<u>Maulik–Okounkov</u>: Constructed $Y_Q$

Yangian, act on equivariant cohomology.

$$\bigoplus_{\alpha} H^*_{G_\beta}\Big(\underbrace{\mathfrak{M}_{\lambda,\theta}(Q,\alpha,\beta)}_{\text{"}H(\alpha)\text{"}}\Big) \circlearrowleft Y_Q$$

FRT construction.

Roughly: Yangian is an enhancement of KM alg.

It is a Hopf algebra, enhancing $U\mathfrak{g}$.

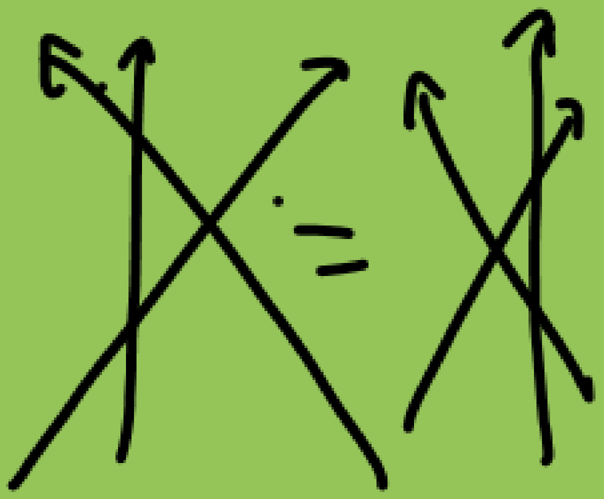
— $U\mathfrak{g}$-mod are monoidal cat under $\otimes$
(mods over group)

$\otimes$ is encoded by $\Delta : U\mathfrak{g} \to U\mathfrak{g} \otimes U\mathfrak{g}$

"quasitriang": have R-matrix $\Rightarrow$ $\otimes$ "braided"

QYBE: $R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$

$R : V \otimes V \to V \otimes V$



NOT necessarily "unitary"

So $V\otimes W \mapsto W\otimes V$ give action of pure **braid** group $B_n$ on $(V_1\otimes\cdots\otimes V_n)$

**Rmk:** Braid eqn is actually $S_{12}S_{23}S_{12}=S_{23}S_{12}S_{23}$

$S = R\circ\sigma$.

$\sigma(V\otimes W)=W\otimes V$.

Maulik–Okounkov: $A$ torus acting on $X$

$\mathrm{Stab}_{\mathcal{C}} : H_A^{\cdot}(X^A) \longrightarrow H_A^{\cdot}(X)$ (opp of restr.)

$\mathcal{C} \subseteq \mathrm{Lie}\, A =: \sigma$ "chamber"

component of $\sigma \setminus$ hyperplanes given by wts of $N_{X^A \subseteq X}$.

. Given $\mathcal{C} \subseteq \sigma$, define "$\mathcal{C}$-stable" $x \in X$.

if $\lim_{z \to 0} \sigma(z) \cdot x$ exists $\in X^A$, $\sigma \in \mathcal{C}$.

$\mathcal{C}$-stable decomposes into components:

attracting loci of components of $X^A$.

Given $\gamma \in H^*(X^A)$, can define $\mathrm{Stab}_{\mathcal{C}}(\gamma)$,

supported in attr. locus of $\gamma$,

restricted to $Z \ni \gamma$, cup with $e(N_-)$.

Thm $\quad H(\alpha) \times H(\alpha') \xrightarrow{\oplus} H(\alpha + \alpha')$

im = A-fixed points

$$R_{e,e'}(u) := \mathrm{Stab}_{e'}^{-1} \circ \mathrm{Stab}_e \in \mathrm{End}(H_A^*(X^A))[\mathfrak{a}^{-1}]$$

$$u \in \mathfrak{a}^*$$

Sat. <u>QYBE</u>.

FRT: How to <u>rebuild</u> $^{\otimes}$cat. (Hopf alg)

from $R \circ \sigma : V \otimes W \to W \otimes V$

$$Y_Q := \mathrm{Span} \underbrace{\mathrm{tr}_V((M \otimes 1) R)}_{T(M,u) :=}, \quad M \in \mathrm{End}\, V$$

$$R : V \otimes W \to V \otimes W.$$

- $Y_\Omega$ acts on cohomology
- $Y_\Omega \supseteq \mathfrak{g}_{KM}$ when no loops.
- $Y_\Omega$ has "Bethe" or "Baxter"

commutative subalgebra, by fixing

$$M \in Z(\text{all } R \text{ matrices}) \xrightarrow{\text{Conj}} \text{Give action of } \underline{\text{quantum}} \text{ cup product.}$$

$$M(R \otimes 1) = (R \otimes 1)M.$$

Reln to Gabriel / Kac:

$\Pi_Q$-mod, unlike Rep $Q$, have nicer reflection functor: (because to reflect in $Q$, reverse orientations)

$\rho$ = rep of $\Pi_Q^\lambda$, $\lambda \in \mathbb{C}^{Q_0}$,

$\lambda_i \neq 0$ some $i$

$\rightsquigarrow$ Refl equiv $\mathrm{Rep}_\alpha \Pi_Q^\lambda \xrightarrow{\sim} \mathrm{Rep}_{s_i\alpha} \Pi_Q^{s_i^\vee \lambda}$.

CB explained:

$\mathrm{Indec}(Q) \overset{\text{extend}}{\rightsquigarrow}$ rep of $\Pi^{\lambda}_{Q}$
suitable $\lambda$.

$\overset{\text{refls}}{\rightsquigarrow}$ dim $= e_i$ OR $\in \mathcal{F}$

$\Rightarrow$ dim(indec) is a pos. root.

$\exists!$ rep of dim $e_i$: all arrows $0$. (no loops if $(e_i, e_i) = 2$)