



A Particulate Perspective on Soil Mechanics

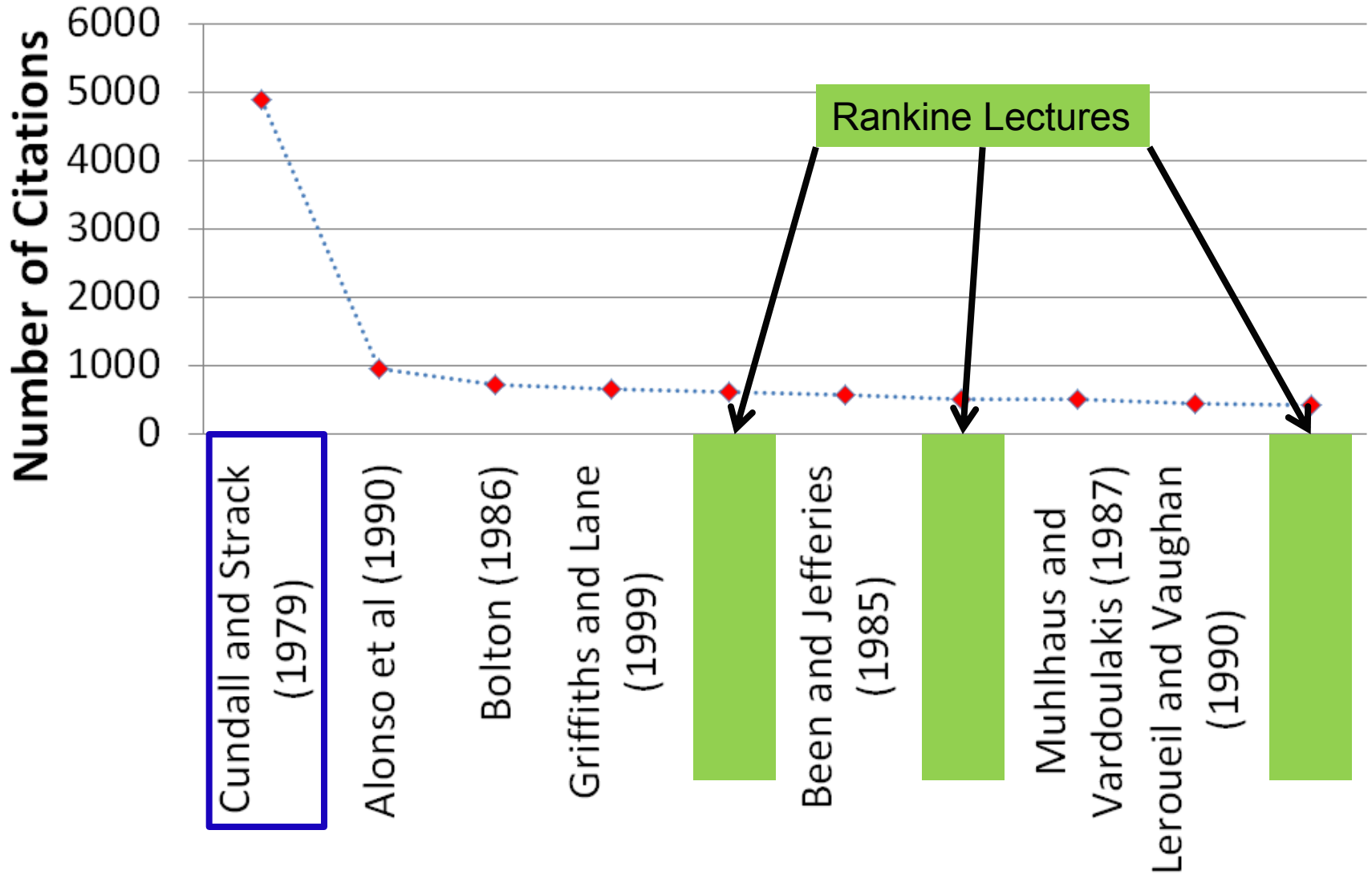
2015 Géotechnique Lecture

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A Particulate Perspective on Soil Mechanics Can...

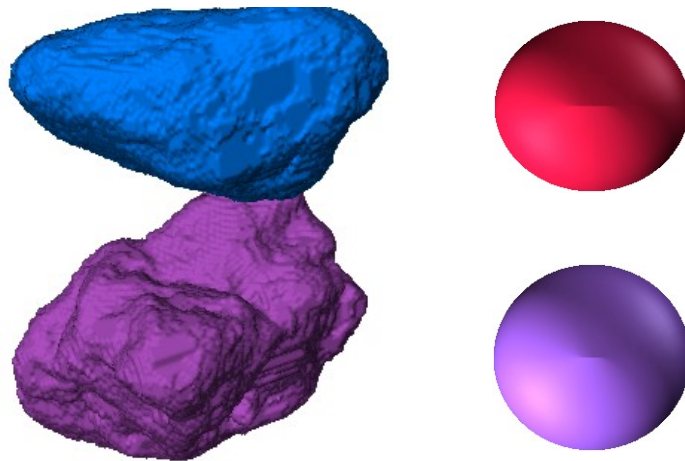
- Reveal soil behaviour under stress states that cannot be attained in experiments **(DEM)**
- Explain perplexing soil behaviour **(microCT)**
- Provide scientific rationale for empirical results used in design **(DEM+
microCT)**

Most Cited Contributions Published in Géotechnique



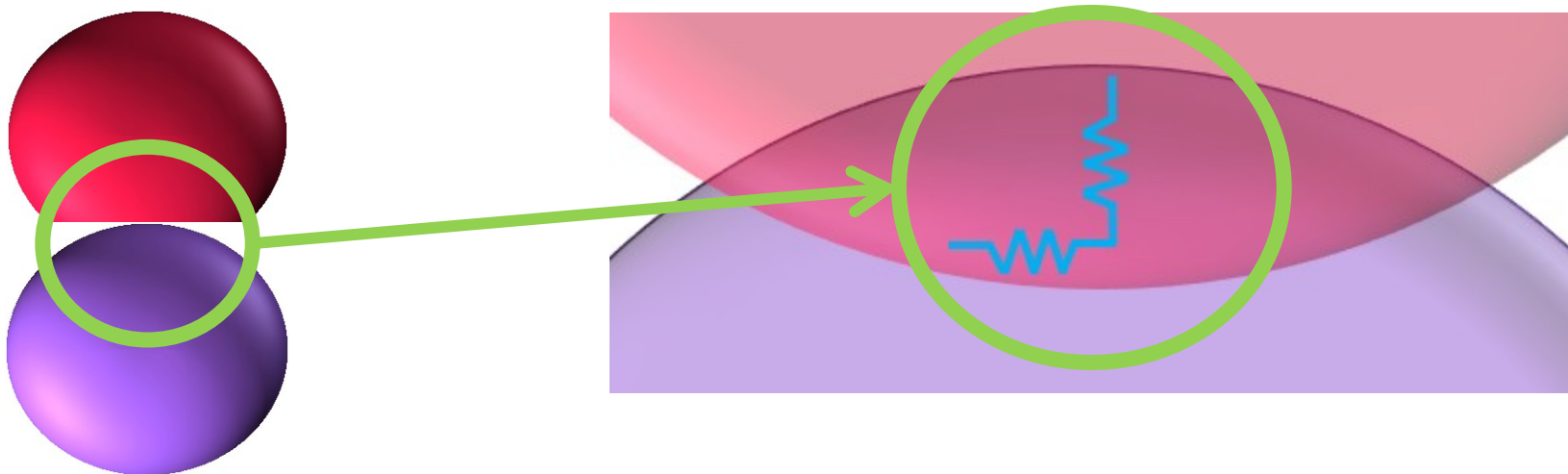
Cundall and Strack (1979)

“A discrete numerical model for granular assemblies”

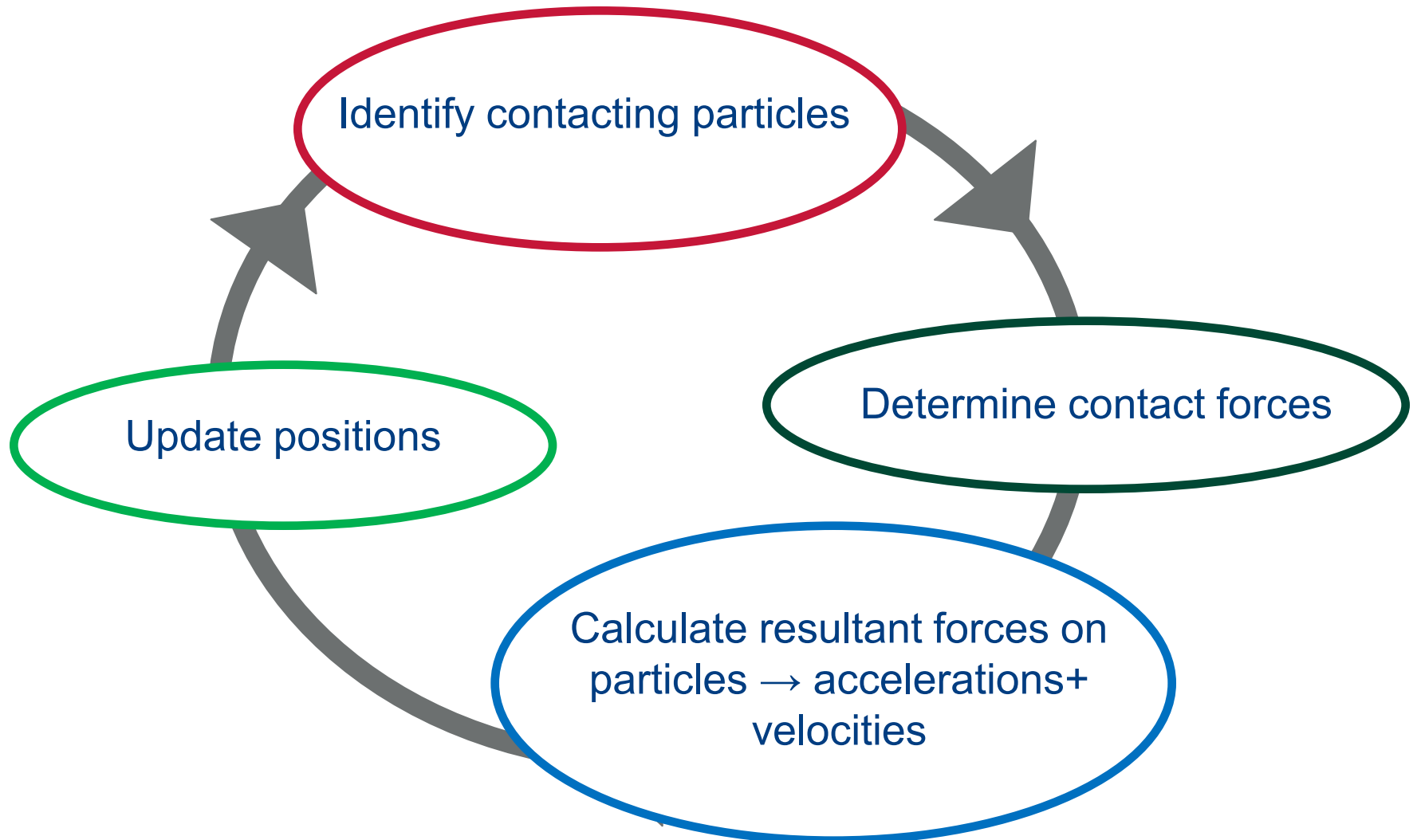


DEM Contact

- Contact force calculated using orthogonal normal and shear springs
- Spring deformation calculated from relative motion of contacting particles
- Sliding governed by Coulomb friction : $T_{max} = \mu F_n$
 - μ = coefficient of friction
 - F_n = contact normal force
 - T_{max} = maximum tangential (shear) force

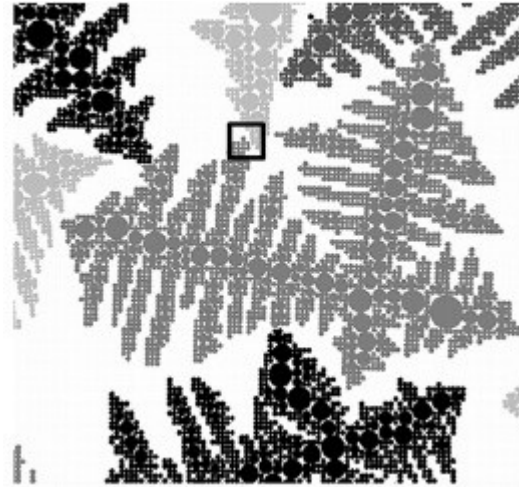
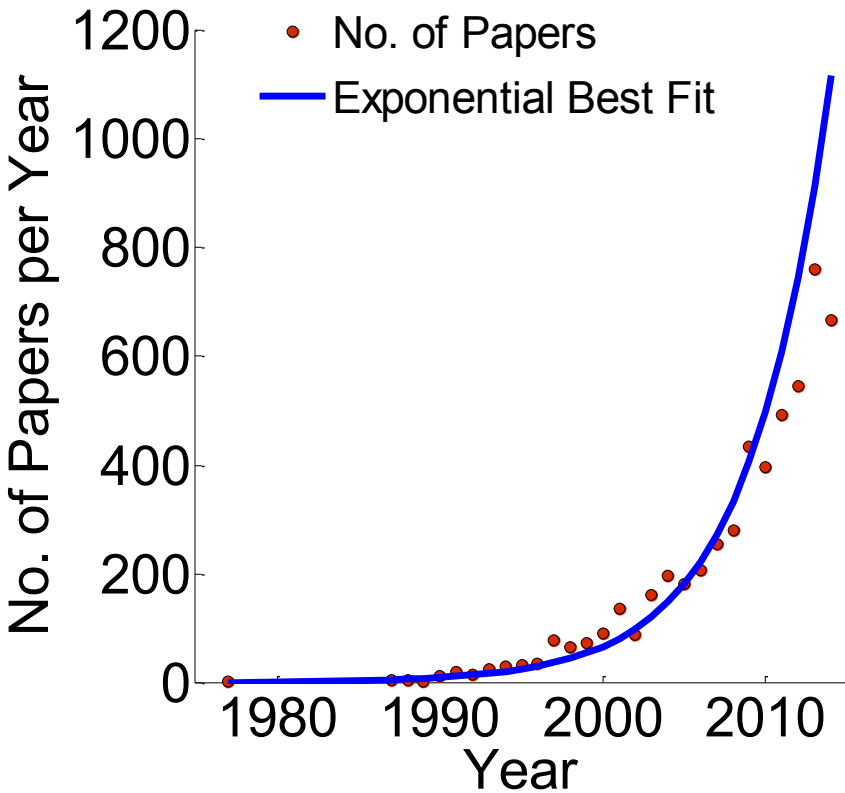


Transient Calculation – In Each Time Step:



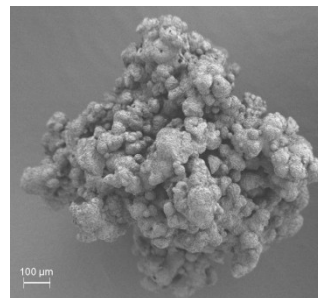
Use of DEM – No of Publications per Year

All Scientific Disciplines

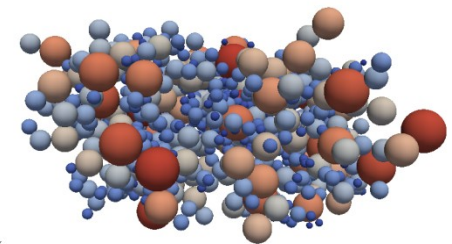


Technical journals

DEM simulation of semi-solid metal
Yuan et al (2012)

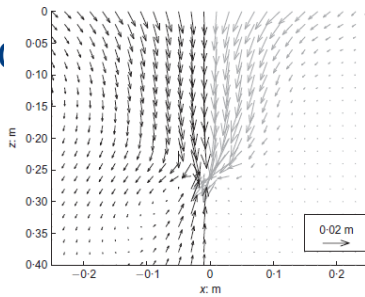
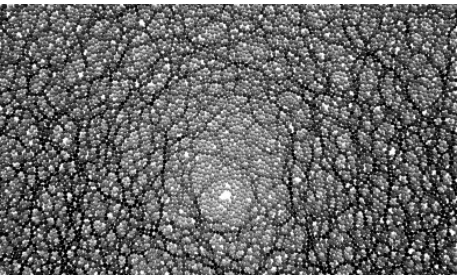


Diameter (mm)
0.125
0.12
0.1
0.08
0.06
0.04
0.02



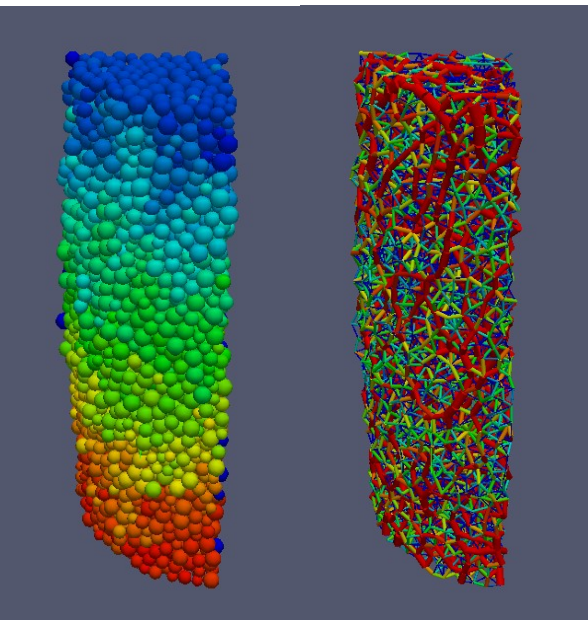
DEM simulation of infant formula
Hanley et al (2012)

Use of DEM – No of Publications per Year

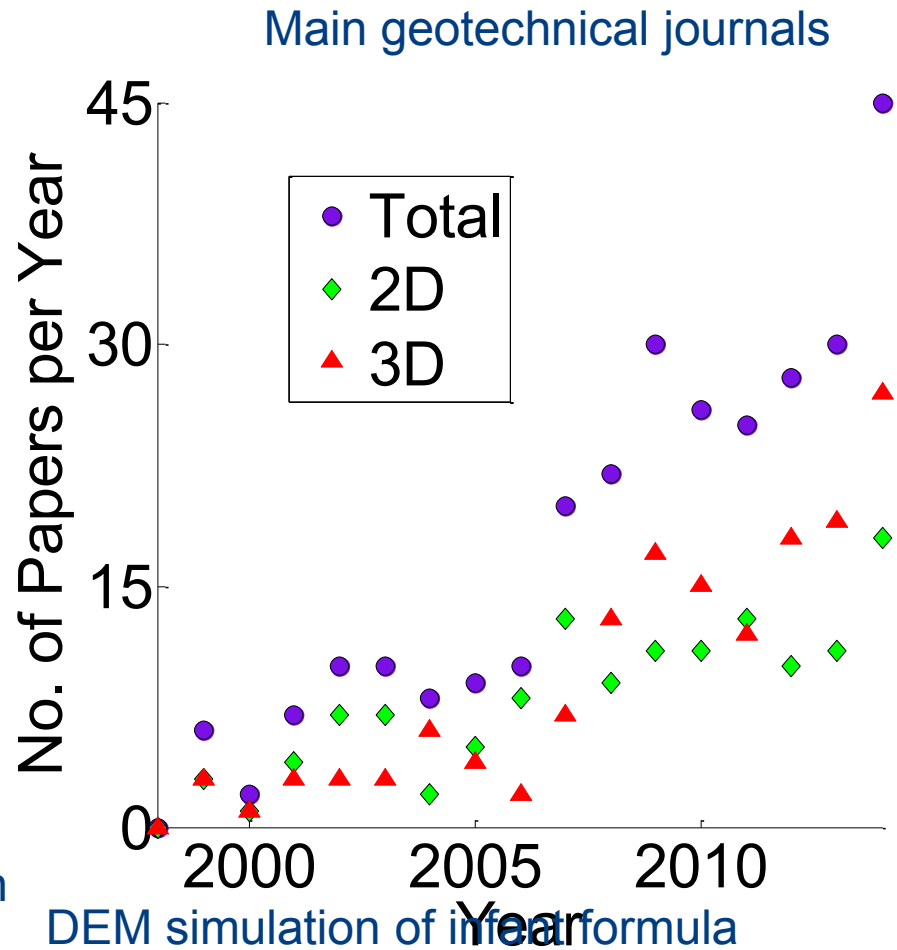


fic

Tunnelling induced displacements
Bym et al. (2013) Géotechnique



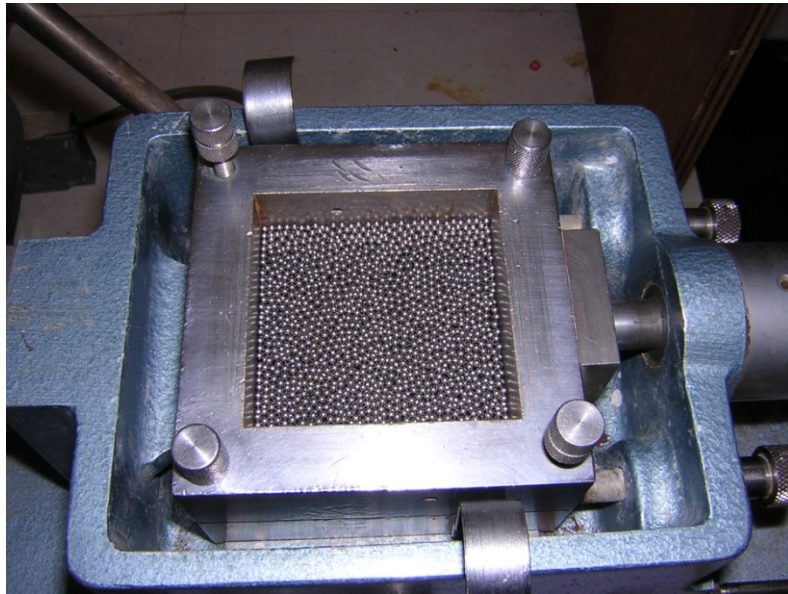
Deformation in
triaxial cell
Cui et al. (2007) Géotechnique



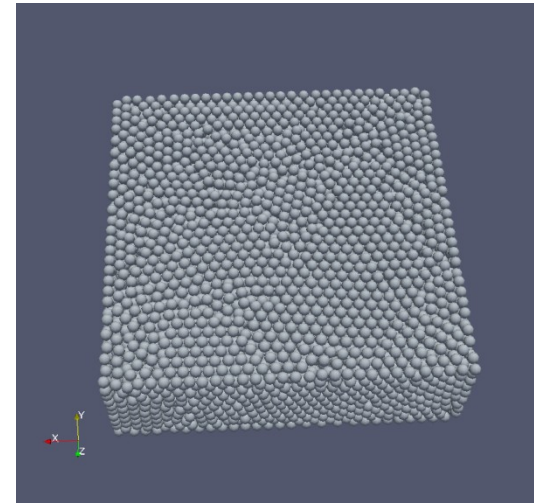
DEM simulation of infill formula

Hanley et al (2012) Géotechnique

Direct Shear Test



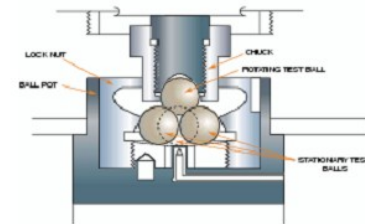
Grade 25 steel precision balls
Diameter 0.992 mm



DEM model: 11700 Spheres



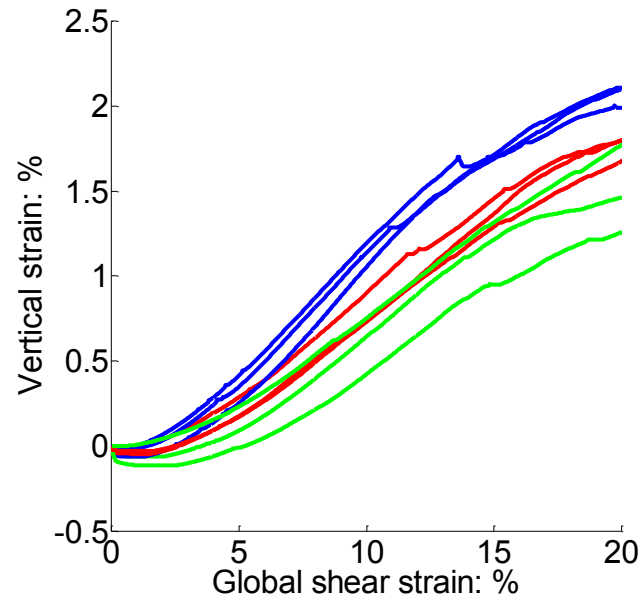
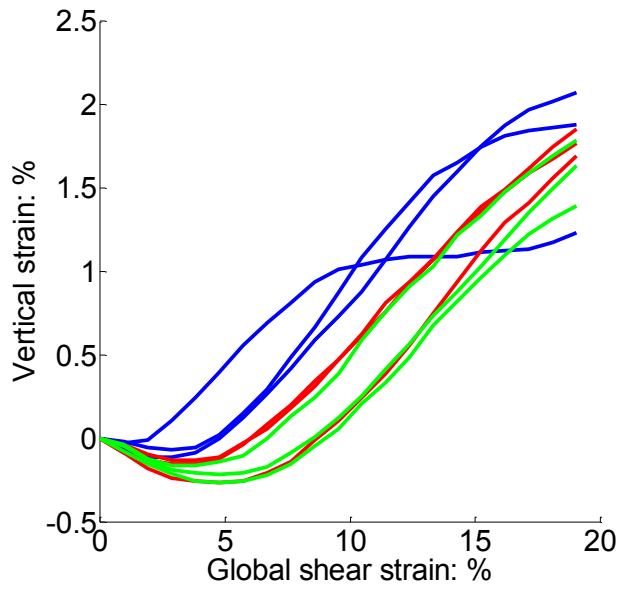
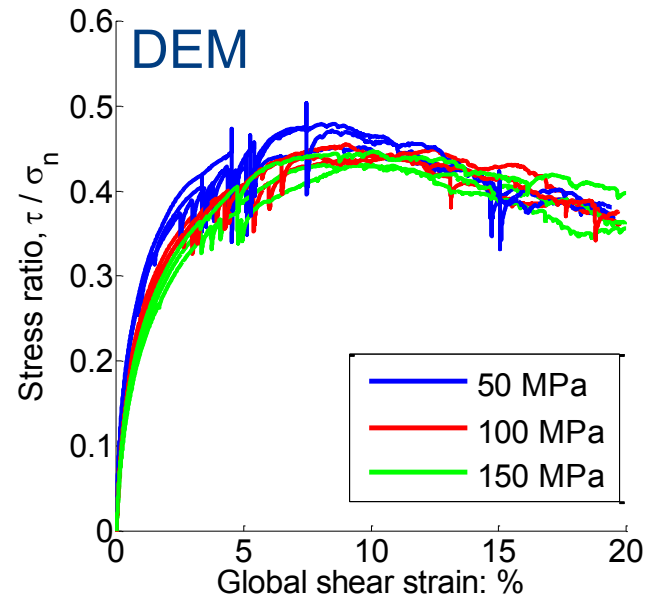
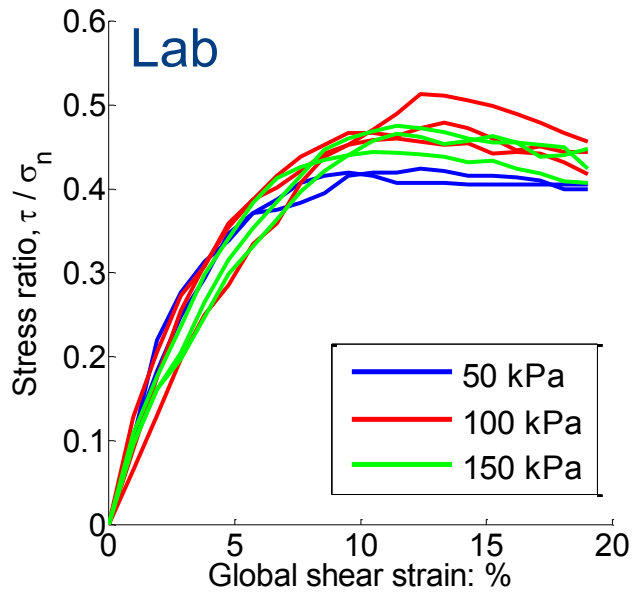
Mean μ_{surface}
5.5°



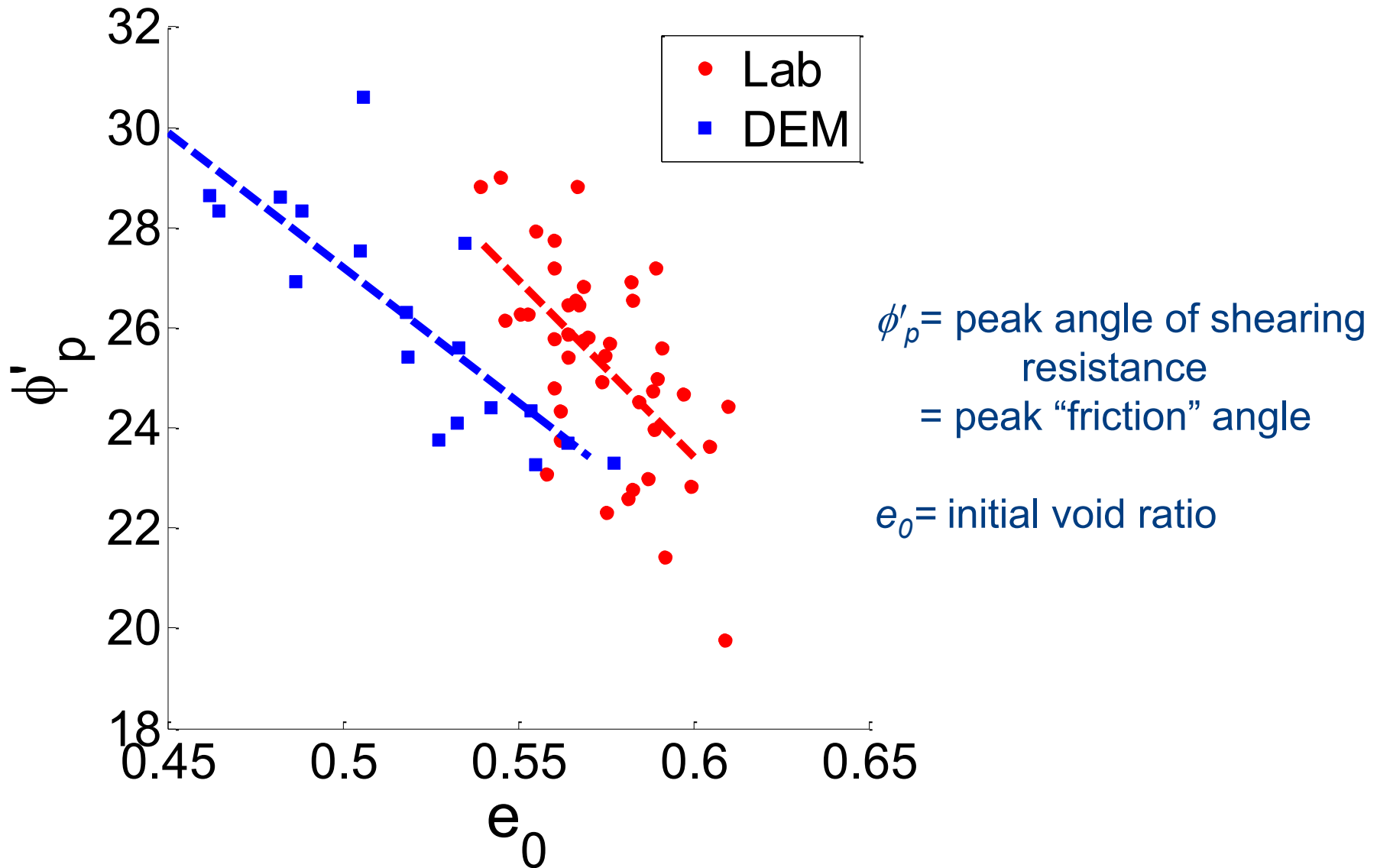
Mean μ_{surface}
5.7°

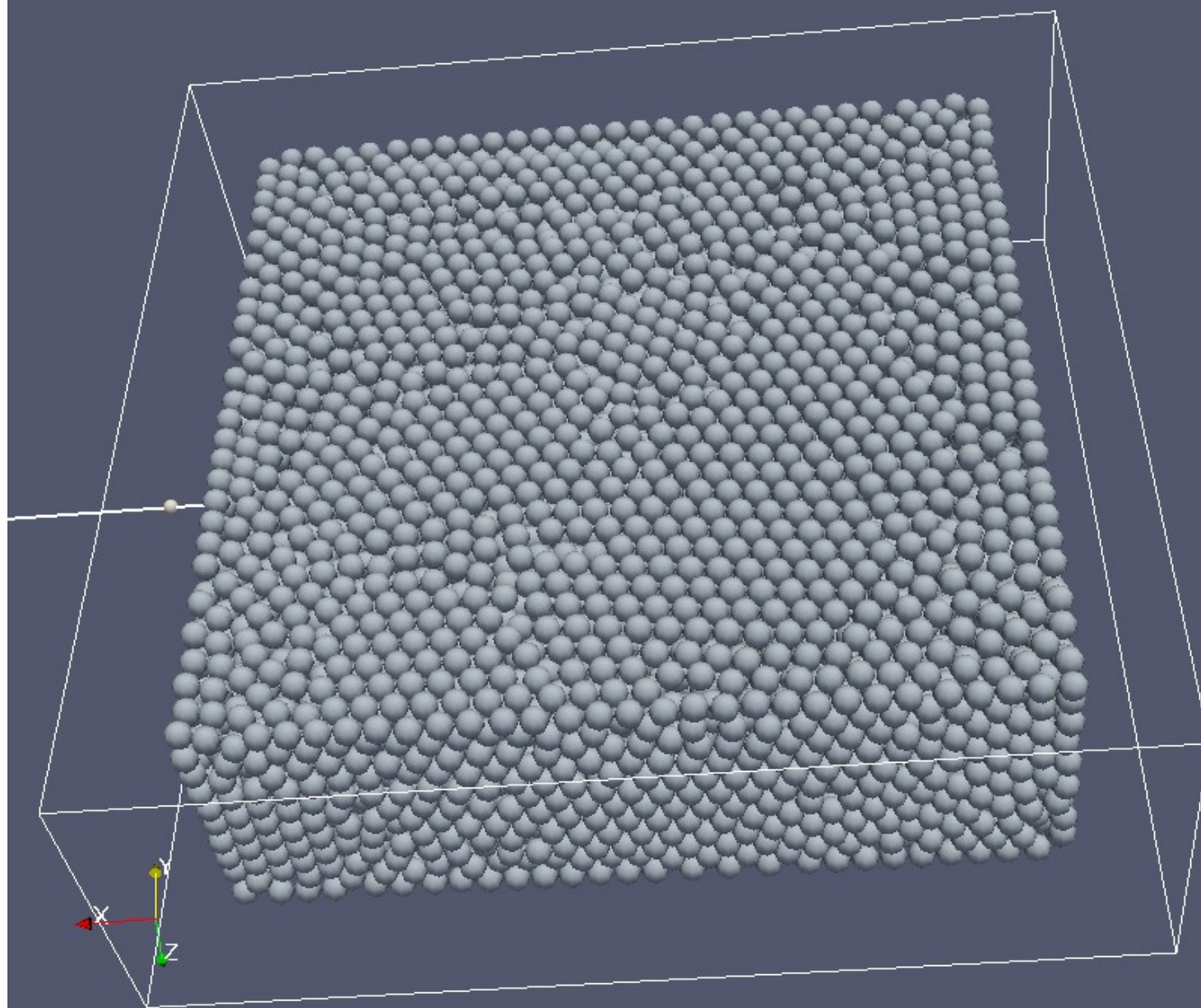
Cui and O'Sullivan (2006)
Géotechnique

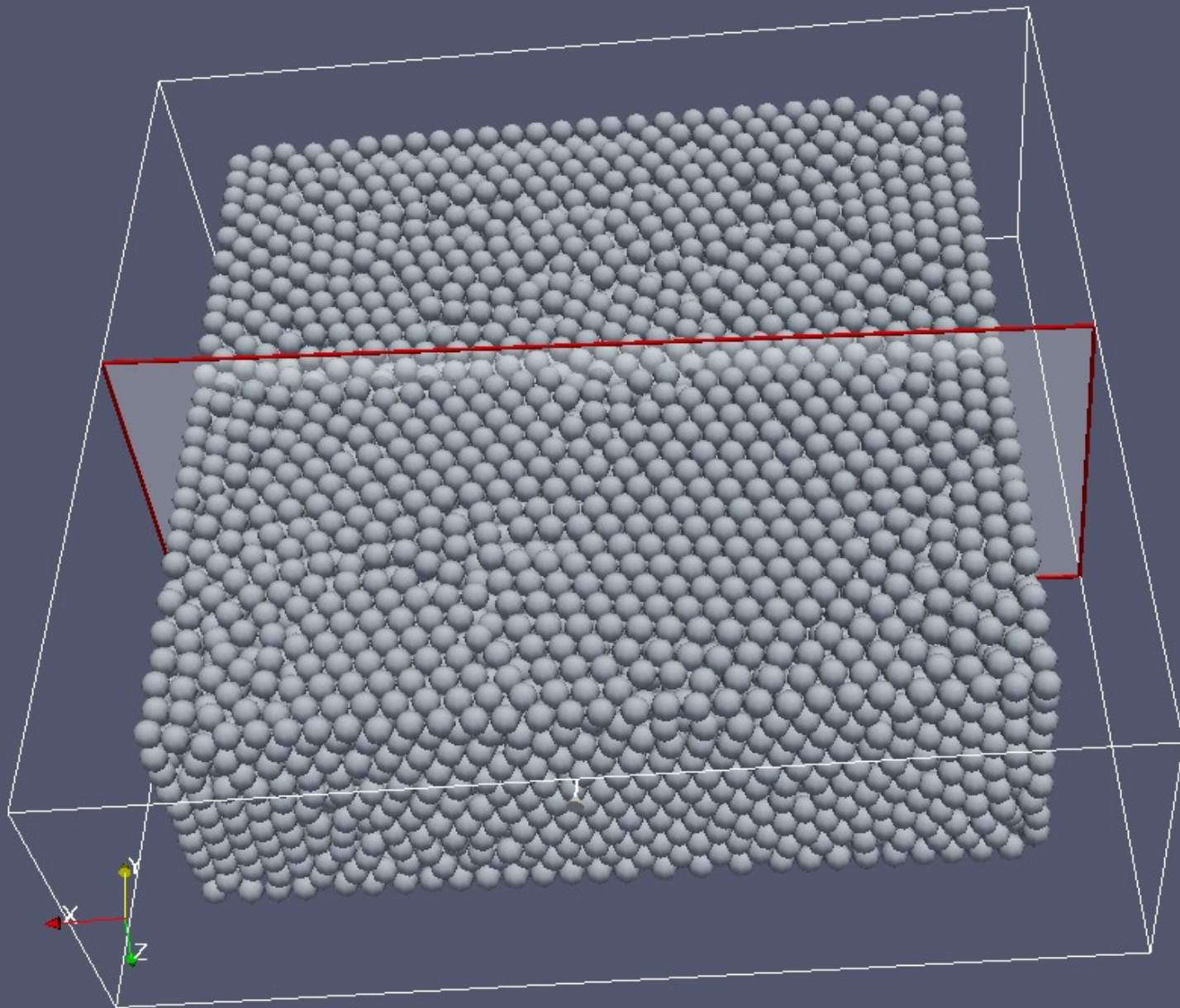
Direct Shear Test

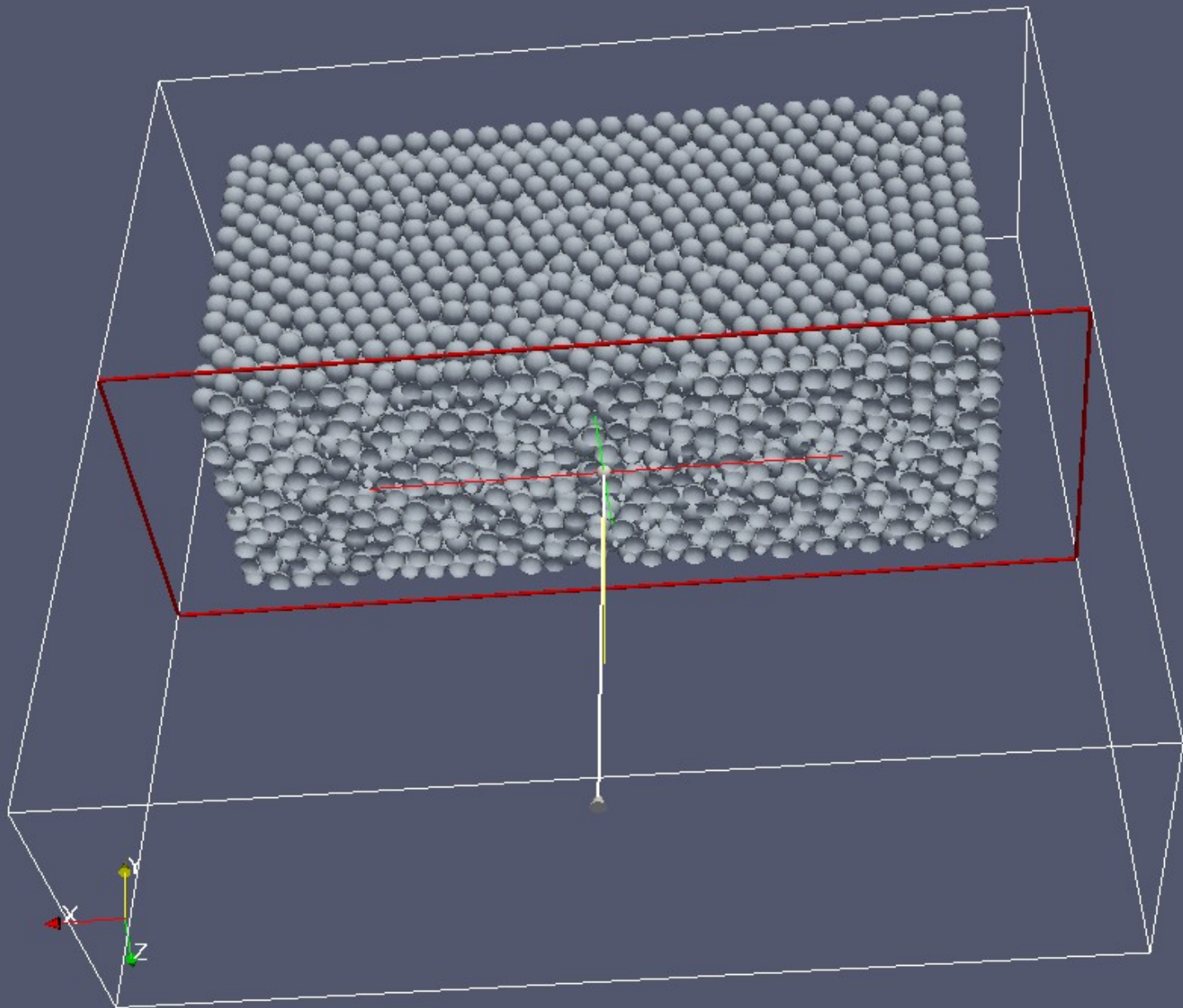


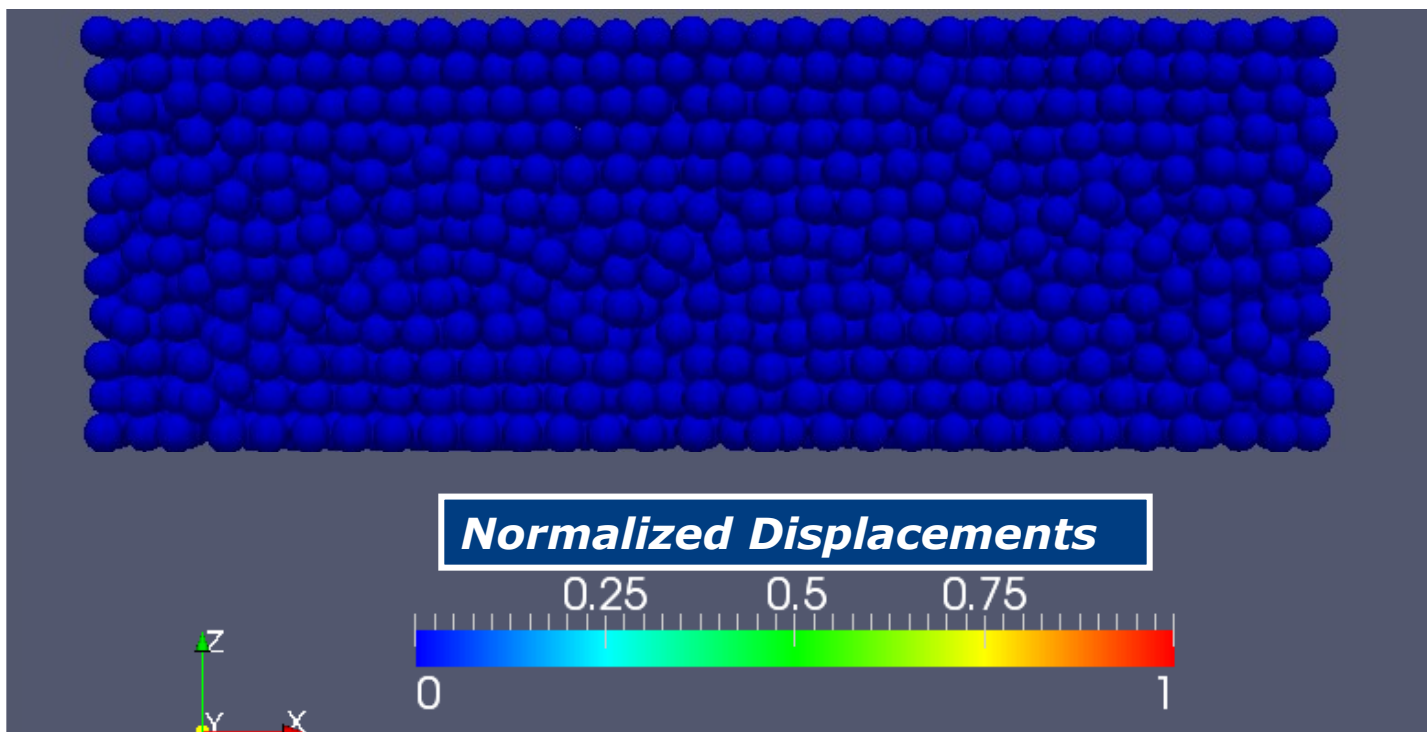
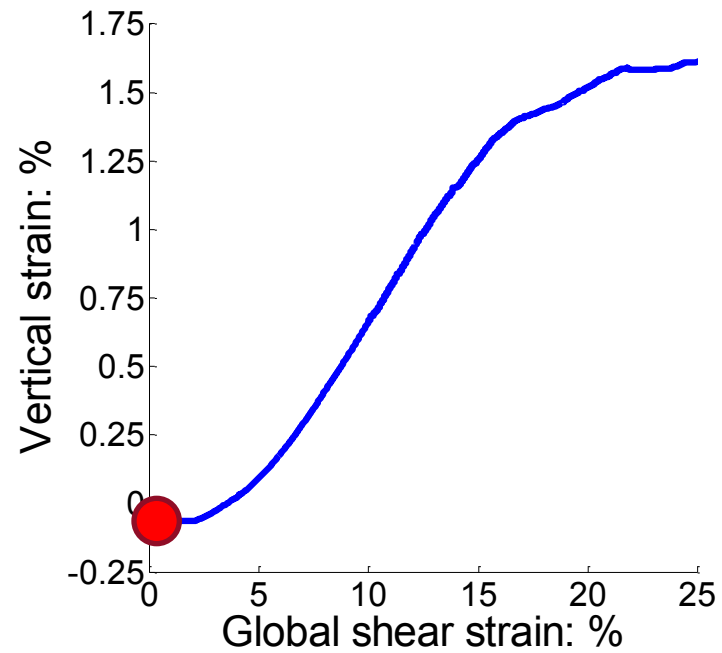
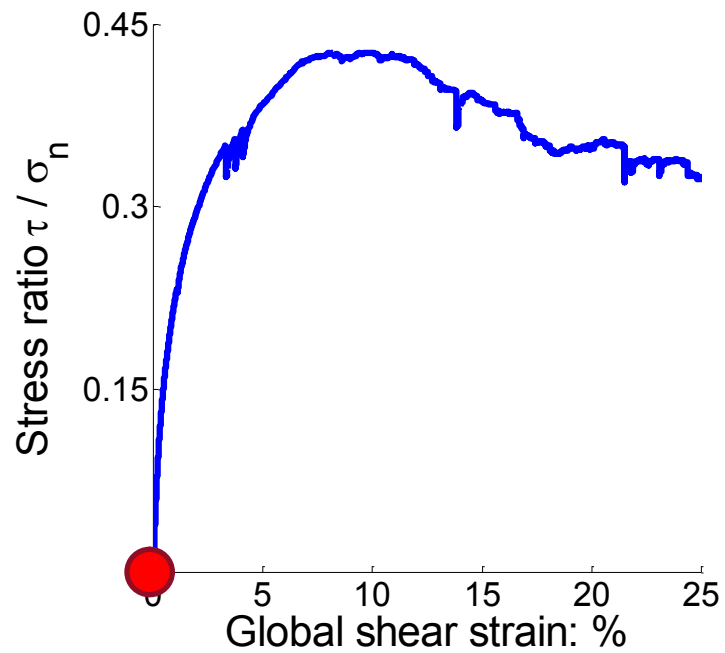
Direct Shear Test

