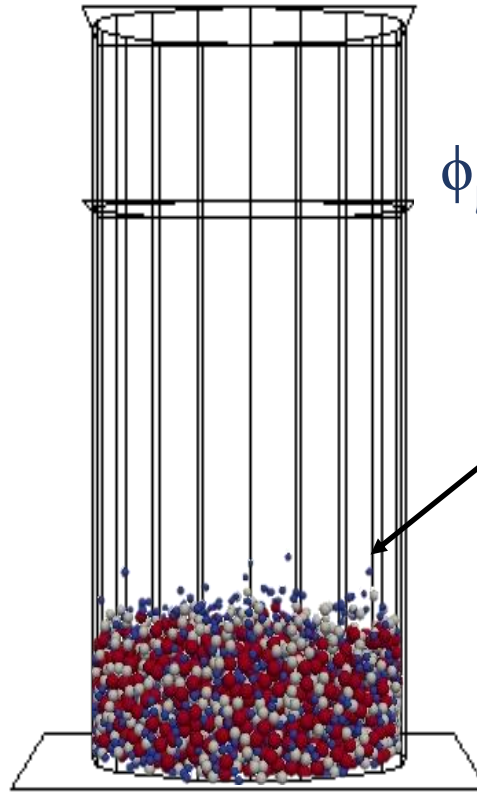


Simple shear simulations: specimen preparation



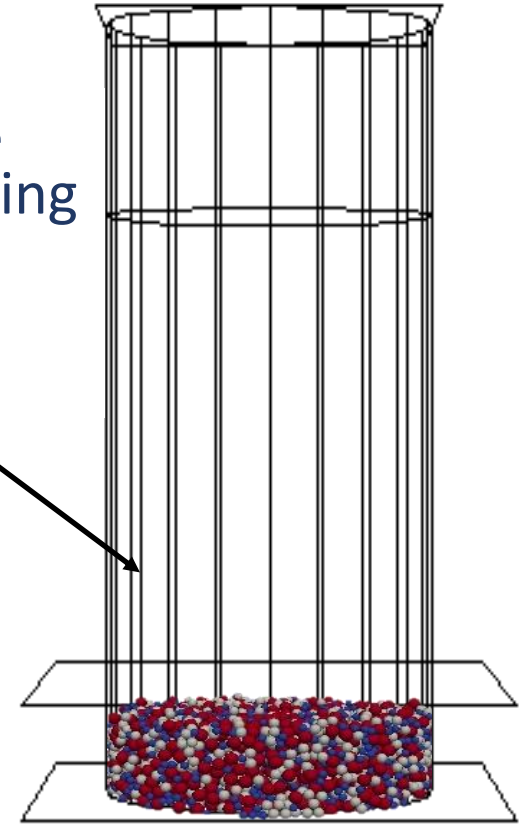
Cloud of non contacting spheres

Bernhardt et al. (2016)



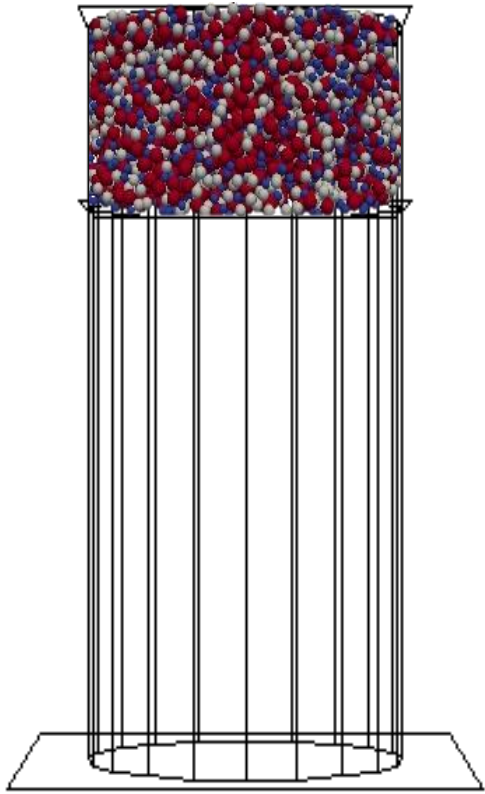
Gravity settling

To create dense
 ϕ_μ reset to 5.5° during
shearing



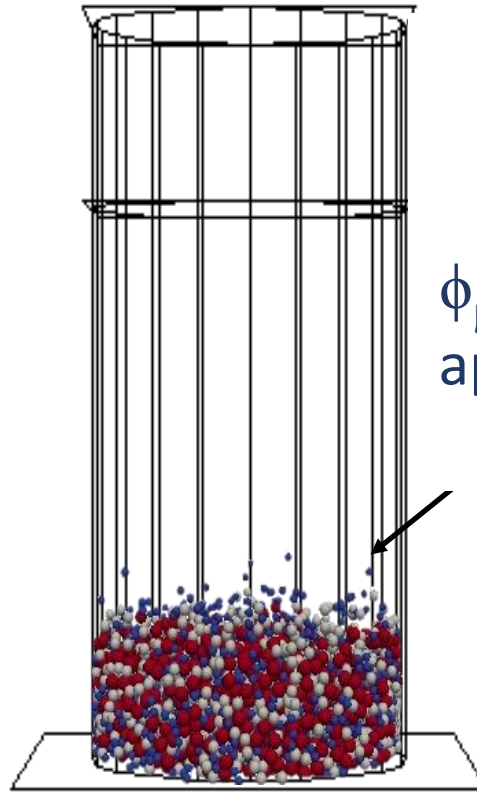
Servo-control walls to
get target vertical
stress

Simple shear simulations: specimen preparation



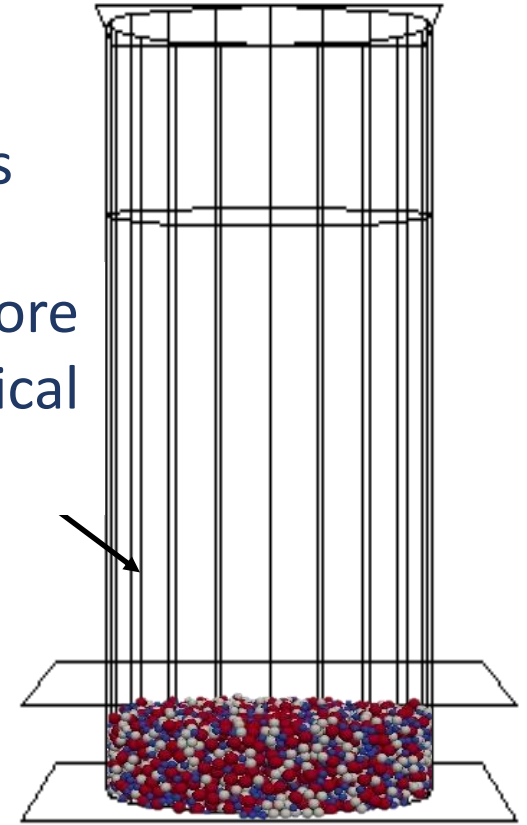
Cloud of non contacting spheres

Bernhardt et al. (2016)



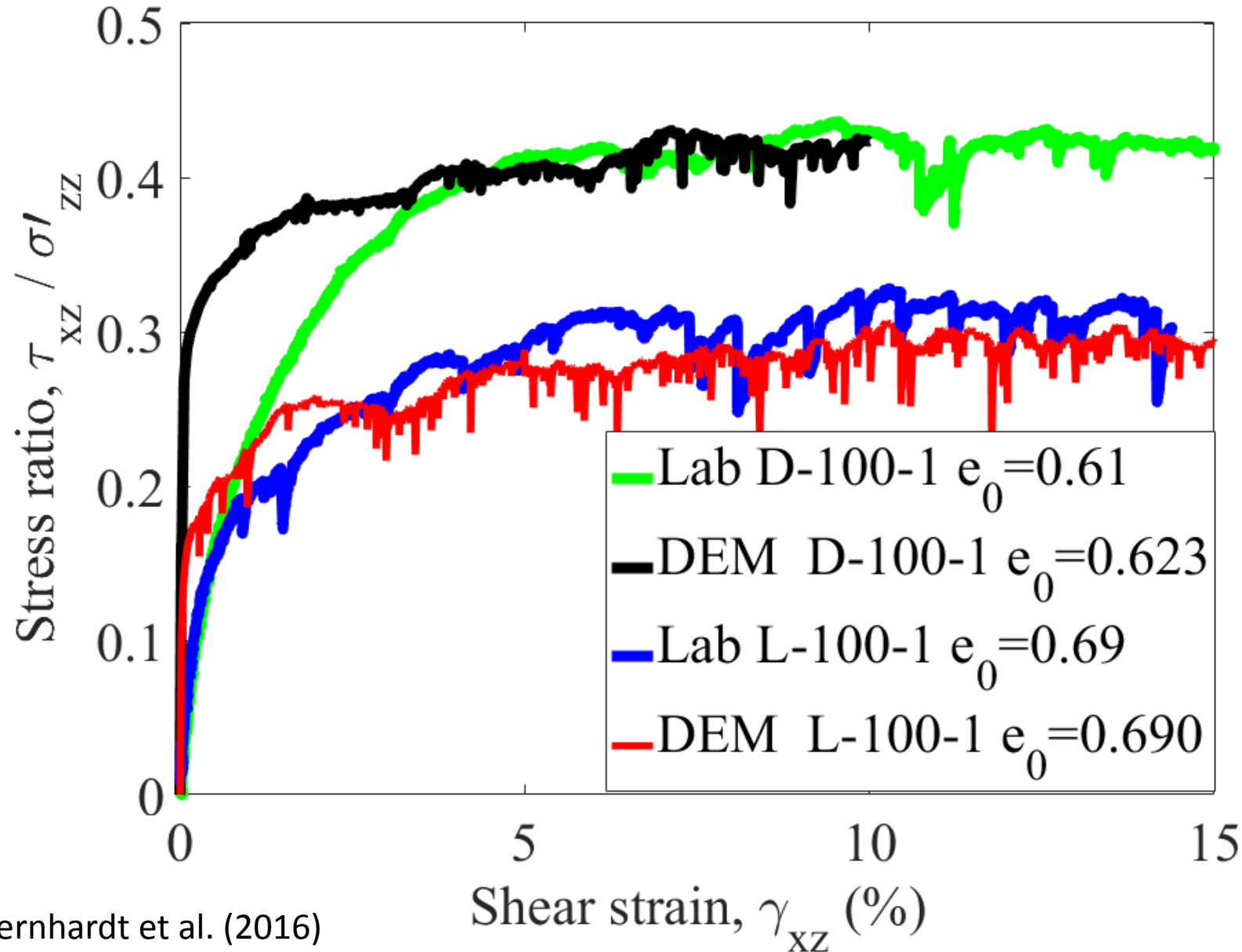
Gravity settling

To create loose samples
 ϕ set to 45°
 ϕ_μ reset to 5.5° before
application of vertical
stress



Servo-control walls to
get target vertical
stress

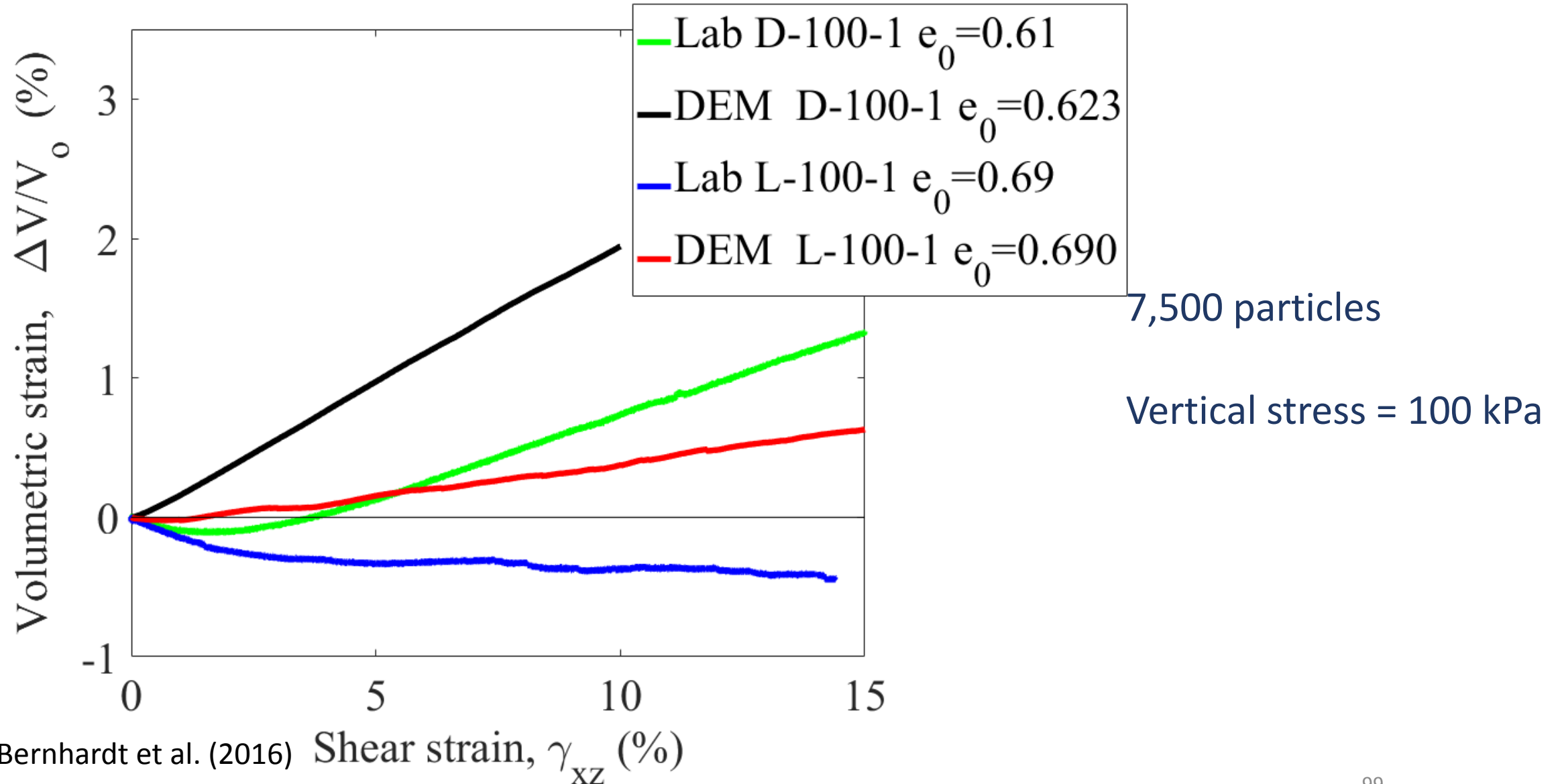
Simple shear simulations: Observed response sample 1



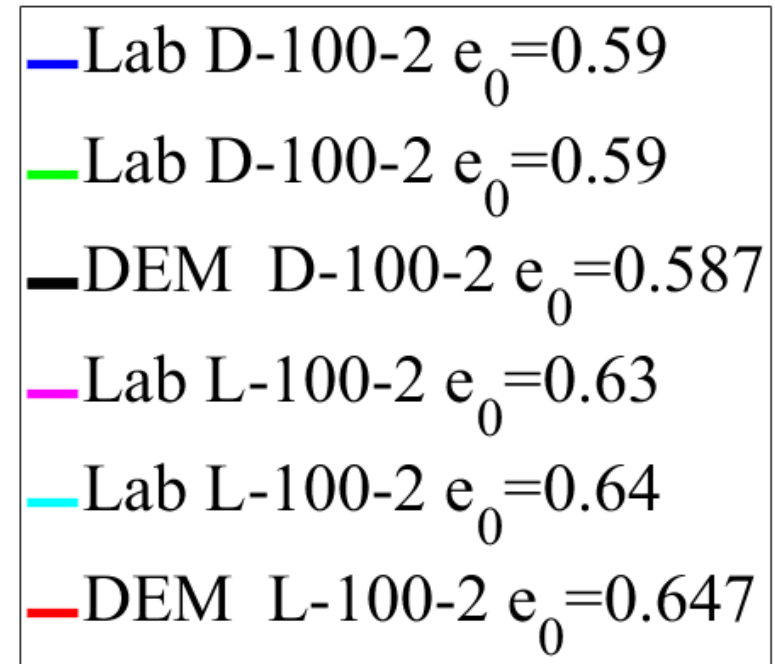
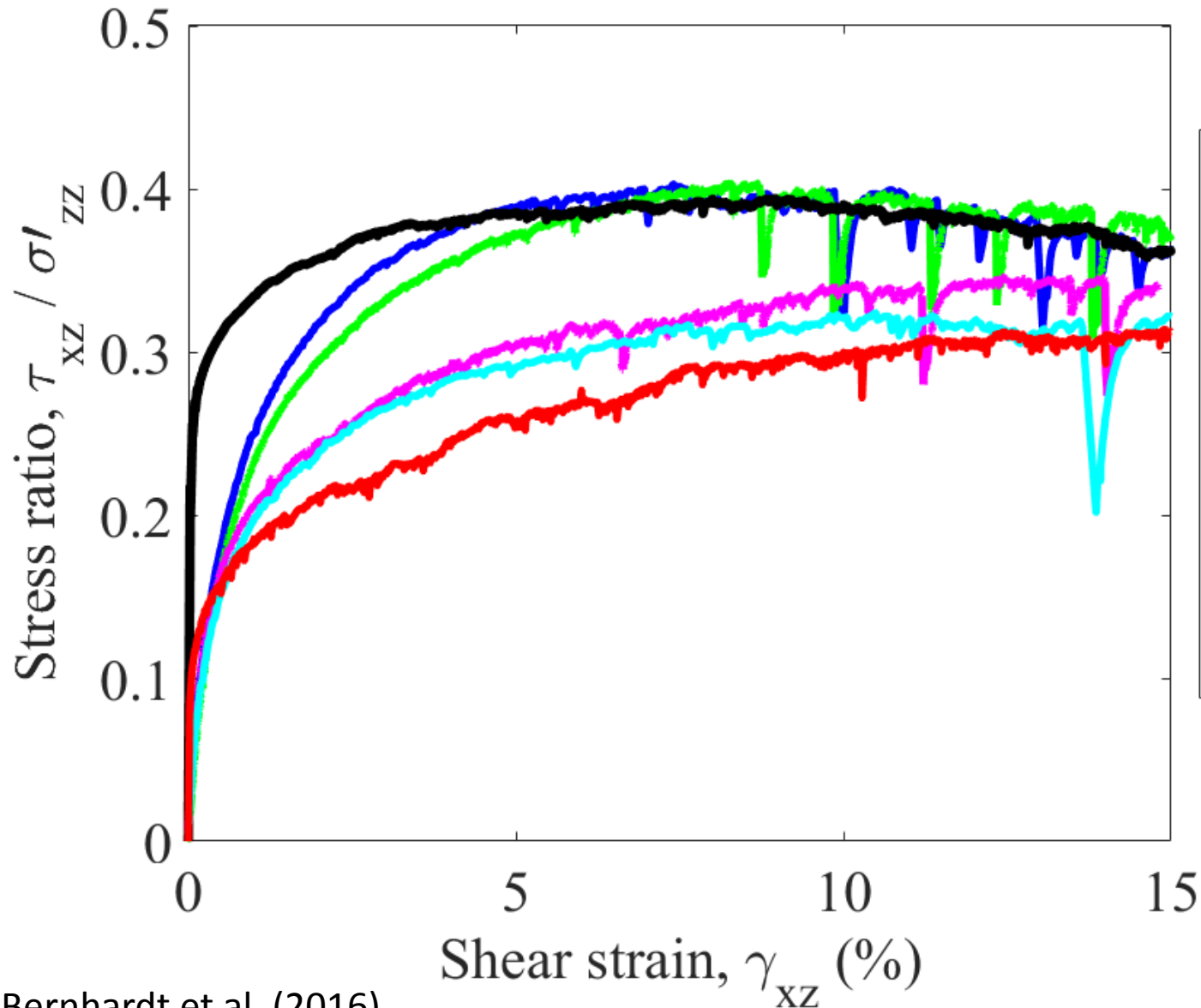
7,500 particles

Vertical stress = 100 kPa

Simple shear simulations: Observed response sample 1



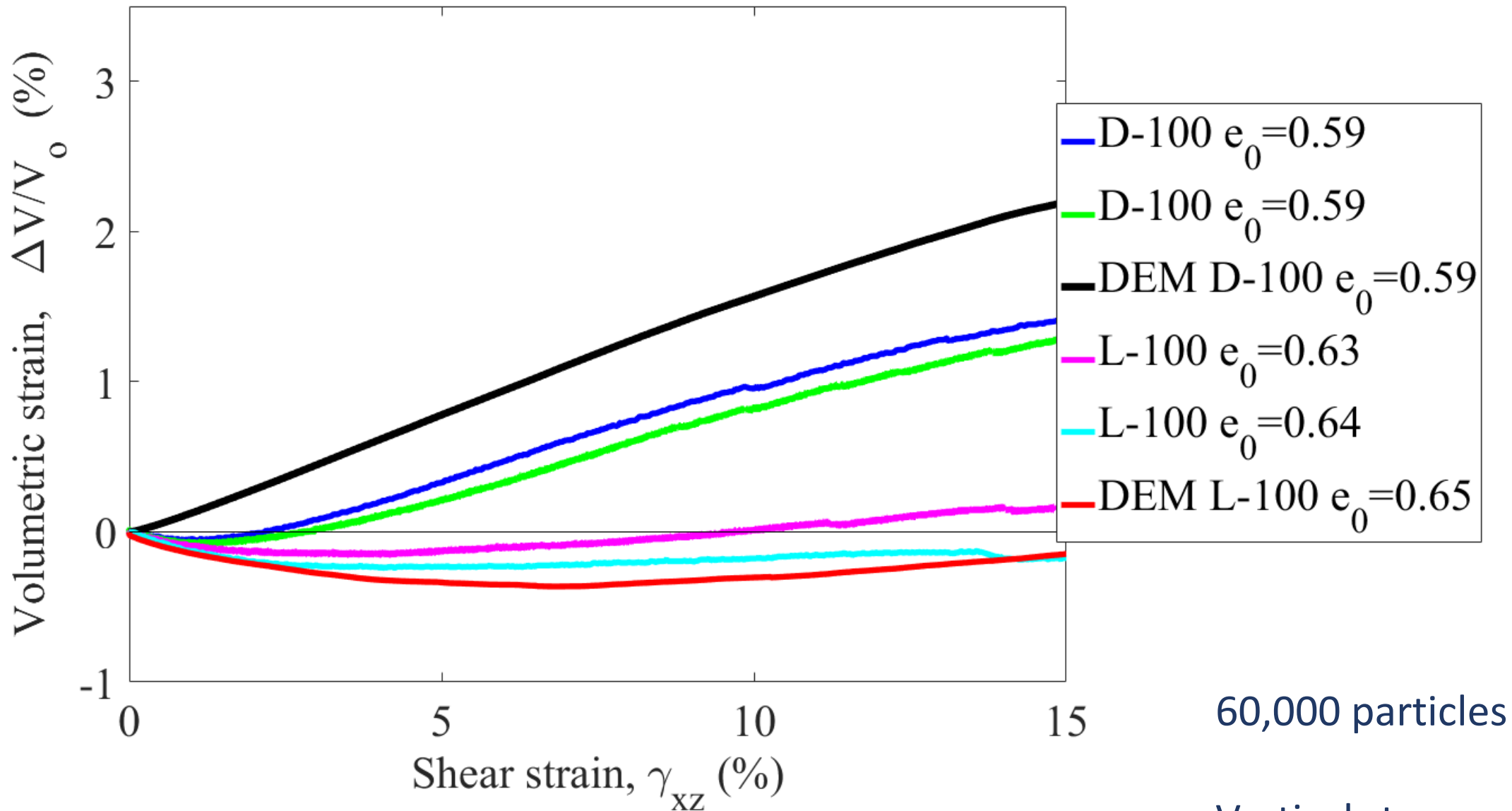
Simple shear simulations: Observed response sample 2



60,000 particles

Vertical stress = 100 kPa

Simple shear simulations: Observed response sample 2



60,000 particles

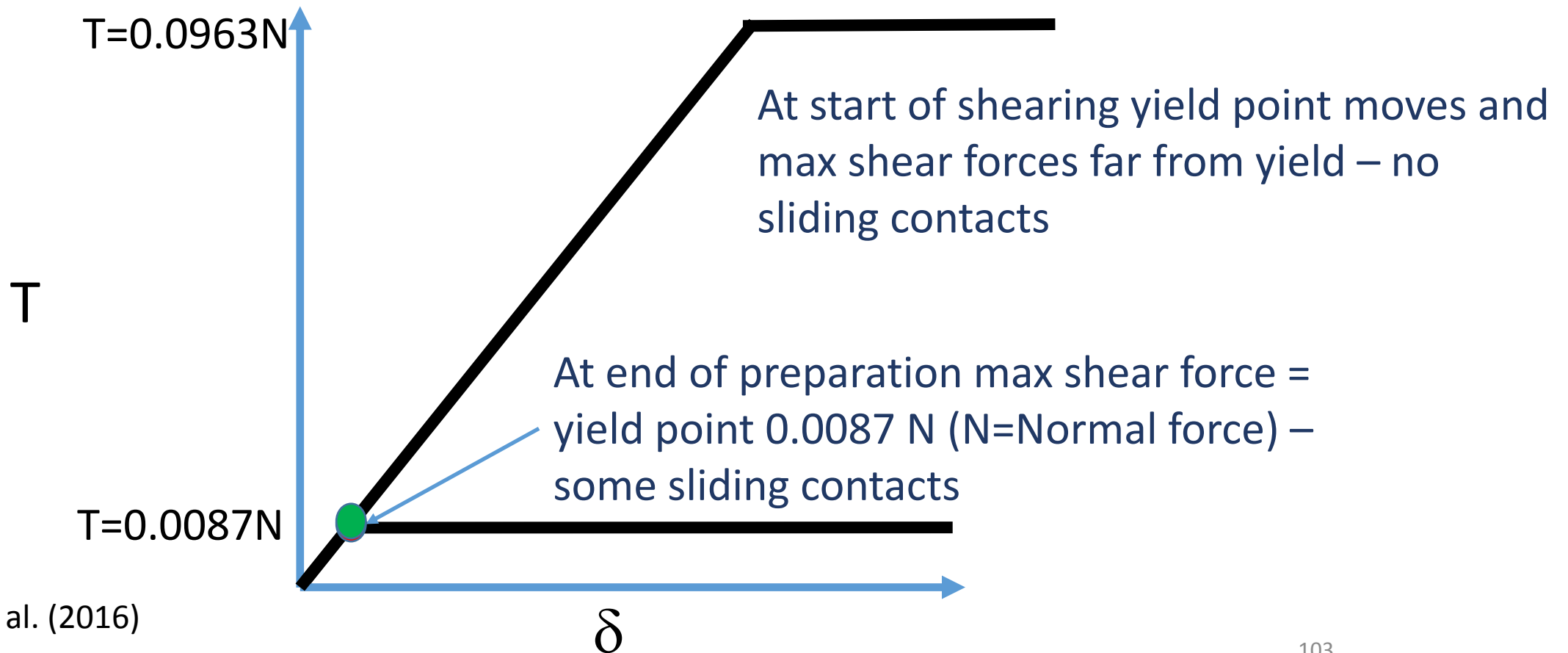
Vertical stress = 100 kPa

Simple shear simulations: Observations

- For both sample types at large strains DEM simulation and lab data agree
- Agreement is more effective for loose samples in both cases
- Very stiff response, dilatant response observed for dense DEM samples

Simple shear simulations: Reduction of ϕ_μ during preparation

- ϕ_μ was reduced only for dense samples where simulation / lab agreement is poor
- For dense samples at start of shearing shear forces far from yield



Conclusions

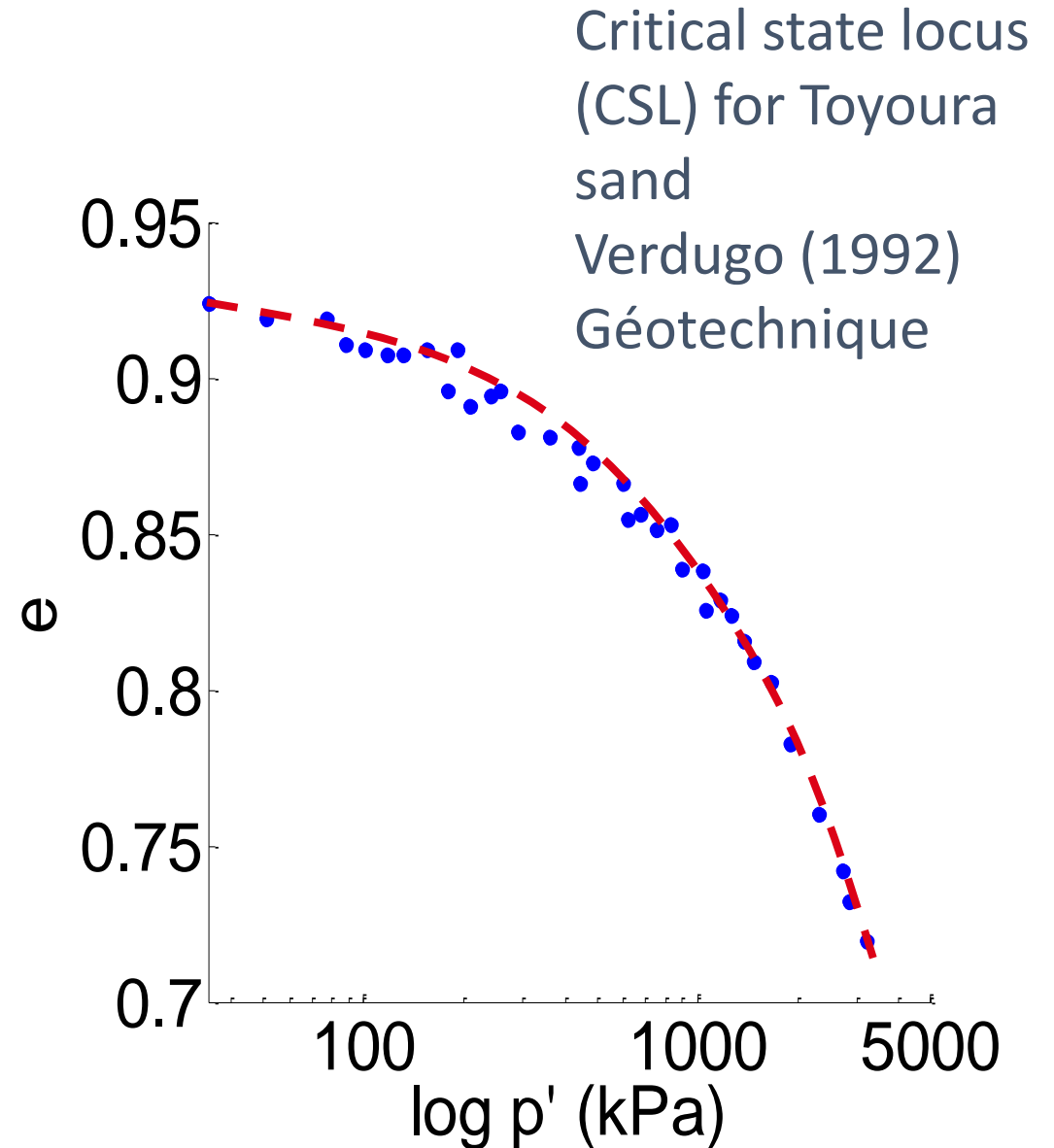
- You can achieve a range of densities by varying particle friction during preparation
- May detrimentally effect pre-peak response

Input parameters

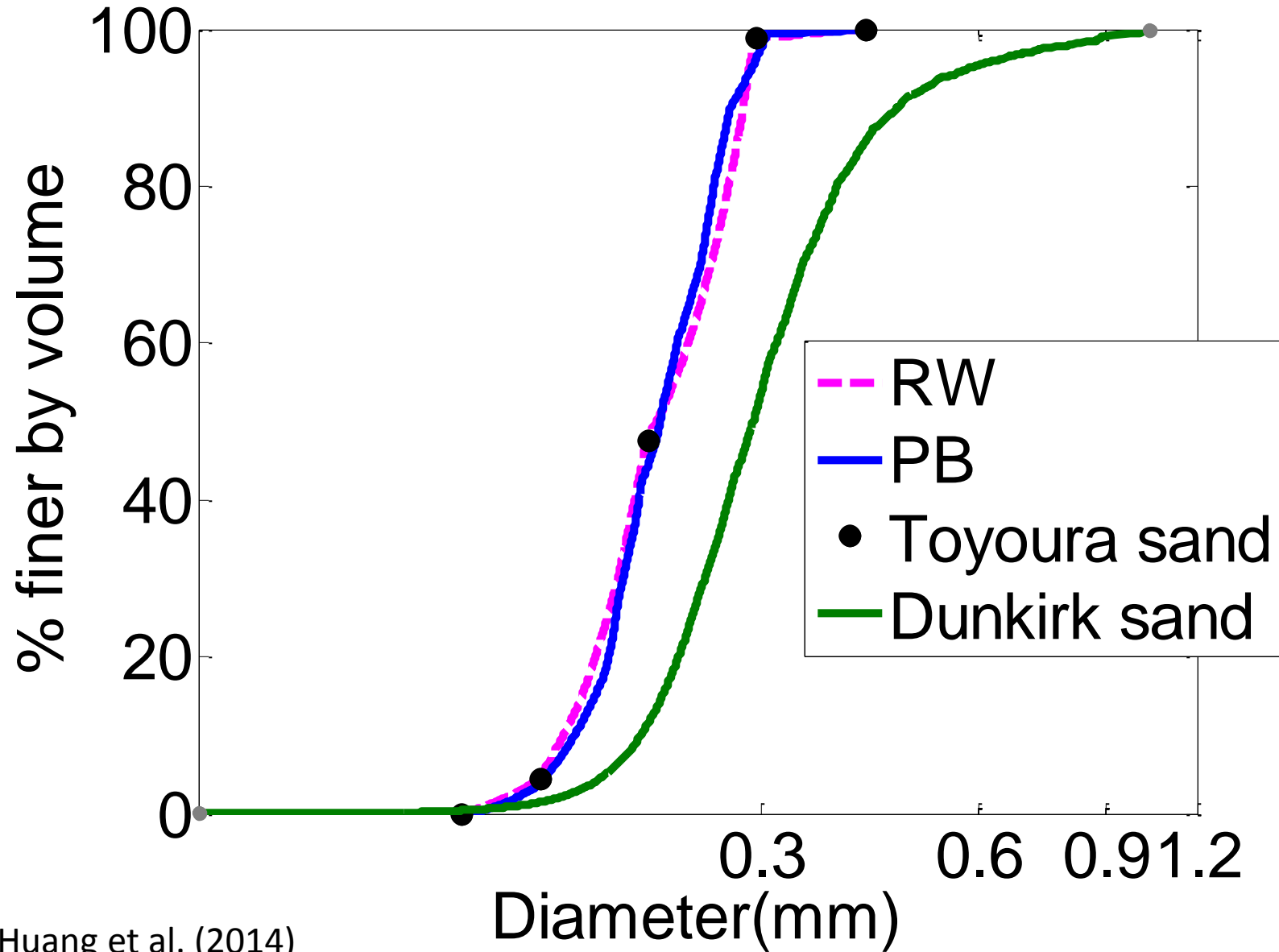
Influence of friction on critical state line

Critical state is a state where the soil deforms at constant stress and constant void ratio

