Topological Defects from Gauge Symmetry Breaking

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Outline

- Global and gauge symmetries
- Topological defects
  - Cosmology
  - Condensed matter
- Defects from phase transitions
  - Global symmetry: Kibble mechanism
  - Gauge symmetry: Flux trapping
- Numerical tests
- Cosmology in the laboratory
Spontaneous Symmetry Breaking
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- Condensed matter
  - Ferromagnets
  - Liquid crystals
  - Superfluids

\[ f[\psi] = \frac{1}{2m} \left| -i\hbar \vec{\nabla} \psi \right|^2 + V(|\psi|) \]
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Superconductors: Charged Cooper pairs

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\[
f[\vec{A}, \psi] = \frac{1}{2\mu_0} \left( \vec{\nabla} \times \vec{A} \right)^2 + \frac{1}{2m} \left| \left( -i\hbar \vec{\nabla} + \frac{2e}{c} \vec{A} \right) \psi \right|^2 + V(|\psi|)
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- Particle physics: Electroweak theory, Grand unification

\[
\mathcal{L}[A_\mu, \phi] = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| (\partial_\mu + ie A_\mu) \phi \right|^2 - V(|\phi|)
\]

- Local gauge symmetry!
Localized, topologically stable field configurations

Exist if $\pi_n(M) \neq 0$ for $n < d$

- $n = 0$: Domain walls
- $n = 1$: Cosmic strings (vortices)
- $n = 2$: Monopoles

Condensed matter:
Vortices in superconductors, superfluids, ...

Cosmology:
Cosmic strings, Magnetic monopoles

Produced in phase transitions
Cosmological phase transitions $\Rightarrow$ Defect formation \textit{(Kibble 1976)}
Cosmic Defects

- Cosmological phase transitions $\Rightarrow$ Defect formation (Kibble 1976)
  - Magnetic monopoles
    - Predicted by all GUTs
    - Monopole problem: Would overclose the universe (Preskill 1979)
  - Domain walls
    - Incompatible with homogeneity
  - Cosmic strings
    - Scaling: Constant fraction of energy density
      $\Rightarrow$ Scale-invariant perturbations of amplitude $\sim G\mu$
    - Tension $G\mu \approx \frac{v^2}{m_{Pl}^2}$ ($\approx 10^{-6}$ for GUTs)
    - Structure formation?
Cosmic Defects

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    - Constraints: $G\mu \lesssim 3.3 \times 10^{-7}$ from CMB (Jeong and Smoot 2004)
      $G\mu \lesssim 10^{-7}$ from gravity waves and pulsar timing
Renewed Interest

- Cosmic strings almost unavoidable
  - Exist in all realistic GUTs (Jeannerot et al 2003)
  - Exist in hybrid inflationary models
  - Exist in superstring theory (Copeland et al 2003)
  - Attempts to build models with no defects (Urrestilla et al, Watari et al, Dasgupta et al 2004)

- Possible observations
  - Lensing of a galaxy by a cosmic string (Sazhin et al 2003,2004)
  - Brightness fluctuations caused by a string loop (Schild et al 2004)
  - Future: Gravitational waves from cosmic strings (LIGO, LISA)
  - Observational view to very high energy physics and very early universe
Lensing Candidate CSL-1

(Sazhin et al., MNRAS 343 (2003) 353)
Lensing by cosmic string? (Sazhin et al. 2003)

- CSL-1: Two seemingly identical galaxies at $z = 0.46 \pm 0.008$ separated by $2''$
- Other possibilities: Single galaxy obscured by dust line
  Actually two identical galaxies
  Lensing by a giant galaxy

  – All ruled out by the authors

- String tension $G\mu \gtrsim 4 \times 10^{-7}$: Marginally compatible with constraints
- Other possible double images nearby (Sazhin et al. 2004)
Brightness Fluctuations of Q0957+561

(Schild et al., Astronomy and Astrophysics 422 (2004) 477)
Brightness Fluctuations of Q0957+561