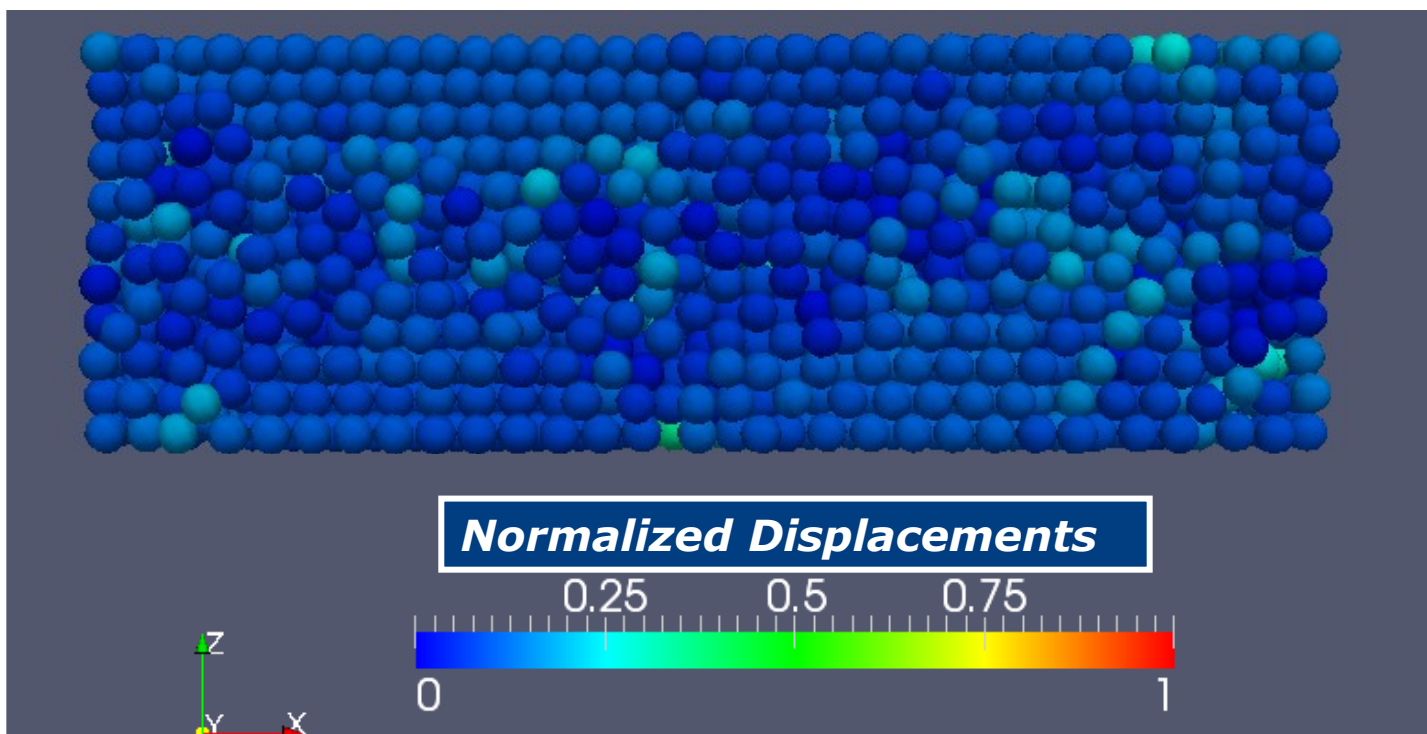
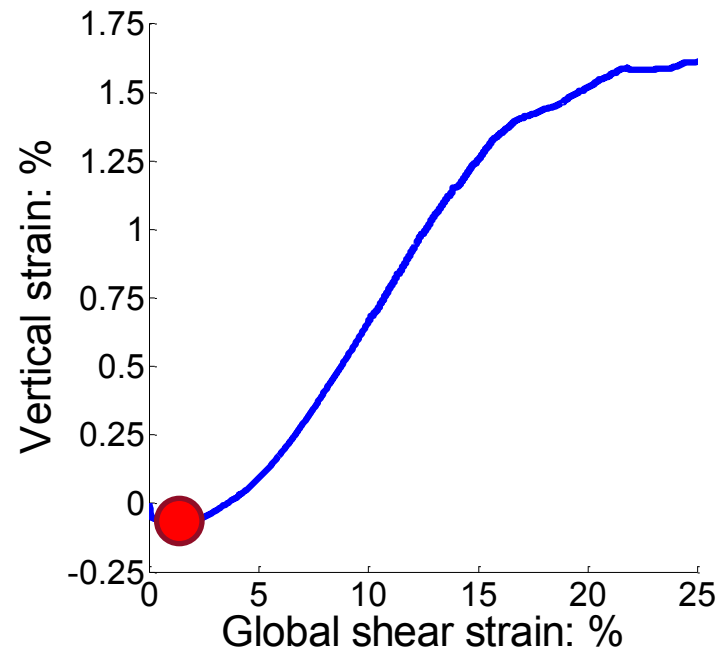
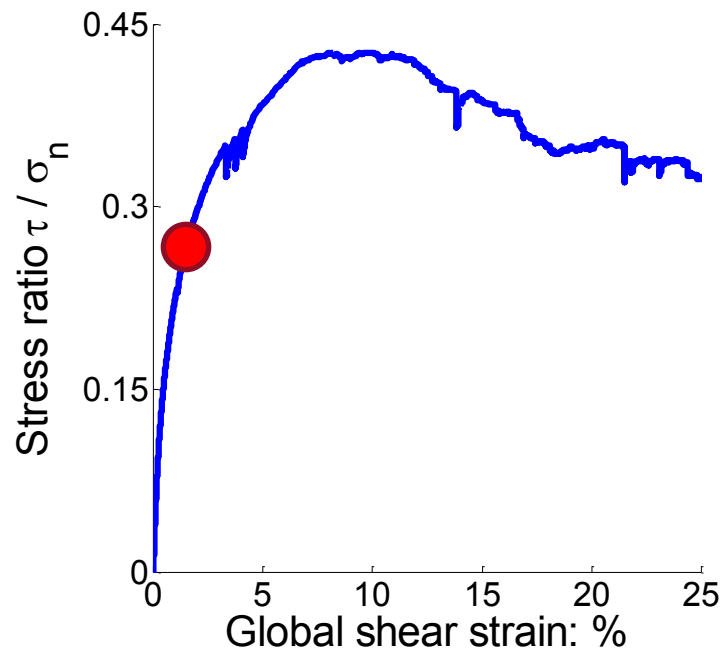


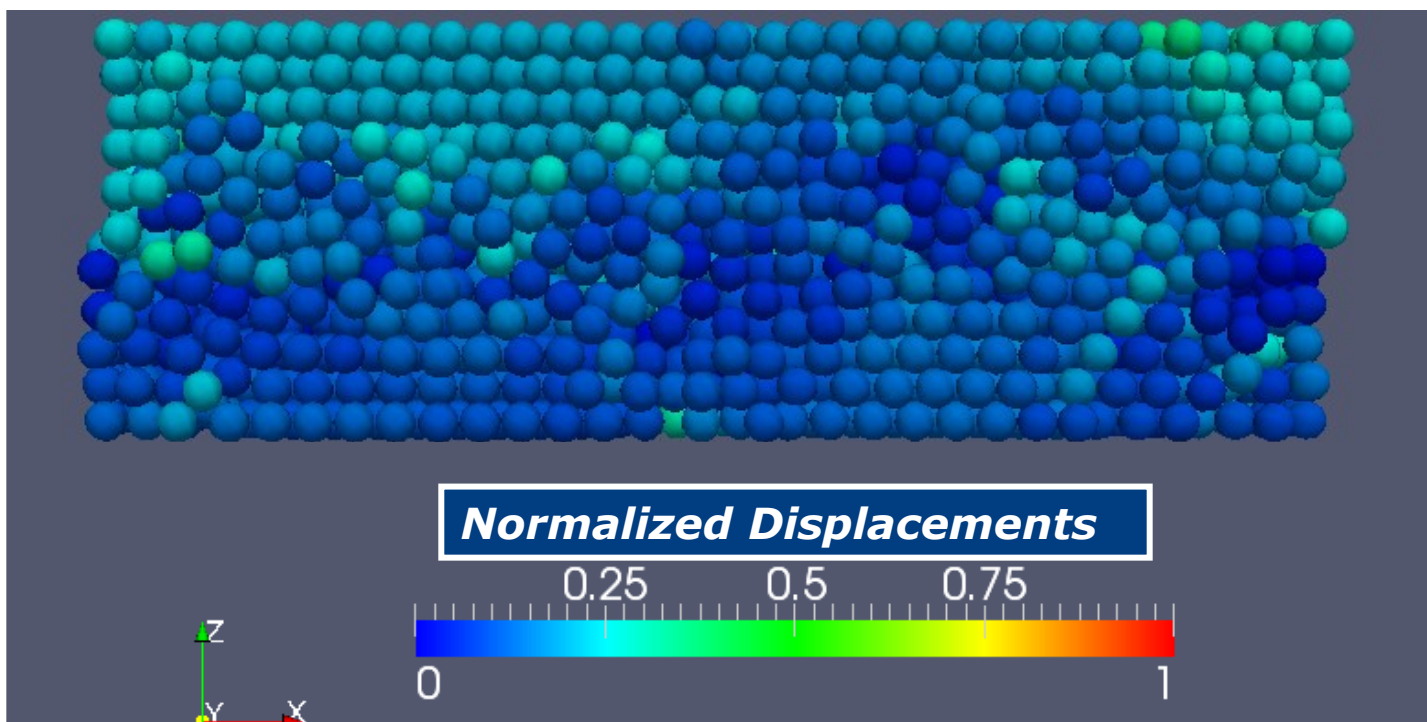
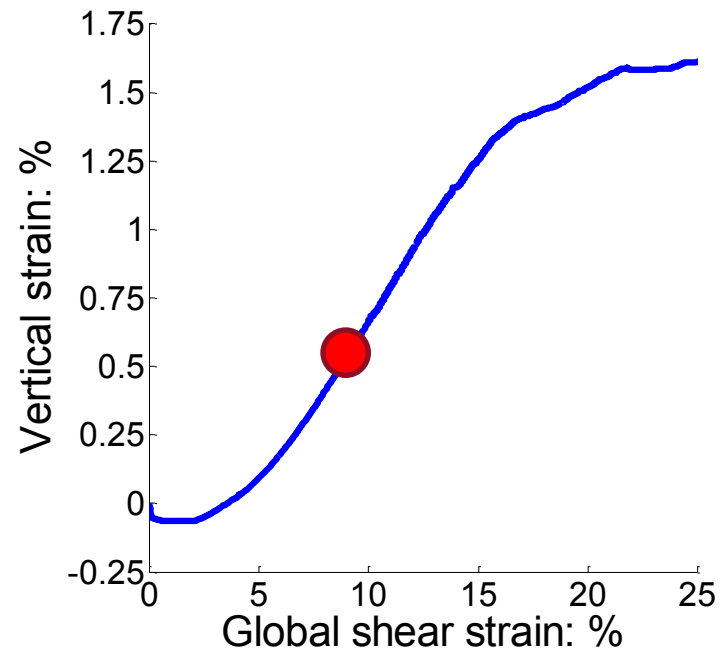
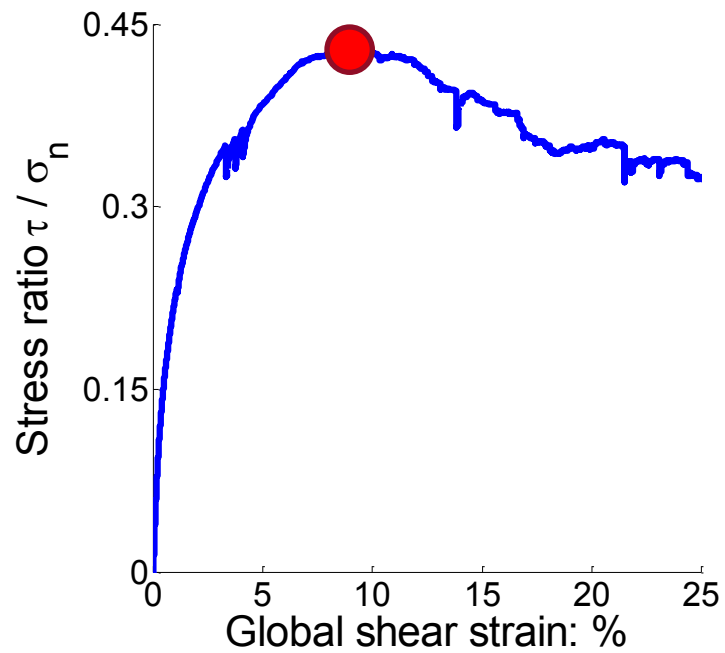


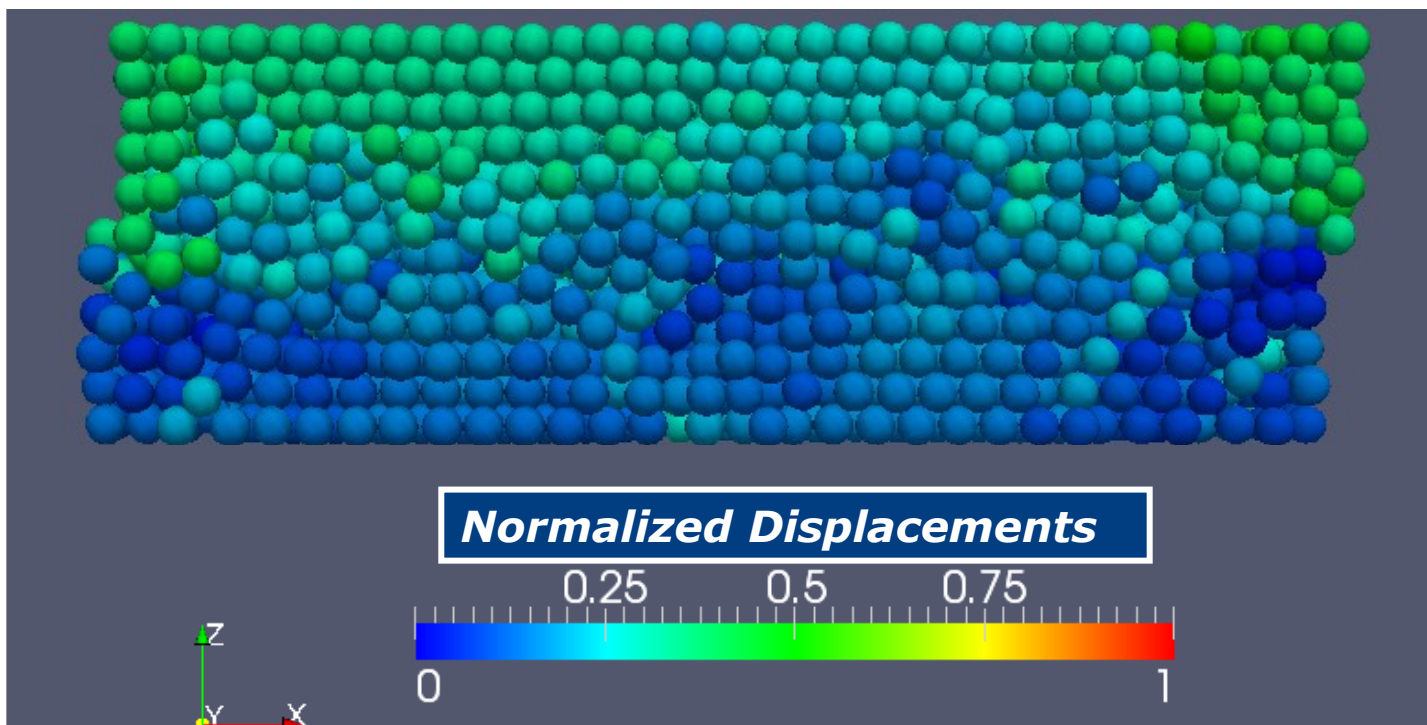
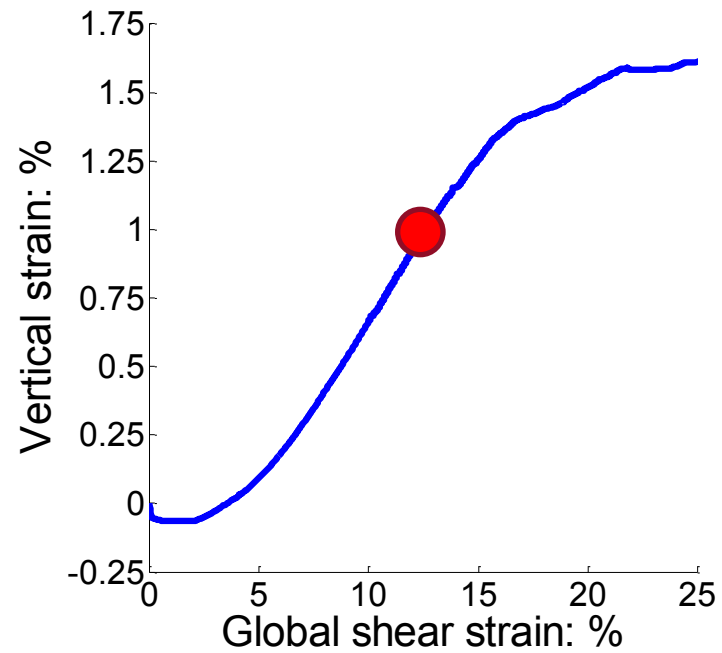
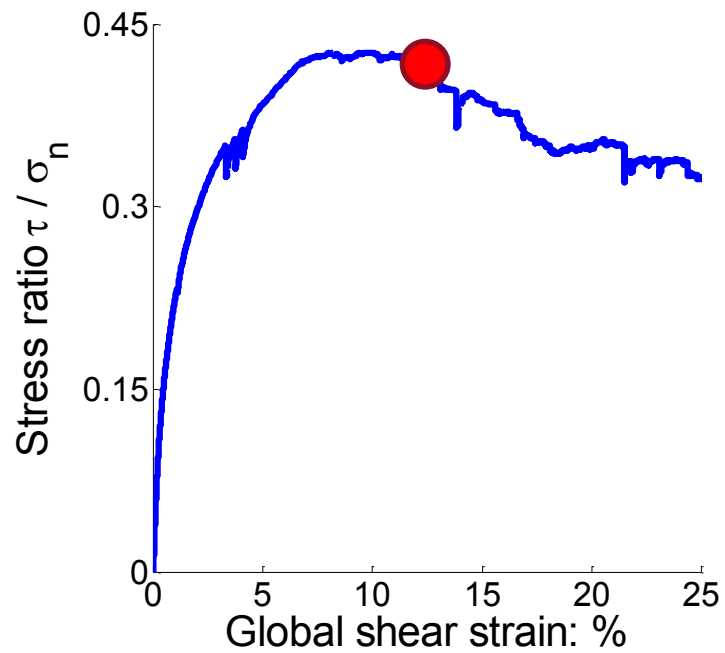


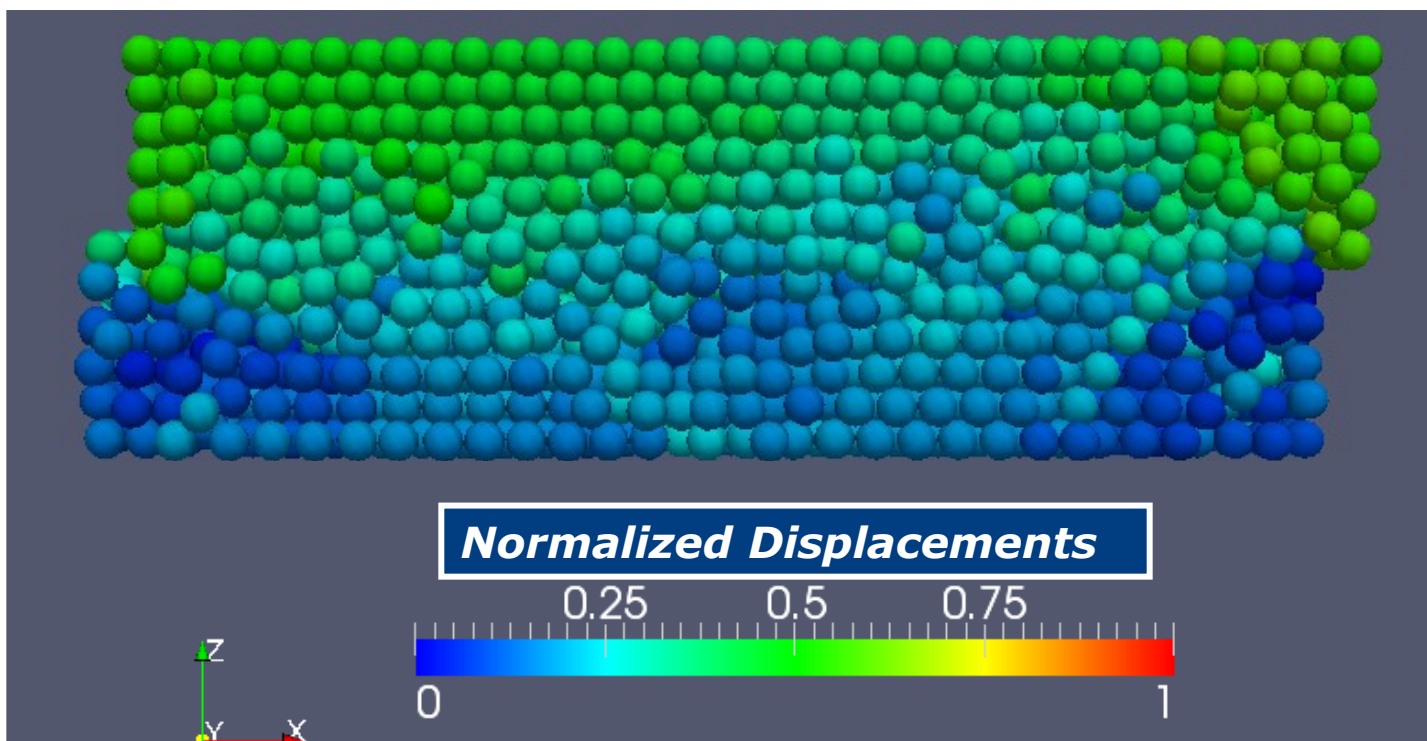
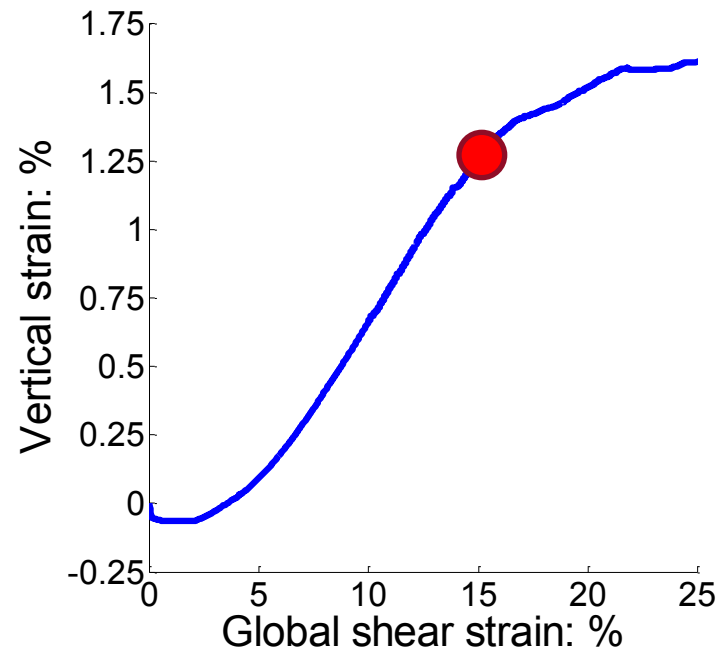
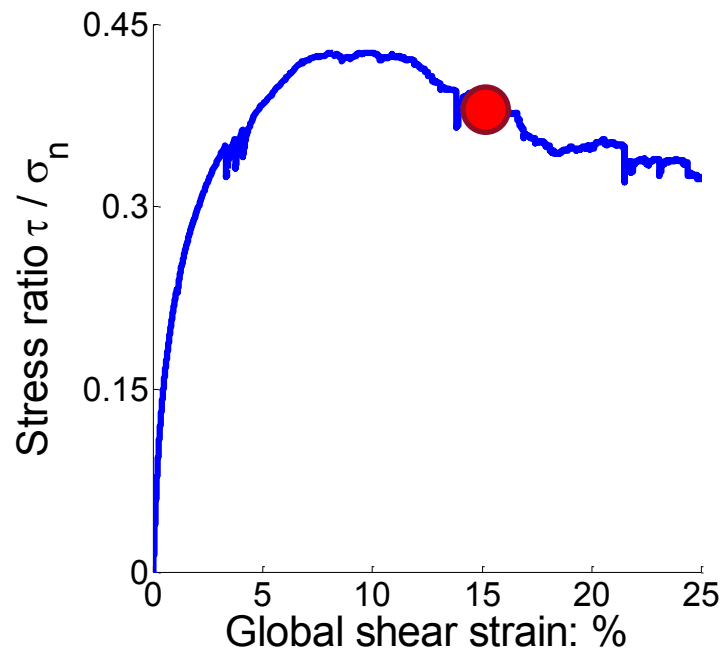


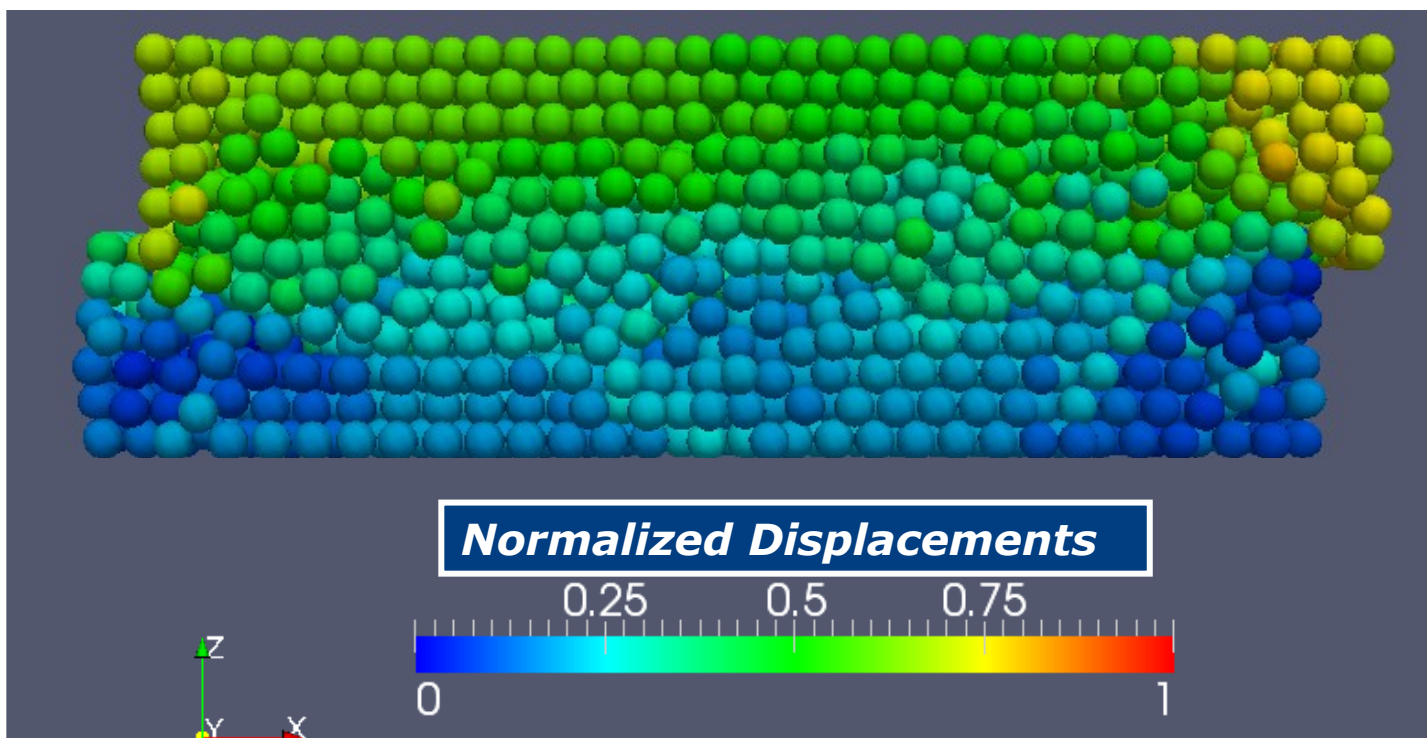
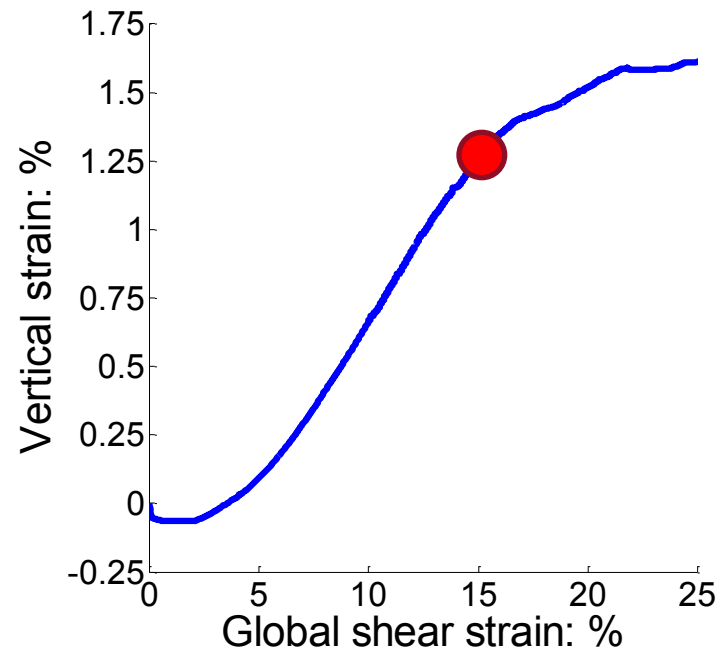
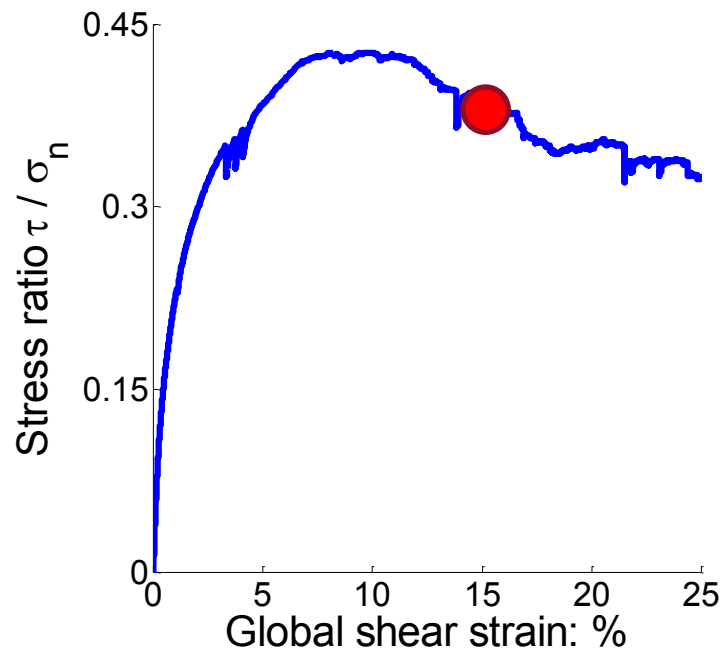


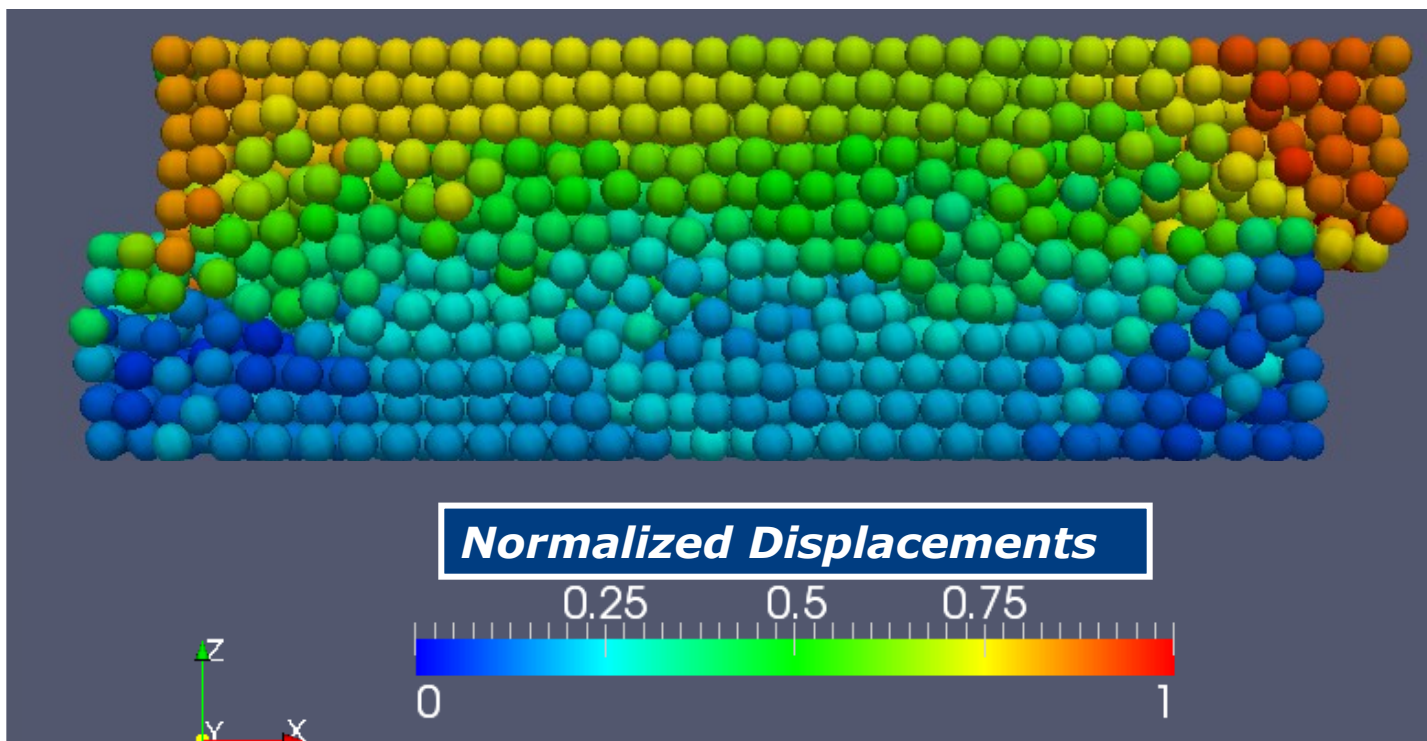
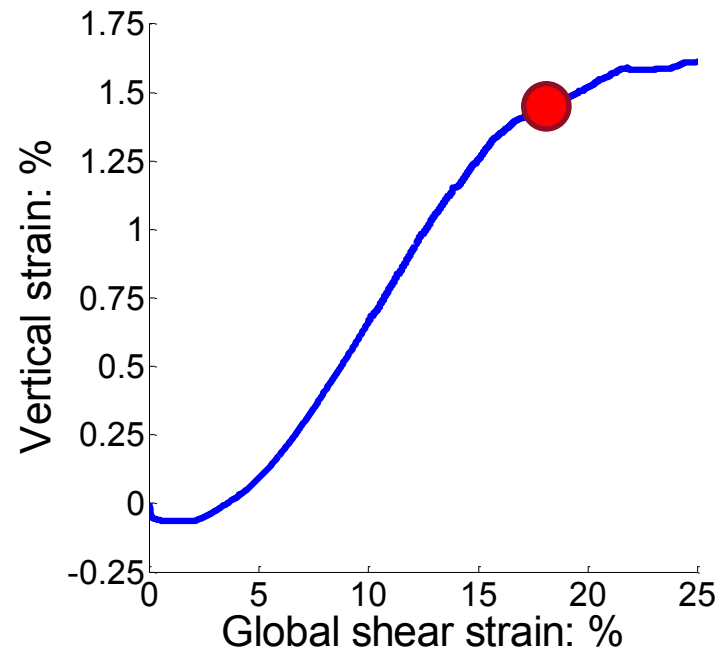
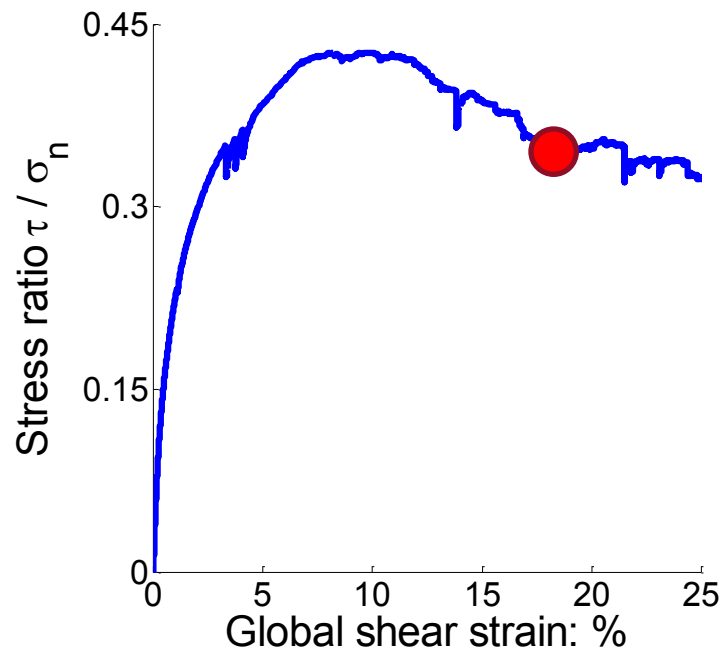