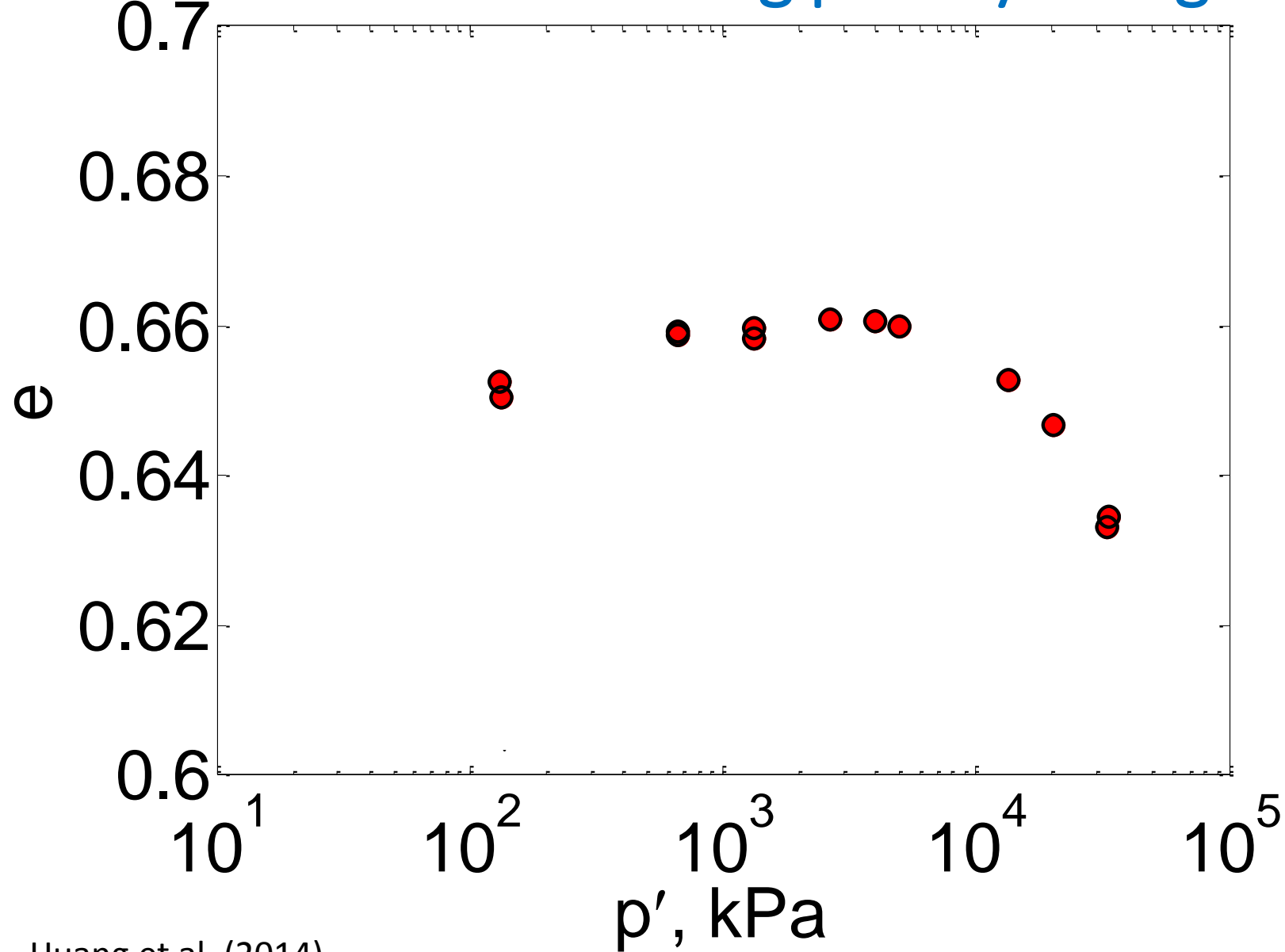
Been and Jefferies (1985)
Géotechnique



Influence of friction on critical state line

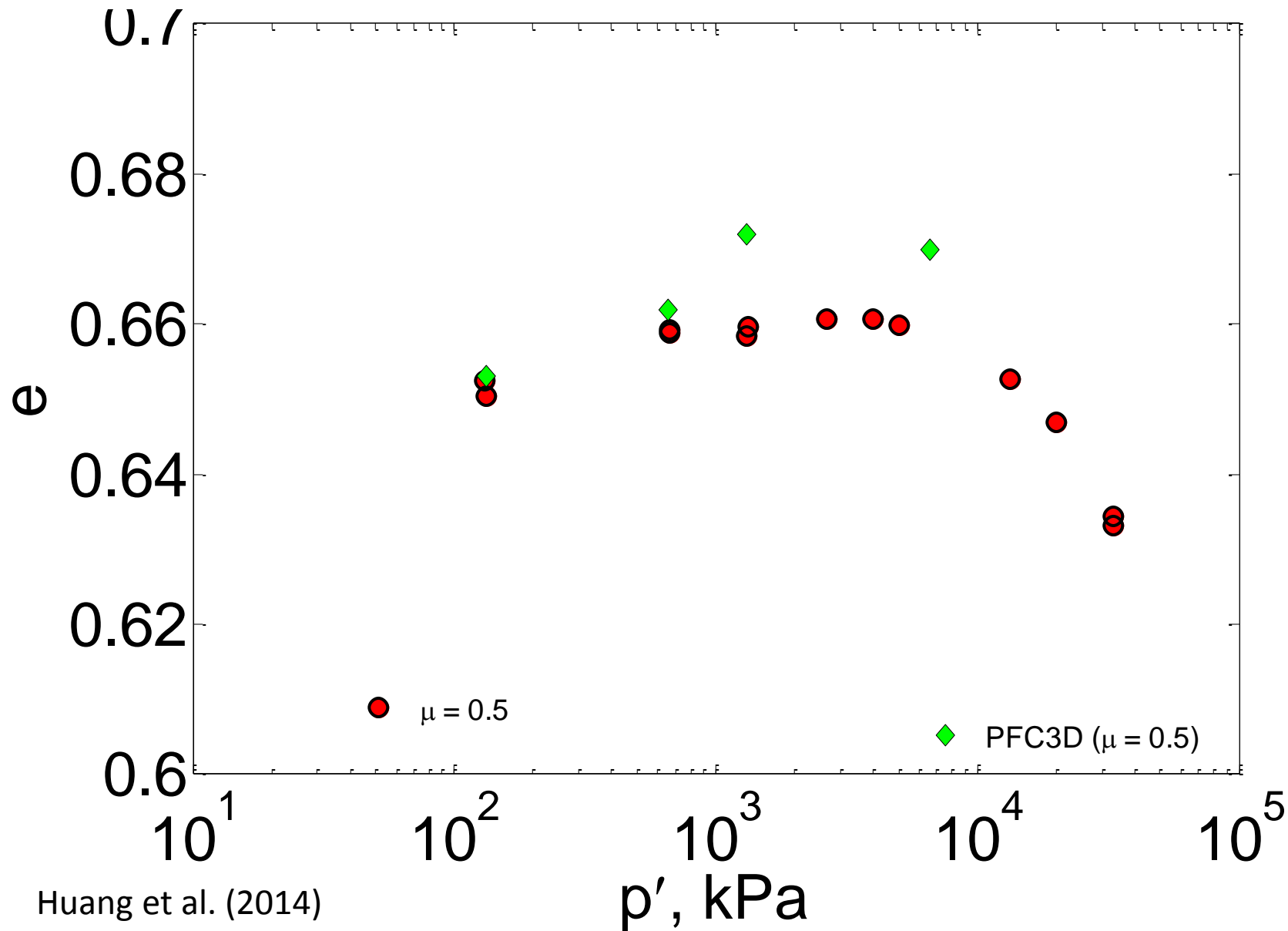


Critical state line e-log p' - Toyoura grading



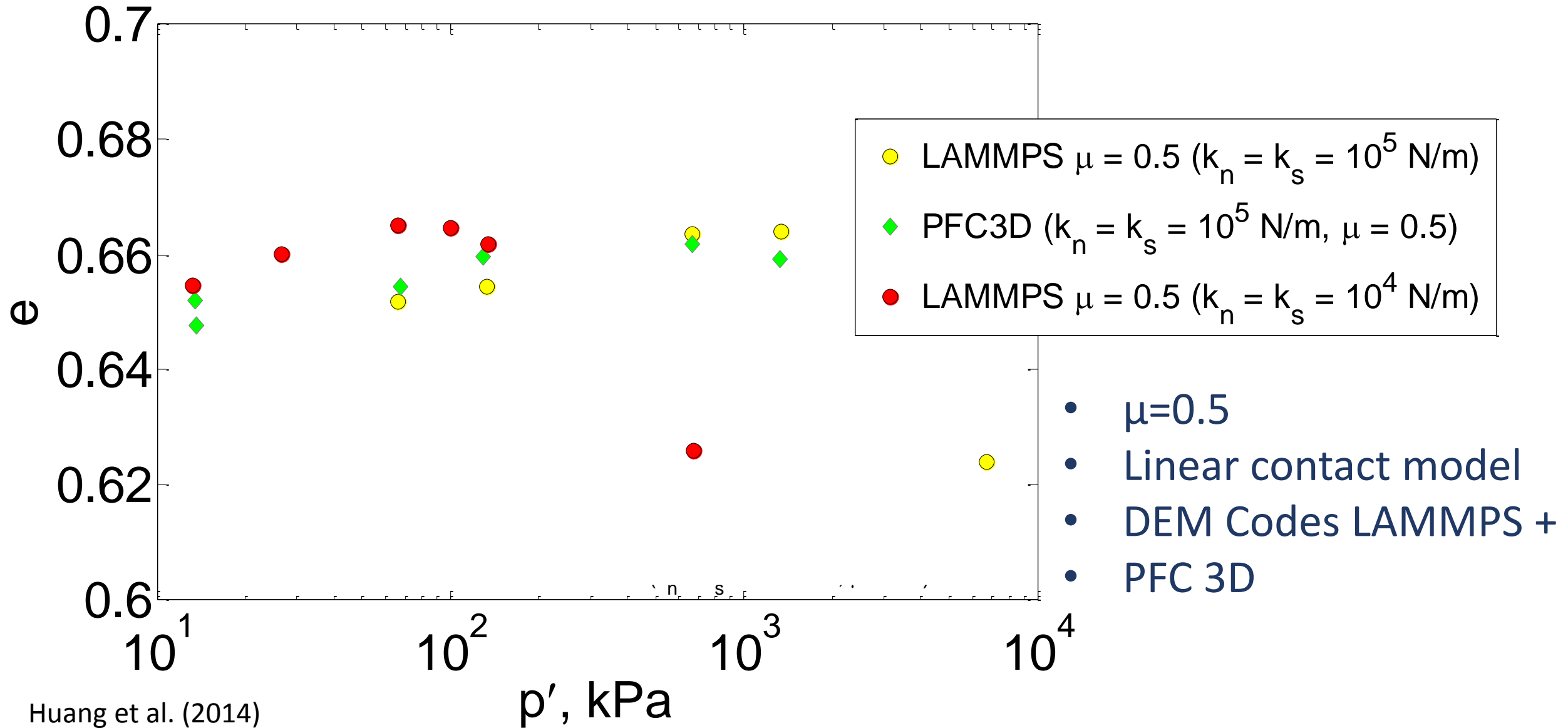
- Hertz-Mindlin contact model
- $\mu=0.5$
- DEM Code LAMMPS

Critical state line e-log p' - Toyoura grading

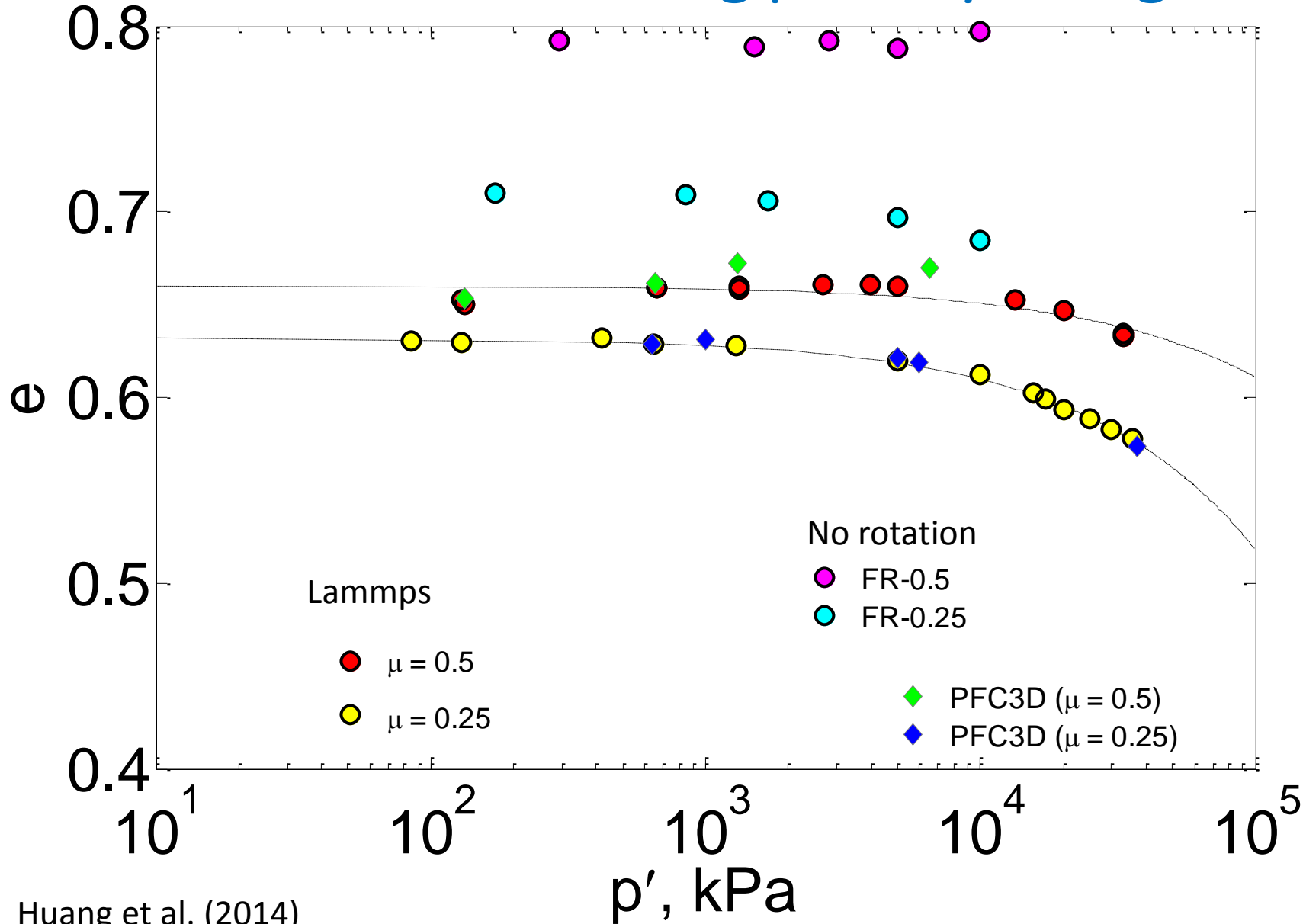


- Hertz-Mindlin contact model
- $\mu=0.5$
- DEM Codes LAMMPS + PFC 3D

Critical state line e-log p' - Toyoura grading

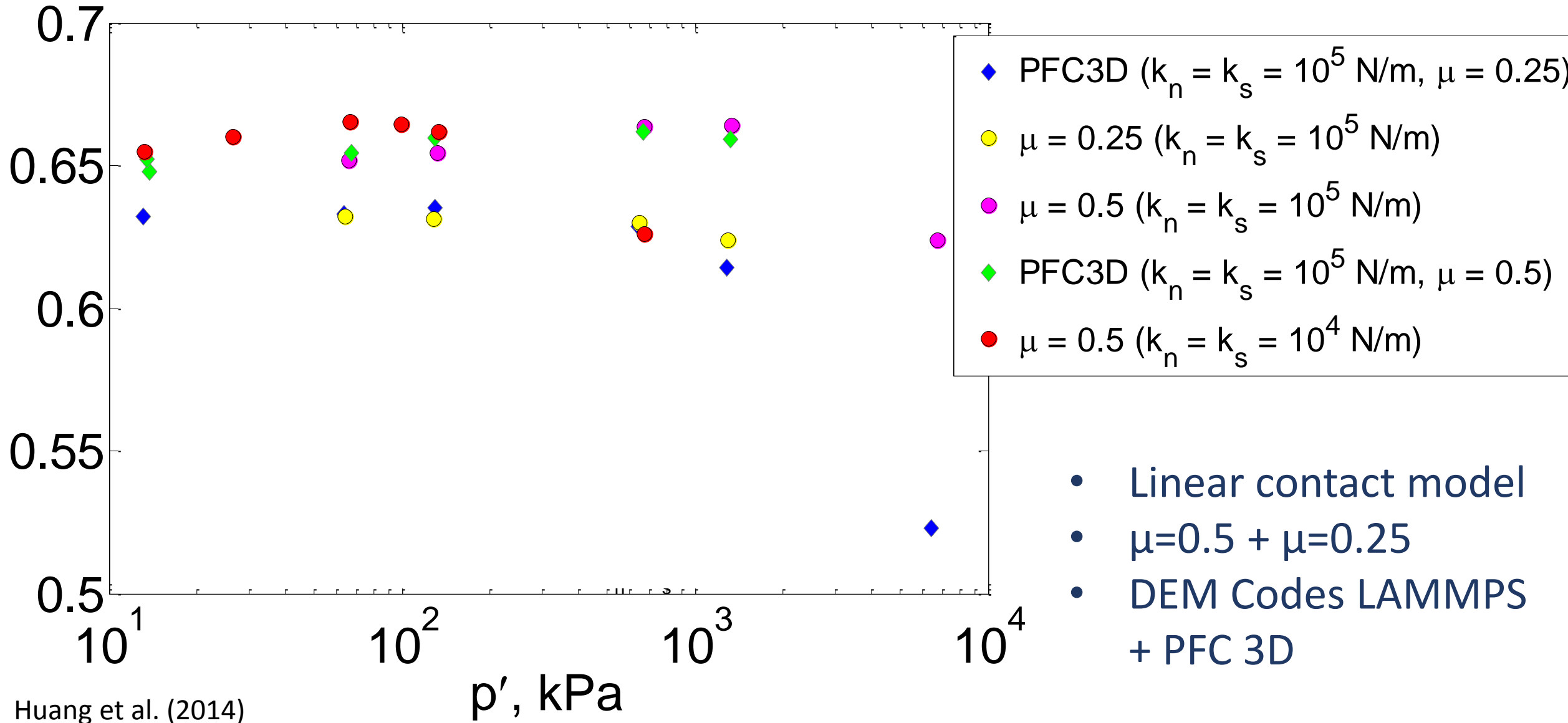


Critical state line e-log p' - Toyoura grading

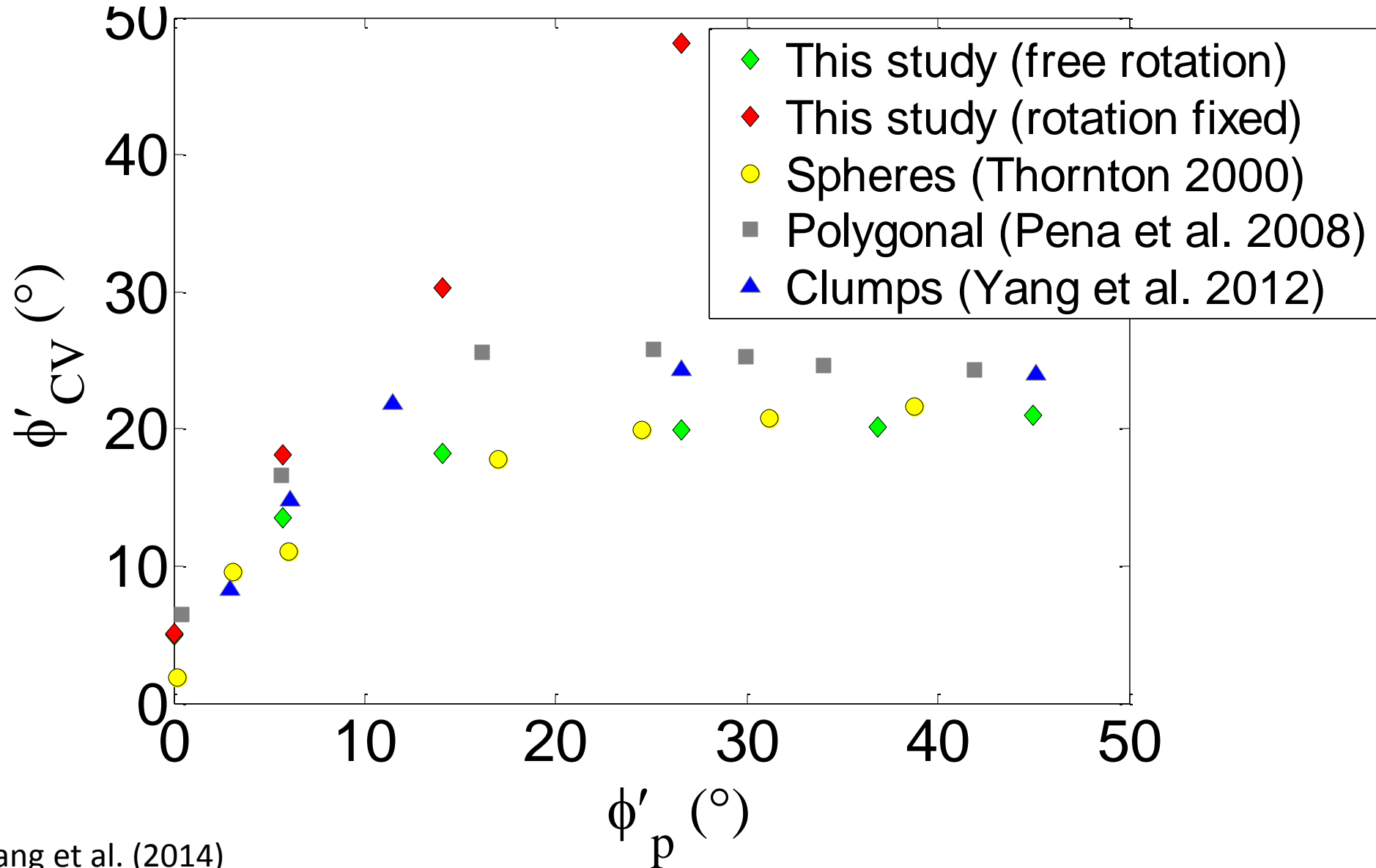


- Hertz-Mindlin contact model
- $\mu=0.5 + \mu=0.25$
- DEM Codes LAMMPS + PFC 3D

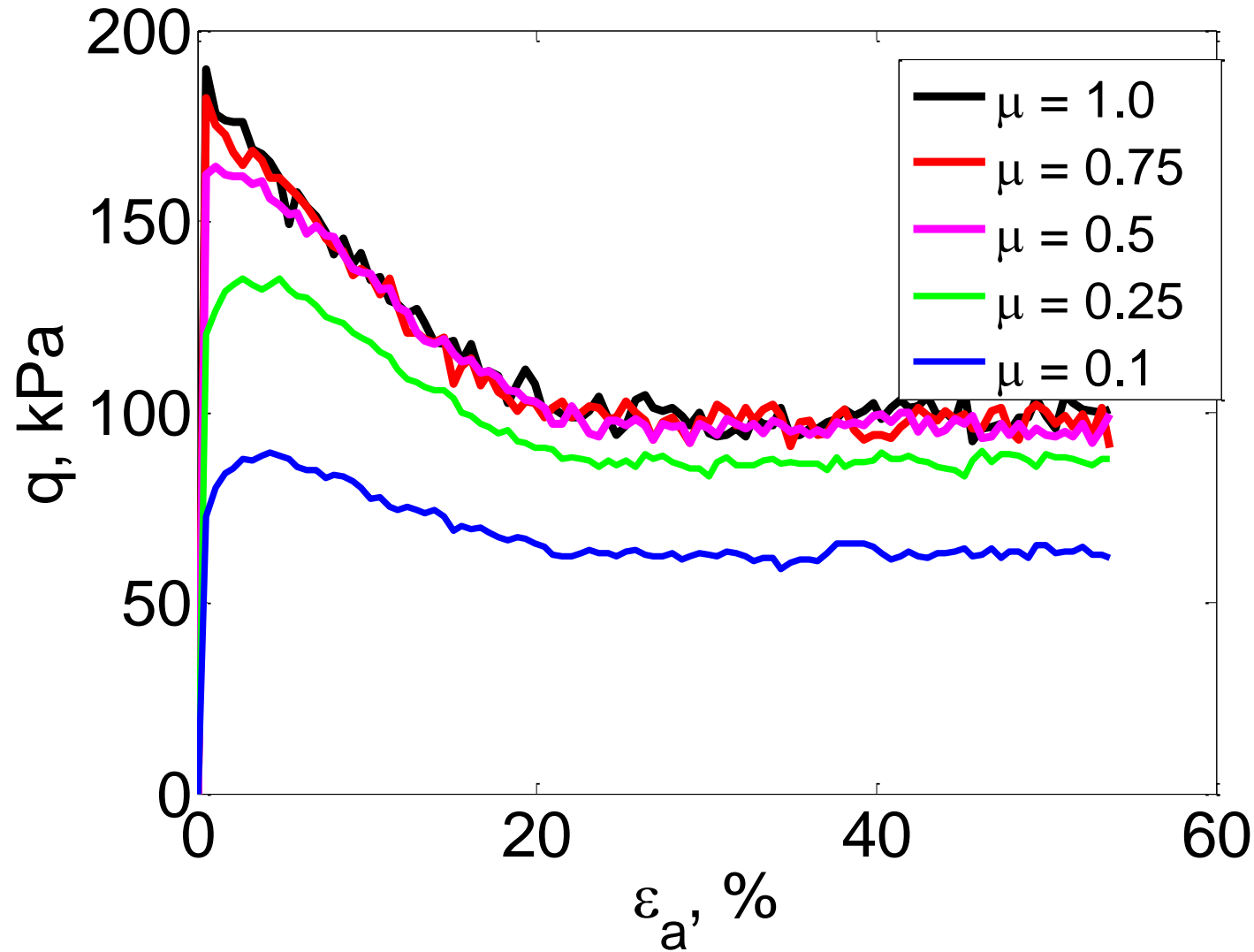
Critical state line e-log p' - Toyoura grading



Critical state angle of shearing resistance (ϕ_{cv})

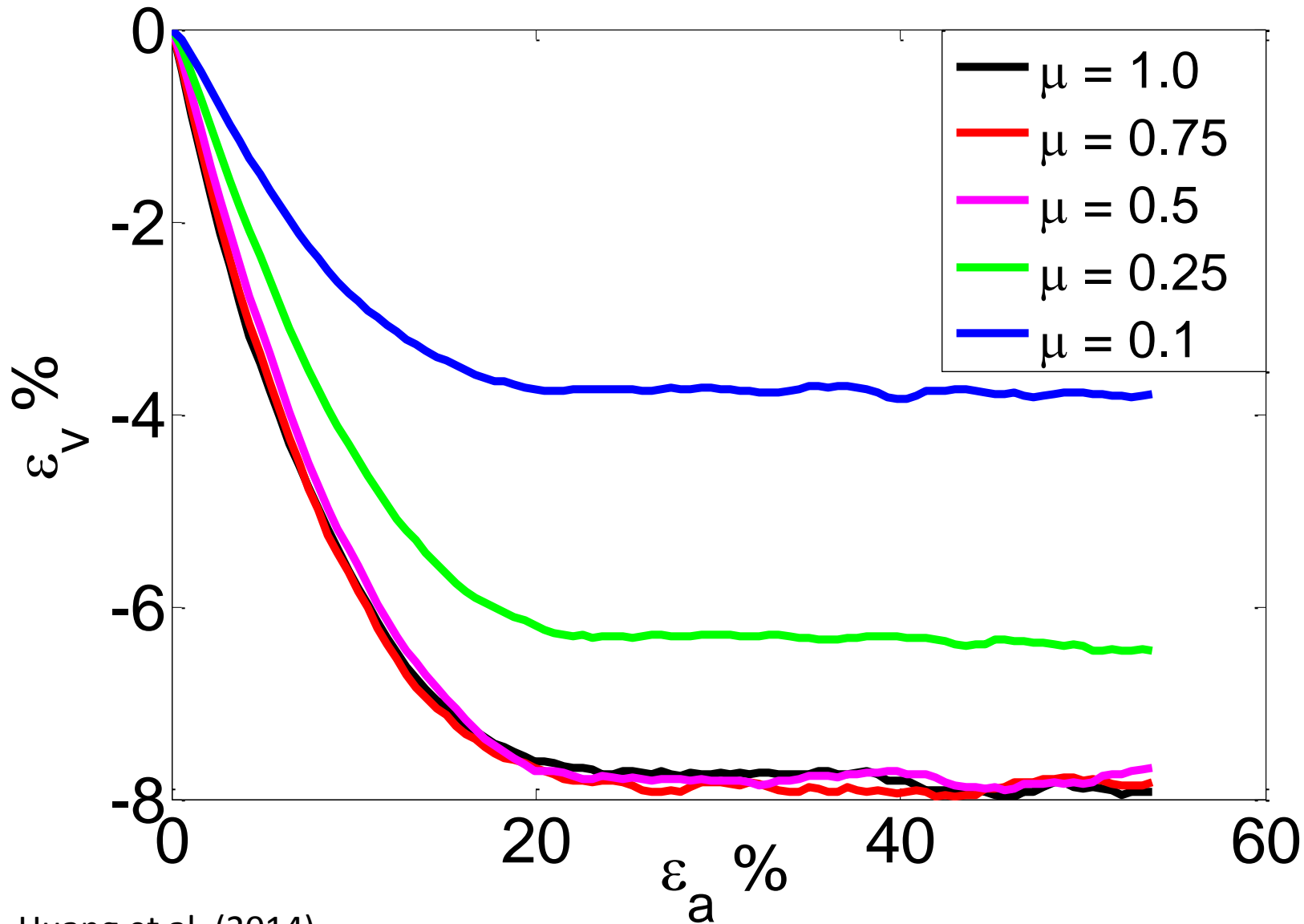


Response of samples consolidated with $\mu=0.1$



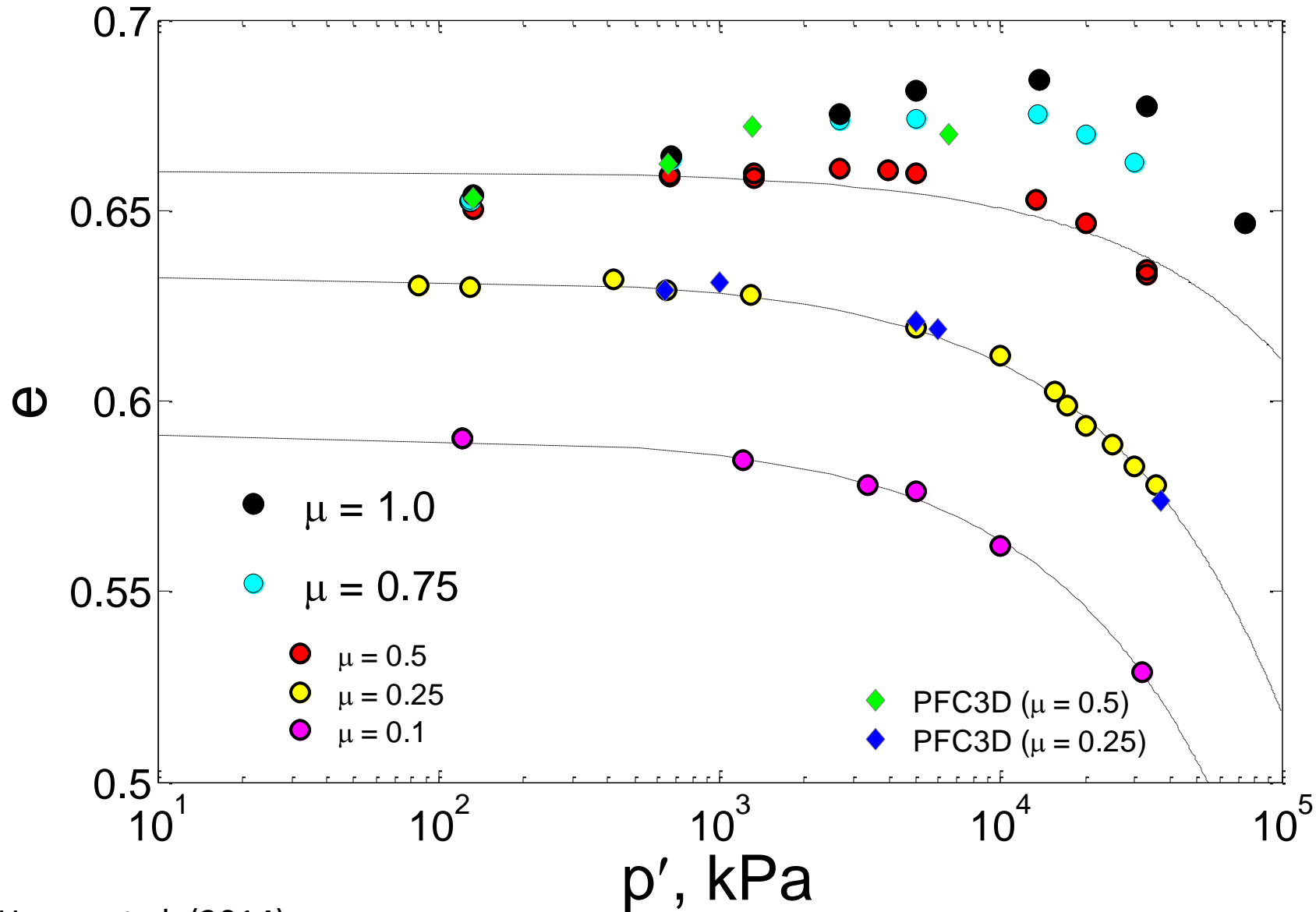
- Initial void ratio, $e_0 = 0.533$
- $\sigma'_3 = 100$ kPa

Response of samples consolidated with $\mu=0.1$



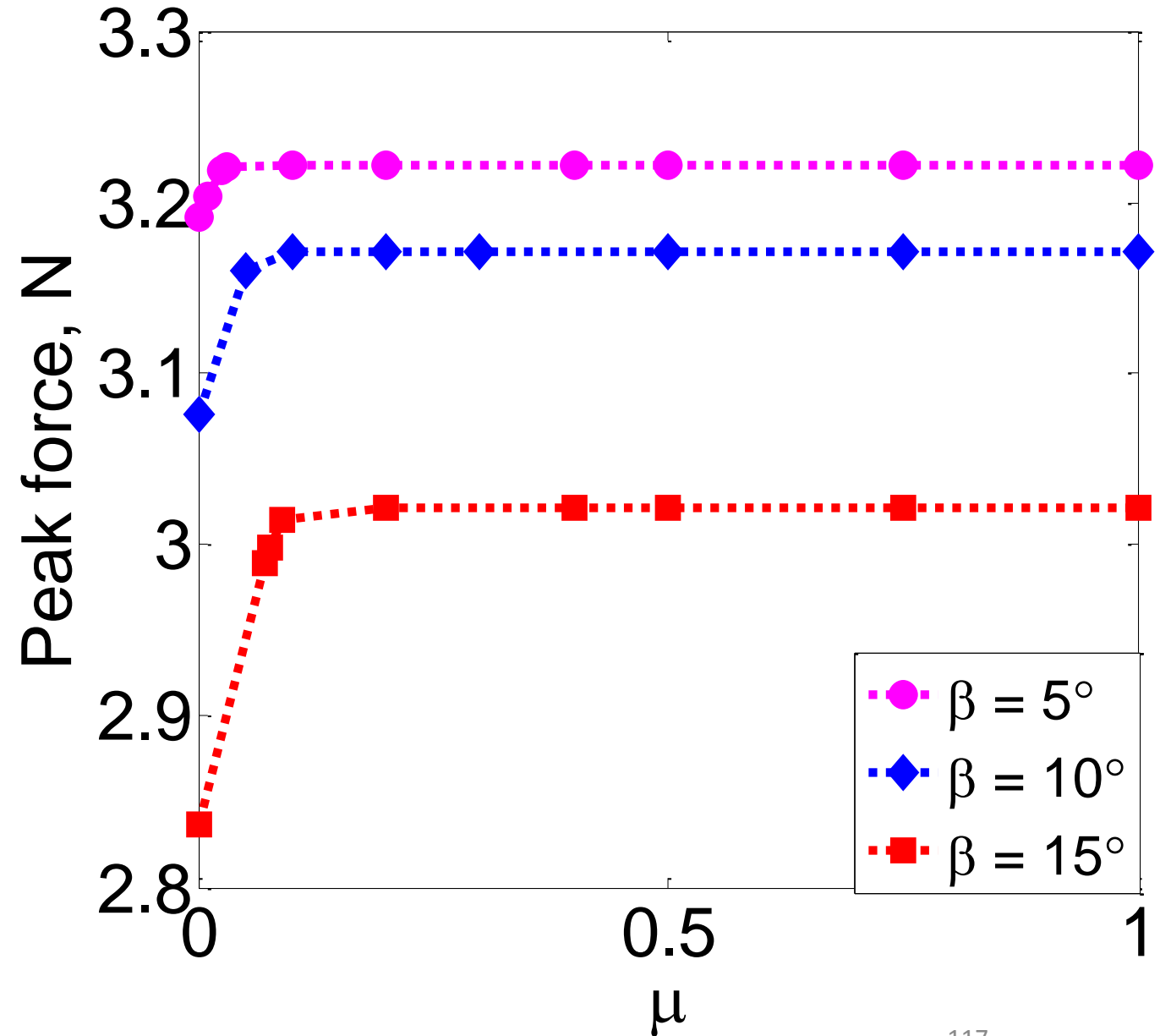
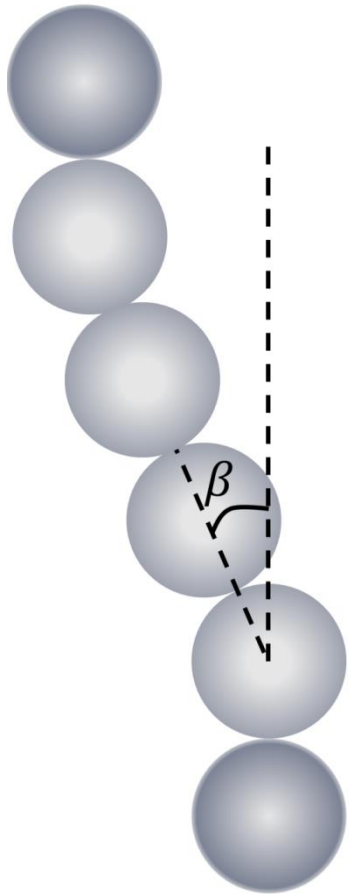
- Initial void ratio, $e_0 = 0.533$
- $\sigma'_3 = 100$ kPa