- Brightness fluctuations due to a string loop? (Schild et al. 2004)
  - Q0957+561: Double image of a quasar due to lensing by a galaxy
    - Time delay between images 417.1 days
    - Synchronous fluctuation component observed in 1994 and 1995
    - Statistical fluctuations or a string loop at 3kpc?
Vortices in Superconductors

- Type II superconductors ($\kappa > 1/\sqrt{2}$)
  - Often high $T_c \sim 100$K
- Carry one magnetic flux quantum $\Phi_0$
  - Meissner effect $\Rightarrow$ No flux elsewhere
- Flux lattice at $H_{c1} \leq H \leq H_{c2}$
- Move slowly
  $\Rightarrow$ Survive some time even at $H = 0$

( Goa et al. 2001)
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Vortices from phase transitions at $H = 0$:
- Carmi & Polturak 1999
- Maniv et al. 2003, 2004
- Kirtley et al. 2003, 2004
- Girard et al. (in progress)

Experimental test for defect formation theories
Global Theories: Kibble Mechanism

(Kibble 1976, Zurek 1985)

Example: Scalar field theory with global U(1) symmetry

\[ f = \left| \vec{\nabla} \phi \right|^2 + V(\phi) \]

= Ginzburg-Landau theory of superfluidity ($^4$He)

- Second-order phase transition at $T_c$
  - Gradient term $\Rightarrow$ Uniform phase $\theta = \arg \phi$ in broken phase
- As $T \to T_c$, correlation length grows $\xi \to \infty$
  - Causality etc. $\Rightarrow$ cannot grow arbitrarily fast
  - Freezes out at a finite value $\hat{\xi}$
- Consider three points separated by $> \hat{\xi}$
  - Phase $\theta$ uncorrelated
  - Interpolates continuously
  - Finite probability for non-zero winding $\Rightarrow$ Vortex
- Number density per unit area $n \approx \hat{\xi}^{-2}$
Example: Scalar field theory with U(1) gauge symmetry

\[
f = \frac{1}{2} \vec{B}^2 + \left| \vec{D} \phi \right|^2 + V(\phi), \quad \vec{D} = \vec{\nabla} + ieA
\]

=Ginzburg-Landau theory of superconductivity

- Normal (Coulomb) phase at \( T > T_c \)
  - Massless photon
  - Thermal fluctuations \( \approx \) Blackbody radiation \( G(k) \approx T \)
    \[
    \langle B_i(\vec{k})B_j(\vec{q}) \rangle = G(k)(\delta_{ij} - k_i k_j / k^2)(2\pi)^3 \delta(\vec{k} + \vec{q})
    \]

- Superconducting (Higgs) phase at \( T < T_c \)
  - Phase \( \theta \) not necessarily uniform but \( \vec{\nabla} \theta + e\vec{A} = 0 \)
  - Equilibrium spectrum \( G(k) = T k^2 / (k^2 + m_\gamma^2) \), \( m_\gamma = 1/\lambda \)
  - Long wavelengths \( k \lesssim m_\gamma \) suppressed
    (Meissner effect)
But: long wavelengths also slow to react:

- $|d \ln G(k)/dt| \lesssim k$ from causality
- $|d \ln G(k)/dt| \lesssim k^2/\sigma$ with conductivity $\sigma$
- Staying in equilibrium at $T_c$ needs
  
  $|d \ln G(k)/dt| \approx (dm^2/\gamma dt)/k^2 \equiv C/k^2$

$\Rightarrow$ Modes with $k \leq k_c \approx (\sigma C)^{1/4}$ freeze out

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Meissner effect: Magnetic fields are repelled
- Uniform magnetic field $\Rightarrow$ Vortex lattice
- Frozen-out fluctuations $\Rightarrow$ Vortex clusters

Estimates:
- Vortices per cluster $N_{cl} \approx (e^2 T/8\pi k_c)^{1/2}$
- Number density per unit area $n \approx (e^2 T k_c^3 / 8\pi^3)^{1/2}$
Simulations

- Thermal initial conditions \( p[\phi(\vec{x})] \propto \exp(-\beta H[\phi(\vec{x})]) \)
  - Generate using lattice Monte Carlo algorithms
  - Correct equilibrium state