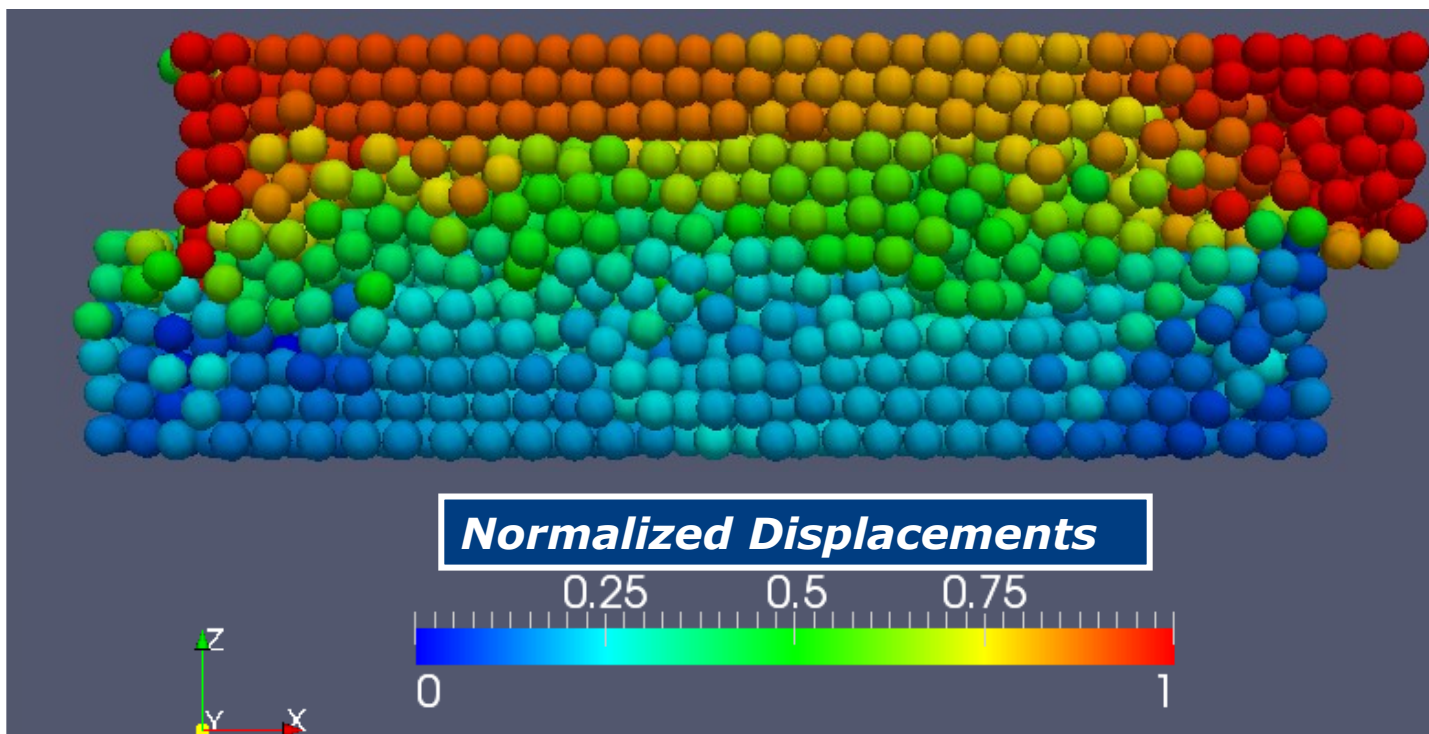
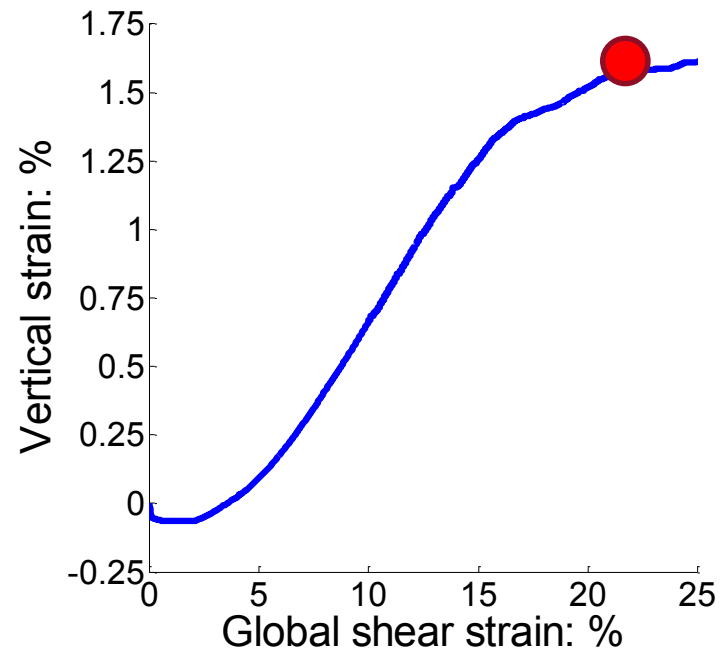
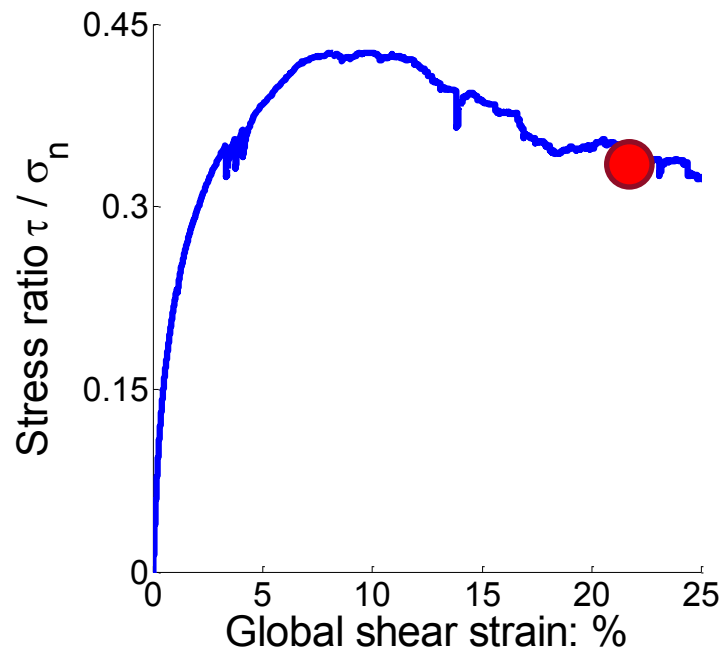


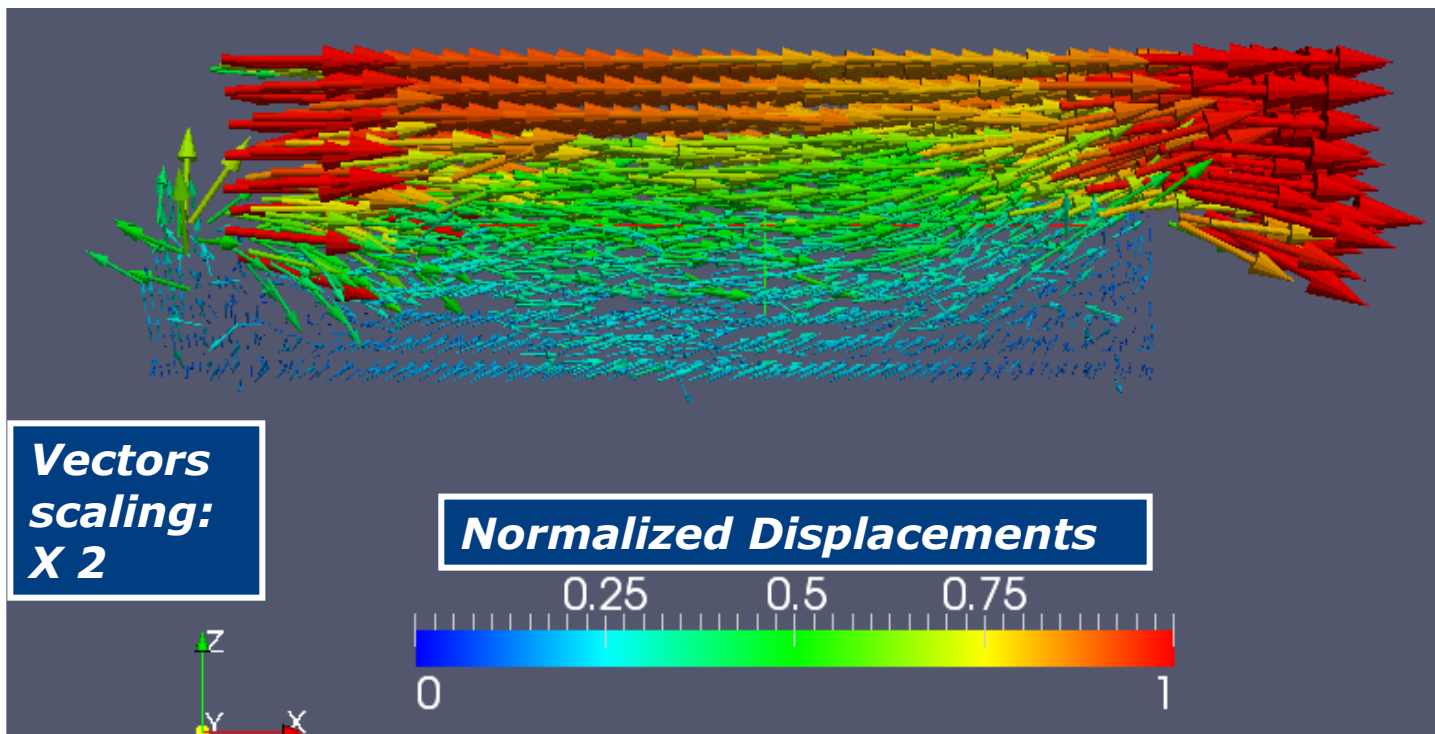
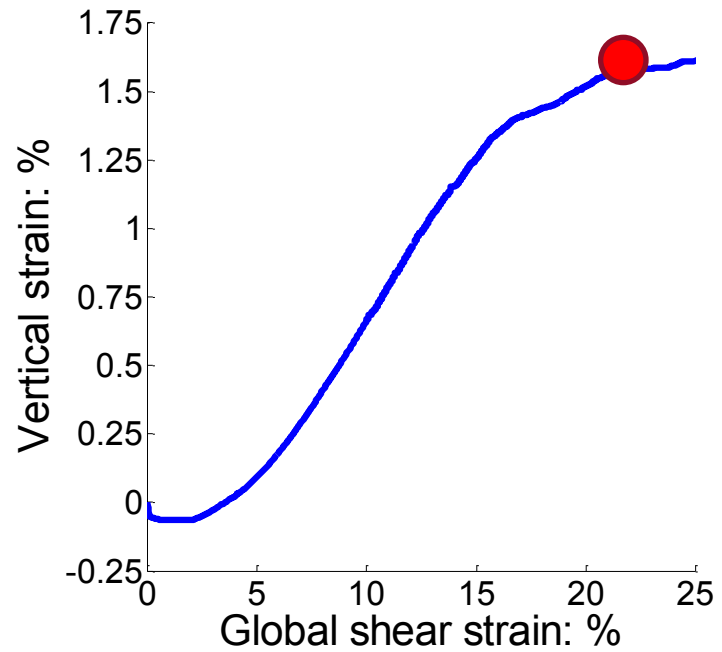
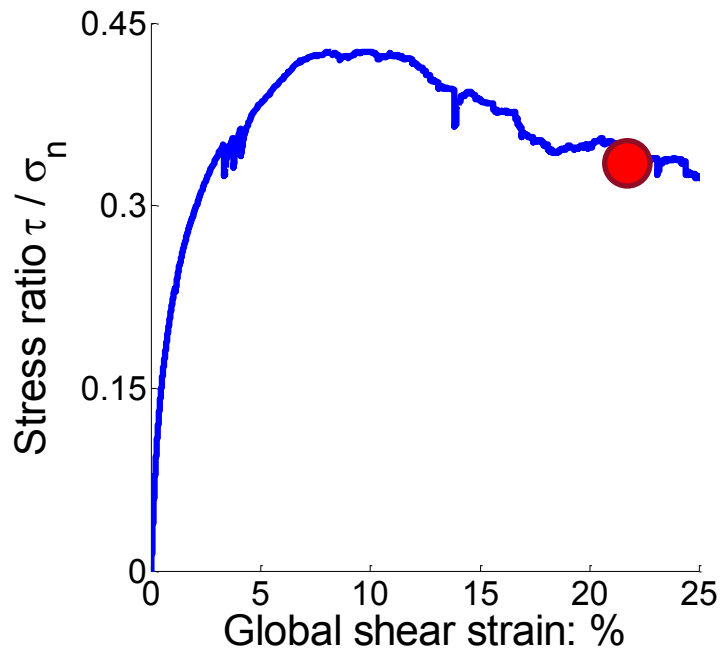


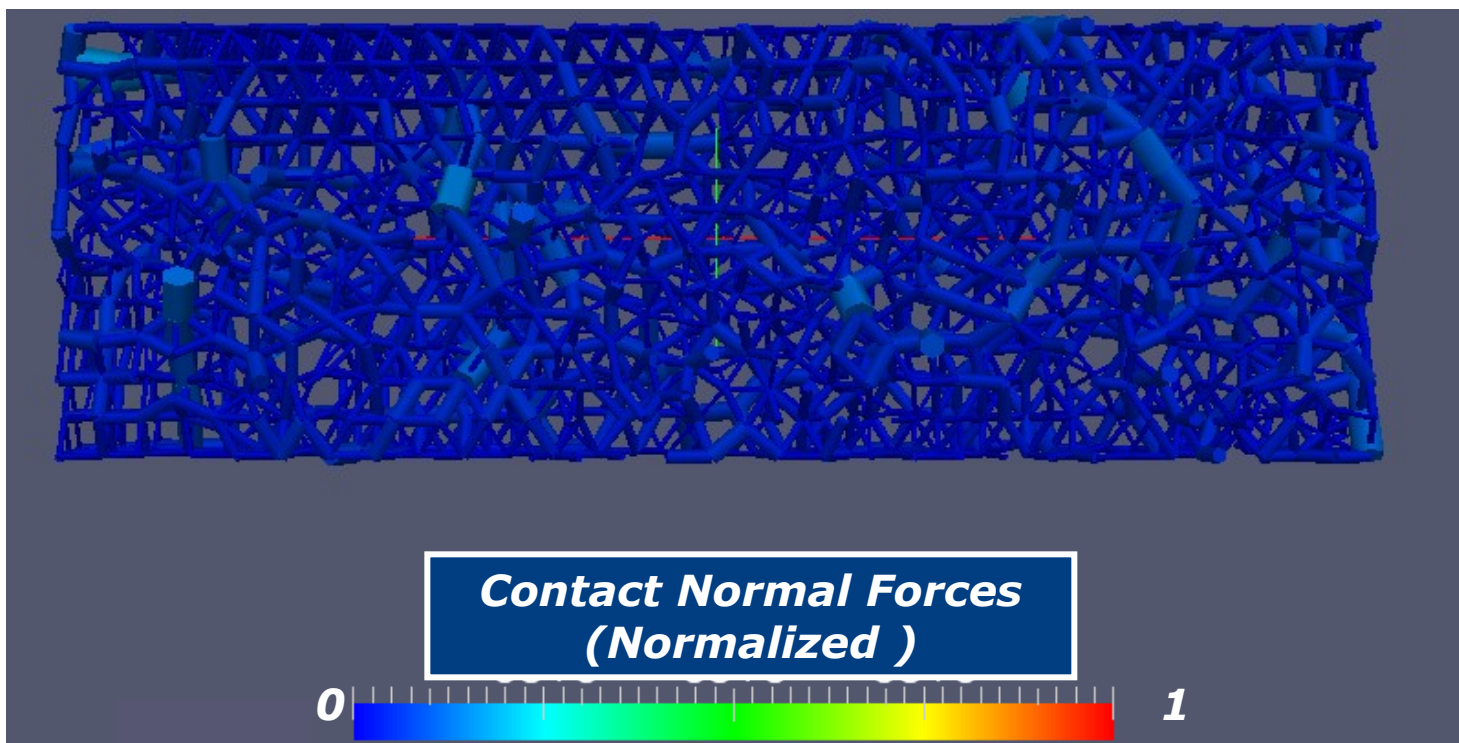
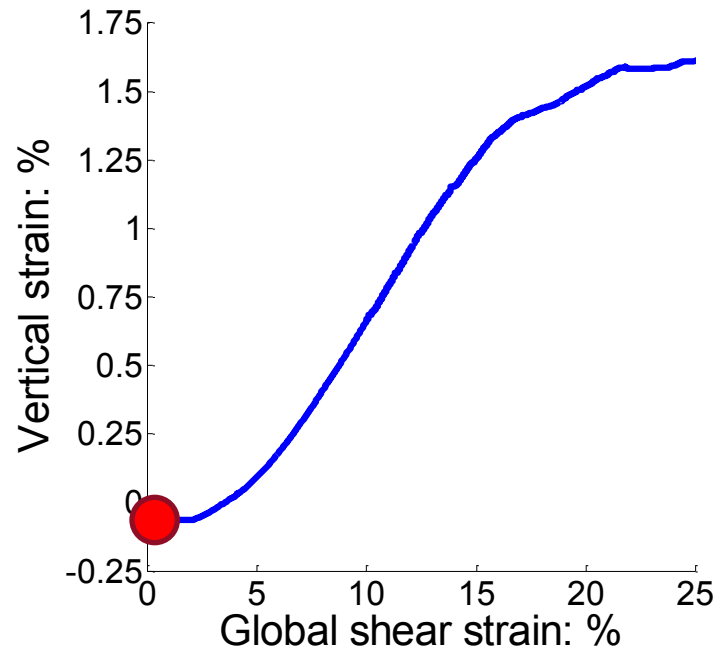
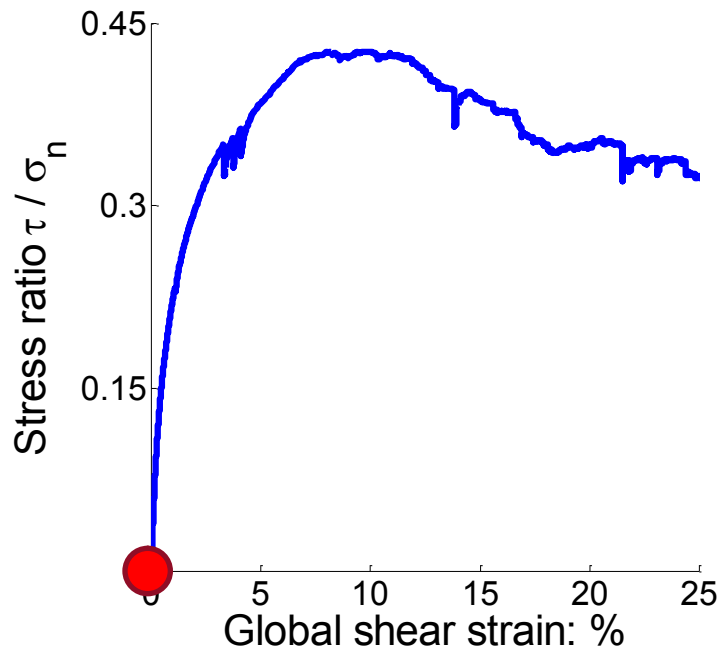


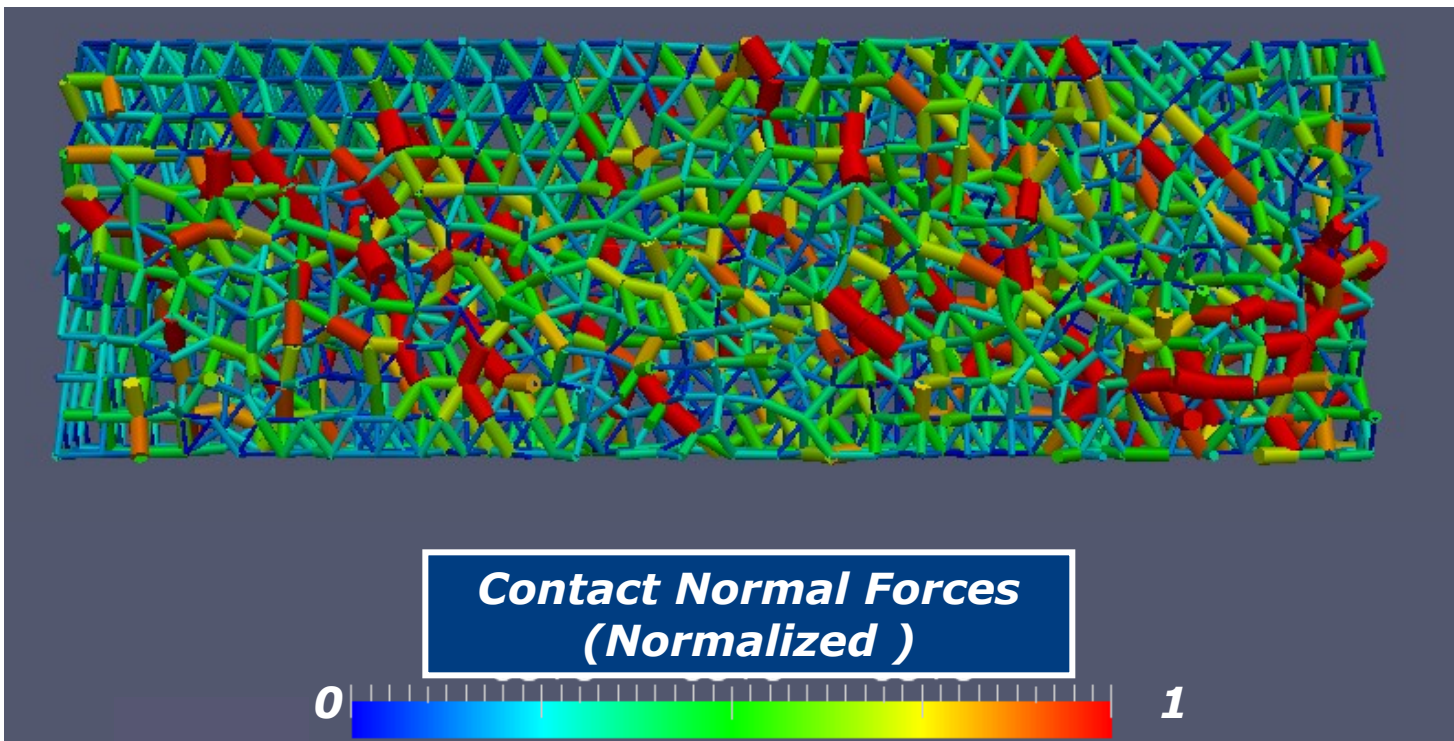
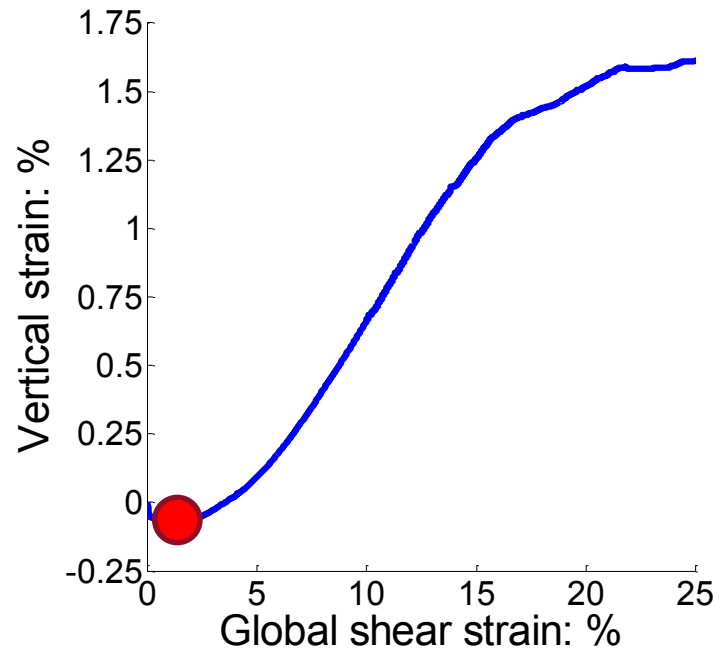
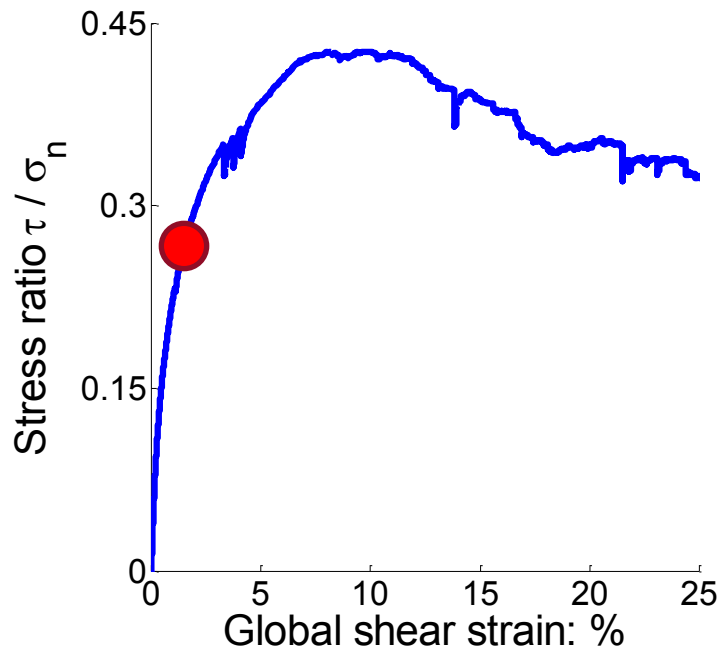


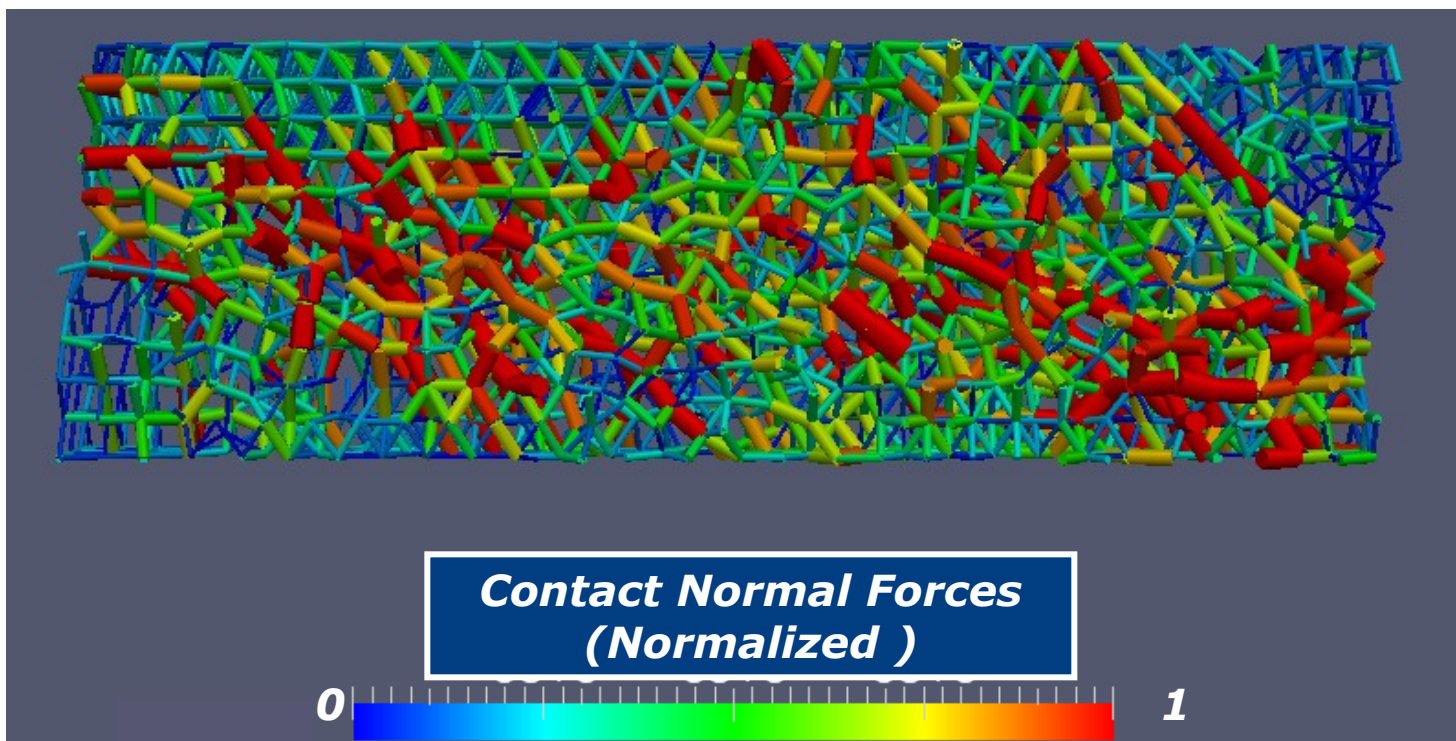
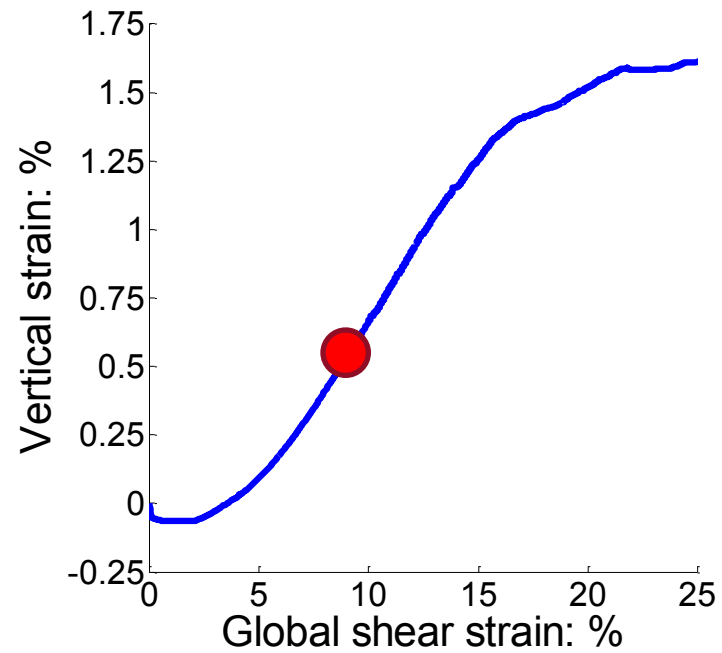
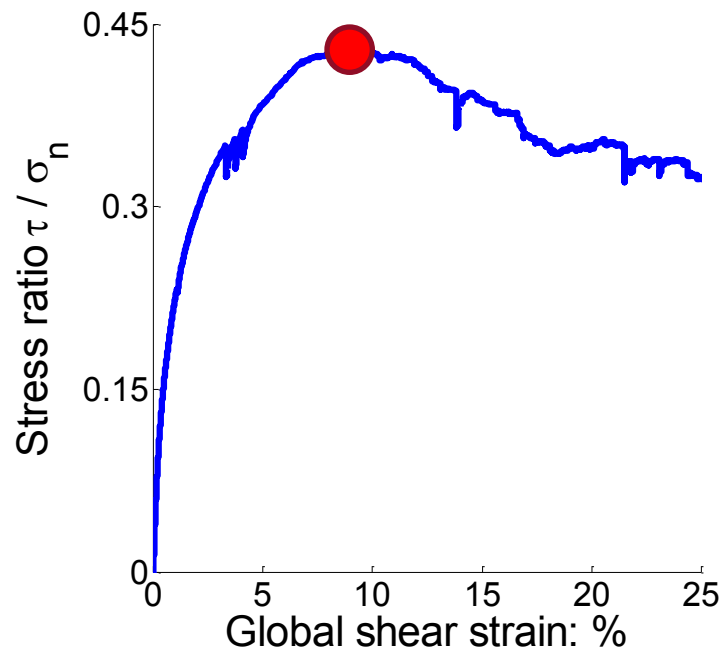


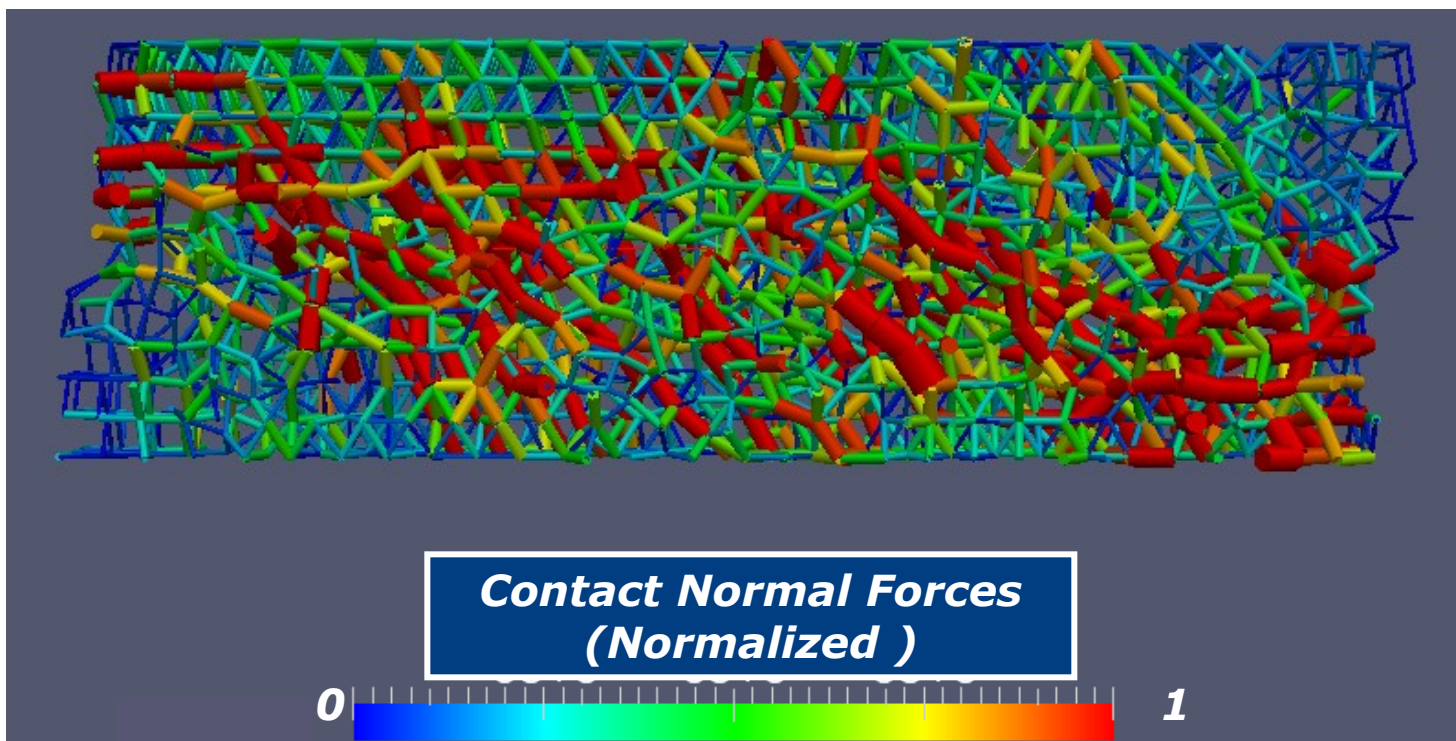
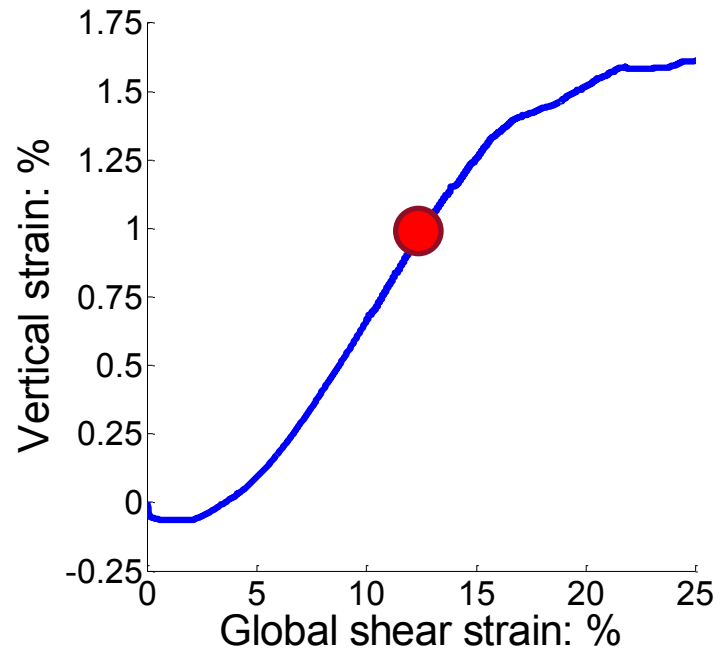
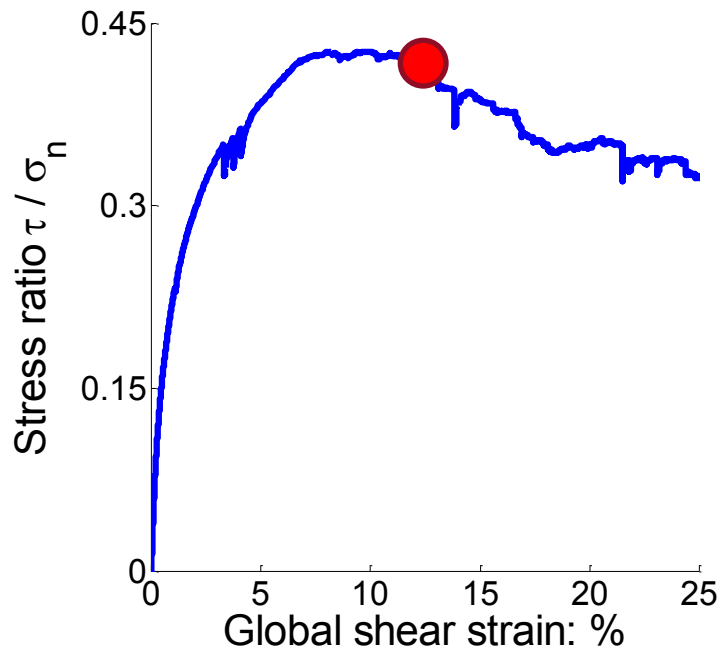


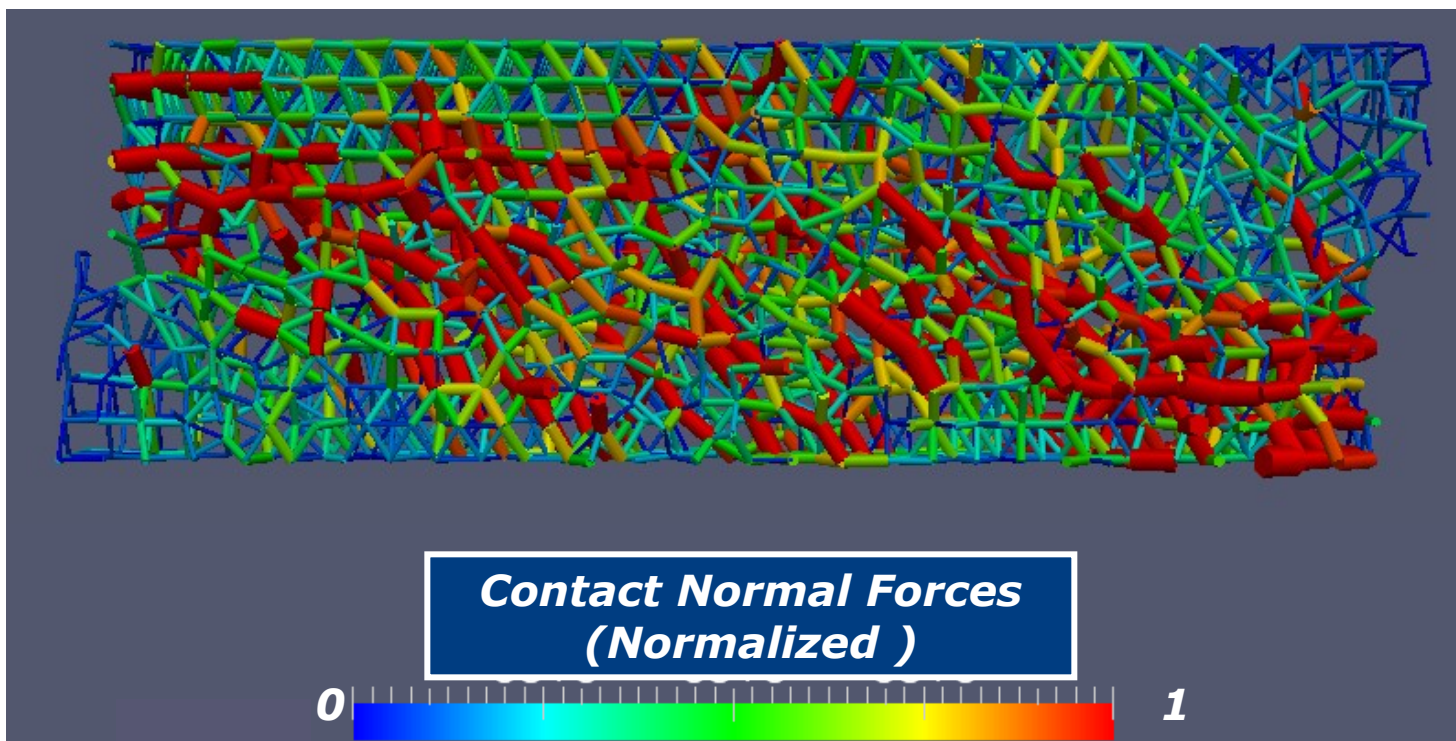
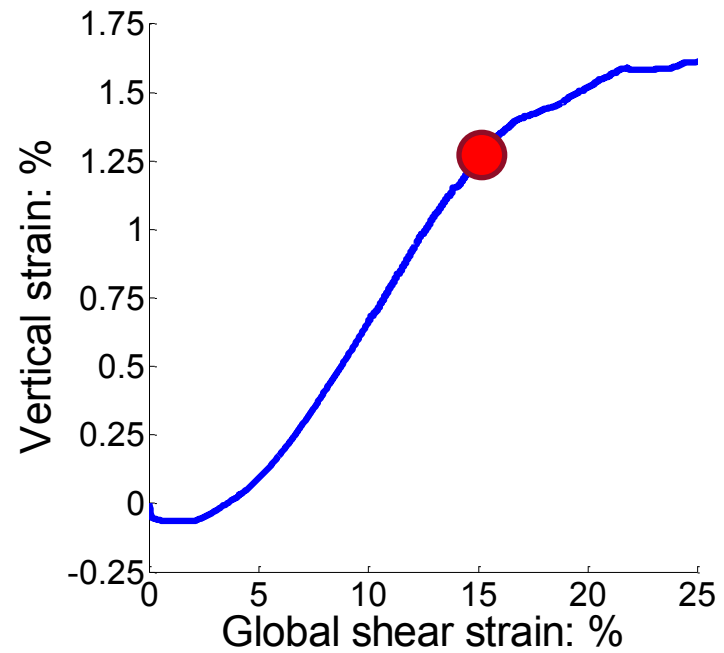
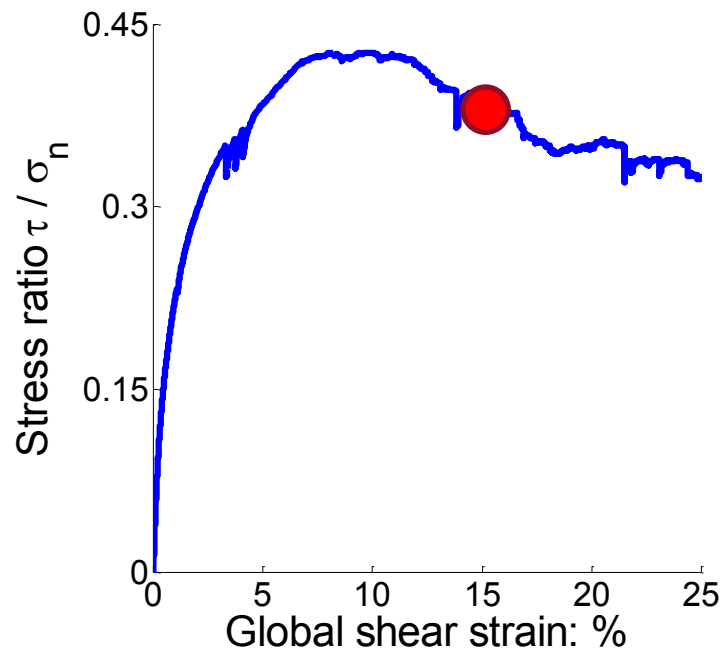