Critical state line in e-log p' space

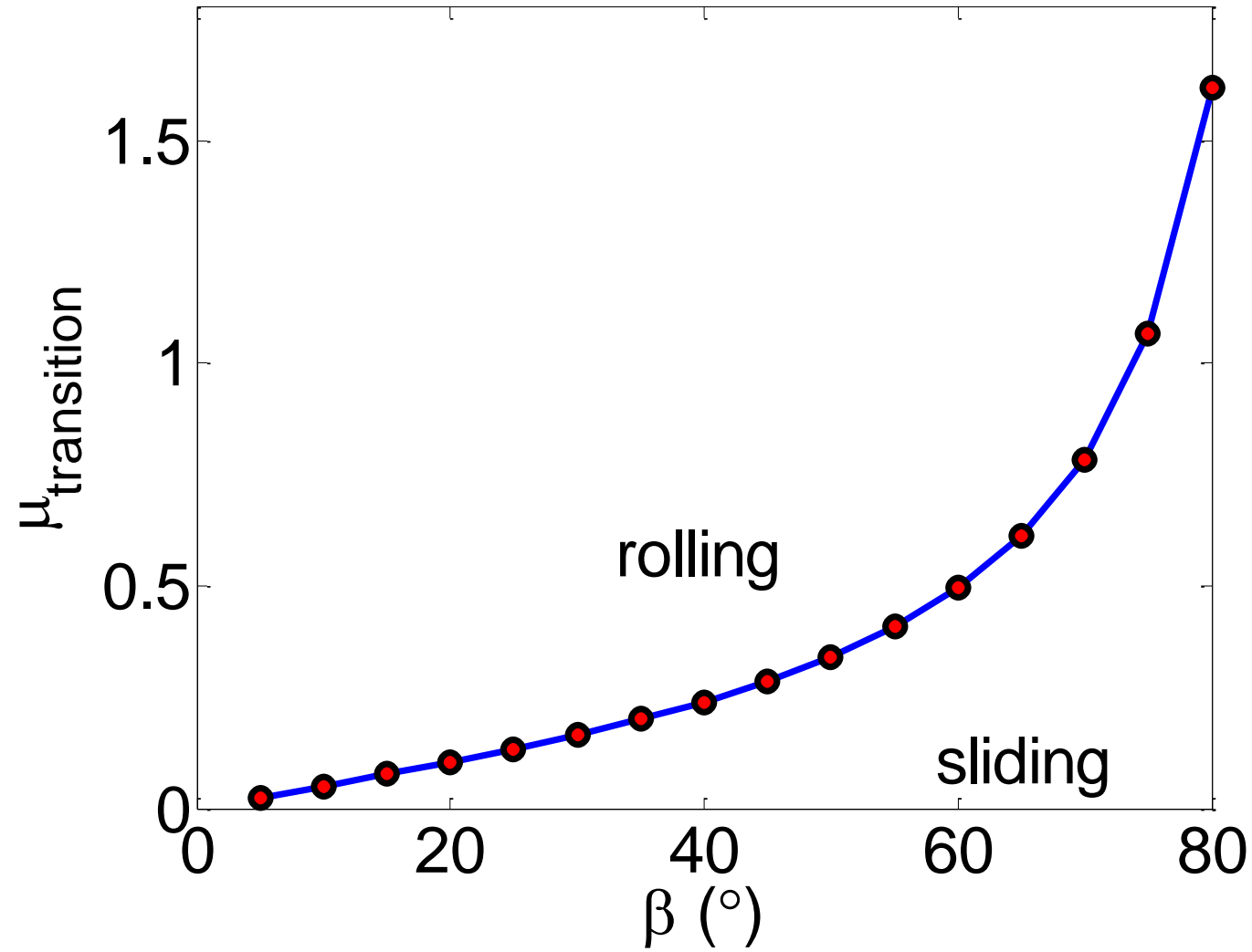
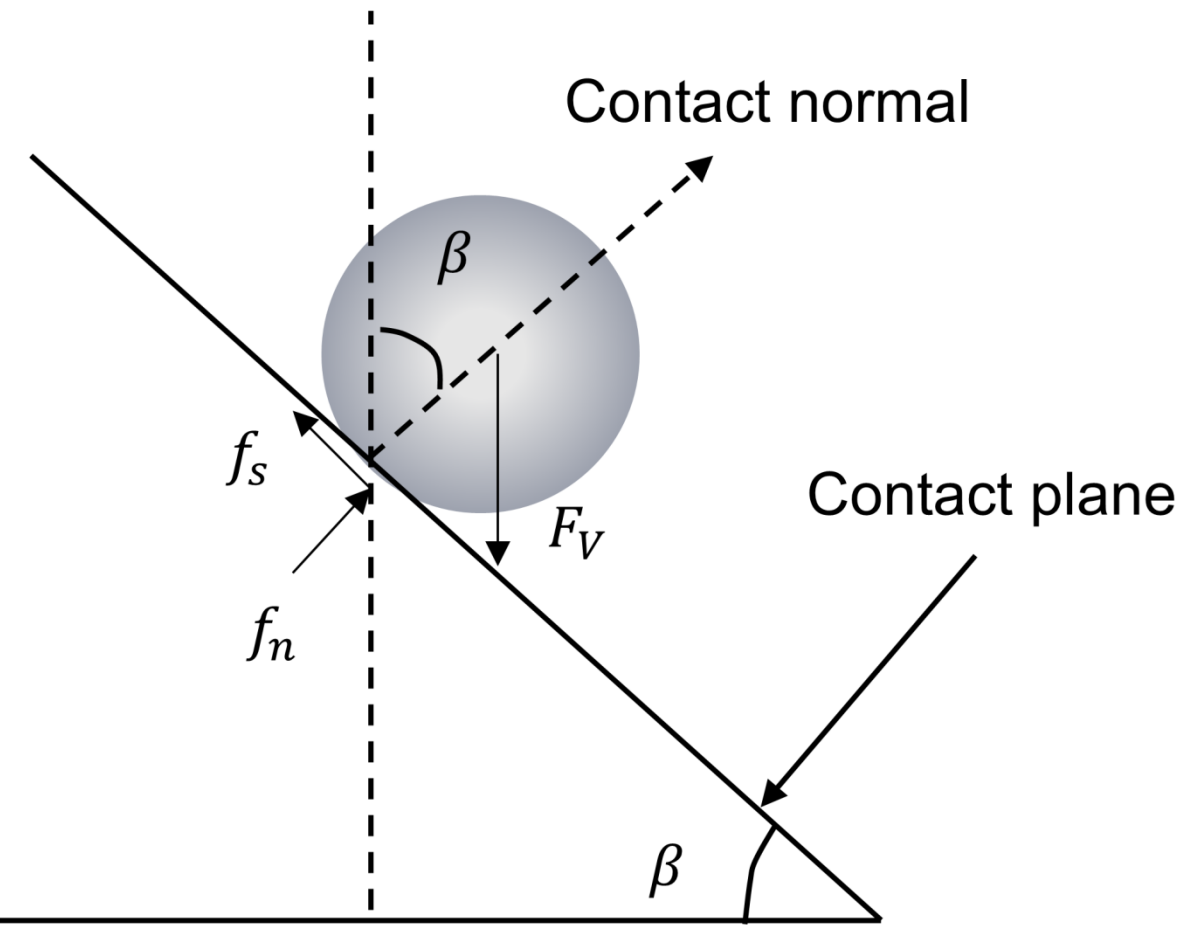


- Hertz-Mindlin contact model
- DEM Codes LAMMPS + PFC 3D

Single force chain stability



Transition from sliding to rolling



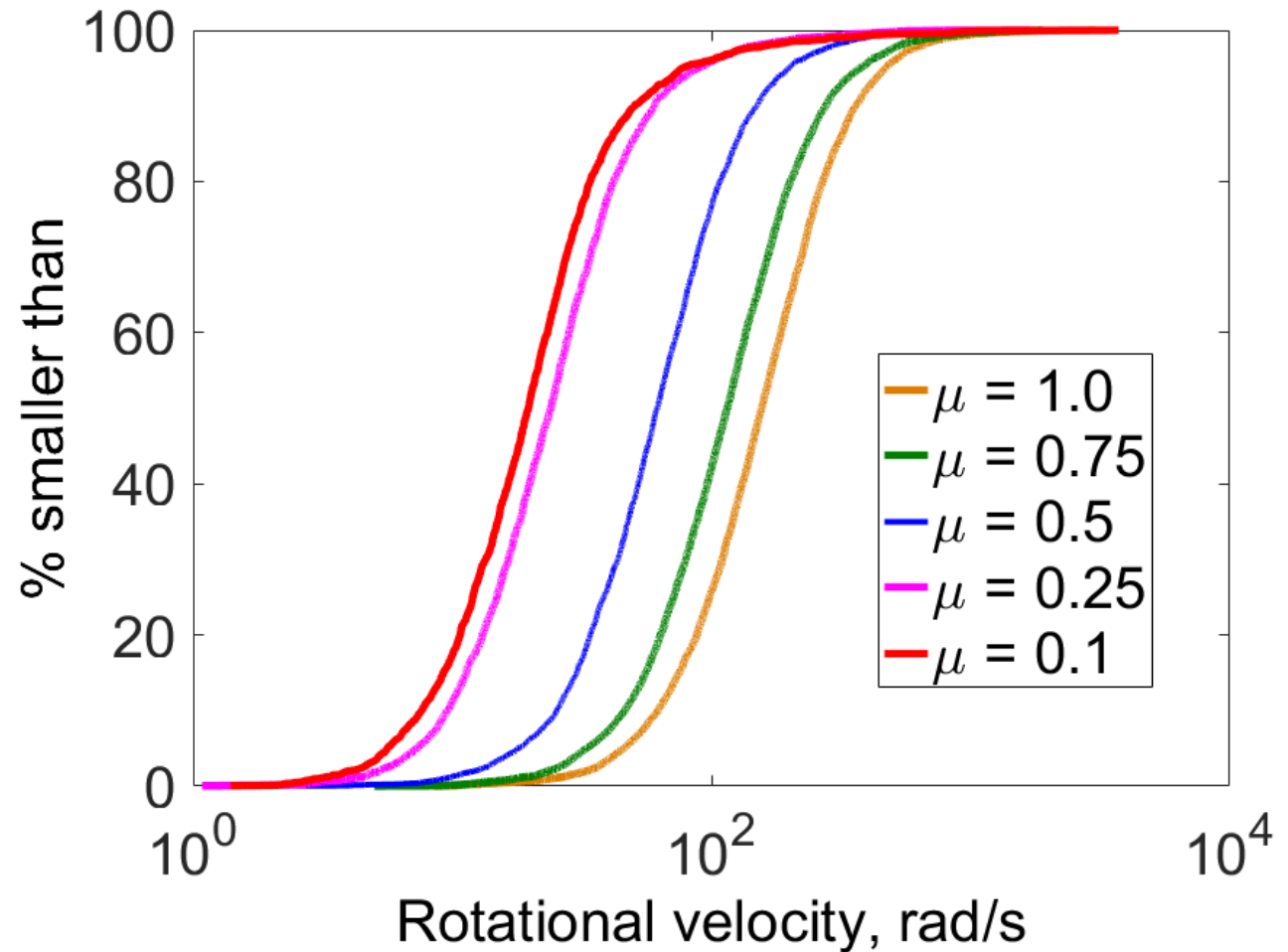
Transition from sliding to rolling

Effect of μ on the angular velocity

LAMMPS simulations

$e_0 = 0.533$, $\sigma'_3 = 100$ kPa

Cumulative distribution
of resultant angular velocities at $\varepsilon_a = 50\%$.



Conclusions

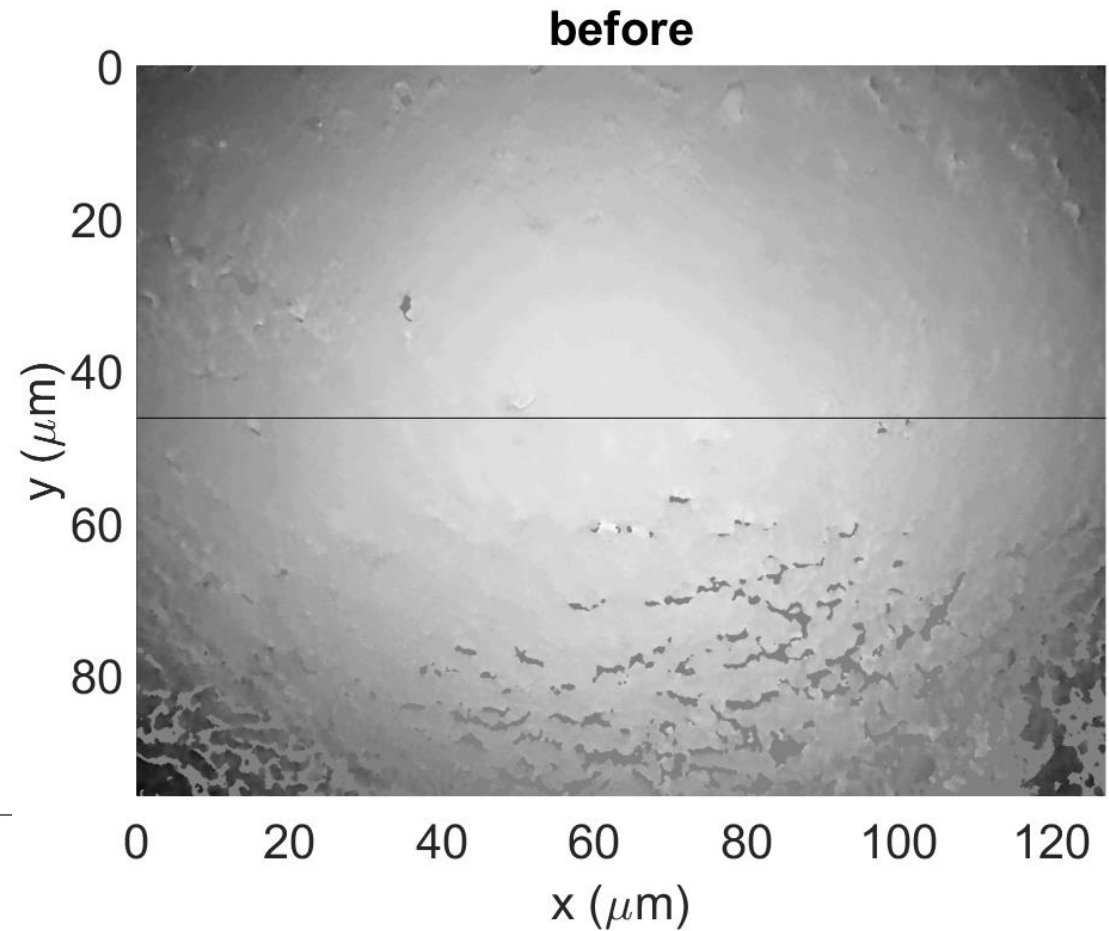
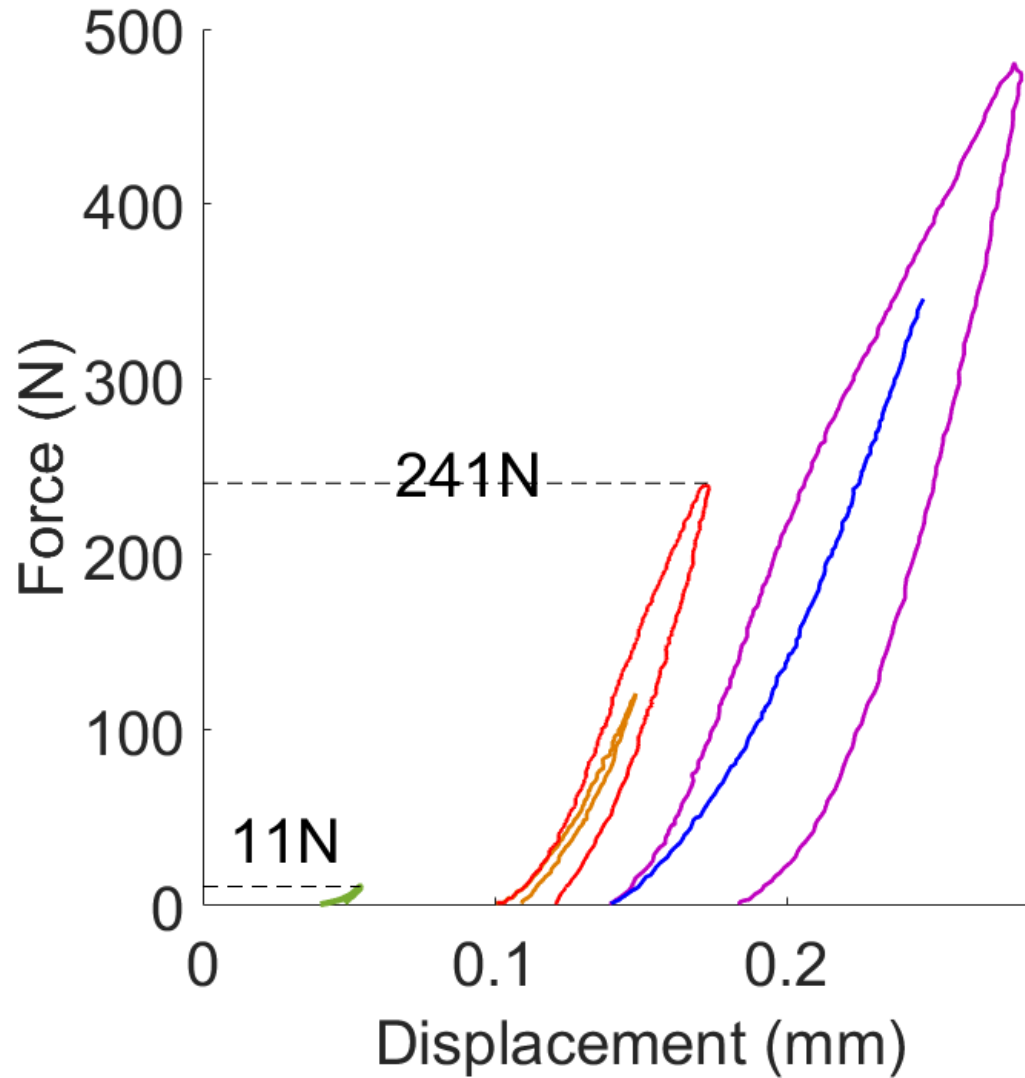
- Using a high coefficient of friction (>0.5) with freely rotating spheres gives a response that is not representative of soil
- Explanation seems to be a transition from sliding to rolling at the contact points
- Use of two codes enhanced our confidence in observations
- Supplemental abstract analyses of small elements of the system were very useful

Contact Model

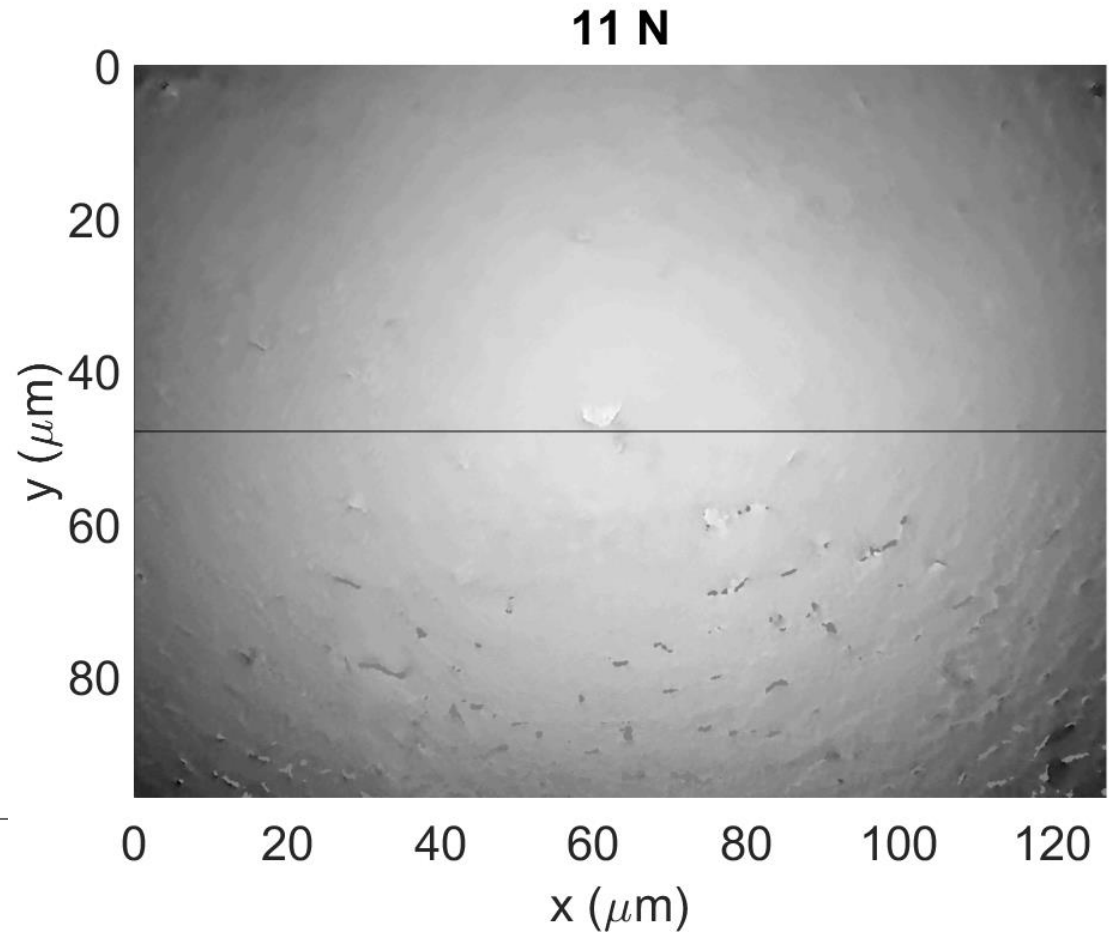
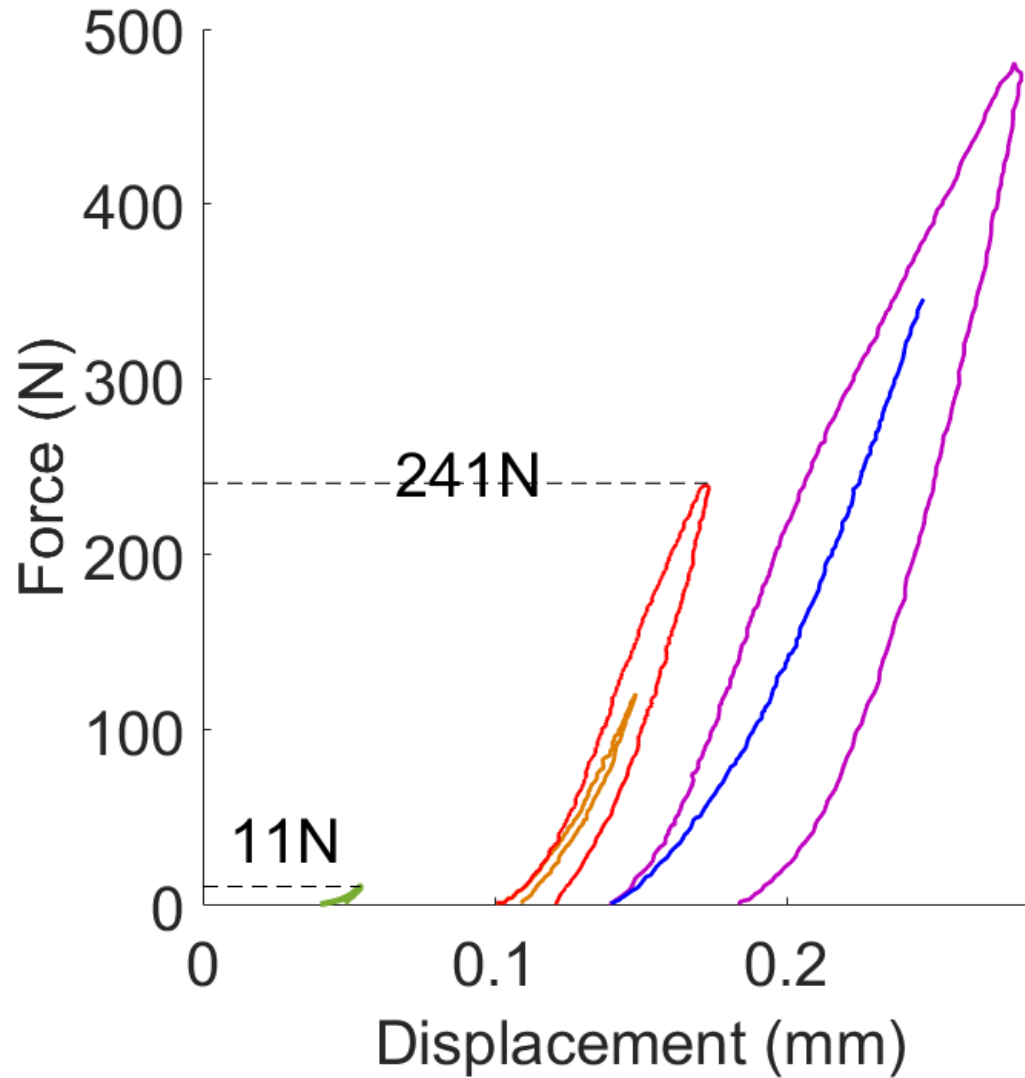
Deviations from Hertzian contact mechanics

- Rough surface effects
- Partial slip effect

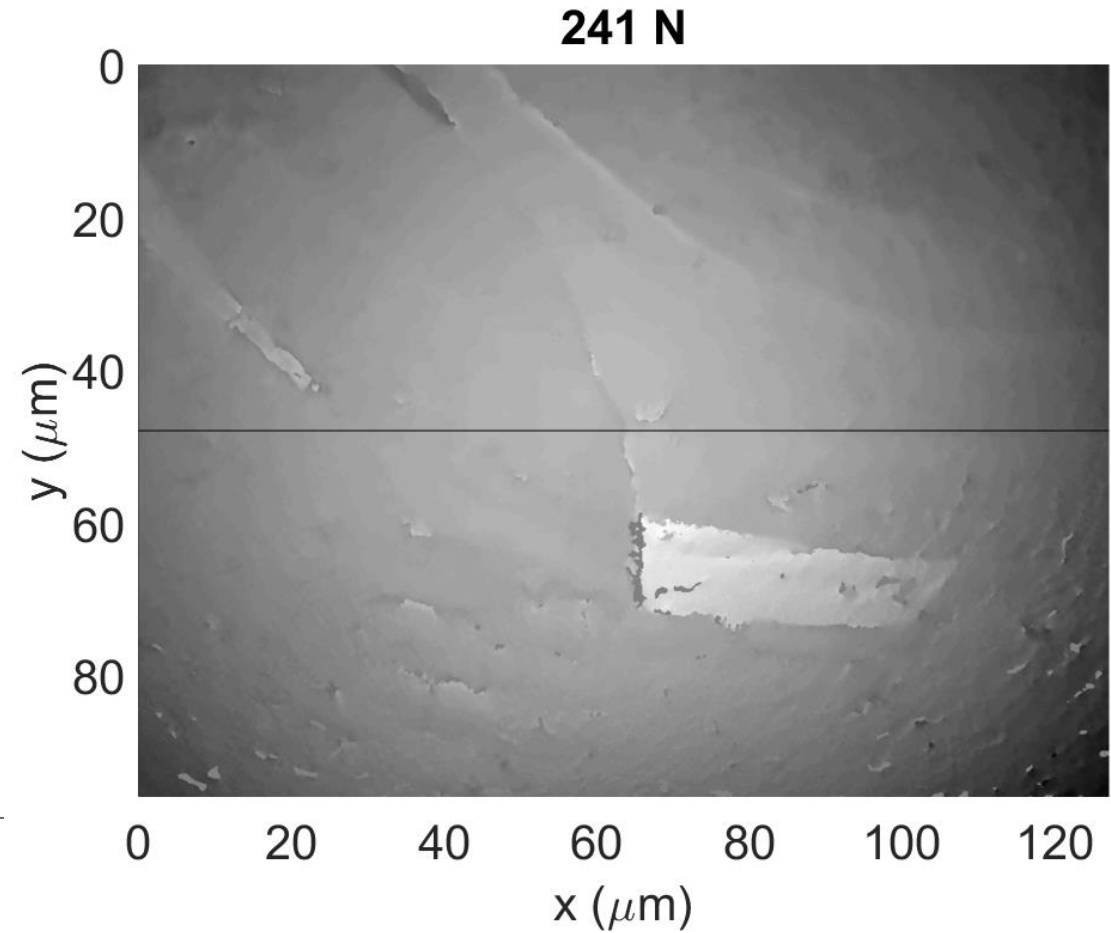
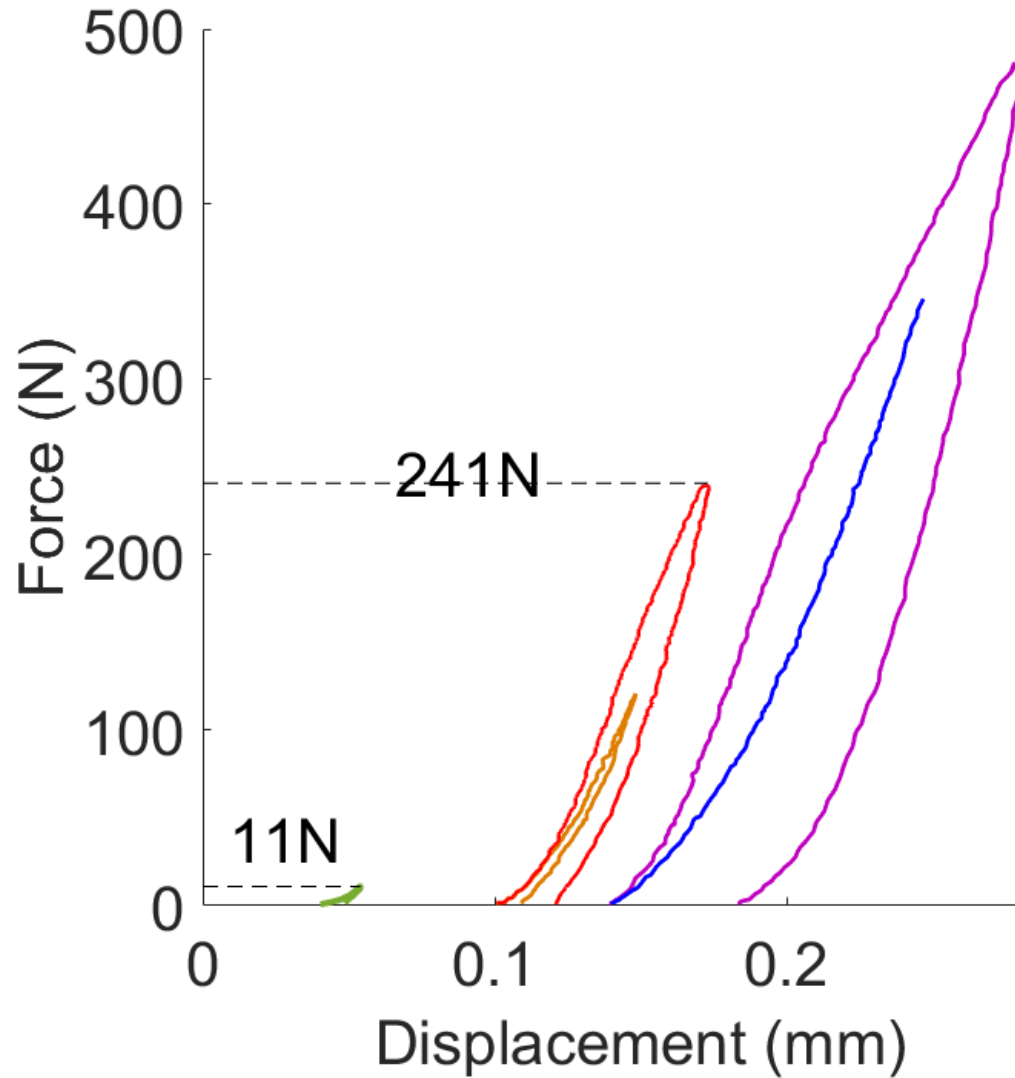
Deviations from Hertzian contact mechanics



Rough surface effects



Rough surface effects



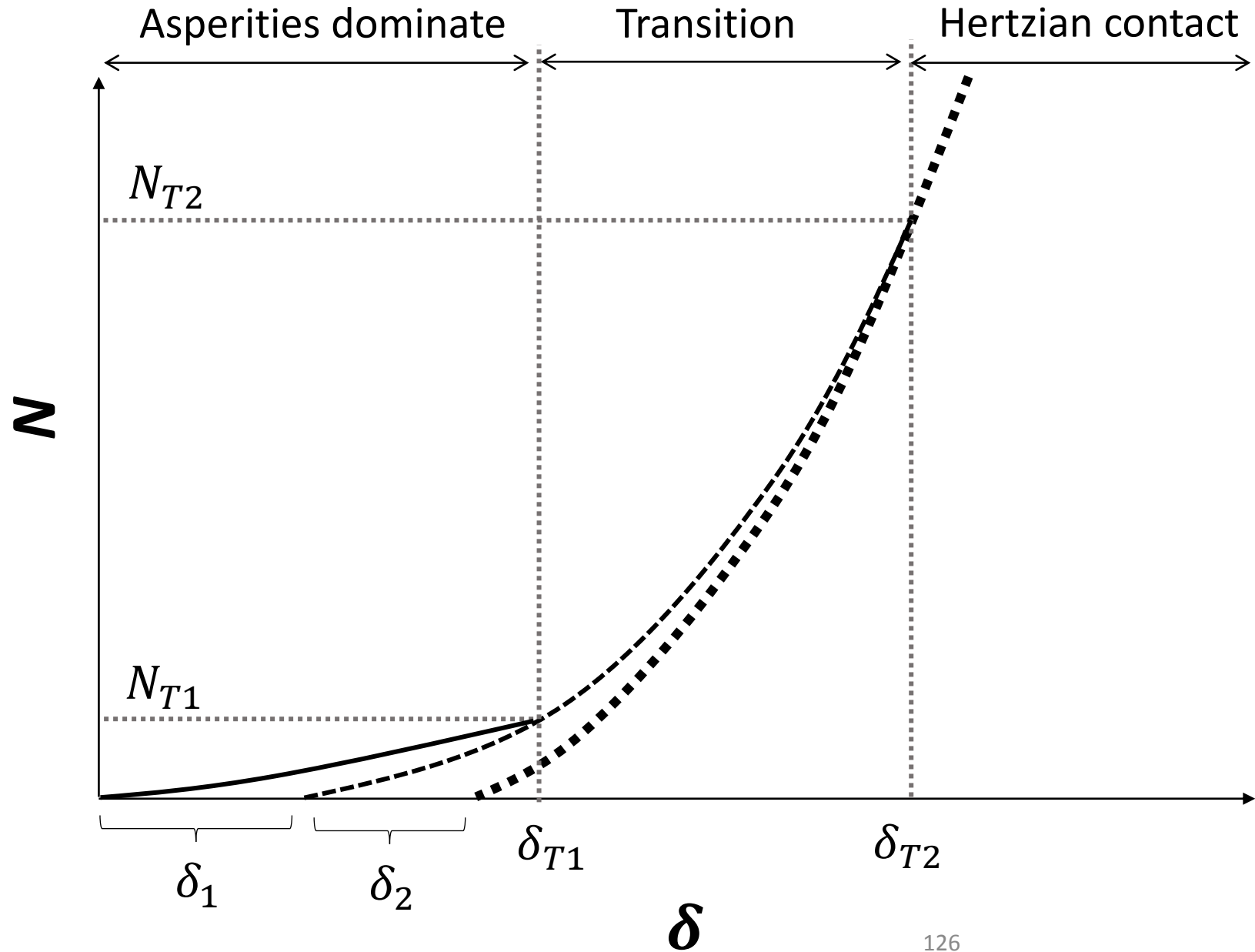
Rough surface effects

Rough surface model

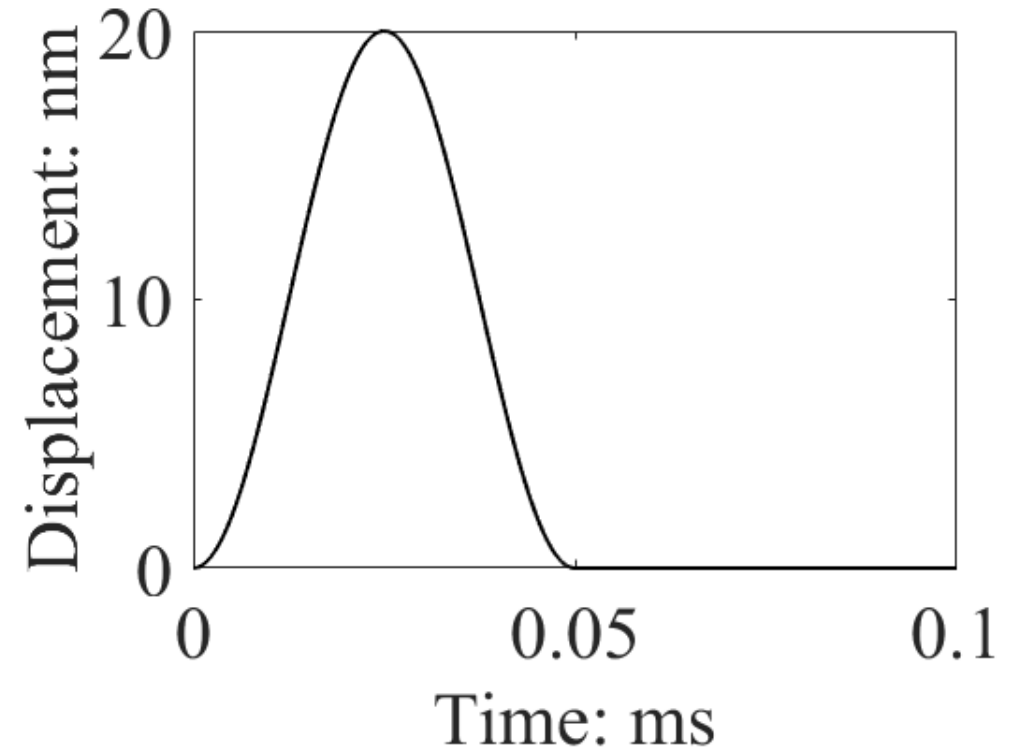
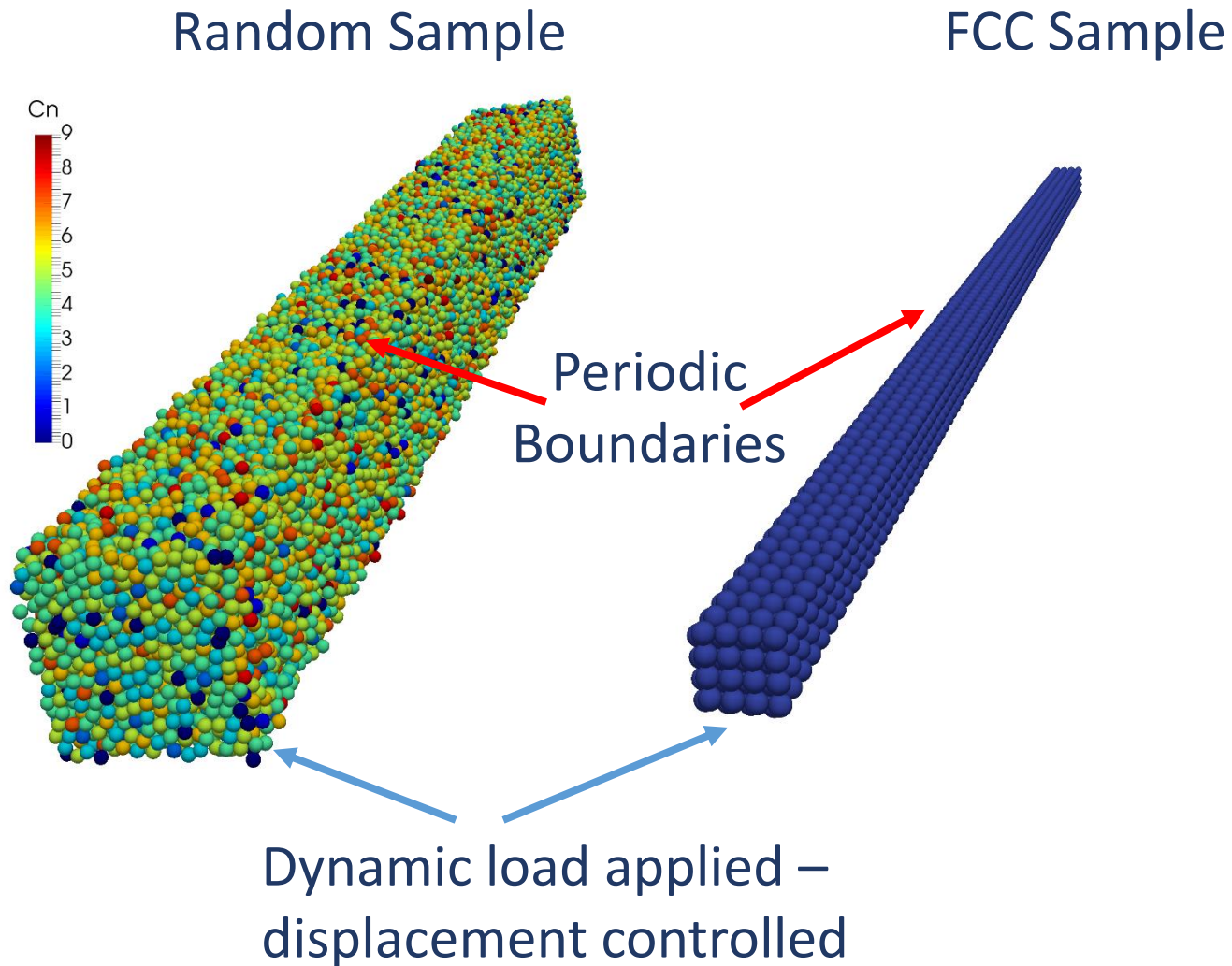
N_{T1} and N_{T2} depend on S_q

