Practical Strategic Mine Planning

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Mining as a staged process

Geological Exploration
From geostatistics to strategic mine planning (inputs)

Mining Extraction
The focus of strategic mine planning

Mineral processing
Crushing, grinding, flotation, leaching, smelting and refining (constraints)
Objectives of strategic mine planning

Multiple and conflicting objectives:

- Annual metal production
- KPIs – increased recovery, mass mined/milled
- Reduce costs
- Increase mine life
- Increase reserves
- Safety

Overarching objective:

- to increase the shareholder value through responsible scheduling of resources (to maximise the cash flow within the long term NPV context)

“create long-term shareholder value” (BHPBilliton, 2015)
“deliver strong and sustainable shareholder returns” (Rio Tinto, 2015)
“deliver superior returns” (Newcrest 2015)
Cost reduction: This type of analysis is normally constrained to only reduce the costs by 10% or within some other production constraints.

Internal rate of return (IRR): We can often get an infinite IRR by not spending any money on such items as waste stripping and maintenance. This tends to bias the results in favour of short term, low cost projects.

Ore resources and reserves: Although we can often keep drilling until we find more resources or keep lowering the cut-off grade to increase the reserves, they will eventually not break even.

Cash flow for the budgeting period: this addresses many important costs, revenues and constraints of the project and is a significant advantage over simple cost reduction. The thing that this ignores is the long term impact of decisions made within the budgeting periods.
Net present value (NPV)

Formulation of the NPV of a project:

\[ NPV = \sum_{n=0}^{N} \frac{F_n}{(1+i)^n} \]

The cash flow function (revenue - costs) is defined as:

\[ F_n = M_n [(s - r) \cdot \bar{g}_n y - m - p - o] \]

The NPV equation can be rearranged into:

\[ NPV = \sum_{n=0}^{N} \frac{M_n [(s - r) \cdot \bar{g}_n y - m - p - o]}{(1+i)^n} \]

The NPV can be considered as:
- Sum of the discounted dividends
- Sum of all the assets
Net present value (NPV)

NPV can be maximised by:
- Changing production rate
- Changing cut-off grade
Overview

Constraints in mining operations

Constraints are often called “opportunity costs” in financial terminology.

- The sequence of mine production: geotechnical slope constraints, preservation of protective areas;
- Capacity of mining equipment: truck hours, shovel mass, phase sinking limits, undercutting;
- Capacity of processing plant: conveyor mass, SAG time, Sulphur mass, concentrate mass;
- Capacity of marketing: metal production and sales constraints;
- Safety and environment.
Overview

Constraints in mining operations: examples

Optimisation of slope angle

Recovery of pillars in room and pillar method
Constraints in mining operations: examples

Delineation of protective areas
A series of analytical, experimental and numerical work on the dynamic stress state around underground cavities by Tao et al. (2017, 2019, 2020) indicated that mine openings are more susceptible to damage under engineering dynamic disturbances, therefore, more mine support and barrier pillars which compromise ore reserves are required in mine planning.
What components do you have significant control over?

<table>
<thead>
<tr>
<th></th>
<th>IMPACT ON NPV</th>
<th>CONTROLABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate</td>
<td>Negligible/significant/large</td>
<td>Negligible/significant/large</td>
</tr>
<tr>
<td>Revenue</td>
<td>Negligible/significant/large</td>
<td>Negligible/significant/large</td>
</tr>
<tr>
<td>Costs</td>
<td>Negligible/significant/large</td>
<td>Negligible/significant/large</td>
</tr>
<tr>
<td>Constraints</td>
<td>Negligible/significant/large</td>
<td>Negligible/significant/large</td>
</tr>
</tbody>
</table>

- **Constraints** normally represent the largest controllable value driver.
- Companies need to put significant energy into both cost reduction and elimination of bottlenecks.
- The **time value** of ore: scheduling the first five years of the resource would be more important than determining the ultimate pit shell limits of a 25-year project.
Pros & cons of working with NPV

Benefits:
- NPV includes a balanced valuation of both short-term value (which receives very little discounting) and longer-term value (which receives greater discounting), making it an appropriate instrument for the commercial valuation of most businesses.

Limitations:
- Risk is not incorporated into a single NPV other than in the discount rate (e.g., surface and underground risk, an existing project compared to a greenfield project, a new country, a new ore body, new processing techniques).
- Risk exists in the field of real options.
- The NPV is rarely a true net present value, it is normally a present value (e.g., not including capital costs, all the resources, realistic costs and constraints).

=> The reason why we need integrated optimisation!
Tactical mine scheduling

Tactical mine scheduling procedure

- **Resource evaluation:** Using a variety of geostatistical techniques with assumptions about mine cut-off grades, spatial grade relationships and selective mining unit dimensions.

- **Ultimate pit design:** Determining the ultimate pit shell to define the scheduling universe, and optimising steady cut-off grade.

- **Pushback design:** Finding the block extraction sequence which produces the best net present value (NPV) whilst satisfying the geotechnical slope constraints. Designing the practically minable mine phases (pushbacks) which are roughly based on the optimal block sequence.

- **Dynamic cut-off grade optimisation:** Optimising the mining schedule and cut-off grades within a set of business and operational constraints. The NPV of this ‘optimal’ schedule is considered as a main criterion of the economical viability of the project. This step requires mine schedule inputs such as pushbacks and their grade-tonnage distribution.
Tactical mine scheduling

Ultimate pit design (UPD)

Block model

Directed graph representing vertical section

Non-cone-based patterns in 3D
The principal focus of an open pit mine design process is to produce a NPV that is positive and robust enough to support the development of the mine and to achieve a high resource recovery.
Pushback design (PBD)

Strategies to design NPV-maximising pushbacks:

- **Ultimate pit limit-based approach**: generate NPV-maximising pushbacks after defining an ultimate pit limit.
- **Comprehensive approach**: simultaneously determine pushbacks and the ultimate pit under constraints (e.g., mixed integer linear programming).
Strategies for cut-off grade optimisation

- Steady cut-off grade optimisation strategy
- Dynamic cut-off grade optimisation strategy: The inputs are a ‘NPV-maximising’ mine schedule based on pushbacks and its material ranking (e.g. grade-tonnage distribution); the output is an adjusted mine schedule additionally maximising project’s NPV.

To illustrate the complexity of dynamic cut-off grade optimisation, consider a simple cut-off grade example:

- Single sequence
- 10 increments
- 62 cut-off grades

\[62^{10} = 10^{17}\] valid options
Dynamic cut-off grade optimisation

Overview

Maximising the NPV of a mine is equivalent to find the optimum exploitation track throughout the life of the mine.
Theoretical formulation

\[ V(T, R) = \max_{0 \leq r \leq R} \left[ r \cdot c(\omega, t) + \frac{V(T + t, R - r)}{(1 + \delta)^t} \right] \]

**V(T,R)** = Maximum value ($M$) at time $T$ with an amount of remaining resource $R$.

$r$ = A small increment of remaining resource ($\text{tonnes}$)

$c$ = Cash flow rate, or cash flow for a unit of resource ($\$/\text{t}$)

$\omega$ = An operating policy (such as cut-off grade) [Unknown]

$t$ = Time taken to mine an increment of resource (years)

$\delta$ = Discount rate (or cost of capital)
Dynamic cut-off grade optimisation

Theoretical formulation

If \( t \) and \( r \) are small, we could approximate the value once the resources consumed using the following relationship.

\[
V[(T + t), (R - r)] = V(T, R) + \frac{t.dV}{dT} - \frac{r.dV}{dR}
\]

The first order of approximation on the binomial theorem will also become useful.