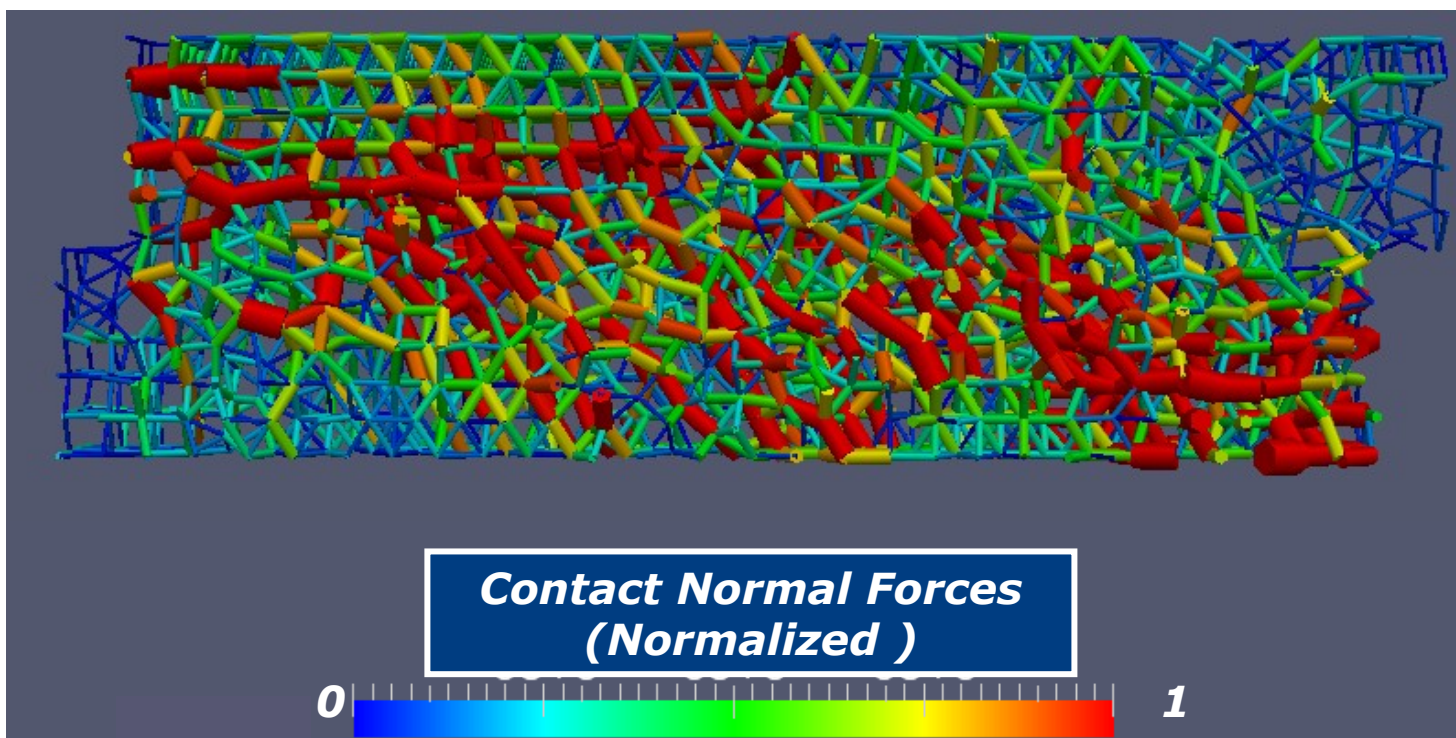
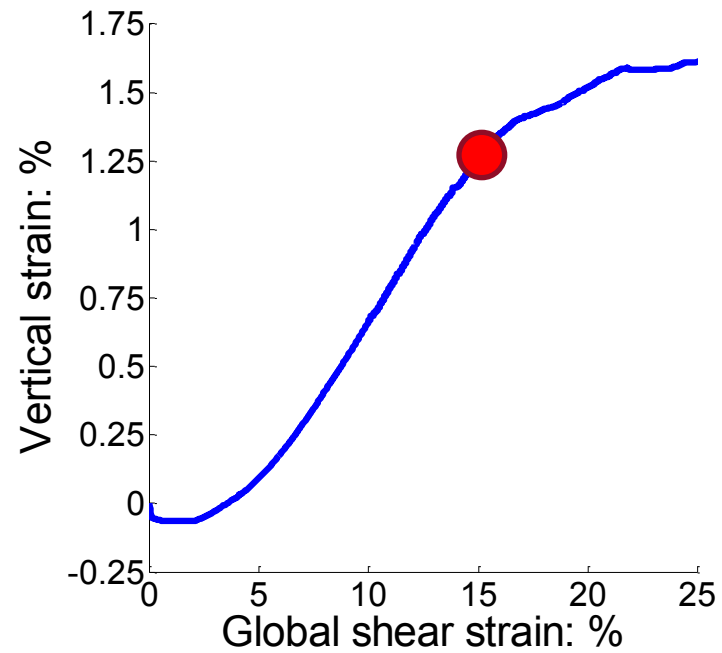
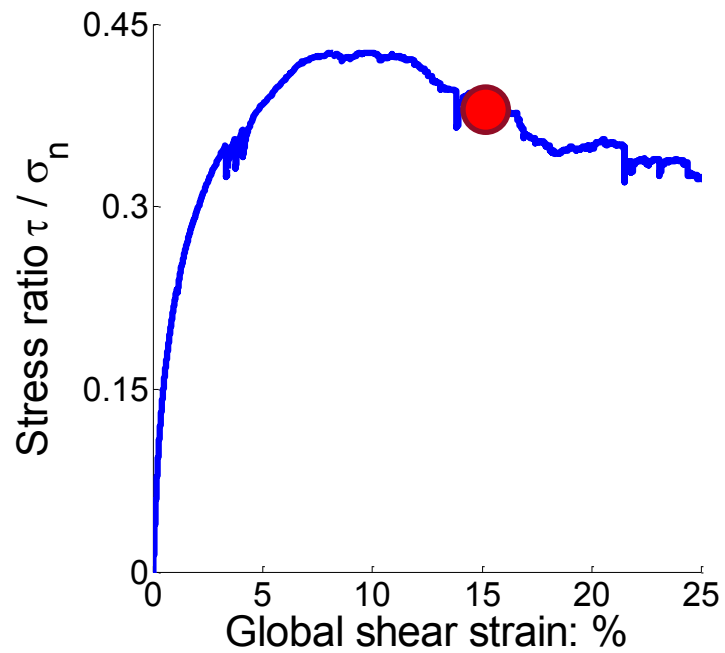


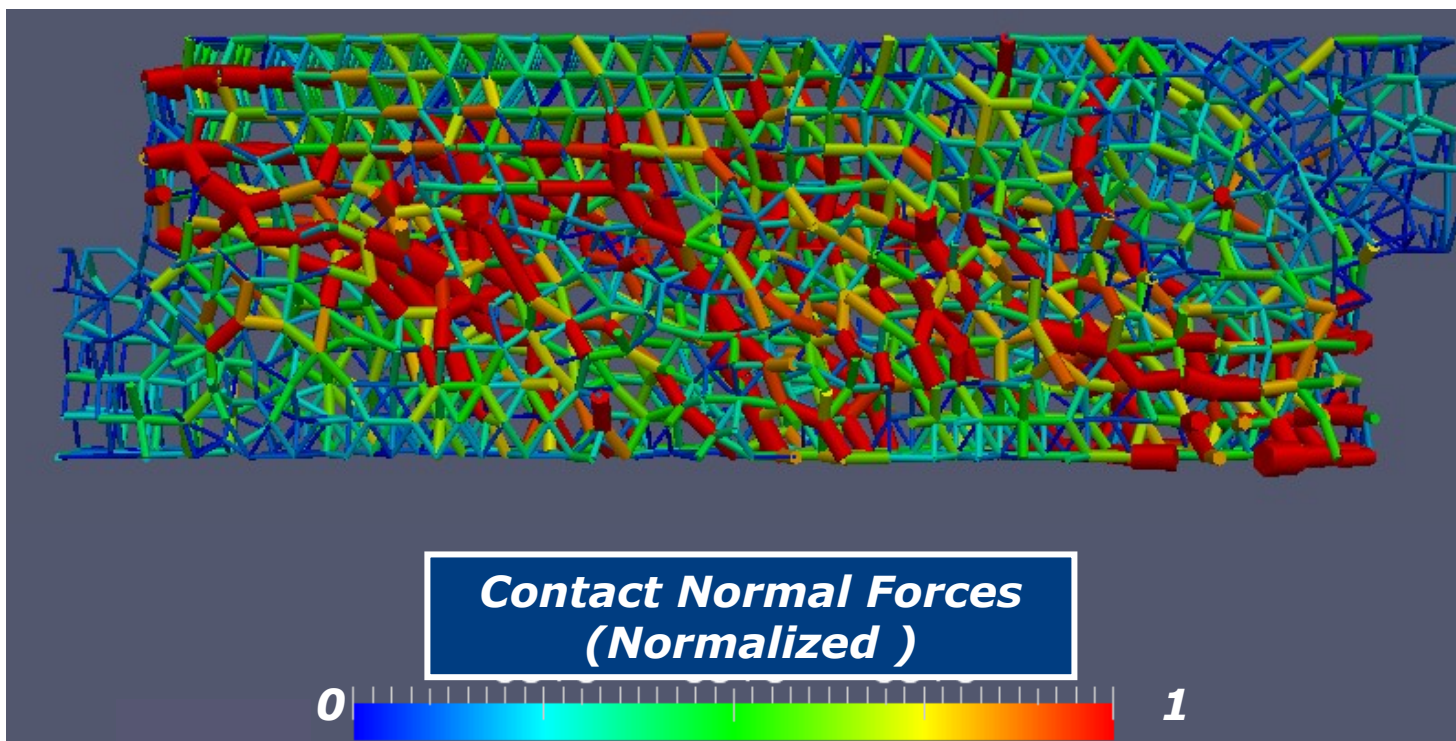
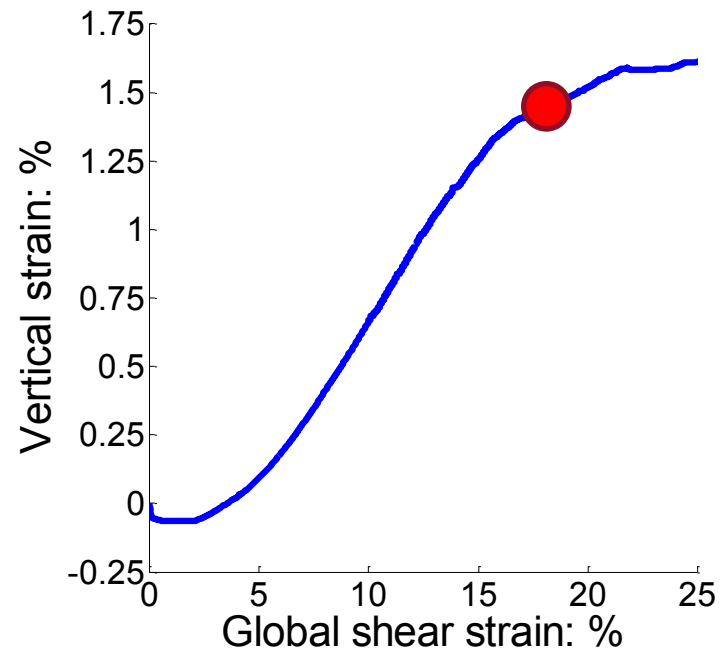
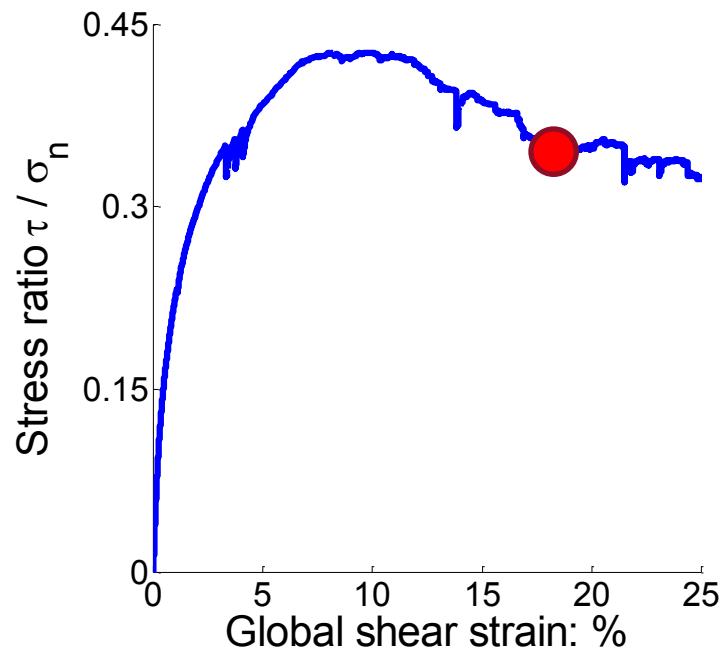


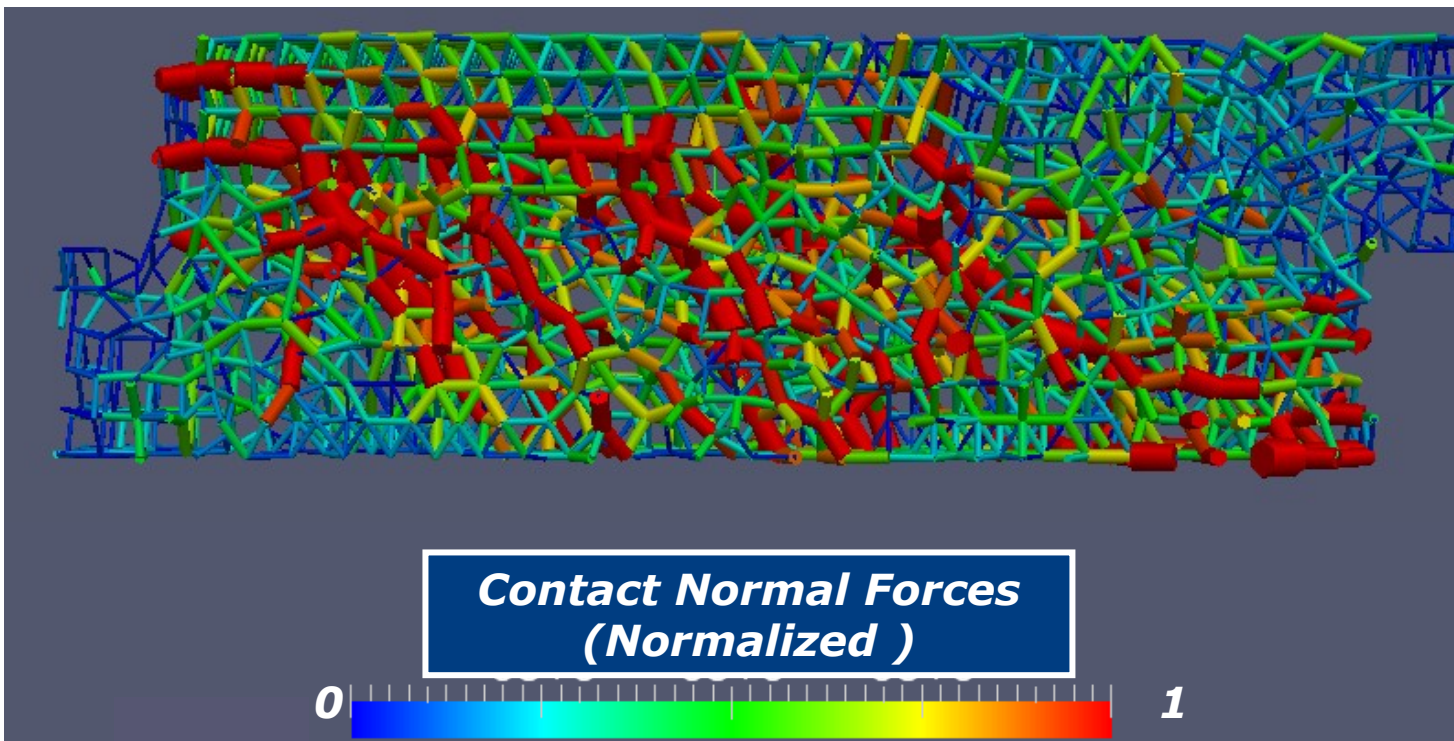
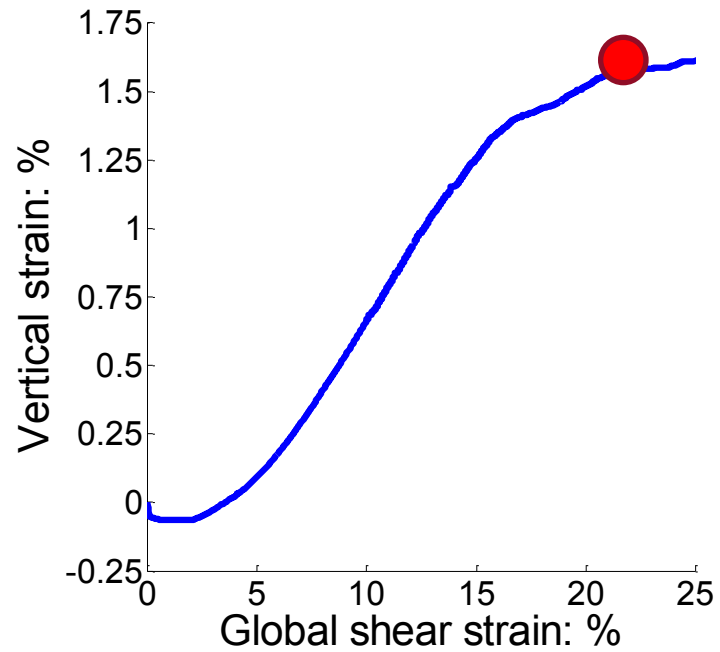
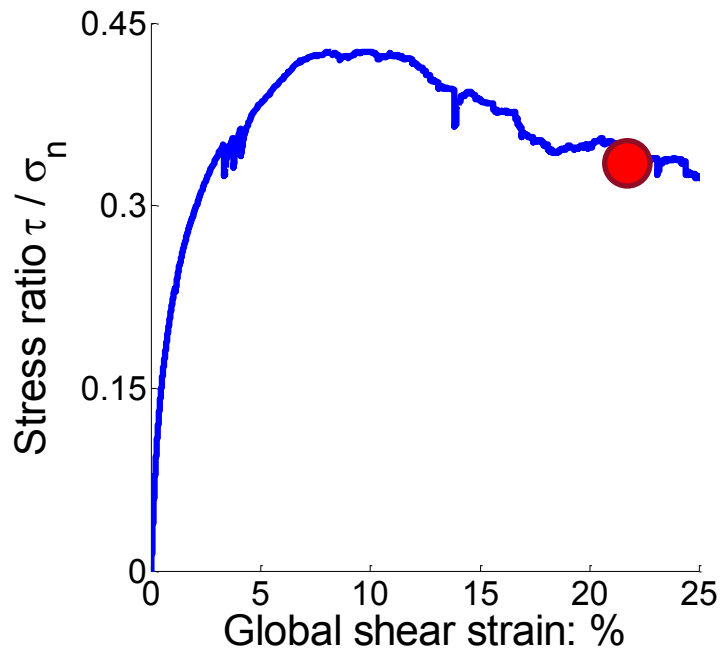


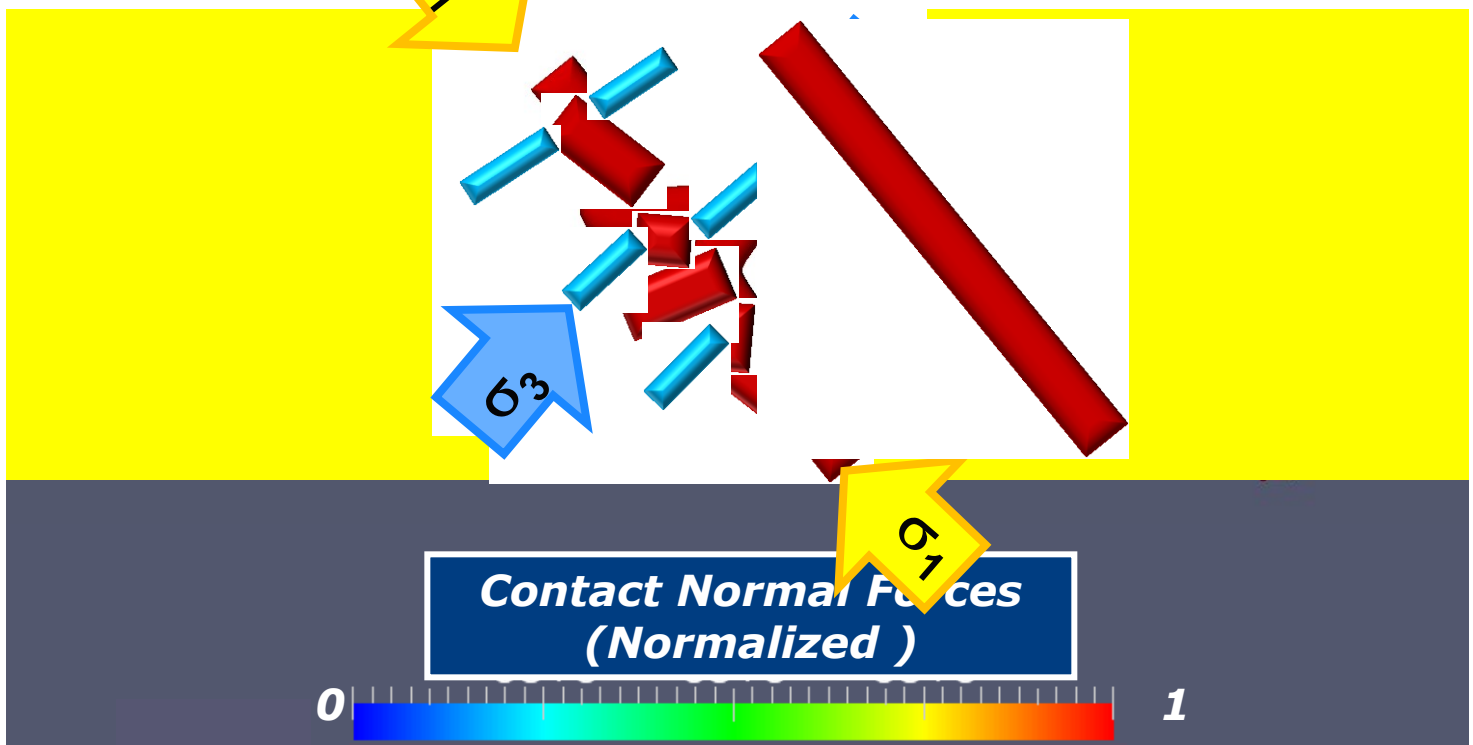
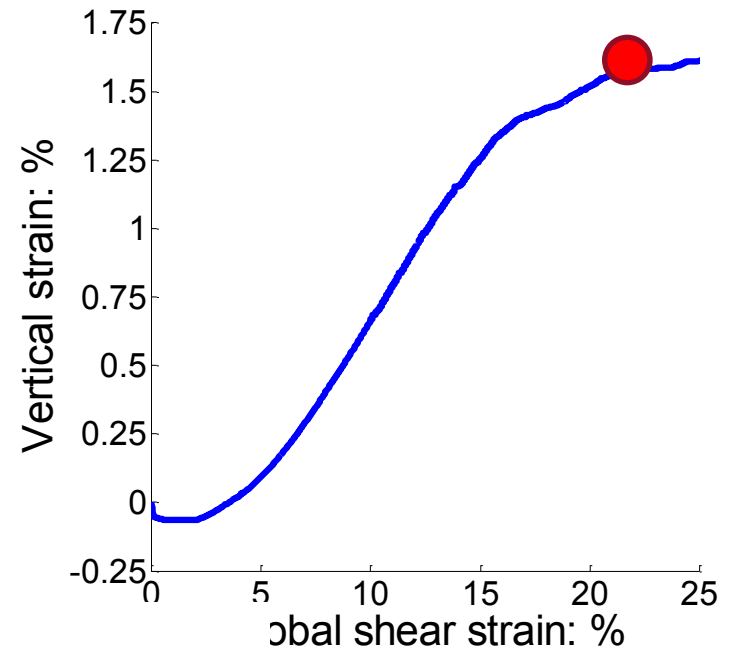
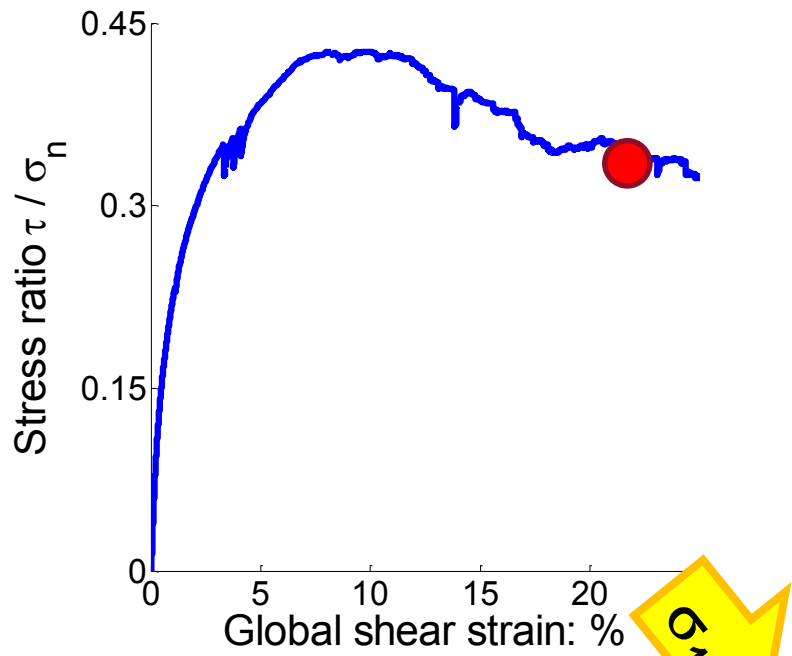












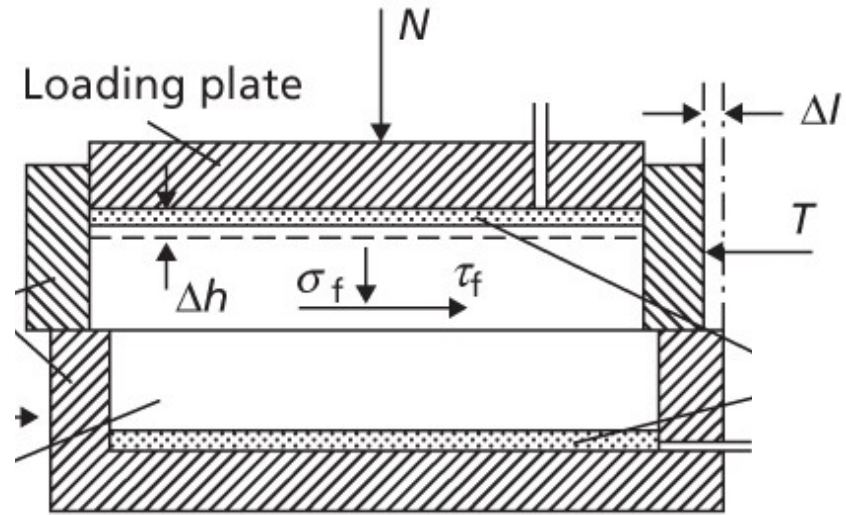
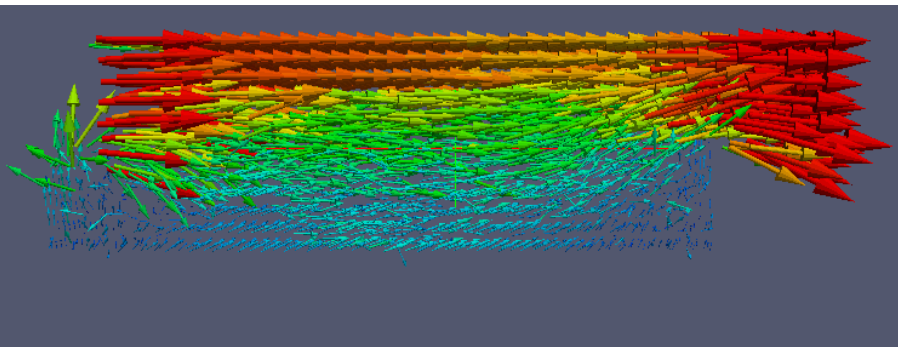
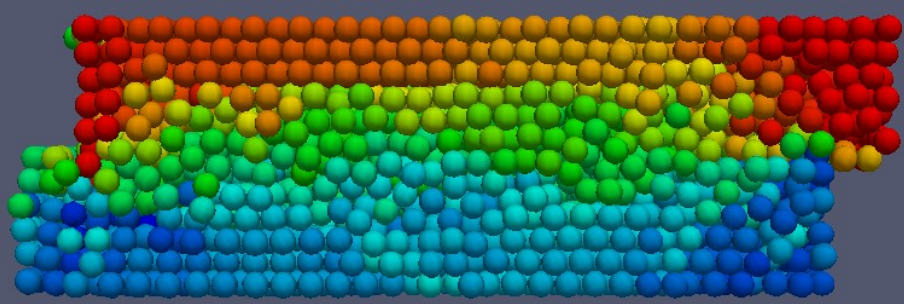
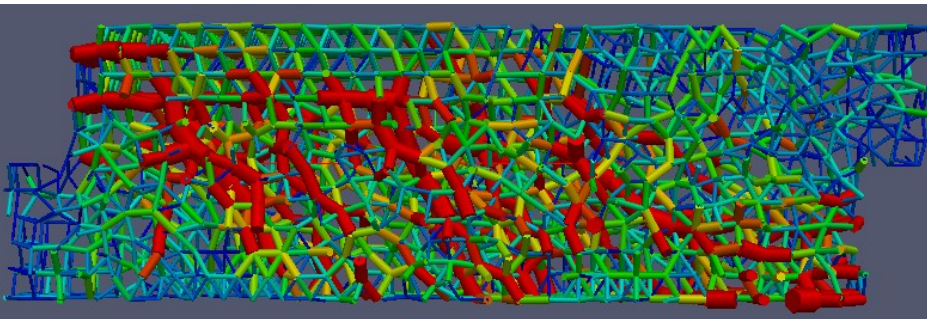
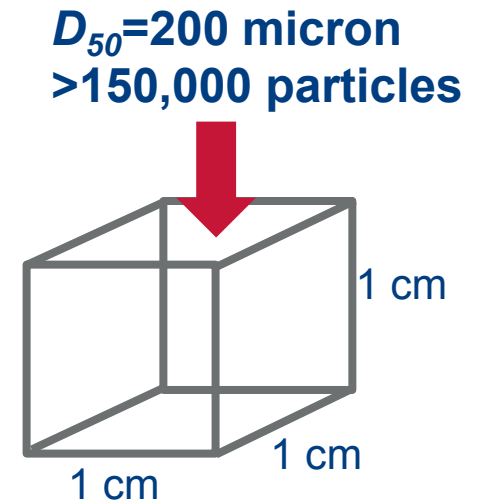
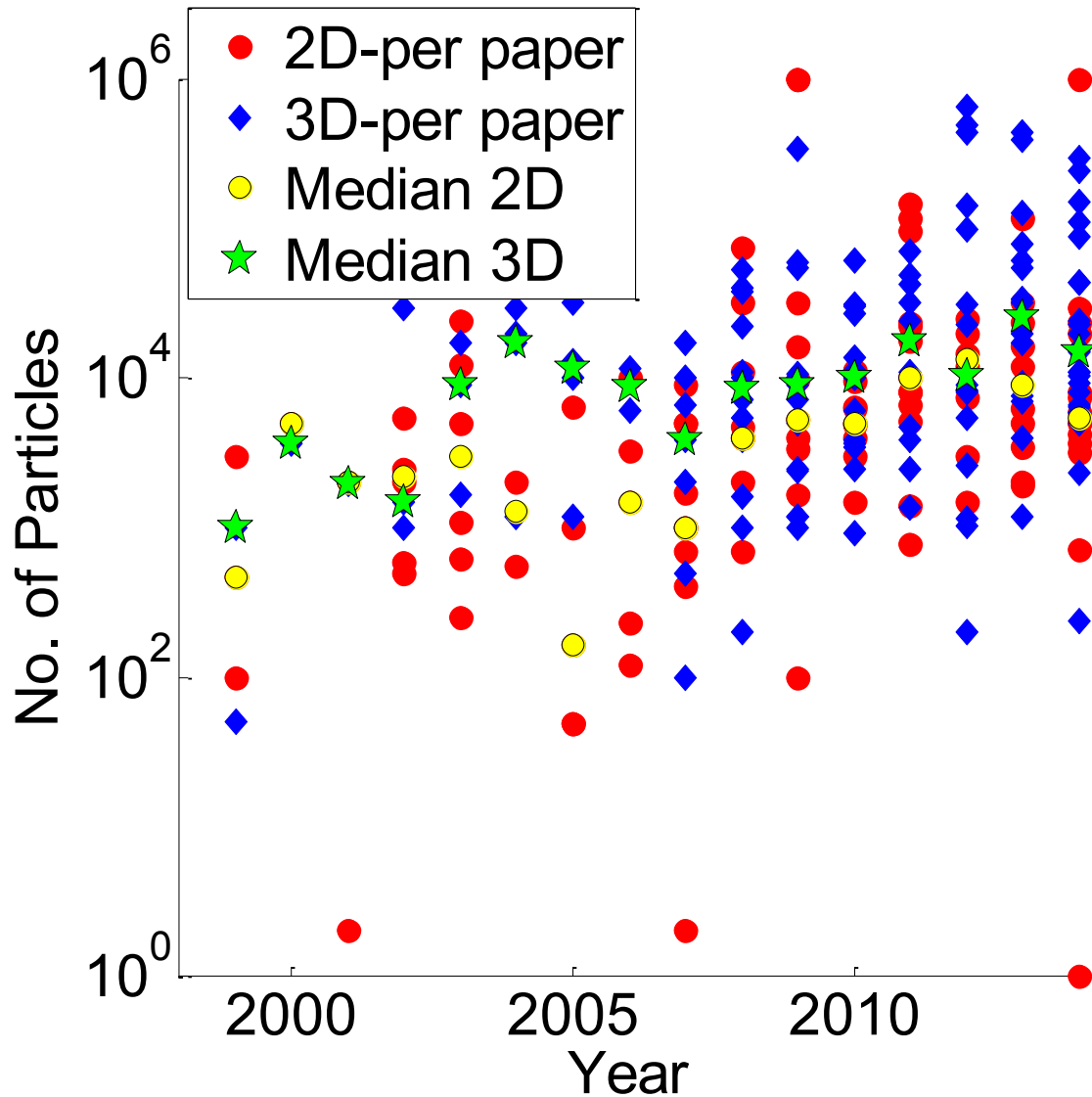


Diagram from
Craig's Soil Mechanics
Knappett and Craig(2012)

Direct Shear Test

- Reasonable correlation with physical test
- DEM gives insight into kinematics and failure mechanisms
- Limited number of particles (11,700)

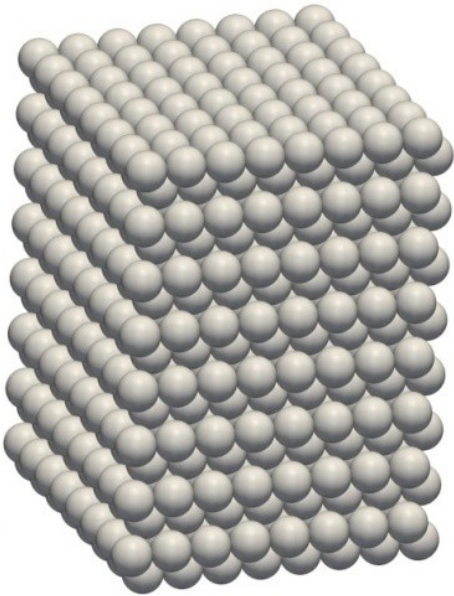
No. of Particles in Geomechanics DEM Simulations



Use of High Performance Computing to Increase Sample Size

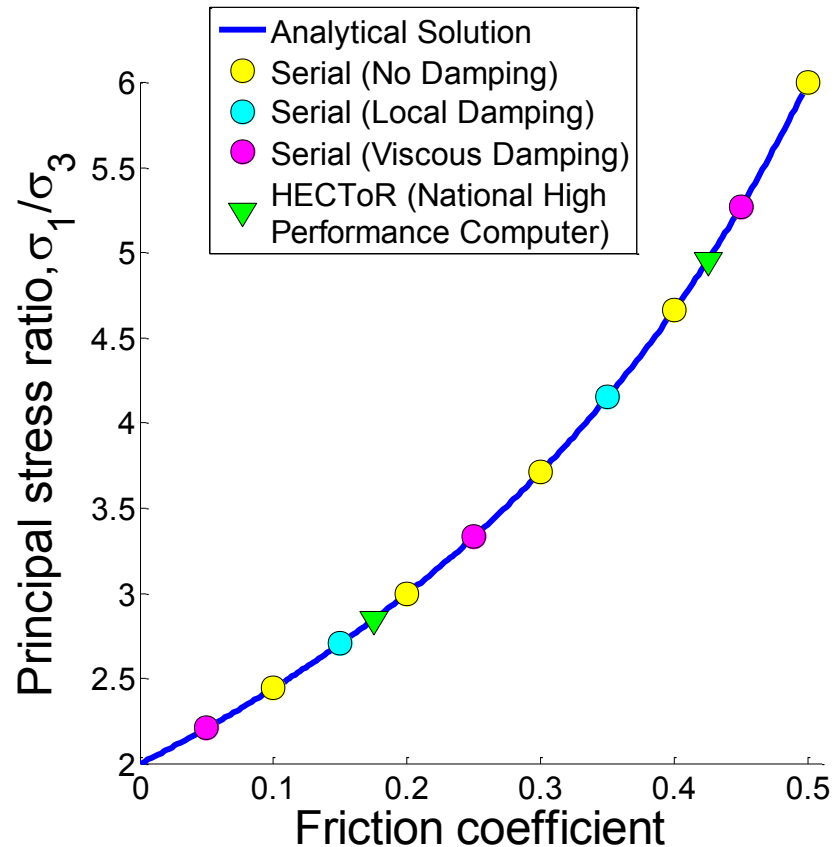
- LAMMPS (<http://lammps.sandia.gov/>)
- Classical molecular dynamics code that can be used for DEM
- Uses spatial decomposition and MPI (message passage interface)
- Can run large high performance computers (hpc) with distributed memory
- Researchers at IC (Marketos, Hanley, Shire, Huang, Otsubo) have been working to modify the granular LAMMPS package

Validation of Granular LAMMPS



Compression of face-centred-cubic array of spheres.

Analytical solution documented in *Thornton (1979) Géotechnique*

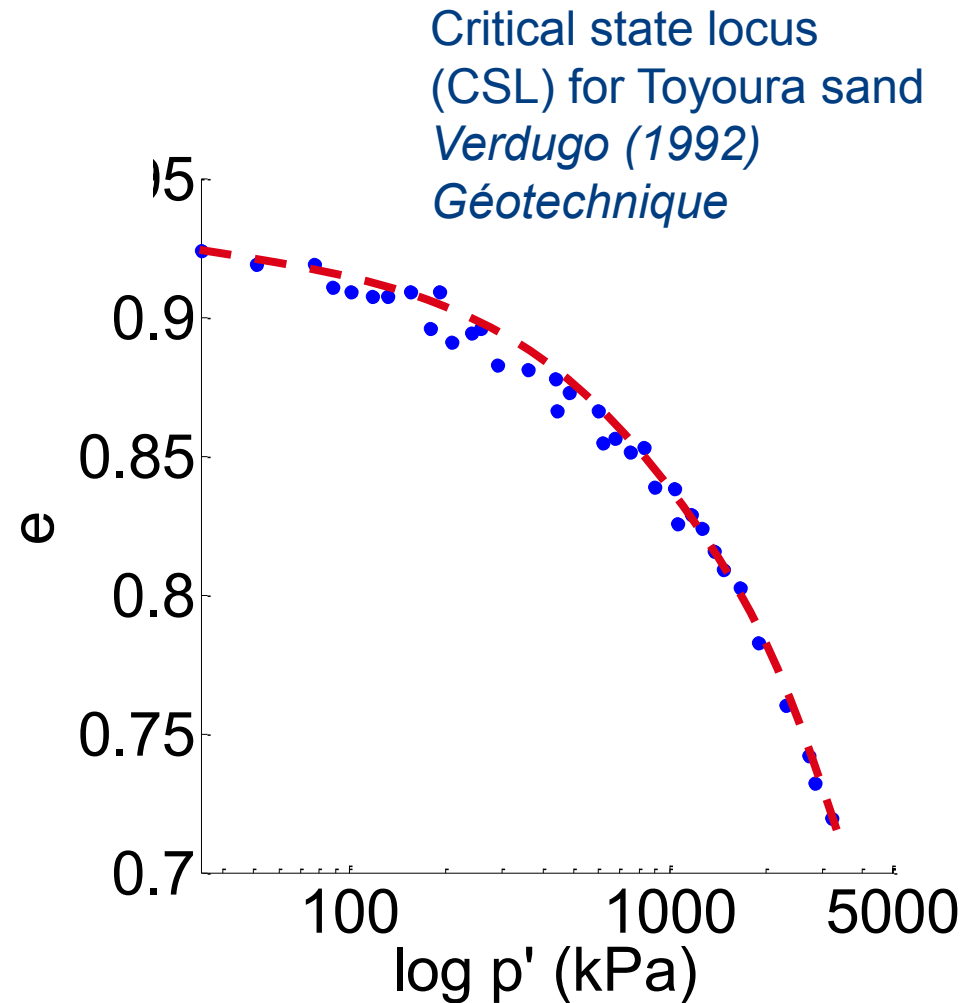


Validations by K. Hanley and X. Huang

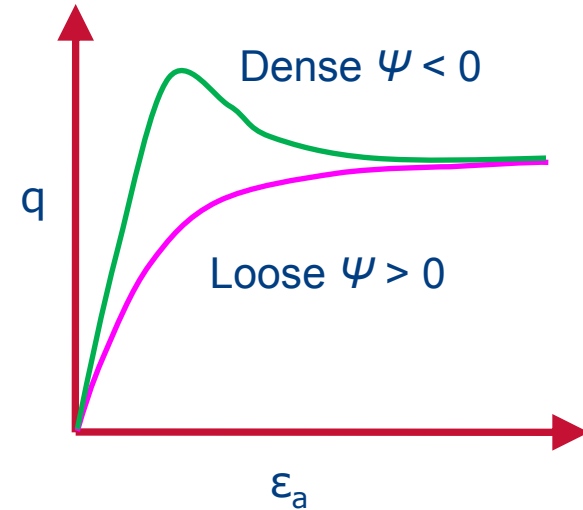
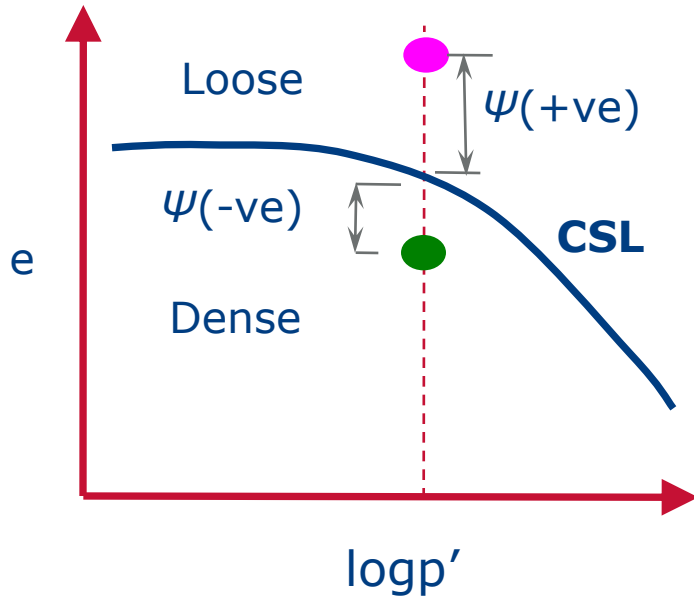
A State Parameter for Sands: ψ

Critical state is a state where the soil deforms at constant stress and constant void ratio