S_q =root mean square of surface roughness

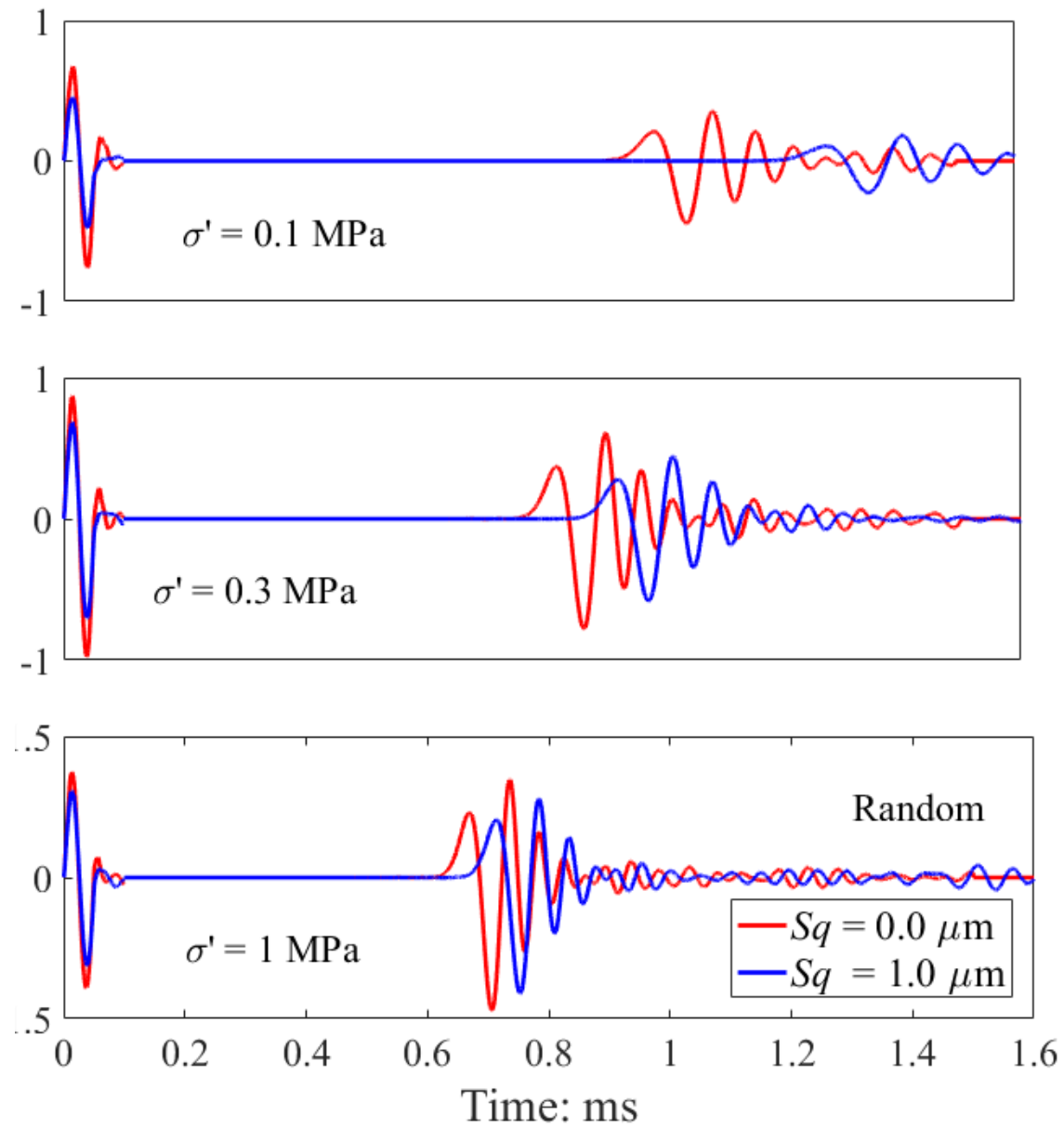
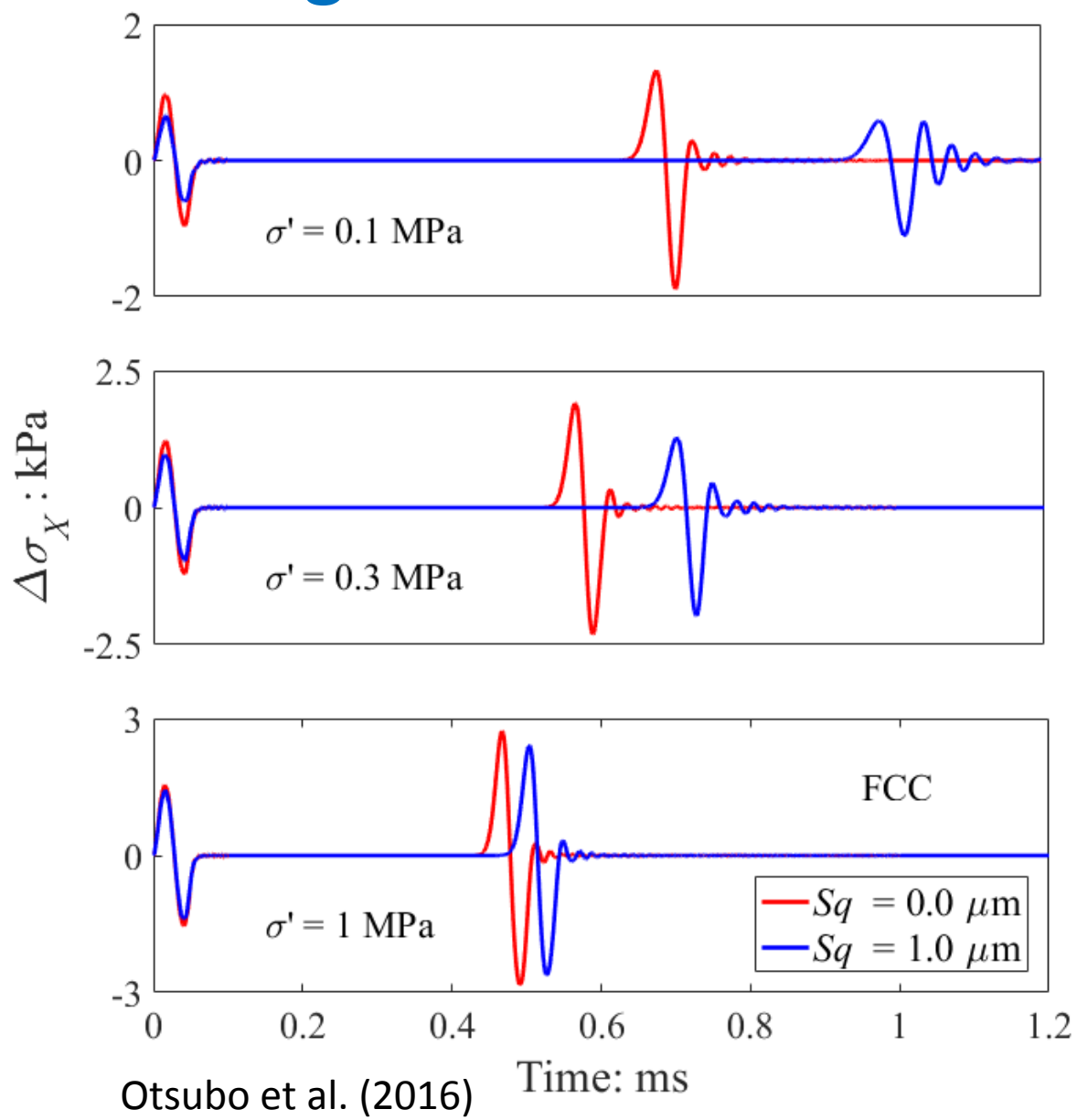
Smooth case $S_q=0.0$



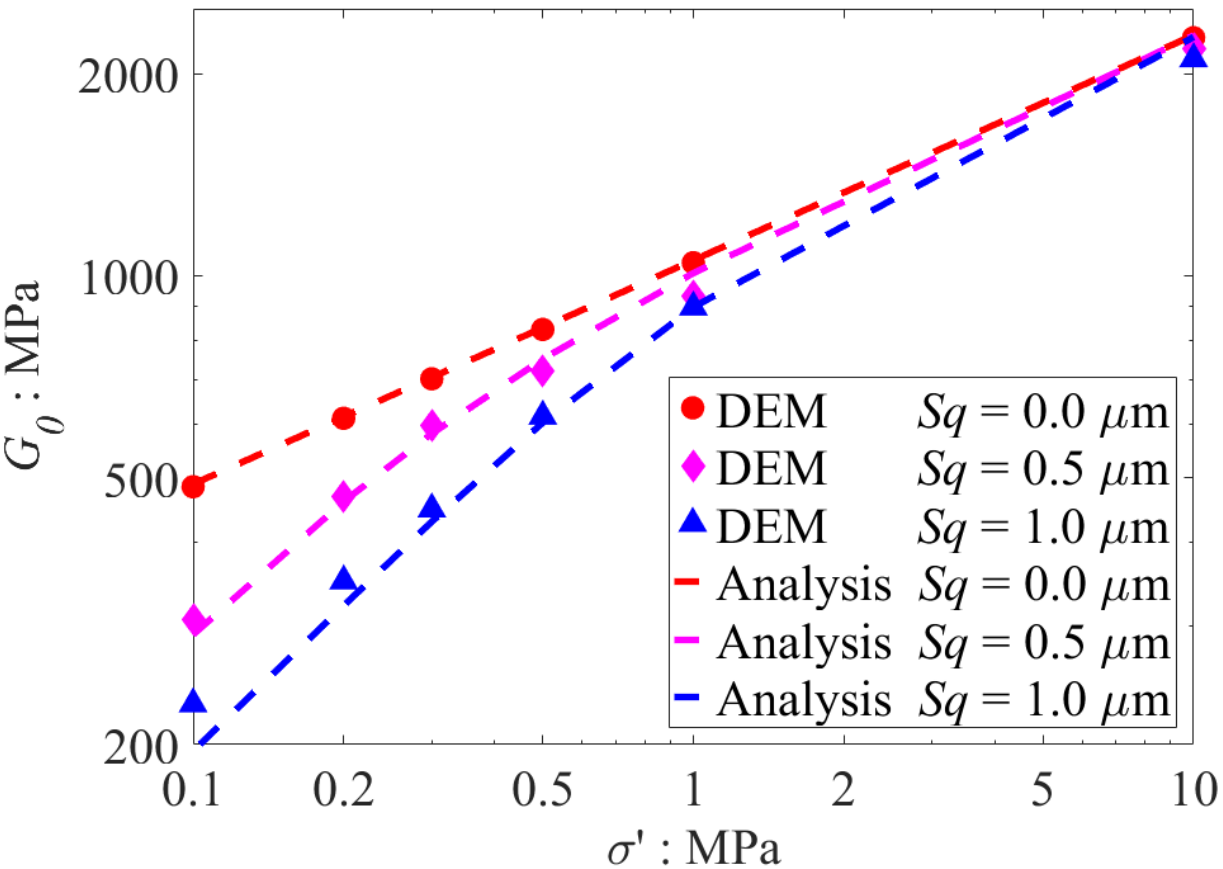
Rough surface effects



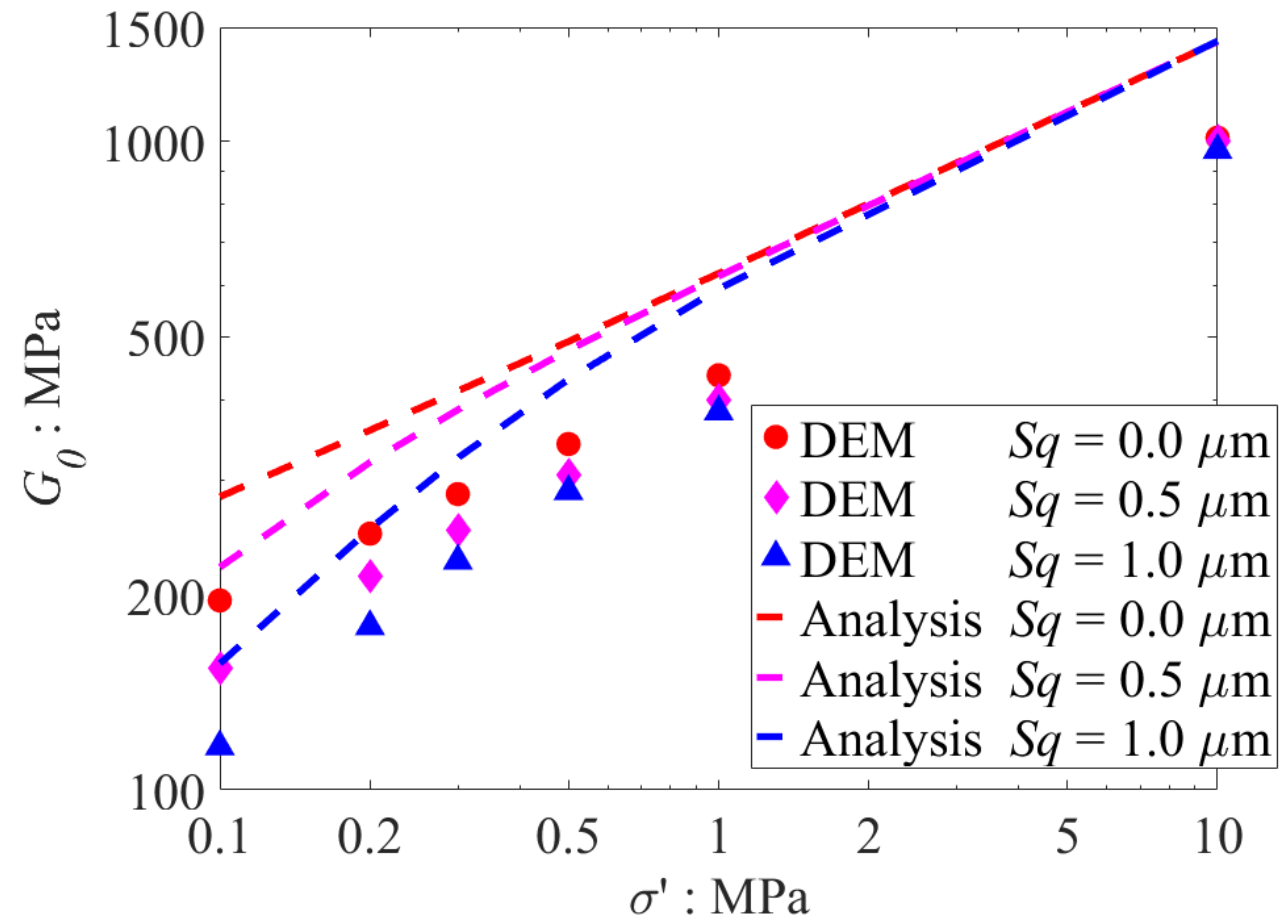
Rough surface effects



Rough surface effects



FCC Sample



Random Sample

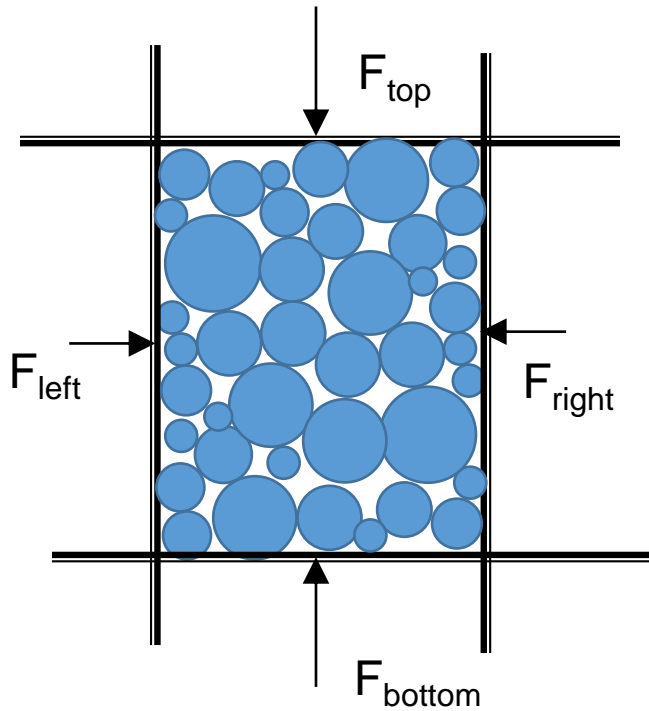
Conclusions

- Hertz-Mindlin contact model ideal – even for glass ballotini
- Surface roughness - measurable effect on stiffness
- Partial slip – more subtle effect, less significant for regular, lattice packings

Running the simulation

Ensuring quasi-static behaviour

Simple approach – check force balance



$$\frac{F_{top} - F_{bottom}}{0.5(F_{top} + F_{bottom})} \leq \varepsilon \quad \varepsilon = \text{tolerance}$$

Challenges:

How to specify tolerance?

What to do with periodic boundaries?

Inertial number / Inertia number

$$I = \dot{\epsilon} d \sqrt{\frac{\rho}{p'}}$$

I quantifies the ratio of the inertial forces to the imposed forces

I = inertial number

$\dot{\epsilon}$ = strain rate

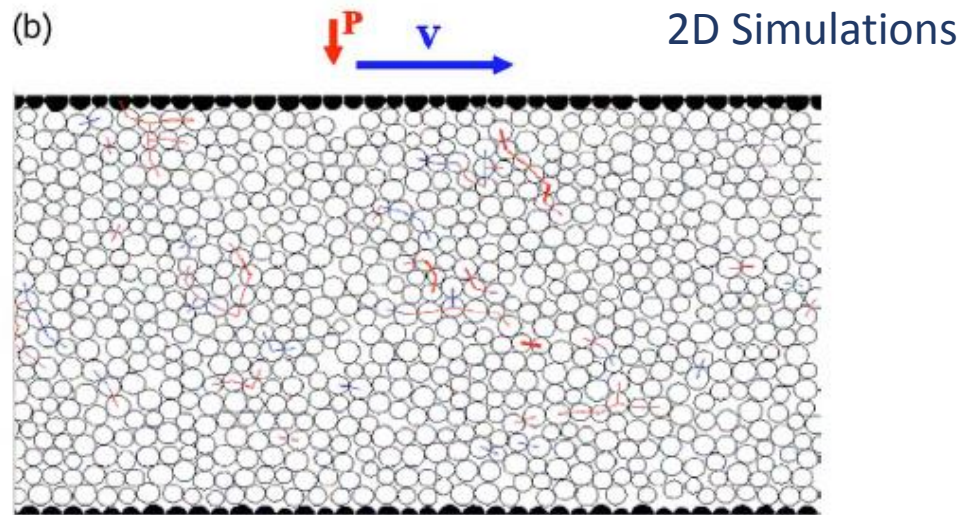
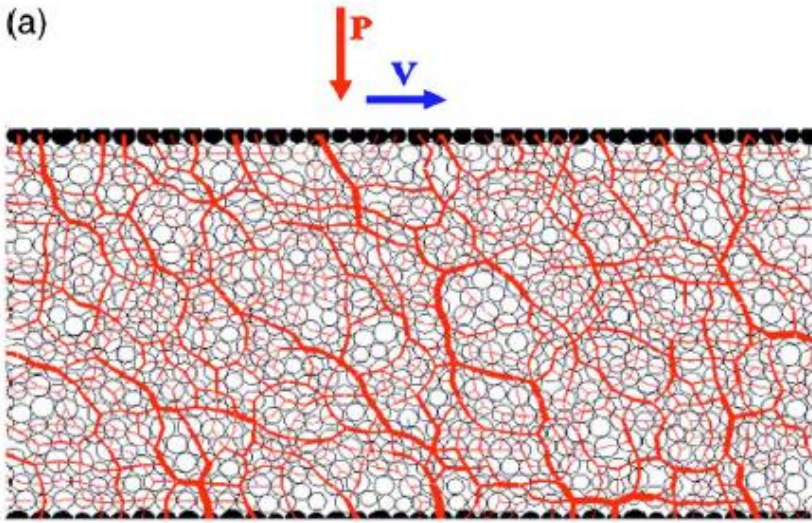
d = mean particle size

ρ = particle density

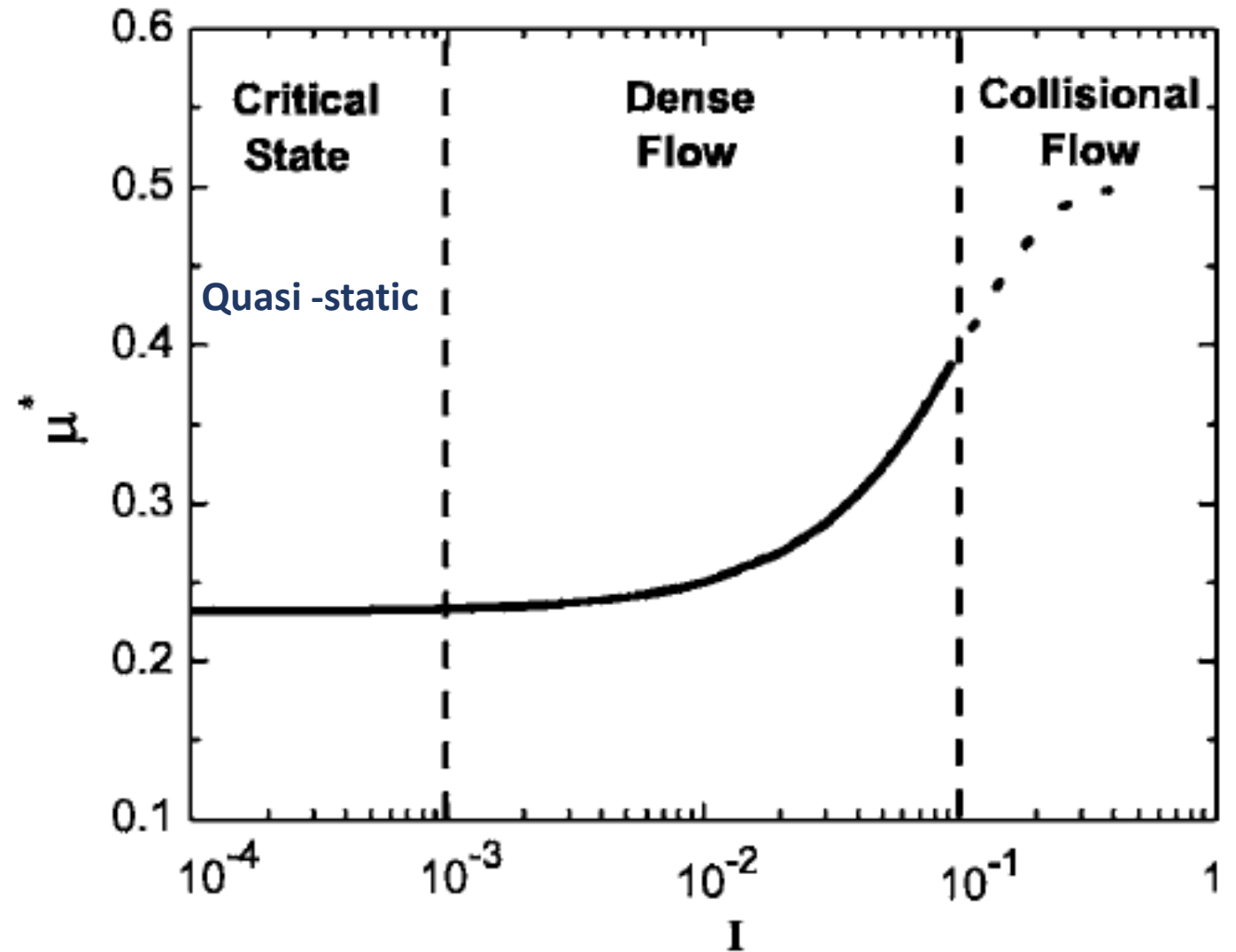
p' = mean effective stress

da Cruz et al. (2005)

Intertial number / Inertia number



da Cruz et al. (2005)



Intertial number / Inertia number

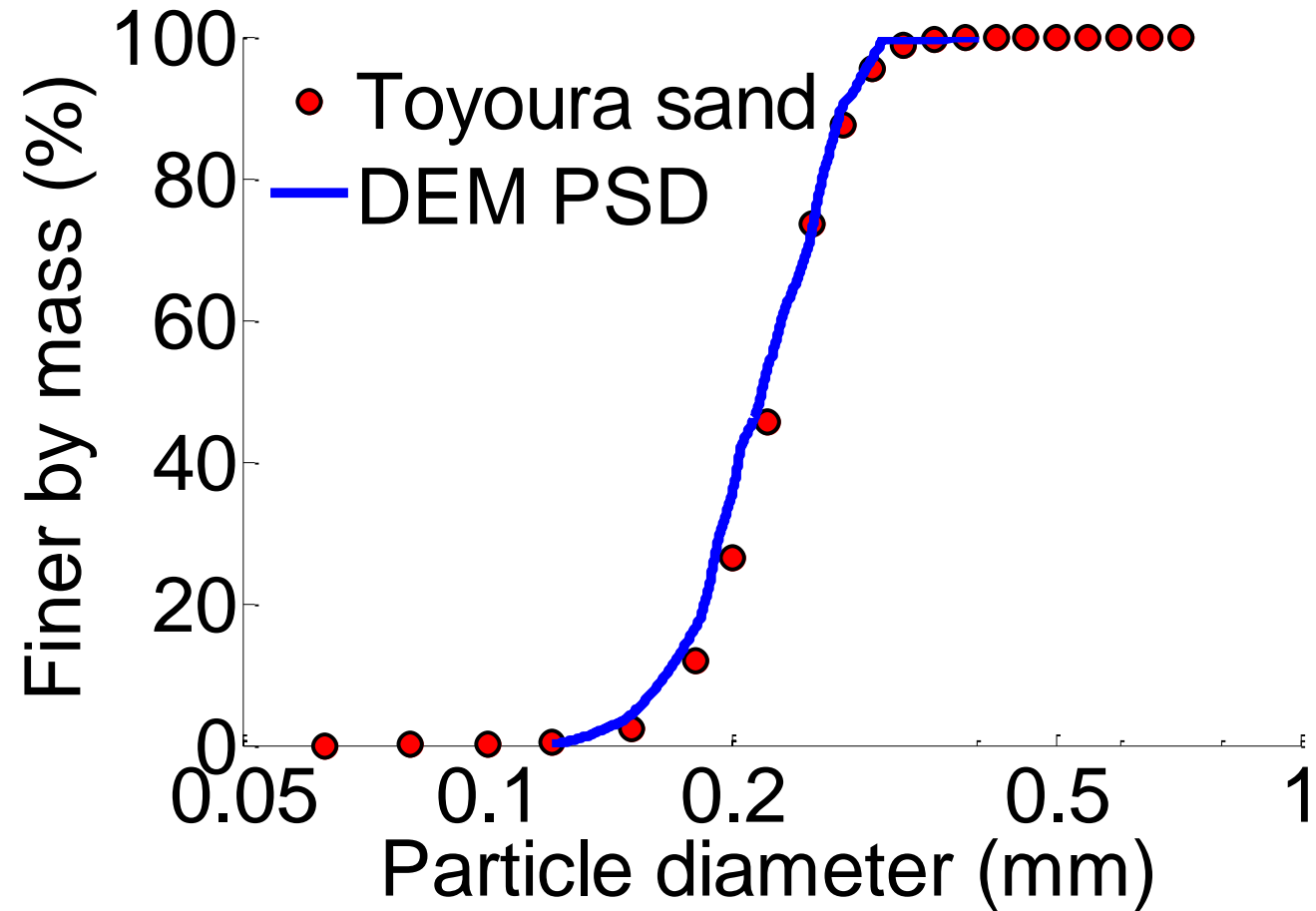
Two key questions

- Can we extrapolate DaCruz findings directly to 3D?
- How does I effect the critical state line?

Influence of I on critical state

Simulation details:

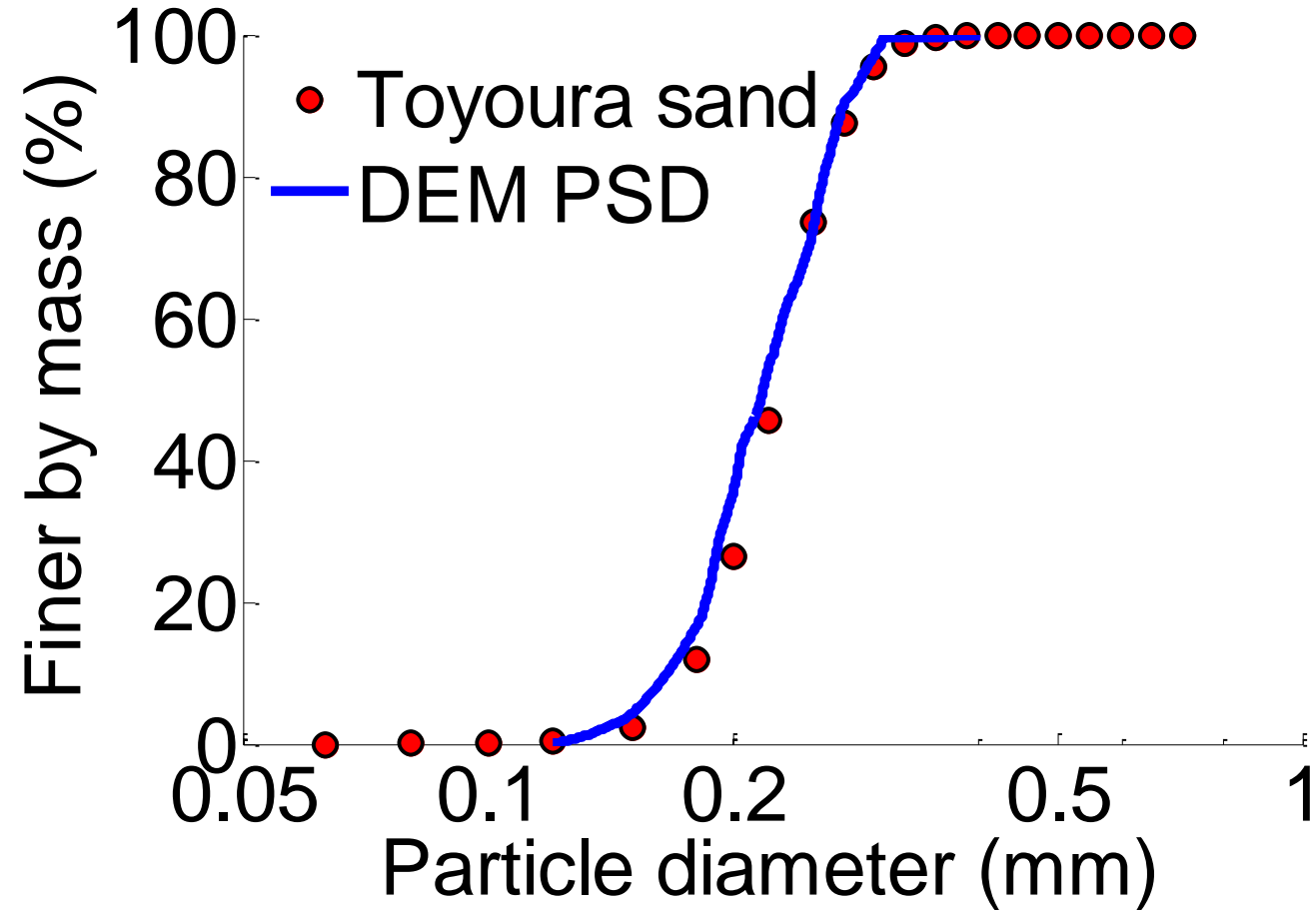
- LAMMPS
- PSD –approximates that of Toyoura sand
- Triaxial compression
- 10,624 particles
- Simplified Hertz-Mindlin contact model ($G=29\text{GPa}$, $\nu=0.12$, $\rho=2650\text{ kg/m}^3$)
- $\mu=0.25$



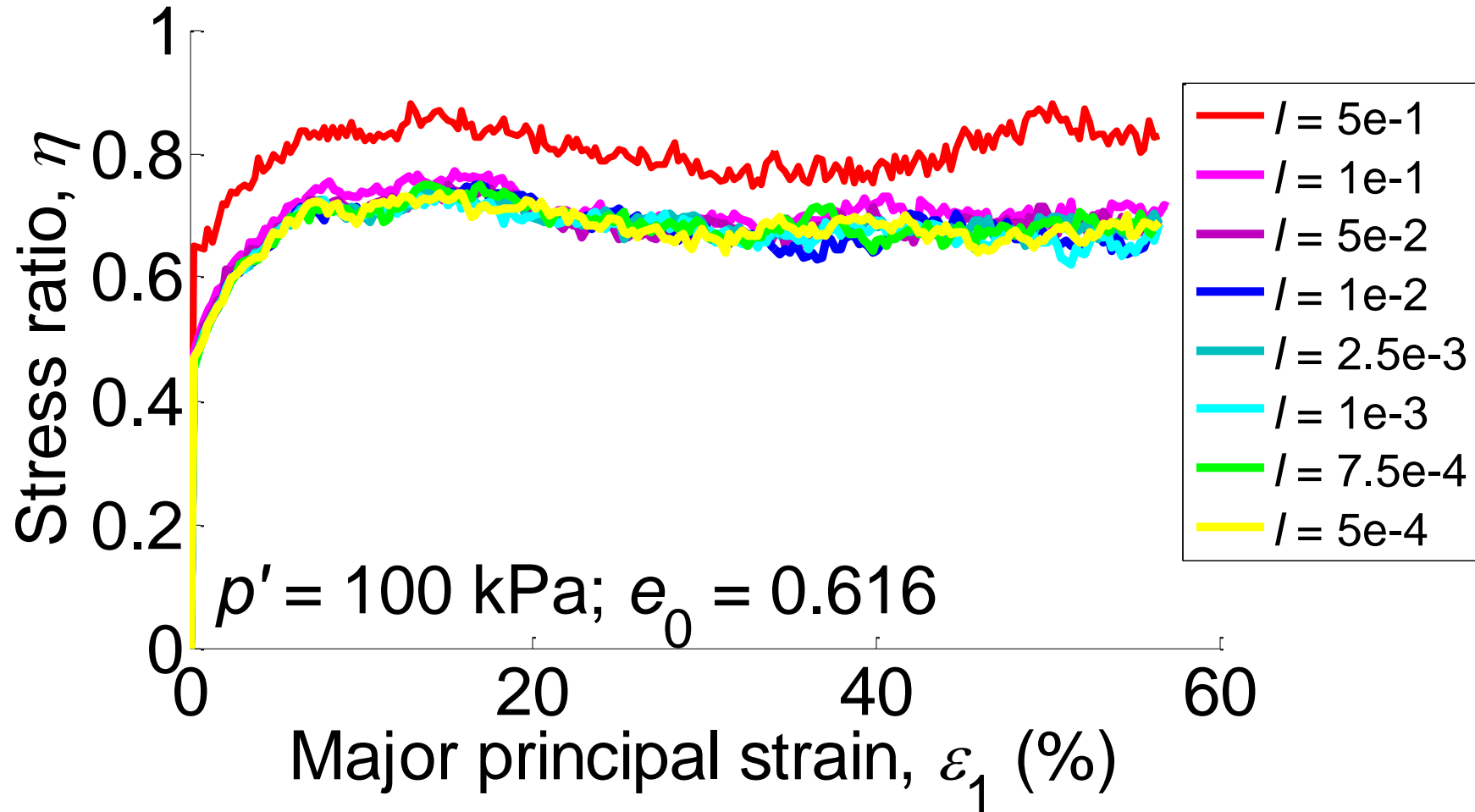
Influence of I on critical state

Simulation details:

- Triaxial compression to large strain
- constant p' (mean effective stress) \rightarrow constant I
- 4 confining pressures:
 $100\text{kPa} \leq p' \leq 5000\text{ kPa}$