\[
\left( \frac{1}{(1 + \delta)^t} \right) = (1 + \delta)^{-t} = (1 - \delta)
\]

Combining these three equations.

\[
V(T, R) = \max_{\omega \in \mathcal{S}_R} \left[ \sum \omega \left( c(\omega, t) + \left[ V(T, R) + \frac{t.dV}{dT} - \frac{r.dV}{dR} \right] (1 - \delta t) \right) \right]
\]

Multiply through by \((1 - \delta t)\).

\[
V(T, R) = \max_{\omega \in \mathcal{S}_R} \left[ r.c(\omega, t) + V(T, R). (1 - \delta t) + \frac{t.dV}{dT} (1 - \delta t) - \frac{r.dV}{dR} (1 - \delta t) \right]
\]

\[
V(T, R) = \max_{\omega \in \mathcal{S}_R} \left[ r.c(\omega, t) + V(T, R) - V(T, R). \delta t + \frac{t.dV}{dT} \delta t - \frac{r.dV}{dR} \delta t + \frac{r.dV}{dR} \right]
\]
Dynamic cut-off grade optimisation

Theoretical formulation

Assume that \[ \frac{t.dV}{dT}.\delta t \] and \[ \frac{r.dV}{dR}.\delta t \] are negligible as both \( r \) and \( t \) are assumed very small numbers.

\[
V(T, R) = \max_{\omega} \left[ r.c(\omega, t) + V(T, R) - V(T, R).\delta t + \frac{t.dV}{dT} - \frac{r.dV}{dR} \right]
\]

Subtract \( V(T,R) \) from both sides of the equation.

\[
0 = \max_{\omega} \left[ r.c(\omega, t) - V(T, R).\delta t + \frac{t.dV}{dT} - \frac{r.dV}{dR} \right]
\]

Divide both sides of the equation by ‘\( r \).’

\[
0 = \max_{\omega} \left[ c(\omega, t) - V(T, R).\frac{\delta t}{r} + \frac{t.dV}{r.dT} - \frac{dV}{dR} \right]
\]

Now add \[ \frac{dV}{dR} \] to both sides of the equation.
Dynamic cut-off grade optimisation

Theoretical formulation

\[
\frac{dV}{dR} = \max_{0 \leq r \leq R} \left[ c(\omega, t) - V(T, R) \cdot \frac{\delta t}{r} + \frac{t}{r} \cdot \frac{dV}{dT} \right]
\]

Simplify by pulling common factor \[\frac{t}{r}\] outside of the equation.

\[
\frac{dV}{dR} = \max_{0 \leq r \leq R} \left[ c(\omega, t) - \frac{t}{r} \left\{ V(T, R) + \frac{dV}{dT} \right\} \right]
\]

Define common terms:

If the time taken to process per resource increment \( \tau = \begin{bmatrix} t \\ r \end{bmatrix} \)

The ‘Opportunity Cost’ can then be defined as \( F = \frac{\delta V(T, R) + \frac{dV}{dT}}{\frac{dV}{dT}} \)

Thus, the way to maximise the value as the resources depleted can be simplified to the following equation:

\[
\frac{dV}{dR} = \max_{0 \leq r \leq R} \left[ c(\omega, t) - F \tau \right]
\]
**Dynamic cut-off grade optimisation**

**Ranking: identifying the best material for each process**

*Question:* Materials need to be ranked before determining the cut-off grade. What is the best ranking factor to rank materials?

**Ranking factors:**

- **Ore grade (%/t)**
- **In-situ value ($/t)**
- **Net value ($/t)**
- **Net value with haulage corrections ($/t)**
- **Net value with haulage and hardness corrections ($/t)**

\[ \text{In-Situ Value}[/t] = \text{Copper Grade}[/\%] \times \text{Copper Price}[/$/t] + \text{Gold Grade}[oz/t] \times \text{Gold Price}[/$/oz] \]

\[ \text{Net Value}[/t] = \text{Recovered Revenue}[/$/t] - \text{Mining Costs}[/$/t] - \text{Processing Costs}[/$/t] \]

\[ \text{NVHaul}[/$/t] = \text{Net Value}[/$/t] + (\text{Ore Haul Costs}[/$/t] - \text{Waste Haul Costs}[/$/t]) \]

\[ \text{NVCorrected}[/$/t] = \text{NVHaul}[/$/t] + \frac{1}{365[d/y]} \times \left( \frac{\text{Fixed Cost}[/$/y]}{\text{Average Throughput}[t/d]} - \frac{\text{Fixed Cost}[/$/y]}{\text{Throughput}[t/d]} \right) \]

\text{Expected unit cost} \quad \text{Actual unit cost}
Dynamic cut-off grade optimisation

Ranking: identifying the best material for each process

Ranking factors:

- Cash Flow Grade ($/hr)

\[ G_{\text{CashFlow}}[$/hr] = (\text{OreValue}[^$/t] - \text{WasteValue}[$/t]) \times \text{OreProcessingRate}[^t/hr] \]

\[ G_{\text{CashFlow}}[$/hr] = \text{OreValue}[$/hr] - \text{WasteValue}[$/hr] \]

\[ \text{WasteValue}[$/hr] = (\text{WasteMassValue}[^$/t]) \times \text{OreProcessingRate}[^t/hr] + \text{FixedValue}[$/hr] \]

\[ \text{OreValue}[$/hr] = (\text{OreMetalValue}[^$/t] + \text{OreConValue}[$/t] + \text{OreMassValue}[$/t]) \times \text{OreProcessingRate}[^t/hr] + \text{FixedValue}[$/hr] \]

\[ \text{OreMetalValue}[^$/t] = \text{MetalGrade}[^t \text{ metal} / t] \times \text{Metal Price}[^$/t \text{ Recovered Metal}] \times \text{Recovery}[^{t \text{ Recovered Metal}} / t \text{ metal}] \]

\[ \text{OreConValue}[$/t] = \text{MetalGrade}[^t \text{ metal} / t] \times \text{Recovery}[^{t \text{ Recovered Metal}} / t \text{ metal}] \times \text{ConValue}[$/t \text{ con}] \]

\[ \text{OreMassValue}[^$/t] = \text{Mass Value}[^$/t] \]
Ranking: identifying the best material for each process

Ranking factors:

- Cash Flow Grade ($/hr)
  - The benefit of processing material as ore instead of waste can be quantified by simply looking at the difference between the ore and waste cash flows.
  - A cash flow grade of zero (0) indicates that there is no difference in cash flow whether the material is processed as ore or waste (or is mined or not).
  - In process constrained operations, the optimum cut-off grade strategy would always be positive. In a mine constrained operation, the Cash Flow Grade cut-off strategy may drop to just above zero (0).
- Maximise the cash flow used in long-term schedule optimisation.
Dynamic cut-off grade optimisation

Example: underground & open pit ranking using cash flow grades

In working with deposits that schedule both underground and open pit resources, a cut-off grade strategy needs to be developed that is consistent across all resources.