Been and Jefferies (1985)
Géotechnique



A State Parameter for Sands: ψ



e void ratio

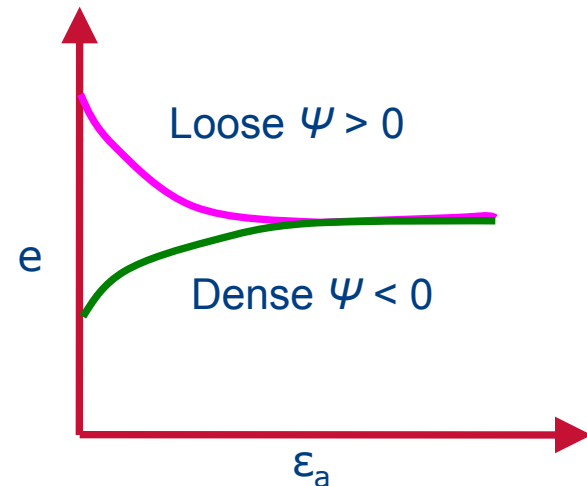
q deviatoric stress

p' mean effective stress

$$q = \sigma'_1 - \sigma'_3$$

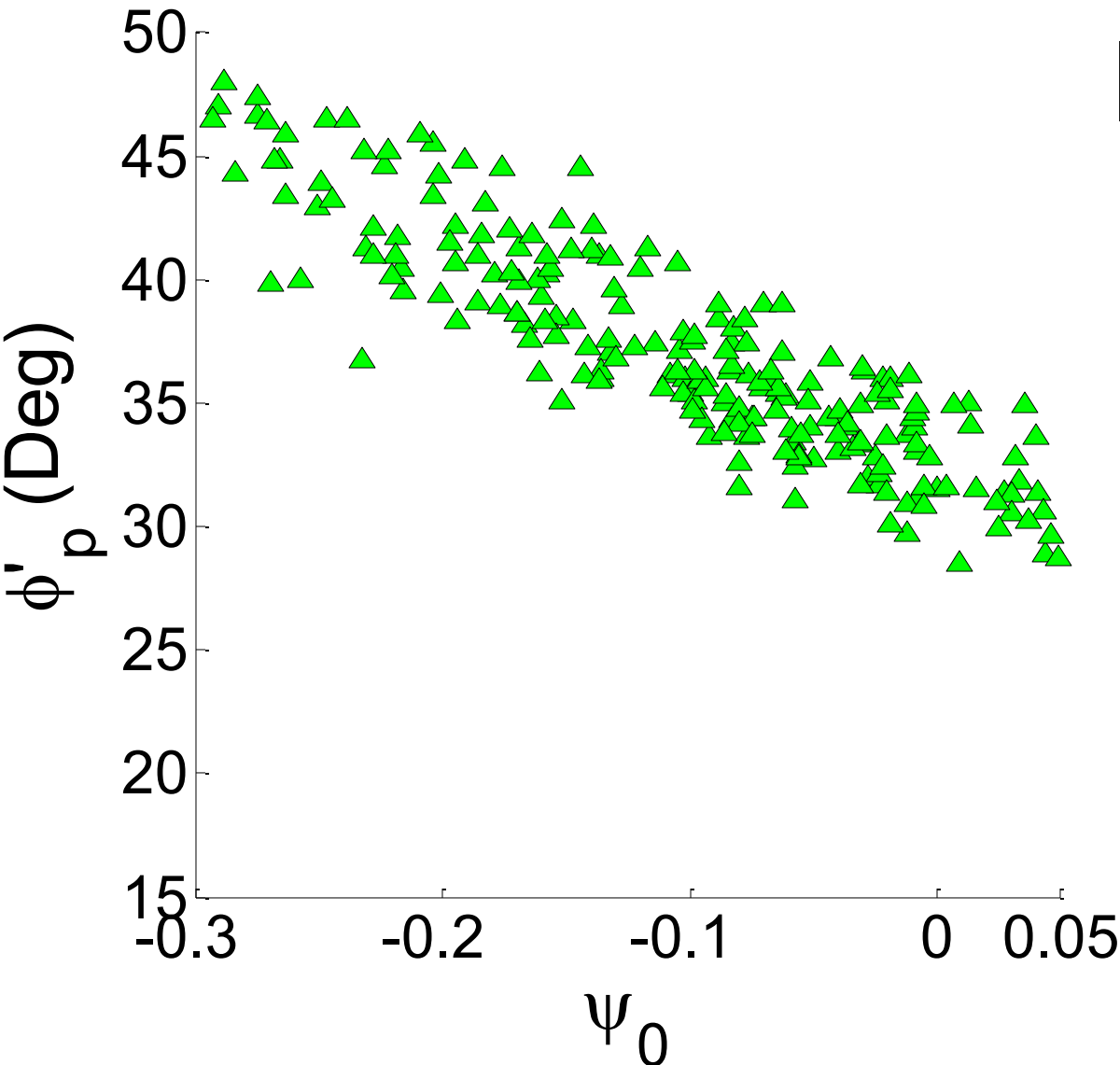
$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$$

(triaxial case)



Been and Jefferies (1985)
Géotechnique

Variation in ϕ'_p with ψ_0



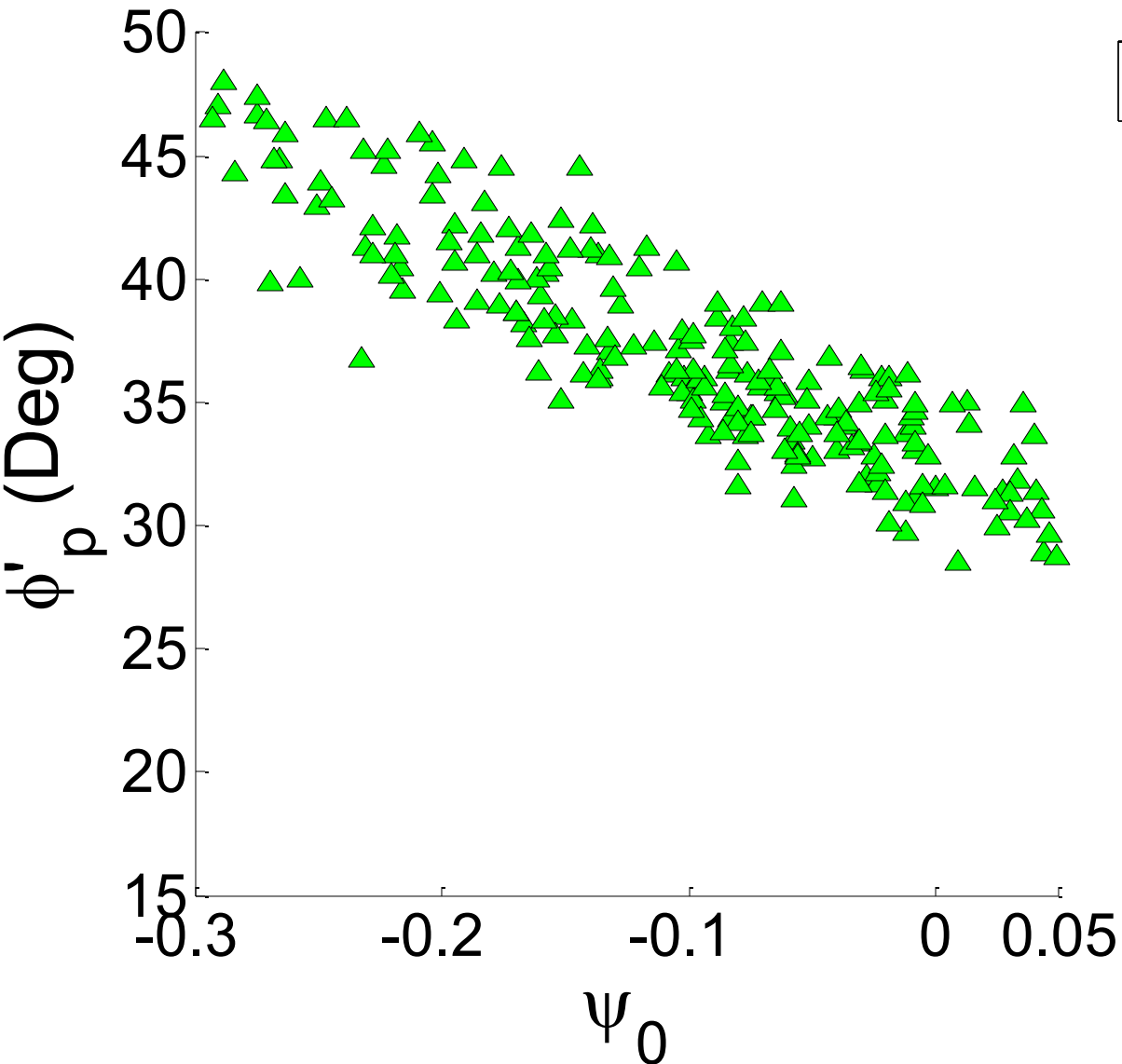
▲ Jefferies and Been

Experimental data
from *Jefferies and
Been (2006)*

ϕ'_p = peak angle of shearing
resistance
= peak "friction" angle

ψ_0 = initial state parameter

Variation in ϕ'_p with ψ_0

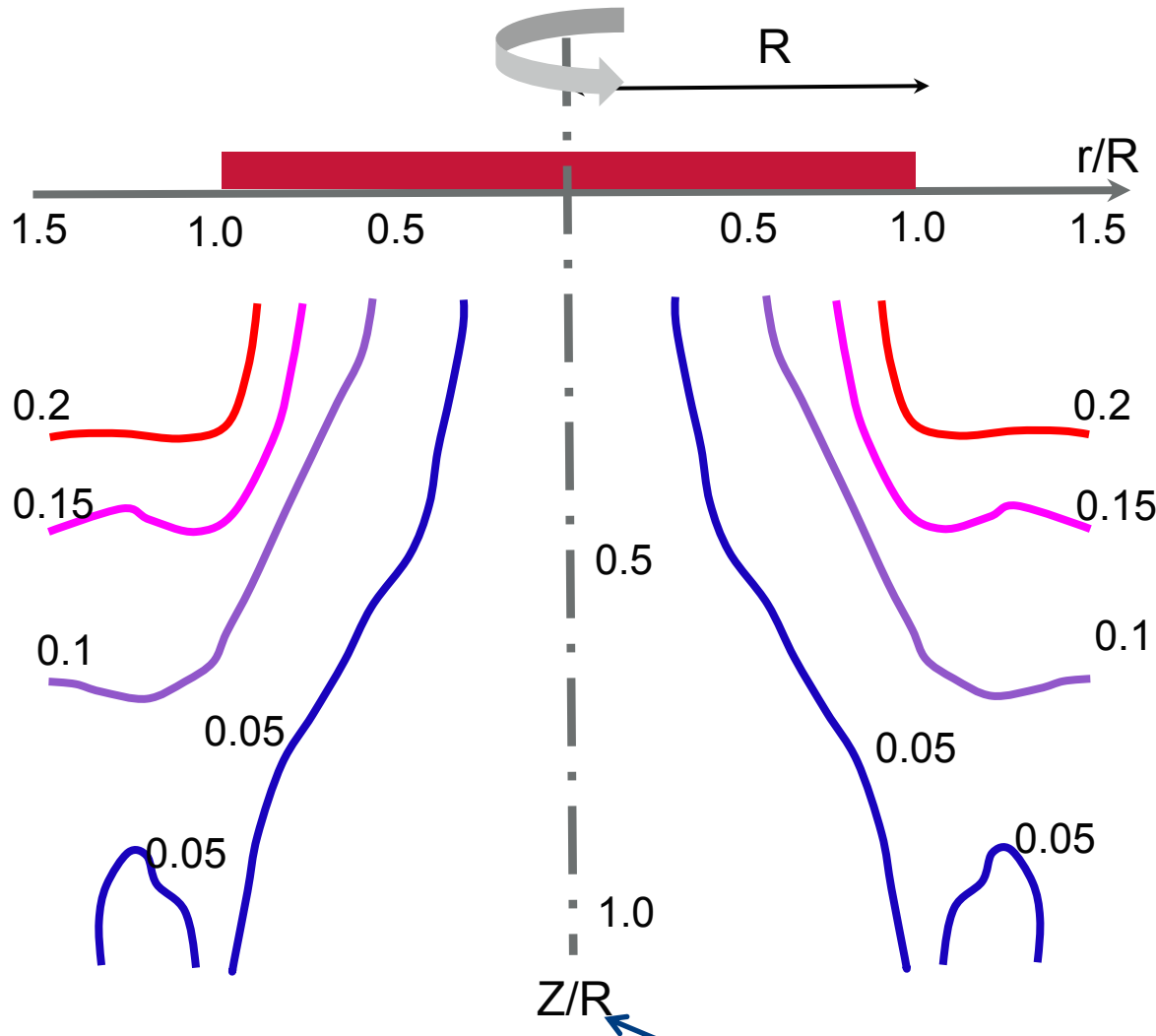


▲ Jefferies and Been



Triaxial cell at IC

In Situ Stresses



Stress increments beneath a circular footing on isotropic, linear elastic foundation

Contours of

$$\frac{\Delta\sigma'_2 - \Delta\sigma'_3}{\Delta\sigma'_1 - \Delta\sigma'_3}$$

σ'_1 =major principal stress
 σ'_2 =intermediate principal stress
 σ'_3 =minor principal stress

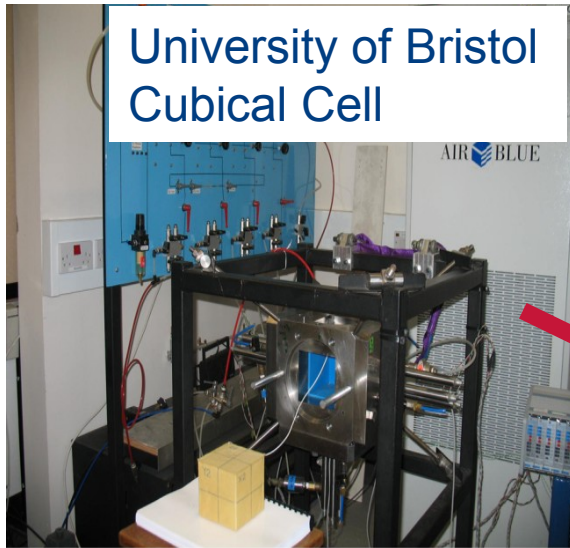
$$\sigma'_1 > \sigma'_2 > \sigma'_3$$

Along CL:

- $\Delta\sigma'_2 = \Delta\sigma'_3$
- Triaxial conditions

Hight (1983)

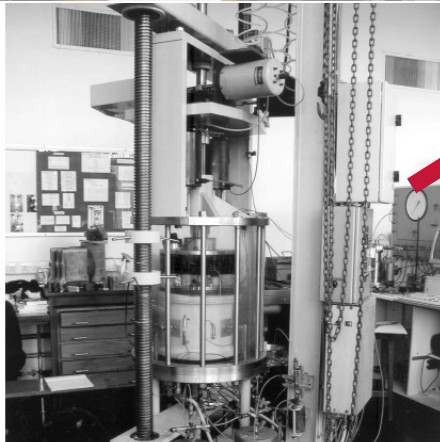
Variation in Soil Strength with Principal Stress Ratio



Can control b

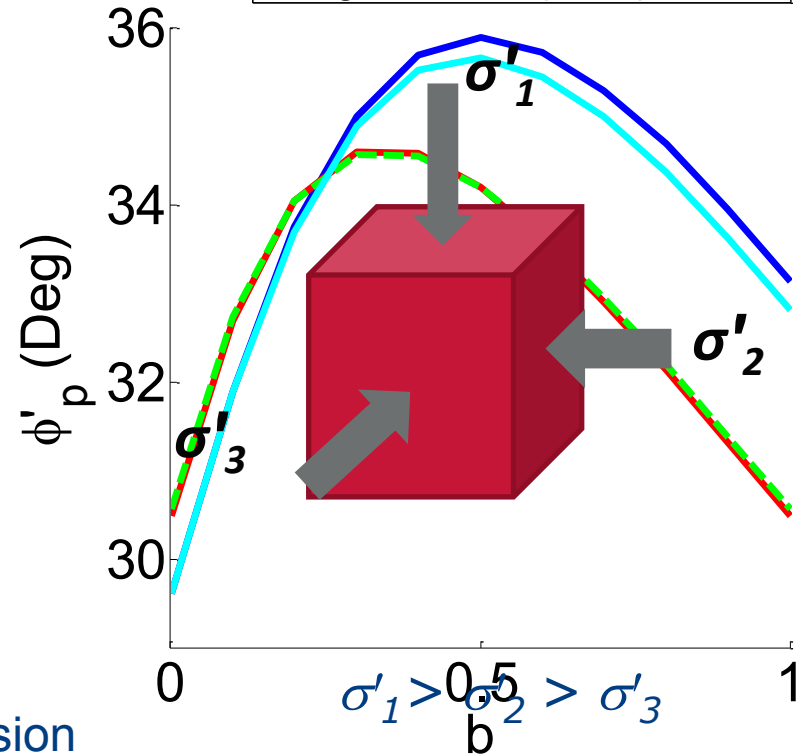
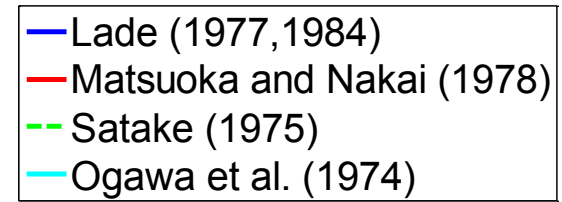
$b =$ intermediate stress ratio

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3}$$



Large Hollow Cylinder Apparatus at IC

- $b=0$: Triaxial compression
- $b=1$: Triaxial extension
- $b \approx 0.5$: Plane strain

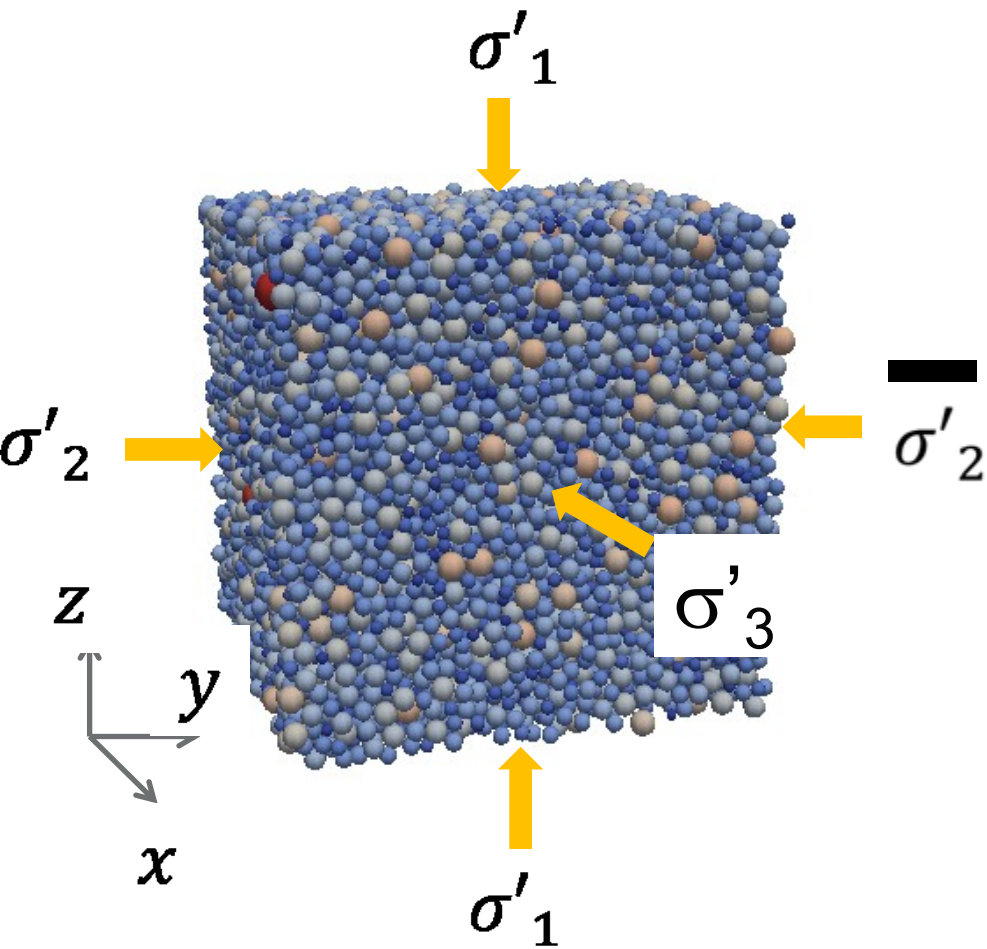


$\phi'_p =$ peak angle of shearing resistance
= peak “friction” angle

O’Sullivan et al. (2015)

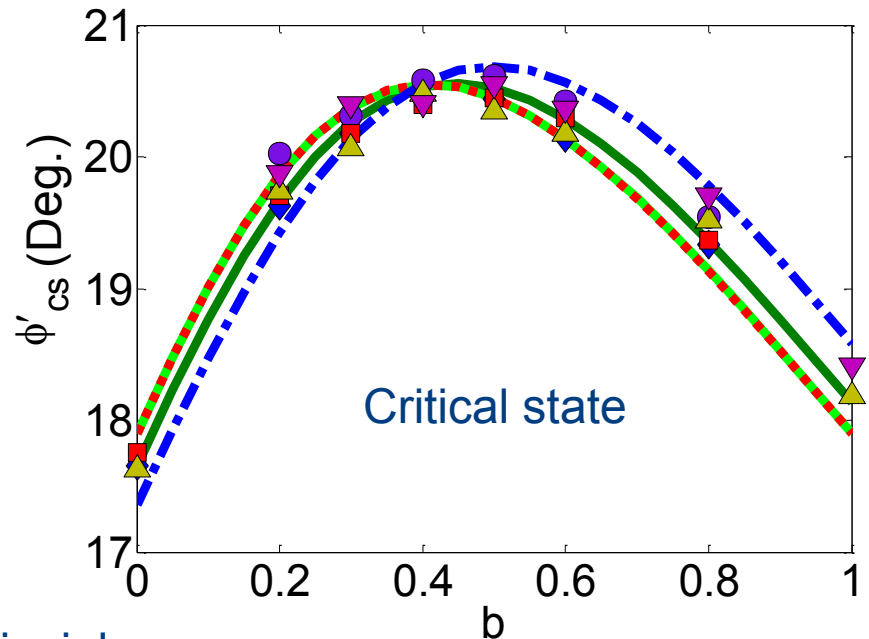
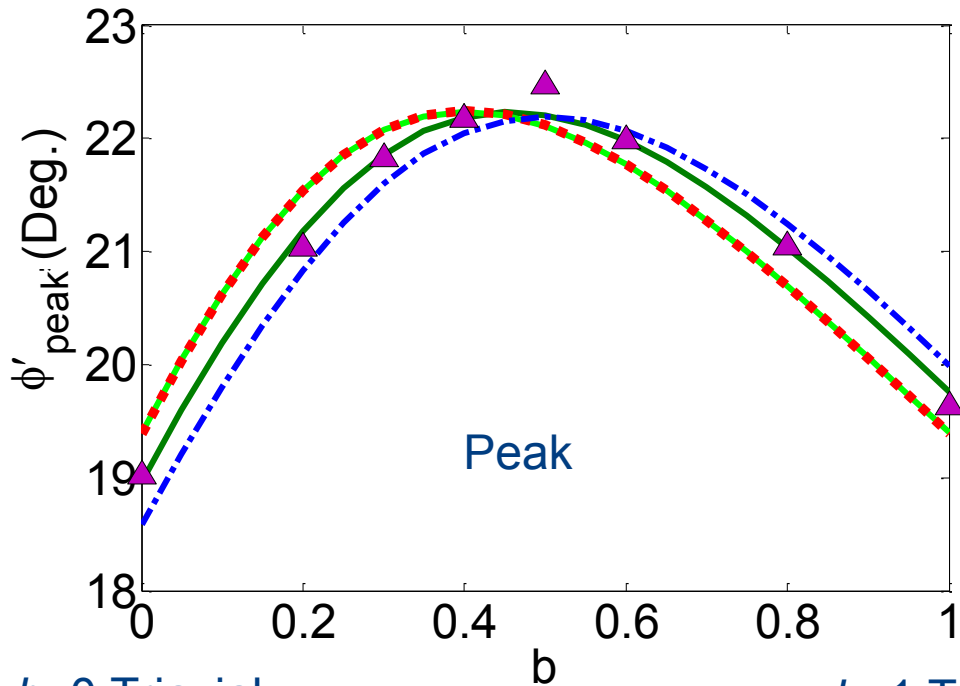
Géotechnique

DEM Analysis of ψ for a 3D Stress State



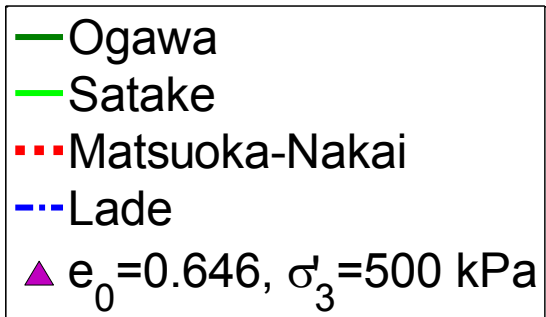
- 20,164 spherical particles
- Periodic boundaries
- Grading similar to Toyoura sand
- Friction coefficient $\mu=0.25$
- Controlled b
- Deformed to large strain
- Varied initial state parameter (ψ_0)
- Most of simulations used LAMMPS

Variation in Peak and Critical State Friction Angles with b



$b=0$: Triaxial
Compression

$b=1$: Triaxial
Extension

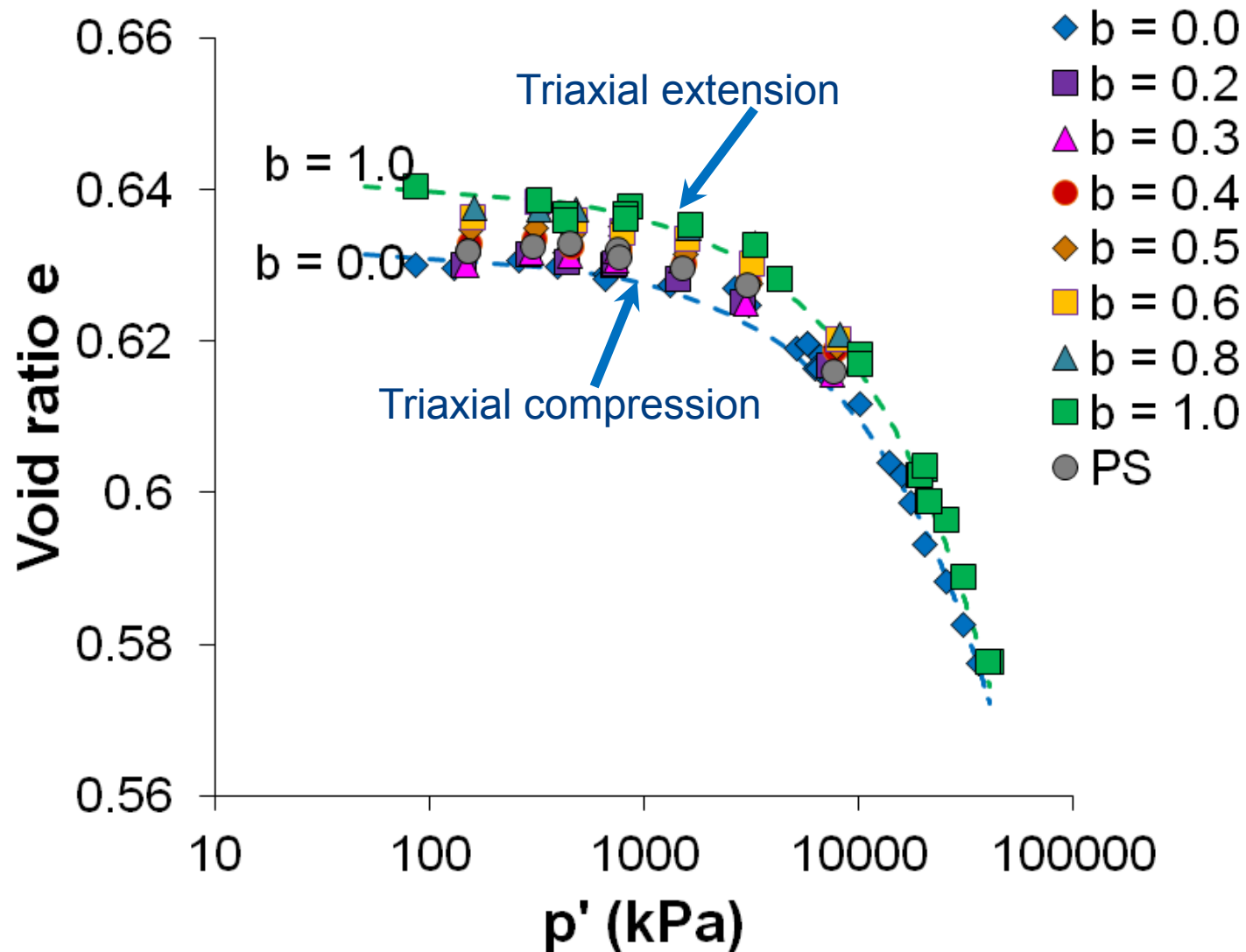


$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3}$$

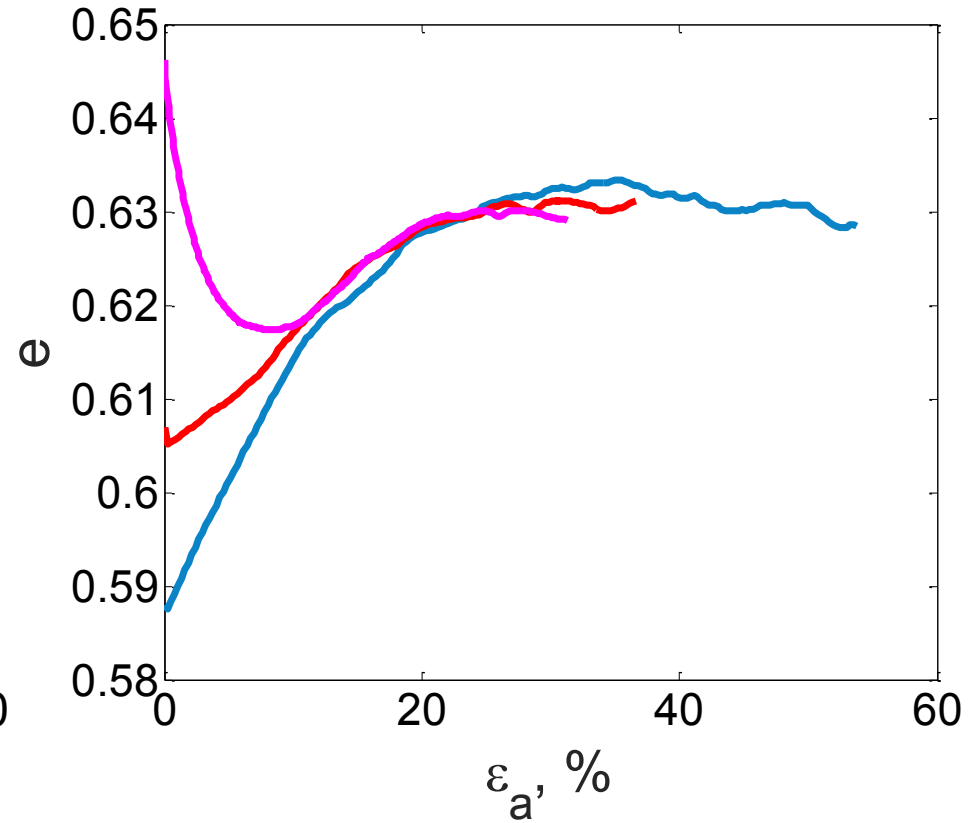
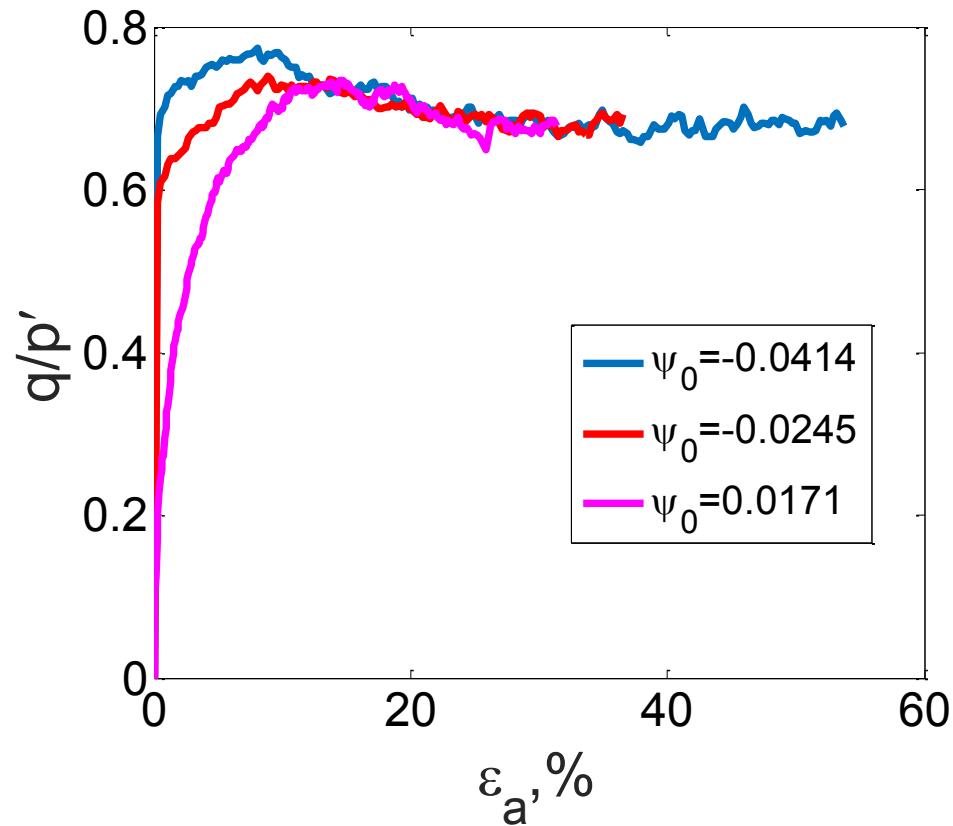
Huang et al. (2014)
Granular Matter

- \blacklozenge $e_0 = 0.533$, $\sigma'_3 = 100$ kPa
- \bullet $e_0 = 0.612$, $\sigma'_3 = 5000$ kPa
- \blacktriangledown $e_0 = 0.625$, $\sigma'_3 = 2000$ kPa
- \blacksquare $e_0 = 0.625$, $\sigma'_3 = 1000$ kPa
- \blacktriangle $e_0 = 0.646$, $\sigma'_3 = 500$ kPa

DEM Analysis of ψ for a 3D Stress State



State Dependent Response

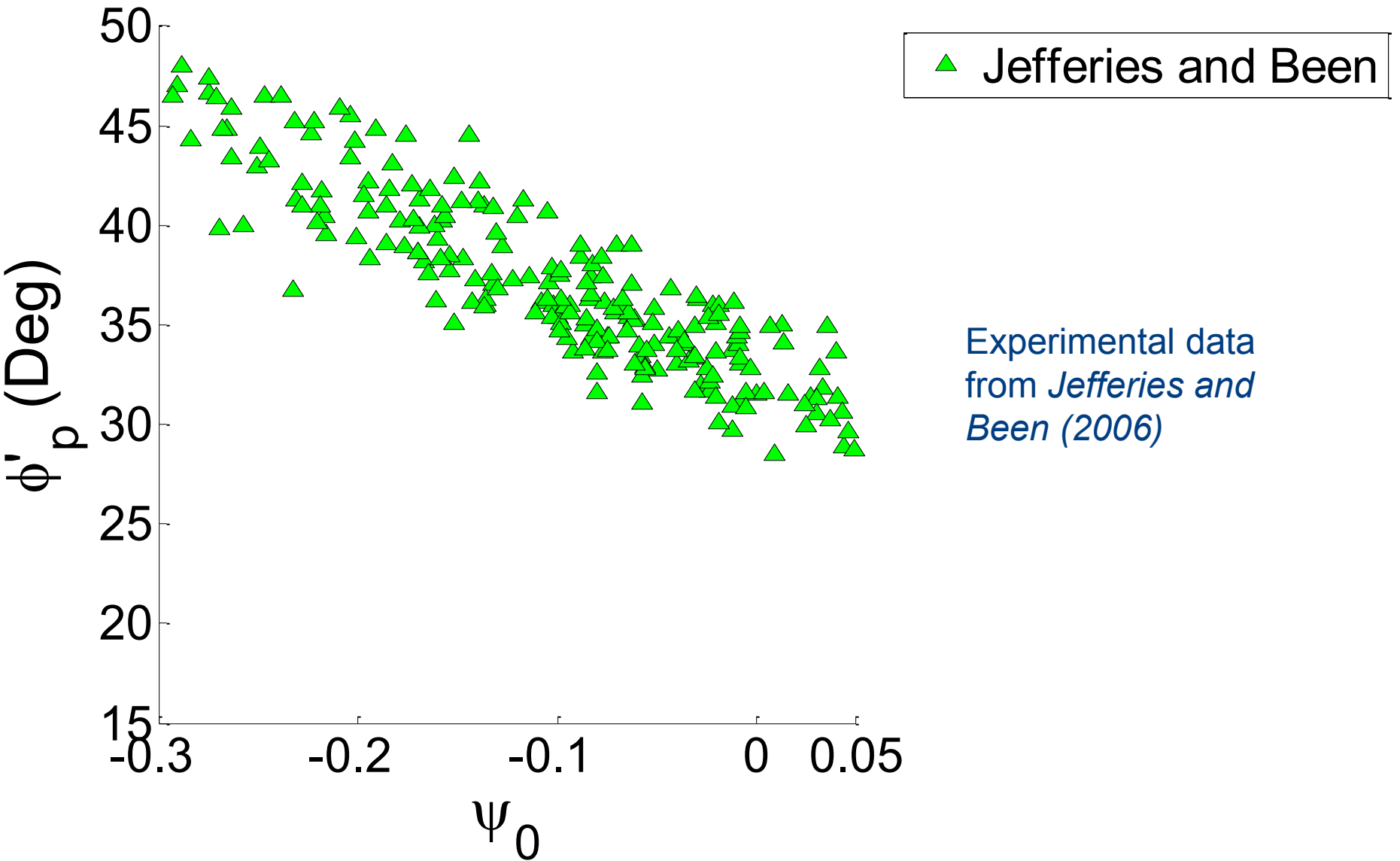


$\psi > 0$ loose $q = \sigma'_1 - \sigma'_3$
 $\psi < 0$ dense $p' = 1/3(\sigma'_1 + 2\sigma'_3)$
 (triaxial)