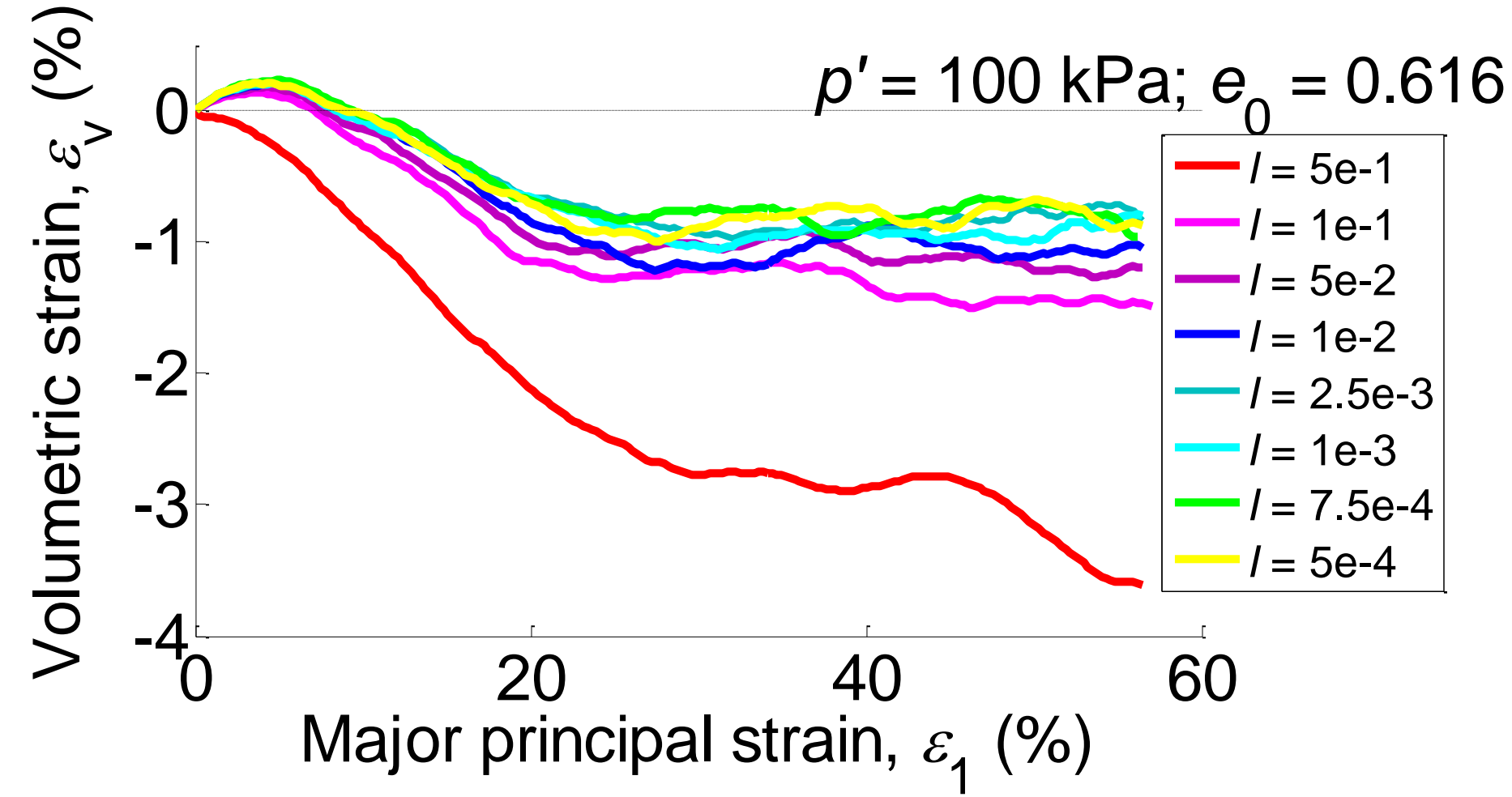


Influence of I on critical state



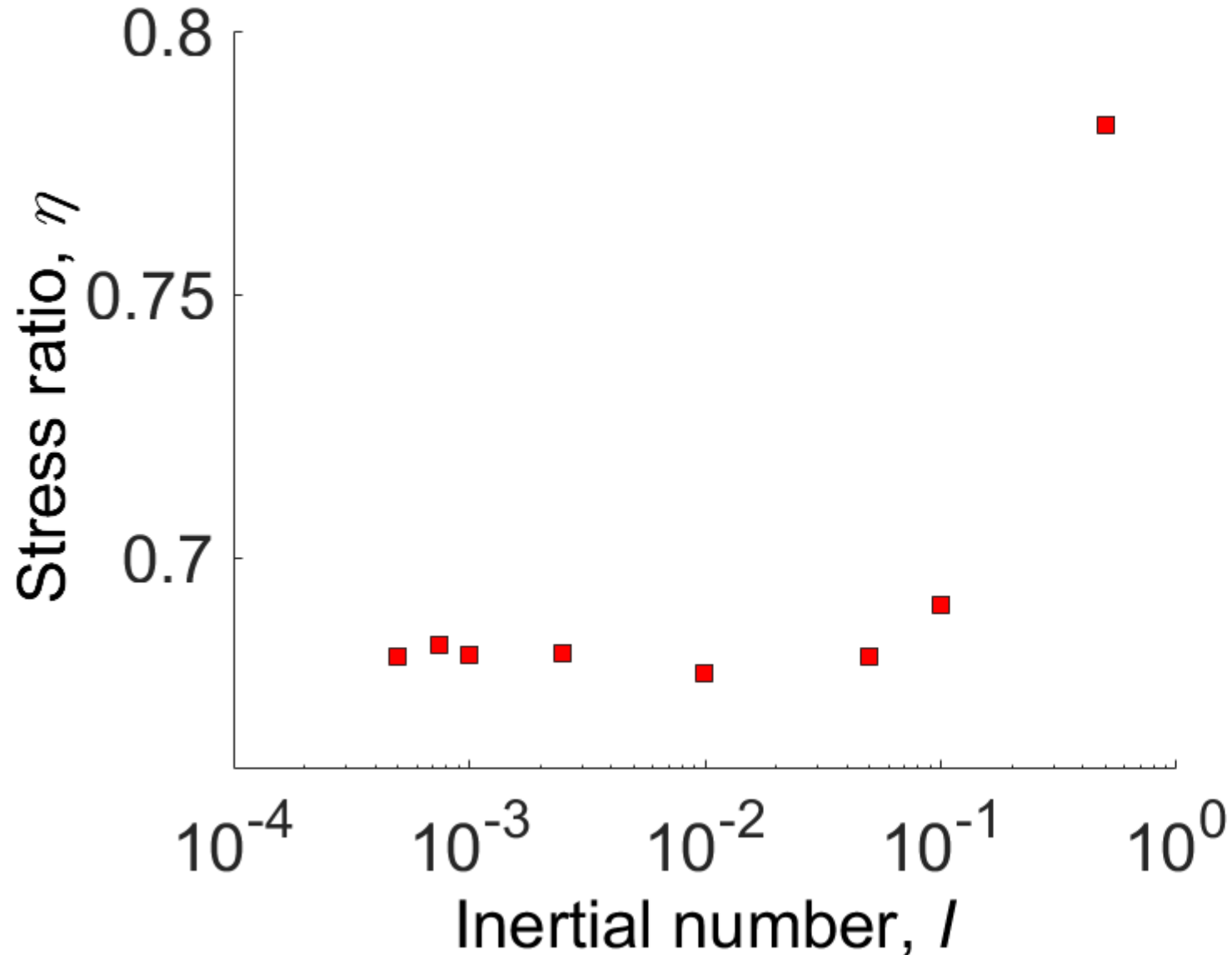
Lopera Perez et al. (2016)

Influence of I on critical state



Lopera Perez et al. (2016)

Influence of I on critical state

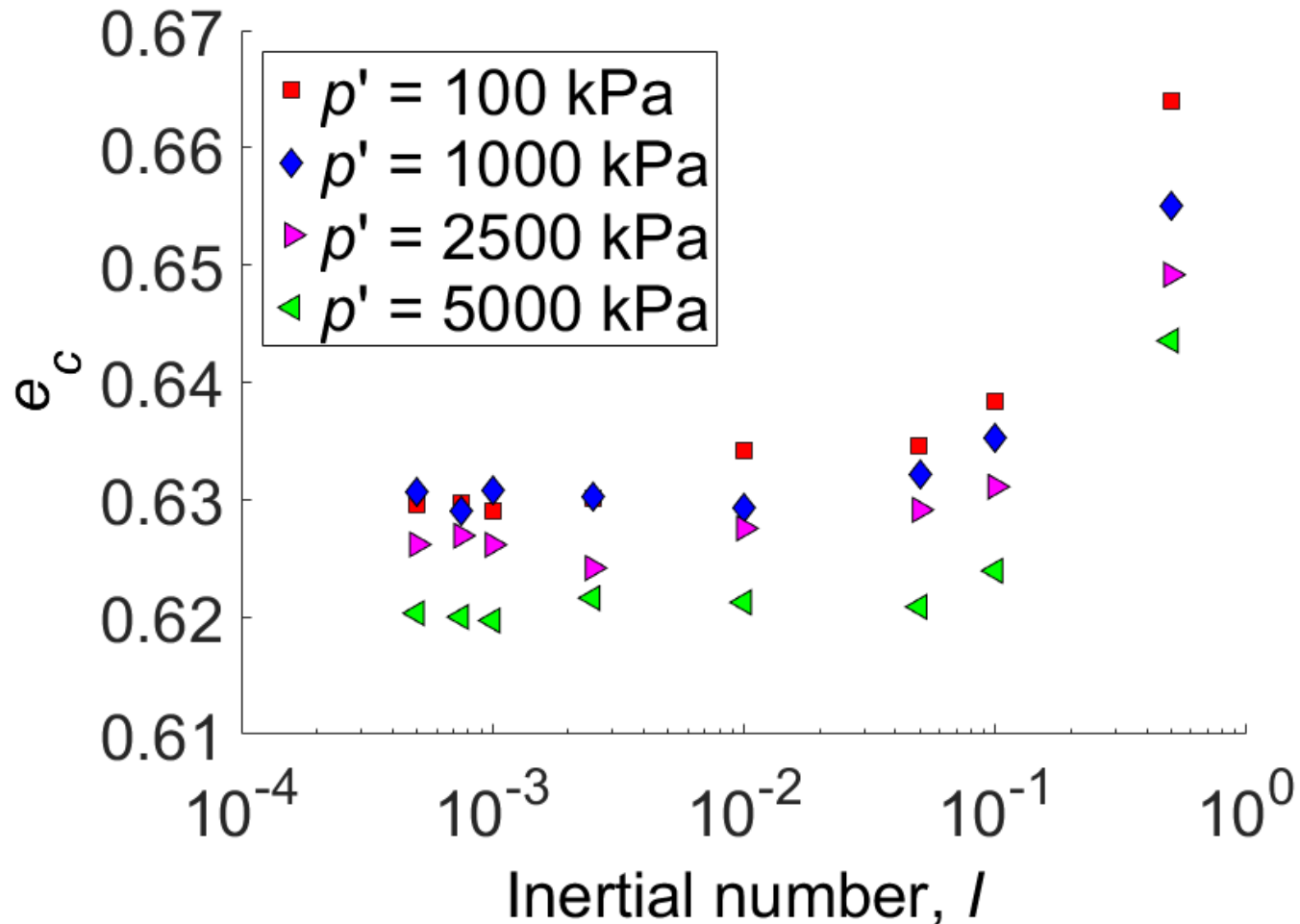


η = stress ratio at critical state = q/p'

$$q = \sigma'_1 - \sigma'_3$$

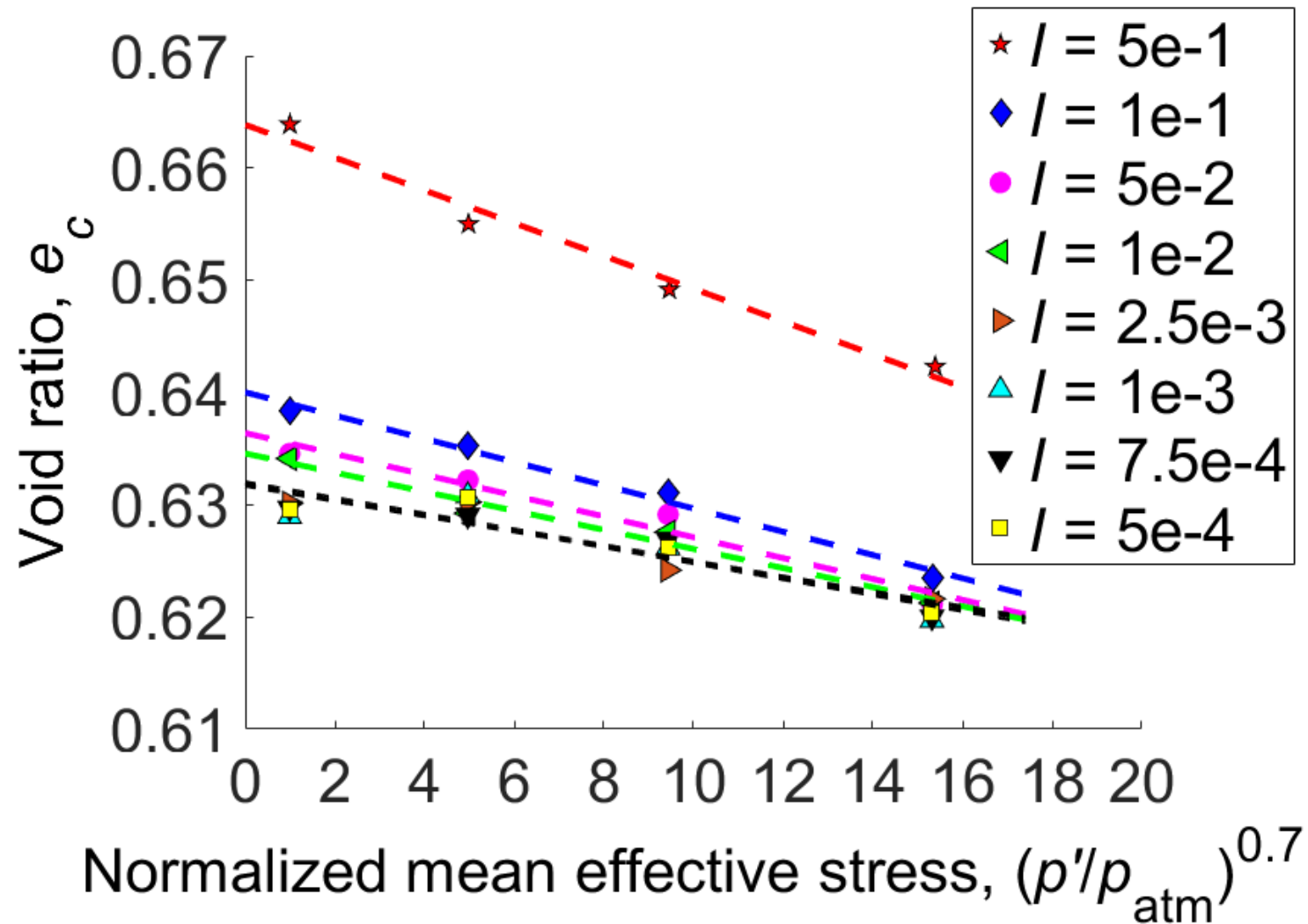
$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$$

Influence of I on critical state



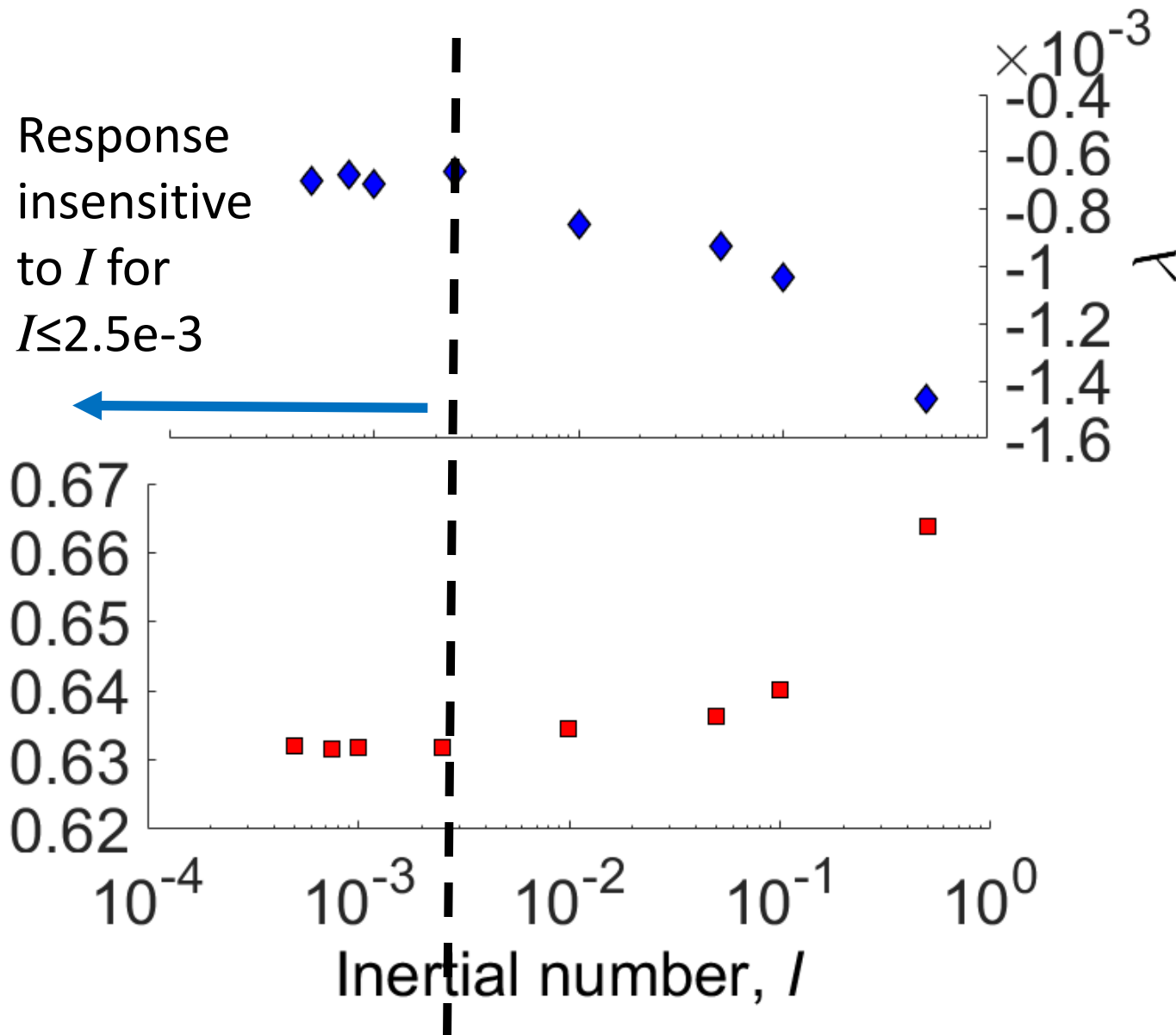
e_c = void ratio at critical state

Influence of I on critical state



e_c = void ratio at critical state

Influence of I on critical state



λ = slope of critical state line in e - p' space

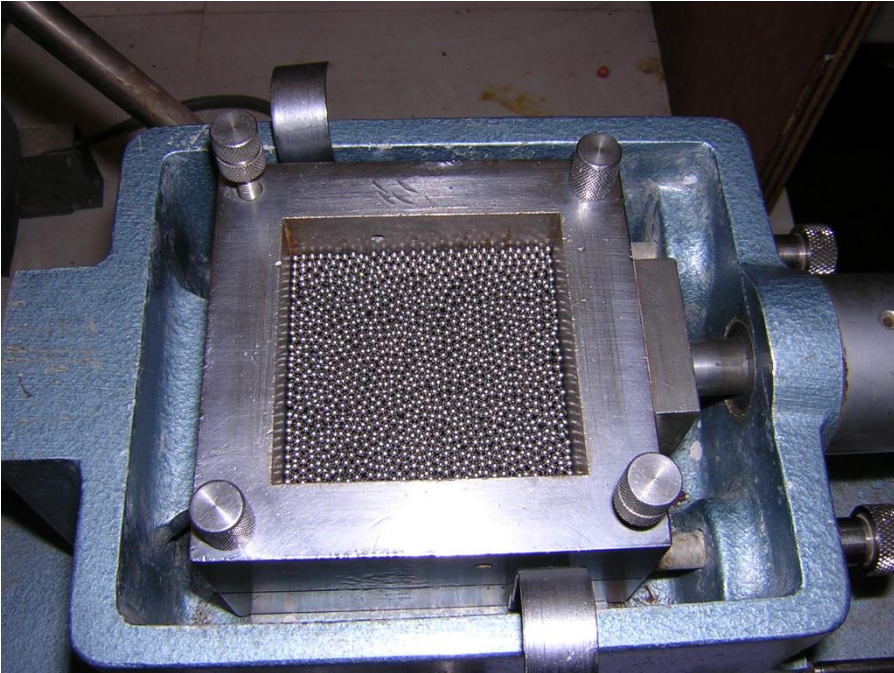
Γ = intercept with $(p'/p_a)^{0.7}=0$

Conclusions

- Inertia number effects position of critical state line
- For quasi-static simulations keep $I \leq 2.5e-3$

Post processing /data interpretation/checks

Direct Shear Test – Coupling Physical Tests and DEM Simulations



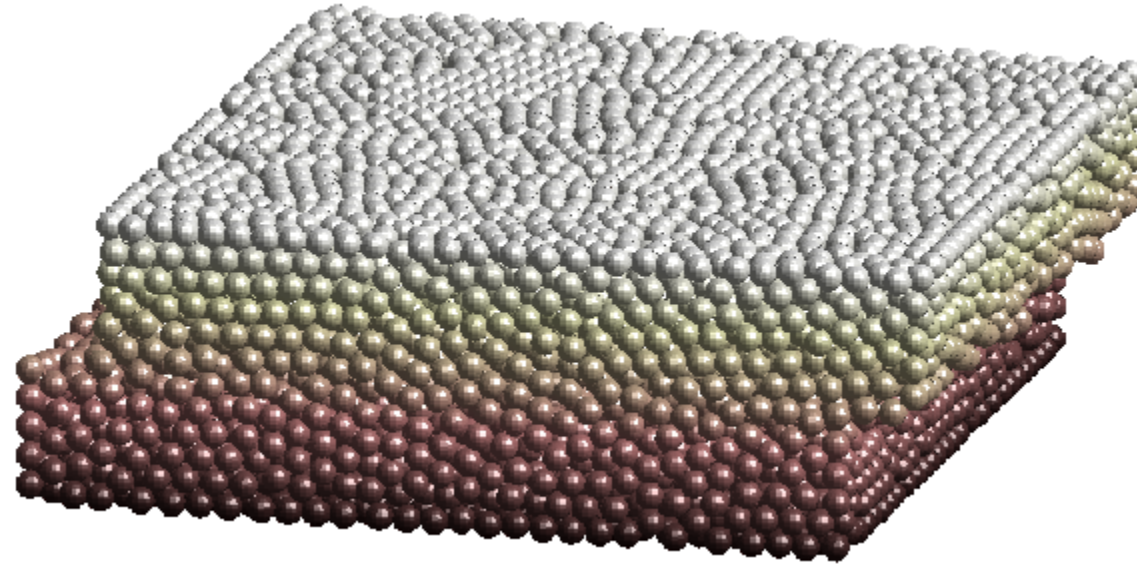
11700 Spheres

Grade 25 steel

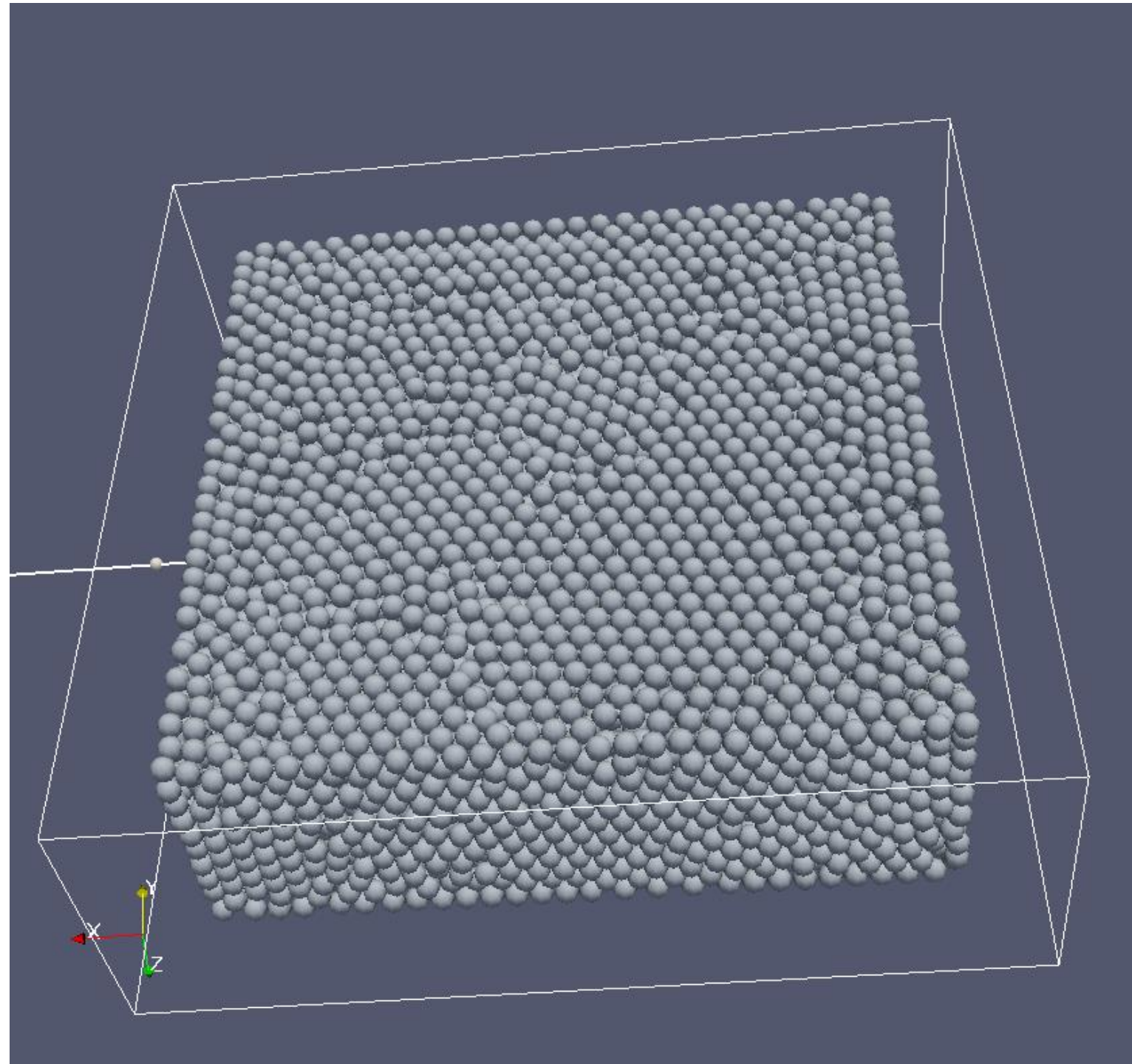
Checks

- Over-penetration – if overlap exceeds 5% - violating small overlap assumption in DEM – discard simulation (Hanley et al., 2014)
- Quasi static? – Up until recently always checked force - now focus on inertial number (I)
- Moving towards energy tracing in simulations
- Always look at coordination number, mechanical coordination number, 2nd order fabric tensor, rose diagrams of contact orientations.

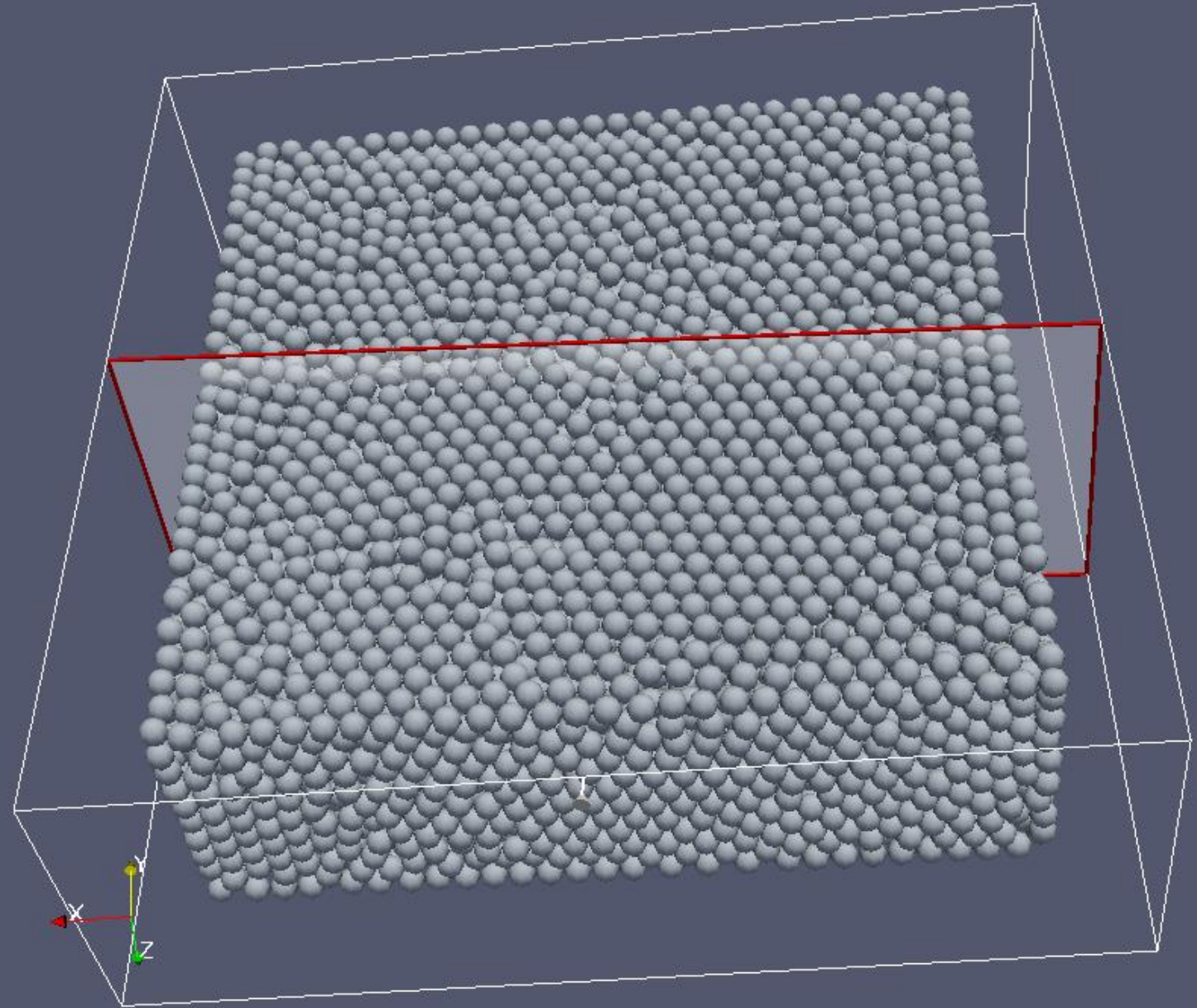
Deformed Specimen – DEM Simulation



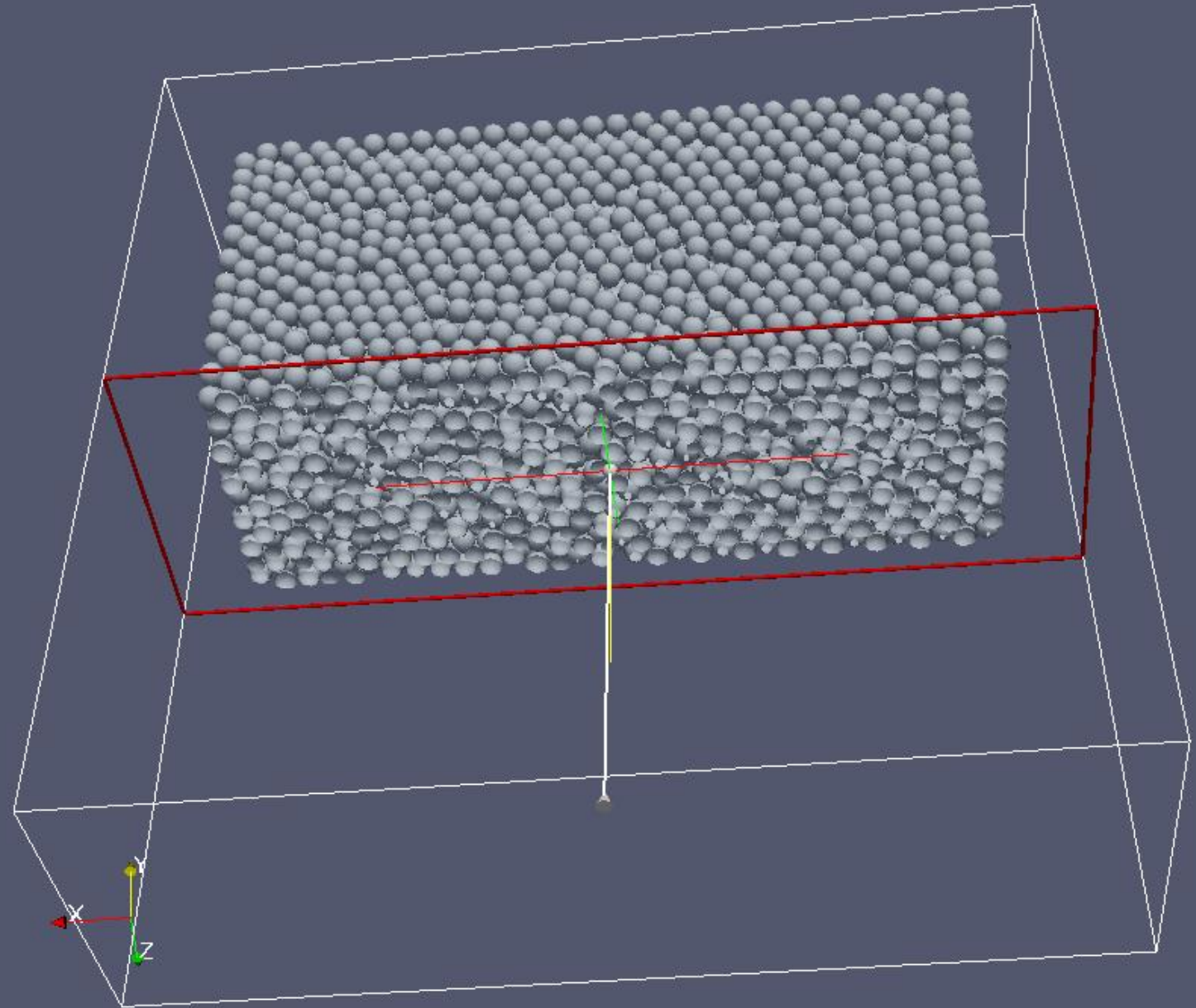
Deformed Specimen



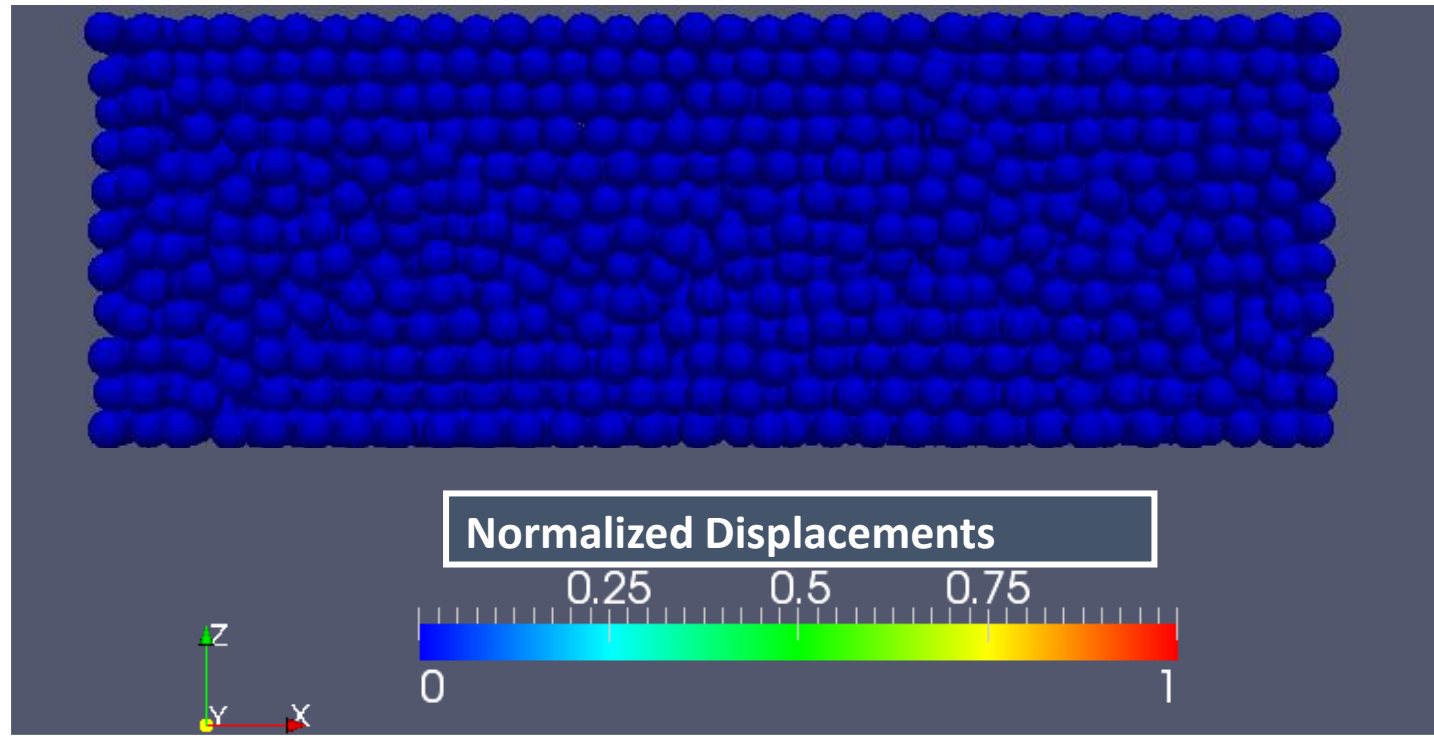
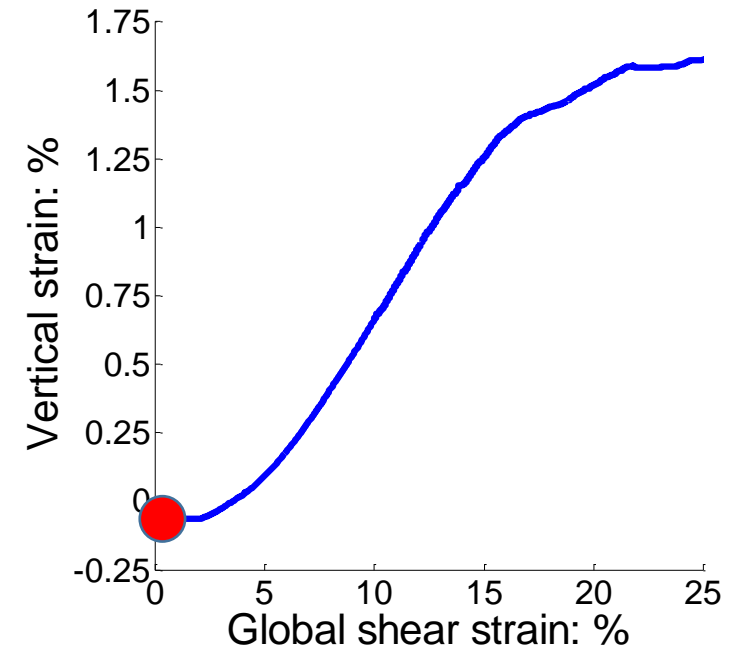
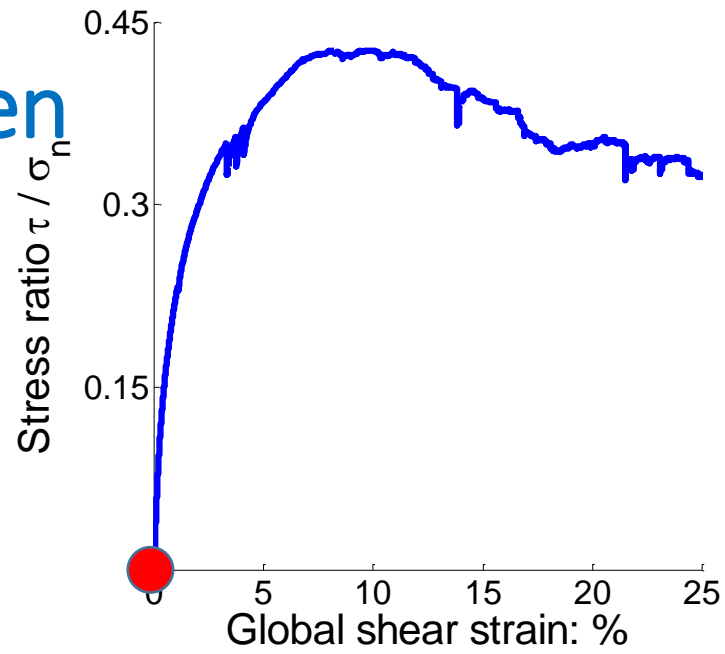
Deformed Specimen