<table>
<thead>
<tr>
<th>Rock</th>
<th>Location</th>
<th>Open Pit</th>
<th>U/G Stoping</th>
<th>U/G Development</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>Ore Revenue</td>
<td>$12.0 /t</td>
<td>$15.0 /t</td>
<td>$25.0 /t</td>
<td>$30.0 /t</td>
</tr>
<tr>
<td>Ore Mining Cost</td>
<td>$1.0 /t</td>
<td>$1.0 /t</td>
<td>$15.0 /t</td>
<td>$15.0 /t</td>
</tr>
<tr>
<td>Ore Processing Cost</td>
<td>$2.0 /t</td>
<td>$2.2 /t</td>
<td>$2.0 /t</td>
<td>$2.2 /t</td>
</tr>
<tr>
<td>Ore Throughput</td>
<td>140 t/hr</td>
<td>80 t/hr</td>
<td>140 t/hr</td>
<td>80 t/hr</td>
</tr>
<tr>
<td>Waste Cost</td>
<td>$1.5 /t</td>
<td>$1.5 /t</td>
<td>$0 /t</td>
<td>$0 /t</td>
</tr>
<tr>
<td>Ore Value ($/t)</td>
<td>$9.0 /t</td>
<td>$11.8 /t</td>
<td>$8.0 /t</td>
<td>$12.8 /t</td>
</tr>
<tr>
<td>Ore Value ($/t) - Waste Value ($/t)</td>
<td>$10.5 /t</td>
<td>$13.3 /t</td>
<td>$8.0 /t</td>
<td>$12.8 /t</td>
</tr>
<tr>
<td>Cash Flow Grade ($/hr)</td>
<td>$1,470 /hr</td>
<td>$1,064 /hr</td>
<td>$1,120 /hr</td>
<td>$1,024 /hr</td>
</tr>
</tbody>
</table>
### Dynamic cut-off grade optimisation

#### Example: illustrative calculations of ranking parameters

Rank 10 rock blocks (A-J) based on different ranking factors. Assume:
- **Copper price**: $0.9/lb
- **Gold price**: $300/oz
- **Mining cost**: $0.5/t
- **Haulage cost**: $100/hr
- **Processing cost**: $2/t
- **Fixed costs**: $20 M/y
- **Processing capacity**: 7884/hr

\[(365 \times 24 \times 90\% \text{ usage})\]
- **Aver. throughput**: 650 t/hr

### Block Properties

<table>
<thead>
<tr>
<th>Block</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper (% Cu)</td>
<td>0.958%</td>
<td>0.925%</td>
<td>0.910%</td>
<td>0.896%</td>
<td>0.881%</td>
<td>0.871%</td>
<td>0.867%</td>
<td>0.865%</td>
<td>0.861%</td>
<td>0.851%</td>
</tr>
<tr>
<td>Gold [oz/t]</td>
<td>0.0189</td>
<td>0.0161</td>
<td>0.0233</td>
<td>0.0179</td>
<td>0.0062</td>
<td>0.0147</td>
<td>0.0113</td>
<td>0.0059</td>
<td>0.0068</td>
<td>0.0064</td>
</tr>
<tr>
<td>Mill Throughput [t/hr]</td>
<td>700</td>
<td>700</td>
<td>625</td>
<td>700</td>
<td>550</td>
<td>700</td>
<td>675</td>
<td>625</td>
<td>625</td>
<td>550</td>
</tr>
<tr>
<td>Copper Recovery (%)</td>
<td>95.7%</td>
<td>93.0%</td>
<td>94.4%</td>
<td>95.7%</td>
<td>94.4%</td>
<td>95.7%</td>
<td>96.0%</td>
<td>94.4%</td>
<td>94.4%</td>
<td>94.4%</td>
</tr>
<tr>
<td>Gold Recovery (%)</td>
<td>75.4%</td>
<td>74.1%</td>
<td>79.1%</td>
<td>74.9%</td>
<td>71.4%</td>
<td>73.5%</td>
<td>68.7%</td>
<td>71.3%</td>
<td>71.7%</td>
<td>71.5%</td>
</tr>
<tr>
<td>Ore Haulage [t/hr]</td>
<td>522.7</td>
<td>497.3</td>
<td>497.3</td>
<td>522.7</td>
<td>461.3</td>
<td>497.3</td>
<td>509.8</td>
<td>497.3</td>
<td>497.3</td>
<td>473.0</td>
</tr>
<tr>
<td>Waste Haulage [t/hr]</td>
<td>203.9</td>
<td>214.3</td>
<td>214.3</td>
<td>203.9</td>
<td>231.0</td>
<td>214.3</td>
<td>209.1</td>
<td>214.3</td>
<td>214.3</td>
<td>45.0</td>
</tr>
</tbody>
</table>

### Costs & Calculations

#### Mining Costs

<table>
<thead>
<tr>
<th>Resource</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ore [$/t]</td>
<td>0.69</td>
<td>0.70</td>
<td>0.70</td>
<td>0.69</td>
<td>0.72</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>Waste [$/t]</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
<td>0.93</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>2.72</td>
<td></td>
</tr>
<tr>
<td>Processing Cost [$/t]</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

#### Fixed Cost Calculations

<table>
<thead>
<tr>
<th>Proportion Of Fixed Costs [$/t]</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.62</td>
<td>3.62</td>
<td>4.06</td>
<td>3.62</td>
<td>4.61</td>
<td>3.62</td>
<td>3.76</td>
<td>4.06</td>
<td>4.06</td>
<td>4.61</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Increase in Fixed Costs [$/t]</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.28</td>
<td>-0.28</td>
<td>0.16</td>
<td>-0.28</td>
<td>0.71</td>
<td>-0.28</td>
<td>-0.14</td>
<td>0.16</td>
<td>0.16</td>
<td>0.71</td>
<td></td>
</tr>
</tbody>
</table>

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## Dynamic cut-off grade optimisation

### Example: illustrative calculations of ranking parameters

<table>
<thead>
<tr>
<th>Ranking Grades</th>
<th>Cu Grade (% Cu)</th>
<th>In-situ Value [$/t]</th>
<th>Net Value [$/t]</th>
<th>Net Value Corrections [$/t]</th>
<th>Cash Flow Grade [$/hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu Grade (% Cu)</td>
<td>0.96% 0.93% 0.91% 0.90% 0.88% 0.87% 0.87% 0.86% 0.85%</td>
<td>24.7 23.2 25.0 23.1 19.3 21.7 20.6 18.9 19.1 18.8</td>
<td>22.5 20.6 22.6 21.0 17.8 19.8 18.8 17.5 17.6 17.3</td>
<td>22.4 20.7 22.2 21.0 16.9 19.8 18.7 17.0 17.2 14.6</td>
<td>14,533 13,239 13,024 13,531 8,823 12,627 11,557 9,827 9,906 9,525</td>
</tr>
</tbody>
</table>

**For illustrative calculations**

### Ranking

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Cu Grade (% Cu)</th>
<th>In-situ Value [$/t]</th>
<th>Net Value [$/t]</th>
<th>Net Value with Corrections [$/t]</th>
<th>Cash Flow Grade [$/hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu Grade (% Cu)</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td>2 3 1 4 7 5 6 9 8 10</td>
<td>2 4 1 3 7 5 6 9 8 10</td>
<td>1 4 2 3 9 5 6 8 7 10</td>
<td>1 3 4 2 10 5 6 8 7 9</td>
</tr>
</tbody>
</table>
Dynamic cut-off grade optimisation

Example: illustrative calculations of ranking parameters

Illustrative calculations:

- **Costs**

  \[
  \text{Proportion of fixed costs} = \frac{\$20M/y}{7884 \text{hr/y}} \times \frac{1}{700 \text{t/hr}} = 3.62\$/t
  \]

  \[
  \text{Increase in fixed costs} = \frac{\$20M/y}{7884 \text{hr/y}} \times \frac{1}{700 \text{t/hr}} - \frac{\$20M/y}{7884 \text{hr/y}} \times \frac{1}{650 \text{t/hr}} = -0.28\$/t
  \]

- **Ranking factors**

  \[\text{In-situ value} = 0.958\% \times \frac{\$0.9}{\text{lb}} \div 0.000453592 \text{t/lb} + 0.0189 \text{oz/t} \times \$300/\text{oz} = 24.7\$/t\]

  \[\text{Net value} = 95.7\% \times 0.958\% \times \frac{\$0.9}{\text{lb}} \div 0.000453592 \text{t/lb} + 75.4\% \times 0.0189 \text{oz/t} \times \$300/\text{oz} = 22.5\$/t\]

  \[\text{Net value corrections} = 22.5\$/t + \left( \frac{\$100/\text{hr}}{522.7 \text{t/hr}} - \frac{\$100/\text{hr}}{203.9 \text{t/hr}} \right) + 0.28\$/t = 22.4\$/t\]

  \[\text{Cash flow grade} = \left[ (22.5\$/t - 0.69\$/t - 2.00\$/t) - (-0.99\$/t) \right] \times 700 \text{t/hr} = 14,533\$/\text{hr}\]
The greatest benefit of using cash flow grade ranking occurs with relatively small processing capacities.