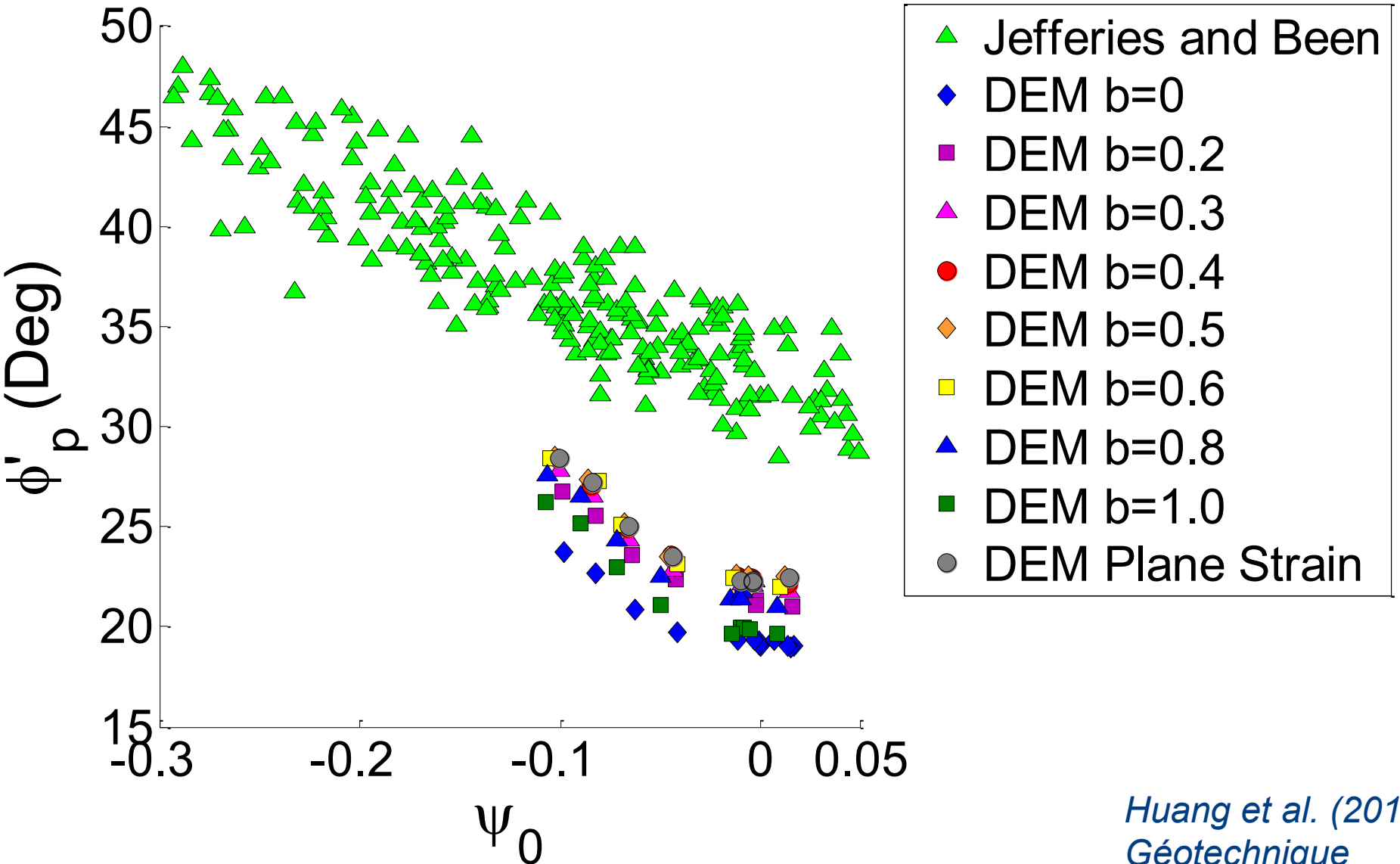
$\sigma'_3 = 500 \text{ kPa}$

Huang et al. (2014)
Géotechnique

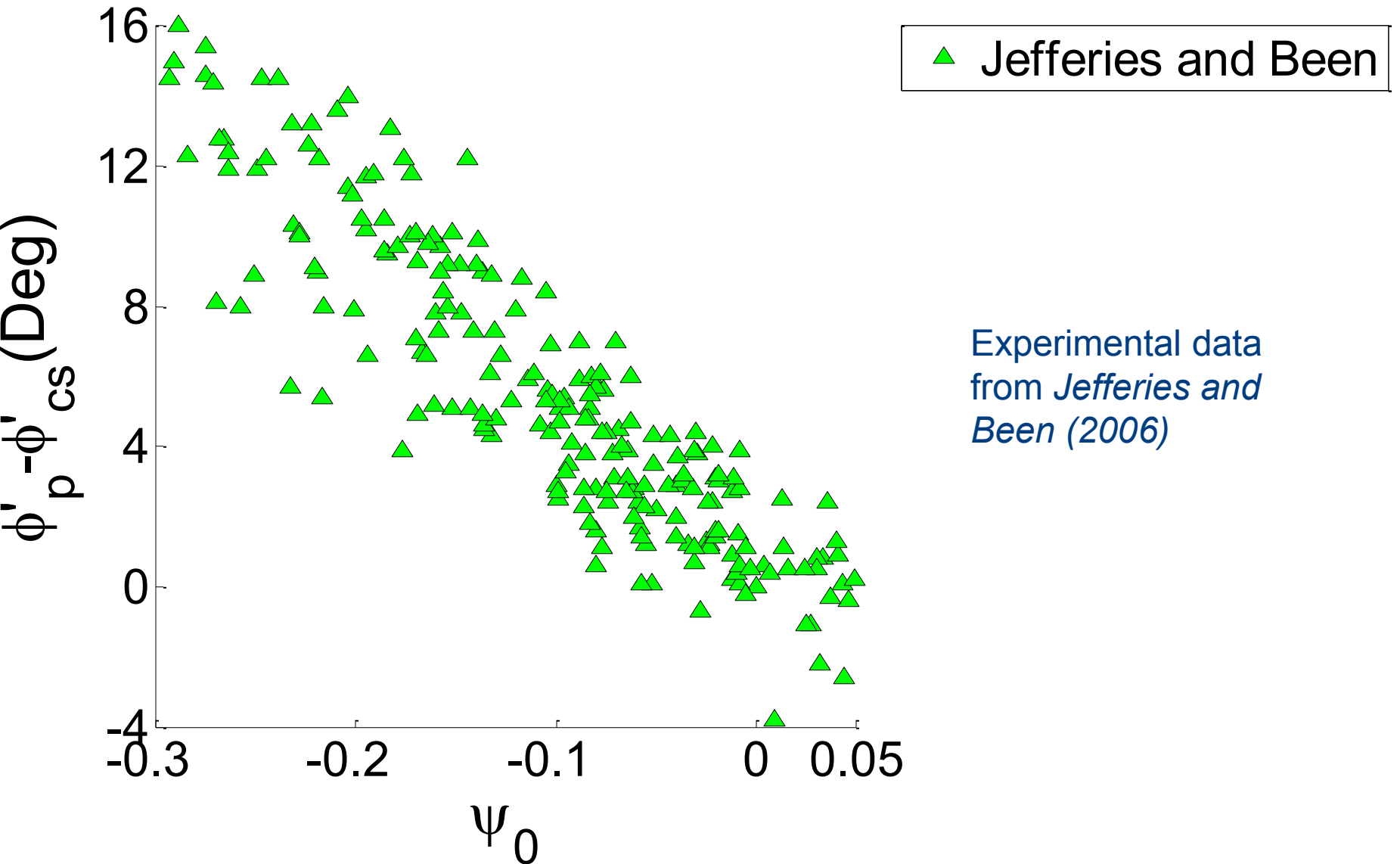
Variation in ϕ'_p with ψ_0



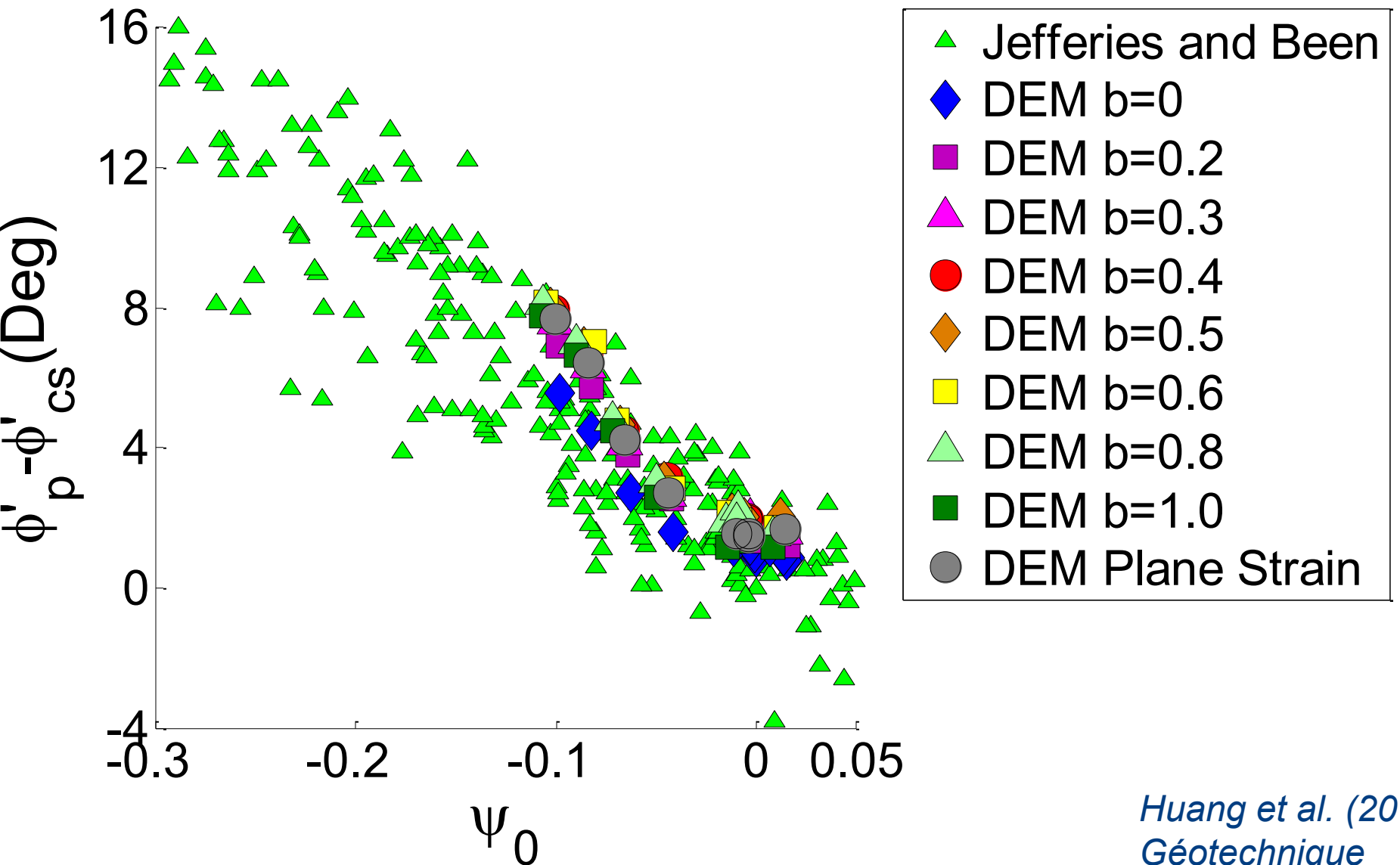
Variation in ϕ'_p with ψ_0



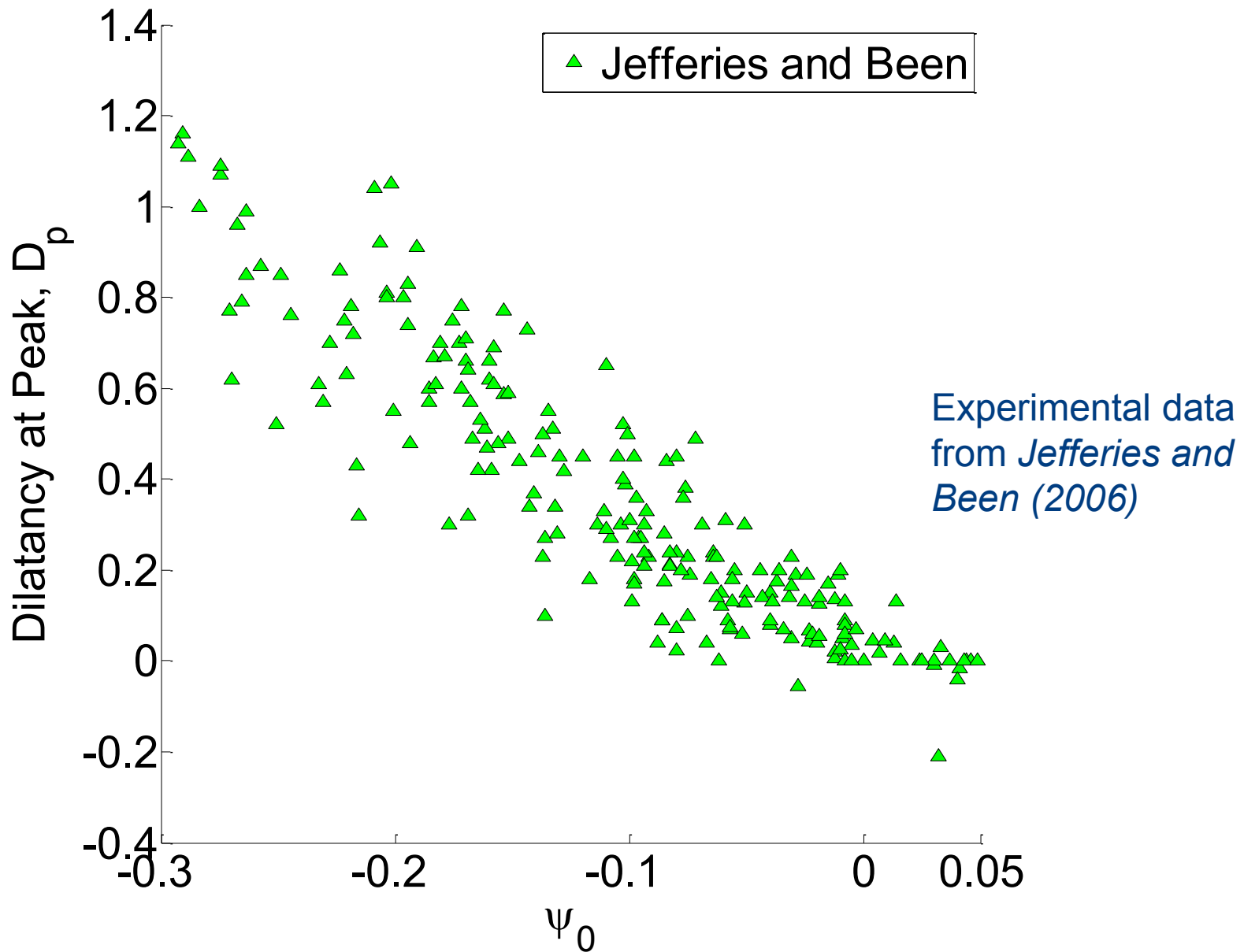
Variation in $\Phi'_p - \Phi'_{cs}$ with Ψ_0



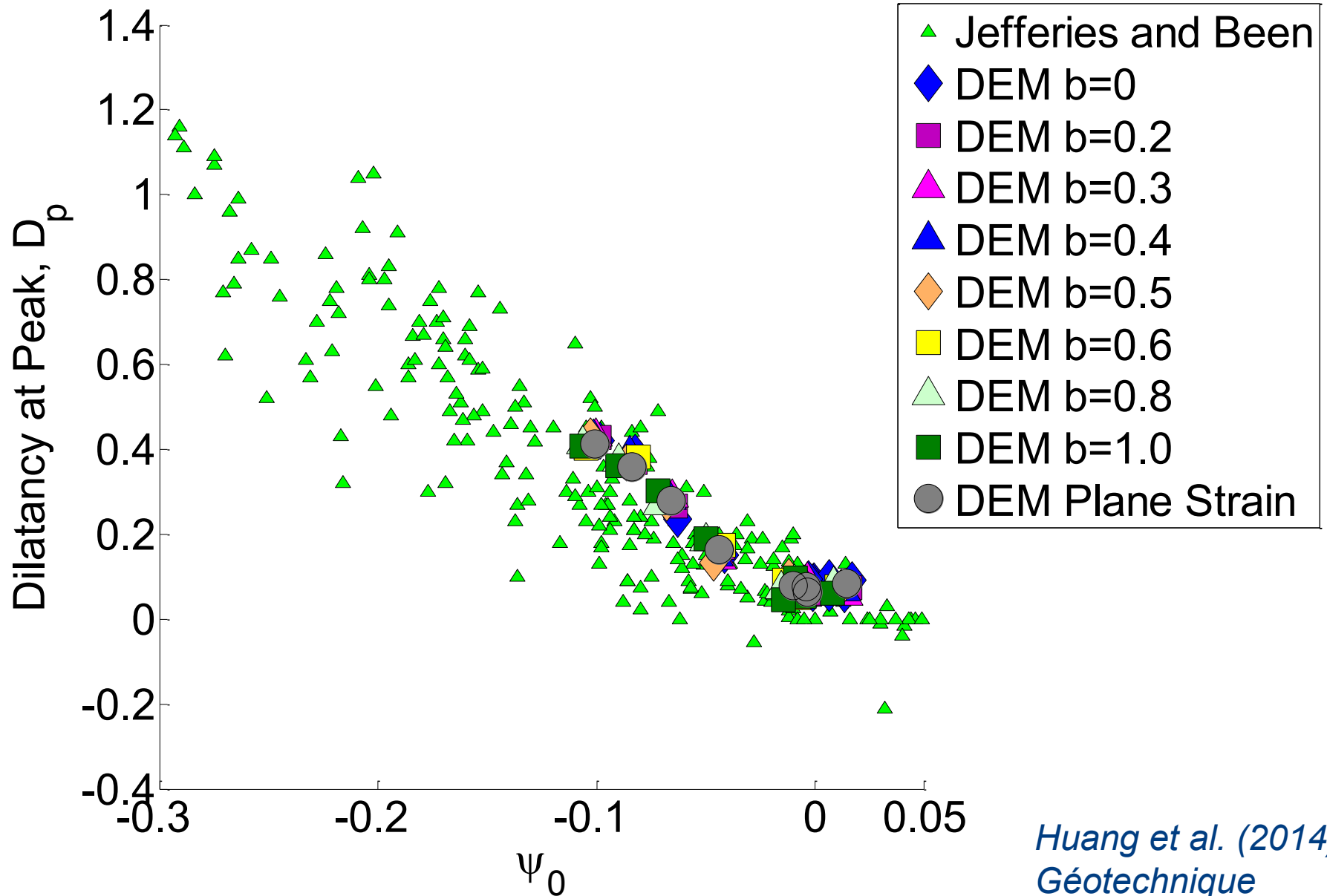
Variation in $\phi'_p - \phi'_{cs}$ with ψ_0



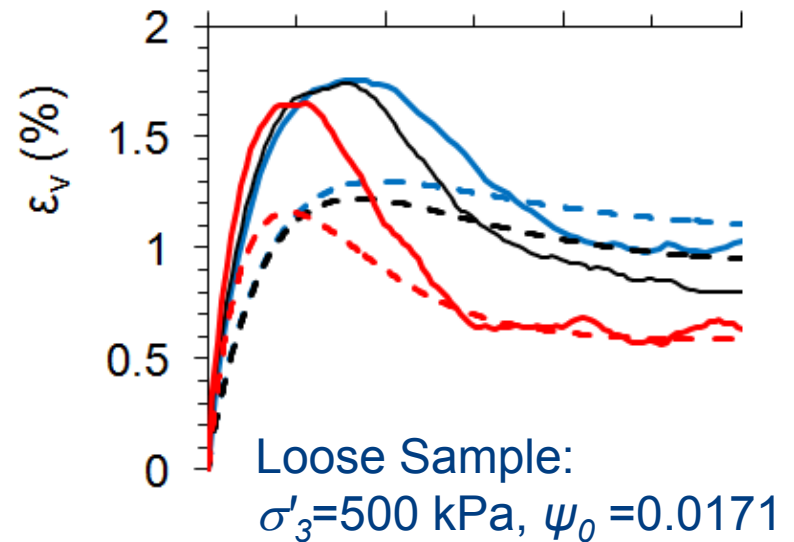
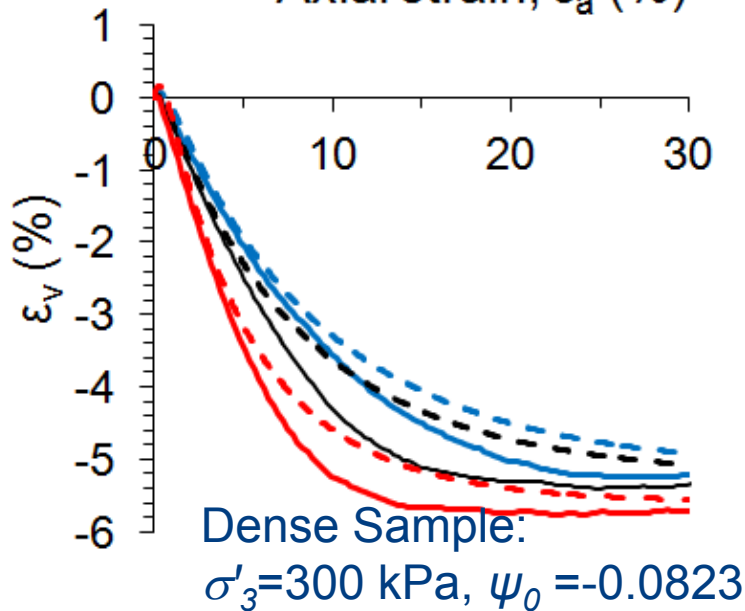
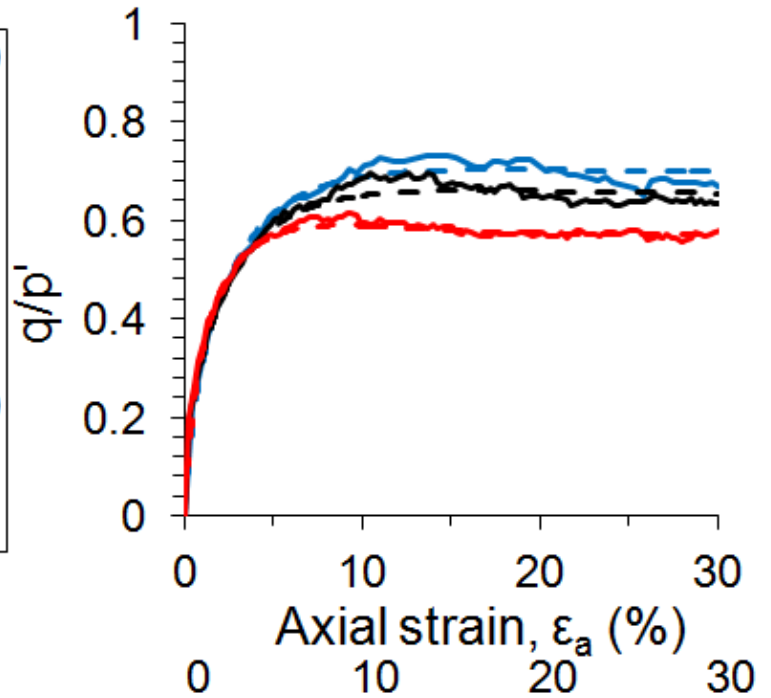
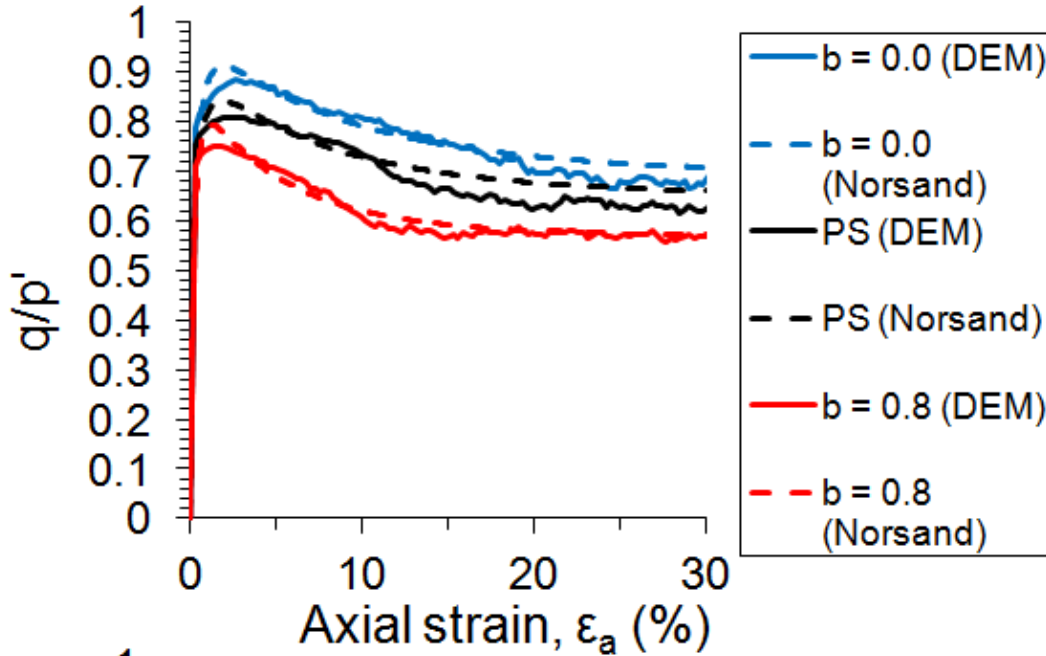
Variation in D_p with ψ_0



Variation in D_p with ψ_0



Performance of Norsand Model



Reminder that Robust Constitutive Models are Needed



The remains of the village of Bento Rodrigues in Brazil following tailings dam collapse, Nov. 2015

<http://www.theguardian.com>

DEM Analysis of ψ for a 3D Stress State

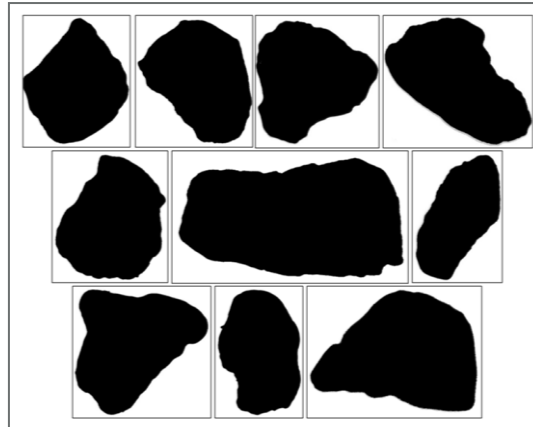
- DEM simulations can capture influence of b on strength and the state-dependency of soil strength and dilation
- DEM data indicate that the correlations between $\phi'_p - \phi'_{cs}$ and ψ_0 and D_p and ψ_0 are independent of particle morphology and b
- DEM simulations can be used to supplement experimental investigations when developing constitutive models for soil behaviour in a general stress space

DEM Limitations

- Spherical particles under-estimate soil strength (ϕ'_{cs} , ϕ'_p)

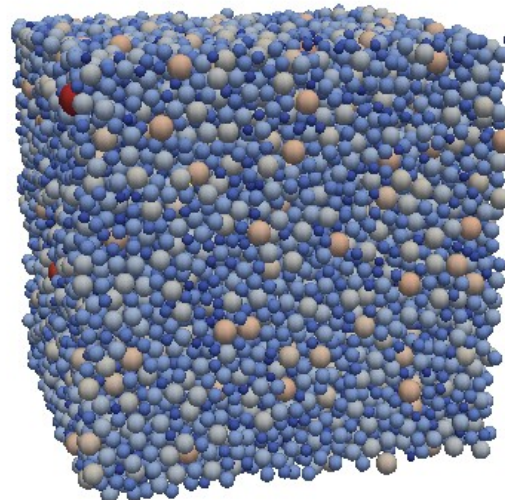
Toyoura sand:

- $\phi'_{cs} = 31^\circ$
- Subangular grains



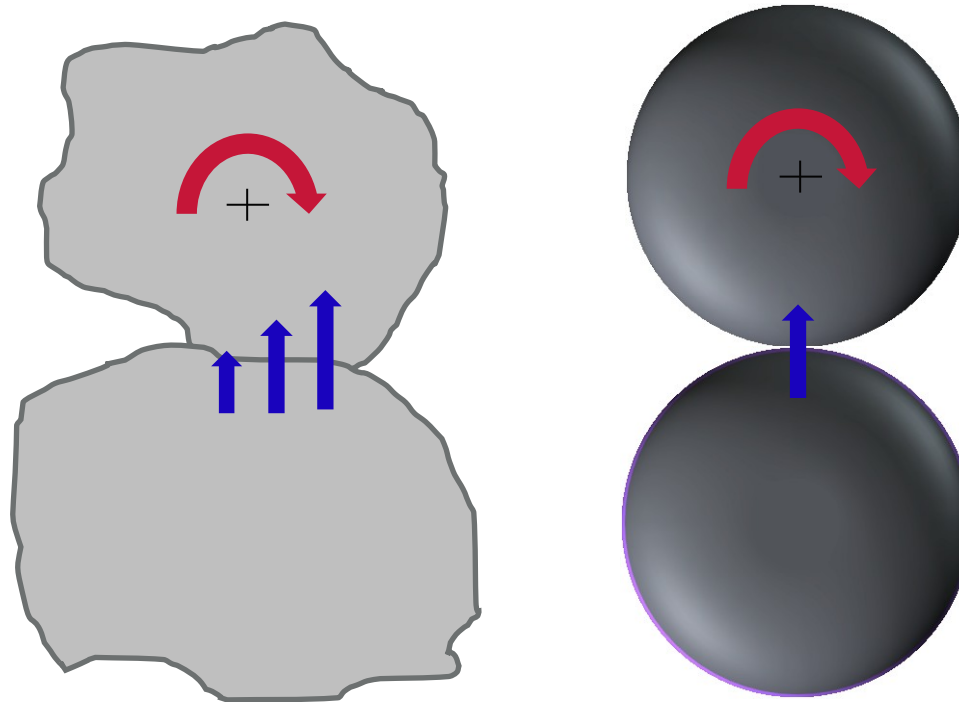
DEM model:

- $\phi'_{cs} = 18.2^\circ$
- Spherical particles



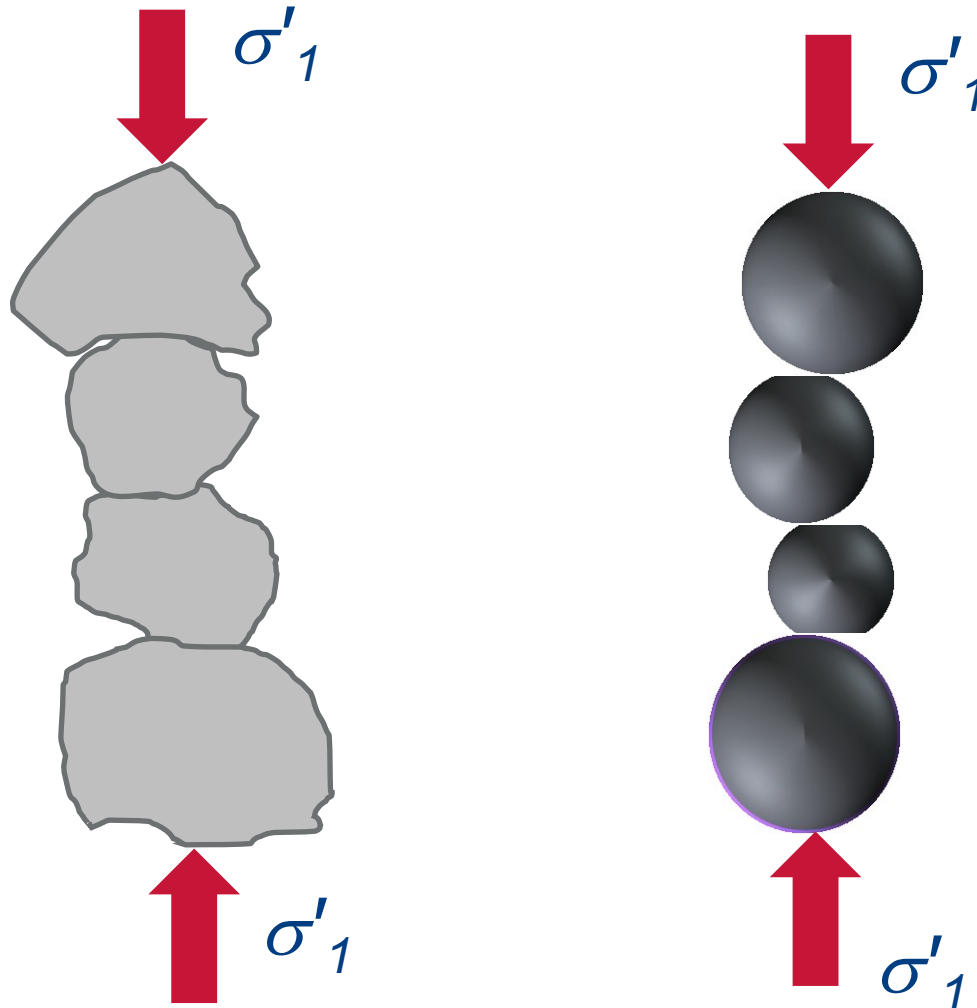
Influence of Morphology on ϕ'_p and ϕ'_{cs}

- Rotation of non-spherical particles will be frustrated by finite contact area



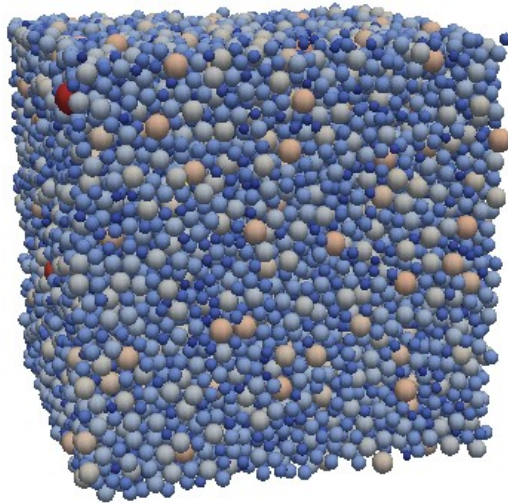
Influence of Morphology on ϕ'_p and ϕ'_{cs}

- Force chains between non-spherical particles are more stable

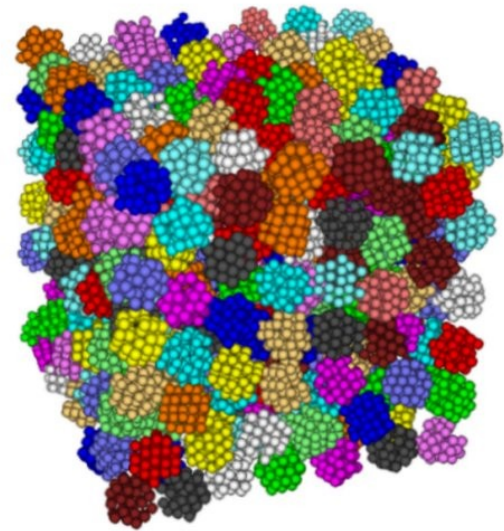


DEM and Particle Geometry

- Options for non-spherical particles include sphere clumps - computational cost increase



389 Agglomerates

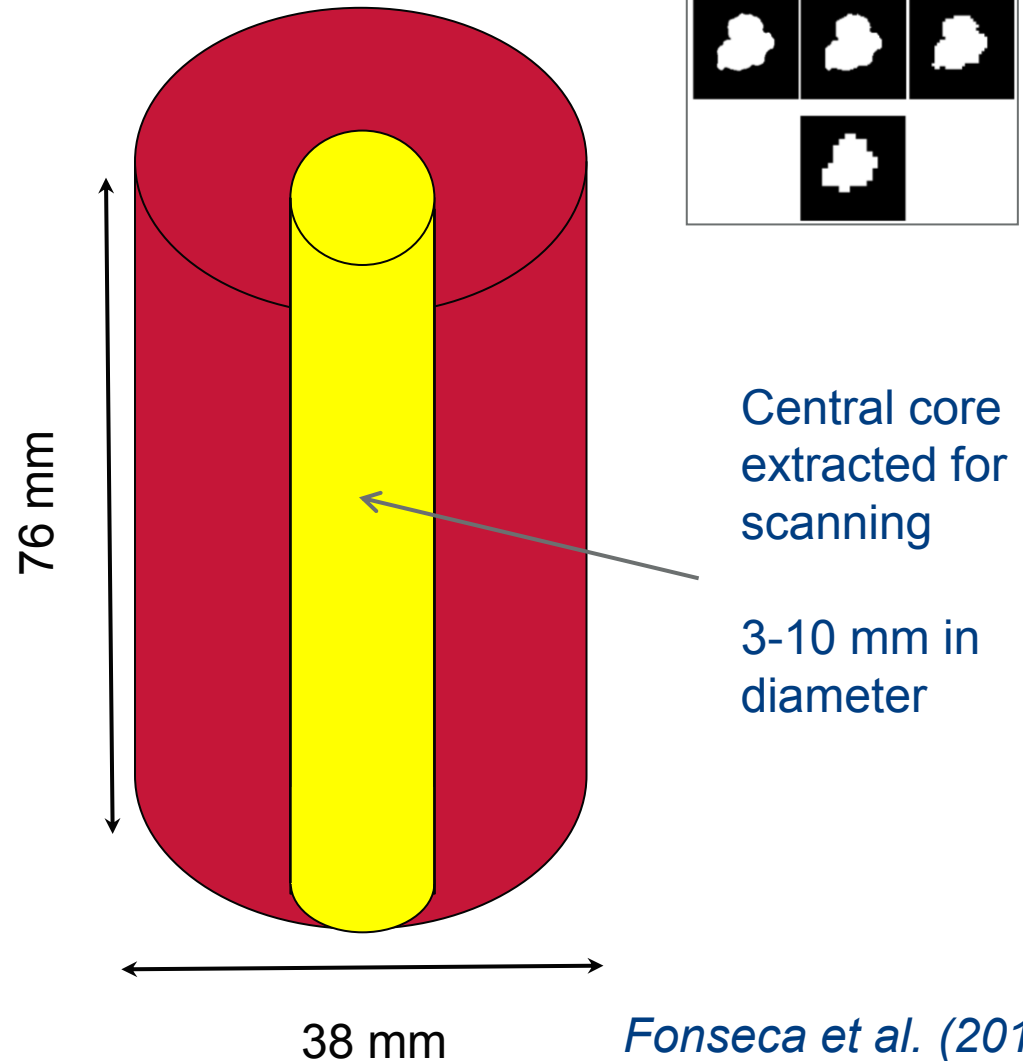
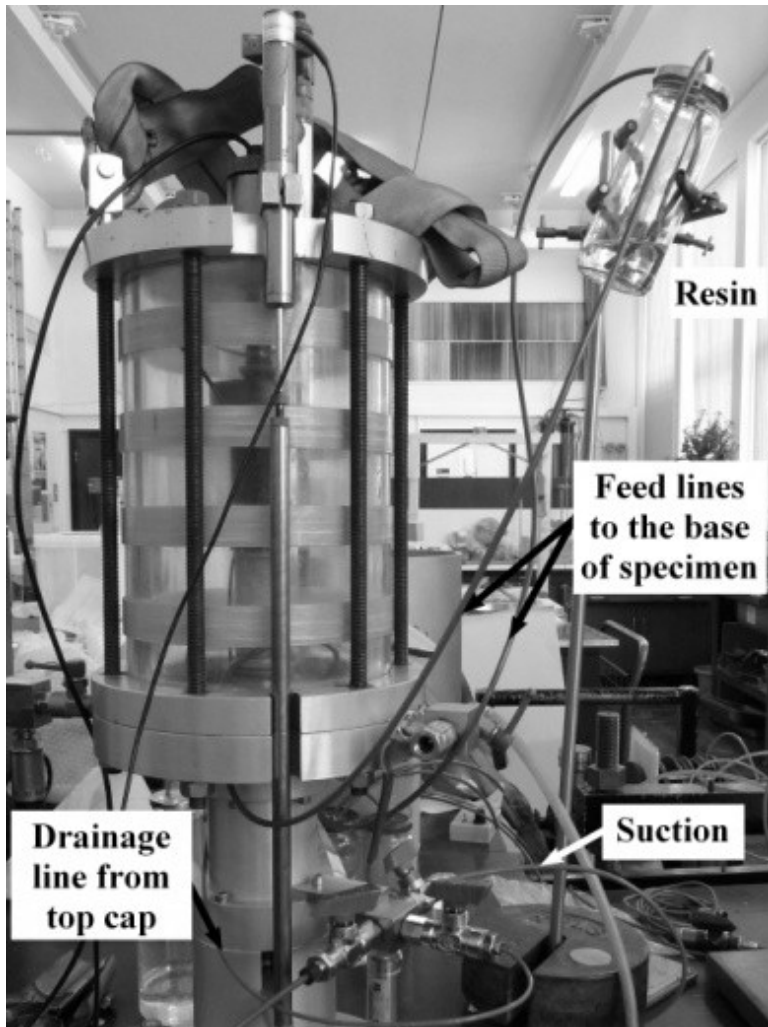


Cheng et al (2003)
Géotechnique

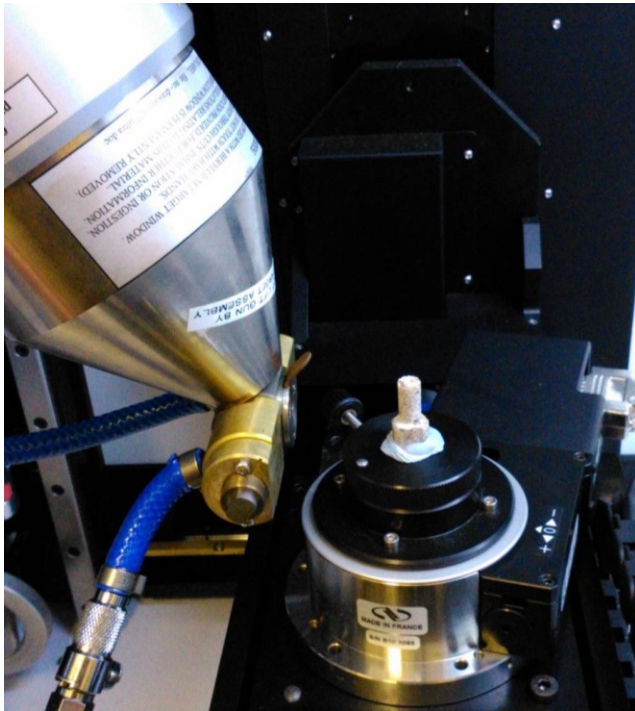
Micro Computed Tomography (microCT)

- High resolution, three-dimensional images created using X-rays
- Non-destructive
- Computed tomography developed by Cormack and Hounsfield – Awarded Nobel Prize in Physiology or Medicine in 1979
- First paper in *Géotechnique* – Oda et al. (2004)

Micro Computed Tomography (microCT)

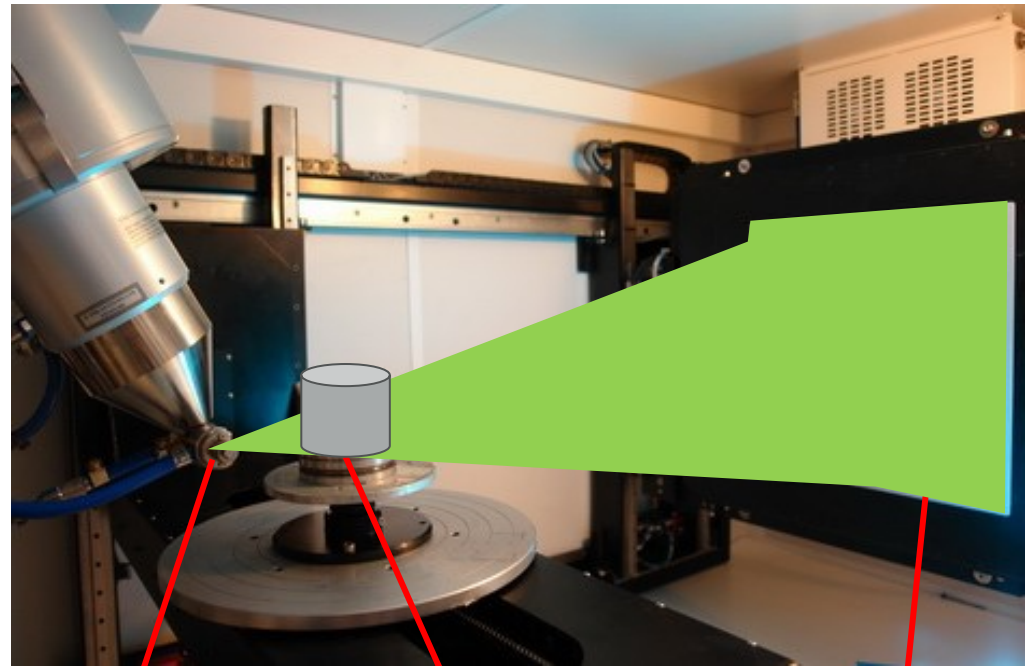


MicroCT scanning



X-Ray source + typical sample for scanning

(H. Taylor, current PhD student)



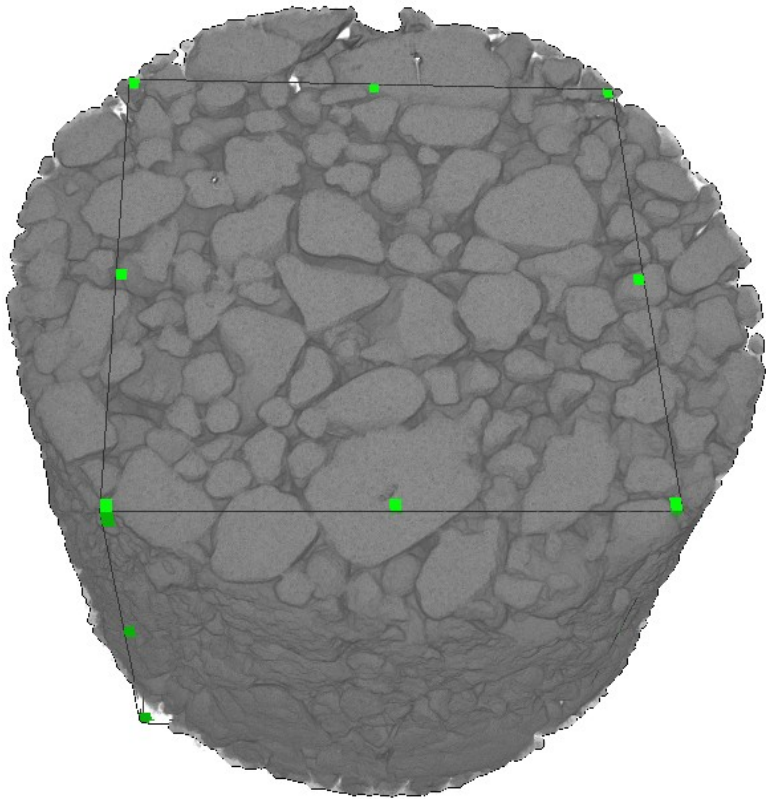
X-ray source

Sample

Detector

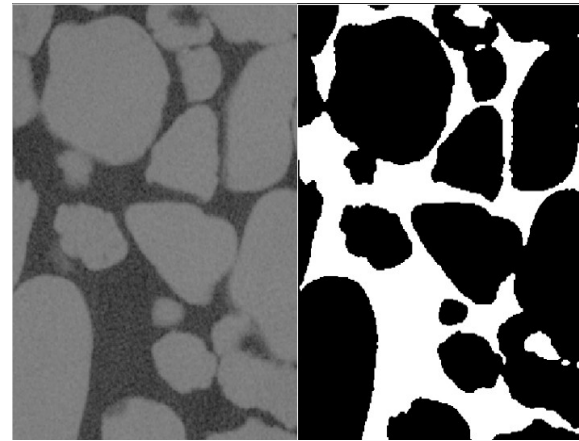
Image source: <http://www.nikonmetrology.com>

MicroCT Data Analysis



Raw output – 3D attenuation map

(H. Taylor)

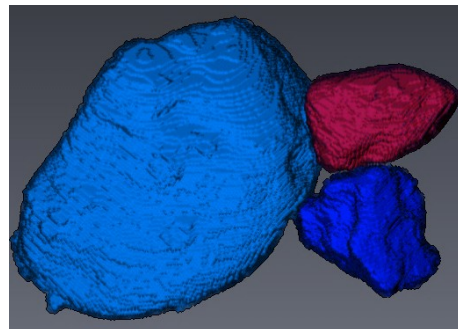
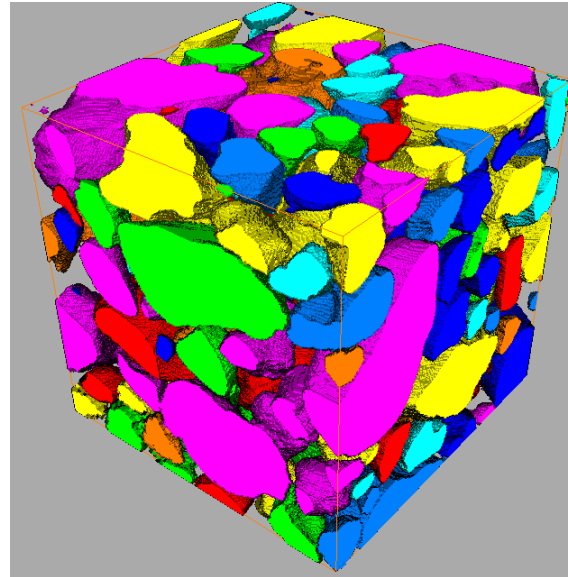
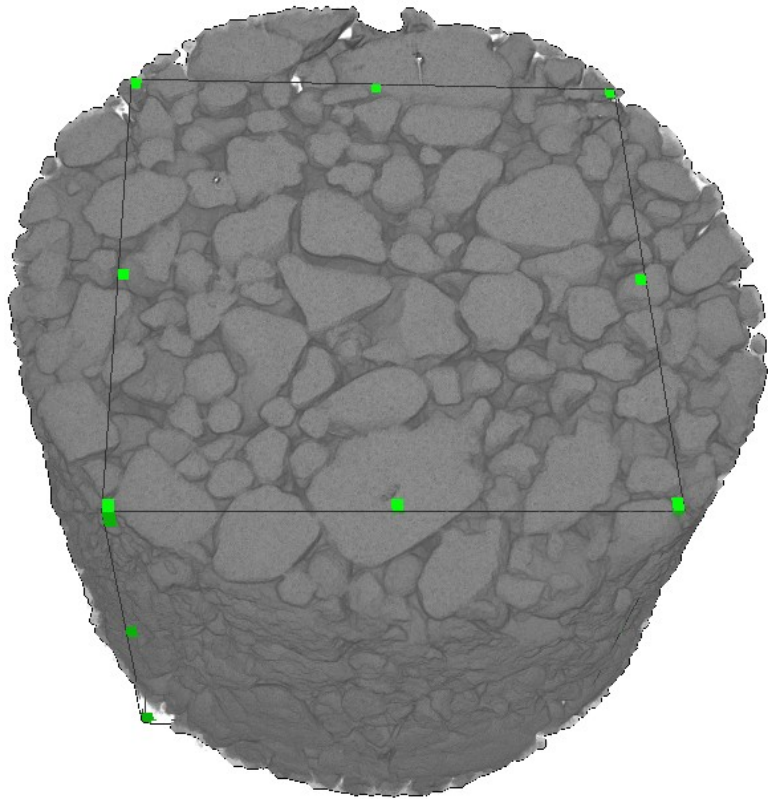