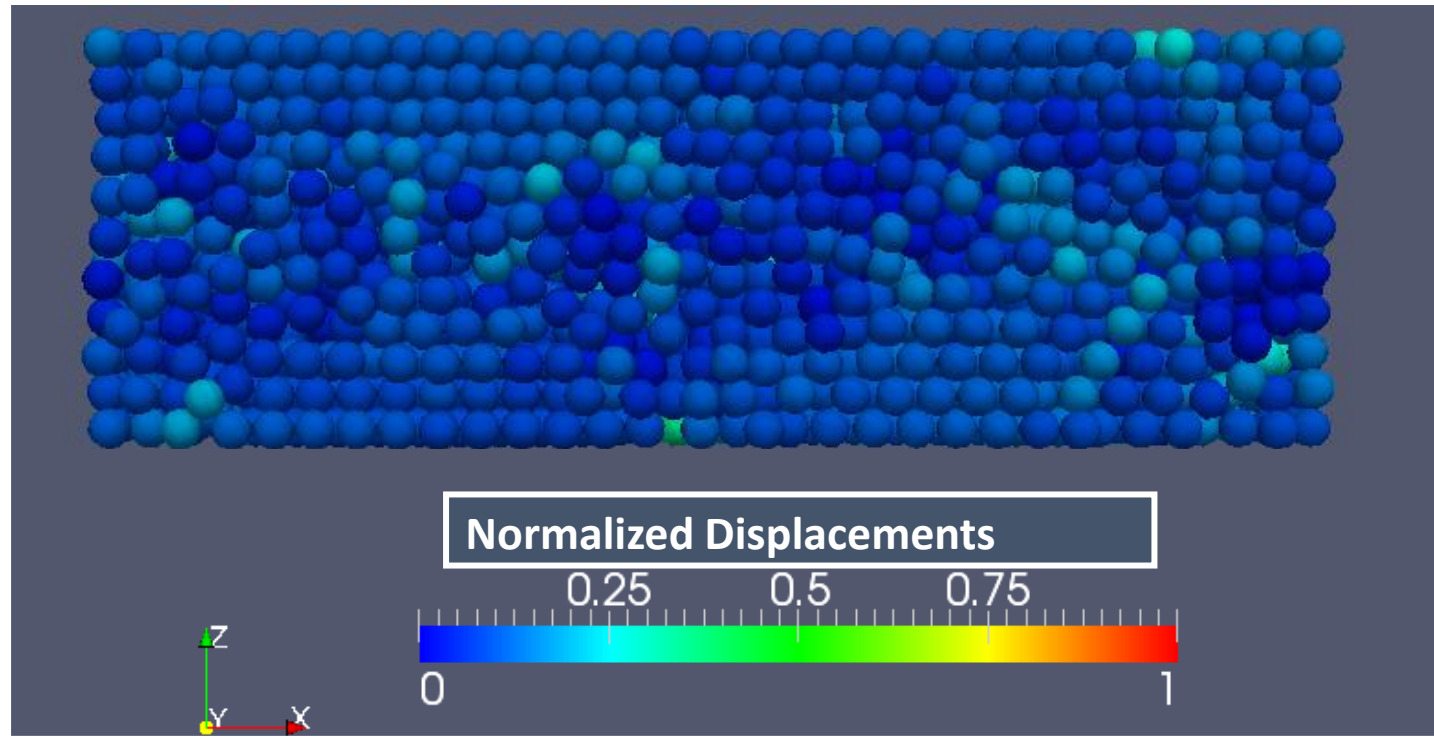
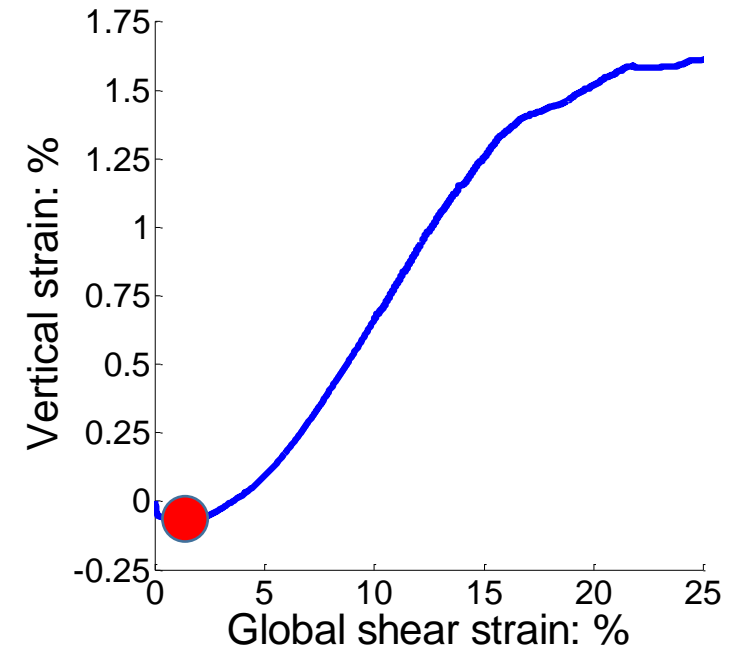
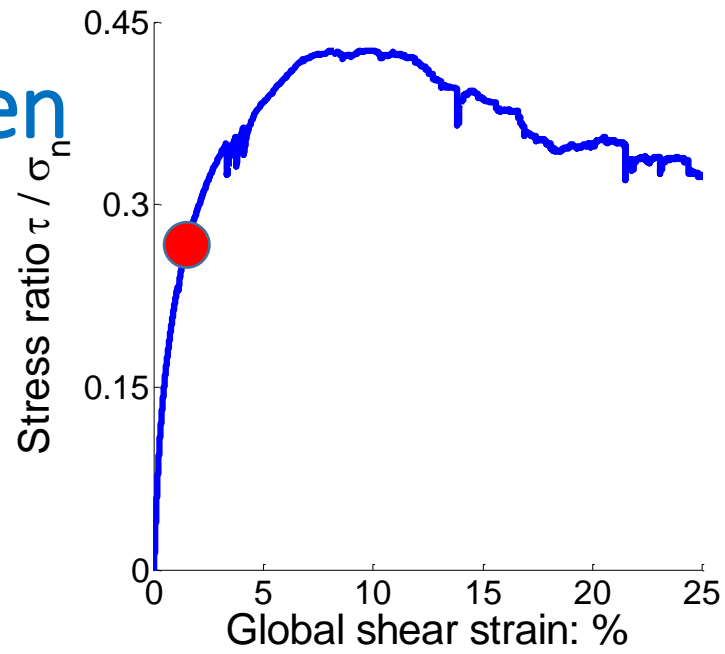
Deformed Specimen



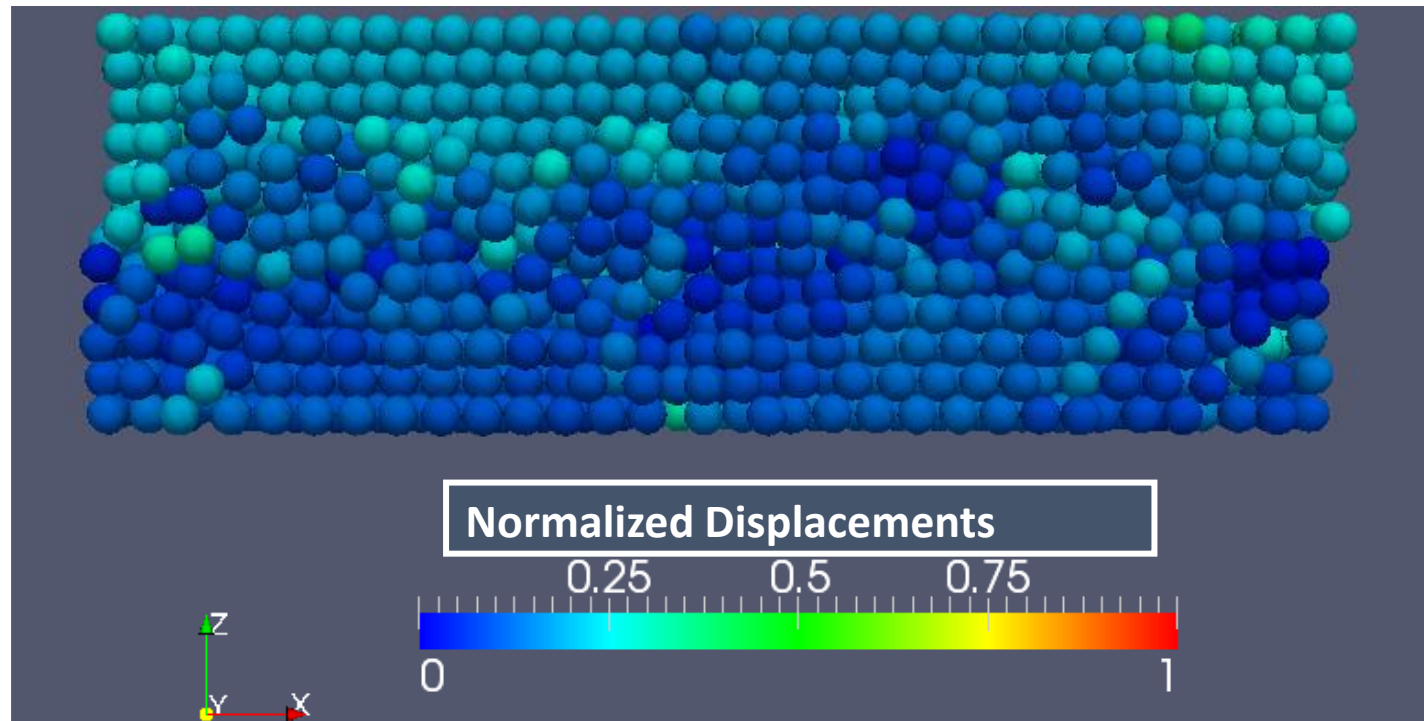
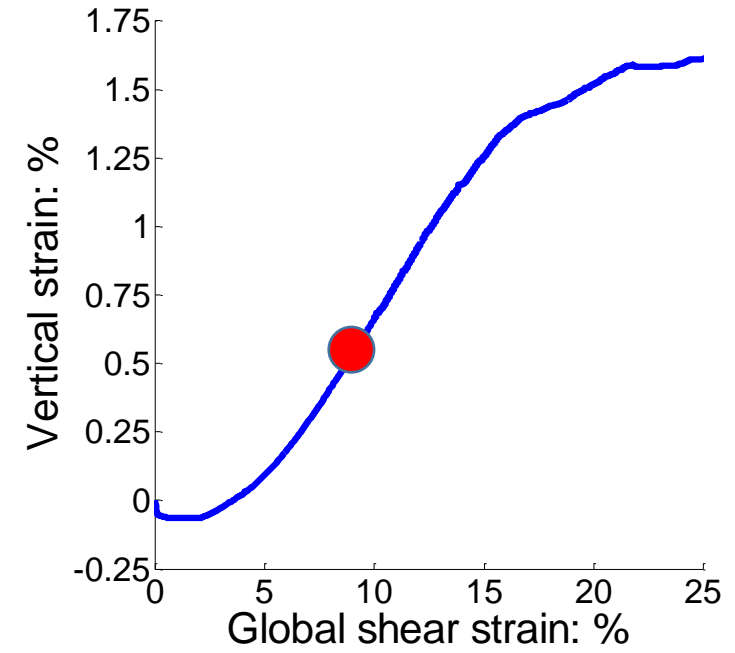
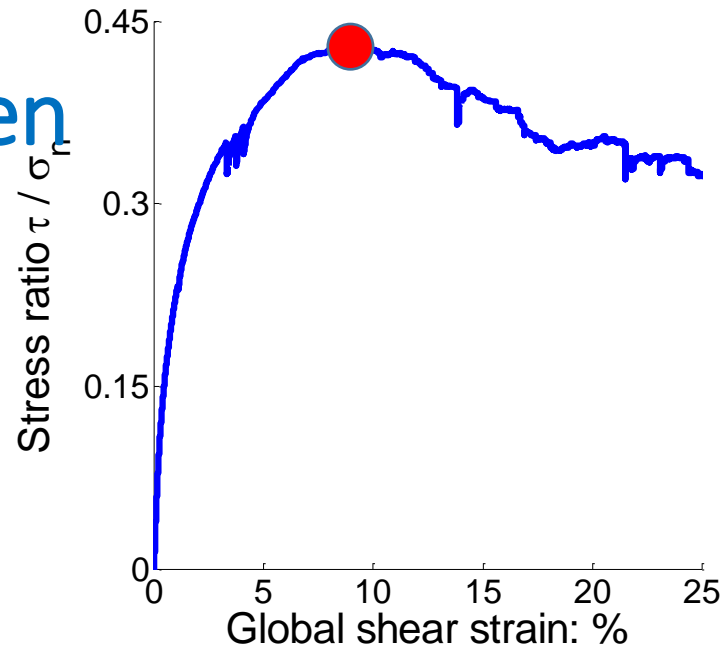
Deformed Specimen



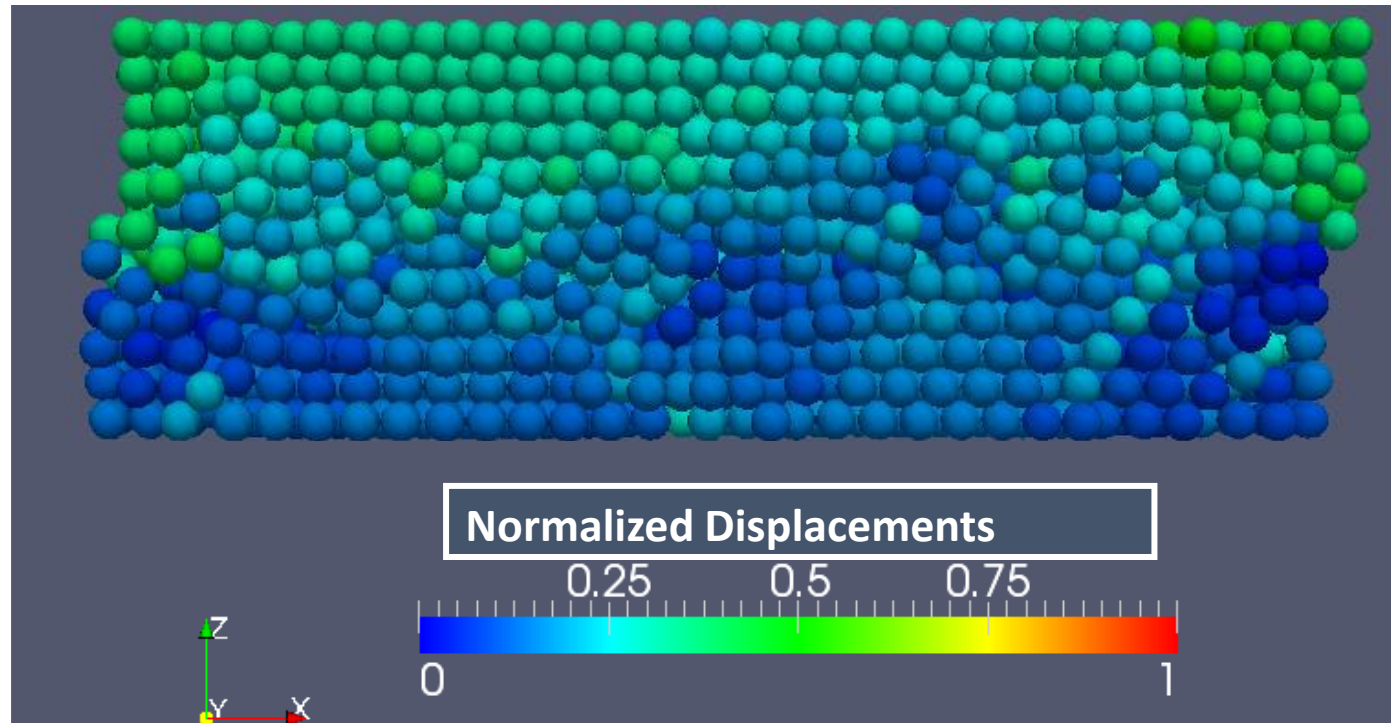
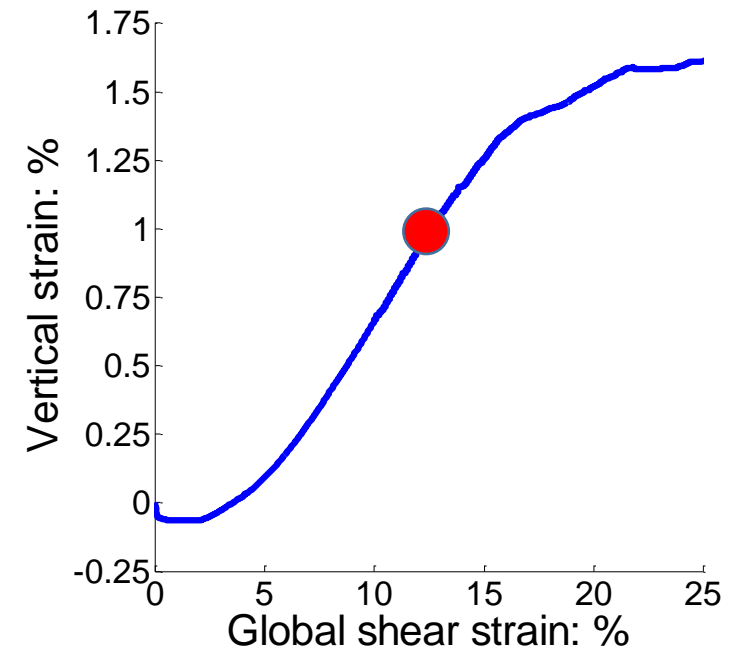
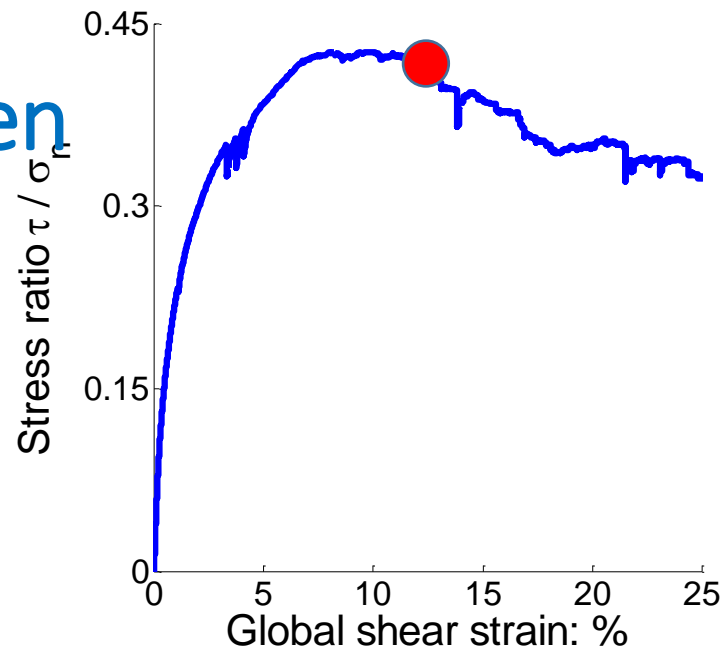
Deformed Specimen



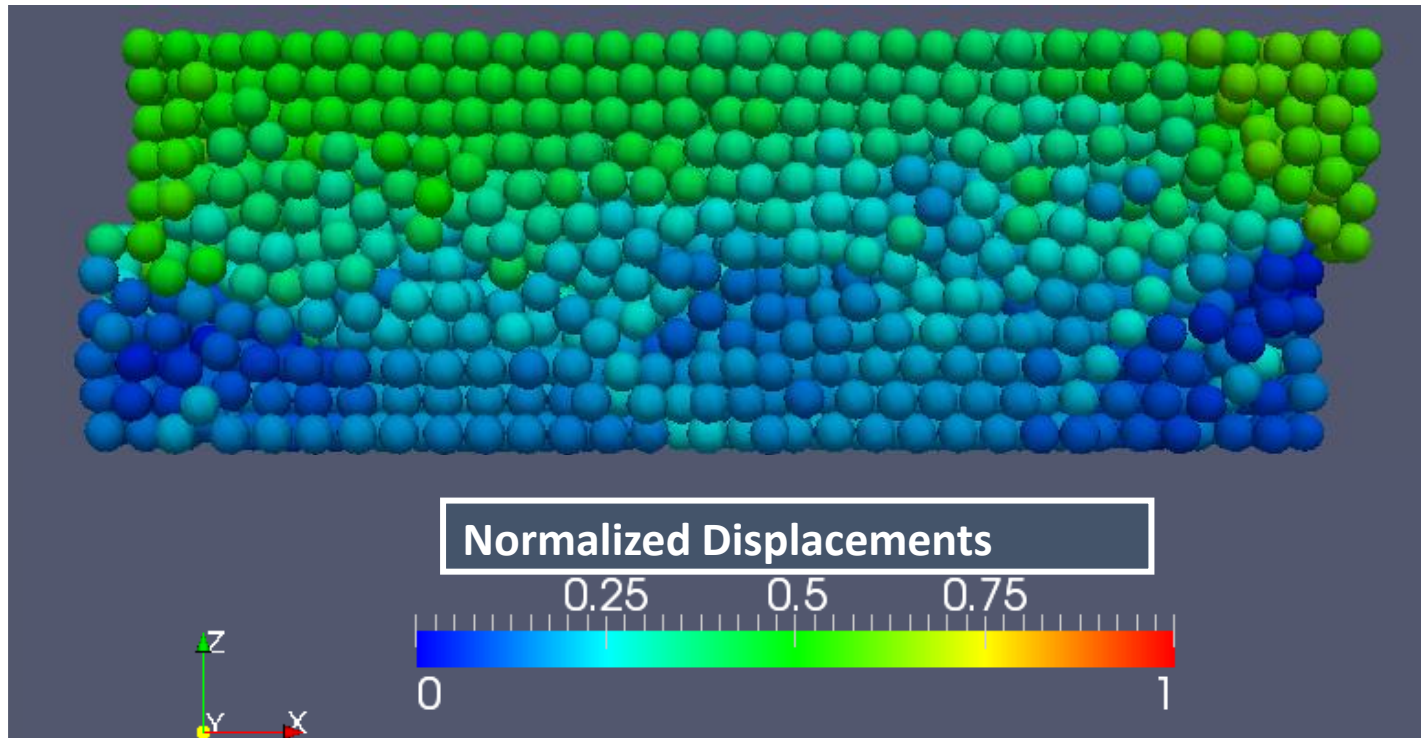
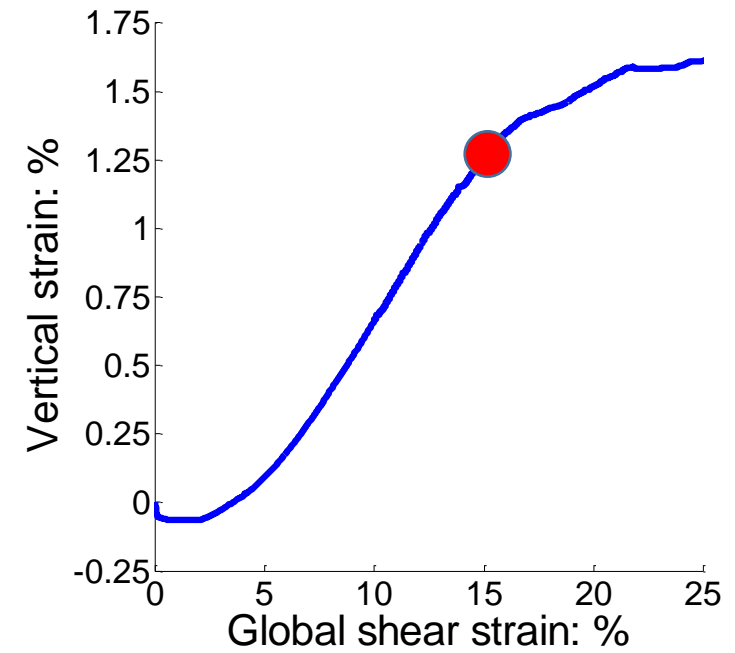
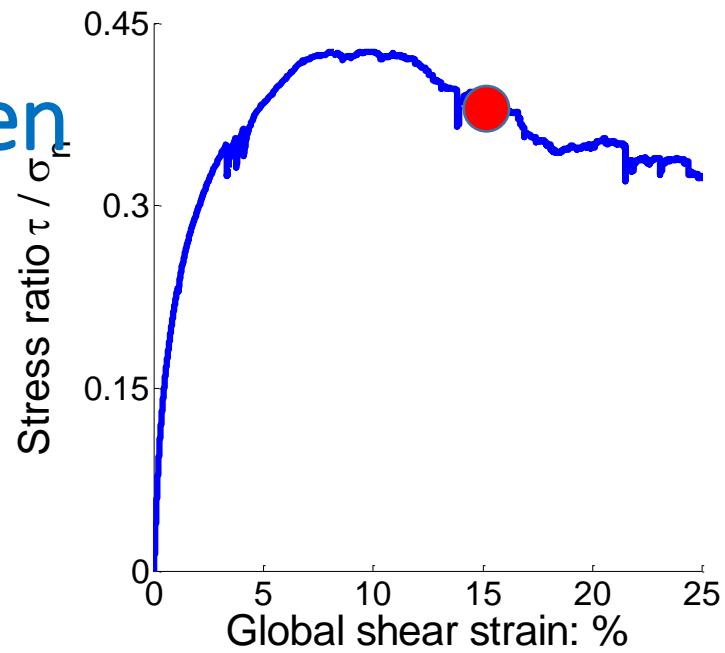
Deformed Specimen



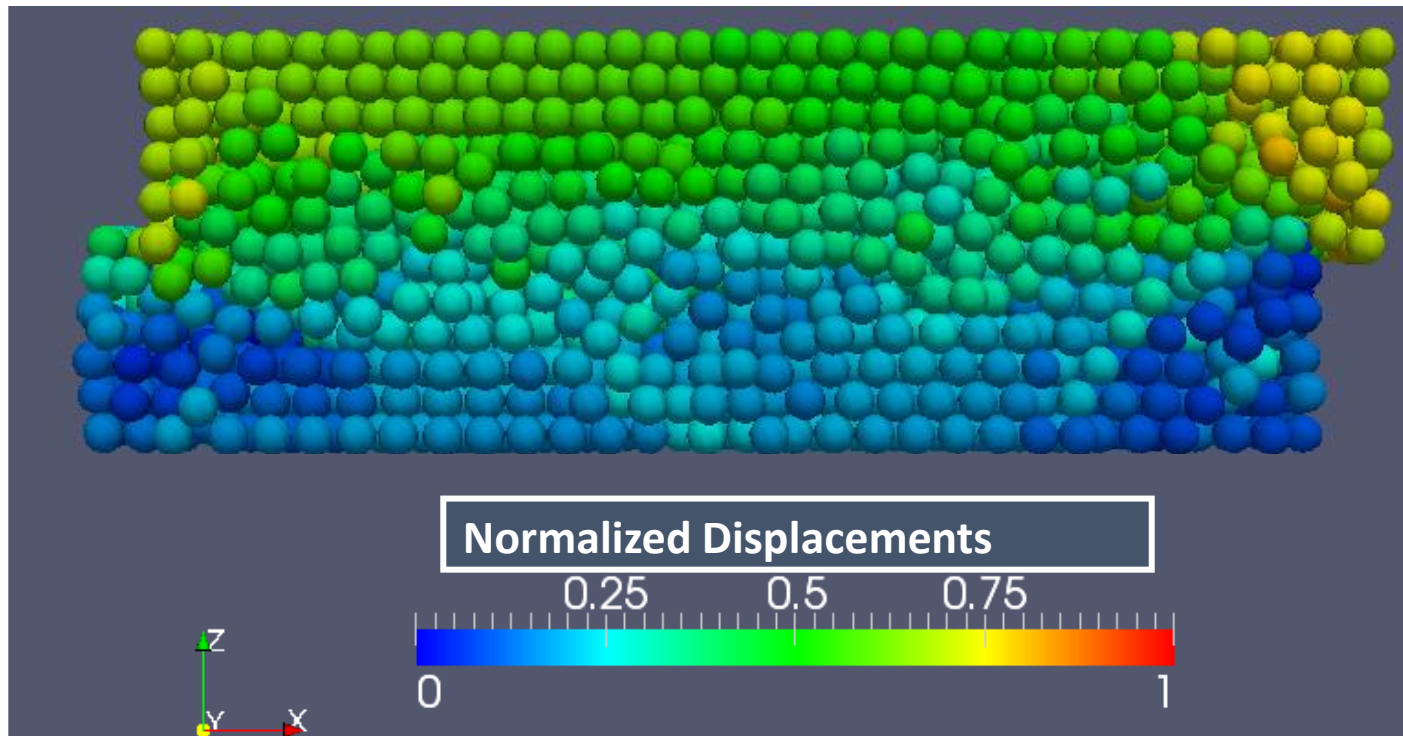
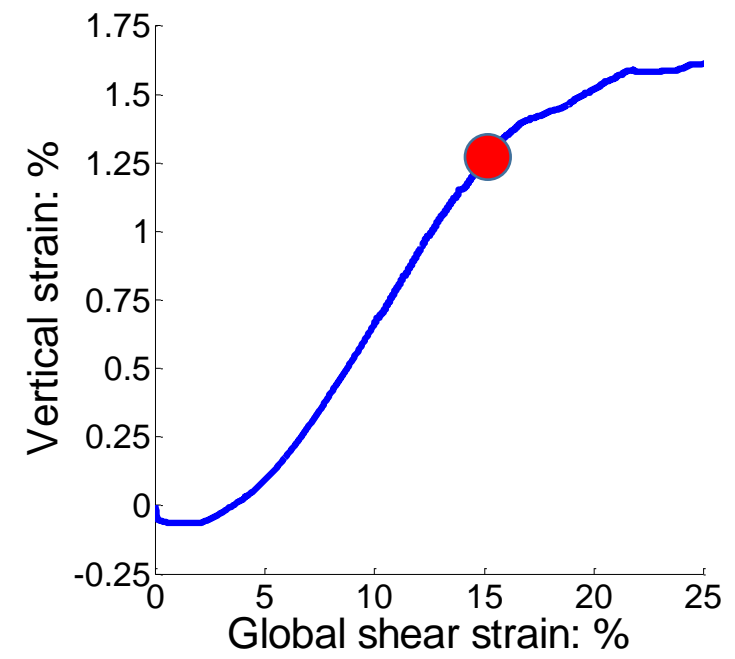
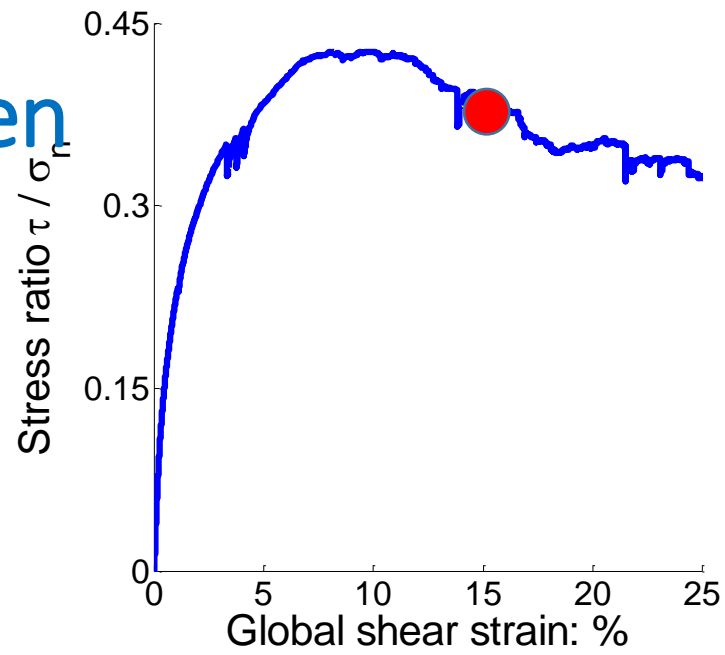
Deformed Specimen



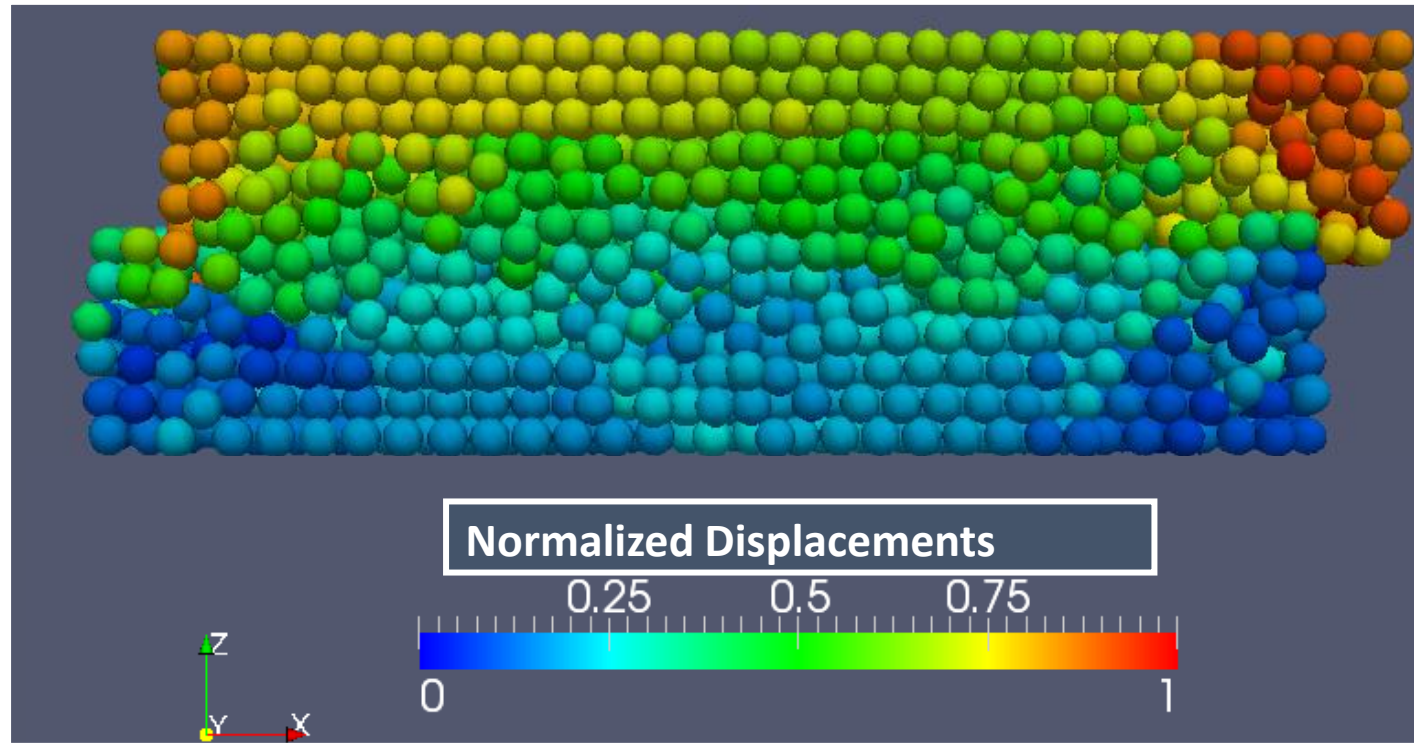
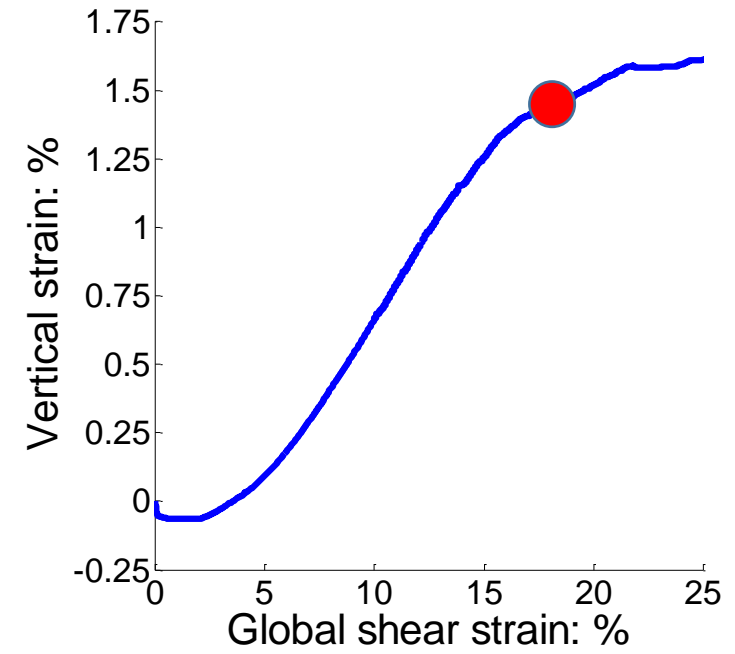
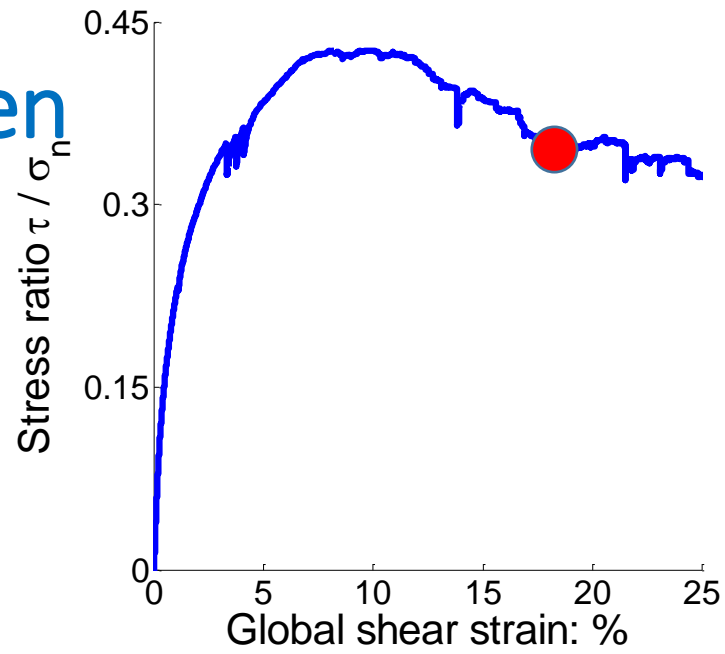
Deformed Specimen