The cash flow increases as more detail is modelled.

High processing capacity (low fixed costs) drives the corrected net value curve closer to the unadjusted curve.
Dynamic cut-off grade optimisation

Exercise: underground & open pit ranking using cash flow grades

Cash Flow Grades for several blocks found in open pit and underground reserves are evaluated at Copper price of $4/lb.

<table>
<thead>
<tr>
<th>Rock</th>
<th>Open Pit</th>
<th>Stockpile</th>
<th>U/G Stoping</th>
<th>U/G Develop.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper Grade (%)</td>
<td>0.33%</td>
<td>0.40%</td>
<td>0.39%</td>
<td>0.39%</td>
</tr>
<tr>
<td>Recovery (%)</td>
<td>75%</td>
<td>80%</td>
<td>82%</td>
<td>72%</td>
</tr>
<tr>
<td>Ore Mining Cost ($/t)</td>
<td>2.50</td>
<td>2.50</td>
<td>2.50</td>
<td>25.0</td>
</tr>
<tr>
<td>Processing Cost ($/t)</td>
<td>4.0</td>
<td>4.40</td>
<td>4.40</td>
<td>4.0</td>
</tr>
<tr>
<td>Throughput (t/hr)</td>
<td>150.0</td>
<td>75.0</td>
<td>110.0</td>
<td>150.0</td>
</tr>
<tr>
<td>Waste Cost ($/t)</td>
<td>3.0</td>
<td>3.0</td>
<td>110.0</td>
<td></td>
</tr>
<tr>
<td>Ore Revenue ($/t)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value Ore ($/t)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Value Ore - Value Waste) ($/t)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Flow Grade ($/hr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From sequential to integrated optimisation

Sequential optimisation

Generic steps for approximating the optimal choice of Policy A:

- Choose a tool that:
  - a. Uses an objective that accurately reflects your business objectives (NPV is assumed)
  - b. Simultaneously optimises as many of the other key policies (Policies B & C) (Sometimes multiple tools are required to model the process)
- Identify the broad options for Policy A
- For each Policy A alternative optimise the remaining policies (Policies B & C)
- Choose the highest value options for some more detailed analyses if the schedule values are close (within the accuracy of the estimate)

Terminology:
- A number of controllable options can make a decision.
- A set of decisions that change over time is called a policy.
- A set of policies that impact the same project is called a strategy.
Example: Surface to underground transition

Factors that impact the transition from surface to underground operations:

- **Surface**: cut-off grades, waste stripping, stockpile generation and reclaim
- **Underground**: access to higher grades, dilution, proportion of resource extracted (due to sterilisation associated with the mining method), production costs and capacities, capital requirements
- **Combined**: tailings capacity, closure cost implications

Each option comprises of:

- **Underground mining design**: development design, production schedules, capital requirements, life and of course value
- **Open pit mining design**: ultimate pit design, pushback design
Example: Surface to underground transition

- Without an underground, the open pit cut-off grades will normally drop down close to break even as the last material is mined.

- With a highly profitable underground that cannot start until the surface operation is complete, open pit cut-off grades will generally increase to bring forward the value from the underground resource.
Example: Surface to underground transition

Highest value integrated surface and underground schedule (Option B, $2,425M) with variable cut-off grade and pushback sequencing
From sequential to integrated optimisation

Example: Surface to underground transition

Best integrated surface and underground schedule (option B, $2,523M) using variable grind (Throughput/Recovery), cut-off grade and pushback sequencing
Example: Pushback optimisation

- Select final pit shell
- Design the final pit shell with access ramps and benches
- Schedule the life of mine following the ramp-down, exposing sufficient ore to feed the process plant capacity allowing the stripping requirements to fluctuate
From sequential to integrated optimisation

Example: Pushback optimisation

Partially simultaneous optimisation:
- Optimise the open pit
- Optimise multiple operations simultaneously

Fully integrated optimisation (incorporating time and capacities)
From sequential to integrated optimisation

Optimisation techniques: Indirect optimising algorithms and rules of thumb

Two rules of thumb:

- Break-even cut-off grade policy
  
  $$\text{Breakeven grade [units/t]} = \frac{\text{cost [$/t]}}{\text{payability [%]} \times \text{product price [$/unit]} \times \text{recovery [%]}}$$

- Maximum mill throughput policy

Main limitations:

- Time discounting of cash flows
- Capital expenditure
- Distribution of different ore grades
- Different constraints on processing ore
From sequential to integrated optimisation

Optimisation techniques: Specific algorithms

Example:
Lerchs-Grossmann (LG) algorithm for pit shell optimisation

Terminology:
- Blocks are replaced by graph “nodes”.
- Each arc goes from one block (A) to a second block (B), indicating that B must be mined first to uncover A.
- p-arc(+): going away from the root
- m-arc(-): going towards the root
- mass: the value of all the nodes a p-arc is supporting or a m-arc is supported by.
- ps/pw: a p-arc supports positive/negative mass
- ms/mw: a m-arc is supported by positive/negative mass
Optimisation techniques: Specific algorithms

Generic steps:

- Connect all nodes to the root node and label the arcs.
- Connect p-strong arcs to all other nodes that must be removed to mine this strong node. When this connection is made, eliminate the strong node going from this node and re-label the arcs.
- Normalise the tree. Root \( (x_0) \) must be common to all strong edges, if not, then:
  - a. Replace the strong p-edge \((x_m - x_n)\) with a dummy arc \((x_0 - x_n)\) to prevent unnecessary support
  - b. Replace the strong m-edge \((x_q - x_r)\) with a dummy arc \((x_0 - x_q)\) to prevent over extending the pit.
- Go to the first step if there is a node overlying another node connected to the root by a strong arc. Otherwise stop.
From sequential to integrated optimisation

Optimisation techniques: Specific algorithms

Example:

Delete dummy arc

\[ \text{arc}(x_0-x_1) = -1 + 4 = +3 \rightarrow p-s \]

\[ \text{arc}(x_0-x_2) = -1 - 2 + 4 = +1 \rightarrow p-s \]

\[ \text{arc}(x_0-x_3) = -1 + 5 = +4 \rightarrow ps \]

\[ \text{arc}(x_0-x_4) = 4 - 1 - 2 + 5 - 1 = +5 \rightarrow ps \]
From sequential to integrated optimisation

Optimisation techniques: Specific algorithms

Example:

\[
\text{arc } (x_0, x_2) = 4 \cdot (-1) - 2 + 5 \cdot (-1) - 2 = +3 \rightarrow p-s
\]

\[
\text{arc } (x_0, x_2) = 4 \cdot (-1) - 2 + 5 \cdot (-1) - 2 = +3 \rightarrow p-s
\]

\[
\text{Mine all the nodes connected to a strong arc.}
G(X, A) = 3
\]
From sequential to integrated optimisation

Optimisation techniques: Specific algorithms

Exercise:
From sequential to integrated optimisation

Optimisation techniques: Specific algorithms

Exercise:

- For $T^4$ (Normalization), the graph shows transitions between states.
- For $T^5$, the graph includes labeled transitions and a highlighted state.
- For $T^6$, the graph represents another set of transitions with an indicated optimum and closure set.