2D Slice from
 μ CT image

Binary image

Taylor et al. (2015)
Computers and Geotechnics

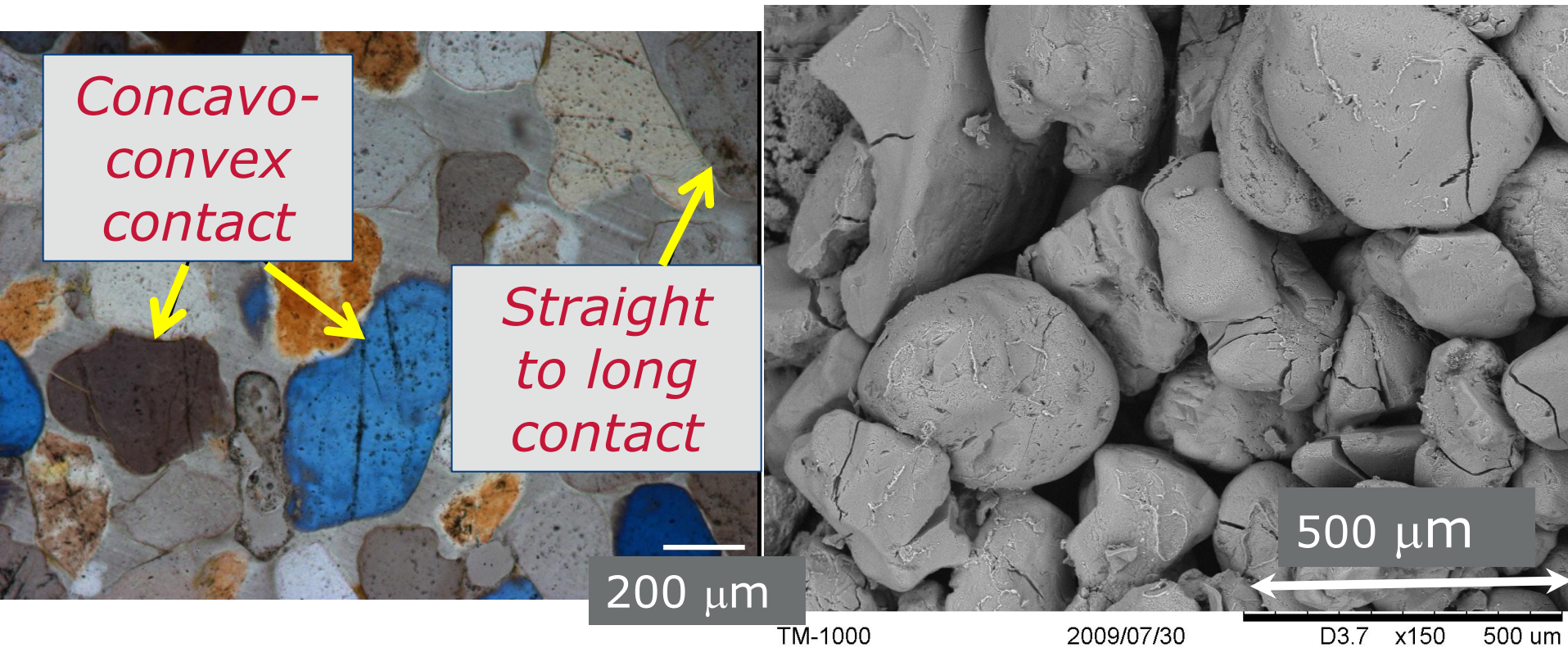
MicroCT Data Analysis



Individual
particles and
contacts from
watershed
segmentation

(H. Taylor)

Reigate Sand: A “Locked Sand”

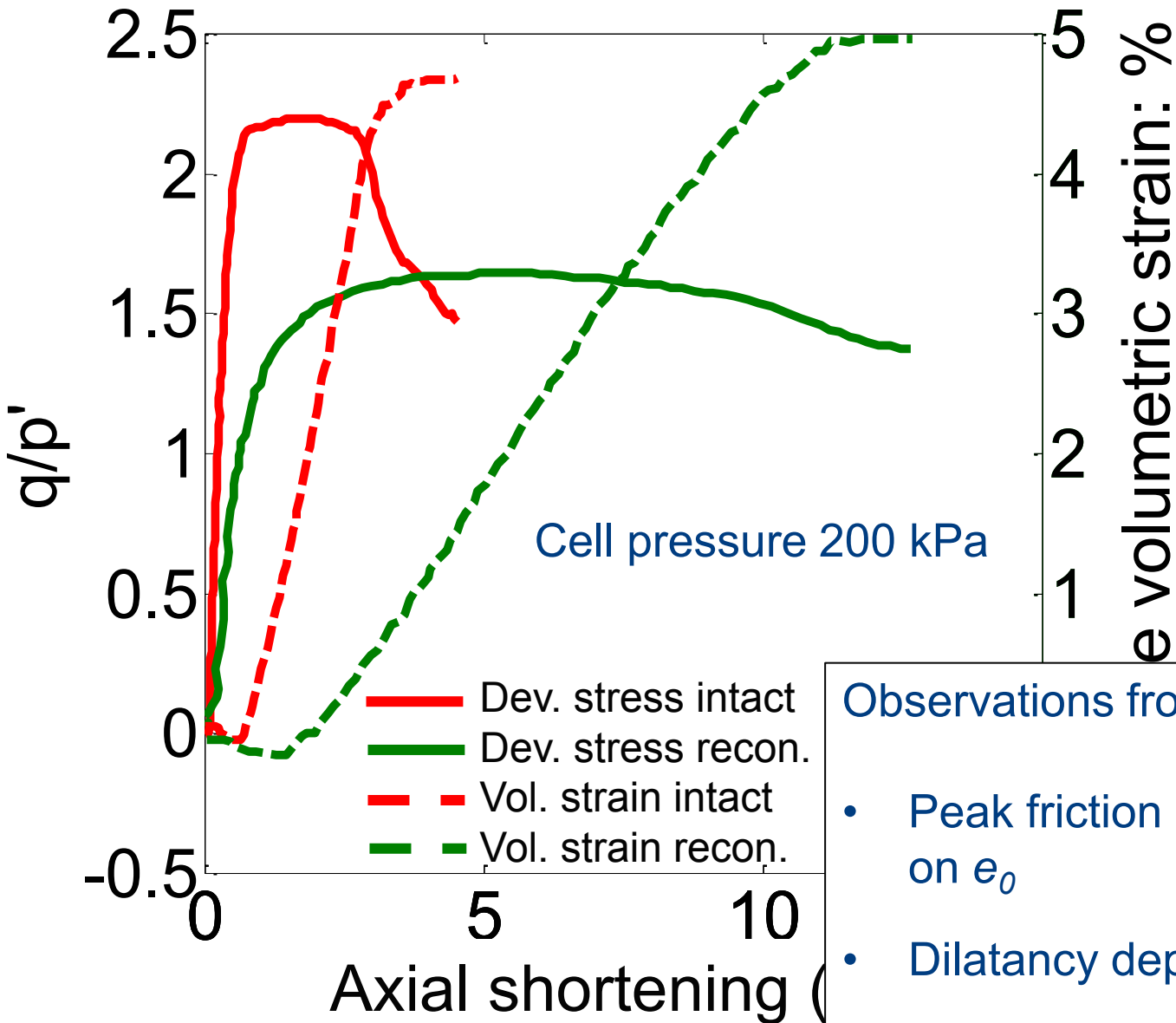


Transmitted light microscope image of thin section (cross-polarized light)
(contact configurations proposed by *Dusseault and Morgensten (1979)*)

Scanning electron microscope (SEM) image

*Doctoral Research of Dr.
J. Fonseca*

Reigate Sand Load: Deformation response



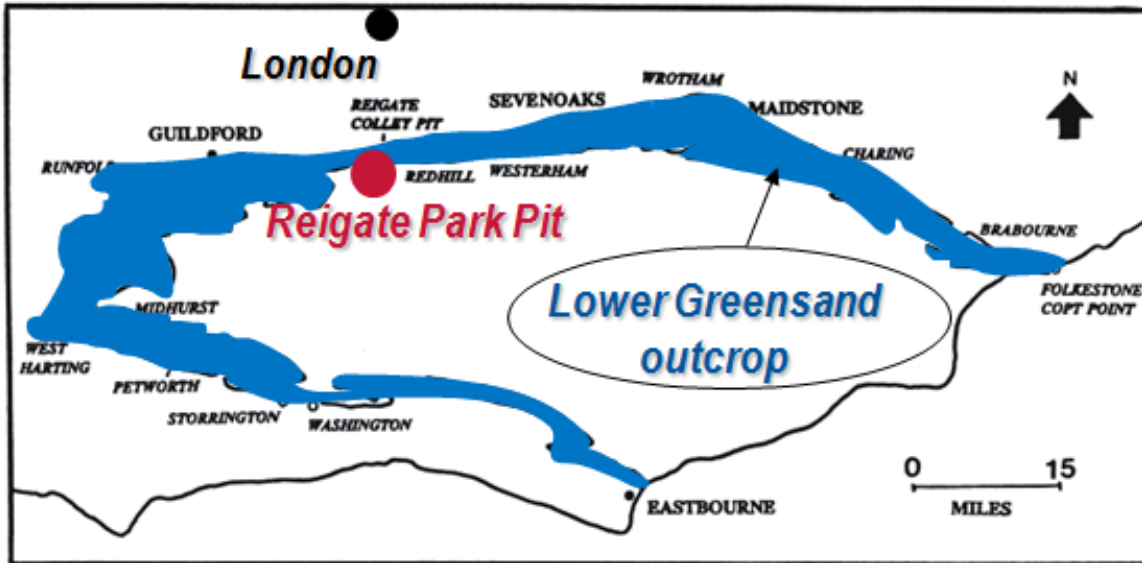
*Cresswell and Powrie
Géotechnique (2004)*

Reconstituted sample
“pluviated to highest
possible density”

Observations from direct shear tests:

- Peak friction angle (ϕ'_p) depends on e_0
- Dilatancy depends on stress level

Reigate Sand – Site Location



*From Richards & Barton
(1999)*

Block Sampling



Triaxial Tests – Sample Preparation



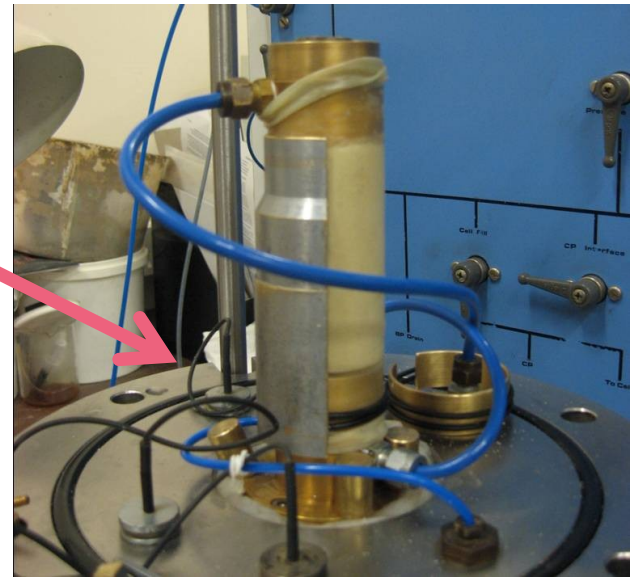
Intact Sample



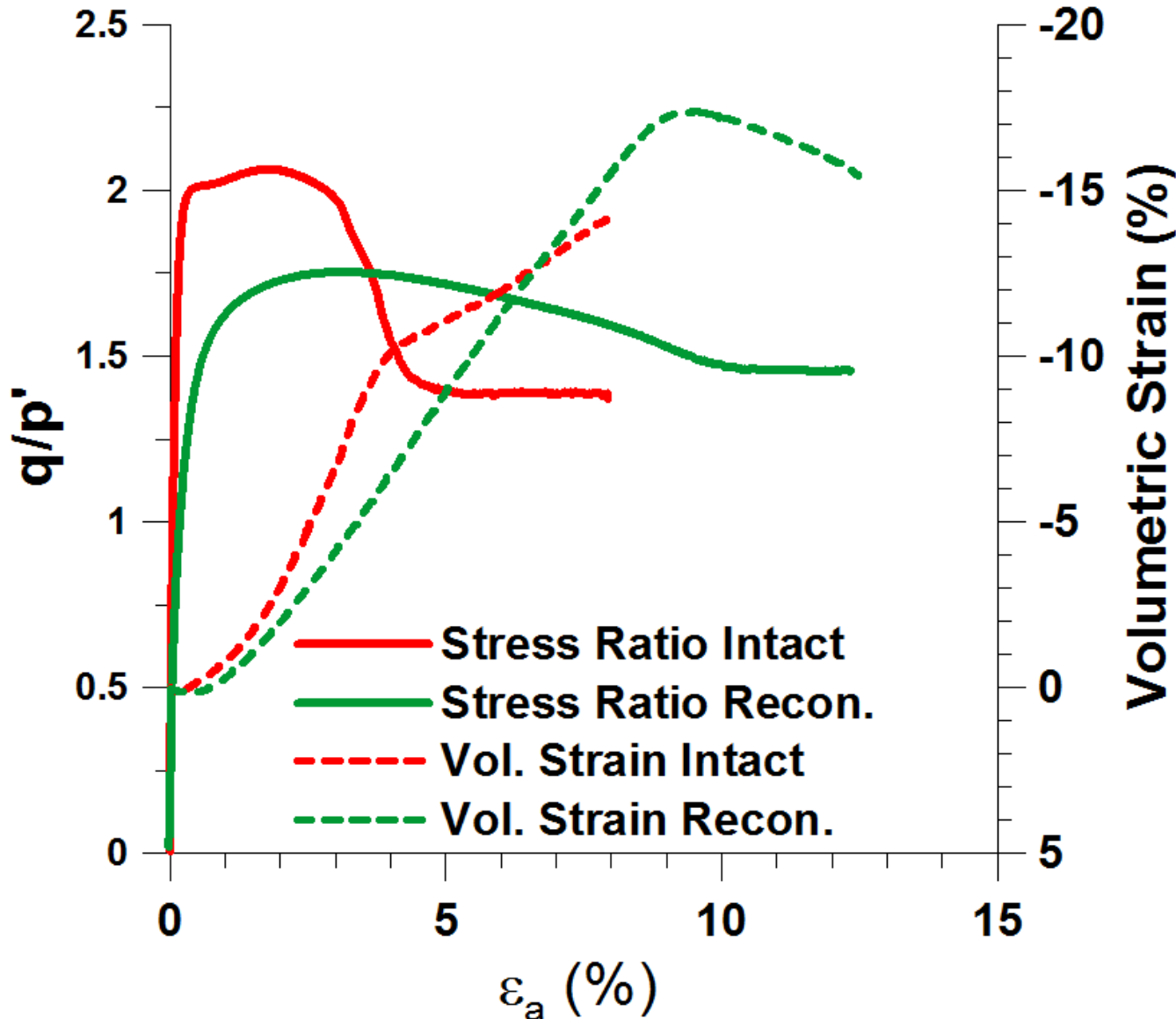
Reconstituted Sample
(Prepared by pouring / tamping/
tapping)



*Doctoral Research of Dr.
J. Fonseca*



Triaxial Test Data



Intact

$$e_0 = 0.48$$

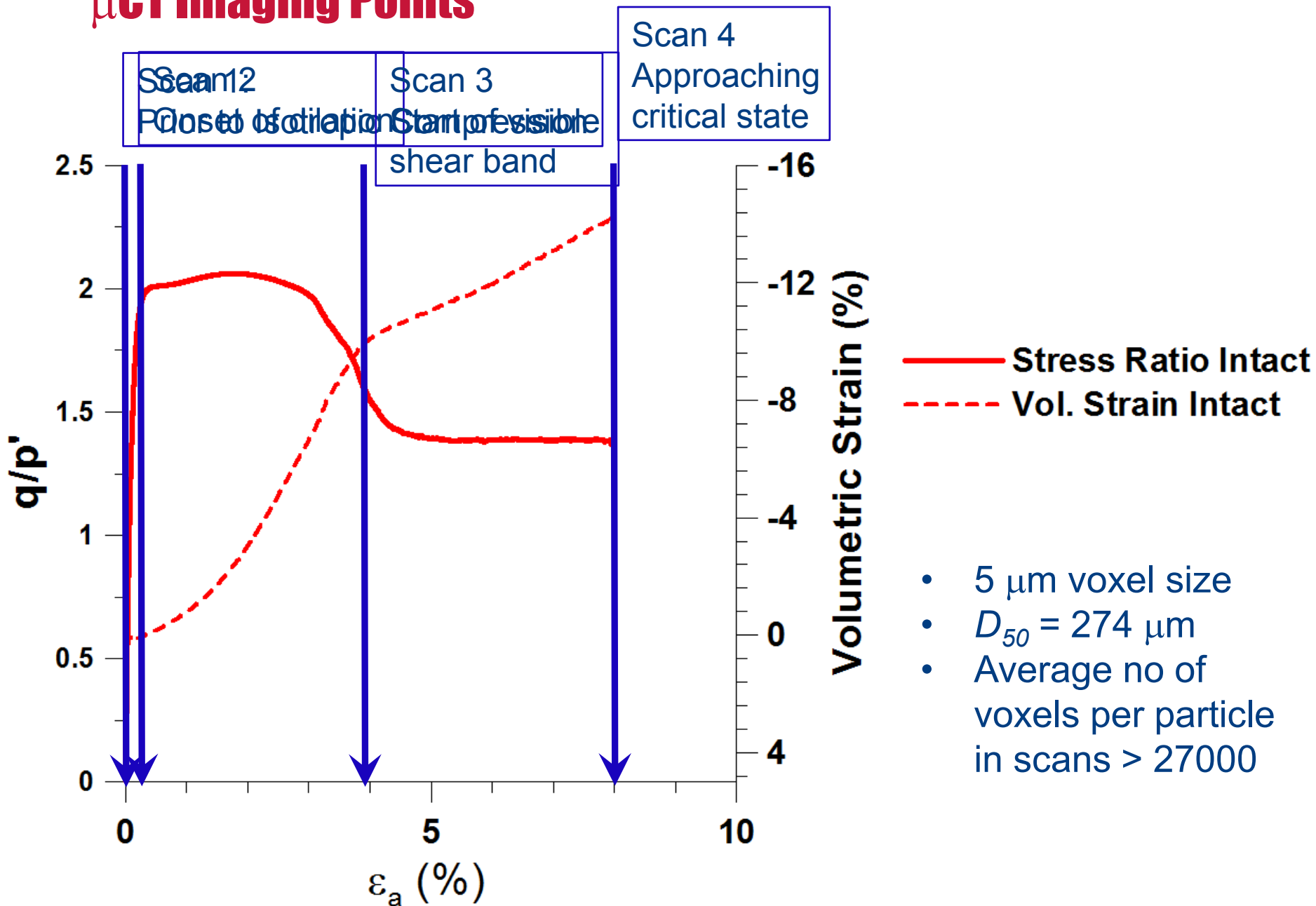
Reconstituted

$$e_0 = 0.49$$

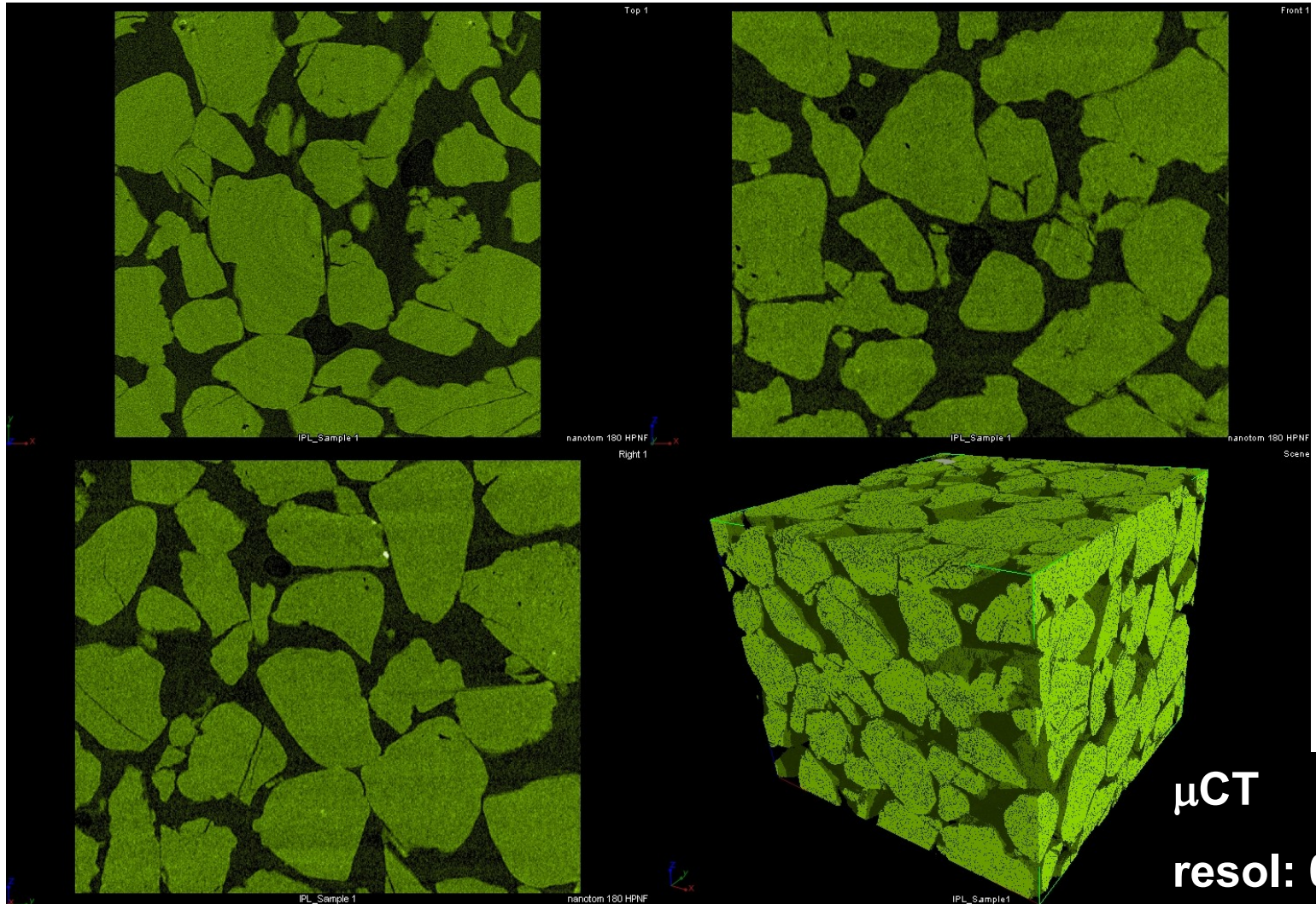
Cell pressure

300 kPa

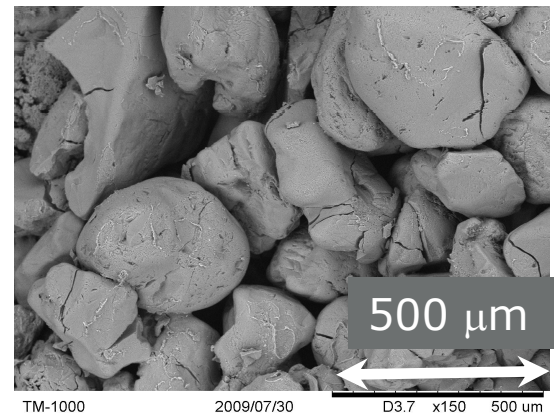
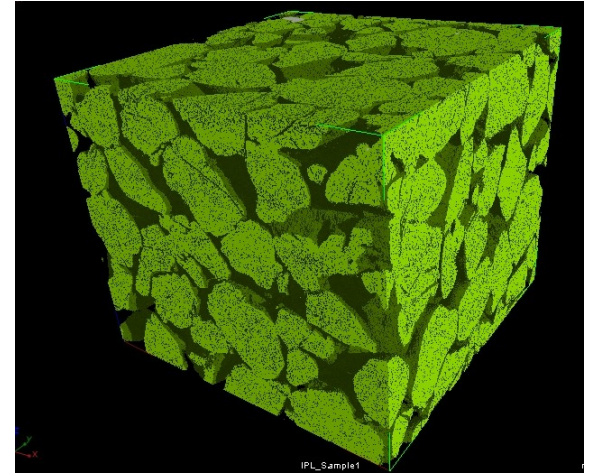
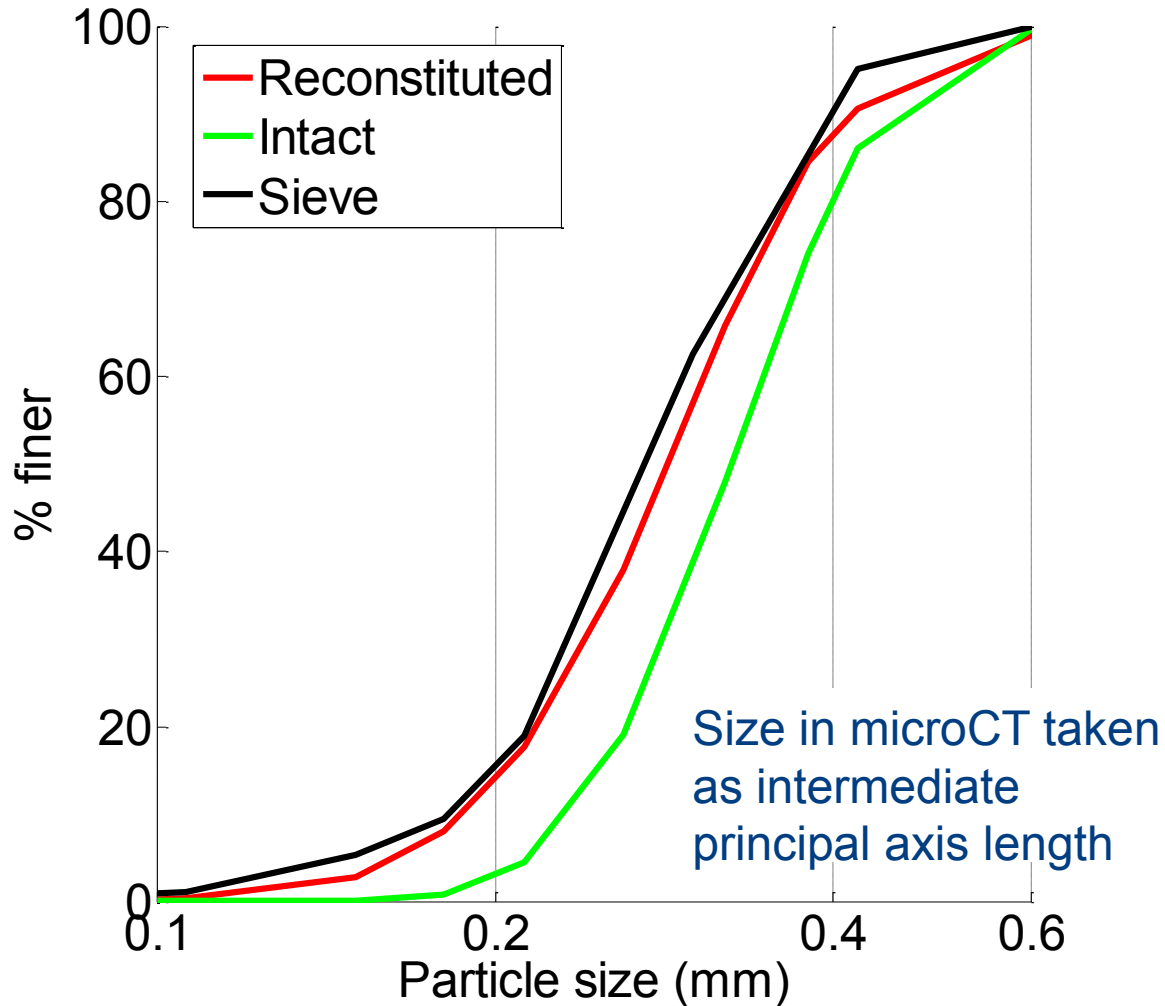
μ CT Imaging Points



MicroCT Images

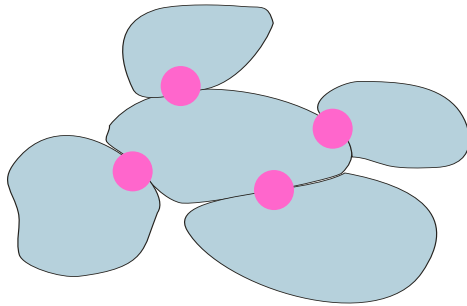


Particle Size Distributions from microCT

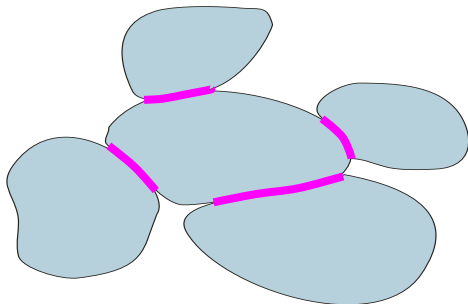


Scalar Fabric Description – Contact Density

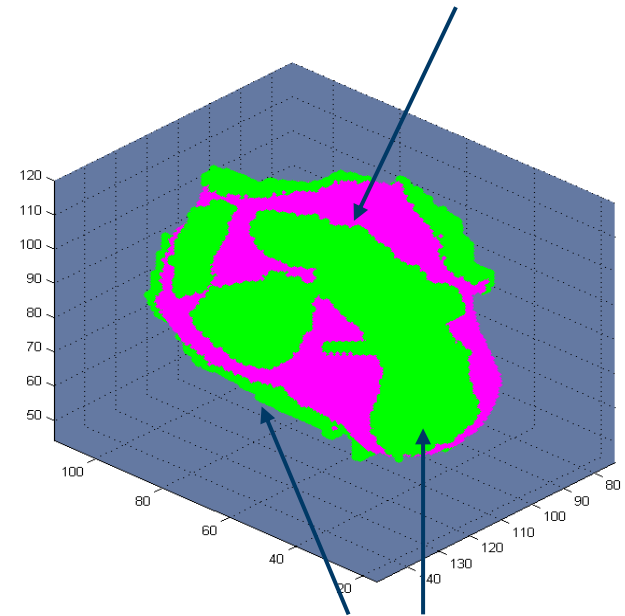
Coordination Number (N_c):
Number of contacts per particle



Contact Index:
Contact area
Particle surface area



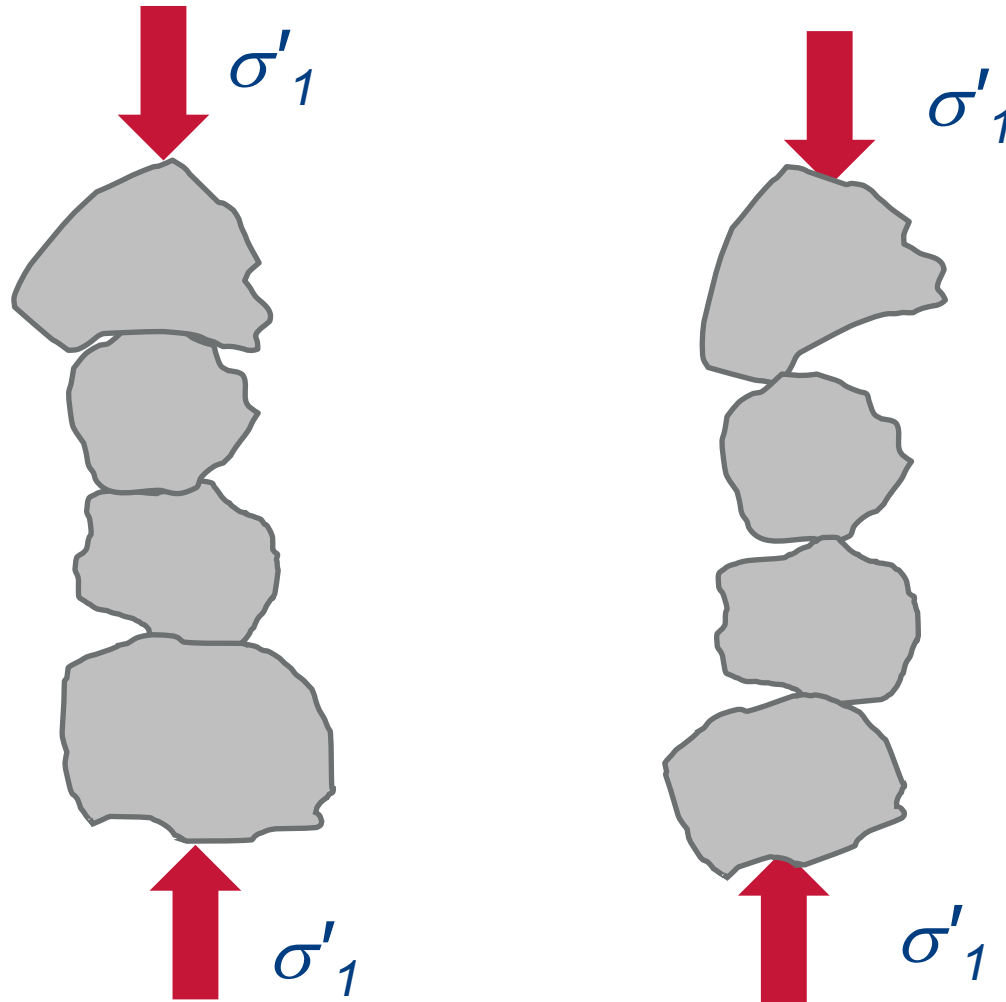
Representative Particle



Contacts

Influence of Contact Area on ϕ'_p

- Force chains more stable when contact area is greater



Particle Scale Analyses of Locked Sand

- Triaxial test data confirmed difference in response of intact and reconstituted materials tested at same void ratio
- Micro CT data revealed a difference between particle size distributions
- At the same void ratio the intact material had a higher coordination number and contact index than reconstituted material
- Contact index better discriminates two materials

Tools of Particulate Soil Mechanics

DEM

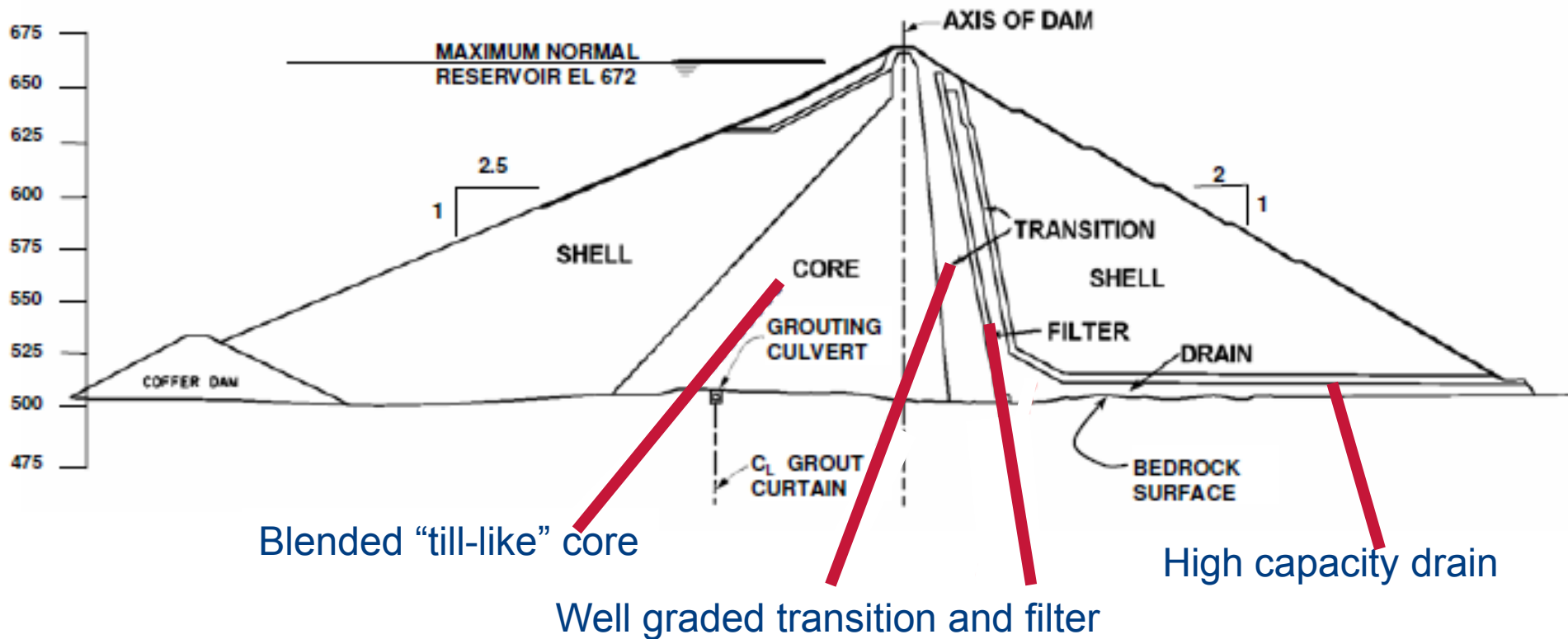
- Model with inherent idealizations - simplified geometries and contact models
- Information on contact forces and kinematics

microCT

- Can observe topology of real materials
- No information on contact forces or particle velocities

Internal Stability of Dam Filters

- WAC Bennett Dam
- British Columbia, Canada
- 183 m high
- Zoned earthfill dam
- Constructed 1968



*BC Hydro as cited by
Muir Wood (2007)*

Internal Instability

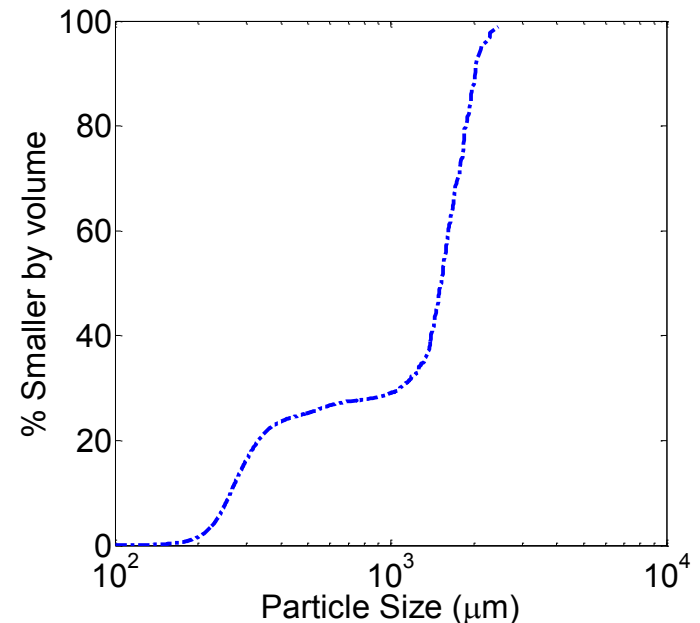
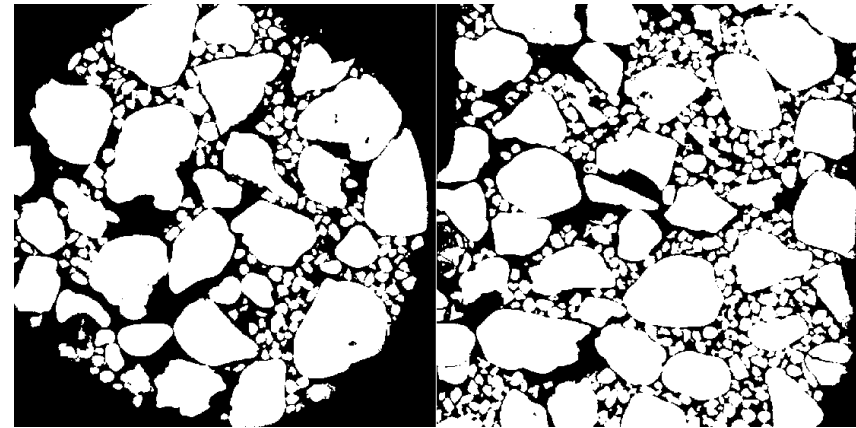
- Internal instability occurs where “sand can migrate within the interstices of a framework or primary fabric formed predominantly of the gravel particles”
- “Sandy gravels having a bimodal (gap-graded) grain-size distribution with less than about 25% of sand exhibit internal instability”
- “Such materials can present serious problems where piping is possible, notably in dams and barrages.”