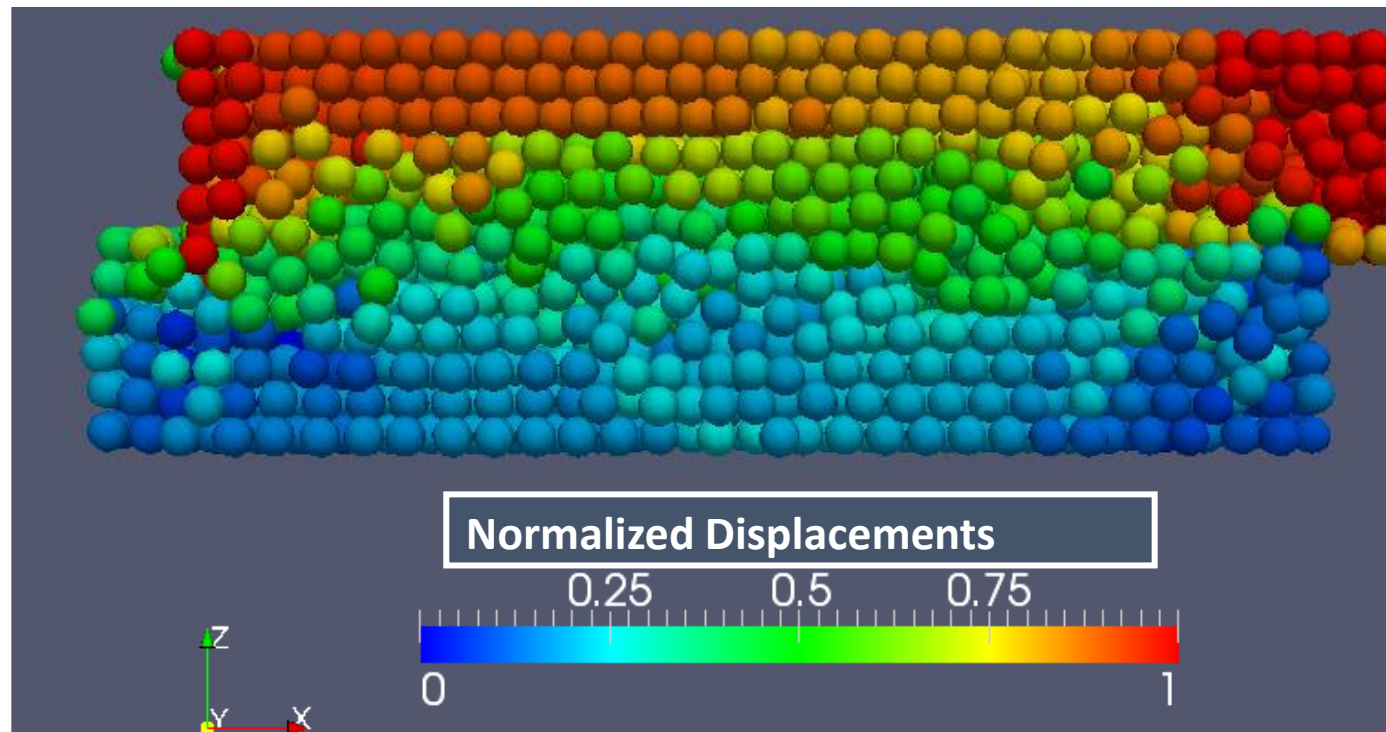
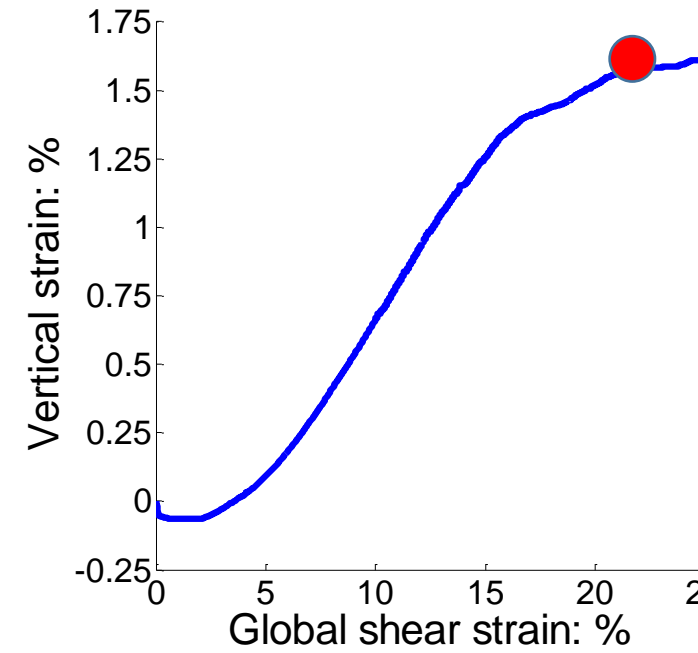
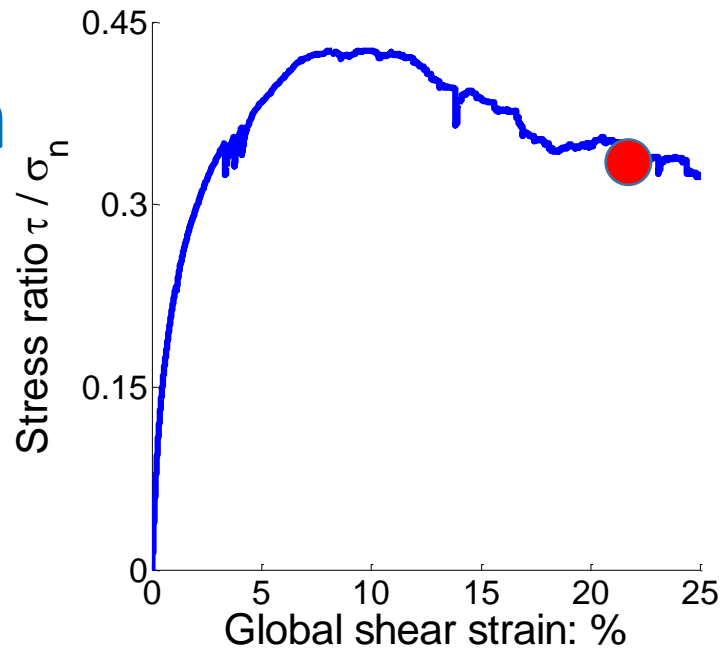
Deformed Specimen



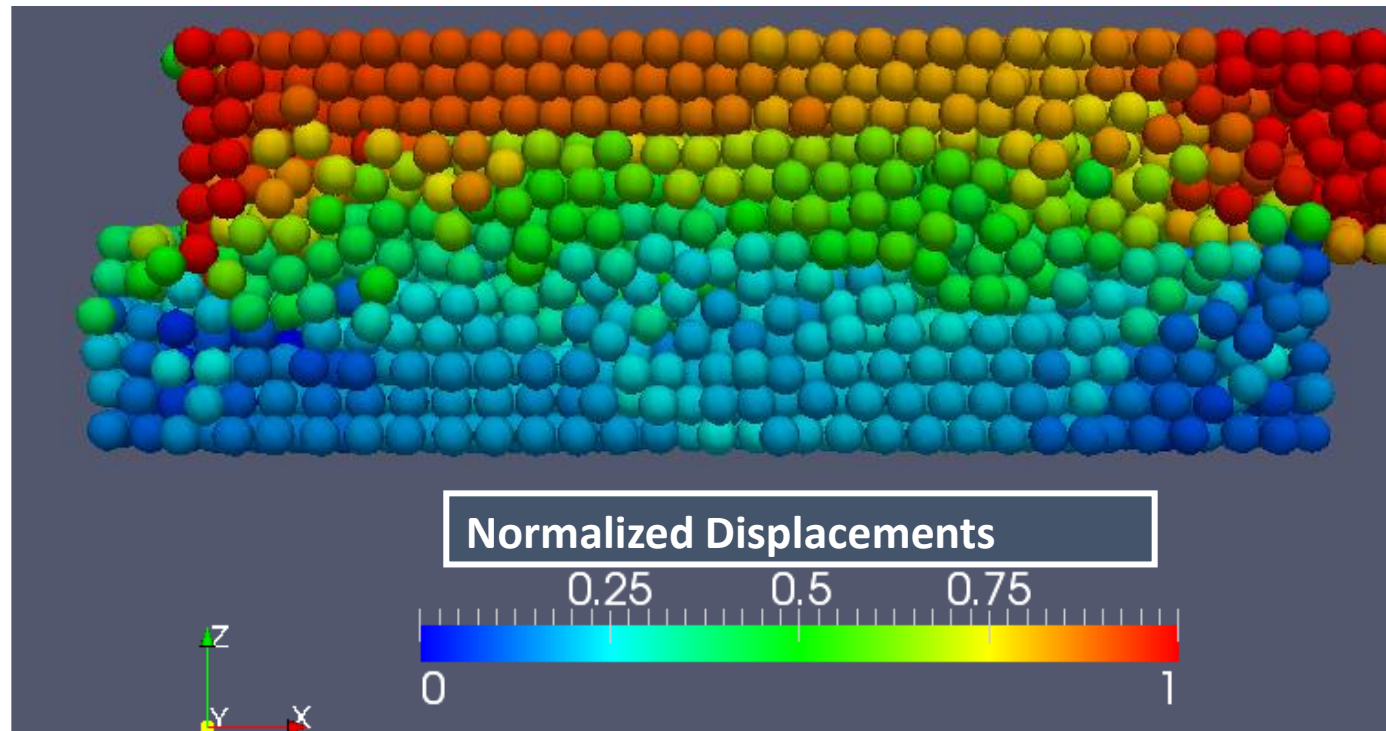
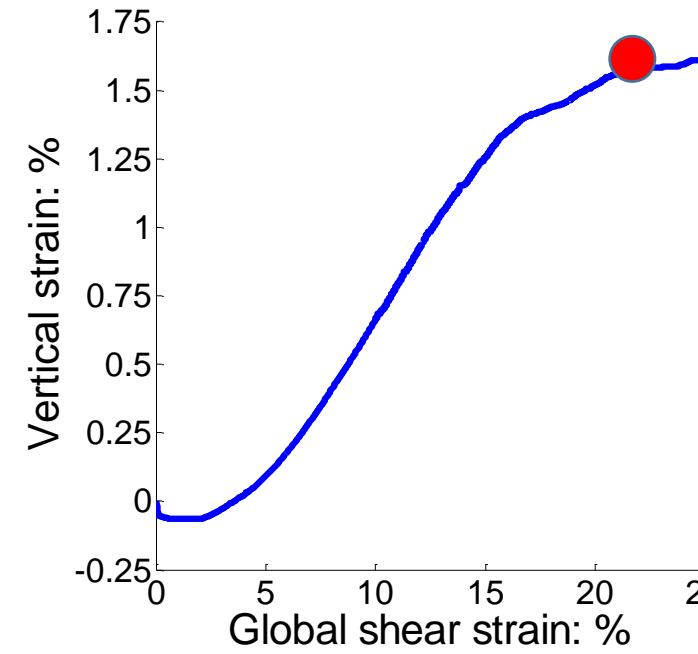
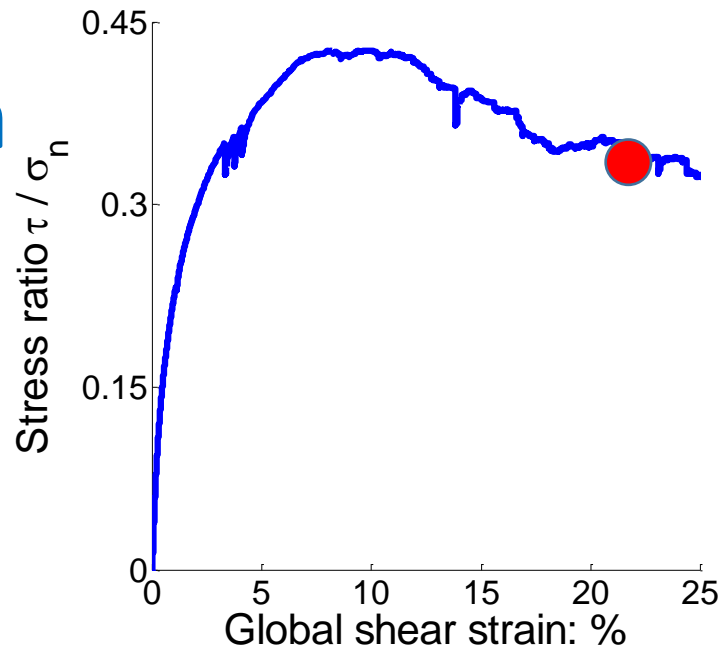
Deformed Specimen



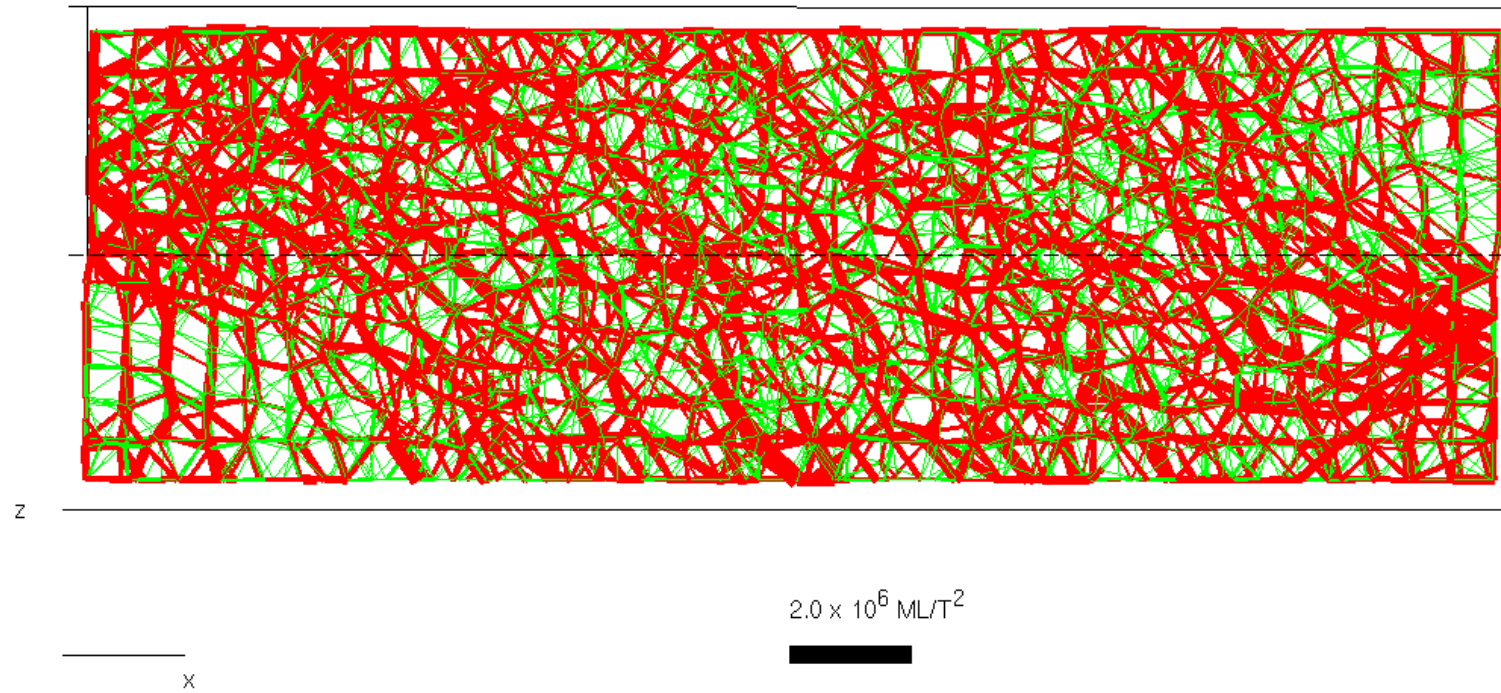
Deformed Specimen



Deformed Specimen



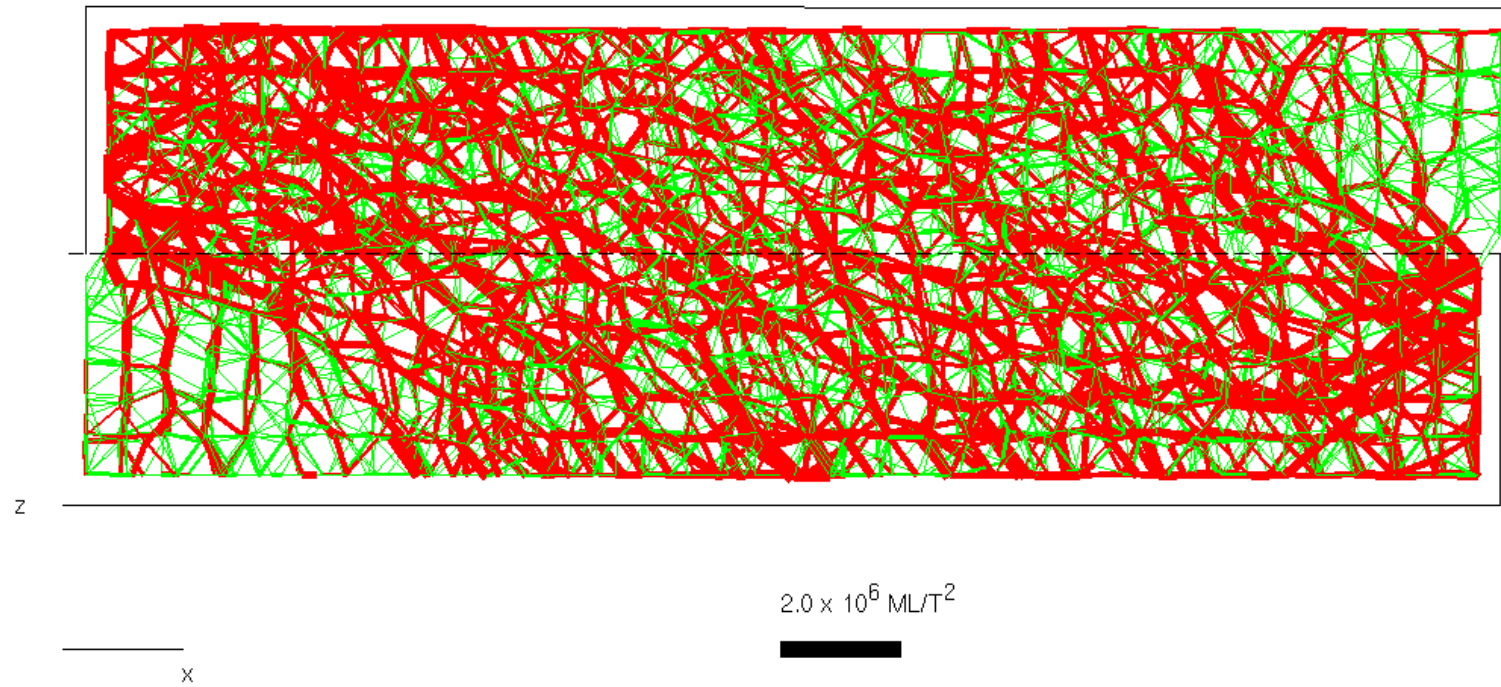
Contact force distribution in $x-z$ plane



$24 L < y < 36 L$

Contact force distribution in x-z plane

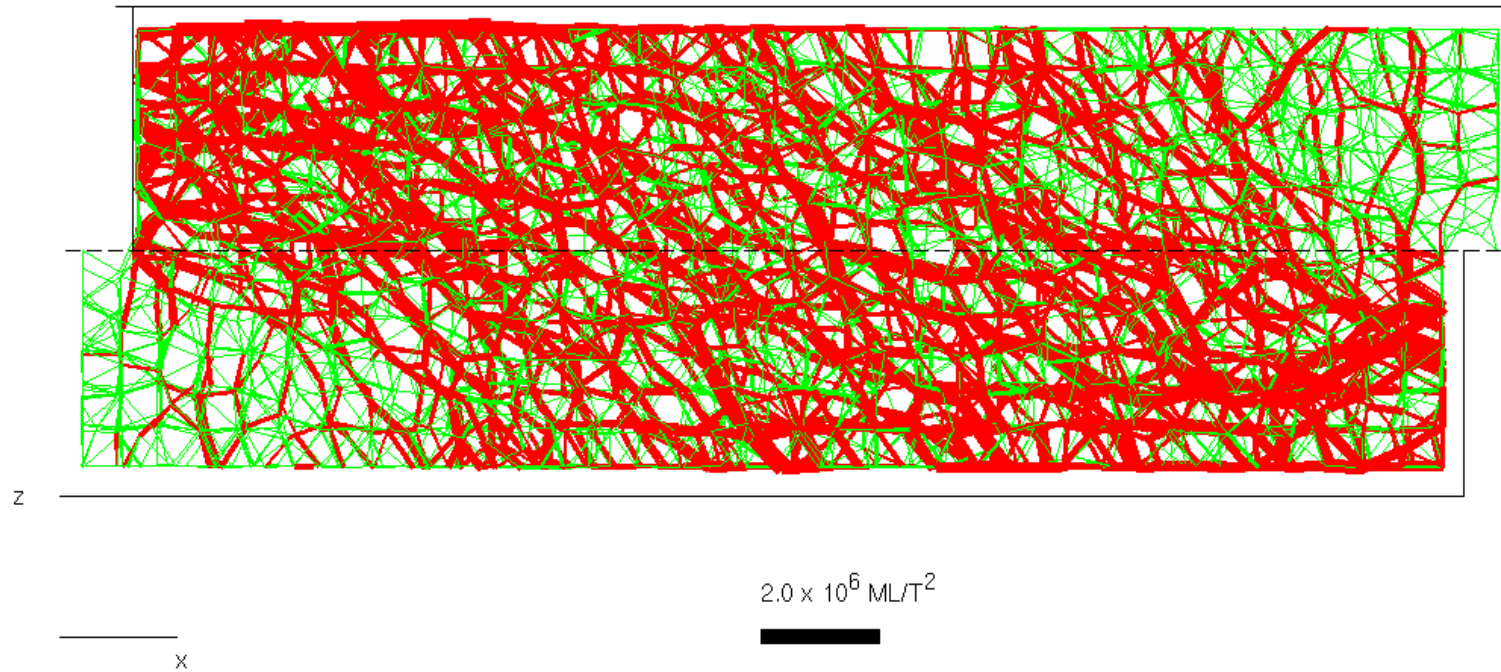
$\gamma=4.9\%$



$24 L < y < 36 L$

Contact force distribution in x-z plane

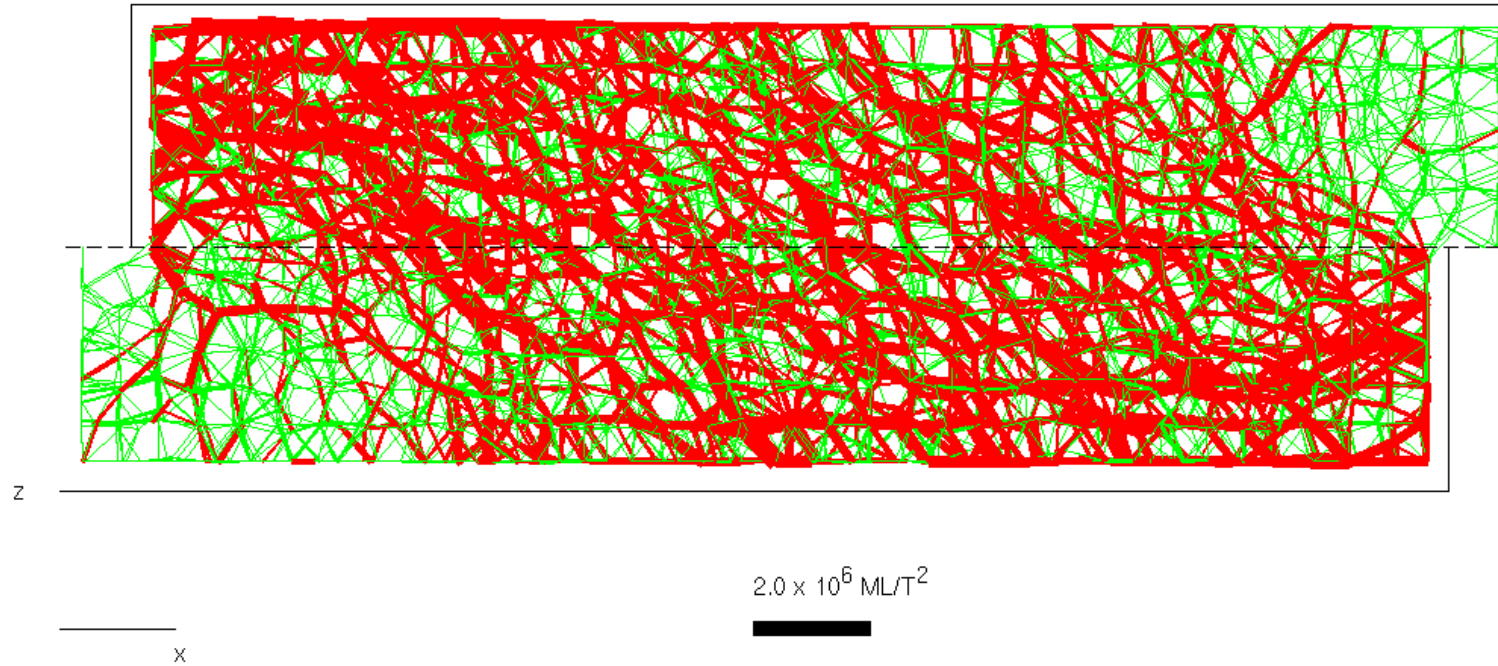
$\gamma=11.8\%$



$24 L < y < 36 L$

Contact force distribution in x-z plane

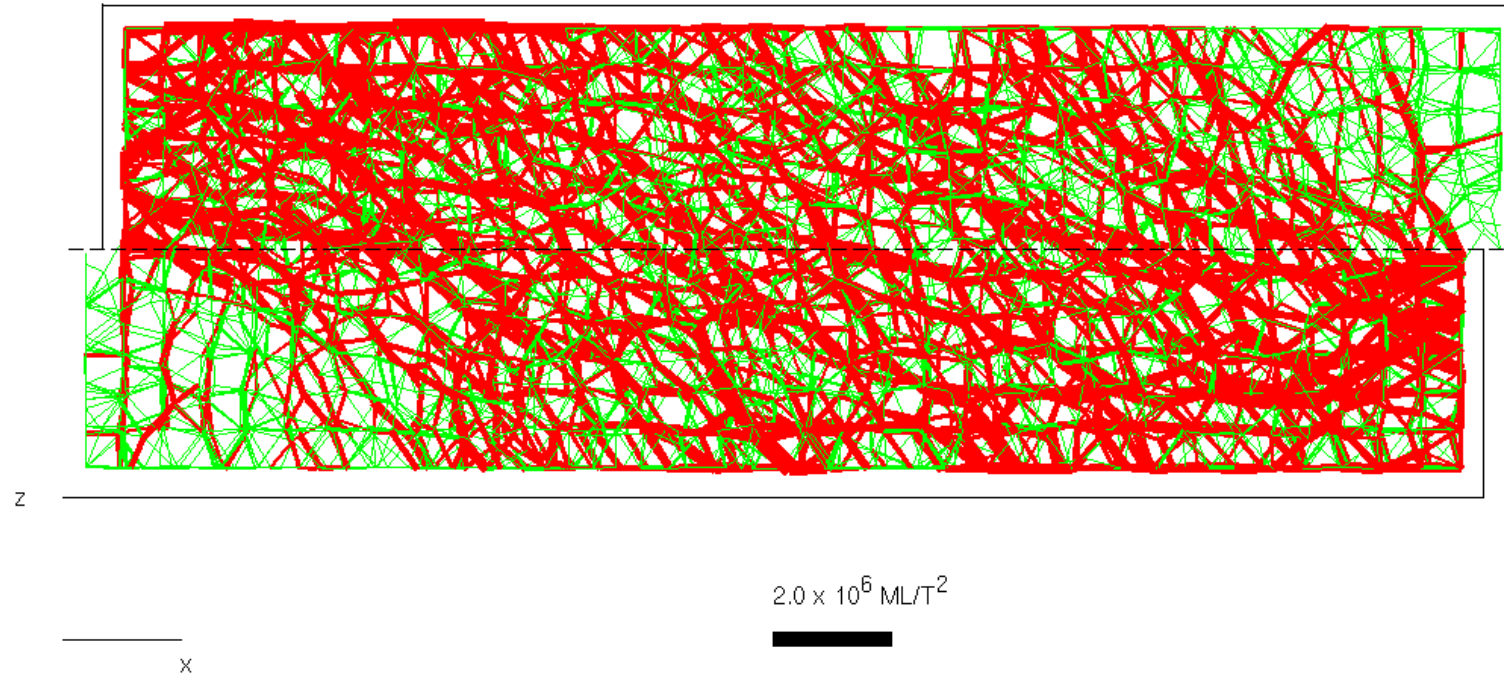
$\gamma=15.3\%$



$24 L < y < 36 L$

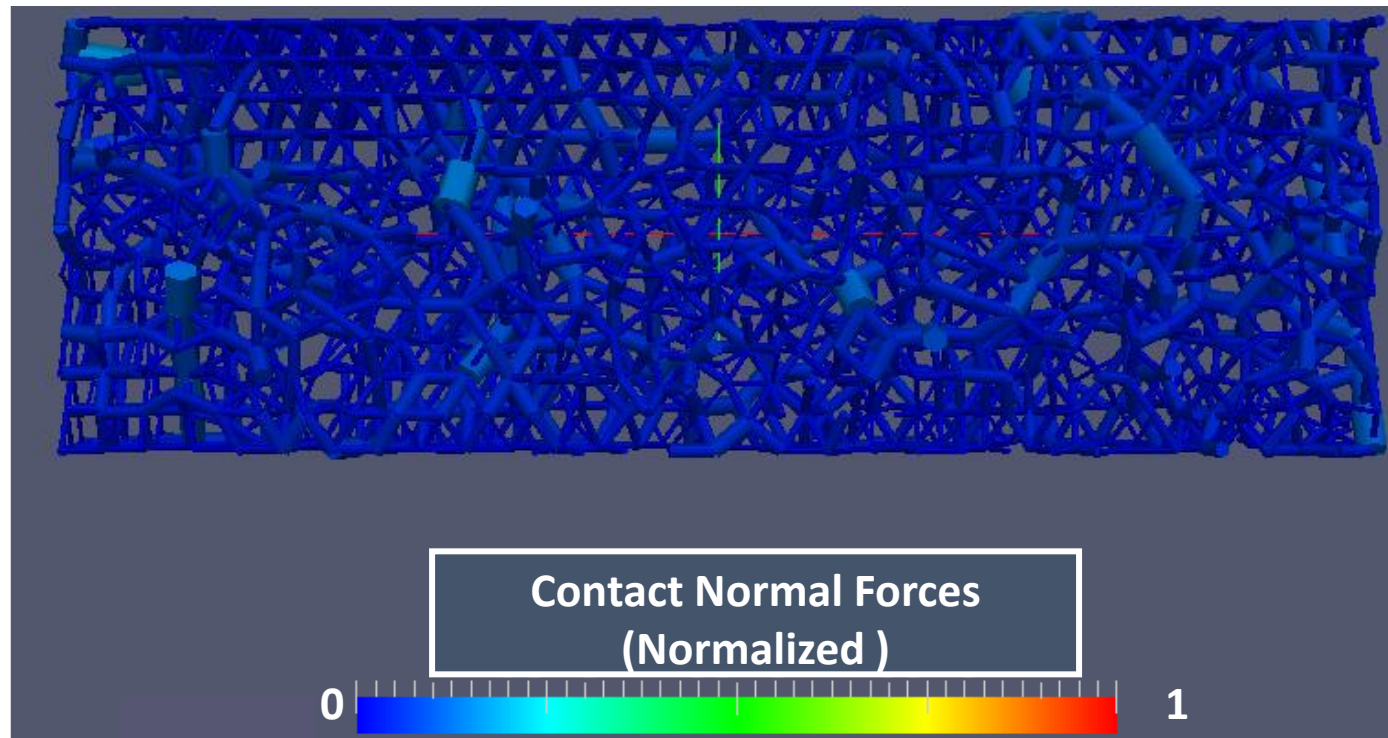
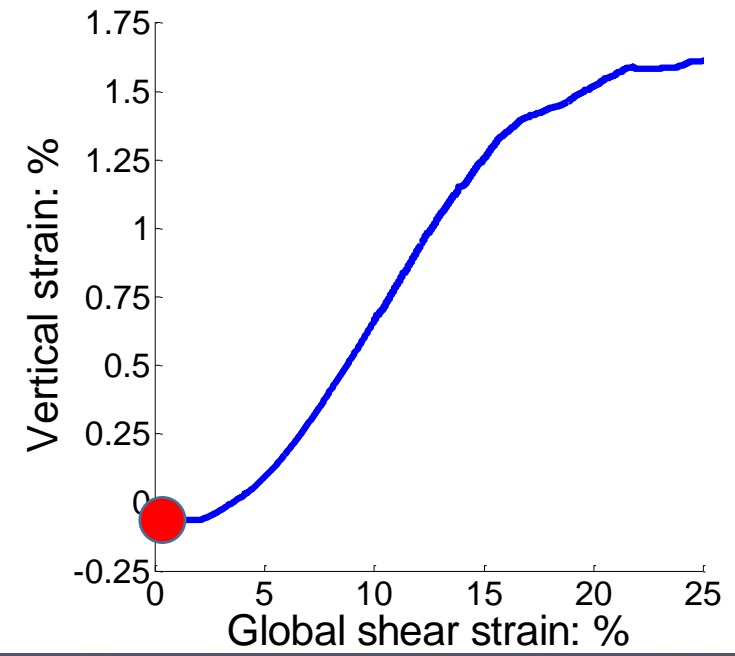
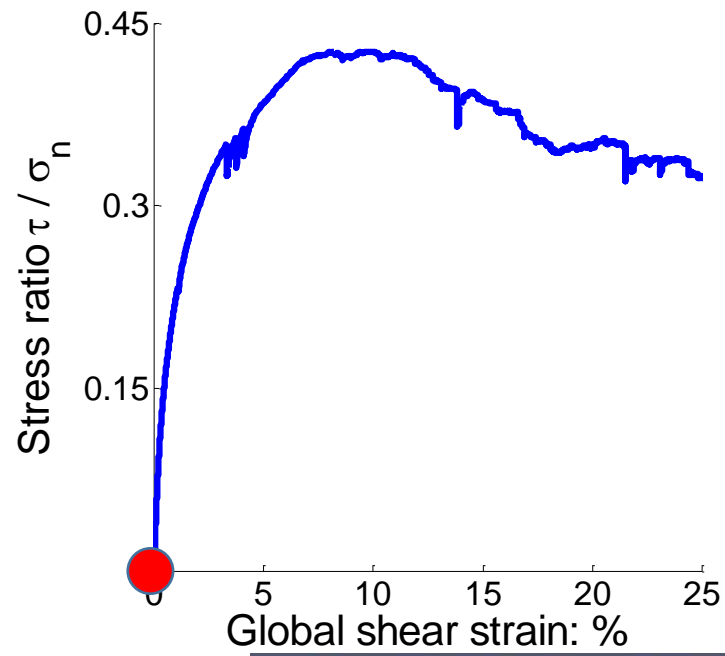
Contact force distribution in x-z plane