The optimum state is $v_2$, with a closure set of $\{v_2, v_3, v_6\}$ and a value of 2.
Main limitation:

- The specific nature of the algorithms limits their application to other problems. For example, the LG algorithm assumes a single block value, precluding ore blending and time-dependent functions (such as product price) from consideration during the ‘optimum’ pit shell analysis.
From sequential to integrated optimisation

Optimisation techniques: Linear, integer and mixed integer programming

Linear programming and other mathematical programming methods, its derivatives are classical analytical techniques for maximising or minimising an ‘objective function’ subject to a number of constraints.

The objective function (the profit function) is defined as the total expected NPV minus the cost of deviations from planned production targets.

- Linear programming
- Integer programming
- Mixed integer programming

Main challenge:
- Converting mining problems into the required linear format.
Consider a copper/cobalt mining operation with six sources of ore (shafts). The budget targets production of 20,000 and 200 tonnes of finished copper and cobalt respectively.

### The primal problem

The LP formulation of this problem is presented as follows:

Let:

\[ x_j = \text{unknown tonnes of ore to be produced from shaft } j \]

The objective is to minimise the total cost of mining. This optimisation criterion will ensure that the mining contribution to profit over the quarter will be maximised. Thus, the objective function is:

\[
\text{Minimise } Z = 7x_1 + 5x_2 + 6x_3 + 8x_4 + 4x_5 + 6.5x_6
\]

Metal production targets are formulated as constraints:

(i) \[0.027x_1 + 0.050x_2 + 0.035x_3 + 0.045x_4 + 0.009x_5 + 0.039x_6 \geq 20000\]

(ii) \[0.004x_1 + 0.007x_2 + 0.0007x_3 + 0.0008x_4 + 0.006x_5 + 0.002x_6 \geq 200\]

Non-negativity constraint ensures that production from each shaft is positive:

\[ x_j \geq 0 \quad \forall j \]

### The dual problem

The dual problem to the above primal problem is formulated as follows:

Let:

\[ y_1 = \text{price of copper on the world market ($/\text{tonne})} \]

\[ y_2 = \text{price of cobalt on the world market ($/\text{tonne})} \]

The objective function is to maximise metal sales value in dollars:

\[
\text{Maximise } v = 20000y_1 + 200y_2
\]

The objective function is subject to the following constraints:

(i) \[0.027y_1 + 0.004y_2 \leq 7.0\]

(ii) \[0.050y_1 + 0.007y_2 \leq 5.0\]

(iii) \[0.035y_1 + 0.0007y_3 \leq 6.0\]

(iv) \[0.045y_1 + 0.0008y_2 \leq 8.0\]

(v) \[0.009y_1 + 0.006y_2 \leq 4.0\]

(vi) \[0.039y_1 + 0.002y_3 \leq 6.5\]

Non-negativity constraints:

\[ y_j \geq 0 \quad \forall j \]
From sequential to integrated optimisation

Optimisation techniques: Example of stochastic integer programming

Conventional approaches to estimating reserves, optimising mine planning and production forecasting result in single, often biased, forecasts. This is largely due to the non-linear propagation of errors in understanding orebodies throughout the chain of mining. A new mine planning paradigm considering stochastic optimisation is considered.

The objective function is:

$$\text{Max} \sum_{t=1}^{p} \left[ \sum_{i=1}^{n} E[(NPV)] \right] b_i - \sum_{s=1}^{m} \left( c_{u}^{d_{su}} + c_{l}^{d_{sl1}} + c_{u}^{d_{su}} + c_{l}^{d_{sl1}} \right)$$

where

- $p$ is the total production periods
- $n$ is the number of blocks
- $b_i^t$ is the decision variable for when to mine block $i$ (if mined in period $t$, $b_i^t$ is 1)

**Conventional optimisation:**

Maximise $(c x_1^{1} + c x_2^{1} + \ldots)$

Subject to...

where

- $c_i$ is the economic value of a block (a constant)
- $x_i$ is a binary variable; mine or not block $i$ in period $p$

The economic value of a block $c_i$ is:

\[ (\text{METAL} \times \text{RECOVERY} \times \text{PRICE} - \text{ORE} \times \text{COST Proc} - \text{ROCK} \times \text{COST Min}) \]

**Stochastic optimisation:**

Maximise $(c x_1^{1} + c x_2^{1} + \ldots + c_n^{x_n})$

Subject to...

where

- $r$ is a given simulation of the deposit
From sequential to integrated optimisation

Optimisation techniques: Genetic algorithms

Rather than have an objective that guides the selection of alternate designs, randomness is used to make changes to designs and then evaluate if these changes improved the objective function.

Tools:
Evolver and RiskOptimizer add-ins for Excel (Paliside Corporation)

Main limitations:
- The optimisation of a single parameter.
- Due to the lack of knowledge applied in the changes, convergence on better and better solutions is not guaranteed.
- As the problem complexity increases the chance of finding optimal solutions or even a valid solution is reduced.
Optimisation techniques: Dynamic programming

Dynamic programming is typically appropriate for the global optimisation of net present value due to the common occurrence of non-linear relationships in these problems.

Tools:
COMET, COMET ONE

Main limitation:
The feasibility of dynamic programming is constrained by the time required to test all the feasible states. For a number of possible pushbacks \( P \) containing a number of feasible elevations \( I \), the number of feasible solutions increases significantly:

\[
N_{states} = \sum_{i=1}^{P} \left( \frac{[(I_i - 1) + (P - i + 1)]!}{(I_i - 1)! (P - i + 1)!} \right)
\]

A “depth first” search algorithm can be used to reduce the feasible schedules investigated.
From sequential to integrated optimisation

Advances in tactical mine planning


- Dynamic Programming (Bellman, 1953-57)
- Cut-off grade (Lane, 1964)
- Production Rate (Roman, 1973)
- Stochastic Parameters (Dowd, 1976)
- Taxation, grain size (Schaap, 1981-84)

Multiple Processes & Stockpiles (Whittle, 1995-6)

Multiple Policies, Sequencing and Closure (King, 1996-2001)

Practical policies and realistic constraints

(COMET Strategy Pty Ltd, 2002-2020)

COMET ONE, free for education (COMET Strategy Pty Ltd, 2017)
Available software packages

- Optimal mine scheduling software
  - Whittle: implemented LG algorithm
  - Commercial Optimal Mine Exploitation Technology (COMET)
  - COMET ONE
- Mine design software
  - Deswik: implemented COMET
  - Surpac
  - 3DMine
  - DiMine
  - Maptek Vulcan
  - Hexagon MineSight
- Own scripting
Introduction to COMET ONE

Overview

- COMET Optimal Scheduler is an advanced multi-policy schedule optimisation tool developed to optimise both single mine and complex combinations of surface and underground mines.

- COMET ONE is an on-line schedule optimisation tool designed to illustrate how to find and exploit the value drivers of a mine.