Quotes from *Skempton and Brogan (1994) Géotechnique*

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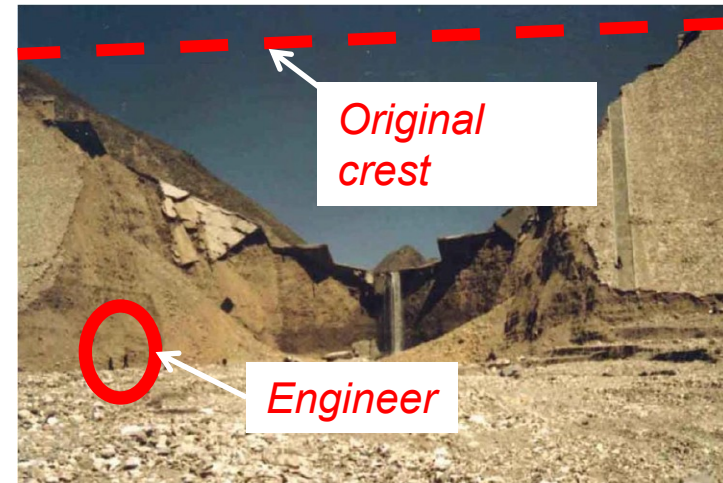
Fonseca et al. (2014) Géotechnique

Internal Instability

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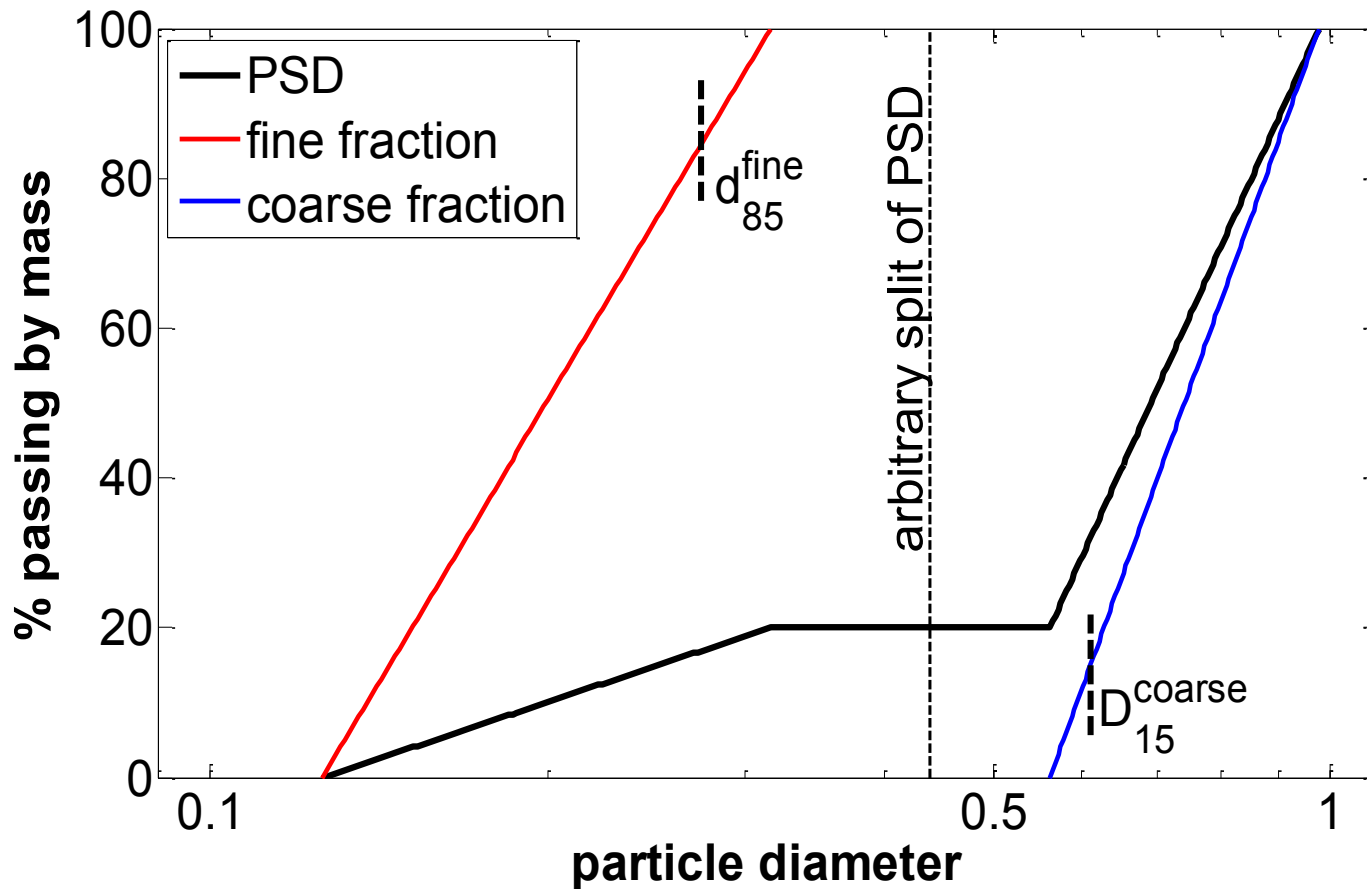
Quotes from *Skempton and Brogan (1994) Géotechnique*

1996 Sinkhole at WAC Bennett Dam
(BC Hydro as cited by Muir Wood, 2007)



Gouhou Dam, China (1993)
71m High; 300 Deaths
(Chang, 2011)

Empirical Filter Criteria: Kézdi (1979)



Relates to Terzaghi filter rule

Split PSD into coarse and fine “PSDs”

Stable if: $d_{85}^{fine} > (D_{15}^{coarse} / 4)$

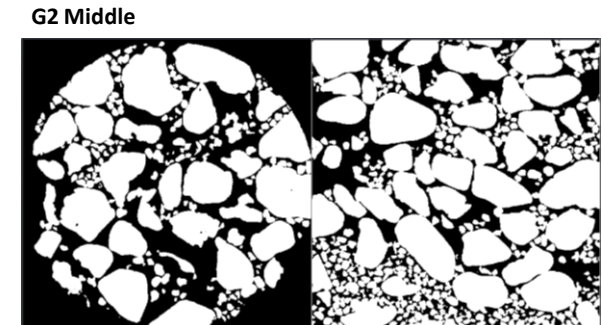
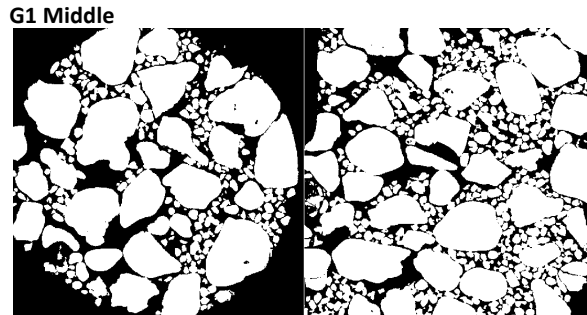
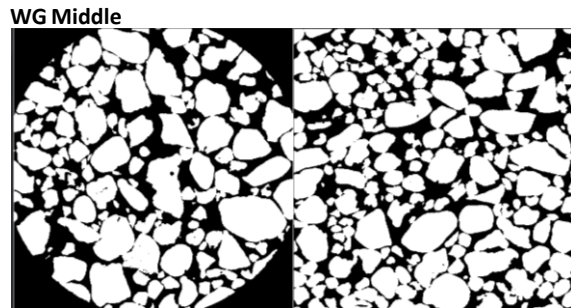
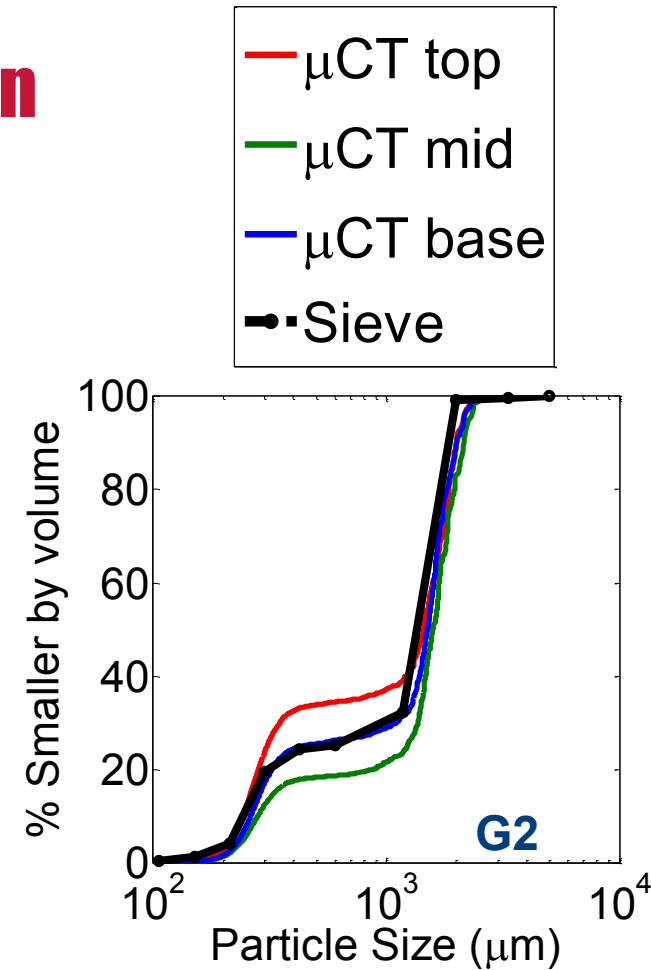
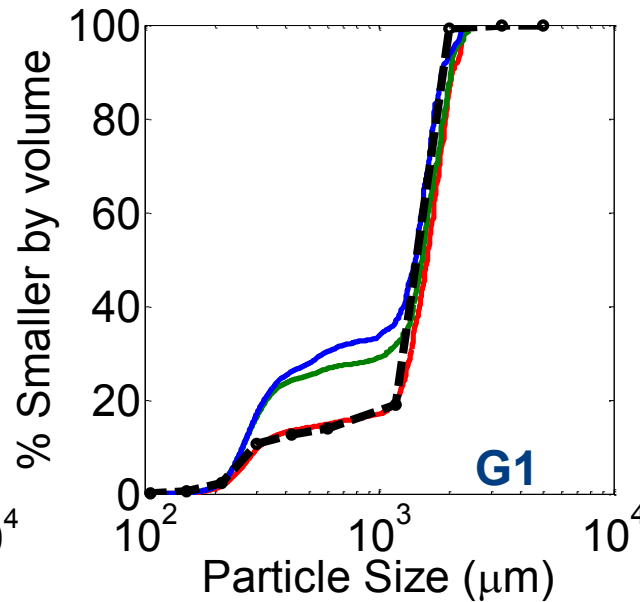
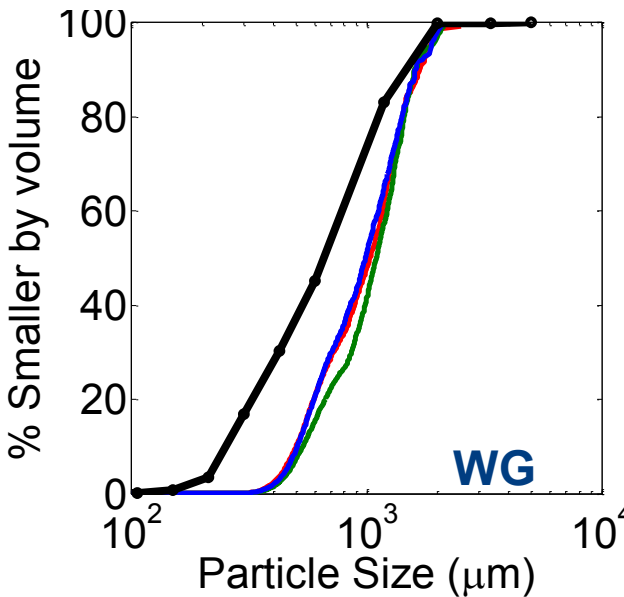
i.e. if

$D_{15}^{coarse} / d_{85}^{fine} < 4$

Particle-Scale Basis for Kézdi Criterion

Samples K_0 consolidated to
 $p'_v=300\text{kPa}$
Voxel size 10 mm
($0.005d_{50}-0.01d_{50}$)

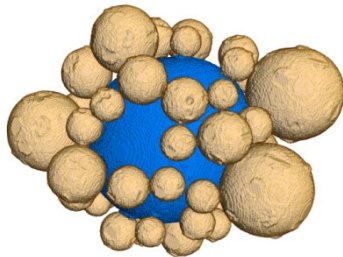
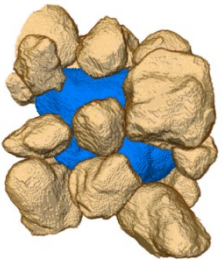
Increasing Kézdi no.
→
Decreasing stability



Variation in Coordination Number with Kézdi Ratio

N_c = Coordination number

No of contacts per particle



Leighton Buzzard Sand
Blue particle
20 contacts

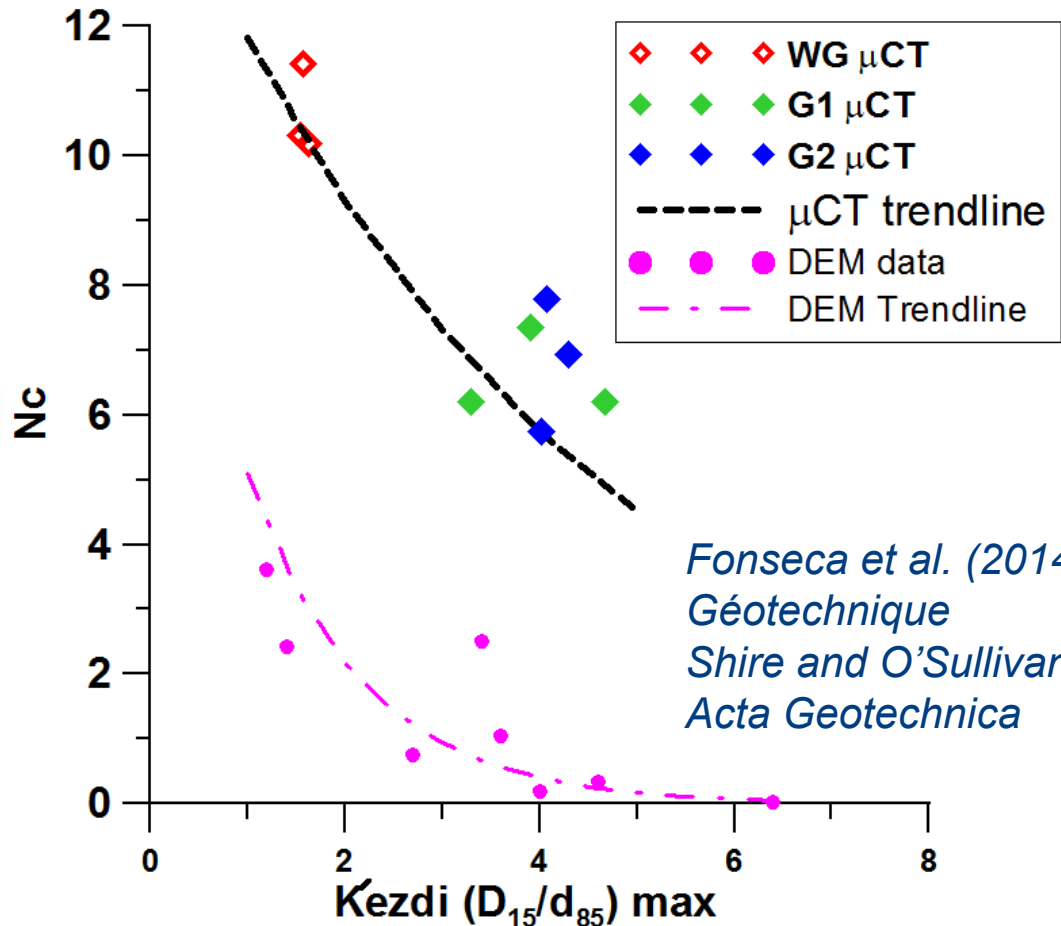
Glass beads
Blue particle
50 contacts

Images from H. Taylor

Increasing Kézdi no.



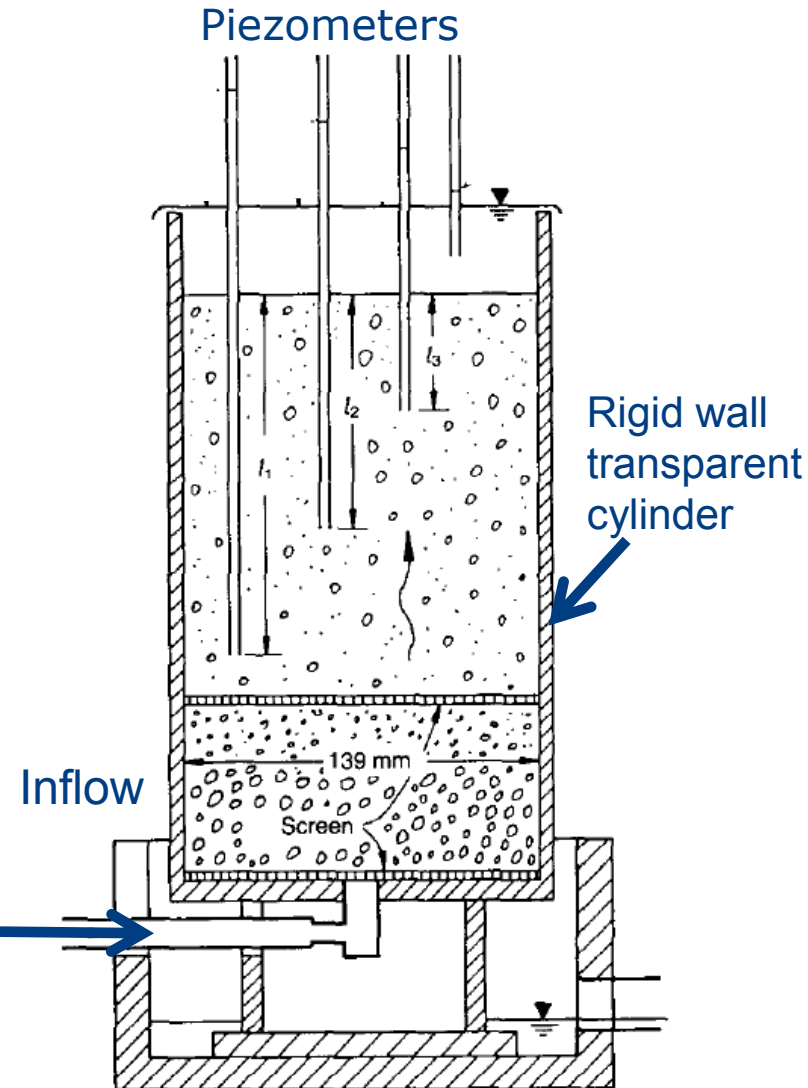
Decreasing stability



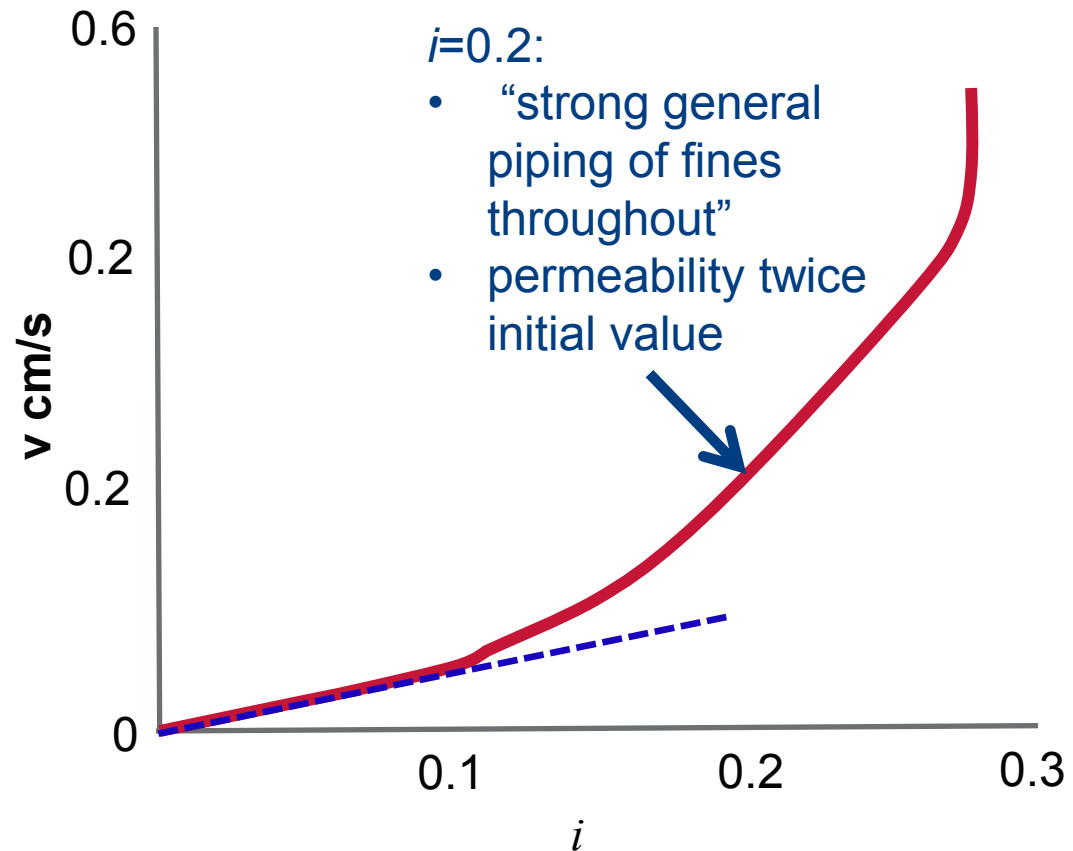
Fonseca et al. (2014)
Géotechnique
Shire and O'Sullivan (2013)
Acta Geotechnica

Skempton and Brogan Permeameter Experiments

Skempton and Brogan (1994)
Géotechnique



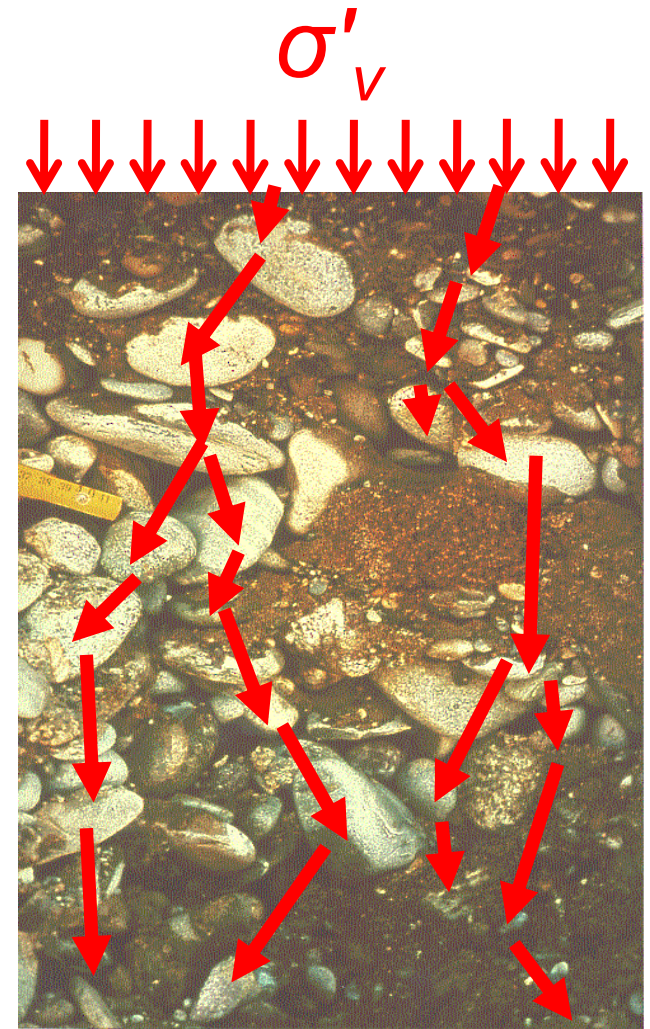
Sample A



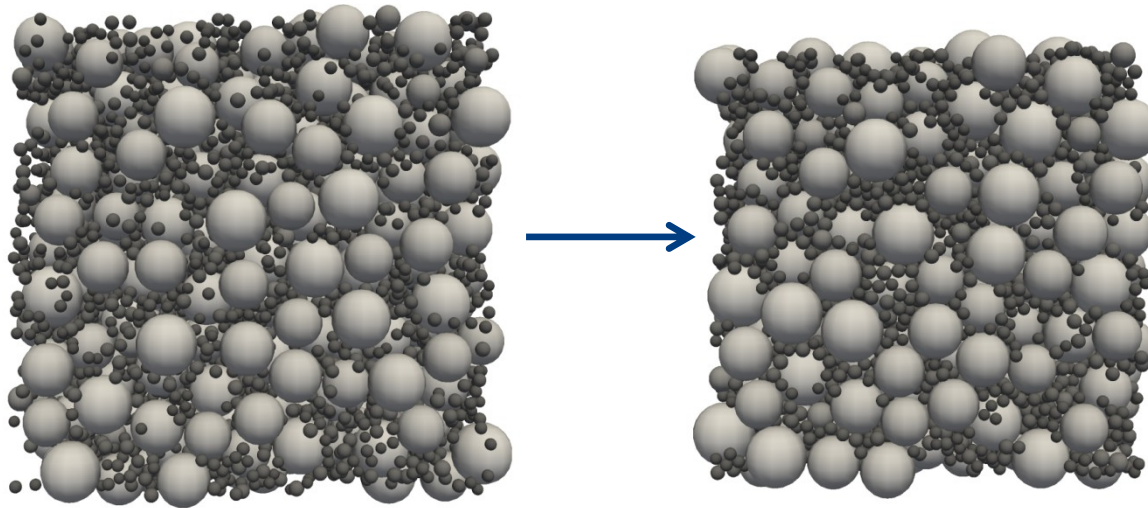
Stress Partition - α

Micromechanical hypothesis to explain $i_{crit} < -1$:

- Coarse matrix transfers overburden
- Fines carry reduced effective stress:
 - $\alpha = i_{crit} / i_{crit(heave)}$
 - $\sigma'_{fines} = \alpha \times \sigma'$



DEM Simulations to Investigate Instability



- DEM code granular LAMMPS with periodic boundaries
- Isotropic compression at to $p' = 50\text{kPa}$
- Sample density controlled using interparticle friction (μ):

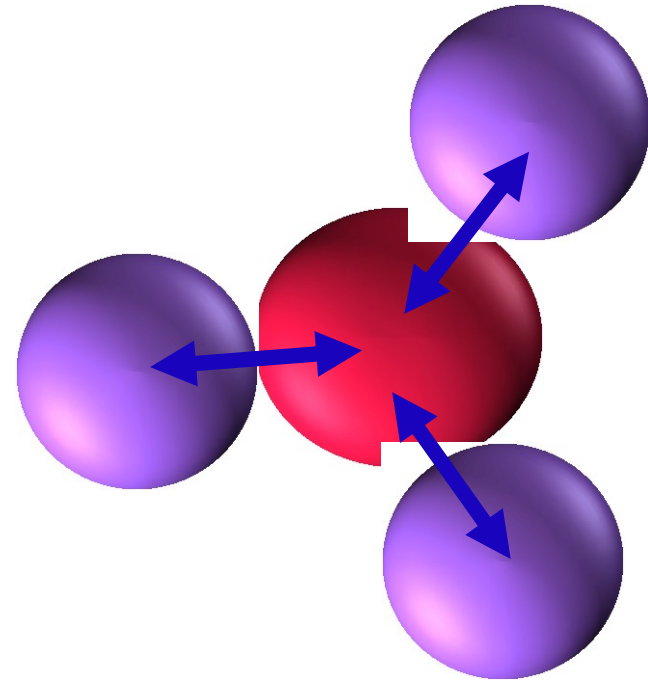
$\mu = 0.0$ (Dense)

$\mu = 0.1$ (Medium dense)

$\mu = 0.3$ (Loose)

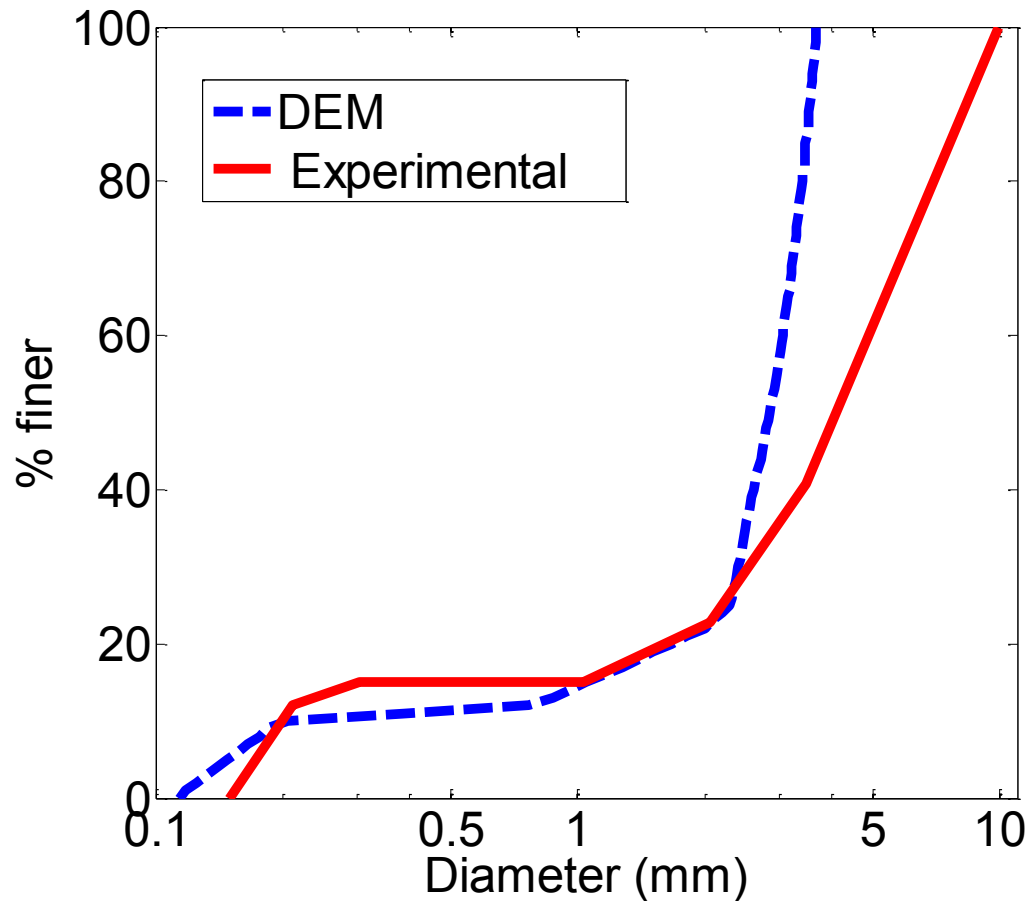
α – DEM Calculations

$$\alpha = \frac{p'_{fine}}{p'}$$



- p' =overall mean effective stress
- p'_{fine} =mean effective stress in finer fraction
- p' and p'_{fine} can be directly obtained from a summation of contact forces in DEM

Skempton and Brogan Sample A: Comparison of α Values

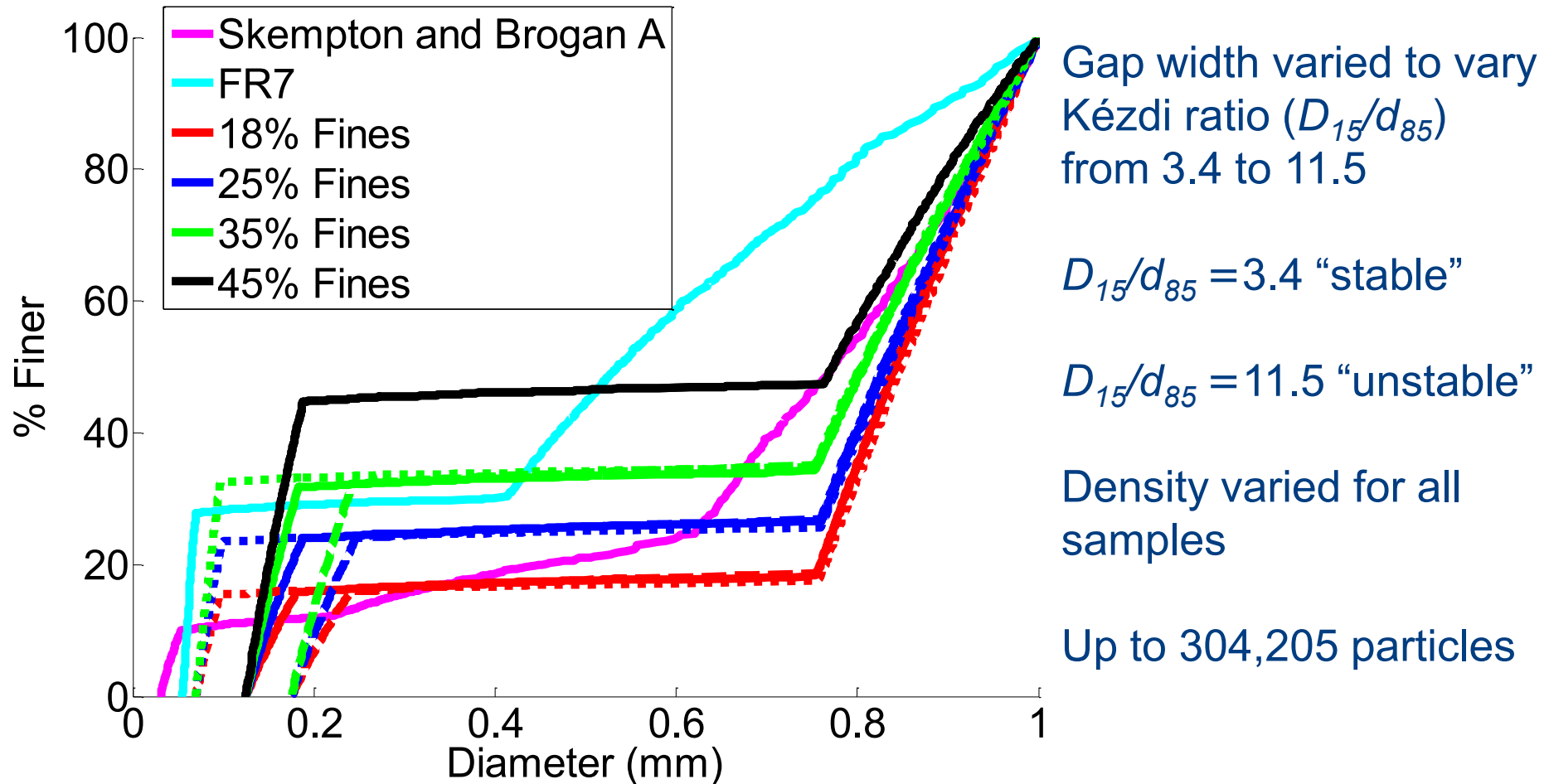


Density	α_{DEM}
Loose	0.15
Medium	0.06
Dense	0.04

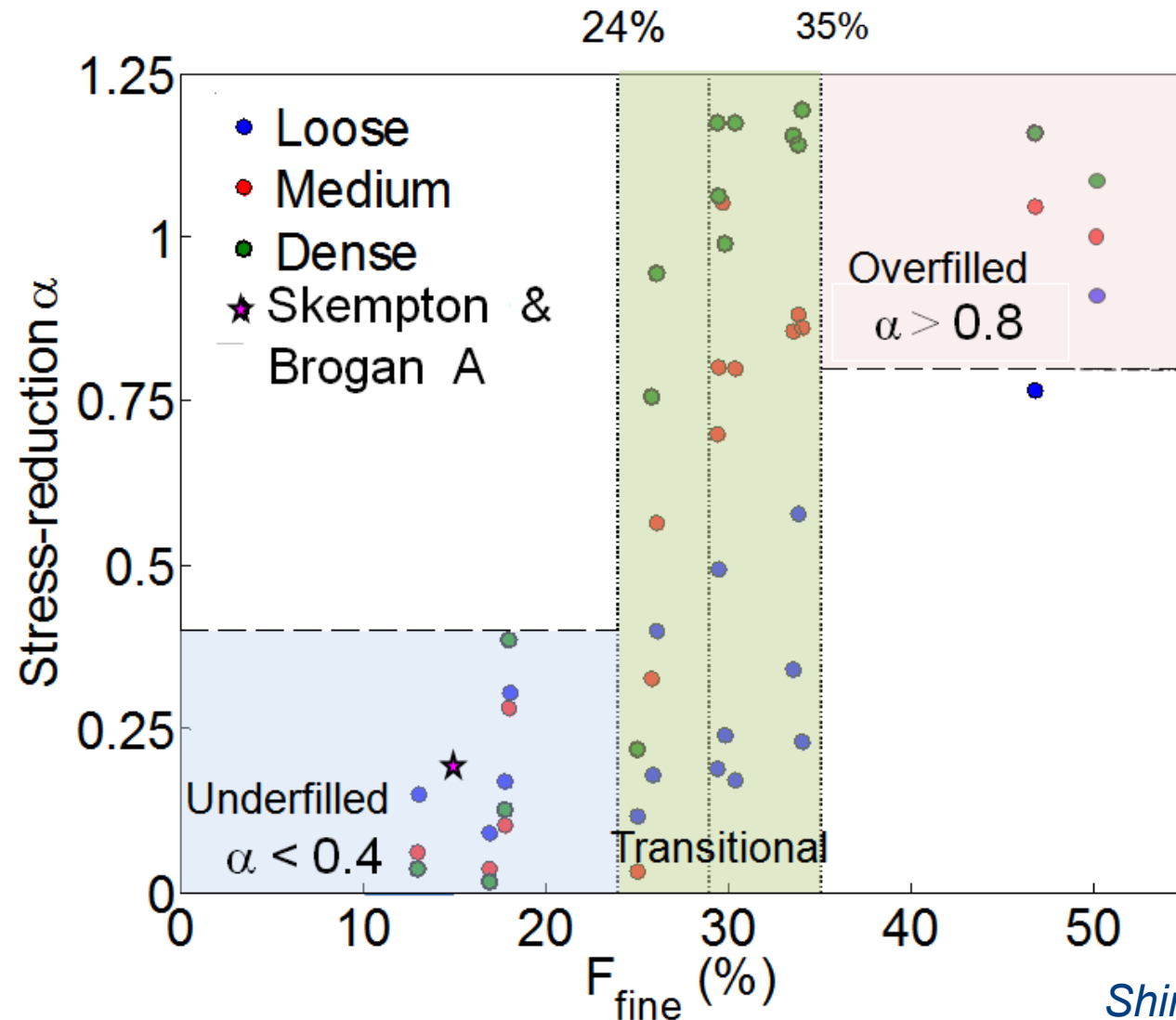
$$\alpha_{experiment} = 0.18$$

Experimental sample placed moist with no densification

Parametric Study on Effect of Gap Width and Fines Content



Variation in α with Fines Content (F_{fine})

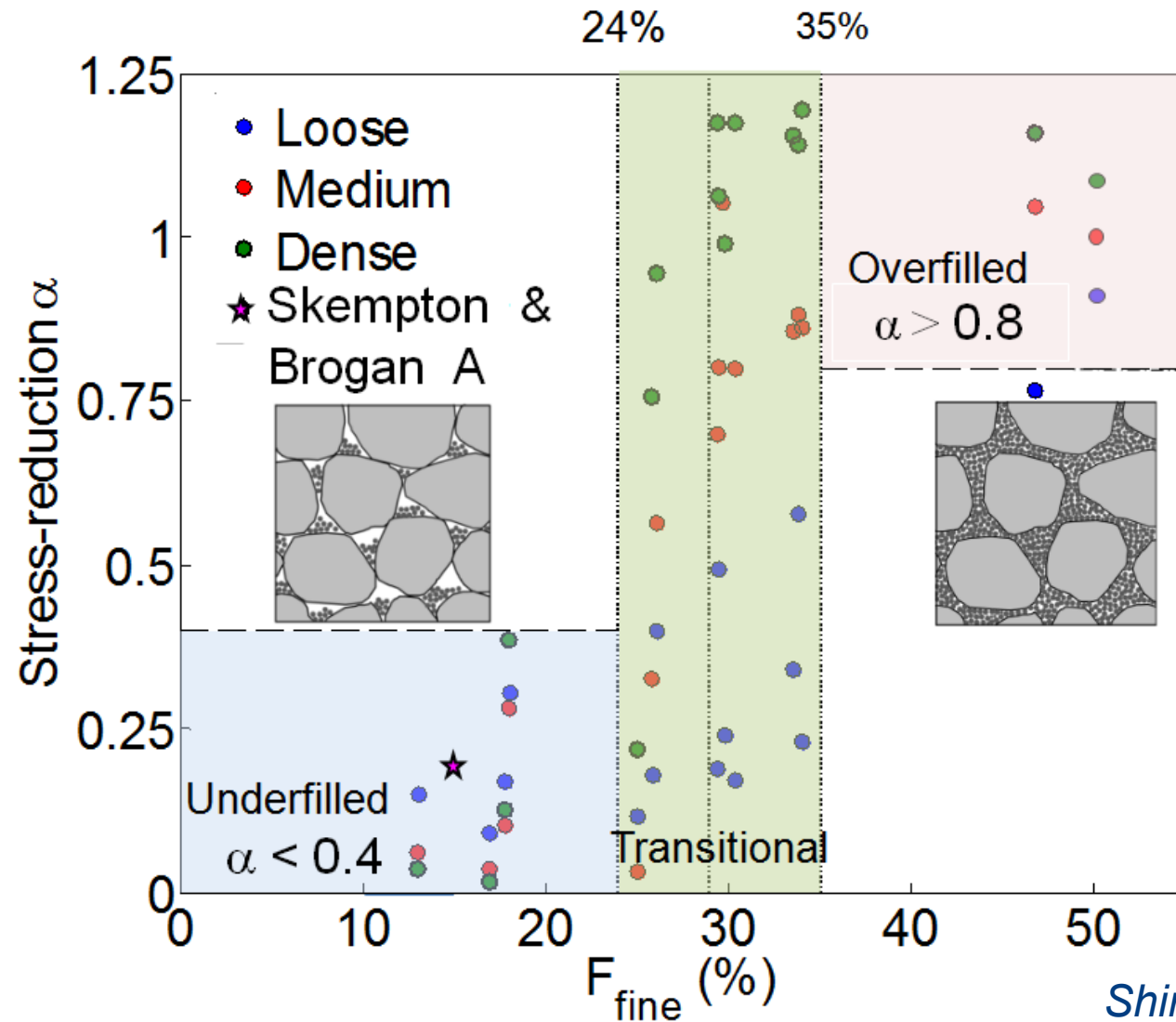


Skempton and Brogan(1994):

Critical fines content where fines just fill voids: $F_{fine} = 24-29\%$

Finer fraction separates coarse fraction particles: $F_{fine} = 35\%$

Variation in α with Fines Content (F_{fine})



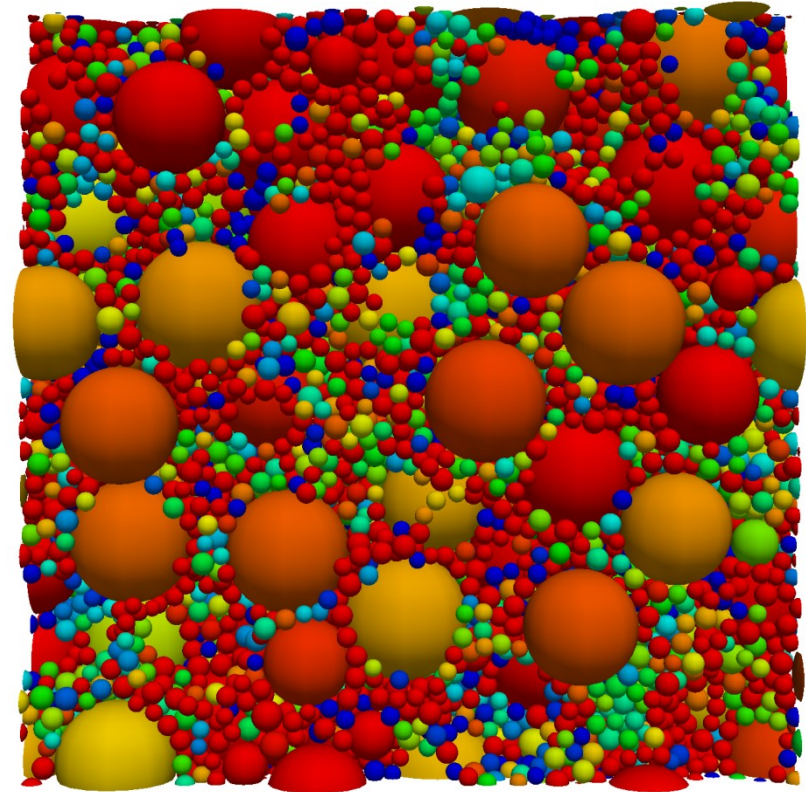
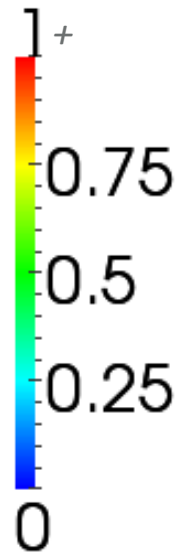
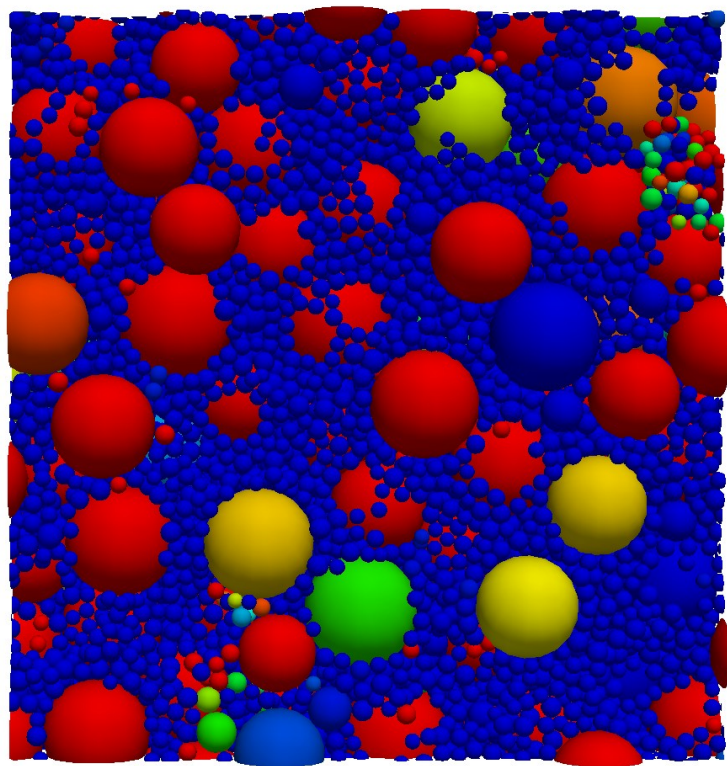
Skempton and Brogan(1994):

Critical fines content where fines just fill voids: $F_{fine} = 24-29\%$

Finer fraction separates coarse fraction particles: $F_{fine} = 35\%$

Visualization of Particle Stresses

Particle Alpha



30% Fines - Loose

→
Relative density increase

30% Fines - Dense

Particle-Scale Analysis of Internal Instability

- Particle scale analyses (microCT and DEM) link between the Kézdi criterion and fabric
- DEM analyses confirm Skempton and Brogan's α hypothesis
- DEM analyses confirm Skempton and Brogan's delineation of underfilled and overfilled fabrics
- DEM analyses highlight the sensitivity of transitional materials to relative density

A Particulate Perspective on Soil Mechanics Can...

- Reveal soil behaviour under stress states that cannot be attained in experiments
- Explain perplexing soil behaviour
- Provide scientific rationale for empirical results used in design

Tools:

Discrete Element Method (DEM)

Micro Computed Tomography

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Huang, X., O'Sullivan, C., Hanley, K. J., Kwok, C.Y. (2014) "DEM Analysis of the State Parameter", *Géotechnique* 64(12) 954-965

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