- Fully non-linear time evolution
  - Relativistic field equations
    (Interactions \(\Rightarrow\) Dissipation)
  - Quench to broken phase: Time-dependent mass term \( m^2(t) \)
    Friction term to cool down the system
  - Count vortices using gauge-invariant winding number (1998)

- Two geometrical setups:
  - Fully two-dimensional
  - 2D film in 3D space
    - Electromagnetic field extends outside the film
    - Closer to experimental setups and fully 3D case
Global theory

- \( \vec{\nabla} \theta = 0 \Rightarrow \) Uniform phase
- Vacua characterised by different \( \theta \)

Gauge theory

- \( \vec{\nabla} \theta + e \vec{A} = 0 \Rightarrow \) Non-uniform phase
- Vacua characterised by different \( \vec{A} \) with \( \vec{\nabla} \times \vec{A} = 0 \)
Gauge theory: Clustering of equal-charge vortices as predicted
Magnetic field extends outside the film
⇒ Long-range interactions
   Inter-vortex potential $1/r$ vs $\exp(-r/\lambda)$
   Affects vortex formation (Kibble & Rajantie 2003)

3D lattice
- Order parameter $\phi$ defined on ($z = 0$) plane
- EM field $A_\mu$ defined everywhere

Empty space outside the film
⇒ Maxwell’s equations
- Add small damping term to cool down
  ~ Ohmic conductivity
- Gauss’s law
  ⇒ Same damping term for $\phi$
Film Simulations

(Donaire, Kibble & Rajantie 2004)
Comparison: Experiment

(Kirtley, Tsuei and Tafuri, 2003)
Simulations: Vortex Number

2D

\[ \rho \text{ vs. } e \]

\[ \rho \text{ vs. } T \]

2D

Film

\[ \text{number of vortices } N \text{ vs. cooling rate } \sigma \]

(Stephens et al. 2002)

(Donaire, Kibble & Rajantie 2004)

- Good agreement with theory (esp. for fast quenches)
  - Dependence on electric charge \( e \)
    (Limit \( e \to 0 \): Kibble mechanism)
  - Dependence on initial temperature \( T \)
  - Dependence on cooling rate \( \sigma \)
First order transitions: Bubble nucleation
- Regions of symmetric phase between bubbles
- Typical flux in such a region $\propto \text{area}^{1/4}$, can be $\gg 1$
  $\Rightarrow$ Thick (and heavy) defects
  with very high winding numbers (Donaire&Rajantie, in progress)
- Explains why $\mu_{\text{obs}} > \mu_{\text{bound}}$?
Monopole Formation

- ’t Hooft-Polyakov magnetic monopoles in any Grand Unified Theory:
  - Similar mechanism – Trapping of magnetic charge (Rajantie 2003)

- Much more complicated
  - Non-Abelian theory: Cannot linearize
  - No real phase transition: Smooth crossover
  - Definition of magnetic charge in symmetric phase?
    - Sources of ’t Hooft magnetic field

- Prediction: Number density \( n \approx g \sqrt{T k_c^5} \)
  - Does not solve the monopole problem

- Future work: Test in simulations
Cosmic Superstrings

- Cosmic strings from superstring theory (Witten 1985)
  - F(undamental) strings
  - D-strings
  - (Semi-)realistic KKLMMT-model (Kachru et al 2003)
    ⇒ Strings stable against breaking and collapse (Copeland et al 2003)
- “Observational test of superstring theory”:
  Several types of strings + junctions
- Form in brane collisions
  - Flux trapping, but no Kibble mechanism (Dvali&Vilenkin 2004)
- Details yet to be worked out
Conclusions

- Defect formation: Two mechanisms
  - Kibble mechanism: global and gauge symmetries
  - Flux trapping: gauge symmetry
- Generic scenarios
  - Superconductors
  - Cosmological phase transitions
  - Superstring theory
  - Detail differ, but principles are the same
- Defect configuration depends on
  - properties of the system
  - dynamics of the transition
- Cosmology in the laboratory