History of development:

- 1960s and 1970s, Rio Tinto group - Optimum Grades for Resource Exploitation (OGRE): optimise cut-off grades within a fixed sequence of material, represented as grade/tonnage data

- 1988, Rio Tinto group - OGREPlus: add a pushback scheduler in an annualised sequence so that successive iterations between the scheduler and OGRE until the process converged on an optimised schedule

- 1996, Rio Tinto group - COnStraint MOdelling Scheduler (Cosmos): a combined schedular and cut-off grade optimiser

- 2001, COMET Strategy - Commercial Optimal Mine Exploitation Technology (COMET)
Introduction to COMET ONE

Getting started – User registration

- [https://www.cometstrategy.com/](https://www.cometstrategy.com/)
Introduction to COMET ONE

Getting started – User interface

Interface:
- Scenarios
- Input Parameters
  - Project
  - Period information
  - Rock properties
  - Cash flow
- Results
  - Summary information
  - Material movement
  - Cash flow components
  - Detailed report
Introduction to COMET ONE

Getting started – input parameters

- Resource model
  - Grade-tonnage distribution within the ultimate pit limits (% or $/ton vs. tonnage)
- Pushback design
- Discounting, cost and revenue drivers
  - Discount rate (%), often in the range 5-15%
  - Mining cost (normally $/t moved)
  - Processing costs (normally $/t milled, also $/oz recovered metal)
  - Selling costs for each metal (normally $/lb or $/oz)
  - Annual ‘Period’ based costs (often G&A, $/year)
  - Revenue for each metal (normally $/lb or $/oz)
- Productivities and capacities (constraints)
  - Mining capacity (typically Mt/year)
  - Processing capacity (typically Mt/year): primary crusher, semi-autogenous mills (SAG mills), ball/rod mills, flotation time, concentrate capacity
  - Processing recovery for each metal (%)
Example: Mina de la Antartida (10 seqs)

- **Objective:**
  - maximising NPV

- **Minerals:**
  - copper, gold

- **Ranking factor:**
  - cash flow grade

- **Cut-off grade bins:**
  - 10 increments

- **Mine sequence:**
  - 10 benches

- **Mining capacity:**
  - 70 Mt/y

- **Processing capacity:**
  - 25 Mt/y
Example: Surface to underground transition

- **Objective:**
  - maximising NPV
- **Minerals:**
  - copper, gold
- **Ranking factor:**
  - cash flow grade
- **Cut-off grade bins:**
  - 10 increments
- **Mine sequence:**
  - surface (1 increment), U/G (1 increment)
## Introduction to COMET ONE

### Comparison of objectives: Maximum reserve cash flow

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>Alternative Maximum Reserve Cash Flow</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>M$ 7,745.27</td>
<td>M$ 6,112.92</td>
<td>M$ -1,632.35</td>
</tr>
<tr>
<td>Life</td>
<td>16.65 years</td>
<td>25.18 years</td>
<td>8.53 years</td>
</tr>
<tr>
<td>Ore</td>
<td>415.63 Mt</td>
<td>629.56 Mt</td>
<td>213.93 Mt</td>
</tr>
<tr>
<td>Finish</td>
<td>2036.05</td>
<td>2045.18</td>
<td></td>
</tr>
</tbody>
</table>

**Lost value from using 'Maximum Reserve Cash Flow' objective**

- Blue: 'Maximum Reserve Cash Flow' schedule value
- Red: Lost value

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Feb 2020
Comparison of objectives: Maximum period cash flow

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>Alternate Period Cash Flow</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV:</td>
<td>M$ 7,745.27</td>
<td>M$ 7,382.44</td>
<td>M$ -362.83</td>
</tr>
<tr>
<td>Life:</td>
<td>16.65 years</td>
<td>12.72 years</td>
<td>-3.93 years</td>
</tr>
<tr>
<td>Ore:</td>
<td>415.63 Mt</td>
<td>313.68 Mt</td>
<td>-101.95 Mt</td>
</tr>
<tr>
<td>Finish:</td>
<td>2036.65</td>
<td>2032.72</td>
<td></td>
</tr>
</tbody>
</table>

Lost value from using 'Maximum Period Cash Flow' objective

- Maximum Period Cash Flow schedule value
- Lost value

Annualised Mining Schedule: Maximum Period Cash Flow

- Rock Mass
- Ore Mass
- Cut-off Grade (Cash Flow Grade) [$/t]

Annualised Cash Flows: Maximum Period Cash Flow

- Revenue [M$/year]
- Costs [M$/year]
- Profit (Cash Flow) [M$/year]
- DCF [M$/year]

2,025.365
Profit (Cash Flow) [M$/year]: 969.834
Introduction to COMET ONE

Comparison of objectives: Maximum life

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>Alternate Maximum Life</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV:</td>
<td>$7,746.27</td>
<td>$3,429.29</td>
<td>$4,316.98</td>
</tr>
<tr>
<td>Life:</td>
<td>16.65 years</td>
<td>35.26 years</td>
<td>18.61 years</td>
</tr>
<tr>
<td>Ore:</td>
<td>415.63 Mt</td>
<td>881.48 Mt</td>
<td></td>
</tr>
<tr>
<td>Finish:</td>
<td>2036.65</td>
<td>2055.26</td>
<td></td>
</tr>
</tbody>
</table>

Lost value from using 'Maximum Life' objective

- Maximum Life schedule value: 55.8%
- Lost value: 44.2%
Comparison of objectives: Maximum cut-off grade

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>Alternative</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>$7,746.27</td>
<td>$6,792.88</td>
<td>-$1,953.39</td>
</tr>
<tr>
<td>Life</td>
<td>10.65 years</td>
<td>12.59 years</td>
<td>-1.94 years</td>
</tr>
<tr>
<td>Ore</td>
<td>415.63 Mt</td>
<td>246.10 Mt</td>
<td>-169.53 Mt</td>
</tr>
<tr>
<td>Finish</td>
<td>2036.65</td>
<td>2032.50</td>
<td>-4.15</td>
</tr>
</tbody>
</table>

Lost value from using ‘Maximum Cut-off grade’ objective

- 25.2% ‘Maximum Cut-off grade’ schedule value
- 74.8% Lost value

Annualised Mining Schedule: Maximum Cut-off grade

- Rock Mass
- Ore Mass
- Cut-off Grade (Cash Flow Grade) [$/t]

Annualised Cash Flows: Maximum Cut-off grade

- Revenue [M$/year]
- Costs [M$/year]
- Profit (Cash Flow) [M$/year]
- DCF [M$/year]
# Introduction to COMET ONE

## Comparison cases: 10% increase in mining capacity

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>10% increase in mining capacity</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV:</td>
<td>£7,746.27</td>
<td>£7,803.16</td>
<td>£57.88</td>
</tr>
<tr>
<td>Life:</td>
<td>16.65 years</td>
<td>16.31 years</td>
<td>-0.34 years</td>
</tr>
<tr>
<td>Ore:</td>
<td>415.63 Mt</td>
<td>407.68 Mt</td>
<td>-7.95 Mt</td>
</tr>
<tr>
<td>Finish:</td>
<td>2036.05</td>
<td>2036.31</td>
<td></td>
</tr>
</tbody>
</table>

### Sensitivity to '10% increase in mining capacity'

- **Base Case**
- **Additional Value from '10% increase in mining capacity'**

### Annualised Mining Schedule: 10% increase in mining capacity

- **Cuto-Off Grade (Cash Flow Grade) [£/t]**
- **Annualised Cash Flows: 10% increase in mining capacity**
  - **Revenue [£/year]**
  - **Costs [£/year]**
  - **Profit (Cash Flow) [£/year]**
  - **DCF [£/year]**
Comparison cases: 10% increase in processing capacity