$$\gamma = 8.4\%$$

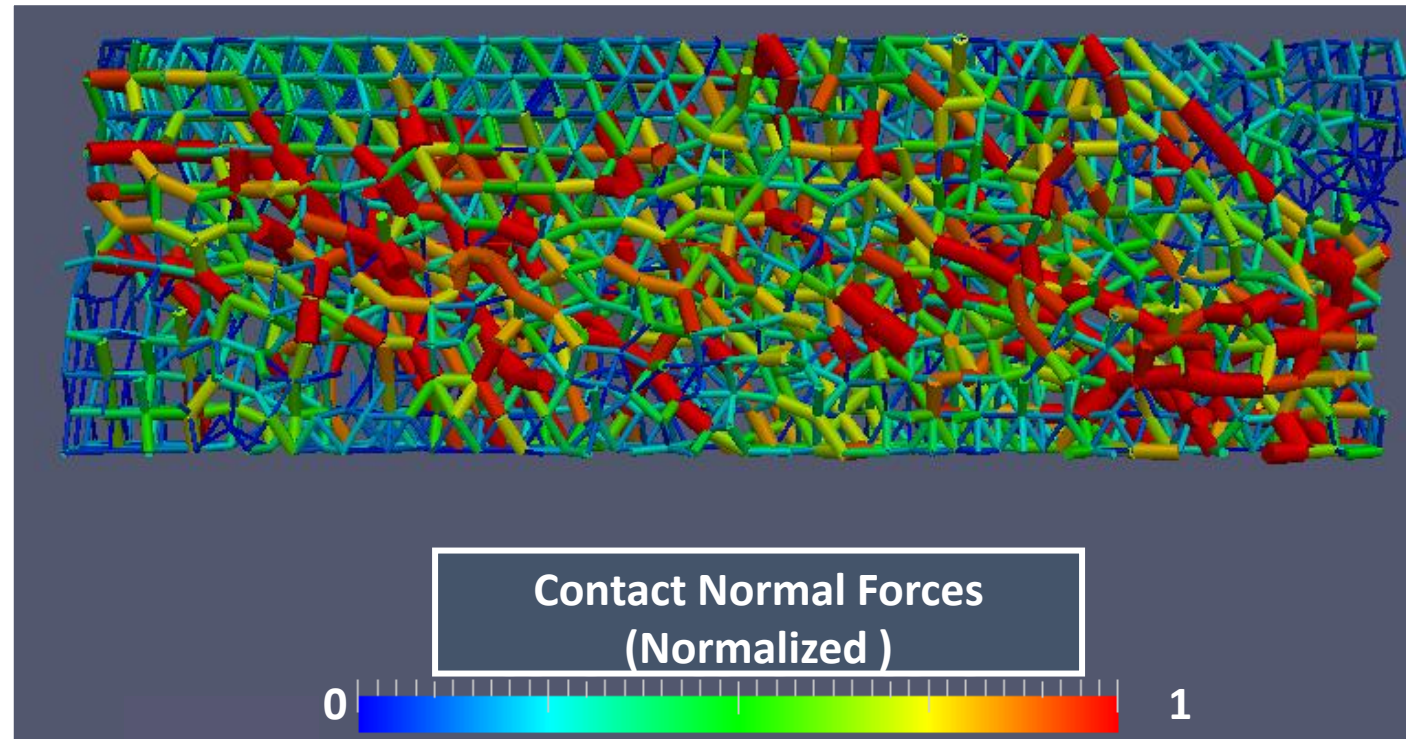
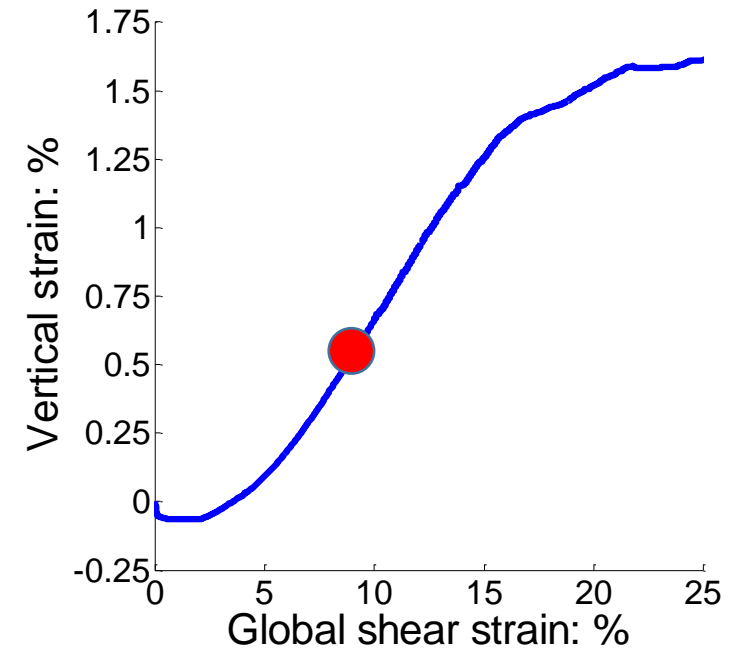
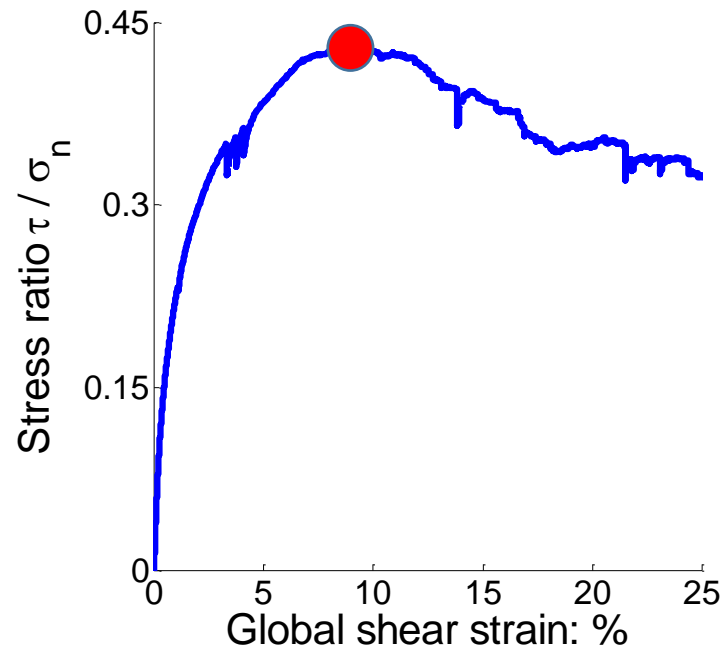


$$24 L < y < 36 L$$

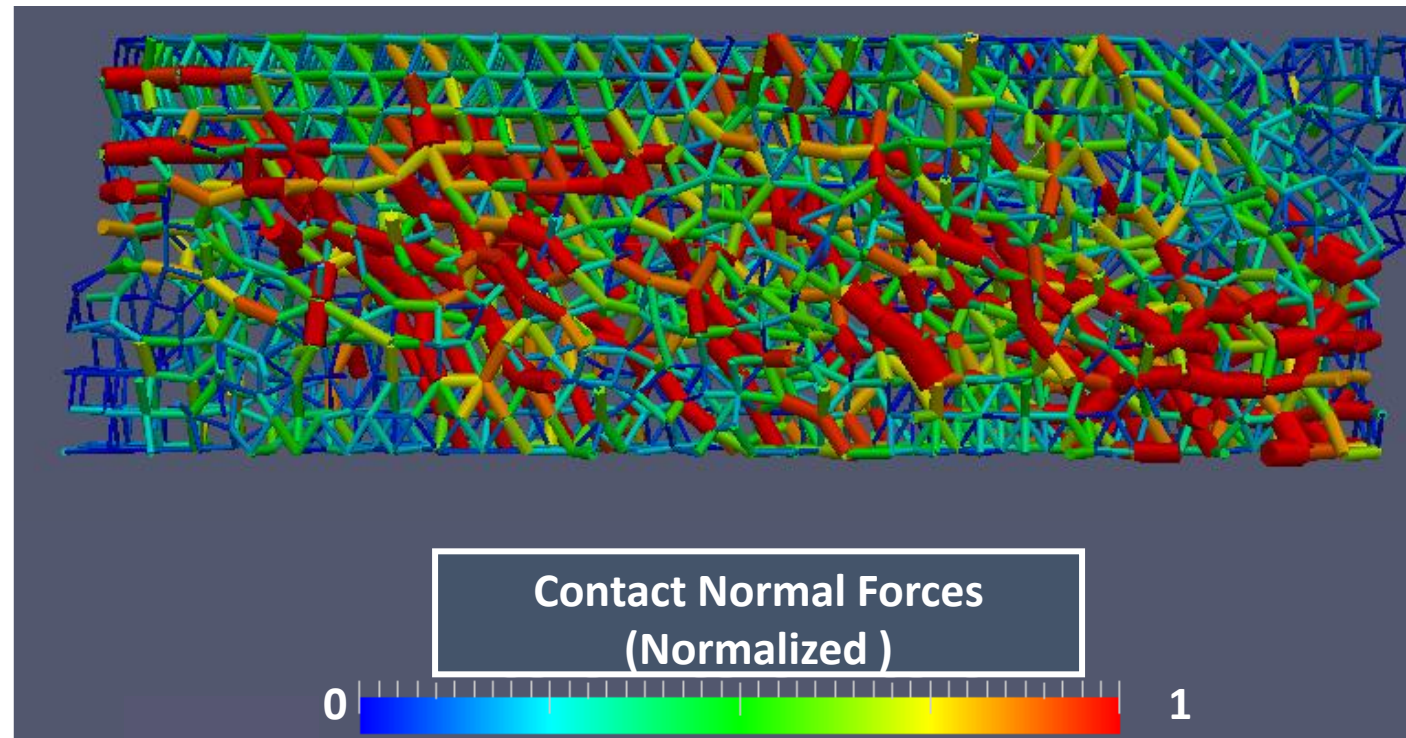
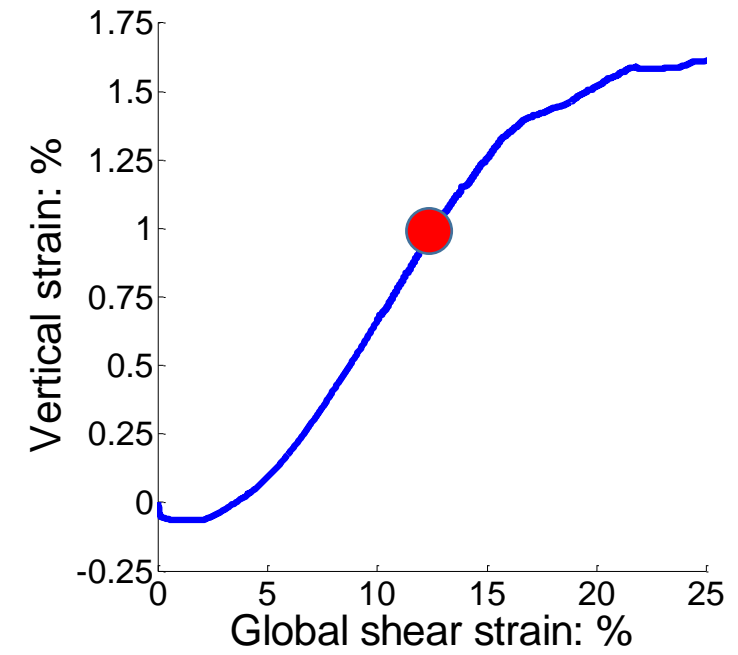
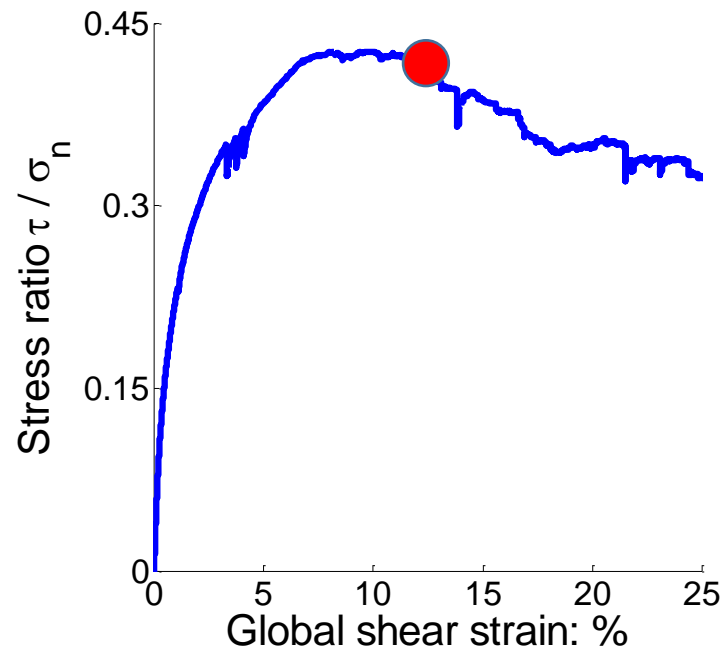
Contact Network



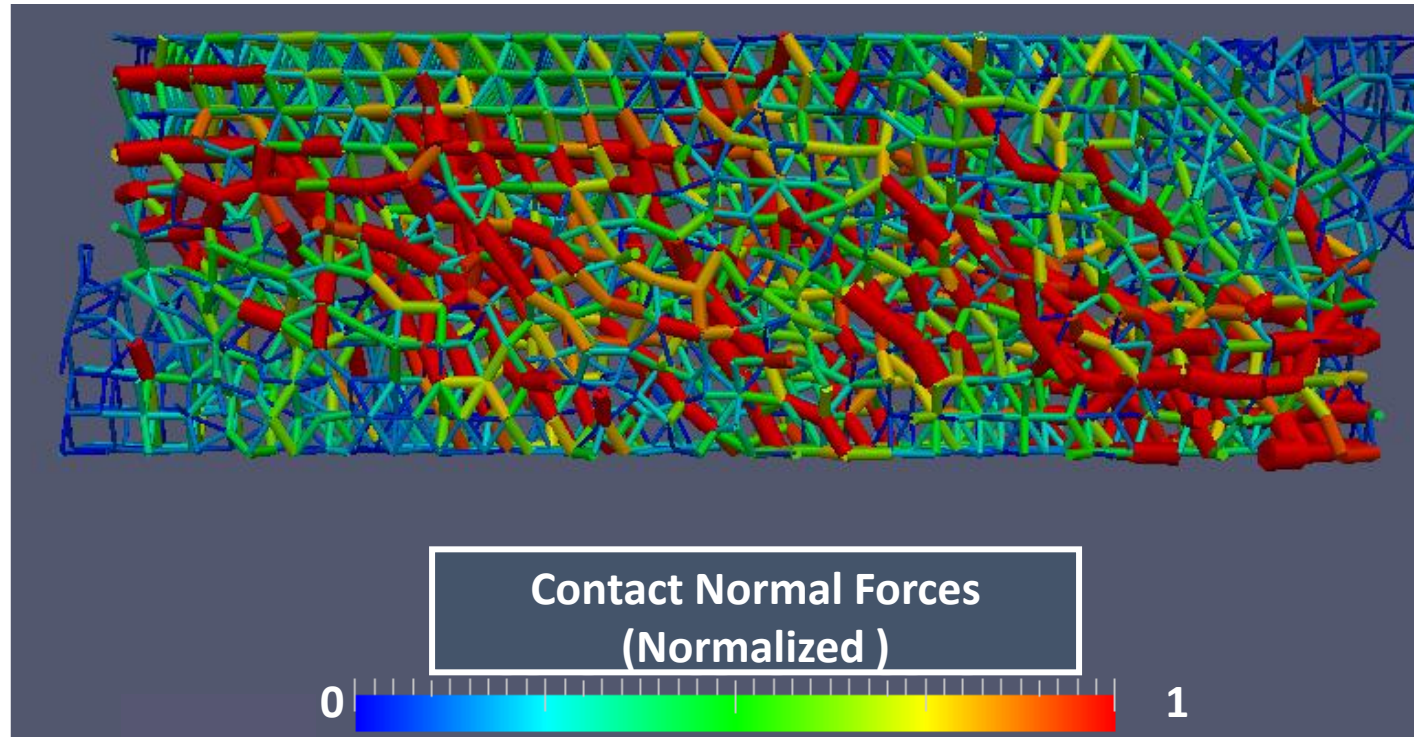
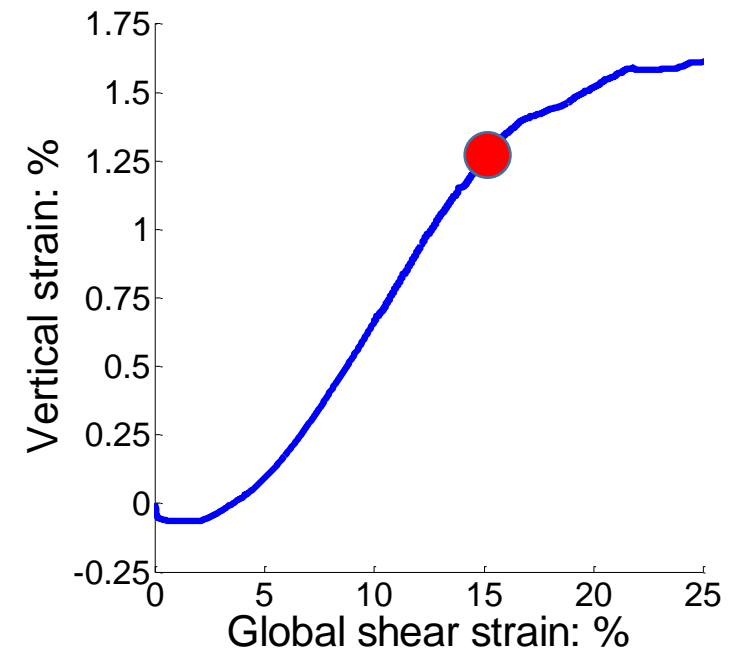
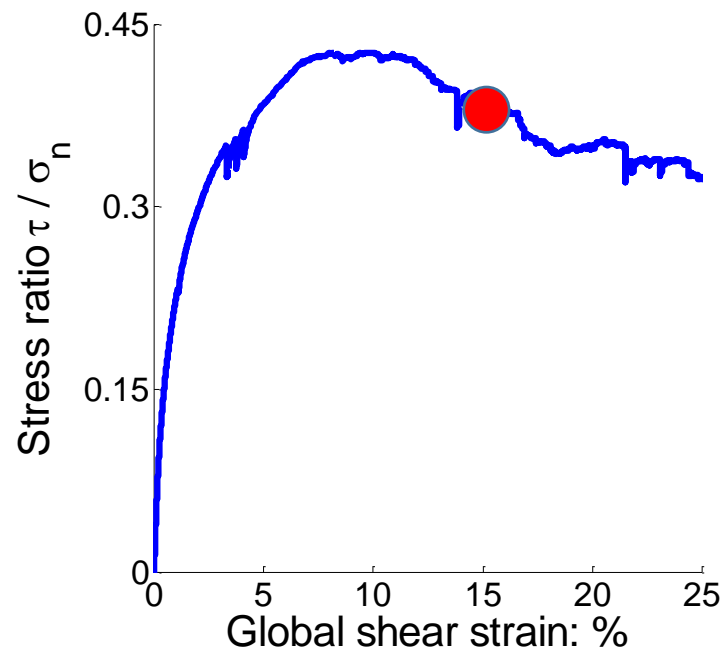
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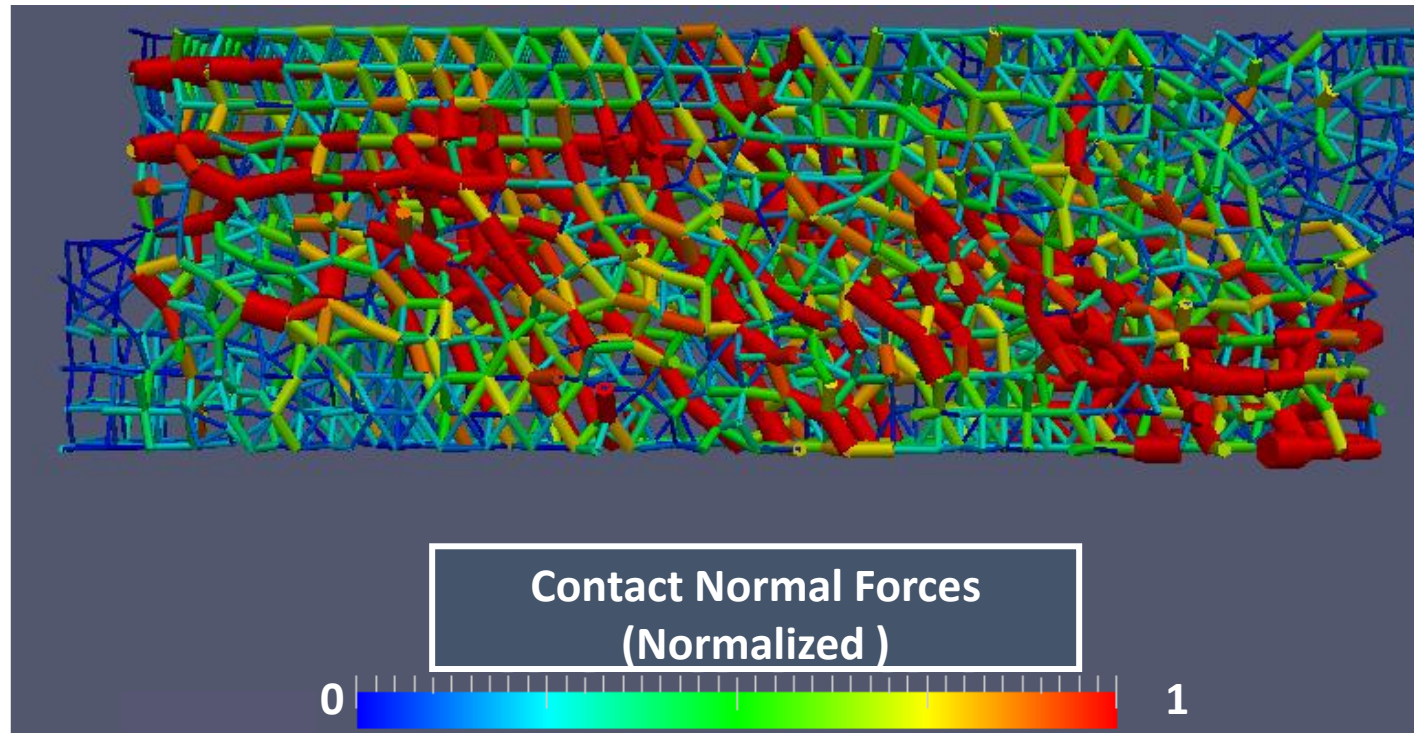
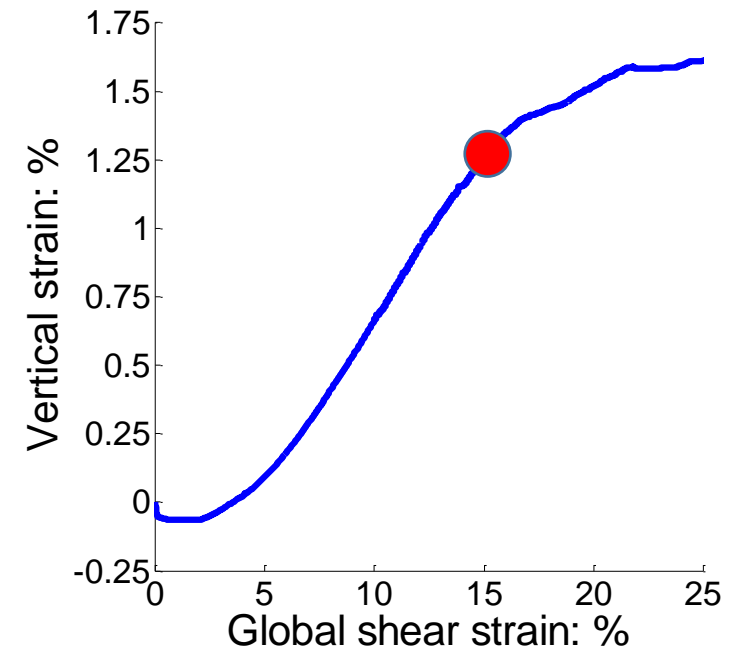
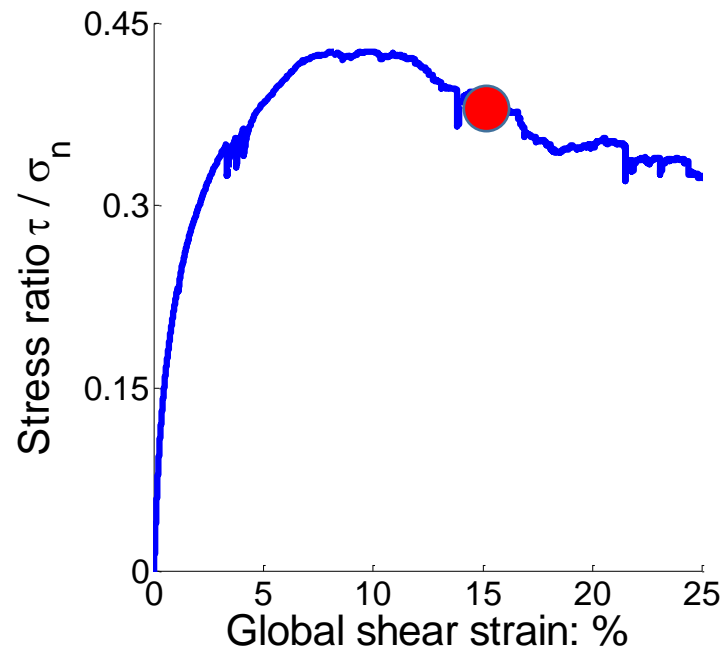
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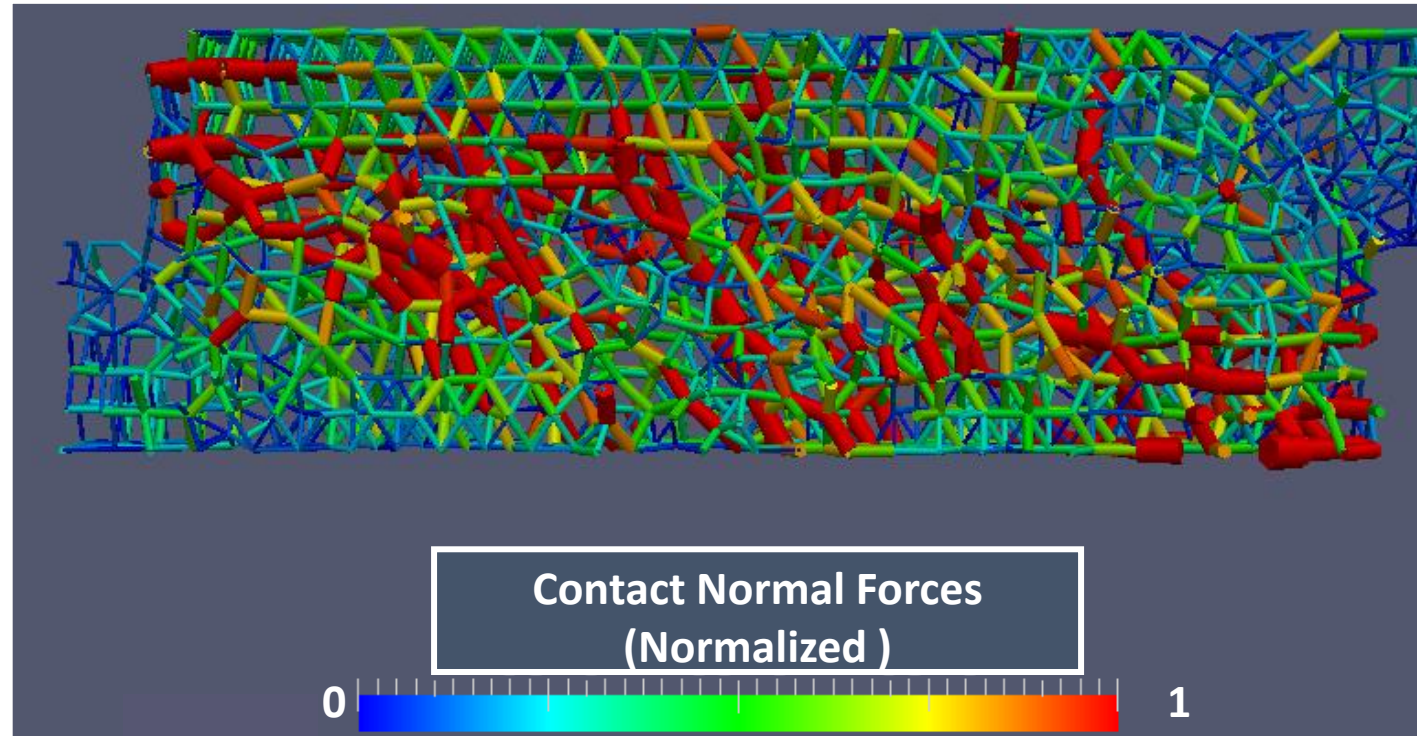
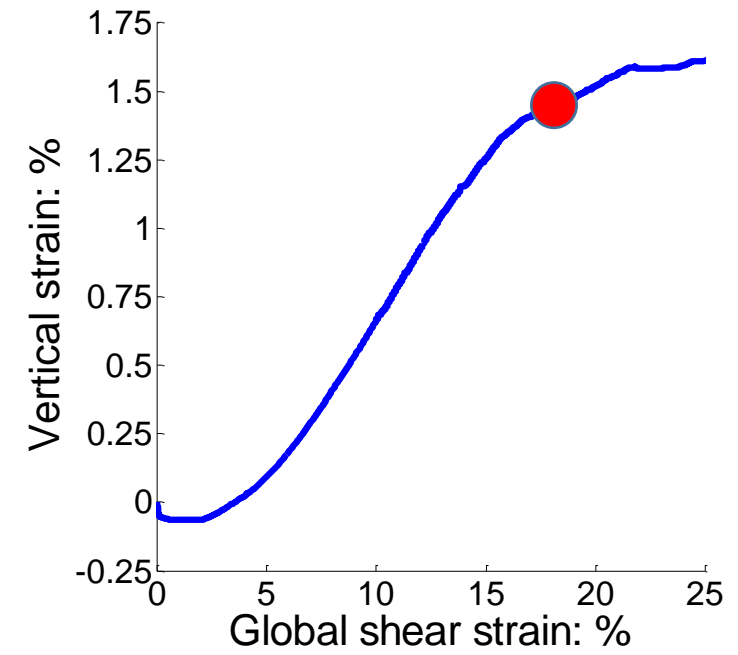
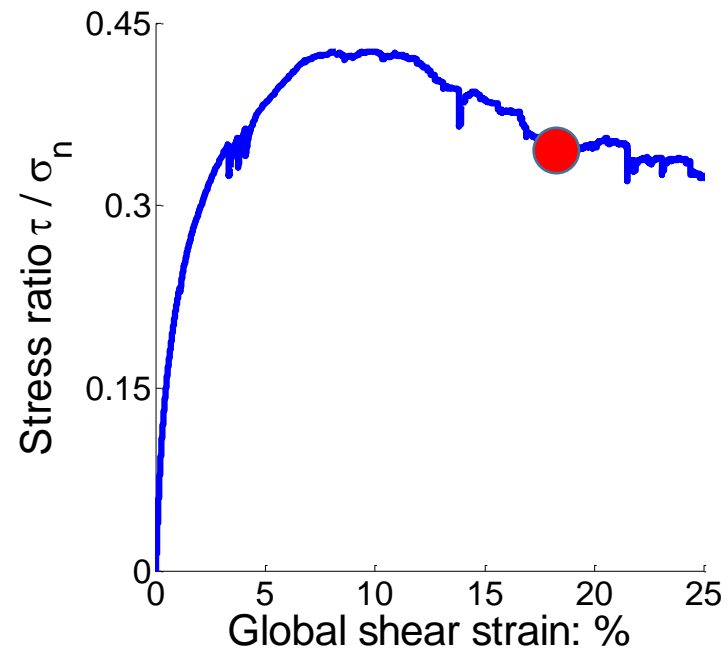
Contact Network



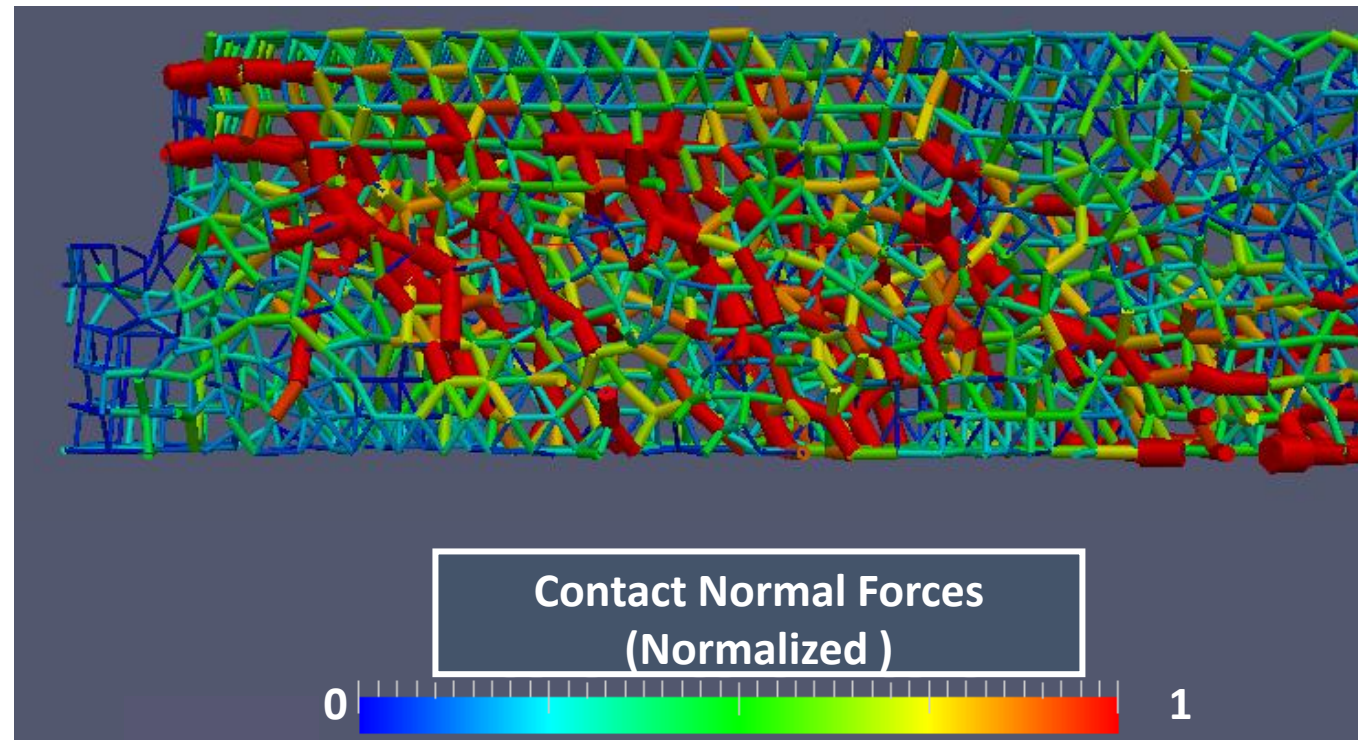
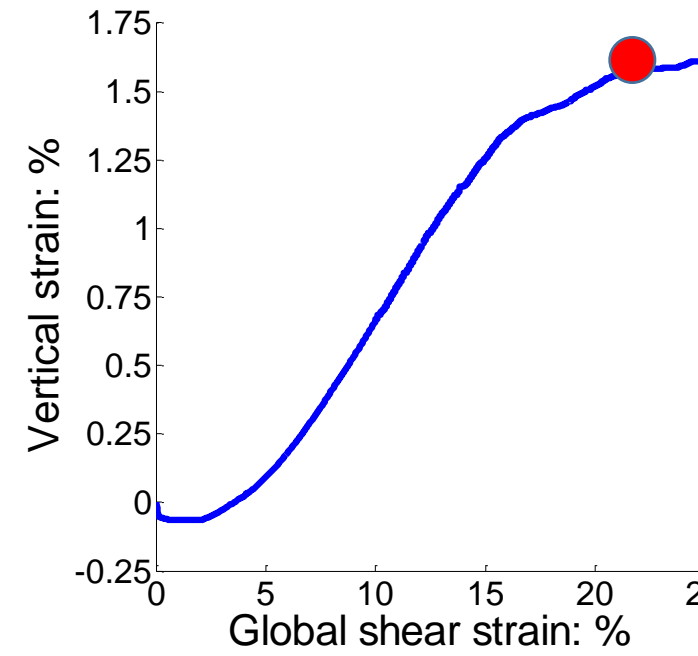
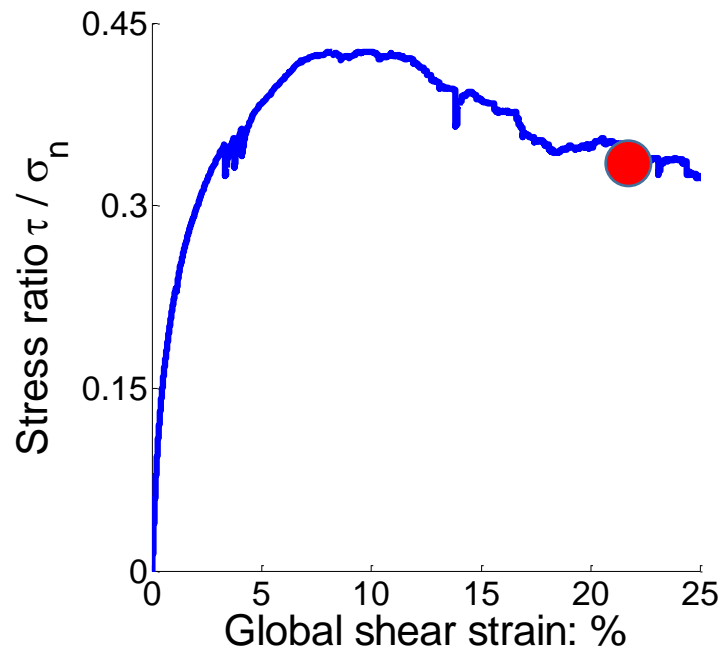
Contact Network



Contact Network



Contact Network



Conclusions

Visualization is essential as a quality check – Paraview is very useful

Conclusions

Choice of code

- PFC is the most popular DEM code based on review of DEM studies published in geomechanics / geotechnical /soil mechanics journals
- Now a good few DEM codes available open source
- Tendency to exploit parallelism.
- YADE / PFC Version 5 – shared memory
- LIGGGHTS / LAMMPS – distributed memory

Validation

- Experimental validation is useful, but not straightforward
- Analytical approaches using regular packing v important
- Benchmarking on random systems is helpful

Time step

- Conditionally stable method
- Essential to be within stability limit, but should not be over conservative

From eigenvalue approach:

- Critical time step depends on contact stiffness (which depends on confining pressure for Hertz-Mindlin model), particle mass and packing density (coordination number)
- Method based on Rayleigh wave speed not always conservative

Boundary Conditions

- Largely three types of boundaries: rigid wall, force, periodic
- Servo control can be used to capture complex conditions
- Use of computationally expensive force / membrane boundaries may not always be justifiable (small strain)
- Post-peak may be more sensitive

Sample size

- V important to establish you have an REV (representative element volume)
- Critical state line position can be affected by boundary conditions – v significant implications for understanding behaviour
- Plots of local void ratio helpful to understand extent of boundary effects
- Periodic cell very useful, but you still need to check an REV is achieved

Specimen generation

- You can achieve a range of densities by varying particle friction during preparation
- May detrimentally effect small strain response

Input parameters

- Using a high coefficient of friction (>0.5) with freely rotating spheres gives a response that is not representative of soil
- Explanation seems to be a transition from sliding to rolling at the contact points
- Use of two codes enhanced our confidence in observations
- Supplemental abstract analyses of small elements of the system were very useful

Contact model

- Hertz-Mindlin contact model ideal – even for glass ballotini
- Surface roughness - measurable effect on stiffness
- Partial slip – more subtle effect, less significant for regular, lattice packings

Running the simulation

- Inertia number effects position of critical state line
- For quasi-static simulations keep $I \leq 2.5e-3$

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