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>10% increase in processing capacity</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>$7,745.27</td>
<td>$8,298.23</td>
<td>$552.96</td>
</tr>
<tr>
<td>Life</td>
<td>16.65 years</td>
<td>16.02 years</td>
<td>0.63 years</td>
</tr>
<tr>
<td>Ore</td>
<td>415.63 Mt</td>
<td>440.58 Mt</td>
<td>24.95 Mt</td>
</tr>
<tr>
<td>Finish</td>
<td>2036.65</td>
<td>2036.02</td>
<td></td>
</tr>
</tbody>
</table>

Sensitivity to '10% increase in processing capacity'

- Base Case: 93.3%
- Additional Value from '10% increase in processing capacity': 6.7%
## Comparison cases: 10% increase in both capacities

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>10% increase in all capacities</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>$7,745.27</td>
<td>$8,406.29</td>
<td>$681.02</td>
</tr>
<tr>
<td>Life:</td>
<td>16.65 years</td>
<td>15.58 years</td>
<td>-1.07 years</td>
</tr>
<tr>
<td>Ore:</td>
<td>415.63 Mt</td>
<td>428.49 Mt</td>
<td>12.86 Mt</td>
</tr>
<tr>
<td>Finish:</td>
<td>2036.65</td>
<td>2035.58</td>
<td></td>
</tr>
</tbody>
</table>

### Annualised Mining Schedule: 10% increase in all capacities

- **Rock Mass**
- **Ore Mass**
- **Cut-off Grade (Cash Flow Grade) [$/t]**

### Annualised Cash Flows: 10% increase in all capacities

- **Revenue [$/year]: 2,022,640**
- **Profit (Cash Flow) [$/year]: 2,022,785**
- **DCF [$/year]**

**Sensitivity to '10% increase in all capacities':**

- 7.9% (Base Case)
- 92.1% (Additional Value from '10% increase in all capacities')
Comparison cases: Mining expansion

- **Mining expansion – no capital:** We will consider increasing the mining capacity from 70Mt/y to 100Mt/y in steps of 10Mt/y.

<table>
<thead>
<tr>
<th>Mill Capacity (Mt/y)</th>
<th>NPV</th>
<th>Mining Capacity (Mt/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>25</td>
<td>7745</td>
</tr>
<tr>
<td>80</td>
<td>7825</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>7829</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>7829</td>
<td></td>
</tr>
</tbody>
</table>

- **Mining expansion – with capital:** For the same increase in mining capacity, add $50M capital costs for each additional 10Mt/y increase in mining.

<table>
<thead>
<tr>
<th>Expansion Capital ($M)</th>
<th>NPV</th>
<th>Mill Capacity (Mt/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
<td>7745</td>
</tr>
<tr>
<td>-50</td>
<td>7875</td>
<td></td>
</tr>
<tr>
<td>-100</td>
<td>7729</td>
<td></td>
</tr>
<tr>
<td>-150</td>
<td>7679</td>
<td></td>
</tr>
</tbody>
</table>
Mill expansion – with capital: Leaving the mining capacity at 70Mt/y, we will consider increasing the mill capacity from 25Mt/y to 40Mt/y in steps of 5Mt/y. Add $1000M capital costs for each additional 5Mt/y increase in processing capacity.

<table>
<thead>
<tr>
<th>Expansion Capital ($M)</th>
<th>Mill Capacity (Mt/y)</th>
<th>NPV ($M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
<td>7745</td>
</tr>
<tr>
<td>-1000</td>
<td>30</td>
<td>7702</td>
</tr>
<tr>
<td>-2000</td>
<td>35</td>
<td>7412</td>
</tr>
<tr>
<td>-3000</td>
<td>40</td>
<td>7220</td>
</tr>
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</table>
Comparison cases: Mine and mill expansions

- **Mine and mill expansions – with capital:** We will consider increasing the mine capacity from 70Mt/y to 100Mt/y in steps of 10Mt/y, and mill capacity from 25Mt/y to 40Mt/y in steps of 5Mt/y. Add $1000M capital costs for each additional 5Mt/y increase in processing capacity and $50M for each 10Mt/y mining capacity increase:

<table>
<thead>
<tr>
<th>Mill Capacity (Mt/y)</th>
<th>Mining Capacity (Mt/y)</th>
<th>Expansion Capital (M$)</th>
<th>Mining Capacity (Mt/y)</th>
<th>NPV (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>70</td>
<td>0</td>
<td>80</td>
<td>-50</td>
</tr>
<tr>
<td>30</td>
<td>25</td>
<td>-1000</td>
<td>30</td>
<td>-1050</td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>-2000</td>
<td>40</td>
<td>-3000</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>-3000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Mining Capacity (Mt/y):** 70, 80, 90, 100

- **Expansion Capital (M$):** 0, -50, -100, -150, -200, -250, -300, -350, -400, -450, -500, -550, -600, -650, -700, -750, -800, -850, -900, -950, -1000

- **NPV (M$):** 7745
Comparison cases: Mill throughput and recovery policy

- We will look at an advanced optimisation technique where we optimise the cut-off grade and the mill throughput and recovery relationship simultaneously throughout the mine life.

Throughput and recovery relationship

The slope of the relationship has different options:

- Compare Advanced (Throughput@Recovery)
  - High (100%@100%)-(110%@98%)-(120%@96%)
  - Med (110%@95%)-(100%@100%)-(90%@105%)
  - Low (80%@120%)-(90%@110%)-(100%@100%)
### Introduction to COMET ONE

#### Comparison cases: Mill throughput and recovery policy (High)

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case</th>
<th>Alternate</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max NPV</td>
<td>High (100%)</td>
<td>(110%)</td>
</tr>
<tr>
<td>NPV:</td>
<td>$7,745.27</td>
<td>$8,200.20</td>
<td>$454.93</td>
</tr>
<tr>
<td>Life:</td>
<td>16.65 years</td>
<td>15.68 years</td>
<td>-0.97 years</td>
</tr>
<tr>
<td>Ore:</td>
<td>415.63 Mt</td>
<td>464.05 Mt</td>
<td>48.42 Mt</td>
</tr>
<tr>
<td>Finish:</td>
<td>2036.65</td>
<td>2035.68</td>
<td></td>
</tr>
</tbody>
</table>

**Sensitivity to (100% @100%) - (110% @98%) - (120% @96%)**

- Base Case: 94.5%
- Additional Value from High: 5.5%

**Annualised Mining Schedule: High (100%) - (110%) - (120%)**

- Rock Mass
- Ore Mass
- Cut-off Grade (Cash Flow Grade) [$/t]

**Annualised Cash Flows: High (100%) - (110%) - (120%)**

- Revenue [$/year]
- Costs [$/year]
- Profit (Cash Flow) [$/year]
- DCF [$/year]
# Introduction to COMET ONE

## Comparison cases: Mill throughput and recovery policy (Medium)

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>Alternate (Med (110%@95%)-(100%@100%)-(90%@105%))</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV:</td>
<td>MS 7,746.27</td>
<td>MS 7,779.76</td>
<td>MS 34.48</td>
</tr>
<tr>
<td>Life:</td>
<td>16.65 years</td>
<td>16.93 years</td>
<td>0.28 years</td>
</tr>
<tr>
<td>Ore:</td>
<td>415.63 Mt</td>
<td>417.56 Mt</td>
<td>1.93 Mt</td>
</tr>
<tr>
<td>Finish:</td>
<td>2036.65</td>
<td>2036.93</td>
<td></td>
</tr>
</tbody>
</table>

### Sensitivity to 'Med (110%@95%)-(100%@100%)-(90%@105%)'

- **Base Case**
- **Additional Value from Med (110%@95%)-(100%@100%)-(90%@105%)**

### Annualised Mining Schedule: Med (110%@95%)-(100%@100%)-(90%@105%)

- **Rock Mass**
- **Ore Mass**
- **Cut-off Grade (Cash Flow Grade) [$/t]**

### Annualised Cash Flows: Med (110%@95%)-(100%@100%)-(90%@105%)

- **Revenue [$/year]**
- **Costs [$/year]**
- **Profit (Cash Flow) [$/year]**
- **DCF [$/year]**
Comparison cases: Mill throughput and recovery policy (Low)

<table>
<thead>
<tr>
<th>Summary</th>
<th>Base Case (Max NPV)</th>
<th>Alternate (Low (80%@120%)-(90%@110%)-(100@100%))</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>$7,745.27</td>
<td>$8,412.81</td>
<td>$667.64</td>
</tr>
<tr>
<td>Life</td>
<td>16.65 years</td>
<td>19.17 years</td>
<td>2.52 years</td>
</tr>
<tr>
<td>Ore</td>
<td>415.63 Mt</td>
<td>363.42 Mt</td>
<td>-52.21 Mt</td>
</tr>
<tr>
<td>Finish</td>
<td>2036.65</td>
<td>2038.17</td>
<td>1.52 days</td>
</tr>
</tbody>
</table>

Sensitivity to ‘Low (80%@120%)-(90%@110%)-(100@100%)’

Annualised Mining Schedule: Low (80%@120%)-(90%@110%)-(100@100%)

Annualised Cash Flows: Low (80%@120%)-(90%@110%)-(100@100%)

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Introduction to COMET ONE

Customised (site specific) planning

- Change the attributes:
  - Rock Properties
    - Attributes
      - Mass
      - Cut-off grade
      - Cash Flow Grade
      - Copper
      - Gold
      - Recovered Copper
      - Mining Cost Factor

- Customise the rock grade distributions:
  - Use custom reserves
  - Number of attributes: 4
  - Number of cut-off grades: 10
  - Number of sequence increments: 10

Tab delimited file:

<table>
<thead>
<tr>
<th>Seq</th>
<th>COG</th>
<th>Mass (MT)</th>
<th>Cut-off Grade</th>
<th>Cash Flow Grade</th>
<th>Copper</th>
<th>Gold</th>
<th>Recovered Copper</th>
<th>Mining Cost Factor</th>
</tr>
</thead>
<tbody>
<tr>
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<td>3.47</td>
<td>1.518783783253</td>
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<td>10</td>
<td>12</td>
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</tr>
</tbody>
</table>
# Case study: Radomiro Tomic Copper Project

## Methodology

### Geological Modelling (Sect. 2)

<table>
<thead>
<tr>
<th>Input</th>
<th>Procedure: Isatis Ver. 2017</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drill-Hole Data</td>
<td>Grid Definition/Discretisation</td>
<td>Geological Block Model</td>
</tr>
<tr>
<td>Regional Geology</td>
<td>Geological Boundaries</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Geostatistical Estimation</td>
<td></td>
</tr>
</tbody>
</table>

### Mining Engineering (Sect. 3)

<table>
<thead>
<tr>
<th>Input</th>
<th>Procedure: Python 3.7, Gurobi Optimizer</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geological Block Model</td>
<td>Mining Block Model (block_modelling.py)</td>
<td>Production Schedule</td>
</tr>
<tr>
<td>Price and Cost Assumption</td>
<td>Cut-Off Grade Optimisation (upd_of_opt.py)</td>
<td>Revenue Cash Flows</td>
</tr>
<tr>
<td></td>
<td>Ultimate Pit Design (udp.py)</td>
<td>Cost Cash Flows</td>
</tr>
<tr>
<td></td>
<td>Pushback Design (opms.py)</td>
<td>CAPEX</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ultimate Pit Surface Blocks</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pushback Volume Blocks</td>
</tr>
</tbody>
</table>

### Design Visualisation

<table>
<thead>
<tr>
<th>Input</th>
<th>Procedure: Blender Ver. 2.79</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate Pit Surface Blocks</td>
<td>Pit/Pushback Visualisation (vis_for_blender.py)</td>
<td>Visualised Ultimate Pit</td>
</tr>
<tr>
<td>Pushback Volume Blocks</td>
<td></td>
<td>Visualised Pushbacks</td>
</tr>
</tbody>
</table>

### Financial Modelling: DCF Analysis, and Real Option Analysis
The Vertical Geological Cross-Section 10,700 N at Radomiro Tomic
Case study: Radomiro Tomic Copper Project

Geostatistics modelling

144 drillhole data
Case study: Radomiro Tomic Copper Project

Ultimate pit design and cut-off grade optimisation

$$\max: NV = \max: \sum_{b \in B} PROF_b x_b$$

such that

- $x_b = \{0, 1\}$ for $\forall b \in B$
- $x_b \leq x_{b'}$ for $\forall b \in B, \forall b' \in B'_{b}$

Ultimate pit design for the Radomiro Tomic project
Case study: Radomiro Tomic Copper Project

Pushback design and production schedule

- PBD option 1

\[ \text{max: } NPV = \max \sum_{b \in B, t \in T} \frac{PROF_b x_{b,t}}{(1 + r)^t} \]

such that

- \( x_{b,t} \in \{0,1\} \) for \( \forall b \in B, \forall t \in T \)
- \( x_{b,t-1} \leq x_{b,t} \) for \( \forall b \in B, \forall t \in T \)
- \( x_{b,t} \leq x_{b',t} \) for \( \forall b \in B, \forall t \in T, \forall b' \in B' \)
- \( L_t \leq \sum_{b \in B} M_b x_{b,t} \leq U_t \) for \( \forall t \in T \)

- PBD option 2

\[ \text{max: } NPV = \max \sum_{b \in B, t \in T} \frac{PROF_b (y_{b,t} - y_{b,t-1})}{(1 + r)^t} \]

such that

- \( y_{b,t} \in \{0,1\} \) for \( \forall b \in B, \forall t \in T \)
- \( y_{b,t-1} \leq y_{b,t} \) for \( \forall b \in B, \forall t \in T \)
- \( y_{b,t} \leq y_{b',t} \) for \( \forall b \in B, \forall t \in T, \forall b' \in B' \)
- \( L_t \leq \sum_{b \in B} M_b (y_{b,t} - y_{b,t-1}) \leq U_t \) for \( \forall t \in T \)

Difference: Assuming block is mined at year 4 in 6-year life of mine, the situation is numerically described as:

- PBD option 1: \( x_{b,1} = 0; x_{b,2} = 0; x_{b,3} = 0; x_{b,4} = 1; x_{b,5} = 0, x_{b,6} = 0 \)
- PBD option 2: \( x_{b,1} = 0; x_{b,2} = 0; x_{b,3} = 0; x_{b,4} = 1; x_{b,5} = 1, x_{b,6} = 1 \)
Case study: Radomiro Tomic Copper Project

Pushback design and production schedule
The production schedule for a 24-Year Radomiro Tomic project
## Dynamic cut-off grade optimisation (10 years)

Grade-tonnage distribution for the first pushback

<table>
<thead>
<tr>
<th>Pushback</th>
<th>Cut-Off Grade ($/tonne)</th>
<th>Mass (Mt)</th>
<th>Cash Flow Grade ($/tonne)</th>
<th>Copper Grade (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-10.0</td>
<td>2.09</td>
<td>-0.94</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.38</td>
<td>1.29</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>0.32</td>
<td>2.54</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>3.2</td>
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<td></td>
<td>10.4</td>
<td>20.05</td>
<td>20.22</td>
<td>0.50</td>
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</tbody>
</table>
Dynamic cut-off grade optimisation (10 years)
# Reading materials


Reading materials


